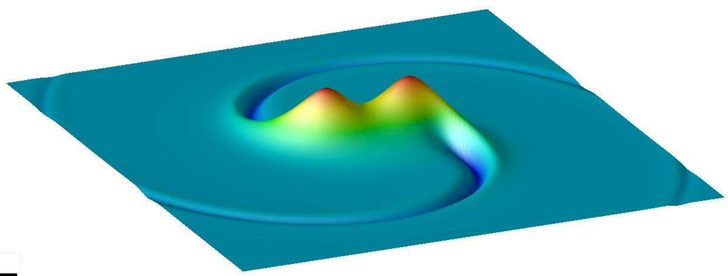
Holographic Mesons:

Adding Flavor to the Gauge/Gravity Duality



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Kruczenski, Mateos, Winters + RCM
hep-th/0304032;
(Hovdebo, Thomson, …..)

Maldecena; Witten; Gubser, Klebanov & Polyakov; …..

Anti-de Sitter/Conformal Field Theory Correspondence:

d=10 Type IIb superstrings in AdS_5 X S^5
with N units of RR flux

equivalent to

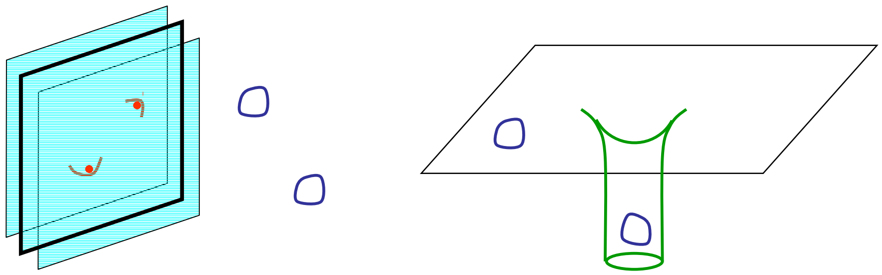
d=4 $\mathcal{N}$ =4 U(N) super-Yang-Mills

$$\left(R^2/\alpha'\right)^2 = g_{YM}^2 N \equiv \lambda$$
$$4\pi\, g_s = g_{YM}^2$$

Maldecena

Decoupling limit of D3-branes provides "derivation" of AdS/CFT correspondence

N coincident D3-branes:



U(N) open strings

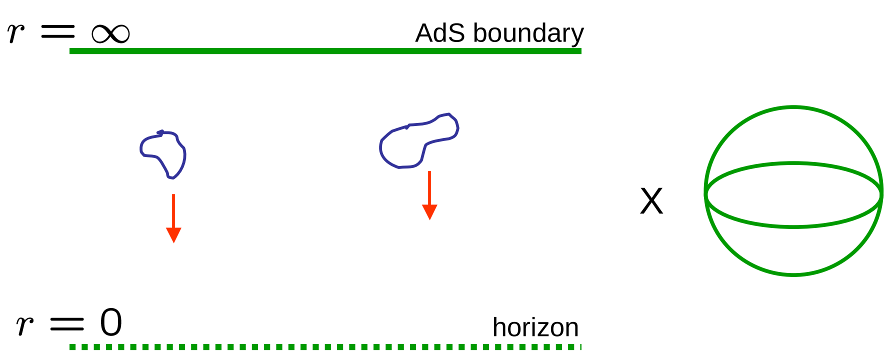
black brane with $R^4 \sim g_s N \alpha'^2$

Low-energy limit with $\alpha' E^2 \to 0$

- decouples asymptotic strings
- throat geometry: AdS_5 X S^5
- reduces brane theory to field theory: $\mathcal{N}$=4 SYM

d=10 Type IIb superstrings in AdS_5 X S^5 with N units of RR flux

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$



Symmetries: SO(4,2) X SO(6)

d=4 $\mathcal{N}$ =4 U(N) super-Yang-Mills

$$
\begin{aligned}
\mathcal{L} = \frac{1}{g^2}\text{Tr}\Big[& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda - i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i \\
& +(D^\mu\phi_i)^\dagger(D_\mu\phi_i) - \frac{1}{2}[\phi_i^\dagger,\phi_i]^2 + [\phi_j^\dagger,\phi_k^\dagger][\phi_j,\phi_k] \\
& -i\sqrt{2}[\lambda,\psi_i]\phi_i^\dagger - i\sqrt{2}[\bar{\lambda},\bar{\psi}_i]\phi_i \\
& -\sqrt{2}i\epsilon_{ijk}(\phi_i\psi_j\psi_k - \bar{\psi}_k\bar{\psi}_j\phi_i^\dagger)\Big]
\end{aligned}
$$

vector:	A_μ
4 Weyl fermions:	$\lambda, \psi_{1,2,3}$
3 complex scalars:	$\phi_{1,2,3}$

} all adjoint fields!

- conformally invariant: SO(3,1) $\rightarrow$ SO(4,2)
- R-symmetry: SO(6)

Maldecena; Witten; Gubser, Klebanov & Polyakov; …..

Anti-de Sitter/Conformal Field Theory Correspondence:

d=10 Type IIb superstrings in $AdS_5 \times S^5$
with N units of RR flux

equivalent to

d=4 $\mathcal{N}$ =4 U(N) super-Yang-Mills

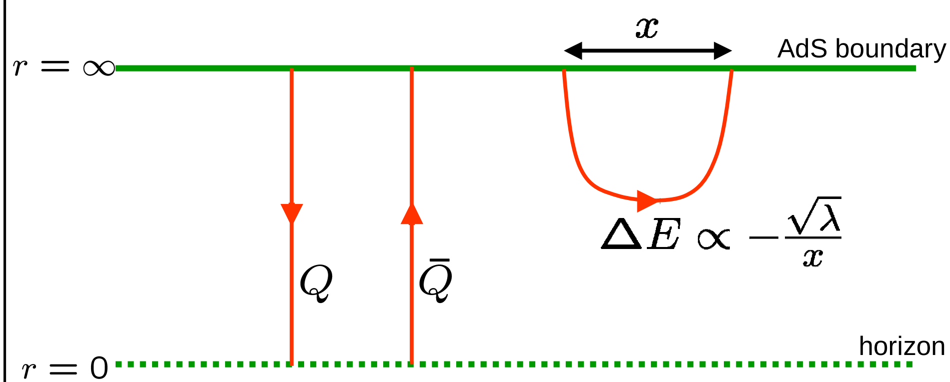
$$
\begin{aligned}
\left(R^2/\alpha'\right)^2 &= g_{YM}^2 N \equiv \lambda \gg 1 \\
4\pi\, g_s &= g_{YM}^2 \ll 1
\end{aligned}
$$

supergravity limit ~ large-N with strong `t Hooft coupling

Maldecena; Rey & Yee

Adding Static “Quarks”:

external sources in fundamental representation



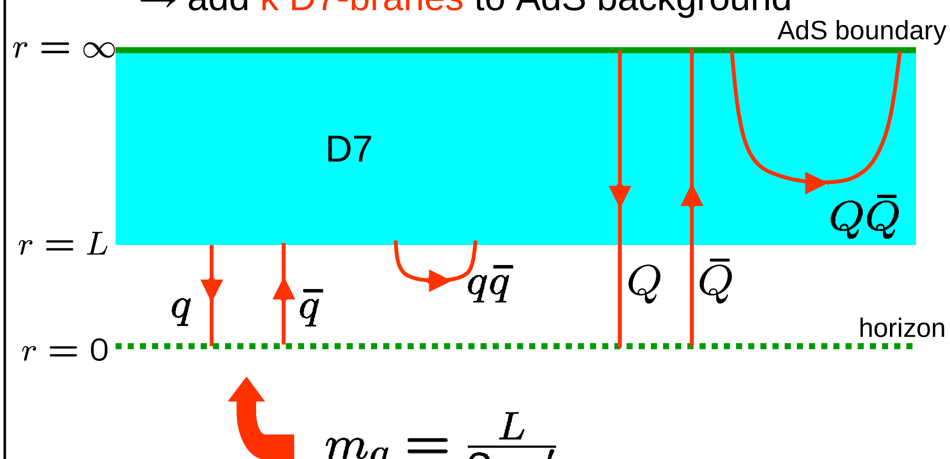
Motivation: remove a D3-brane to infinity breaking U(N+1) → U(N) X U(1); massive gauge bosons transform as bifundamental of residual gauge group

Karch & Katz

Adding Dynamical “Quarks”:

need a place/brane where strings can end at finite r

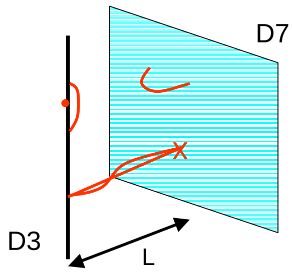
→ add k D7-branes to AdS background

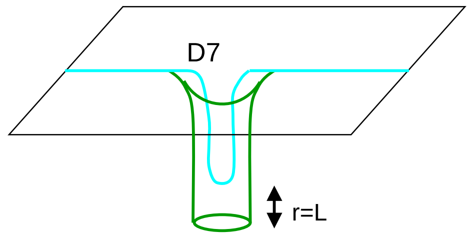


$$m_q = \frac{L}{2\pi\alpha'}$$

Decoupling limit of N D3-branes with k D7-branes

N coincident D3-branes:





black brane with $R^4 \sim g_s N \alpha'^2$

d=4 U(N) o.s., d=8 U(k) o.s.
& bifundamental 3-7 o.s.

Low-energy limit with $\alpha' E^2, L^2/\alpha' \to 0$

Minimum mass for 3-7 strings: $m_q = \frac{L}{2\pi\alpha'}$

Gauge coupling on D7's: $g^2_{YM} \sim g_s \alpha'^2 \to 0$

- throat geometry: AdS_5 X S^5 with k D7's
- brane theory → $\mathcal{N}$=4 U(N) SYM with fundamental matter

Field theory:

$\mathcal{N}$=4 U(N) super-Yang-Mills

vector:	A_μ	adjoint in U(N)
4 Weyl fermions:	$\lambda, \psi_{1,2,3}$	
3 complex scalars:	$\phi_{1,2,3}$	

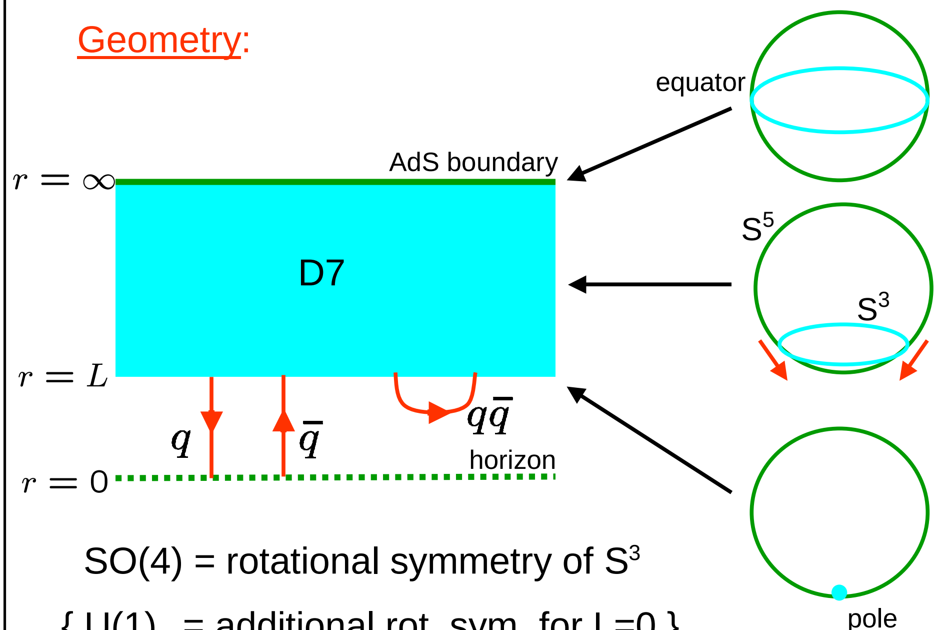
coupled to: k massive $\mathcal{N}$=2 hypermultiplets

2 complex scalars :	$q_\pm$	fund. in U(N) & global U(k)
2 Weyl fermions:	$\chi_\pm$	

- SUSY: $\mathcal{N}$=4 → $\mathcal{N}$=2
- SO(6) → SO(4) = $SU(2)_L$ X $SU(2)_R$ $\{ X\ U(1)_R \}_{m=0}$

$$Q_i \quad (0,\tfrac{1}{2},1); \quad q_\pm \quad (0,\tfrac{1}{2},0); \quad \chi_\pm \quad (0,0,\mp 1)$$

Geometry:



SO(4) = rotational symmetry of S^3

{ $U(1)_R$ = additional rot. sym. for L=0 }

Probe approximation:

This construction does not take into account the "gravitational" back-reaction of the D7-branes!

$\rightarrow$ considering large-N limit with k fixed
(in fact, k=1 in following)

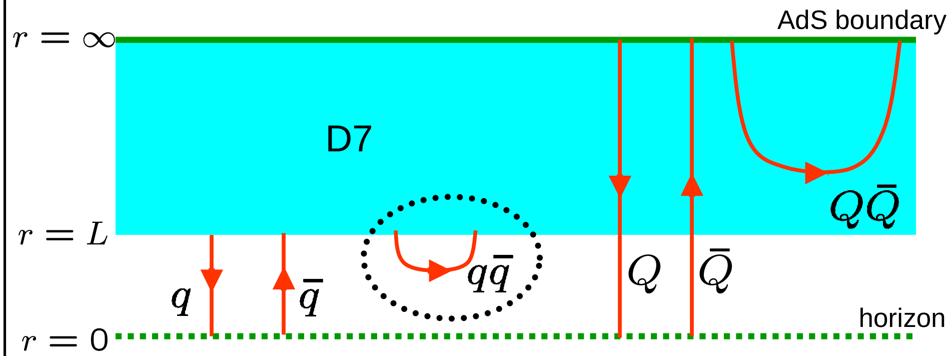
(see, however: Burrington et al)

Note: even with m_q=0, hypermultiplets introduce non-vanishing β-function - however, running of `t Hooft coupling vanishes in large-N

$$\beta(g) \sim k\, g^3, \quad \beta(\lambda) \sim \frac{k}{N}\lambda^2 \rightarrow 0$$

"Mesons":

bound states of fundamental fields dual to open string states supported by D7-brane



Consider two cases:

- mesons with low J (and arbitrary R-charge)
- mesons with large J (and no R-charge)

Mesons with J=0,1 (and arbitrary R-charge)

lowest lying open string states are excitations of the massless modes on D7-brane: vector, scalars (& spinors)

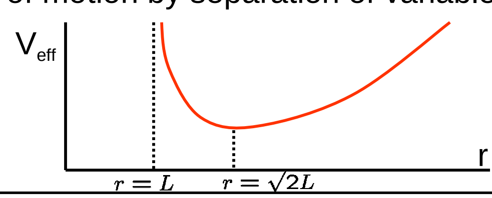
→ their dynamics is governed by usual worldvolume action:

$$S_{D7} = -\frac{1}{(2\pi)^7 g_s \alpha'^4}\int d^8\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{1}{2(2\pi)^5 g_s \alpha'^2}\int P[C^{(4)}]\wedge F\wedge F$$

free spectrum:

- expand action to second order in fluctuations
- solve linearized eq's of motion by separation of variables

→ discrete spectrum



Meson spectrum:

$$M^2(n,\ell) = \frac{4L^2}{R^4}(n+\ell+1)(n+\ell+2)$$

n = radial AdS quantum #
ℓ = angular quantum # on S³= R-charge

A_μ	:	one vector in the	(ℓ,ℓ)	with	$n\geq 0$,	$\ell\geq 0$
		one scalar in the	(ℓ,ℓ)	with	$n\geq 0$,	$\ell\geq 1$
		one scalar in the	$(\ell,\ell+2)$	with	$n\geq 0$,	$\ell\geq 0$
		one scalar in the	$(\ell,\ell-2)$	with	$n\geq 0$,	$\ell\geq 2$
Φ^a	:	two scalars in the	(ℓ,ℓ)	with	$n\geq 0$,	$\ell\geq 0$
Ψ^α	:	Dirac fermion in the	$(\ell,\ell+1)$	with	$n\geq 0$,	$\ell\geq 0$
		Dirac fermion in the	$(\ell,\ell-1)$	with	$n\geq 0$,	$\ell\geq 1$

massive supermultiplets with $8(\ell+1)$ bosons and fermions

$$n=\ell=0\ :\quad m_{\rm gap} = 2\sqrt{2}\frac{L}{R^2} = 2m_q\sqrt{\frac{2\pi}{g_sN}}$$

→ dominate low energy physics

Meson interactions:

Continue expansion of D7-brane action beyond 2nd order

• substitute $\Phi = \phi(x)\psi_{n,\ell}(r)Y_\ell(\Omega)$ and integrate out r and S³

$$\mathcal{L}_{eff} \simeq -\frac{1}{2}\left[(\partial\phi)^2 + M^2\phi^2\right] + \underbrace{\frac{2\pi\alpha'}{L}\frac{1}{\sqrt{N}}}\phi(\partial\phi)^2 + \ldots$$

$$g_{\phi(\partial\phi)^2} \simeq \frac{2\pi\alpha'}{L}\frac{1}{\sqrt{N}} = \frac{1}{m_q}\frac{1}{\sqrt{N}}$$

4th order: $g_{\phi^4} \sim \frac{1}{\lambda}\frac{1}{N}$, $g_{\phi^2(\partial\phi)^2} \sim \frac{1}{N}\frac{1}{m_q^2}$, and $g_{(\partial\phi)^4} \sim \frac{\lambda}{N}\frac{1}{m_q^4}$

→ agrees with standard large N

of course, for fixed $\lambda \sim g_sN$:

glueballs → $g_s \sim \frac{1}{N}$, $g_o \sim \sqrt{g_s} \sim \frac{1}{\sqrt{N}}$ ← mesons

Very large R-charge:

radial profile $\sim \frac{(r^2-L^2)^{\ell/2}}{r^{2\ell}}$

with narrow peak at $r=\sqrt{2}L$

null geodesics or time-like geodesics with large J/M orbiting S^3 in induced D7-brane geometry sit at precisely this radius

($\rightarrow$ PP-wave analog for open strings??)

Mega-mesons: for very large ℓ, also expect to see brane expansion effects as for giant gravitons

Mesons with large J (and no R-charge)

Classical rotating open strings attached to D7-brane

Nambu-Goto action: $S=-\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-det\,G_{\mu\nu}\partial_a X^\mu\partial_b X^\nu}$

Ansatz: $t=\tau,\ \ \rho=\sigma,\ \ \theta=\omega\tau,\ \ r=R^2/z(\sigma)$

$$S=-\frac{R^2}{2\pi\alpha'}\int d^2\sigma\,\frac{1}{z^2}\sqrt{\left(1-\omega^2\sigma^2\right)\left(1+z'^{\,2}\right)}$$

eom: $\frac{z''}{1+z'^{\,2}}+\frac{2}{z}-\frac{\omega^2\sigma z'}{1-\omega^2\sigma^2}=0$

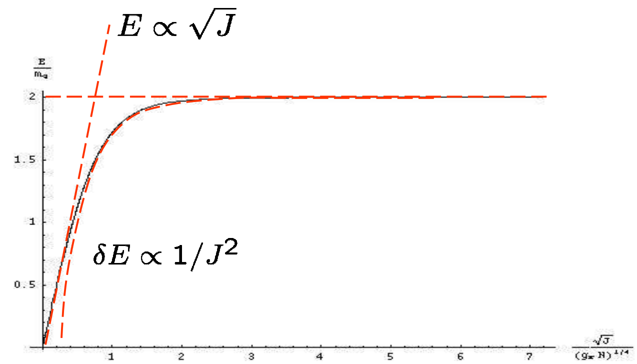
bc: $z'\rightarrow\infty$ (string ends $\perp$ to D7)

$$E\ =\ \omega\frac{\partial L}{\partial\omega}-L=\frac{R^2}{2\pi\alpha'}\int d\sigma\,\frac{1}{z^2}\sqrt{\frac{1+z'^{\,2}}{1-\omega^2\sigma^2}}$$

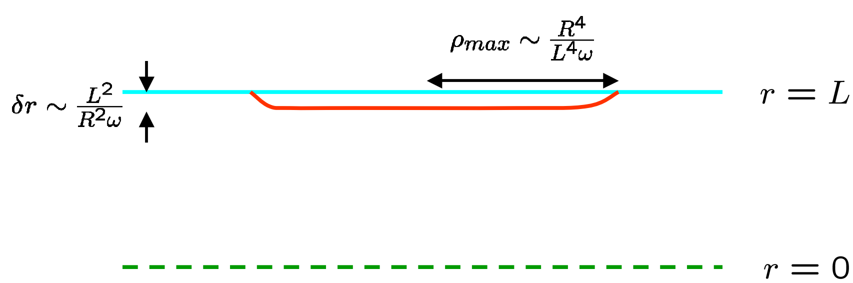
$$J\ =\ \frac{\partial L}{\partial\omega}=\frac{R^2}{2\pi\alpha'}\int d\sigma\,\frac{\sigma^2}{z^2}\sqrt{\frac{1+z'^{\,2}}{1-\omega^2\sigma^2}}$$

Meson spectrum:

solve E(J) numerically:



Case I: proper size << R $(\omega \rightarrow \infty)$

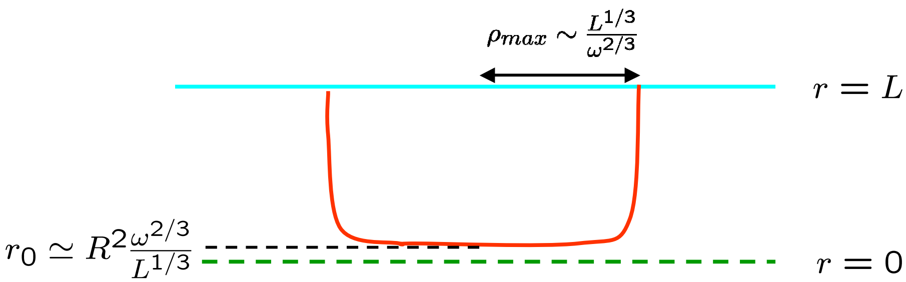


Regge behavior: $J^2 \simeq \frac{R^2 \alpha'}{L^2} E^2 = \frac{\sqrt{g_s N}}{2\pi^{3/2}} \frac{1}{m_q^2} E^2$

$$T_{eff} = \frac{1}{2\pi\alpha'} \frac{L^2}{R^2} = \sqrt{\pi} \frac{m_q^2}{\sqrt{g_s N}} \quad \left(\simeq \sqrt{g_s N} m_{gap}^2\right)$$

Analysis requires: $\frac{L}{R} \rho_{max} \ll R$ → $1 \ll J \ll \sqrt{g_s N}$

Case II: proper size >> R ($\omega \to 0\,, J \gg \sqrt{g_s N}$)

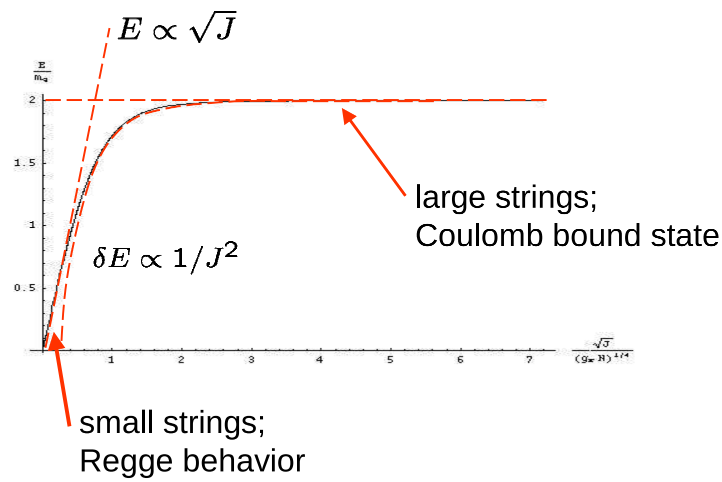


$$E \simeq 2m_q - \kappa^4 \frac{m_q}{4J^2} \quad \text{with} \quad \kappa^2 = \frac{8\pi^2\sqrt{2\pi}}{\Gamma(1/4)^4}\sqrt{g_s N}$$

Binding energy for Coulomb-like potential, $V = -\frac{\kappa^2}{r}$

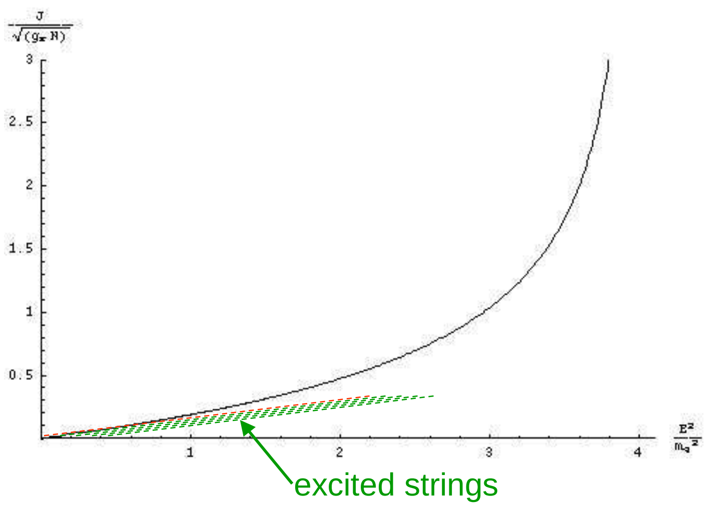
→ matches precisely static quark potential!

Maldecena; Rey & Yee



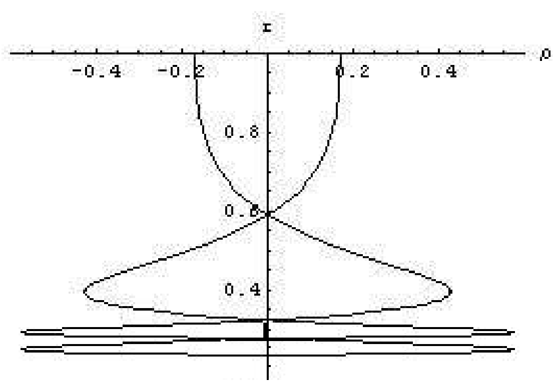
Cross-over scale: $|x| \sim \frac{\sqrt{g_s N}}{m_q} \sim \frac{1}{m_{gap}}$

Regge behavior:



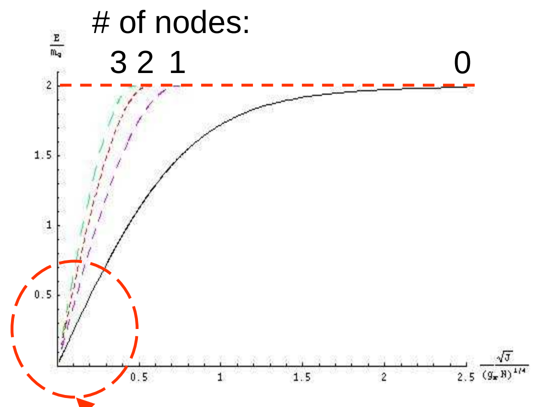
Exotic states:

Numerical integration also finds solutions with radial oscillations



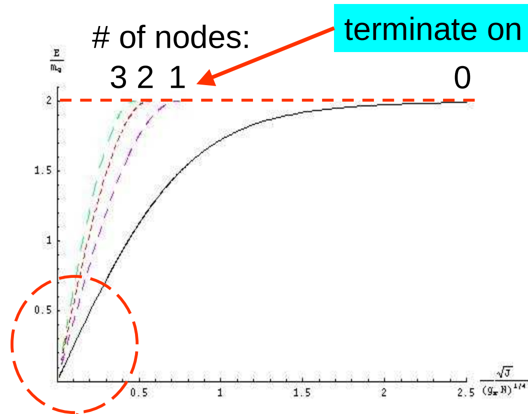
decay suppressed in large-N: $g_s \sim 1/N$

→ hybrid states or glue-excited mesons



Regge behavior: compare slopes

$$(m_1/m_0)^2 = 2.99 \sim 3$$
$$(m_2/m_0)^2 = 4.97 \sim 5$$
$$(m_3/m_0)^2 = 6.90 \sim 7$$

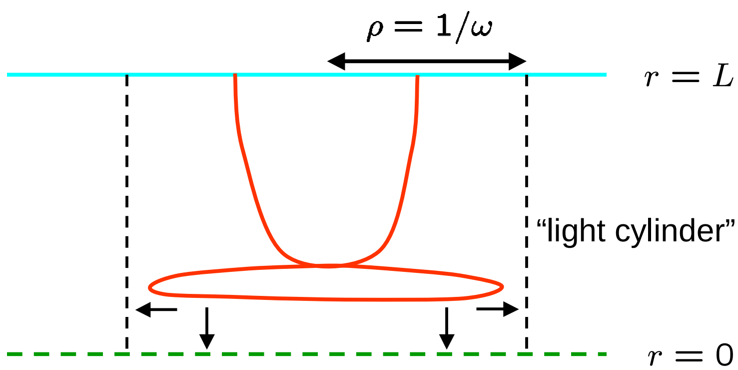


terminate on “light cylinder”

Regge behavior

1 node gives essentially 3 strings stretching between quarks

1 node, ω small

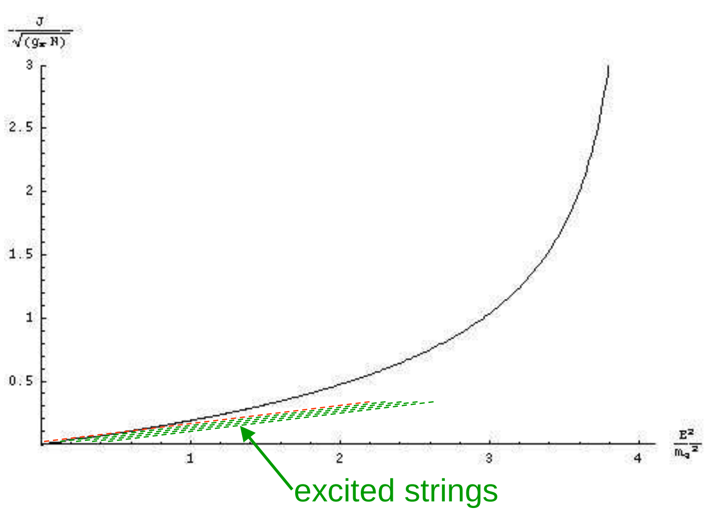


trajectory terminates when loop hits "light cylinder"

interesting structure (loops) sink towards horizon (infrared) as $\omega \to 0$

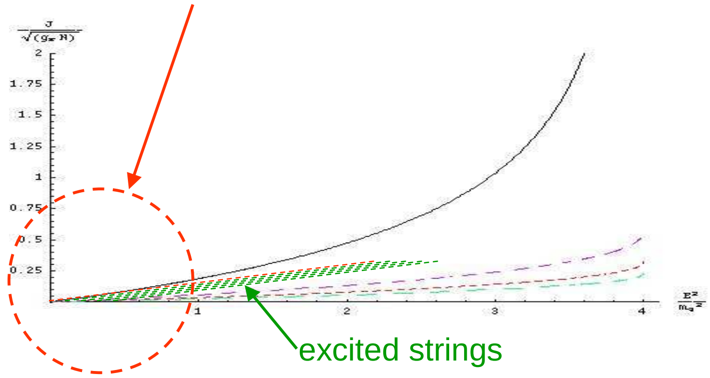
→ <u>limits</u> applicability to QCD

<u>Regge behavior</u>:



Regge behavior:

standard Regge trajectories "polluted" by "hybrid states" (amongst other things?)



excited strings

Danielsson, Keski-Vakkuri & Kruczenski

Holography: "shadows on the wall"

AdS/CFT dictionary allows us to see "glue clouds" associated with quark/meson states

eg, SUGRA dilaton is dual to $\langle F^2 \rangle$; so evaluate asymptotic dilaton sourced by particular (macroscopic) string configuration

for "static quark" (infinite straight string):

$$\langle F^2 \rangle = \frac{1}{8\pi} \frac{\sqrt{g_s N}}{|x|^4}$$

in agreement with the Wilson loop calculation for the potential between such external quarks

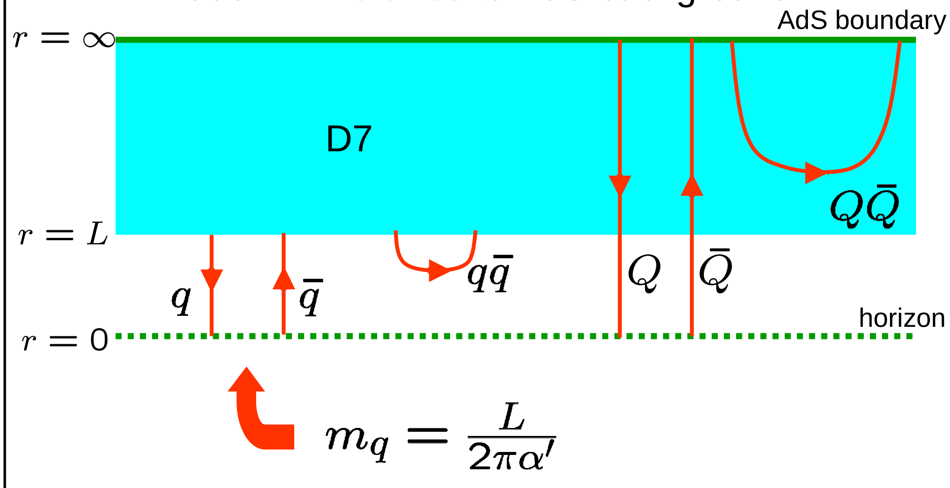
Maldecena; Rey & Yee

Karch & Katz

Adding Dynamical "Quarks":

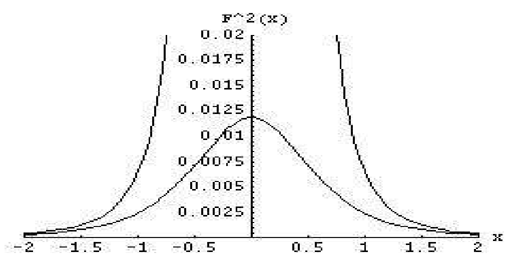
need a place/brane where strings can end at finite r

→ add k D7-branes to AdS background



$$m_q = \frac{L}{2\pi\alpha'}$$

Holography:



for finite mass "quark" (straight string ending on D7):

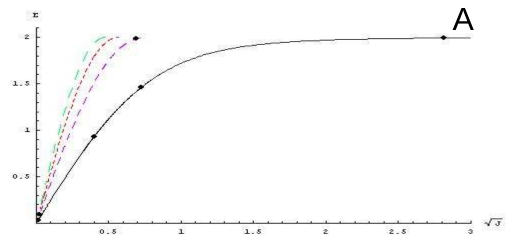
$$\langle F^2\rangle = \frac{15}{64\pi}\frac{\sqrt{g_sN}}{|x|^4}\left(1+\frac{1}{3M^2|x|^2}+\cdots\right) \quad \text{for large } |x|$$

$$\langle F^2\rangle = \frac{1}{8\pi}\sqrt{g_sN}\,M^4\left(1+\frac{7}{2}M^2|x|^2+\cdots\right) \quad \text{for small } |x|$$

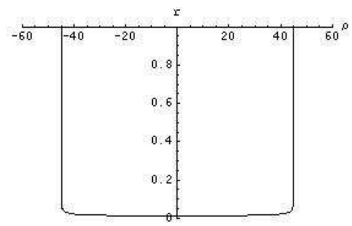
where $M = L/R^2 = \frac{\sqrt{\pi}m_q}{\sqrt{g_sN}} \simeq m_{gap}$

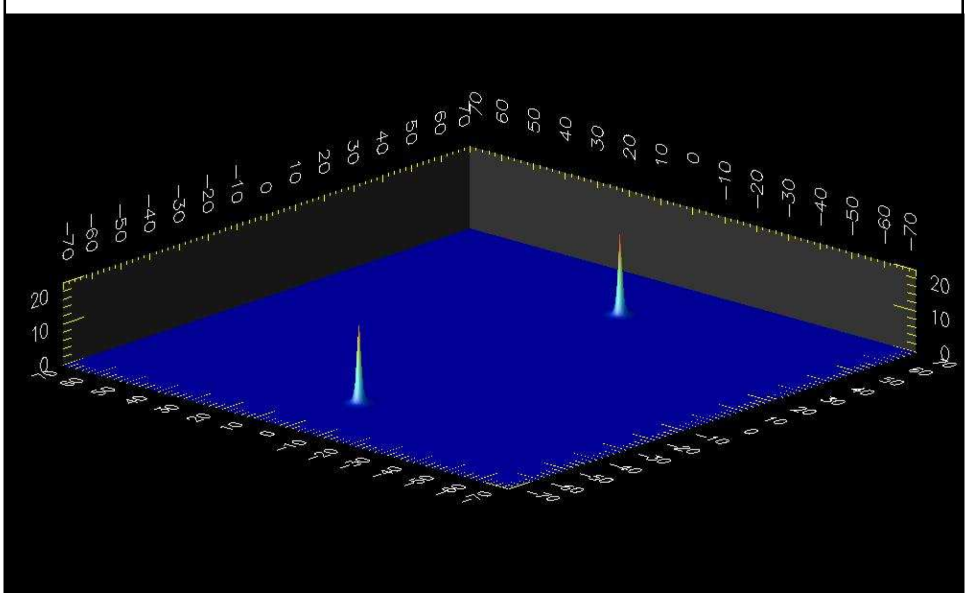
— ? → $T_{eff} \sim m_q^2/\sqrt{g_sN} \sim \sqrt{g_sN}\,m_{gap}^2$

Holographic Mesons:

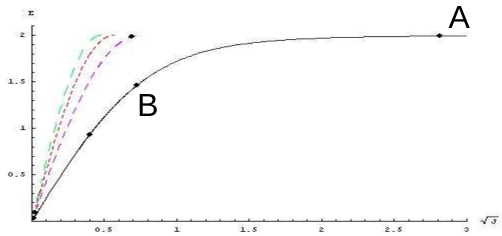


Profile A:

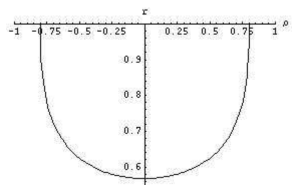


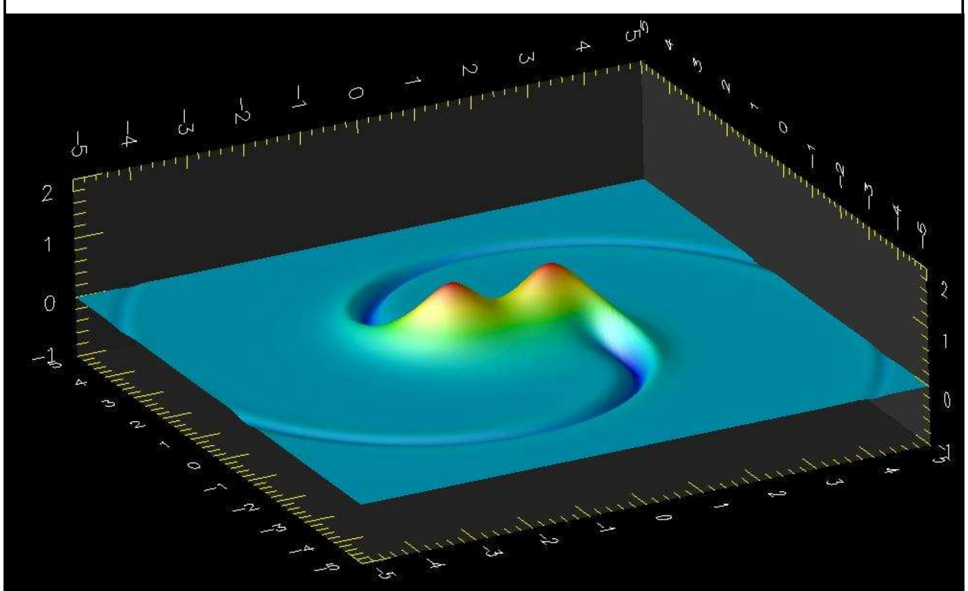


Holographic Mesons:



Profile B:





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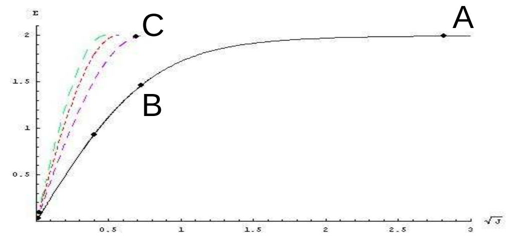
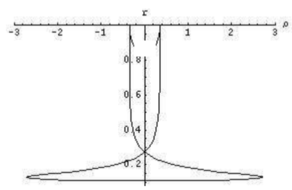
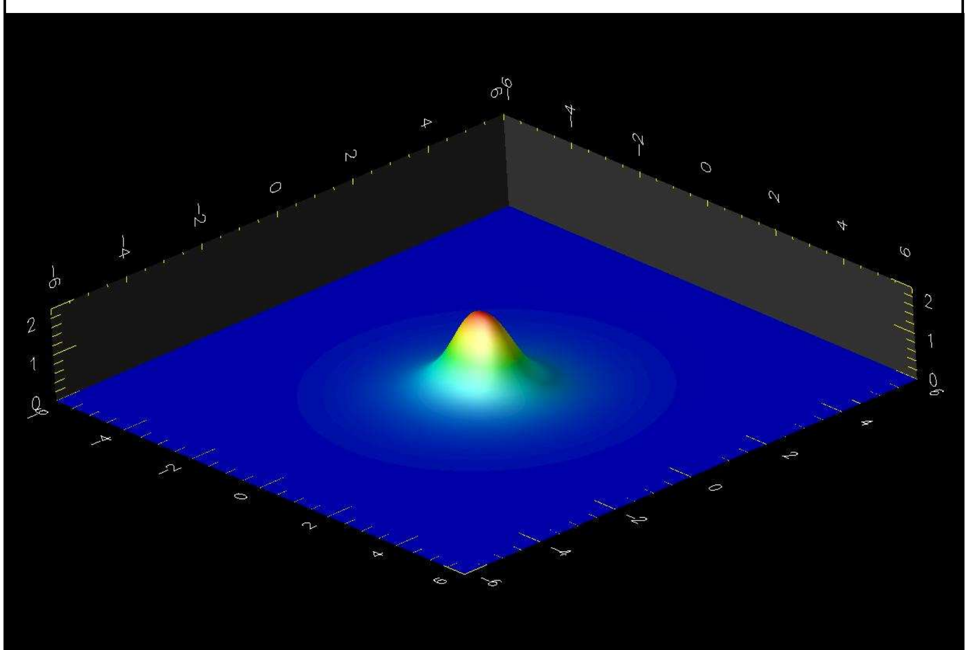
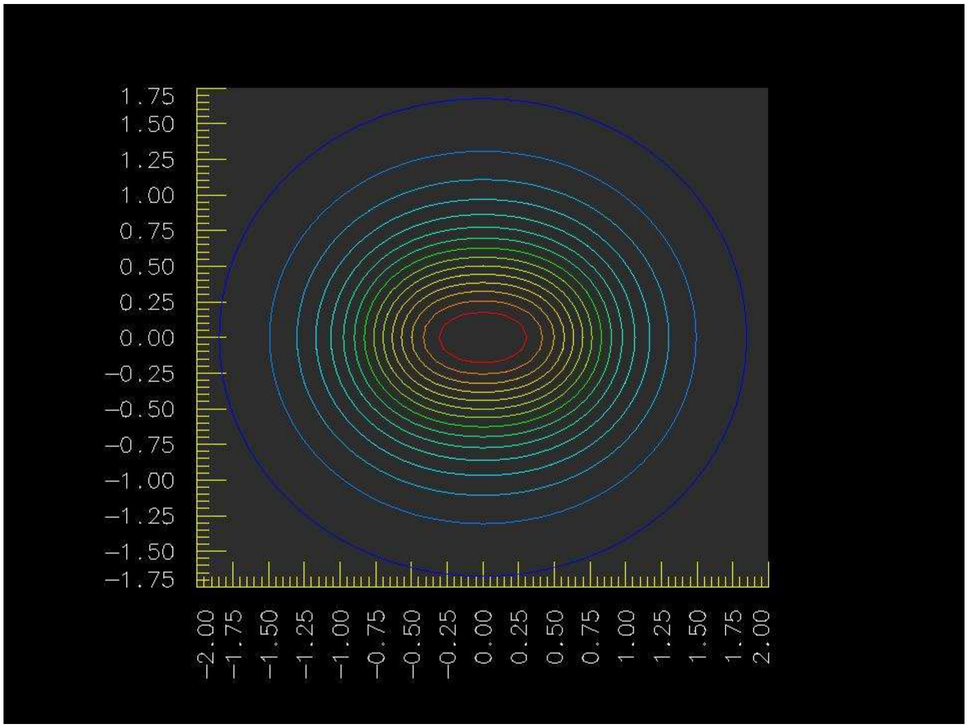


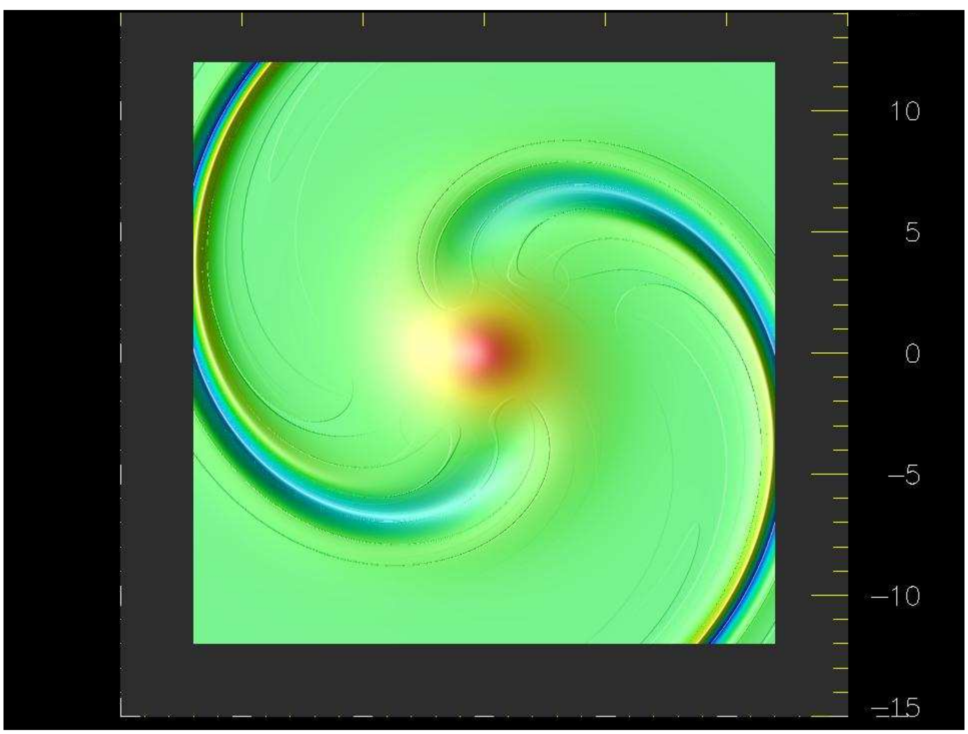
Profile C:





Holographc Mesons





Conclusions:

Flavor physics is fun!

New directions:

- Lower dimensions:
  new test of gauge/gravity duality
  (work in progress)
- Include back-reaction of D7-branes
  (Burrington et al; Erdmenger and Kirsch)
- Adding flavors to more interesting theories
  (Babington et al; Kruczenski et al; Evans & Schock; Nunez, Paredes & Ramallo; Sakai & Sonnenschein; . . .)

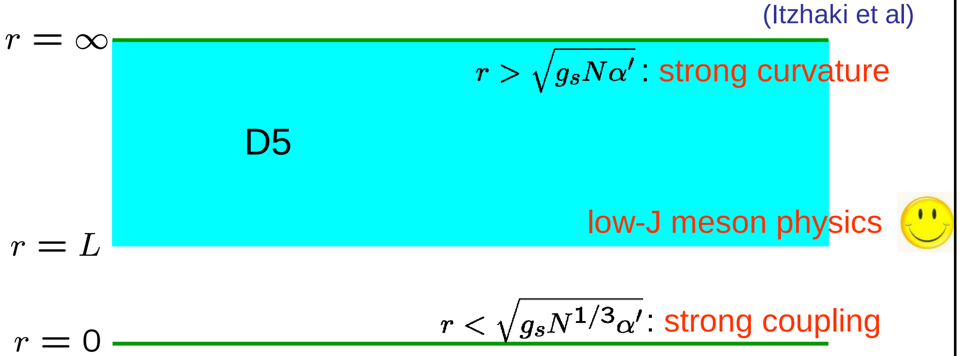
(Pinsky et al)

Large-N SUSY gauge theories with DLCQ
in lower dimensions!

d=2 $\mathcal{N}$=(4,4) U(N) SYM coupled to fundmental matter

— dual to → D5-brane probes in D1-background

(Itzhaki et al)



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- Lower dimensions:
  new test of gauge/gravity duality
  (work in progress)
- Include back-reaction of D7-branes
  (Burrington et al; Erdmenger and Kirsch)
- Adding flavors to more interesting theories
  (Babington et al; Kruczenski et al; Evans & Schock; Nunez, Paredes & Ramallo; Sakai & Sonnenschein; . . .)

→ See you at the discussion session!