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S-Duality, Confinement and Large-N String Theory Near $\mathcal{N}=4$ SUSY Yang-Mills

hep-th/0310117
hep-th/0406104

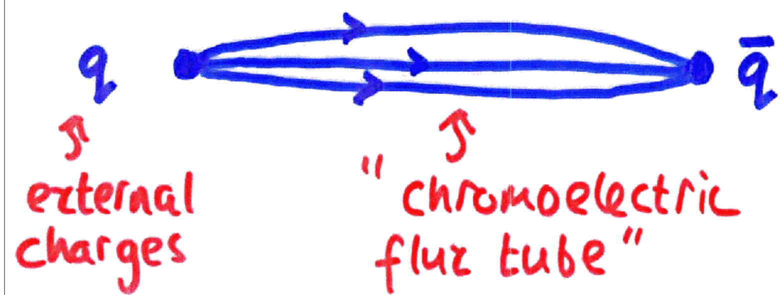
+ work to appear with
T.J. Hollowood (Swansea)

confinement in pure Yang-Mills

$G = SU(N)$

Two ideas:

I) Non-abelian dual Meissner effect



Electric flux/charge: $\mathbb{Z}_N \subset SU(N)$ ← center

II) Large-N

Yang-Mills $G = SU(N)$ $\overset{N\to\infty}{\sim}$ closed string theory

$g_s^2 \sim 1/N^2$

Overview

$\mathcal{N}=4$ SCFT $\xrightarrow[\text{deformation}]{\text{marginal}}$ $\mathcal{N}=1$ SCFT

VEVs

Higgs $\xleftrightarrow[\text{EM duality}]{}$ Confinement

- New confining phases

spontaneously broken conformal symmetry

$$G = U(N) \longrightarrow U(1)$$

$$G = U(N) \longrightarrow U(M) \longrightarrow U(1)^M$$

$M | N$

- Semiclassical description of confinement via EM duality

BPS flux tubes

- AdS duals

M NS5 branes wrapped on $T^2 \subset S^5$

- Interesting relation to a six-dimensional theory

Deconstruction of Little String Theory

<u>claim</u> 't Hooft limit in phase where $G = U(N) \xrightarrow{\text{confined}} U(M)$ M/N

$\equiv$ Type IIB LST on $\mathbb{R}^{3,1} \times T^2$

\+ decoupled states

volume $\propto \lambda = g^2 N$

- <u>Application</u>:

Deconstruction of Double-Scaled LST

$$G = U(N) \longrightarrow U(1)^M \quad M/N$$

Exact string theory description of part of large-N spectrum

The Model

$\mathcal{N}=1$ superfields:

$\mathcal{N}=4$ SUSY Yang-Mills $G=U(N)$ $\supset$ V vector

Φ_i adjoint chiral $i=1,2,3$

Superpotential:

$$W = \mathrm{Tr}_N\left(\Phi_1\left[\Phi_2,\Phi_3\right]_\beta\right)$$

$$[\Phi_i,\Phi_j]_\beta = e^{+i\beta/2}\Phi_i\Phi_j - e^{-i\beta/2}\Phi_j\Phi_i$$

deformed commutator

- $\beta=0 \Rightarrow \mathcal{N}=4$ SUSY Yang-Mills
- $\beta\neq 0 \Rightarrow \mathcal{N}=1$ SUSY

central $U(1)$ photon decouples but $a_i = \mathrm{Tr}_N(\Phi_i)$ do not

Classical Vacua satisfy

$$[\Phi_1,\Phi_2]_\beta = 0 \quad + \text{ cyclic}$$

F-flatness

- Generic $\beta\neq 0$

$\langle\Phi_1\rangle = \mathrm{diag}(\lambda_1,\lambda_2,\ldots,\lambda_N)$, $\langle\Phi_2\rangle=\langle\Phi_3\rangle=0$

$\lambda_i\neq\lambda_j \;\forall i,j \Rightarrow G=U(N) \xrightarrow[\text{Mechanism}]{\text{Higgs}} U(1)^N$

$\Rightarrow$ Coulomb Branch

- Special $\beta\in 2\pi\mathbb{Q}$ eg $\beta=\frac{2\pi}{N}$

$\langle\Phi_1\rangle = \alpha_1 U_{(N)}$, $\langle\Phi_2\rangle = \alpha_2 V_{(N)}$ "shift"

"clock" $\alpha_1,\alpha_2\in\mathbb{C}$

$\alpha_1,\alpha_2\neq 0 \Rightarrow$

$G=U(N) \longrightarrow U(1)$

$\Rightarrow$ Higgs Branch

$$U_{(N)} = \begin{pmatrix} \omega & & & 0 \\ & \omega^2 & & \\ & & \ddots & \\ 0 & & & \omega^N \end{pmatrix}$$

$$V_{(N)} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 & \vdots \\ \vdots & & \ddots & 1 \\ 1 & 0 & \cdots & 0 \end{pmatrix}$$

$\omega = e^{2\pi i/N}$

Effective Theory on Higgs Branch

$\langle \Phi_1 \rangle = \alpha_1 U_{(N)}\,,\ \langle \Phi_2 \rangle = \alpha_2 V_{(N)}$

Classical Spectrum:

$$M^2_{\ell_1,\ell_2} = 4|\alpha_1|^2 \sin^2\left(\frac{\ell_1 \pi}{N}\right) + 4|\alpha_2|^2 \sin^2\left(\frac{\ell_2 \pi}{N}\right)$$

$\ell_1, \ell_2 = 1, 2, \dots, N$

$\Rightarrow$ 2 Kaluza-Klein towers at large-N

Deconstruction

Adams + Fabinger

classical equivalence:

β-deformed theory $G = U(N)$ on $\mathbb{R}^{3,1}$

$\cong$ $\mathcal{N}=(1,1)$ SUSY Yang-Mills $\hat{G} = U(1)$

on $\mathbb{R}^{3,1} \times \mathcal{L}$ ← $N \times N$ lattice

Non-commutative lattice gauge theory

Square torus

$|\alpha_1| = |\alpha_2| = |\alpha|$

Lattice spacing

$\hookrightarrow \varepsilon = \dfrac{1}{|\alpha|}$

radius ↗ $R = \dfrac{N}{2\pi|\alpha|}$

$G_6^2 = \dfrac{g^2 N}{|\alpha|^2}$

↑ $D=6$ coupling

$\Theta = \dfrac{N}{2\pi|\alpha|^2}$

↑ NC parameter

- length scales ($N \gg 1$)

	$\ell\|\alpha\|$			
		$D=4$	$G=U(N)$	$\mathcal{N}=1$
ε	1		↓	
$\sqrt{\Theta}$	$\sqrt{N}$	$D=6$	$\hat{G}=U(1)$	$\mathcal{N}=(1,1)$
			↓	
R	N	$D=4$	$\hat{G}=U(1)$	$\mathcal{N}=4$

- Weak coupling $g^2 N \ll 1$

$\Rightarrow G_6 \ll \varepsilon$

Lattice theory far from continuum

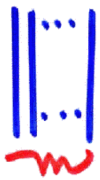
IIB String Theory Picture

Myers
Polchinski + Strassler

$\mathcal{N}=4$ SUSY Yang-Mills, $G=U(N)$

+ β-deformation $\beta\in\mathbb{R}$ $\beta\ll 1$

$\Rightarrow F^{(3)}_{RR} \propto \beta$

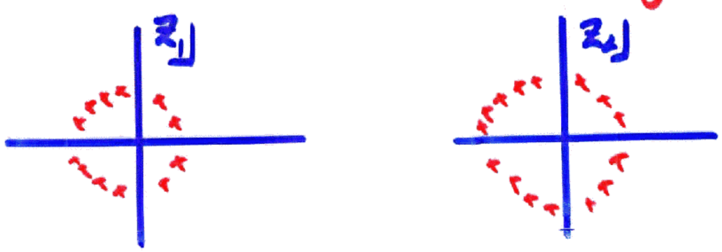


$N\times D3$

Higgs Branch vacuum: $\langle\Phi_1\rangle=\alpha_1 U_{(N)}$

$\langle\Phi_2\rangle=\alpha_2 V_{(N)}$

D3 brane positions ≡ scalar eigenvalues



$z_1=x_4+ix_5$
$z_2=x_6+ix_7$

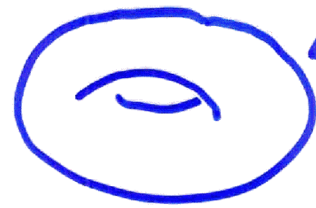
D3 branes distributed on a torus $T^2\subset\mathbb{R}^6$

radii: $r_1=|\alpha_1|(2\pi\alpha')$
$r_2=|\alpha_2|(2\pi\alpha')$

$[\Phi_1,\Phi_2]\neq 0 \Rightarrow$ D5 brane charge density

Large-N description

Myers



D5 brane wrapped on $T^2\subset\mathbb{R}^6$
+ N units of D3 charge

$$\Rightarrow \int_{T^2} B_{NS} = 2\pi N$$

- Energetically stable for $\beta=\frac{2\pi}{N}$ due to D5 coupling to $F^{(3)}_{RR}$
- World volume theory – Seiberg + Witten

$\mathcal{N}=(1,1)$ SUSY Yang-Mills, $\hat{G}=U(1)$ on $\mathbb{R}^{3,1}\times\hat{T}^2_\Theta$ dual NC torus

agrees with QFT analysis

On Higgs branch:

$$G = U(N) \simeq \frac{U(1)\times SU(N)}{\mathbb{Z}_N} \xrightarrow{\text{Higgs}} U(1)$$

expect <u>magnetic flux tubes</u> classified by $\pi_1(SU(N)/\mathbb{Z}_N) \simeq \mathbb{Z}_N$

.... these show up as <u>BPS strings</u> in low-energy theory

$\equiv$ $\hat{G} = U(1)$ Non-commutative Yang-Mills instantons on $\mathbb{R}^2 \times T^2_\Theta$

D-strings bound to D5 brane

Tension: $T = \frac{8\pi^2}{G_6^2} = \frac{8\pi^2 |\alpha|^2}{g^2 N}$

Core-size: $\rho \sim \sqrt{\Theta} = \frac{\sqrt{N}}{|\alpha|} \gg \varepsilon$

<u>More Higgs Branches</u> $G = U(N)$

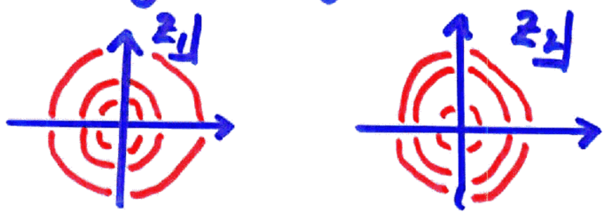
$\beta = \frac{2\pi}{n}$, $N = mn$

$\langle \Phi_1 \rangle = \Lambda^{(1)} \otimes U_{(n)}$, $\langle \Phi_2 \rangle = \Lambda^{(2)} \otimes V_{(n)}$

$\Lambda^{(i)} = \mathrm{diag}(\mu^{(i)}_1, \ldots, \mu^{(i)}_M)$ $i = 1,2$

$$G = U(N) \xrightarrow{\Lambda^{(i)} \not\propto \mathbb{1}} U(M) \xrightarrow{\Lambda^{(i)} \not\propto \mathbb{1}} U(1)^M$$

string theory picture:



- M D5 branes wrapped on M concentric tori radii controlled by $\mu^{(i)}_a$
- World volume theory

$\mathcal{N} = (1,1)$ SYM $\hat{G} = U(M)$ on $\mathbb{R}^{3,1} \times \hat{T}^2_\Theta$

- **BPS flux tubes** $T = 8\pi^2/G_6^2$

$\equiv$ Non-commutative $\hat{G} = U(m)$ Yang-Mills instantons on $\mathbb{R}^2 \times T^2_\Theta$

$\hat{G} = U(M) \Rightarrow$ variable core-size $e \gtrsim \sqrt{\Theta}$

$\hat{G} = U(1)^M \Rightarrow$ fixed-core size $e \sim \sqrt{\Theta}$

- **Deconstruction** $\hat{G} = U(m)$ $N = mn$

$\Lambda^{(1)} = \Lambda^{(2)} = \alpha\, \mathbb{1}_{m \times m}$

$$\epsilon = \frac{1}{|\alpha|}, \quad R = \frac{n}{2\pi|\alpha|}$$

$$G_6^2 = \frac{g^2 n}{|\alpha|^2}, \quad \Theta = \frac{n}{2\pi|\alpha|^2}$$

$$T = \frac{8\pi^2|\alpha|^2}{g^2 n}$$

- Continuum limit?
- Light string limit?

Quantum Theory. $G = U(N)$

complexified coupling $\tau = \frac{4\pi i}{g^2} + \theta/2\pi$

$\beta = 0$ $\Rightarrow$ $\mathcal{N}=4$ SUSY Yang-Mills

- Exact conformal invariance

τ is marginal

- Exact $SL(2,\mathbb{Z})$ duality

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \qquad a,b,c,d \in \mathbb{Z}, \quad ad - bc = 1$$

- Large-N string theory:

IIB string on $AdS_5 \times S^5$

$$g_s = g^2 \qquad R^4/\alpha'^2 = \lambda = g^2 N$$

string coupling, radius, 't Hooft coupling

IIB SUGRA valid for $\lambda \gg 1$

$|\beta| \ll 1 \Rightarrow \mathcal{L} \simeq \mathcal{L}_{\mathcal{N}=4} + \beta \hat{O} + \text{h.c.}$

chiral primary

- $\mathcal{N}=4$ superconformal algebra

$\Rightarrow \dim[\hat{O}] \equiv 4$

$\Rightarrow$ β is marginal $\forall \tau$

- AdS/CFT dictionary

$$\hat{O} \longleftrightarrow G^{(3)} = F^{(3)}_{RR} + \tau H^{(3)}_{NS}$$

chiral primary SUGRA field

$\Rightarrow$ $\hat{O}$ transforms with modular weight $(+1,0)$ under $SL(2,\mathbb{Z})$

....hence restore $SL(2,\mathbb{Z})$ duality by assigning β weight $(-1,0)$

$$\beta \to \beta/(c\tau+d), \quad \tau \to \frac{a\tau+b}{c\tau+d}$$

$\beta \neq 0$ beyond linear order

- β and τ are exactly marginal

$\Rightarrow$ 2 parameter family of $\mathcal{N}=1$ SCFTs

Leigh + Strassler

- holomorphic observables from planar limit of matrix model

Dijkgraaf + Vafa

$$Z_{DV} = \oint \prod_{i=1}^{3} [d\Phi_i]\, e^{-1/\mu\, W_{cl} + \text{source terms}}$$

$$W_{cl} = \mathrm{Tr}\left(\Phi_1 [\Phi_2, \Phi_3]_\beta\right)$$

- Exact superpotential (deformed theory)

ND + Hollowood + Kumar

- "Seiberg-Witten" curve of Coulomb branch $\cong$ spectral curve of Z_{DV}

ND + Hollowood

Coulomb Branch

$\langle \Phi_1 \rangle = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$

$\langle \Phi_2 \rangle = \langle \Phi_3 \rangle = 0$

$\lambda_i \neq \lambda_j \Rightarrow G = U(N) \xrightarrow{\text{Higgs}} U(1)^N$

massless $\mathcal{N}=1$ chiral superfields:

N photons $\subset W_\alpha^a$ $a = 1, 2, \ldots, N$

N scalars $\subset u_n = \mathrm{Tr}_N[\Phi_1^n]$ $n = 1, 2, \ldots, N$

F-term effective action:

$$\mathcal{L}_F = \frac{1}{8\pi} \mathrm{Im}\left[\int d^2\theta \, \tau_{\mathrm{eff}}^{ab}[\{u_n\}] W_{\alpha a} W_b^\alpha \right]$$

- τ_{eff}^{ab} holomorphic in τ, β, u_n
- $Sp(2N, \mathbb{Z})$ monodromies around singular points ← new massless d.o.f
- $\mathrm{Im}[\tau_{\mathrm{eff}}^{ab}] \geq 0$ unitarity

conditions satisfied if, Seiberg + Witten

τ_{eff}^{ab} = Period matrix of Riemann surface Σ ← genus N

U(2) case ND + Hollowood

Scale + $U(1)_R$ ⇒ curve depends on a single dimensionless modulus

$$u = \frac{u_1}{\sqrt{2u_1^2 - u_2}} \times \frac{i}{\sin(\beta/2)}$$

DV proceedure yields curve:

$$\Sigma: \lambda^2 - u\lambda + \wp(i\beta) - \wp(z) = 0$$

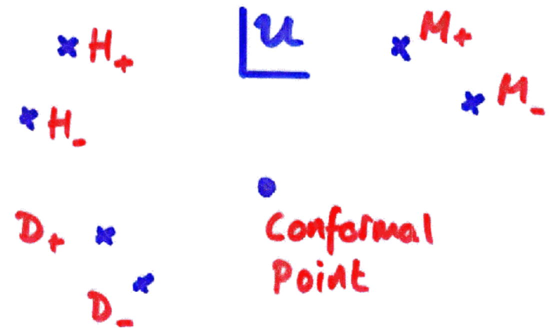
Weierstrass function

≡ Double-cover of torus genus 2

$$E(\tau) \simeq \mathbb{C}/\mathbb{Z} \oplus \tau\mathbb{Z}$$

⇒ expected $SL(2,\mathbb{Z})$ action on β, τ, u

$G = U(2)$ Moduli Space



Six singular points:

$u = H_{\pm} \Rightarrow$ massless charged hypermultiplet

$u = M_{\pm} \Rightarrow$ massless ~~charged~~ monopoles

$u = D_{\pm} \Rightarrow$ massless dyons

permuted by $SL(2,\mathbb{Z})$

special points in parameter space:

$\beta = \pi \Rightarrow H_+ = H_- = 0 \Rightarrow$ Higgs branch root

$\updownarrow S$

$\beta = \pi\tau \Rightarrow M_+ = M_- = 0 \Rightarrow$ confining branch root

New

$G = U(N)$ $(\theta = 0)$

$\beta = \frac{2\pi}{N} \Rightarrow$ Higgs branch

$$G = U(N) \xrightarrow{\text{Higgs}} U(1)$$

moduli: $\frac{1}{N}\langle \mathrm{Tr}_N \Phi_1^N \rangle = \alpha_1^N$

$\frac{1}{N}\langle \mathrm{Tr}_N \Phi_2^N \rangle = \alpha_2^N$

$\updownarrow S$

$\tilde{\beta} = \frac{8\pi^2 i}{\tilde{g}^2 N} \Rightarrow$ Confining branch

$$G = U(N) \xrightarrow{\text{confined}} U(1)$$

moduli: $\frac{1}{N}\langle \mathrm{Tr}_N \Phi_1^N \rangle = \tilde{\alpha}_1^N$

$\frac{1}{N}\langle \mathrm{Tr}_N \Phi_2^N \rangle = \tilde{\alpha}_2^N$

$\tilde{g}^2 = \frac{4\pi}{g^2}$ $\qquad \tilde{\alpha}_i = \left(\tilde{g}^2/4\pi\right)\alpha_i \quad i=1,2$

Higgs **Confinement**

magnetic flux tubes $\xleftrightarrow{S}$ electric flux tubes

$\pi_1\left(\frac{SU(N)}{\mathbb{Z}_N}\right) \cong \mathbb{Z}_N$ $\mathbb{Z}_N \subset SU(N)$ ← center

- Semiclassical description of confining phase for $g^2 >> N$ via S-duality to Higgs phase
  N >> 1 BPS flux tubes in low energy theory
- Confinement + spontaneously broken conformal invariance
  ⇒ massless composite dilaton
- New branch occurs for $\beta = \frac{8\pi^2 i}{g^2 N} \in i\mathbb{R}$
  ⇒ invisible in classical theory
- $|\beta| << 1$ in SUGRA regime $g^2 N >> 1$

$G = U(N)$ $N = mn$ $\beta = 2\pi/n$

Higgs Branch: $\langle \Phi_1 \rangle = \alpha \, \mathbb{1}_{(m)} \otimes U_{(n)}$

$\langle \Phi_2 \rangle = \alpha \, \mathbb{1}_{(m)} \otimes V_{(n)}$

$\alpha \neq 0 \Rightarrow G = U(N) \xrightarrow{\text{Higgs}} U(m)$

6d Theory. $\hat{G} = U(m)$ on $\mathbb{R}^{3,1} \times \mathcal{L}$ ← n x n lattice

lattice spacing → $\varepsilon = \frac{1}{|\alpha|}$, $R = \frac{n}{2\pi|\alpha|}$ ← radius

$G_6^2 = \frac{g^2 n}{|\alpha|^2}$, $\theta = \frac{n}{2\pi|\alpha|^2}$

(G_6: 6d coupling; θ: NC parameter)

Continuum Limit ??? $N = mn \to \infty$, m fixed

$|\alpha| \sim n$ $g^2 \sim n$

$\Rightarrow \varepsilon, \theta \to 0$ with G_6, R fixed

Now apply S-duality:

$$g^2 \to \tilde{g}^2 = \frac{4\pi}{g^2}, \quad |\alpha| \to |\tilde{\alpha}| = \frac{\tilde{g}^2}{4\pi}|\alpha|$$

$$\beta = \frac{2\pi}{n} \longrightarrow \tilde{\beta} = \frac{8\pi^2 i}{\tilde{g}^2 n}$$

Higgs — Confinement

limit becomes: $N = mn \to \infty$, m fixed

$$\tilde{g}^2 \sim 1/n = \frac{m}{N} \qquad |\tilde{\alpha}| \sim n^0$$

$\Rightarrow$ 't Hooft limit

- choose $\tilde{g}^2 n \gg 1$

then i) $\tilde{g}^2 N = \tilde{g}^2 nm \gg 1$

ii) $|\tilde{\beta}| \ll 1$

$\Rightarrow$ use AdS duality.

Partial Confinement $G = U(N)$ $N = mn$

New branches occur at $\beta = \frac{8\pi^2 i}{g^2 n}$

Moduli: $\langle \mathrm{Tr}_N \Phi_1^n \rangle = \alpha_1^n$ $\quad \alpha_1, \alpha_2 \in \mathbb{C}$

$\langle \mathrm{Tr}_N \Phi_2^n \rangle = \alpha_2^n$

$$\alpha_1, \alpha_2 \neq 0 \Rightarrow G = U(N) \xrightarrow{\text{"confined"}} U(m)$$

't Hooft limit $N = mn \to \infty$

$\lambda = g^2 N, m, \alpha_1, \alpha_2$ held fixed

Expect:

- $U(m)$ neutral states obey standard large-N scaling

interactions suppressed by $1/N^2$

- $U(m)$ charged states need not

eg. IR $U(m)$ gauge coupling $\sim g^2 n = \lambda/m$

$D=4$ SCFT $\xrightarrow{\text{RG flow}}$ confinement / conformal S.B.

UV IR

UV theory: $g^2 n \gg 1$

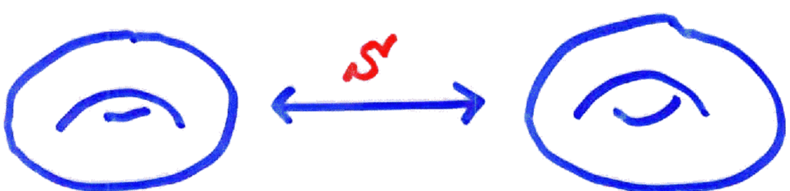
$\Rightarrow g^2 N \gg 1$ and $|\beta| = \dfrac{8\pi^2}{g^2 n} \ll 1$

$\Rightarrow$ dual to IIB SUGRA on $AdS_5 \times S^5$ with boundary source for

$$G^{(3)} = F^{(3)}_{RR} + \tau H^{(3)}_{NS} \propto \beta$$

IR theory (n int)

Higgs **Confinement**

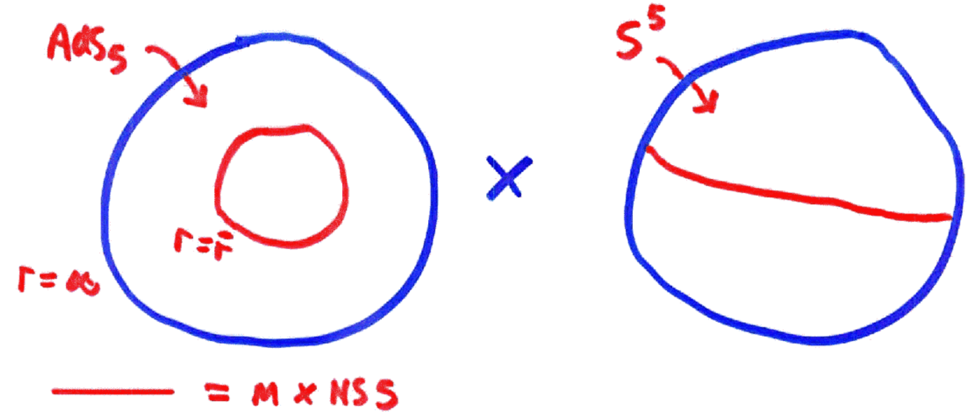


$M \times D5$ wrapped on T^2 + N D3 branes

$M \times NS5$ wrapped on T^2 + N D3 branes

AdS-dual a la Polchinski-Strassler

M NS5 branes wrapped on $T^2 \subset S^5$ at fixed radial distance $\bar{r} \sim |\alpha_i| \alpha'$



- **Probe analysis** DBI action

NS5 branes stabilized by boundary source for $H^{(3)}_{NS}$

- **Patched solution** interpolates between

A) $AdS_5 + S^5$ + source far from branes

and

B) Near horizon geometry of M NS5 branes near branes

Spectrum

- bulk states neutral under $U(m)$
- NS5 brane states charged under $U(m)$

m^2 gluons
instanton strings

't Hooft limit $g^2 \sim 1/N \to 0$

$\Rightarrow g_s \to 0$

As in flat space these two sectors decouple in this limit

$\Rightarrow$ "charged" sector $\equiv$ Type A_m Little String Theory (LST)

String Tension $= \dfrac{8\pi^2 |\alpha_1||\alpha_2|}{g^2 n}$

m coincident NS5 branes $g_s \to 0$

Near horizon geometry:

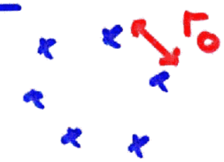
$$\mathbb{R}^{5,1} \times \mathbb{R}_\sigma \times S^3$$

S^3: SU(2) WZW level $= m$

$\mathbb{R}_\sigma$: Linear dilaton

$$\varphi = \frac{2}{\sqrt{m\alpha'}}\sigma$$

m seperated NS5 branes Giveon + Kutasov



Double-scaling limit

$g_s \to 0, \; r_0 \to 0$

$\bar{M} = g_s / r_0$ held fixed

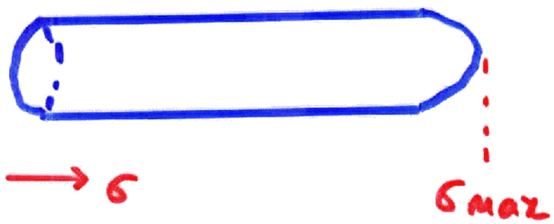
Near horizon geometry:

$$\mathbb{R}^{5,1} \times \left(\frac{SL(2)}{U(1)} \times \frac{SU(2)}{U(1)} \right) \Big/ \mathbb{Z}_m$$

$\frac{SL(2)}{U(1)}$: semi-infinite cigar

$SL(2)/U(1)$ WZW at level m

= semi-infinite cigar



maximum string coupling $\hat{g}_s = e^{\varphi(\sigma_{max})}$

$\sim \bar{M}\alpha'$

- tree-level string theory ok for $\bar{M}\alpha' \ll 1$
- Exact world-sheet CFT Giveon + Kutasov

$\Rightarrow$ Spectrum

Hagedorn density

$\mathcal{S}$-matrix

Regge behaviour

Deconstruction of Double-Scaled LST

$G = U(N)$ $N = mn$ $\beta = 8\pi^2 i/g^2 n$

moduli: $\langle \mathrm{Tr}\,\Phi_1^n \rangle = \alpha_1^n$ $\langle \mathrm{Tr}\,\Phi_3^N \rangle = \mu^N$

$\langle \mathrm{Tr}\,\Phi_2^n \rangle = \alpha_2^n$

$\alpha_1, \alpha_2, \mu \neq 0 \Rightarrow$

$$G = U(N) \xrightarrow{\text{confined}} U(1)^m$$

AdS-dual m seperated NS5 branes wrapped on $T_i^2 \subset S^5$ $i = 1, \ldots, m$

seperation $r_0 \sim \mu\alpha'$

't Hooft limit $N \to \infty$

$\lambda = g^2 N$, m, α_1, α_2, μN fixed

$\to$ IIB on

$$\mathbb{R}^{3,1} \times T^2 \times \left(\frac{SL(2)}{U(1)} \times \frac{SU(2)}{U(1)} \right) / \mathbb{Z}_m$$