Chiral Symmetry Breaking and Light Mesons

From Generalizations of the AdS/CFT Correspondence

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hep-th/0306018, hep-th/0408113

Motivation

A new description of $U(1)_{\rm A}$ chiral symmetry breaking from string theory

Starting point: AdS/CFT correspondence

Duality: Anti-de Sitter supergravity in 5d / conformal field theory in 4d

Generalization: 5d gravity description of

4d non-supersymmetric confining $SU(N)$ gauge theory ($N \to \infty$) with quark degrees of freedom (conjecture)

$U(1)_{\rm A}$ chiral symmetry

Gravity description of chiral symmetry breaking

Non-perturbative calculation of meson spectra by solving 2nd order gravity equations of motion.

Motivation

The standard AdS/CFT correspondence is an equivalence (duality) of

i) Type IIB string theory on $AdS_5 \times S^5$

ii) $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory

Special features of the field theory side of the correspondence:

- $N \to \infty$
- $\mathcal{N} = 4$ Supersymmetry
- Conformal Invariance ($\beta = 0$)
- All fields in the adjoint representation of the gauge group

Generalizations of the AdS/CFT correspondence:

i) Go beyond supergravity approximation (e.g. plane waves)

ii) break conformal invariance and (some of) supersymmetry: Deform $AdS_5 \times S^5$ geometry (holographic RG flows)

iii) Add FLAVOUR (i.e. fundamental matter) to AdS/CFT by introducing probe branes into the correspondence

Deformations of AdS/CFT

The original $AdS_5 \times S^5$ metric is deformed such as to reduce the original $SO(4,2) \times SO(6)$ symmetry

$\to$ conformal symmetry and (some of the) supersymmetry is broken

Holographic RG flows

Renormalization group equation from supergravity gradient flow

Radial direction in deformed AdS $\Leftrightarrow$ energy scale

Example:

metric of deformed AdS space: $ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu - dr^2$

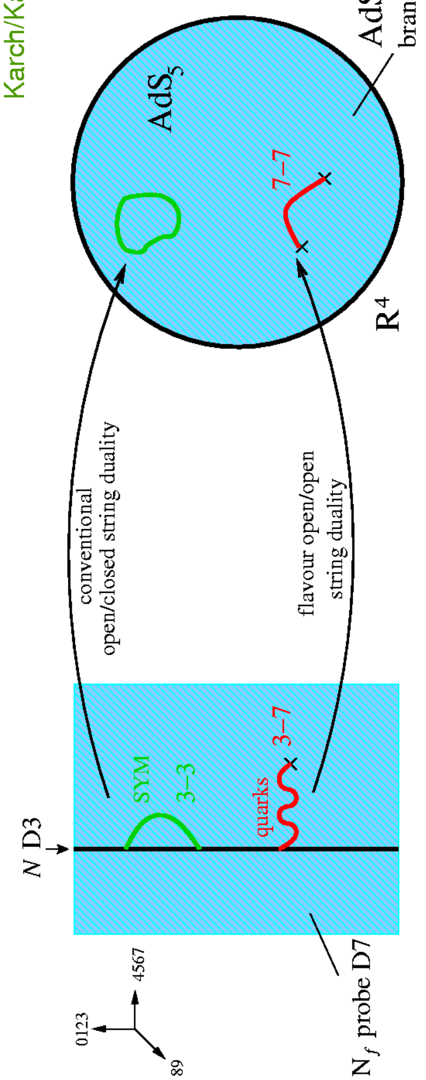
AdS: $A(r) = \frac{r}{L}$

field theory: addition of relevant operators to $\mathcal{N} = 4$ $SU(N)$ Super Yang-Mills

flows to:
- $N = 2$ SUSY
- $N = 1$ SUSY (conformal or confining)
- no SUSY (pure gauge theory, confining)

Adding probe D7-branes to (standard) AdS/CFT

Karch/Katz, 2002



limit: $N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

Duality acts twice!

4d $\mathcal{N}=4$ SU(N) Super Yang-Mills theory coupled to 4d $\mathcal{N}=2$ hypermultiplet $\longleftrightarrow$ type IIB SUGRA on $AdS_5 \times S^5$ + Dirac-Born-Infeld theory on $AdS_5 \times S^3$

Motivation

Main result:

Combine the addition of a D7 brane probe with the deformation of the $AdS_5 \times S^5$ space (holographic RG flow to IR theory without supersymmetry)

$\Rightarrow$ gravity dual description of spontaneous chiral symmetry breaking by quark condensate and Goldstone bosons

Outline:

1. Fundamental matter in standard AdS/CFT
2. Flavour in the AdS black hole geometry
3. Myers-Constable deformation and chiral symmetry breaking

Overview: Flavour in AdS/CFT

- Meson spectrum for Karch/Katz background ($\mathcal{N} = 2$ SUSY), $M_{\rm meson} \propto m_{\rm quark}$
  Kruczenski, Mateos, Myers, Winters I
- Meson spectrum for Klebanov/Strassler background
  Sakai, Sonnenschein
- Non-supersymmetric background,
  spontaneous χSB by quark condensate,
  $M_{\rm meson} \propto \sqrt{m_{\rm quark}}$
  Babington, J.E., Evans, Guralnik, Kirsch
- D5 brane probes in Maldacena/Nunez background
  Nunez, Paredes, Ramallo
- Spontaneous chiral symmetry breaking in non-supersymmetric D4/D6 brane configuration
  Kruczenski, Mateos, Myers, Winters II

Overview: Flavour in AdS/CFT

Stringy corrections:

- Veneziano-Witten formula, $1/N$ corrections
  Barbon, Hoyos, Mateos, Myers
- Backreaction in D3/D7 system with $\mathcal{N} = 1$ SUSY
  Burrington, Liu, Pando Zayas, Vaman
- Spinning string spectra Pons, Russo, Talavera
- D2/D6 brane configuration: 3d Yang-Mills Theory J.E., Kirsch

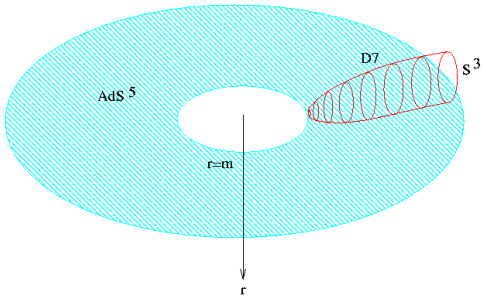
Applications:

- Gasser-Leutwyler parameters (in D3/D7 system without SUSY)
  Evans, Shock

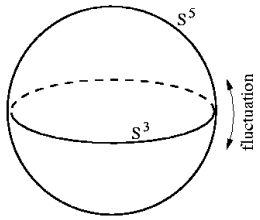
Fluctuations in the probe-supergravity background

RG flow: separate D7 probe from D3's in one perpendicular direction $\Rightarrow$ gives rise to mass for fundamental hypermultiplet in dual field theory ($m \neq 0$)

D7-brane "ends" at $r = m$ (due to S^3 collapse)



D7-branes wrap a topological trivial cycle: $S^3 \subset S^5$



- mass spectrum of the scalar fluctuations:

$$m_l^2 = l(l+2) - 3$$

($m_l^2 \geq -4$ BF bound $\rightarrow$ stable)

fluctuation-operator map

lowest fluctuation mode		bilinear operator $\bar{q}q$
$m_0^2 = -3$	$\longleftrightarrow$	$\Delta = 3$

In the following we use this map to find the meson spectrum by a supergravity computation.

D7 brane probe in deformed backgrounds

Consider a D7 brane probe in gravity backgrounds dual to confining gauge theories without supersymmetry.

1. AdS black hole background:

- horizon
- $\mathcal{N} = 4$ SYM at finite temperature
- $\Rightarrow 3d$ pure gauge theory for $T \rightarrow \infty$

2. Constable-Myers background:

- non-constant dilaton
- non-constant S^5 radius
- naked singularity in IR
- dual field theory confining

In UV limit, both geometries return to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$.

General strategy

i) start from Dirac-Born-Infeld action for a D7-brane

ii) derive equations of motion for transverse scalars (say w_5, w_6)

iii) solve equations of motion numerically using shooting techniques, the solutions (e.g. $w_5 = 0, w_6 = w_6(\rho)$) determine the embedding of the D7-brane

iv) meson spectrum: consider fluctuations δw_5, δw_6 around a background solution obtained in iii) and solve equations of motion linearized in $\delta w_5, \delta w_6$

UV asymptotic behaviour of solutions to equation of motion:

$$w_6 \propto m\, e^{-r} + c\, e^{-3r}$$

m quark mass, $c = \langle \bar{q} q \rangle$ quark condensate

$m \neq 0$: explicit breaking of $U(1)$ chiral symmetry

$c \neq 0$: spontaneous breaking of $U(1)$ chiral symmetry

Flavour in the AdS Black Hole geometry

Consider $\mathcal{N} = 4$ $SU(N)$ SYM at finite temperature (Witten, 1998)

The dual string theory background is the Euclidean AdS-Schwarzschild solution defined by the metric (in an appropriate coordinate system)

$$ds^2 = \left(w^2 + \frac{b^4}{4w^2}\right) d\mathbf{x}^2 + \frac{(4w^4 - b^4)^2}{4w^2(4w^4 + b^4)} d\tau^2 + \frac{1}{w^2} \sum_{i=1}^{6} dw_i^2$$

with radial coordinate $w^2 = \rho^2 + w_5^2 + w_6^2$

b a deformation parameter, τ periodic (period $\pi b = T^{-1}$)
horizon: S^1 collapses at $w = \frac{1}{2} b$

Embedding of the D7-brane

D7-brane dynamics is captured by the DBI action:

$$S_{D7} = -T_7 \int d^8\xi \sqrt{-\det(P[G_{ab}])}$$

(use static gauge)

assume $w_5 = 0$, $w_6 = w_6(\rho)$

solve equation of motion for w_6 numerically using shooting technique

AdS Black Hole: Karch-Katz vs condensate solutions

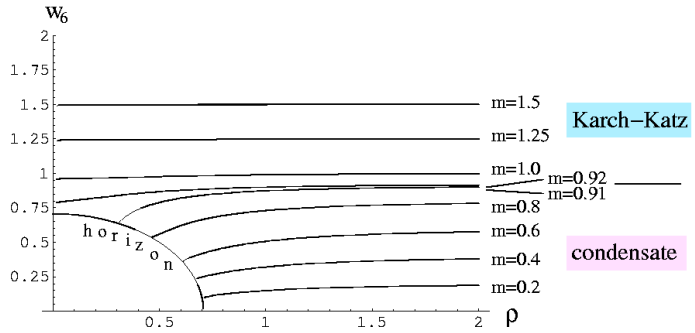
For the regular solutions the D7-brane either ends at the horizon,

$$w^2 = w_6^2 + \rho^2 = \tfrac{1}{2}b^2 \quad (S^1 \text{collapses})$$

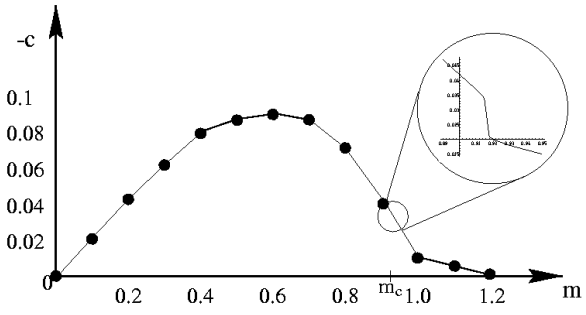
or ends at a point outside the horizon,

$$\rho = 0, w_6^2 \geq \tfrac{1}{2}b^2 \quad (S^3 \text{collapses})$$

⇒Two classes of regular solutions in the AdS black hole background:



Condensate c versus quark mass m plot:



phase transition expected at

$$m_c \approx 0.92$$

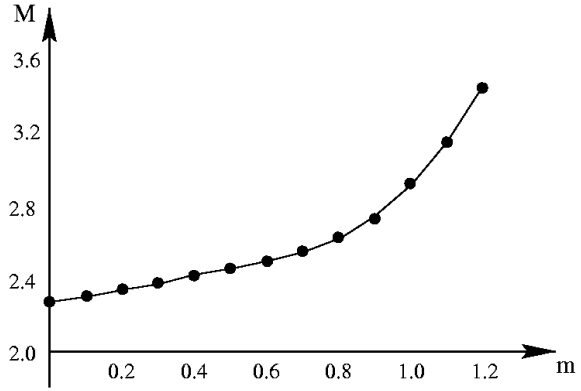
AdS BH: Meson spectrum

Consider fluctuations of w_5 about the embedding $w_5 = 0, w_6(\rho)$:

$$\delta w_5 = f(\rho)\sin(k \cdot \mathbf{x}) \ , \qquad M^2 = k^2$$

Solve the linearized (in δw_5) equation of motion for diffent values M and search for regular solutions

Plot of the w_5 meson mass M vs bare quark mass m:



$U(1)$ chiral symmetry

$U(1)$ isometry in $w_5 - w_6$ plane $= U(1)$ chiral symmetry in FT

$c = \langle \bar{q}q \rangle \neq 0$ breaks $U(1)$

$c = 0$ for $m = 0 \Rightarrow$ no spontaneous chiral symmetry breaking

mass gap ($M \neq 0$ for $m = 0$)

The Myers-Constable deformation

$\mathcal{N} = 4$ super Yang-Mills theory can be deformed by the presence of a vacuum expectation value for an R-singlet operator with $\Delta = 4$ (such as $\mathrm{tr}\, F^{\mu\nu} F_{\mu\nu}$) $\rightarrow$
a non-supersymmetric QCD-like field theory

The Myers-Constable background is given by the metric

$$ds^2 = H^{-1/2} \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta/4} dx_4^2 + H^{1/2} \left(\frac{w^4+b^4}{w^4-b^4}\right)^{(2-\delta)/4} \frac{w^4-b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta} - 1 \qquad (h^2+\delta^2 = 10)$$

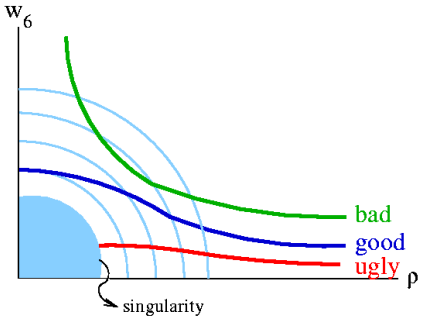
and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4+b^4}{w^4-b^4}\right)^{h}, \qquad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz$$

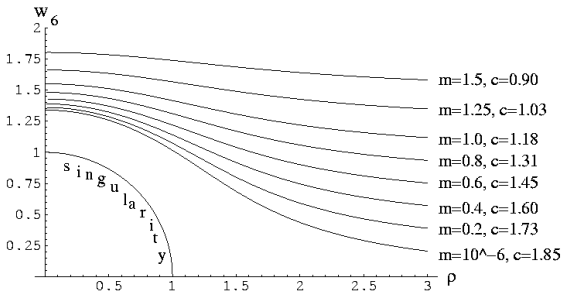
This background has a singularity at $w = b$.

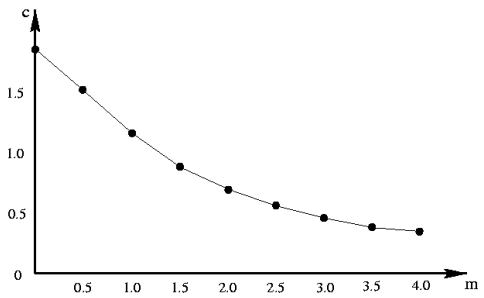
MC deformation: D7-brane embedding & chiral symmetry breaking

Different possibilities for solutions of the D7-brane equations of motion:



Regular solutions in the Constable-Myers background (b=1):





Numerical results:

screening effect: regular solutions terminate before reaching the singluarity!

spontaneous breaking of $U(1)$ chiral symmetry: in the limit $m \rightarrow 0$ we have $c \neq 0$

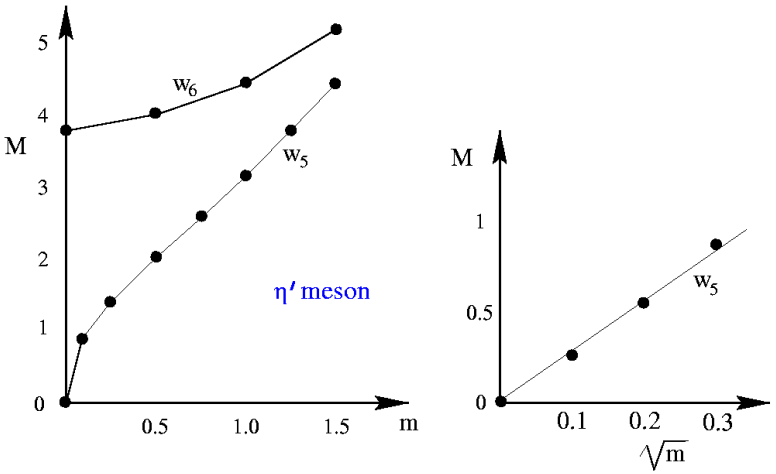
Meson spectrum and large N Goldstone boson (η')

Spontaneous chiral symmetry breaking (via a condensate in the $m \to 0$ limit) $\Rightarrow$ expect a Goldstone boson in the meson spectrum

Consider fluctuations

$$\delta w_5 = f(\rho)\sin(k\cdot x)\ , \qquad \delta w_6 = h(\rho)\sin(k\cdot x)$$

around $w_5 = 0$, $w_6 = w_6(\rho)$

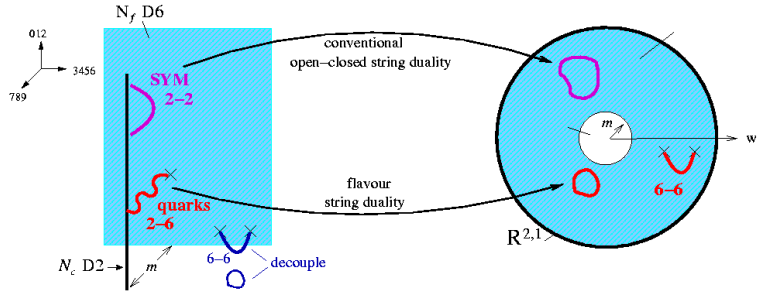


Note:

- U(1) chiral symmetry is non-anomalous in the limit $N \to \infty$
- m = 0: η' is a true (massless) Goldstone boson in a large N QCD-like theory (a pseudo-Goldstone boson for $m \neq 0$)

D2/D6 system

J.E., Kirsch

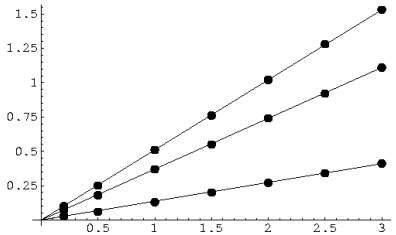


Supergravity metric known (Cherkis+Hashimoto, Pelc+Siebelink)

No probe approximation necessary

Dual gauge theory: three-dimensional $\mathcal{N} = 4$ SUSY gauge theory

Meson operators dual to closed string states!



Discussion

D7 probe in Constable-Myers background:

- spontaneous chiral symmetry breaking
- Goldstone boson: η' meson

AdS black hole geometry dual to 3d gauge theory:

- quark condensate vanishes for vanishing quark mass
- geometric transition corresponding to a first order phase transition in the dual gauge theory

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In all scenarios (D3/D7, D4,D6, . . .) of gauge/gravity duals:

$$\Lambda_{\rm SUSY} \sim \Lambda_{\rm QCD}$$

due to strong-coupling nature of gauge/gravity duality.

- D4/D6: UV theory five-dimensional, regular in IR.
- D3/D7: UV theory four-dimensional (conformal for N_f finite and small, $N_c \to \infty$),
  IR singularity in background with screening effect

D2/D6: beyond probe approximation, mesons dual to closed strings

Challenges

- $N_f \sim N_c$ in $d = 4$
- $SU(N_f) \times SU(N_f)$ chiral symmetry
- Stability of non-supersymmetric backgrounds
- Stringy corrections ($1/N$ corr. in field theory)
- Further applications to physical observabes

Gauge/gravity duals with flavour:

Interesting string-theoretical tool for non-perturbative description of low-energy dynamics!