6D conformal theory as a candidate for TOE

- A.S., hep-th/0407231
- E. Ivanov + B. Zupnik + A.S., work in progress

Motivation

- Problems with quantum gravity:
  - no universal flat time
  - no hermitean hamiltonian
  - breaking of unitarity / causality
  - problems even in classical theory – wormholes

Main idea: 1. our Universe is a soap film (brane) in a higher-dimensional $\boxed{\text{flat}}$ bulk

Rubakov + Shaposhnikov, 1983

2. Gravity is induced
3. $\boxed{\text{Field theory}}$ as a fundamental theory of soap

- $\sim \mathrm{Tr}\{F_{\mu\nu}^2\}$ has dimensionful coupling. Not renormalizable
- $\sim \mathrm{Tr}\{F_{\mu\nu}\,\Box\,F_{\mu\nu}\}$ has dimensionless coupling in 6D. Renormalizable
  - higher derivatives $\Rightarrow$ ghosts
  - Breaking of unitarity/causality in quantum theory
  - Seen at the classical level as instabilities of vacuum

Toy models

-3-

A) $\mathcal{L} = \frac{1}{2}\ddot{q}^2 - \frac{\Omega^4}{2} q^2$

- disp. eq. $\lambda^4 - \Omega^4 = 0$ has real and imaginary solutions. Unstable mode $q(t) \sim e^{\Omega t}$

B) $\mathcal{L} = \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 - \frac{\alpha}{4} q^4$

eq. mot. $$\left(\frac{d^2}{dt^2} + \Omega^2\right)^2 q - \alpha q^3 = 0$$

- linearized: only real frequencies $q(t) \sim e^{i\Omega t}$ and $q(t) \sim t e^{i\Omega t}$
- nonlinear system is susceptible to collapse

example

falling on the centre for $V(r) = -\frac{A}{r^2}$

-4-

- usually $E = \frac{\dot{q}^2}{2} + V(q) = \text{const}$

if $V(q)$ is bounded from below then $|\dot{q}|$ is bounded from above

- In our case

$$E = \dot{q}(\dddot{q} + \Omega^2 \dot{q}) - \dot{q}(\dddot{q} + \Omega^2 \dot{q})$$
$$- \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 + \frac{\alpha}{4} q^4 = \text{const}$$

- positive and negative terms
- derivatives can grow without bound

Conjecture for any nonlinear system with HD Lagrangian, some trajectories hit singularity

-4a-

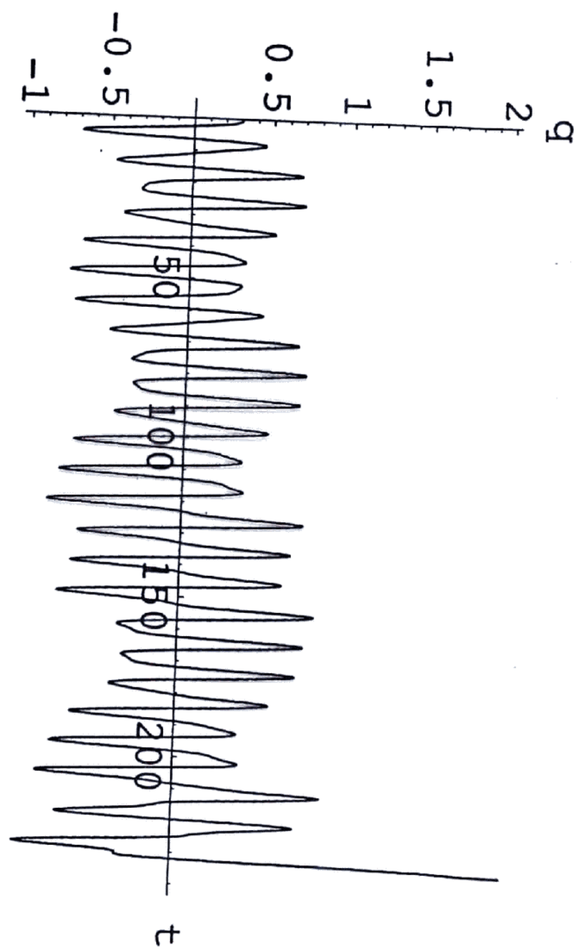

Figure 1: Oscillating and collapsing

-5-

- $\alpha > 0 \Rightarrow$ small fluctuations do not grow
- Separatrice
- vacuum $q = 0$ is perturbatively stable

c) $\mathcal{L} = \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 - \frac{\alpha}{4} q^4 - \frac{\beta}{2} \dot{q}^2 q^2$

- perturbatively stable for $\alpha, \beta > 0$
  $\Downarrow$
  ghosts are benign
- perturbatively unstable for $\alpha, \beta < 0$
  $\Downarrow$
  ghosts are malicious

$\boxed{\text{picture}}$

-5a-

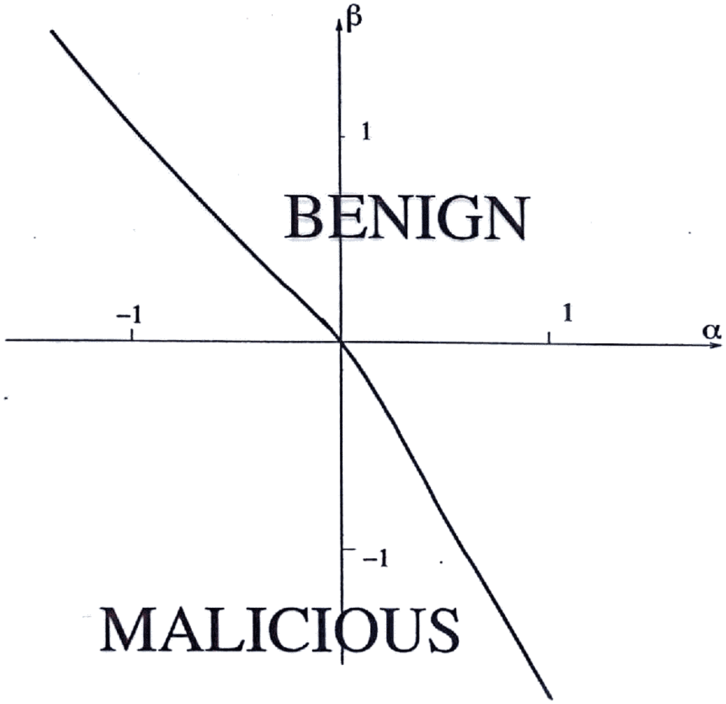

Figure 2: Benign and malicious parameter regions

-6-

Field theory

A) $\mathcal{L} = \frac{1}{2}\varphi\Box^2\varphi - \frac{\lambda}{6!}\varphi^6$ $\boxed{m=0}$

- Modes with nonzero momentum behave as for toy models. Perturbatively stable
- Perturbative instability for the zero momentum model
- creates proper initial conditions for inflation ?
- mass is generated by perturbative corrections

$m^4 \sim \lambda \Lambda_{uv}^4$

-7-

B) HD 6D SQED

$$\mathcal{L} = \frac{1}{2e^2} \int d^8\theta \, \bar{W} W$$

- $W(x_L, \theta, \hat\theta) = \phi + i\sqrt{2}\,\hat\theta^\alpha W_\alpha - \frac{\hat\theta^2}{4} \bar{D}^2 \bar\phi$

is a $\mathcal{N}=2$ chiral superfield

in components

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} \Box F_{\mu\nu} + \bar\varphi \Box^2 \varphi - \frac{1}{2} D \Box D - \bar{F} \Box F - i(\sigma_\mu)_{\alpha\dot\beta} \sum_f \lambda_f^\alpha \partial_\mu \Box \bar\lambda_f^{\dot\beta}$$

- $D, F, \bar{F}$ are now dynamic

-8-

non-Abelian HD theories

- SW effective theory

$$\mathcal{L} = \int d^4\theta \, F(W)$$

- HD theory

$$\mathcal{L} = \frac{1}{g^2} \mathrm{Tr} \int d^8\theta \, \bar{W} e^{-ib} W e^{ib}$$

b – bridge

gauge transformations:

$$\begin{cases} W \to e^{i\Lambda} W e^{-i\Lambda} \\ \bar{W} \to e^{i\bar\Lambda} \bar{W} e^{-i\bar\Lambda} \\ e^{ib} \to e^{i\Lambda} e^{ib} e^{-i\bar\Lambda} \end{cases}$$

$\mathcal{N}=1 \Rightarrow b \equiv V$

$\mathcal{N}=2 \Rightarrow$ no simple expression for b

Adequate technique — harmonic superspace

Galperin + Ivanov + Ogievetsky + Sokatchev

HSS in 6D

- start with θ^i_α, $\alpha = 1,2,3,4$; $i = 1,2$

$$\theta^i = C\overline{\theta_i}\,,$$

C – charge conjugation

- conventional superfields $\Phi(x_\mu, \theta^i_\alpha)$ have too many components; should be subject to constraints
- Introduce an extra $S^2 \equiv CP^1$ parametrized by u^+_i, $i = 1,2$:

$$u^+_i (u^+_i)^* \equiv u^+_i u^{-,i} = 1$$

- Introduce $\theta^\pm_\alpha = \theta^i_\alpha u_i^\pm$

Analytic superfield

$$V^{++}(x_\mu, \theta^+, u)$$

- in Wess-Zumino gauge

$$V^{++} \sim \theta^+ \Gamma_\mu \theta^+ A_\mu + \\ + \varepsilon^{\alpha\beta\gamma\delta} \theta^+_\alpha \theta^+_\beta \theta^+_\gamma \psi^-_\delta + \\ + (\theta^+)^4 D^{--},$$

$$\psi^-_\delta = \psi^i_\delta u_i^-, \quad D^{--} = D^{ik} u_i^- u_k^-$$

- D^{ik} – triplet of "auxiliary" fields

$$D^{ik} = \begin{pmatrix} -F & D \\ D & \bar{F} \end{pmatrix}$$

in components

-11-

$$\mathcal{L} = \frac{1}{g^2}\Big\langle -\nabla_\mu D_{jk} \nabla_\mu D^{jk} - 2 D_{kl} D^{jk} D_{jl} + 2i D_{jk} \psi^j \Gamma_\mu \overleftrightarrow{\nabla}_\mu \psi^k$$

$$- \nabla_\mu F_{\mu\nu} \nabla_\rho F_{\rho\nu} - i \psi^j \Gamma_\mu \nabla_\mu \nabla^2 \psi_j$$

$$+ \frac{1}{2} \psi^j \Gamma_\mu \nabla_\mu \sigma^{\nu\rho} [F_{\nu\rho}, \psi_j] +$$

$$+ 2 \nabla_\mu F_{\mu\nu} \psi^j \Gamma_\nu \psi_j -$$

$$- \frac{1}{2} (\psi^j \Gamma_\mu \psi_j)(\psi^k \Gamma_\mu \psi_k) \Big\rangle$$

- ∇_μ – covariant derivative
- conformally invariant at the classical level

conformal anomaly

-12-

$$\frac{1}{g^2(\mu)} = \frac{1}{g_0^2} + \frac{1}{12\pi^3} \ln \frac{\Lambda}{\mu}$$

- Landau pole

(as. freedom for $\int d^6x \left[\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\lambda}{6}\varphi^3\right]$)

- malicious ghosts

c) $\mathcal{N}=2$ 6D theory

- add an adjoint hypermultiplet (4 bos. degree of freedom)
- Suppose that $\mathcal{L}$ is obtained by reduction from 10D

$$g^2\mathcal{L} = -\langle \nabla_M F_{MN} \nabla_S F_{SN} \rangle =$$

$$= \frac{1}{2}\langle F_{MN} \nabla^2 F_{MN} \rangle +$$

$$+ 2i \langle F_{MN} F_{NP} F_{PM} \rangle ,$$

$M, N, P = 0, 1, \ldots, 5$ – gauge potentials

$M, N, P = 6, 7, 8, 9$ – scalar fields

Scalar potential

$$g^2 V = -\frac{1}{2}\langle [A_P, [A_M, A_N]] \cdot$$

$$\cdot [A_P, [A_M, A_N]] \rangle -$$

$$-2 \langle [A_M, A_N][A_N, A_P][A_P, A_M] \rangle$$

vacuum valley

$$[A_M, A_N] = 0 \Rightarrow A_M = C_M T^3$$

- not lifted by quantum corrections

conjecture : β function vanishes

lessons from 4D

$\mathcal{N}=1$: background field supergraph formalism

- quantum gauge superfield loop vanishes
- 3 chiral multiplets of ghosts: two Faddeev-Popov ghosts and one Nielsen-Kallosh ghost.

$$\Rightarrow \beta^{1\text{-loop}}_{\mathcal{N}=1} = -3$$

in some units

-15-

- $\beta^{1\text{-loop}}_{\mathcal{N}=2} = -3_{\text{ghosts}} + 1_{\text{mat.}} = -2$
- $\beta^{1\text{-loop}}_{\mathcal{N}=4} = -3_{\text{ghosts}} + 3_{\text{mat.}} = 0$
- non-renormalization theorems for higher loops

$\mathcal{N}=2$ background field harmonic supergraph formalism | Buchbinder×2 + Kuzenko + Ovrut, 1997

- two FP hypermultiplets
  one NK hypermultiplet
- NK contributes with the opposite sign compared to FP

$$\beta^{1\text{-loop}}_{\mathcal{N}=2} = 2\left[(-2)_{FP} + 1_{NK}\right] = -2$$

-16-

$$\beta^{1\text{-loop}}_{\mathcal{N}=4} = 2\left[(-2)_{FP} + 1_{NK} + 1_{MAT.}\right] = 0$$

our guess is that the background field technique for 6D HD SYM (to develop!) will give the same result

- The only scale comes from

$$\langle A_{6,7,8,9} \rangle = C_{6,7,8,9}\, T^3 \neq 0$$

- Scalar v.e.v.s enter as momenta. For a transverse fluctuation Φ

$$\mathcal{L}^{(2)} \approx \Phi\left[\Box + \bar{C}^2\right]^2 \Phi$$

☺ no instabilities. Probably also, if taking interaction into account

- zero-momentum valley mode is unstable

Toy model

$$L = \frac{1}{2}\left(\ddot q + y^2 q\right)^2 + \frac{1}{2}\ddot y^2 - \frac{1}{4}q^4$$

transverse variable

valley variable

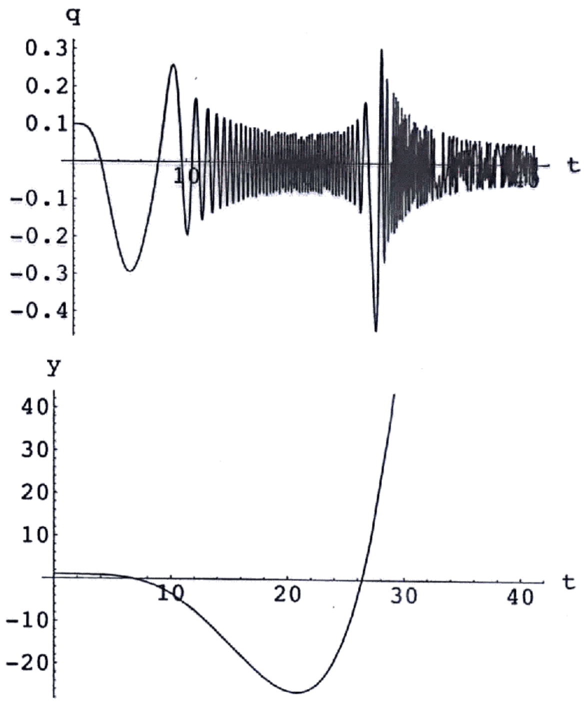

Figure 3: $q(t)$ and $y(t)$ for the system (29) with the initial conditions (30).

Classical brane solution

- 3-branes appear naturally in Abelian Higgs model:

  Hughes + Liu + Polchinski, 1983

  AND strings in 4D → 3-branes in 6_D
- not seen in our case
- 2-branes appear (≡ BPS monopoles in 4D)

energy density

$$\mathcal{E} = -\int d^3x_\perp \langle \nabla_M F_{MN} \nabla_S F_{SN} \rangle$$

($A_{6,7,8,9}$ – scalar fields

$A_{1,\dots,5}$ — spatial gauge potentials

- $\mathcal{E}$ vanishes for class. solutions

$$\nabla_M F_{MN} = 0$$

- No induced cosmological term

- Extended (1,1) SUSY algebra in 6D has the group of automorphisms

$$SO(4) \equiv SU(2) \times SU(2)$$

- Our theory should be dual to $AdS_7 \times S^3$

I ♥ it!