Inclusive B–decay Spectra and Infrared Renormalons

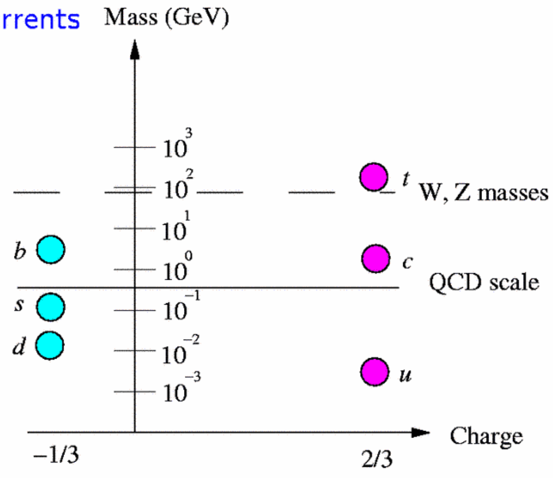
Einan Gardi (Cambridge)

plan of the talk

- Experimental motivation: Flavor physics $\Longrightarrow$ Inclusive decays $\Longrightarrow$ Decay spectra
- Introduction: Sudakov–logs and renormalons
- The endpoint region in inclusive decays and the quark distribution in the meson
- Factorization and Dressed Gluon Exponentiation
- Renormalon ambiguity and its resolution
- Comparison with inclusive DIS
- Open theoretical problems and directions

The Standard Model

- Weak interaction $\Longrightarrow$ flavour–changing currents
- Decays probe short–distance dynamics
- Transition between generations: CKM

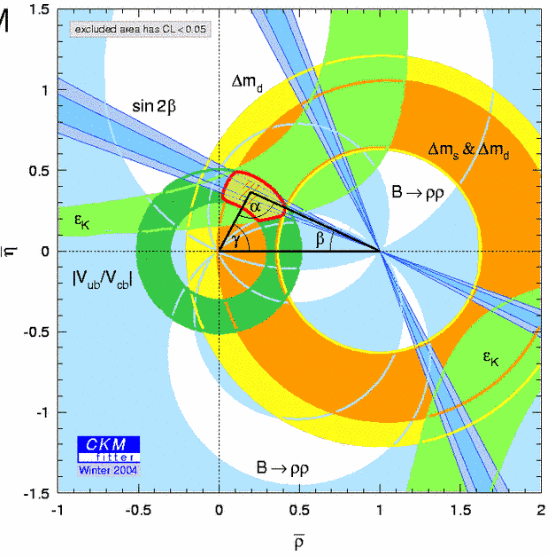


$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

Flavor Physics

motivation: identify deviations from SM

- (over)constraining the unitarity triangle,
- branching ratios of rare decays,



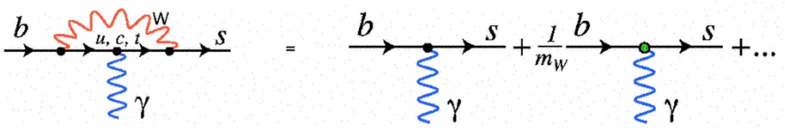
However

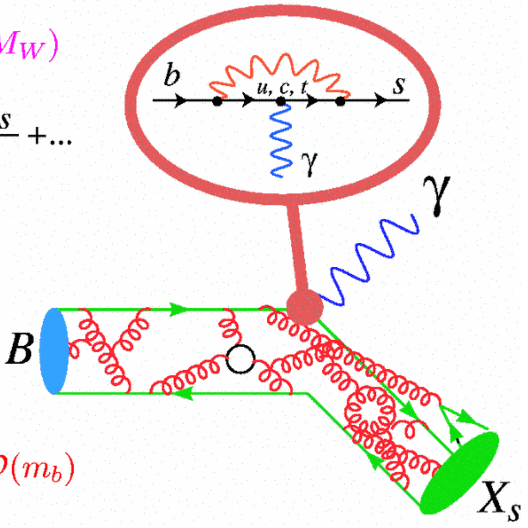
- quarks are *confined* into mesons
- both exclusive measurements (specific final states) and inclusive ones are useful
- advantage of inclusive measurements: absolute rate is computable in spite of QCD e.g. $B \to X_c l\nu$, $B \to X_u l\nu$, $B \to X_s\gamma$

Hierarchy of scales and effective field theories

separation between short– and long–distance physics

First step: integrate out fields with virtuality $\mathcal{O}(M_W)$





To leading order in $1/m_W$ and in α_s (only O_7):

$$H = \sqrt{2}G_F V_{tb}V_{ts}^* A(m_t^2/m_W^2)\,\frac{em}{16\pi^2}\,\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}$$

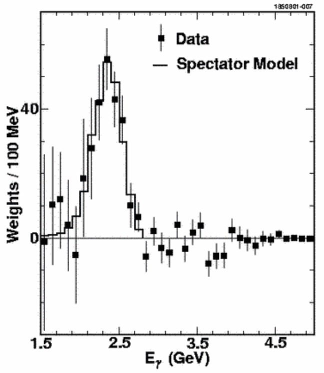
Second step: integrate out fields with virtuality $\mathcal{O}(m_b)$
Heavy–Quark Effective Theory (HQET)

To leading order in $1/m_b$ the interaction with the heavy quark is
$\mathcal{L} = \bar{h}_v iv \cdot D h_v$, where $v = p_B/M$ and $h_v(x) = e^{imv\cdot x}\,\frac{1+\not{v}}{2}\,\Psi(x)$.

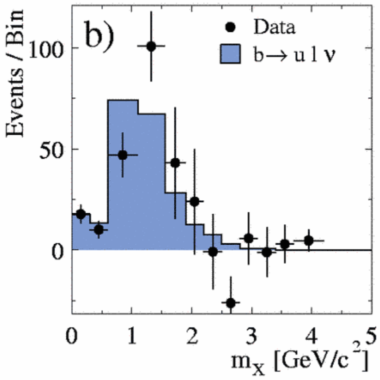
Inclusive B–decay Spectra

radiative decay: $\bar{B} \longrightarrow X_s\,\gamma$

semi-leptonic decay: $\bar{B} \longrightarrow X_u\, l\bar{\nu}_l$



CLEO

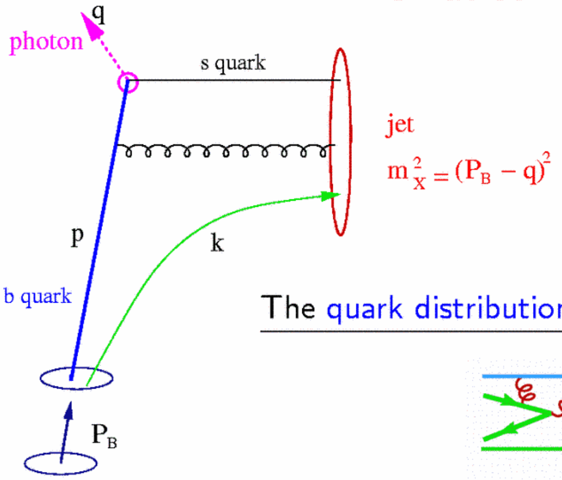


BABAR

The distribution peaks close to the endpoint $(E_\gamma \longrightarrow \frac{M}{2};\quad m_X \longrightarrow 0)$

Example: extracting $|V_{ub}|$ from the semi-leptonic decay
Experimental measurements are restricted to the endpoint region (charm background)
Theoretical description of the spectrum near the endpoint is necessary!

Kinematics in $\bar{B} \longrightarrow X_s\,\gamma$

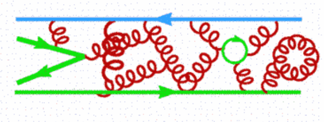


$x \equiv 2E_\gamma/M$

scales:

Hard: M
Jet: $m_X^2 = (P_B - q)^2 = M^2(1-x)$
Soft: $M(1-x)$

The quark distribution in the B meson $f(z,\mu)$



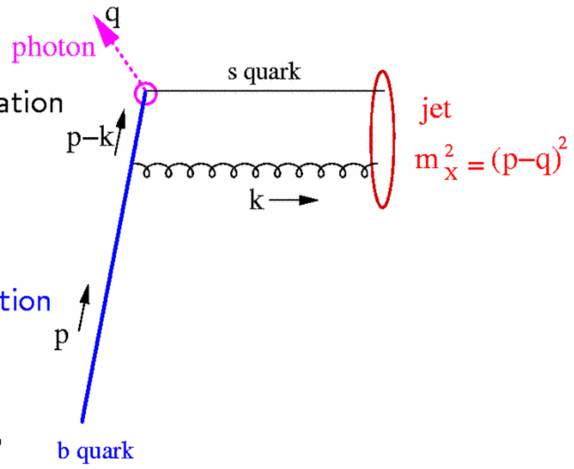
In the B rest frame all components of k are $\mathcal{O}(\Lambda)$ so usually $f(z,\mu)$ is $\mathcal{O}(\Lambda/M)$

But near the endpoint $M(1-x) \sim \Lambda$ the effect of $f(z,\mu)$ is $\mathcal{O}(1)$

$$m_X^2 = (k+p-q)^2 \simeq (p-q)^2 + 2k\cdot(p-q) = \mathcal{O}(\Lambda M)$$

Decay spectra: the endpoint region in perturbation theory

Consider b–quark decay for x near 1: **Hard:** m $\gg$ **Jet:** $m\sqrt{1-x}$ $\gg$ **Soft:** $m(1-x)$
but such that **Soft:** $m(1-x)$ $\gg$ Λ



Infrared– and collinear–safe observable,
dominated by multiple, soft and collinear radiation

Logarithms:
e.g. soft emission from the heavy–quark line:
$(p-k)^2 - m^2 \simeq 0$
singular propagator & phase–space integration
$\Downarrow$
$\ln(1-x)$

$d\Gamma/dx$ diverges at $x \longrightarrow 1$ at any order in α_s,
but actually it should vanish there...

Exponentiation: factorization of matrix element & additivity of energy fractions
$\Downarrow$
exponentiation in moment space

Decay spectra: some history

Both PT and NP aspects have been addressed ~ 10 years ago

- Sudakov resummation Korchemsky & Sterman ('94)
- HQET Jaffe & Randall; Bigi, Shifman, Uraltsev & Vainshtein; Neubert ('93–'94)

However, this is not sufficient to compute the spectra...
The reason: lack of mechanism for **PT/NP separation**

The endpoint region beyond PT theory: parametrically–enhanced power corrections
Indeed the Sudakov exponent has power–like ambiguity from infrared renormalons.

The solution: power–like **PT/NP separation** in the Sudakov exponent
Possible if PT is treated as asymptotic $=$ Dressed Gluon Exponentiation (DGE)

Sudakov Logs and Renormalons

hierarchy of scales $\longrightarrow$ logs

Running coupling logs in loops $\xrightarrow{\int d^4k}$ Renormalons

Soft and collinear gluon radiation
(nearly on-shell partons) $\xrightarrow{\int d^4k}$ Sudakov logs

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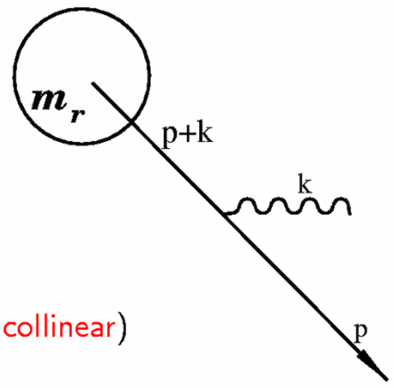
Sudakov Logs



The quark propagator
(for $k^2 = 0$ and $p^2 = 0$)

$$\frac{1}{(p+k)^2} = \frac{1}{2pk} = \frac{1}{2E_gE_q(1-\cos\theta_{qg})}$$

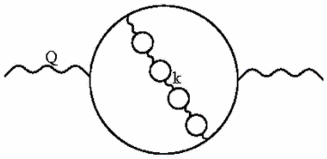
is singular at $E_g = 0$ (soft) and at $\theta_{qg} = 0$ (collinear)

incomplete cancellation between real and virtual corrections
For infrared and collinear safe observables,
phase-space integration transforms the singularity into logs

Renormalons

Example: vacuum polarisation

$$D(Q^2) \;=\; C_F\alpha_s \sum_n \int_0^\infty \frac{dk^2}{k^2}\,\phi(k^2/Q^2)\,\left[-\frac{\beta_0\alpha_s}{\pi}\,\ln(k^2/Q^2)\right]^n$$
$$= \; C_F \int_0^\infty \frac{dk^2}{k^2}\phi(k^2/Q^2)\,\frac{\alpha_s(k^2)}{\pi}$$



For small momenta (IR) $\phi(\epsilon) \sim \epsilon^2$

For large momenta (UV) $\phi(\epsilon) \sim \ln\epsilon/\epsilon$

$$A^n \int_0^{Q^2} \frac{dk^2}{k^2}\left(\frac{k^2}{Q^2}\right)^p \left[-\ln\left(k^2/Q^2\right)\right]^n = A^n\,\frac{n!}{p^{n+1}} \qquad A \equiv \frac{\beta_0\alpha_s}{\pi}$$

Minimal term at $n \sim n_m = p/A$. Ambiguity $\sim\; n_m! n_m^{-nm} \;\sim\; \exp(-n_m) \;=\; (\Lambda^2/Q^2)^p$

Dressed Gluon Exponentiation

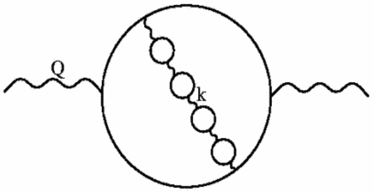
expansion in $\alpha_s(Q^2)$

large order n

Single Dressed Gluon

Renormalons $\longrightarrow$ $1/Q^2$

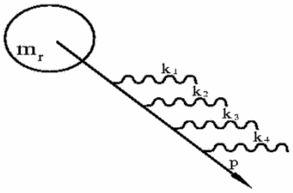
$n! C_F \beta_0^{n-1} \alpha_s^n$



$x \longrightarrow 1$ (large N)

Dressed Gluon Exponentiation (DGE)

(Λ^2/W^2 is not negligible)



multiple emission

Sudakov Double Logs $C_F^n \alpha_s^n L^{2n}$

$\alpha_s L \ll 1 \;\; (W^2 \gg \Lambda^2)$

Coefficients in the exponent – the thrust distribution

$$\ln J_\nu^{\mathrm{PT}}(Q^2) = -\frac{C_F}{\beta_0}\sum_{n=1}^{\infty}\sum_{m=1}^{n+1} r_{n,m} A(Q^2)^n \,(\ln\nu)^m$$

$$A \equiv \alpha_s\beta_0/\pi$$

The coefficients $r_{n,m}$ in the exponent

$$m \longrightarrow$$

$$\begin{array}{c} n \\ \downarrow \end{array}\begin{bmatrix} -.35 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 3.82 & .98 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 19.89 & 8.44 & 2.19 & 1.17 & 0.00 & 0.00 & 0.00 & 0.00 \\ 113.17 & 61.99 & 17.69 & 3.95 & 1.50 & 0.00 & 0.00 & 0.00 \\ 955.64 & 466.61 & 168.43 & 36.18 & 6.86 & 2.07 & 0.00 & 0.00 \\ 9659.52 & 4839.38 & 1578.62 & 424.98 & 73.16 & 11.87 & 3.00 & 0.00 \\ 116592.70 & 58314.36 & 19479.89 & 4770.74 & 1024.61 & 147.12 & 20.73 & 4.54 \\ -1637813.50 & 818688.19 & 272967.28 & 68393.72 & 13406.87 & 2396.22 & 295.04 & 36.61 \end{bmatrix}$$

Subleading Sudakov logs

$$\ln J_\nu^{\mathrm{PT}}(Q^2) = \frac{C_F}{2\beta_0}\sum_{k=1}^{\infty} A^{k-2}\, f_k(\lambda) \qquad \text{with} \quad A \equiv \beta_0\alpha_s(Q^2)/\pi \qquad \lambda \equiv A\ln\nu$$

$$\begin{aligned} f_1(\lambda) &= 2(1-\lambda)\ln(1-\lambda) - (1-2\,\lambda)\ln(1-2\,\lambda) \\ f_2(\lambda) &= -2\,\gamma\,(\ln(1-\lambda) - \ln(1-2\,\lambda)) - \frac{3}{2}\ln(1-\lambda) \end{aligned}$$

$$\begin{aligned} f_3(\lambda) &= 0.804/(1-\lambda) - 2.32/(1-2\lambda) \\ f_4(\lambda) &= 0.779/(1-\lambda)^2 - 2.68/(1-2\lambda)^2 \\ f_5(\lambda) &= 2.32/(1-\lambda)^3 - 5.00/(1-2\lambda)^3 \\ f_6(\lambda) &= 6.12/(1-\lambda)^4 - 15.32/(1-2\lambda)^4 \\ f_7(\lambda) &= 23.8/(1-\lambda)^5 - 61.12/(1-2\lambda)^5 \\ f_8(\lambda) &= 120.9/(1-\lambda)^6 - 305.52/(1-2\lambda)^6, \\ f_9(\lambda) &= 721.54/(1-\lambda)^7 - 1833.55/(1-2\lambda)^7 \end{aligned}$$

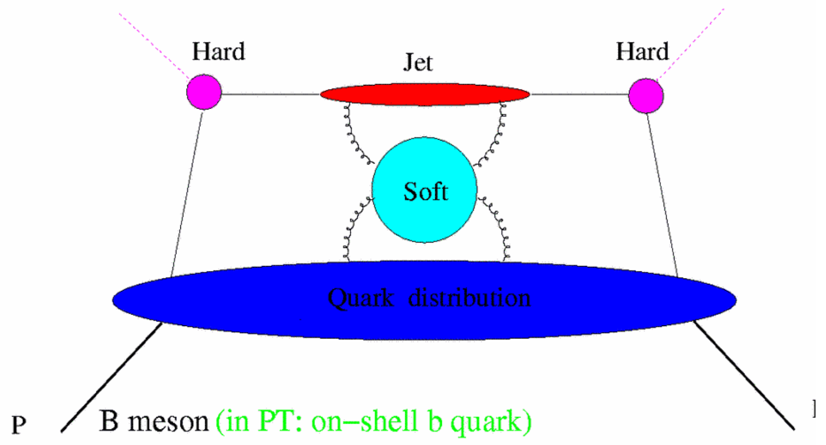
Sub-leading logs f_k are enhanced by renormalons:

increase factorially with k
have an increasing singularity at $A\ln\nu = 1/2$

Large–x factorization in inclusive B decays

Spectral moments

$$M_N \equiv \int_0^1 dx x^{N-1} \frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma}{dx}$$



Perturbation theory: $M_N = H(m) J(m^2/N;\mu) F_{\text{PT}}(m/N;\mu) + \mathcal{O}(1/N)$

Non-perturbatively: $M_N = H(m) J(m^2/N;\mu) F(m/N;\mu) + \mathcal{O}(1/N)$

From the quark distribution function to power corrections

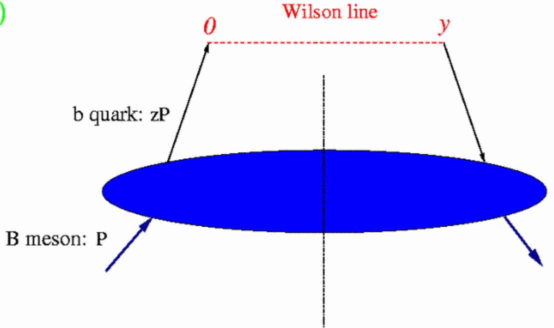
- $F(m/N;\mu)$ is (the large–N limit of) the b–quark distribution in the **meson** $B(p_B)$

$$f(z;\mu) \equiv \frac{1}{4\pi} \int_{-\infty}^{\infty} dy^- \, \mathrm{e}^{izp_B^+ y^-} \left\langle B(p_B) \left| \bar{\Psi}(0)\gamma^+ \, P\mathrm{e}^{-i\int_0^1 dsy^- A^+(sy)} \, \Psi(y) \right| B(p_B) \right\rangle_\mu$$

$$F(m/N;\mu) \equiv \lim_{N\to\infty} \int_0^1 dz z^{N-1} f(z,\mu)$$

Asymptotically $N \Longleftrightarrow -ip_B^+ y^-$.

- $F_{\text{PT}}(m/N;\mu)$ is defined similarly, but with an **on-shell heavy quark** state $\langle b(p)|$.
- The two distributions differ by $F_{\text{NP}}(m/N)$:



$$F(m/N;\mu) = F_{\text{PT}}(m/N;\mu) \, F_{\text{NP}}(m/N) + \mathcal{O}(1/N),$$

which resums power corrections on the soft scale.
These powers can be analysed using HQET.

Heavy quark effective theory and $\bar{\Lambda}$

Meson: $p_B = Mv$
Quark: $p = mv + k$. The components of k are much smaller than m.

The heavy quark field is near its mass shell, so the hard component can be scaled out:

$$h_v(x) = \mathrm{e}^{imv\cdot x}\,\frac{1+\not{v}}{2}\,\Psi(x).$$

$$f_{\mathrm{NP}}(\xi) = \int_{-\infty}^{\infty}\frac{d(v\cdot y)}{4\pi}\,\mathrm{e}^{iv\cdot y(\bar{\Lambda}-M\xi)}\left\langle B(Mv)\left|\bar{h}_v(0)\,P\mathrm{e}^{-i\int_0^1 dsy^- A^+(sy)}\,h_v(y)\right|B(Mv)\right\rangle,$$

where $\bar{\Lambda} \equiv M - m$

$$F_{\mathrm{NP}}(m/N) = \int_0^{\infty} d\xi\,\mathrm{e}^{-(N-1)\xi}\,f_{\mathrm{NP}}(\xi) \;\equiv\; \mathrm{e}^{-(N-1)\bar{\Lambda}/M}\,\mathcal{F}\left(\frac{(N-1)\Lambda}{M}\right)$$

$$= \mathrm{e}^{-(N-1)\bar{\Lambda}/M}\left[1 + \frac{f_2}{2!}\left(\frac{N-1}{M}\right)^2 + \frac{f_3}{3!}\left(\frac{N-1}{M}\right)^3 + \cdots\right]$$

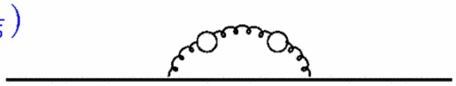
Here f_n are defined by local matrix elements in the HQET. They are m independent.
f_1 vanishes (equation of motion); f_2 is related to the kinetic energy (μ_π^2, λ_1).

Renormalon ambiguity in the pole mass

Infrared renormalons: a perturbative probe of large–distance effects

The propagator: $\dfrac{i}{\not{p} - m_{\overline{\mathrm{MS}}} - \Sigma(p, m_{\overline{\mathrm{MS}}})}$

Computed in the large–N_f limit



Off shell $\Sigma(p, m_{\overline{\mathrm{MS}}})$ is not problematic
But applying the on–shell condition (inverse propagator vanishes at the pole $p^2 = m^2$):

$$\frac{m}{m_{\overline{\mathrm{MS}}}} = 1 - \frac{C_F}{\beta_0}\int_0^{\infty} du\left(\frac{\Lambda^2}{m_{\overline{\mathrm{MS}}}^2}\right)^u\left[\frac{3}{2}\mathrm{e}^{\frac{5}{3}u}\,\frac{(1-u)\Gamma(u)\Gamma(1-2u)}{\Gamma(3-u)} - \frac{3}{4u} + R_{\Sigma_1}(u)\right].$$

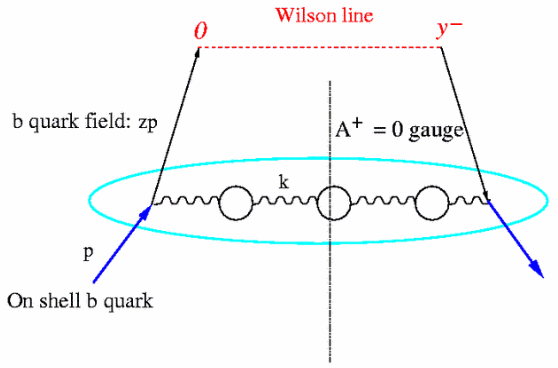
Beyond PT the pole mass is ambiguous...

Beneke & Braun; Bigi, Shifman, Uraltsev & Vainshtein (94)

$F_{\mathrm{NP}}(m/N)$ has the **renormalon ambiguity** of the pole mass through $\bar{\Lambda} = M - m$.
The ambiguity cancels in observables: e.g. total decay rate Beneke, Braun & Zakharov (94)

The quark distribution in an on-shell heavy quark

$F_{\mathrm{PT}}(m/N;\mu)$ is the quark distribution in an on-shell heavy quark:



It should be computed taking into account

- **Sudakov logs:** $\ln N$ enhanced terms
- **Renormalons:** power accuracy on m/N

Dressed Gluon Exponentiation (DGE) yields:

$$F_{\mathrm{PT}}(m/N;\mu) = \exp\left\{\int_0^\infty \frac{du}{u}\left(\frac{\Lambda^2}{m^2}\right)^u \left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) + \left(\frac{m^2}{\mu^2}\right)^u B_{\mathcal{A}}(u)\ln N\right]\right.$$

$B_{\mathcal{S}}(u)$ and $B_{\mathcal{A}}(u)$ are anomalous dimensions (renormalon free),

$$B_{\mathcal{S}}(u) = \frac{C_F}{\beta_0}\mathrm{e}^{cu}\,(1-u) + \mathcal{O}(1/\beta_0^2),$$

$B_{\mathcal{A}}(u)$ is the cusp anomalous dimension.

Cancellation of renormalon ambiguities in the Sudakov exponent

Combining the separately ambiguous perturbative and non-perturbative components:

$$\begin{aligned} F(m/N;\mu) &= F_{\mathrm{NP}}(m/N) \times F_{\mathrm{PT}}(m/N;\mu) \\ &= \exp\left\{-(N-1)\bar{\Lambda}/M\right\}\,\mathcal{F}\left(\frac{(N-1)\Lambda}{M}\right) \times \\ &\times \exp\left\{\int_0^\infty \frac{du}{u}\left(\frac{\Lambda^2}{m^2}\right)^u \left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) + \left(\frac{m^2}{\mu^2}\right)^u B_{\mathcal{A}}(u)\ln N\right]\right\} \end{aligned}$$

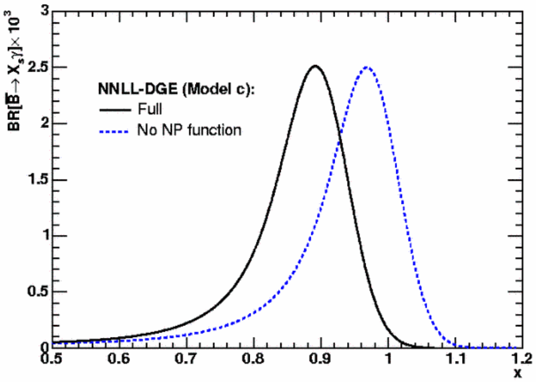
the $\mathcal{O}(N\Lambda/M)$ ambiguities in the exponent cancel out. Gardi (2004)

Leading NP effect: **shift** of the spectrum by $\bar{\Lambda}\big|_{\mathrm{reg}}/M$ towards smaller values of x.
The magnitude of the shift depends on the **prescription** $|_{\mathrm{reg}}$ used in summing the perturbative exponent (e.g. Principal value of the Borel sum).

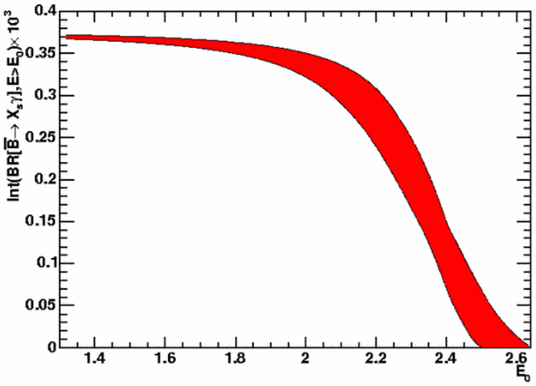
But the final answer for $F(m/N;\mu)$ is prescription independent!
$\mathcal{F}\left(\frac{(N-1)\Lambda}{M}\right)$ sums up higher power corrections on the soft scale.

$\bar{B} \longrightarrow X_s\,\gamma$ decay spectrum by Dressed Gluon Exponentiation

The PT result and the NP effect



The computed spectrum



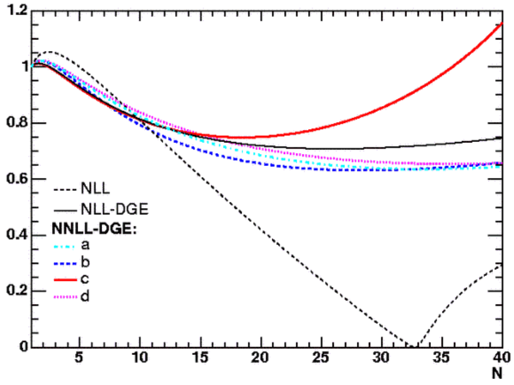
J.Andersen & Gardi (to be published)

parameters: $\alpha_s(m) = 0.216$, $m = 4.874$ GeV, $M = 5.279$ GeV, $\bar{\Lambda}_{\text{PV}} = 405$ MeV

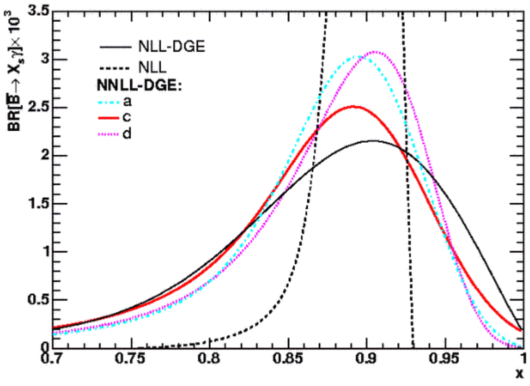
$\bar{\Lambda}_{\text{PV}}$ is **computed** based on M and the relation of the pole mass with $m_{\overline{\text{MS}}}\,(m_{\overline{\text{MS}}}\,) = 4.17$ GeV

$\bar{B} \longrightarrow X_s\,\gamma$ Spectrum

Different approx. for the Sudakov factor



The resulting spectra



Qualitative differences between DGE and fixed logarithmic accuracy (NLL):

- No Landau singularity
- Possibility to systematically include power corrections (parametrized / computed)

Conclusions

- Inclusive decay spectra can be systematically described even near the endpoint by extending the concept of **large-x factorization** beyond logarithms.
- Soft dynamics (M/N) requires **power-like separation** between PT and NP. This can be achieved only if renormalons in the Sudakov exponent are resummed. DGE with Principal–Value Borel summation of the exponent provides practical implementation of such separation.
- In spite of the remormalon ambiguity, the "pole mass" can be defined and used beyond perturbation theory. In a given regularization of the soft Sudakov exponent, the leading non-perturbative effect is a shift with a **known magnitude** $\bar{\Lambda}\big|_{\rm reg}$.

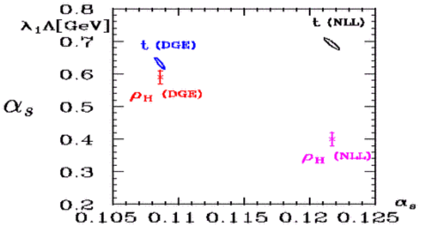
"Radiative and semi-leptonic B-meson decay spectra: Sudakov resummation beyond logarithmic accuracy and the pole mass," JHEP **0404** (2004) 049 [hep-ph/0403249]

B–decay spectra (radiative, semileptonic) are being computed. Prospects for comparison with Belle and BaBar. J.Andersen & Gardi (work in progress)

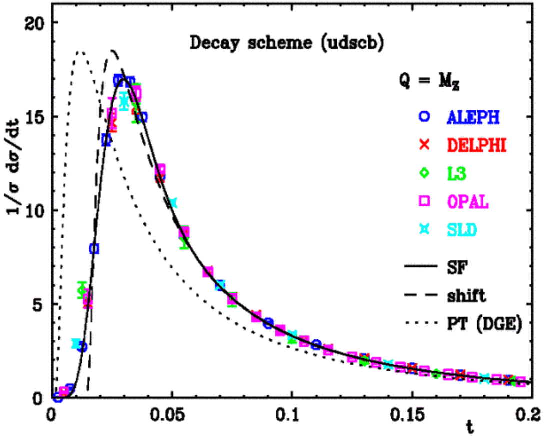
Dressed Gluon Exponentiation – Event-shape Distributions

- enhancement of subleading Sudakov logs ⟷ Renormalons
- information on non-perturbative shape-functions
- renormalon resummation is essential for the determination of α_s
- consistent description of the thrust and the heavy-jet mass

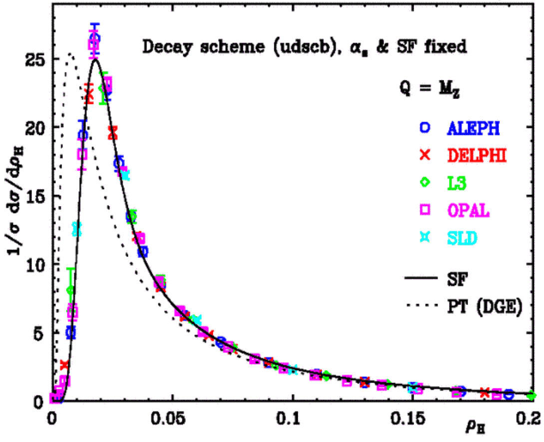
Gardi & J. Rathsman (2002)



thrust (fit)

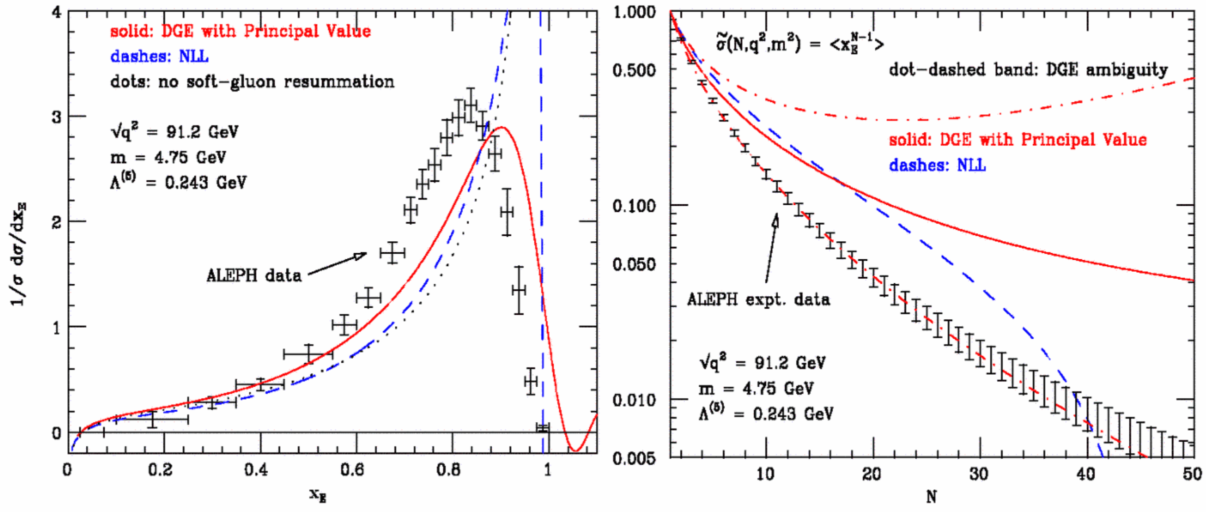


heavy-jet mass (prediction)



The DGE result for heavy–quark fragmentation

Single–particle inclusive cross section, $e^+e^-(Q) \longrightarrow B(E) + X$ at LEP1, $x \equiv 2E/Q$:



M. Cacciari & Gardi (2003)

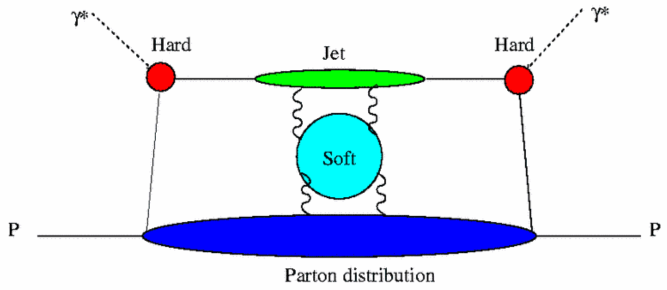
Open problems and possible directions

Vacuum expectation value of operators involving Wilson lines — going beyond PT

Higher renormalon ambiguities in the quark–distribution function and thier cancellation

All–order calculations beyond the large–β_0 limit

DIS – Factorization in the semi-inclusive region



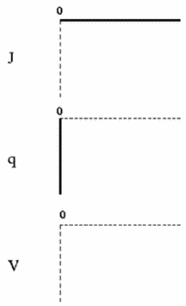
$$F_2(x,Q^2) = H\left(Q^2\right)\; J\left(Q^2(1-x)/x;\mu_{F_1}^2\right) \otimes V\left(x;\mu_{F_1}^2/\mu_{F_2}^2\right) \otimes q(x;\mu_{F_2}^2)$$

$$J\left(Q^2(1-\xi)/\xi;\mu_F^2\right) = \frac{1}{2\pi}\,\mathrm{Im}\int d^4\tilde{y}\,\mathrm{e}^{-i\,(q+zp)\tilde{y}}\left\langle 0\left|\mathrm{Tr}\left(\Psi(\tilde{y})\overline{\Psi}(0)\not{p}\right)\right|0\right\rangle$$

with $A_- = 0$ and $\xi \equiv x/z$

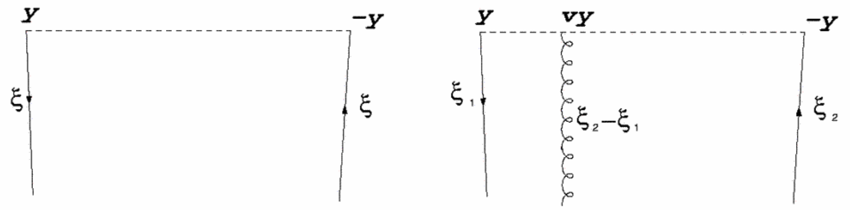
$$q\left(z;\mu_F^2\right) = \int_{-\infty}^{\infty}\frac{d(py)}{py}\,\mathrm{e}^{-i\,z\,py}\left\langle p\left|\overline{\Psi}(y)\not{y}\Psi(0)\right|p\right\rangle$$

with $y^2 = 0$ and $A_+ = 0$



OPE for DIS structure functions

$$\int_0^1 dx\,x^{N-1}F_2(x,Q^2) = C^{(2)}(N,\mu_{\mathrm{F}})\,\langle O^{(2)}(N)\rangle_{\mu_{\mathrm{F}}} + \frac{1}{Q^2}\sum_j C_j^{(4)}(N,\mu_{\mathrm{F}})\,\langle O_j^{(4)}(N)\rangle_{\mu_{\mathrm{F}}} + \ldots$$



advantages:

systematic expansion in $1/Q^2$
higher-twist operators are defined : calculable non-perturbatively
evolution equations

but for $x \sim 1$ $(N \longrightarrow \infty)$

coefficient functions are Sudakov enhanced by powers of $\ln(1-x)$
power corrections are enhanced at large x: $\sim 1/(Q^2(1-x))^n$
at twist 4 and beyond: the number of local operators increases with N

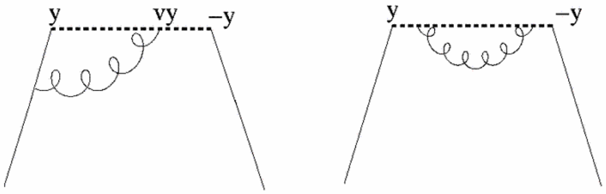
Cancellation of IR renormalon ambiguity

Beneke & Braun
Gardi, Korchemsky, Ross and Tafat

$$\int_0^1 dx\, x^{N-1} F_2(x,Q^2) = \left[C^{(2)}(N,\mu_{\mathsf{F}})\Big|_{\mathsf{reg}} + \frac{\delta_{\mathsf{reg}}\Lambda^2}{Q^2}\right] \langle O^{(2)}(N)\rangle_{\mu_{\mathsf{F}}}$$
$$+\frac{1}{Q^2}\sum_j C_j^{(4)}(N,\mu_{\mathsf{F}}) \left[\langle O_j^{(4)}(N)\rangle_{\mu_{\mathsf{F}}}\Big|_{\mathsf{reg}} + \delta_{\mathsf{reg}}^{(j)}\Lambda^2 \langle O^{(2)}(N)\rangle_{\mu_{\mathsf{F}}}\right]$$
$$+\ldots$$

- $1/Q^2$ IR renormalon ambiguity at twist two cancels against UV ambiguity in the definition of twist-four matrix elements

- the twist four ambiguity is due to mixing under renormalization of the twist-four operators with twist two



(a) (b)

- There remains no $1/Q^2$ ambiguity in the OPE provided
  - twist-two renormalon resummation is performed
  - twist-four UV-divergent contribution is included

- "renormalon dominance" is "ultraviolet dominance"