

①

## Magnetic quasi-particles in hot QCD

and

### k - tensions

- Why magnetic degrees of freedom?
  - × Ancient idea of dual superconductivity
  - × Strings between  $q\bar{q}$  pair an electric Abrikosov flux tube
- Why look at high  $T$  plasma?
  - × gluons are AF at high  $T$ , "e-quasi-particle"
  - × guess: i) magnetic quasi-particle = "lump" of magnetic gluons
    - ii) "lumps" form dilute gas at high  $T$
- Why/what are k-tensions?
  - × k-tensions govern area law of k-loops = Wilson loops formed by k fundamental loops
  - × spatial k-loops average magnetic flux of m-q-p's.
  - Lattice results within 1 to 1.5% of  $\langle \dots \rangle_{\text{lattice}}$

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1. Forces, screening in the plasma
2. Pressure DR
3. Simple example: electric flux loop in QED
4. Electric fluxloop in QCD (= 't Hooft) loop)
  - i) analytic
  - ii) quasi-particle approach
5. Magnetic fluxloop (spatial Wilsonloop)
  - i) analytic
  - ii) quasi-particle approach
6. Verdict of the lattice
7. Epilogue

## Prologue

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P<sub>1</sub>

- "QCD" stands for "gluodynamics", i.e. no quarks
- In discussing high T QCD we discuss only equilibrium quantities  
 $\text{Tr}_{\text{phys}} e^{-H/T}, \dots$
- For calculational purpose (perturbation theory, lattice calculations) these quantities are transcribed in path-integral language. Gauge potentials periodic in the Euclidean time, with period  $1/\tau$ :

$$A(\vec{x}, \tau) = \sum_n \exp(i n \tau 2\pi T) A_n(\vec{x})$$

(4)

P<sub>2</sub>

- As  $T$  grows large w.r.t. hadron scales

$$\times \tilde{n}(T) \equiv g^2(T) N \sim \frac{1}{\log T / \Lambda_T}$$

- integrate out perturbatively the heavy modes  $A_{n \neq 0}$ , so that:

$$\times L_{\text{QCD}} \longrightarrow L_{\text{EQCD}} \text{ (only } A_{n=0} \text{ fields)}$$

$$L_{\text{EQCD}} = (D_i A_0)^2 + m_E^2 A_0^2 + \lambda_E A_0^4 + F_{ij}^2 + \delta L_E$$

$$D_i A_0 \equiv \partial_i A_0 - ig_E [A_i, A_0]$$

$$m_E^2 = \frac{\gamma T^2}{3} (1 + O(g^2)), \dots$$

physics  $\sim m_E \sim gT$

- if  $A_0$  mode gets very heavy,  $m_E \gg g_E^2 N$ ,

$$L_{\text{EQCD}} \longrightarrow L_{\text{MQCD}} = F_{ij}^2 + \delta L_M$$

c.c. is now:

$$g_M^2 = g_E^2 \left( 1 - \frac{1}{48} \left( \frac{g_E^2 N}{\pi m_E} \right) - \frac{19}{4608} \left( \frac{g_E^2 N}{\pi m_E} \right)^2 \right)$$

(5)

 $P_3$ 

- Thus:  $\times 3d$  lattice calculations needed  
for the coefficients of  $g^{\alpha\beta}$  (depending  
on observable).

$\times 4d$  lattice can check the sum of  
the series!

## ⑥ Pressure

$$\frac{P}{P_{SB}} = 1 + P_2 \tilde{\lambda} + P_3 \tilde{\lambda}^{3/2} + (P_4 + \ell_4 \log \tilde{\lambda}) \tilde{\lambda}^2 + P_5 \tilde{\lambda}^{5/2} + (P_6 + \ell_6 \log \tilde{\lambda} + P_6^{NP}) \tilde{\lambda}^3 + \dots$$

$\tilde{\lambda} \equiv g^2(T) N$

- $P_6^{NP}$  is non-perturbative, is coefficient of 3d YM partition function with magnetic coupling  $\tilde{g}_m^2 = g^2 N T$

Ex. 1

1.

1.

0.5

0

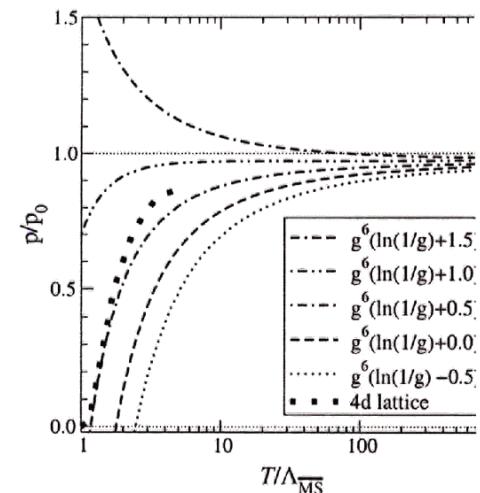
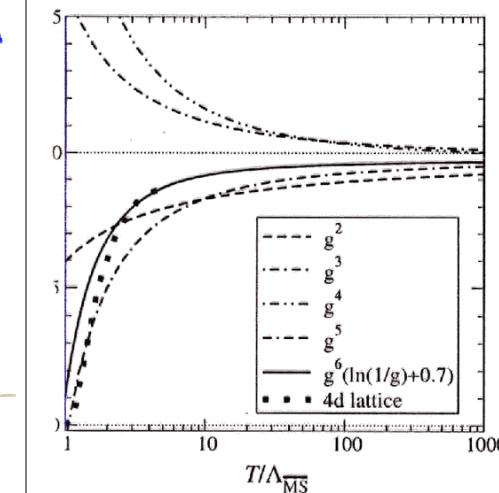


Figure 13: Left: perturbative results at various orders, including  $\mathcal{O}(g^6)$  for an optimist. Right: the dependence of the  $\mathcal{O}(g^6)$  result on the (not yet computed) constant, which contains both perturbative and non-perturbative contributions. The lattice results are from [33]. From ref. [20].

The pressure is normalized by  $p_0$  and consists of three parts:

$$\frac{p}{p_0} = p_h + p_E + p_M$$

The hard modes are cut-off in the infrared by  $\Lambda_E$  and equal  $p_h$ . Schematically we get:

$$p_h = 1 + a^2 + a^4 \log \frac{T}{\Lambda_E} + a^4 + a^6 \log \frac{T}{\Lambda_E} + a^6 \dots$$

↑  
not  
down  
from this

②

Ex 2

Spatial Wilson loop

- Exact opposite of pressure: leading term is 3d NP!

$$\langle W_p(L) \rangle_T = \int D\vec{A} \text{Tr} \exp i g d\vec{l} \cdot \vec{A} \exp(F_{ij}^2 + \delta L_M)$$

- hard modes  $\rightarrow$  only perimeter  $L$
- soft modes only in action
- only modes  $\sim g^2 T$  left.

Write  $\langle W_p(L) \rangle_T \sim \exp(-\sigma_s \text{Area})$

Dimensional argument:

$$\sigma_s = c (g_M^2)^2$$

$$c = 0.5530 \text{ (2d)} \quad \text{Karsch, Teper (3d)}$$

③

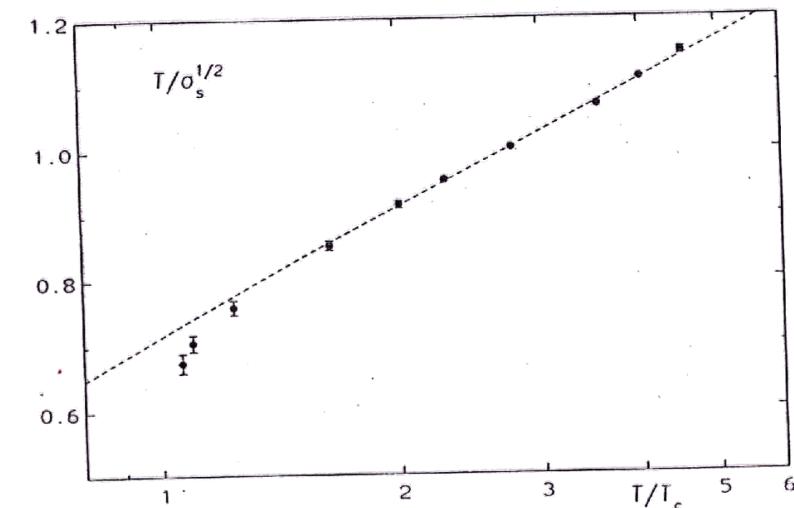
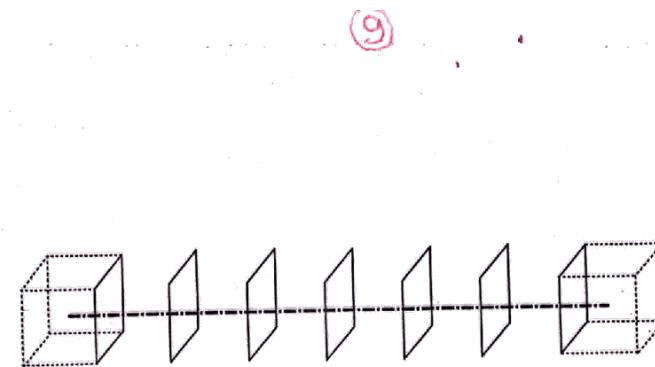


Fig. 7. The temperature over the square root of the spatial string tension  $\sigma_s$  versus  $T/T_c$  for SU(3). The dashed line shows a fit according to a two loop scaling formula for the coupling, see text below Eq. (5.8). From Ref. [15].

Data points are 4d,  $c$  determined from 3d.  
 $\delta L_M$  is only contributing significantly for  $T$  below  $\sim 1.5 T_c$ .

Magnetic screening mass



7: Monopole antimonopole pair induced by twisting the plaquettes pierced by Dirac string.

$$\exp -F_M(r)/T = \left( \int DA \exp -S_{(k)}(A) \right) / \int DA \exp -S(A).$$

The action  $S_{(k)}$  is the usual action, except for those plaquettes pierced by the Dirac string. Those plaquettes are multiplied by a factor  $\exp ik\frac{2\pi}{N}$ .

Screening is expected in both confined and deconfined phases:

$$F_M(r) = F_{M0} - c_M \frac{\exp -m_M r}{r}.$$

5 (1)

(3) 5

### 3.1 Magnetic screening

For progress of knowledge of magnetic sector: we need a g.i. definition of magnetic screening mass  $m_M$ .

Let us start from the lattice definition.

Our path integral with twisted plaquettes can be rewritten as a Hamiltonian matrix element

$$\text{SDA exp} - S_{(k)}(\mathbf{A})$$

$$= \langle 0 | V_k e^{-H_T^r} V_k^\dagger | 0 \rangle$$

Here  $\vec{z}$  is the "time" direction and  $H_T^r$  is the Hamiltonian of  $x, y$  and periodic  $\vec{z}$  space.  $P$  ( $y \leftrightarrow -y$ ),  $C$  and  $R$  ( $z \rightarrow -z$ ) are symmetries, together with  $J_T = x\partial_y - y\partial_x$

6 (1)

(3) 7

$$\text{Under } C, P: V_k \rightarrow V_k^\dagger, R: V_k \rightarrow V_{-k}, J_T = 0$$

Then:

$m_M$  is mass of  $J_{T=0}^{PC} = 0_+^{++}$  state of  $H_{T=0}$

Two extremes:

$$T=0: m_M = 0_+^{++} \text{ glueball} \quad \text{P.de F., C.H. O.P., CPKA}$$

$$T \rightarrow \infty: m_M = 0_+^{++} \text{ state of } H_{T=\infty} = H_{xy}$$

$$\frac{m_M}{\sqrt{g}} = 4.718(43) \text{ SU}(2)$$

$$= 4.329(41) \text{ SU}(3)$$

$$= 4.065(55) (1 + 0.634(g_0)/N_2 + \dots)$$

M.Teperek, hep-ph/9804008

Note  $\frac{\sigma}{m_M^2} \approx l_H^2 \sigma \approx 0.06$  small!

- Both  $\sigma$  and  $m_M^2 \sim (g^2 N T)^2$  so no parametric reason to be small.

1

② 2

3a

2.3 A Gedanken experiment in a plasma of protons and electrons.

Take a closed loop  $L$ , area  $A(L)$ ,  
in an ionised plasma, and look at  
the electric flux through it.

$$\Phi(L) = \int dS \cdot \vec{E} / e$$

$$V(L) = \exp(i 2\pi \Phi(L))$$

"electric flux loop"

$\rightarrow$  If  $T < T_{\text{ionization}}$  atoms neutral :

$$V(L) = 1$$

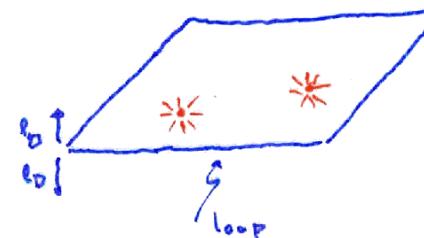
$\rightarrow$  If  $T > T_{\text{ionization}}$

$$V(L)|_{\text{electron}} = \exp(i 2\pi \cdot \frac{1}{2}) = -1$$

$$\langle V(L) \rangle = \sum_e P(e) V(L)|_e = \sum_e P(e) e^e$$

$$= \exp(-2) \quad \text{if } \underbrace{\text{Poissonian}}$$

$T > T_{\text{ionization}}:$  Debye screening  $\frac{e_D}{k_B n} \sim 10^6$



slab of thickness  $b_D$

2

(2) 3

$\downarrow$  caveat  
 The "tension"  $g$  in units of Debye length  
 $\cdot l_D^2 g = 2 l_D^3 \pi = \text{big number} (10^6)$

NB: If we do not neglect size of atom  
 (relevant below Tionization) then

$$T < T_{\text{ionization}} : \frac{g}{l_D^2} = 2 l_{\text{at}}^2 \pi$$

$$\text{and } l_D^2 \frac{g}{l_{\text{at}}^2} = \text{small number} \\ (l_{\text{at}} \ll l_D)$$

 $\therefore$ 

electric flux loop  $V(L)$  is order parameter

Question: Can we do better?  
 ↓  
 Coulomb gas  
 ↓  
 QED

$$\begin{array}{l} \text{(i) } m_D \\ \text{(ii) } P(L) \end{array}$$

First: a different look at  $V(L)$

3

(2) 3

A closer look at the flux loop

A little thought:

\*  $V(L) = \text{closed Dirac flux line along } L$

Follows from considering a gauge transf.  
 . with discontinuity  $2\pi$  through  
 $w_1(x) \nearrow$   
 . falls off fast at  $|x| \rightarrow \infty$

$$\int_{S(L)} d\vec{S} \cdot \vec{E} = \frac{1}{2\pi} \int_{\text{all space}} dV \vec{\nabla}(\vec{E} w_1) \quad \text{Gauss}$$

$$= \frac{1}{2\pi} \left[ \int dV [(\vec{\nabla} \cdot \vec{E} - e j_0) w_1 + (\vec{E} \cdot \vec{\nabla} + e j_0) w_1] \right] \quad \text{charge density}$$

$$= \frac{1}{2\pi} \int_{\text{on physical subspace}} (\vec{E} \cdot \vec{\nabla} + e j_0) w_1 dV$$

\*  $V(L)$  is singular gauge transf., discontinuity  $2\pi$  through surface.



$$\int d\vec{S} \cdot \frac{1}{e} \vec{\nabla} w_1 = \frac{2\pi}{e}$$

4

Electric flux loop in  $SU(N)$ 

$$\text{In QED: } V(L) = \exp i \frac{2\pi}{e} \int dS \cdot \vec{E}$$

$$\text{In QCD: } V_k(L) = \exp i \frac{2\pi}{g} \int dS \cdot \text{Tr} \vec{E} Y_k$$

 $N \times N$  matrix

$$Y_k \equiv \frac{1}{N} \begin{pmatrix} N-k & & & \\ & N-k & & 0 \\ & & N-k & \\ 0 & & & N-k \end{pmatrix} \quad \exp i 2\pi Y_k = e^{-ik\frac{2\pi}{N}} \mathbb{1}$$

- $V_k(L)$  creates  $\mathbb{Z}(N)$  flux loop of strength  $e^{ik\frac{2\pi}{N}}$
- gauge invariant in physical Hilbert space ( $Y_k \rightarrow S^1 Y_k S^1$  invariant)
- $Y_1$  hypercharges:  $2(N-1)$  gluons have charge  $\pm g$ , remaining ones charge  $\approx 0$
- $Y_k$ :  $2k(N-k)$  gluons have charge  $\pm g$
- So one single gluon shines a flux  $\pm \frac{1}{2}g$  through  $L$ :  $V_k(L)|_{\text{1 gluon}} = \exp(i \frac{2\pi}{g} \pm \frac{1}{2}g) = -1$

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- assume: i) gluons are independent  
ii) have a Poisson distribution around mean number  $\bar{L}$  in the slab

- Then for one species with charge  $\pm g$ :

$$\times \langle V_k \rangle_{\text{1 species}} = \sum_l P(l) (-)^l = \exp - 2\bar{L}$$

x independence:

$$\begin{aligned} \langle V_k \rangle_{\text{all species}} &= \exp - 4\bar{L} k(N-k) \\ &= \exp \left( -8 \ell_D \ln k(N-k) \text{ Area} \right) \\ &= g_k \end{aligned}$$

$\ell_D$  is Debye screening length  $\left(\frac{\pi T^2}{3}\right)^{\frac{1}{2}}$   
 $n$  is single gluon density  $\frac{g(3)}{\pi^2} T^3$

$$\times g_k = \# k(N-k) \frac{1}{\sqrt{\pi}} T^2$$

Same parametric dependence as from 1 and 2 loop calculation

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Ratio of  $\frac{g_2}{g_1} = 1.333 \dots$  in  $SU(4)$ .

LATTICE:  $T_{T_c}$      $\frac{g_2}{g_1}$

2.3    1.342 (13)

1.5    1.300 (18)

1.2    1.310 (30)

P.de Forcrand, B.Lucini, M.Vettorazzo, lat/0409148

M.Tepер et al., to appear

7

Analytic method to compute

't Hooft tension  $g_k$

Essential is  $Z(N)$  invariance

× Perform a gauge transfn  $S_{[k]}$ :

$$S_{[k]}(\vec{x}, \tau + \frac{1}{T}) = e^{ik\frac{\pi\tau}{N}} S_{[k]}(\vec{x}, \tau)$$

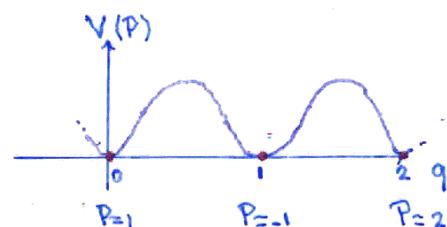
- action  $S$  is invariant
  - measure in path integral invariant
  - Polyakov loop  $P(A_\theta) \rightarrow e^{ik\frac{\pi\theta}{N}} P(A_\theta)$
- ∴ global  $Z(N)$  invariance, order parameter  
is Polyakov loop.

$\langle P \rangle = 0$      $T < T_c$     unbroken

$\langle P \rangle \neq 0$      $T > T_c$     broken

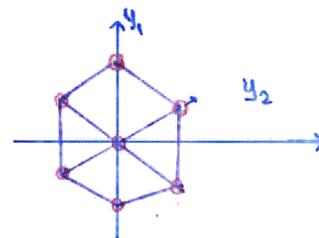
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As ever, compute effective action  
as function of  $P$ :

 $SU(2)$ 

$$P = \frac{1}{2} \text{Tr} e^{i 2\pi \frac{Y_3 q}{2}}$$

$0 < q < 1$

 $SU(3)$ 

$$P = \frac{1}{3} \text{Tr} e^{i 2\pi Y_3 q}$$

or  $= \frac{1}{3} \text{Tr} e^{i 2\pi Y_2 q}$

Tunnel from  $P=1$  to  $P=e^{ik2\pi/N}$

$$\rho_k = \min_q \int_{-\infty}^{\infty} dz (\partial_z q)^2 + V(q)$$

$V_k(q)$  along straight path  $1 \rightarrow e^{i k 2\pi/N}$

" computed for large  $T$  in  $\mathbb{R}^4$

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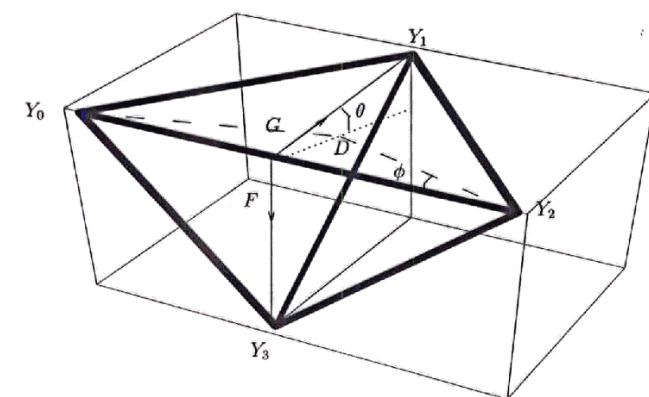


Figure 6: Elementary cell for  $SU(4)$ . The broken line connecting the corners  $Y_0$  and  $Y_2$  is a typical path on which we minimized the effective action  $U(C)$ . Indicated are the directions of the  $F$  and  $G$  axes defined in the text.

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1 ~ 2 loop result

$$S_k(T) = g_k^{(1)}(T) \left( 1 - \frac{a_s N}{4\pi} (15.2785 \dots - \frac{11}{3} (\chi_8 + \frac{1}{22})) \right)$$

$$+ O(g^3)$$

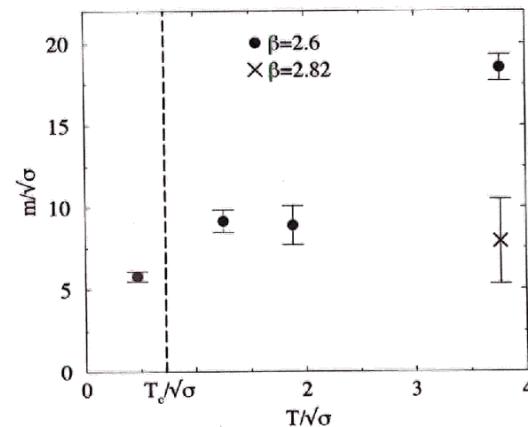
$\underbrace{\phantom{0}}$   
 k-dependence

$$g_k^{(1)}(T) = k(N-k) \frac{4\pi^2}{3\sqrt{3}(g^2 N)^{1/2}} T^2$$

Compare to  $g^{qq}(T) = \frac{2k(N-k)}{\text{multiplicity}} L_D n(T) 6.568 \dots$   
 $n(T)$  density of one  
 single gluon species

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(12)



SU(2) hep-lat/0003010  
 Figure 5: Screening mass vs. temperature at  $\beta = 2.6$ . Because of the large systematic errors introduced by the small temporal lattice extent  $N_t = 2$  for the last data point, we also included results from simulation at the same physical volume and half the lattice spacing ( $\beta = 2.82$ ). The dashed line indicates the critical temperature (taken from [23]).

11	$W_1$
Wilsonloops at high T	
• High T = 3d for W-loop	
Model for 3d YM.	
① magnetic gluons form lumps of size	
$\ell_M = m_M^{-1}$	
② lumps are dilute: $\ell_M^3 n_M \approx \delta \ll 1$	
③ lumps are monopoles	
• GNO classification of monopoles in unbroken gauge theory: monopoles are in multiplets of magnetic global $SU(N)$ .	
④ Choose the adjoint	

12	$W_2$
• Wilsonloop given by JRRREP R	
$W_R(L) = \frac{1}{d_R} \text{Tr} \exp[i g \int dT \cdot \vec{B}_R]$	
Need a flux representation!	
	$W_R(L) = \int dS \exp[i g \int dS \text{Tr} \vec{B} S L_R S^\dagger]$
	DP
	$H_R$ highest weight of R.
	If R is totally AS with k boxes:
	$H_R = y_k$
	• Compute for k-AS the averages in the gas, with adjoint monopoles.
	$\langle W_{k,AS}(L) \rangle = \langle \exp[i g \int dS \text{Tr} \vec{B} S y_k] \rangle$

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- Since thermal de Broglie length  $\sim \frac{1}{T} \ll \frac{1}{g^2 T} \sim \frac{1}{M}$

we can take the gas classical, so

Poisson distribution for l bumps inside our slab of thickness  $\ell_M$  around the loop.

- Like in the gluon case:

$$2k(N-k) \text{ monopoles have charge } \pm \frac{2\pi}{9}$$

- one charged monopole shines  $\pm \frac{1}{2} \frac{2\pi}{9}$  through loop

$$\exp i g \cdot \pm \frac{1}{2} \frac{2\pi}{9} = -1$$

- one charged species averages loop to:

$$\langle W_k(L) \rangle_{\text{species}} = \sum_l P(l) (-)^l e^{-\bar{l}} = e^{-2\bar{l}}$$

- independent of the  $2k(N-k)$  species:

$$\langle W_k(L) \rangle_{\text{all}} = e^{-4\bar{l} k(N-k)}$$

$$\underline{\sigma_k = 4 \ell_M n_M k(N-k)}$$

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## 6 Comparison to lattice simulations

We have been discussing a model at very high temperature. Hence it is tested in 3d lattice simulations. The ratios found<sup>14</sup> for the totally antisymmetric irreps are close — within a percent for the central value — as far as the adjoint multiplet of magnetic quasi-particles is concerned :

$SU(4) : \sigma_2/\sigma_1 = 1.3548 \pm 0.0064$	adjoint : 1.3333	fundamental : 1.8182
$SU(6) : \sigma_2/\sigma_1 = 1.6160 \pm 0.0086$	adjoint : 1.6000	fundamental : 1.9686
$\sigma_3/\sigma_1 = 1.808 \pm 0.025$	adjoint : 1.8000	fundamental : 2.3635

The results are that precise, that you see a two standard deviation from the adjoint, except for the second ratio of  $SU(6)$ . This deviation is natural, since the diluteness of the magnetic quasi-particles is small, on the order of a couple of percent, as we will explain at the end of this subsection. So we expect corrections on that order to our ratios.

There is a less precise determination of the ratio  $\sigma_2/\sigma_1 = 1.52 \pm 0.15$  in  $SU(5)$ <sup>20</sup>. But the central value is within 1 to 2% of the predicted value  $3/2$  from the adjoint. The fundamental gives a ratio 1.8231.

The  $SU(8)$  ratios are known on a rather coarse lattice<sup>20</sup> and using a different algorithm:

$\sigma_2/\sigma_1 = 1.692(29)$	adjoint : 1.714	fundamental : 2.106
$\sigma_3/\sigma_1 = 2.160(64)$	adjoint : 2.143	fundamental : 2.958
$\sigma_4/\sigma_1 = 2.26(12)$	adjoint : 2.286	fundamental : 3.256

In conclusion: the seven measured ratios are consistent with the quasi-particles being independent, as in a dilute gas and in the adjoint representation. The number of quasi-particle species contributing to the k-tension is  $2k(N-k)$ . This number happens to coincide with the quadratic Casimir operator of the anti-symmetric representation.

The fundamental monopoles are clearly disfavoured by the data.

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## Epilogue

- ① adjoint multiplet of monopoles gives 1 to 2% deviation from lattice results. WHY so small?

$$\frac{\sigma_k}{\sigma_0} = k(N-k)(1 + O(\delta))$$

- ② Our model gives generically

$$\sigma \sim l_H n_H$$

hence

$$\delta \equiv l_H^3 n_H = l_H^2 \sigma = \frac{\sigma}{m_M^2} \approx 0.05$$

- ③ ratio's sensitive to choice of monopole rep. Fundamental rep. off by ~30%.

- ④  $n_H \sim (g^2 T)^3 \sim \left(\frac{T}{\log T}\right)^3$  so SB limit is recovered.

- ⑤ 3d calculation of C. Herzog gives Casimir scaling as well.

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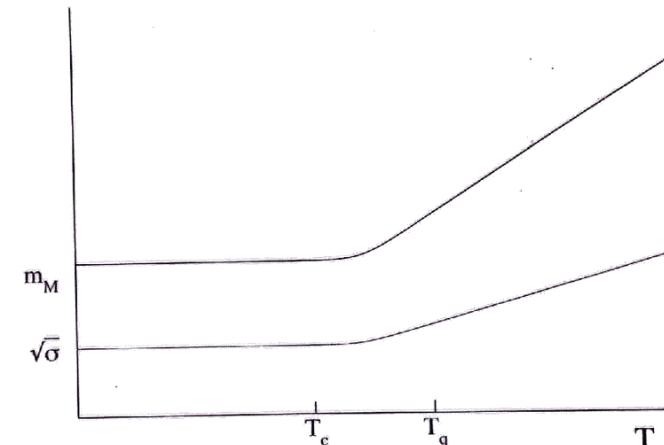


Figure 2: Magnetic mass  $m_M$  and tension  $\sigma$  as function of temperature, schematically.  $m_M(0) = m_{0++}$  and from ref. (13):  $T_c = 0.174 m_{0++}$ . The temperature  $T_q \leq m_{0++}$  is where the de Broglie thermal wave length becomes equal to the magnetic screening length. For the calculation of the tension it is below  $T_q$  that quantum statistics applies, above classical statistics applies as in section 5.

$$\delta(T) = \frac{\sigma}{m_M^2}(T) = \begin{cases} 0.05 & T = 0 \\ 0.09 & T \neq 0 \end{cases}$$

Is our monopole gas a dilute Bose gas, with BE condensation at  $T_c$ ?