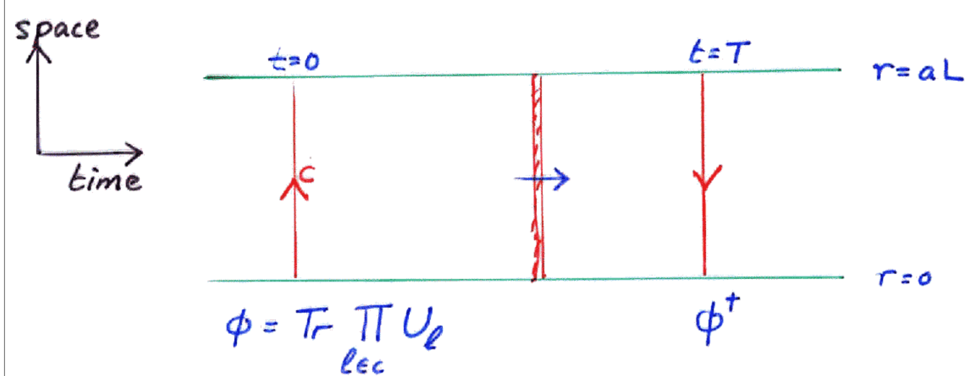
$SU(N_c)$ gauge theories
– the view from the lattice

M. Teper (Oxford)

- confinement ? (old) strings ?
- spectrum $SU(3) \simeq SU(\infty)$?
- $g^2 N_c$ fixed ? non-perturbative ?
- k-strings new glueballs ?
- topology instantons ? interlaced θ-vacua?
- chiral symm. breaking } ?
- deconfinement multiple transitions ?
- space-time reduction non-analyticity ?

- MT + B. Lucini, H. Meyer, U. Wenger
- L. Del Debbio, H. Panagopoulos, P. Rossi, E. Vicari
- R. Narayanan, H. Neuberger, J. Kiskis

- string tension:



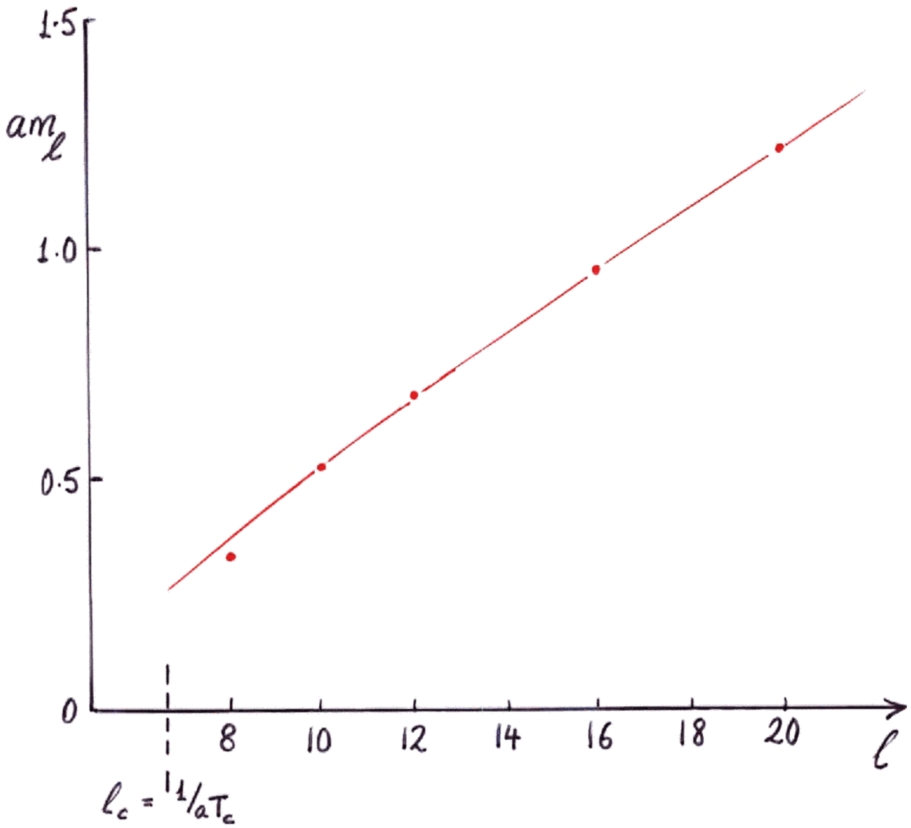
then $\langle \phi^\dagger(T)\, \phi(0) \rangle$ gives lightest energy of flux tube winding around torus:

$$m_l = aL \cdot \sigma - \frac{\pi}{6} \frac{D-2}{aL} c_s + O\left(\frac{1}{L^3}\right)$$

aL: length; σ: energy per unit length; $\frac{\pi}{6} \frac{D-2}{aL} c_s$: universal string correction: $c_s = 1$ for bosonic string universality class

- calculate am_l, assume $c_s = 1$ } → the string tension $a^2\sigma$

SU(6)



fit to $\ell \geq 10$

$$am_\ell = a^2\sigma\ell - \frac{1.06(11)}{\ell}$$

↑ linear ↑ bosonic string

$a\sqrt{\sigma} \simeq 0.252$

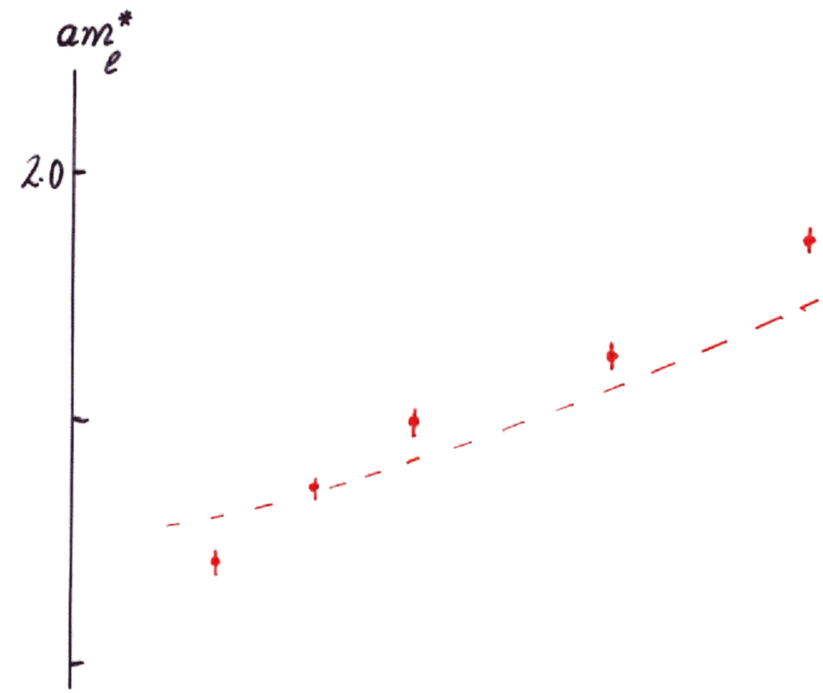
Nambu - Goto closed string:

$$am_\ell = a^2\sigma\ell\left\{1 + \frac{8\pi}{a^2\sigma\ell^2}\left[-\frac{1}{24}(d-2) + n\right]\right\}^{1/2}$$

$$\xrightarrow[\ell\to\infty]{} a^2\sigma\ell - \frac{\pi}{3}\frac{1}{\ell} + \frac{4\pi n}{\ell} + O\left(\frac{1}{\ell^3}\right)$$

Luscher-Weisz hep-th/0406205

↑ excited strings



we can calculate masses

$$H\,|G\rangle = m_G\,|G\rangle$$

from correlation functions:

$$\langle \Phi^\dagger(t)\Phi(0)\rangle = \langle \Phi^\dagger e^{-iHt}\Phi\rangle = \sum_n |\langle\Omega|\Phi^\dagger|n\rangle|^2\, e^{-iE_n t}$$

$$\sum_n |n\rangle\langle n| \qquad \sum_{n'} |n'\rangle\langle n'| \quad : \quad H|n\rangle = E_n|n\rangle$$

$t \to -it$ | Minkowski

$x^2 - t^2 \to x^2 + t^2$ | ↓ Euclidean

$$\langle \Phi^\dagger(t)\Phi(0)\rangle_E = \langle \Phi^\dagger e^{-Ht}\Phi\rangle = \sum_n |\langle\Omega|\Phi^\dagger|n\rangle|^2\, e^{-E_n t}$$

↑ lowest E_n

$\Rightarrow$ lightest glueballs with quantum numbers of Φ

$$\langle \Psi(A)\rangle = \frac{1}{Z}\int \prod_{x,\mu} dA_\mu(x)\; \Psi(A)\; e^{-\frac{1}{g^2}\int \mathrm{Tr}\, F_{\mu\nu}^2\, d^4x}$$

↓ continuum, $V=\infty$

↓ lattice: × × × hypercubic; × × × finite, periodic V (spacing a)

$$\approx \frac{1}{Z_L}\int \prod_{n,\mu} dU_\mu(n)\; \Psi_L(U)\; e^{-\frac{1}{g^2}S_L[U]}$$

where:

$$dU_\mu(n) \xrightarrow[a\to 0]{} dA_\mu(x=an)$$

$$S_L(U) \xrightarrow[a\to 0]{} \int \mathrm{Tr}\, F_{\mu\nu}^2\, d^4x$$

$$\Psi_L(U) \xrightarrow[a\to 0]{} \Psi(A)$$

↓ Monte Carlo

$$\approx \frac{1}{N}\sum_{I=1}^{N} \Psi_L(U^I) \pm O\left(\frac{1}{\sqrt{N}}\right)$$

good if:

$a \ll 1\,\mathrm{fm}$ — renormalisable

$V \gg (1\,\mathrm{fm})^4$ — no massless state

theoretical ✓ ...

FT	LFT
x	$n \xrightarrow{\ \ell\ } n+\hat\mu \qquad \rightarrow \mu$
$A_\mu(x)$ ∈ SU(N) Lie Algebra	$U_\ell \equiv U_\mu(n)$ ∈ SU(N) group

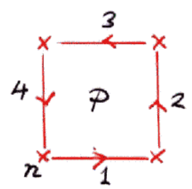
gauge trans ↓ $g(x)$

$g(x)\,A_\mu(x)\,g^\dagger(x) + \frac{1}{e}\, g\,\partial_\mu g^\dagger(x)$ $\qquad$ $g(n)\,U_\mu(n)\,g^\dagger(n+\hat\mu)$

gauge-inv. ↓ action

x

$Tr\, F^2_{\mu\nu}(x)$



$Tr\, U_p(n) \equiv Tr\, U_1 U_2 U_3^\dagger U_4^\dagger$

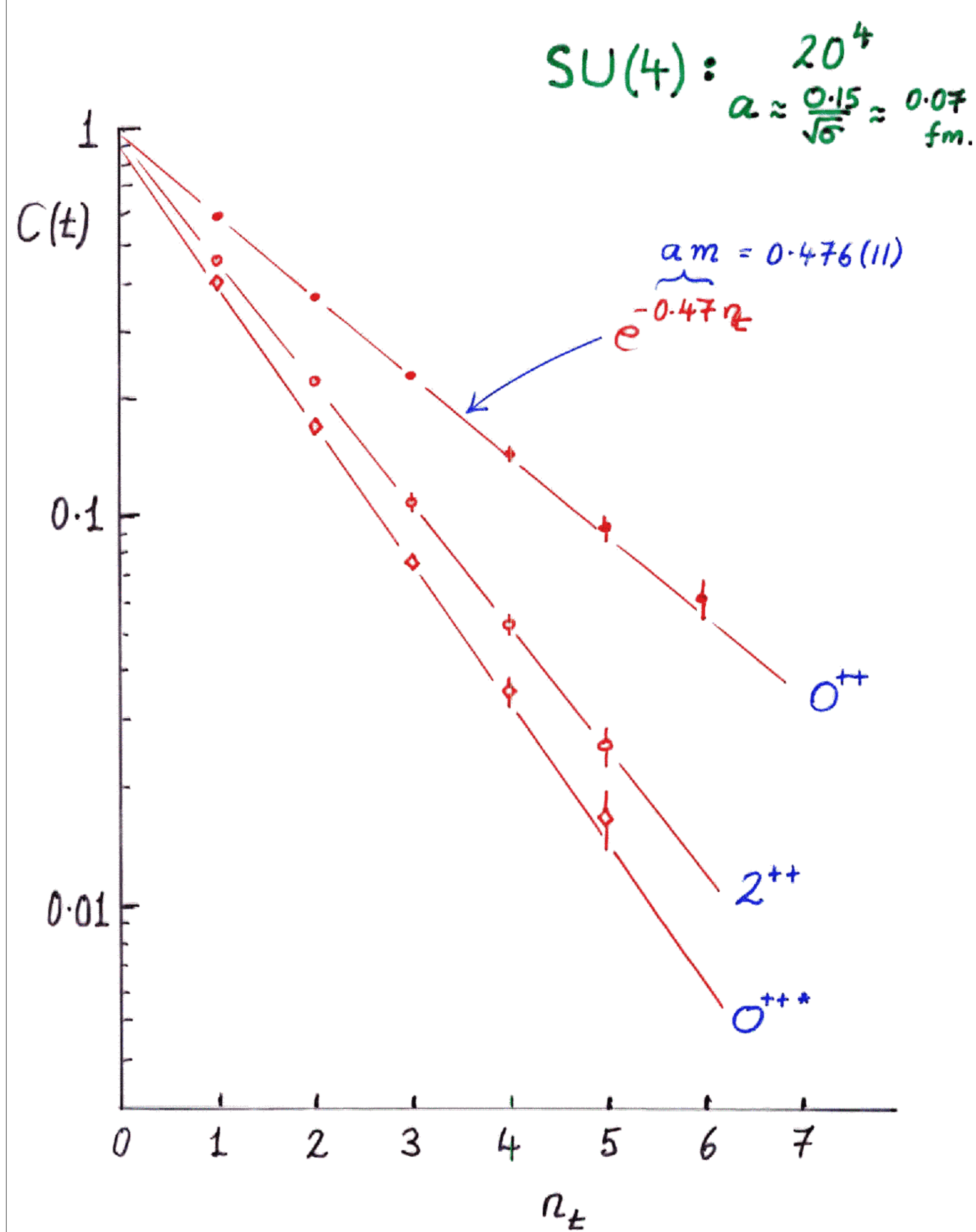
EFPI ↓

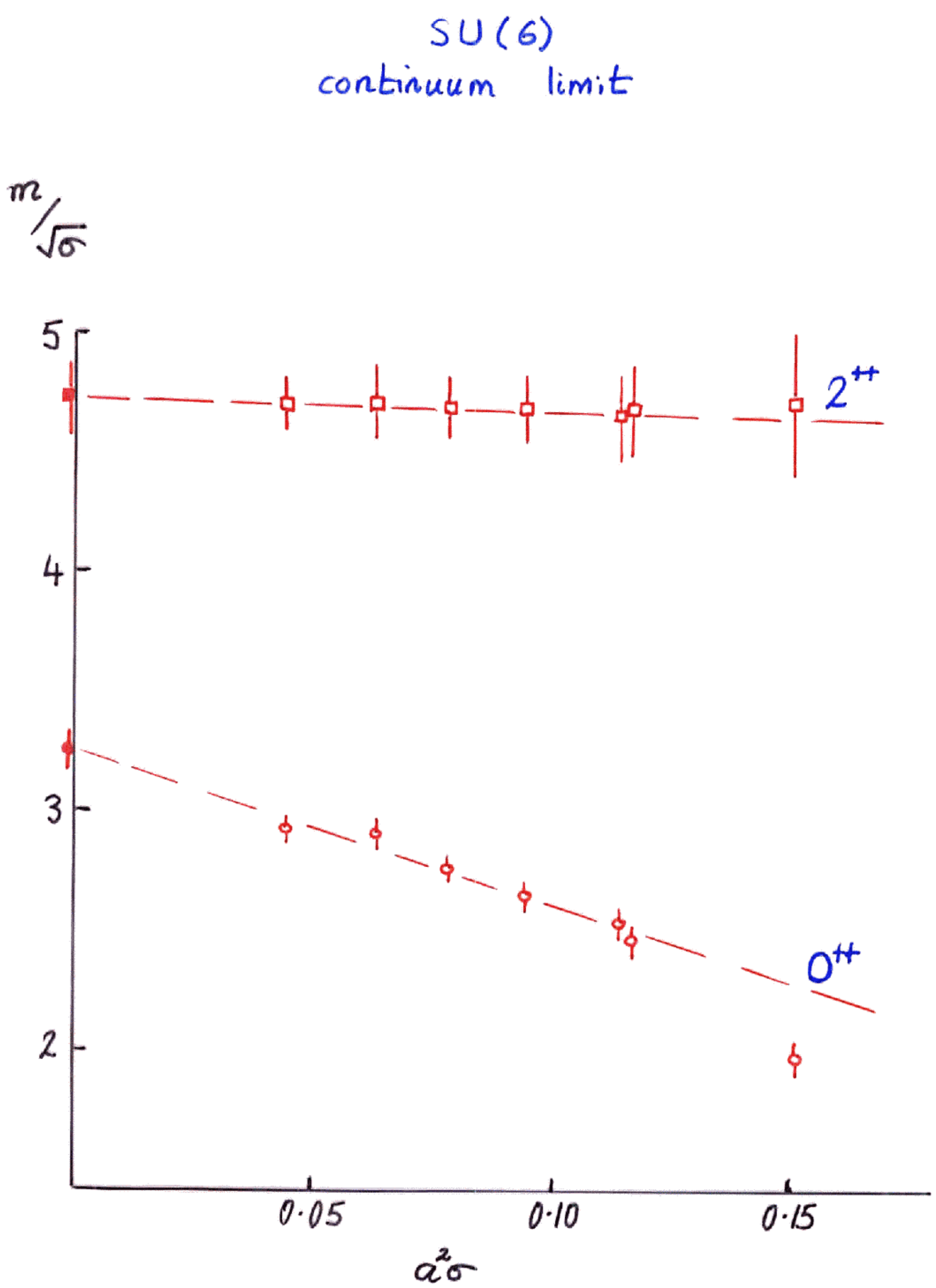
$$\int \prod_{x,\mu} dA_\mu(x) \ldots e^{-\frac{1}{g^2}\int Tr F^2_{\mu\nu} d^4x}$$

$$\int \prod_{n,\mu} dU_\mu(n) \ldots e^{-\beta \sum_p \left(1-\frac{1}{N} ReTr U_p\right)}$$

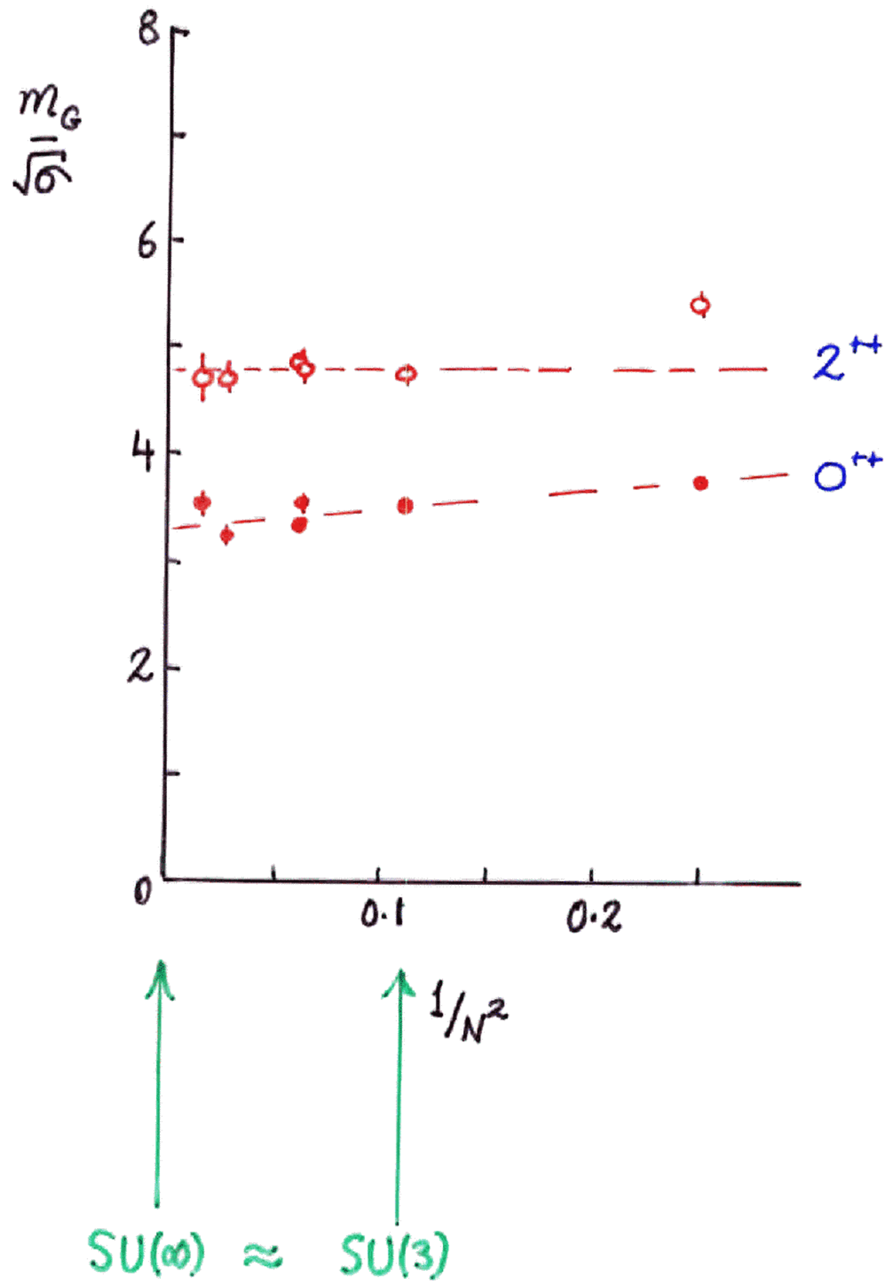
$2N_c$

$$\beta = \frac{const}{g^2} \equiv \frac{const}{g_L^2(a)}$$

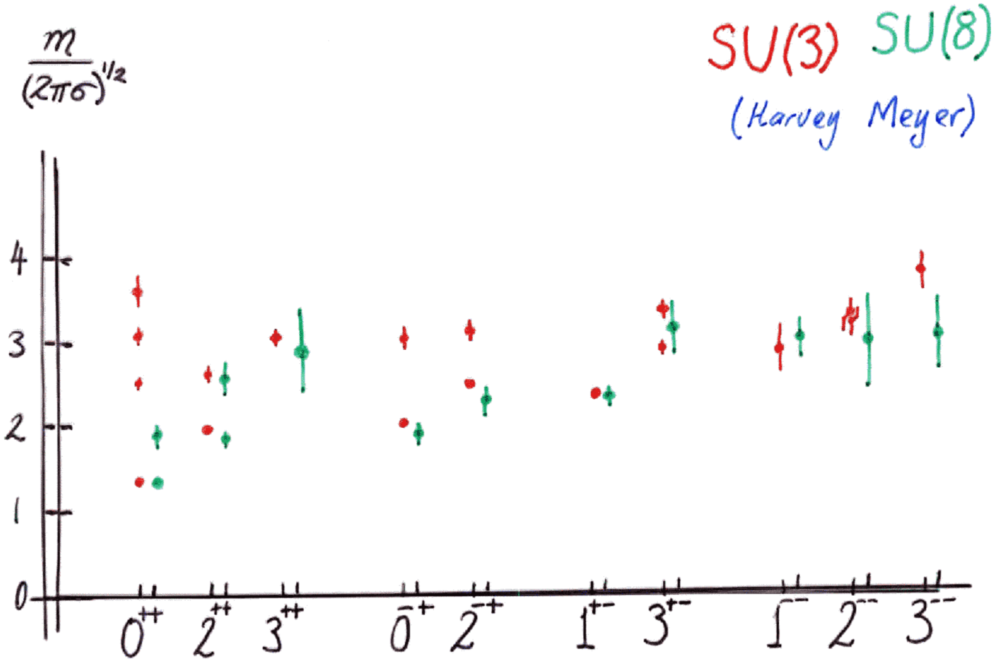




SU(6)
continuum limit
m/√σ
5
4
3
2
2++
0++
0·05
0·10
0·15
a²σ



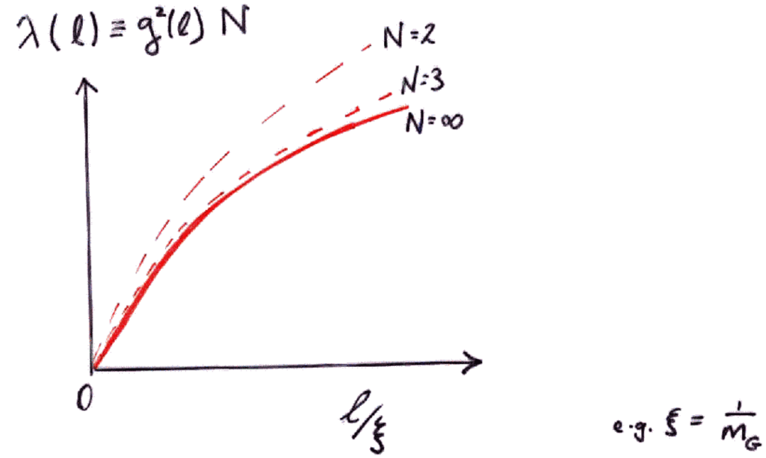
8
mG/√σ
6
4
2
0
2++
0++
0·1
0·2
1/N²
SU(∞) ≈ SU(3)



glueball spectra for SU(3) and SU(8)
- very similar except for lighter 0^{++*} for $N_c = 8$
↓
first signal of glueballs built on $k=2$ flux loops ?

$\lambda \equiv g^2 N$ fixed ?

i.e.



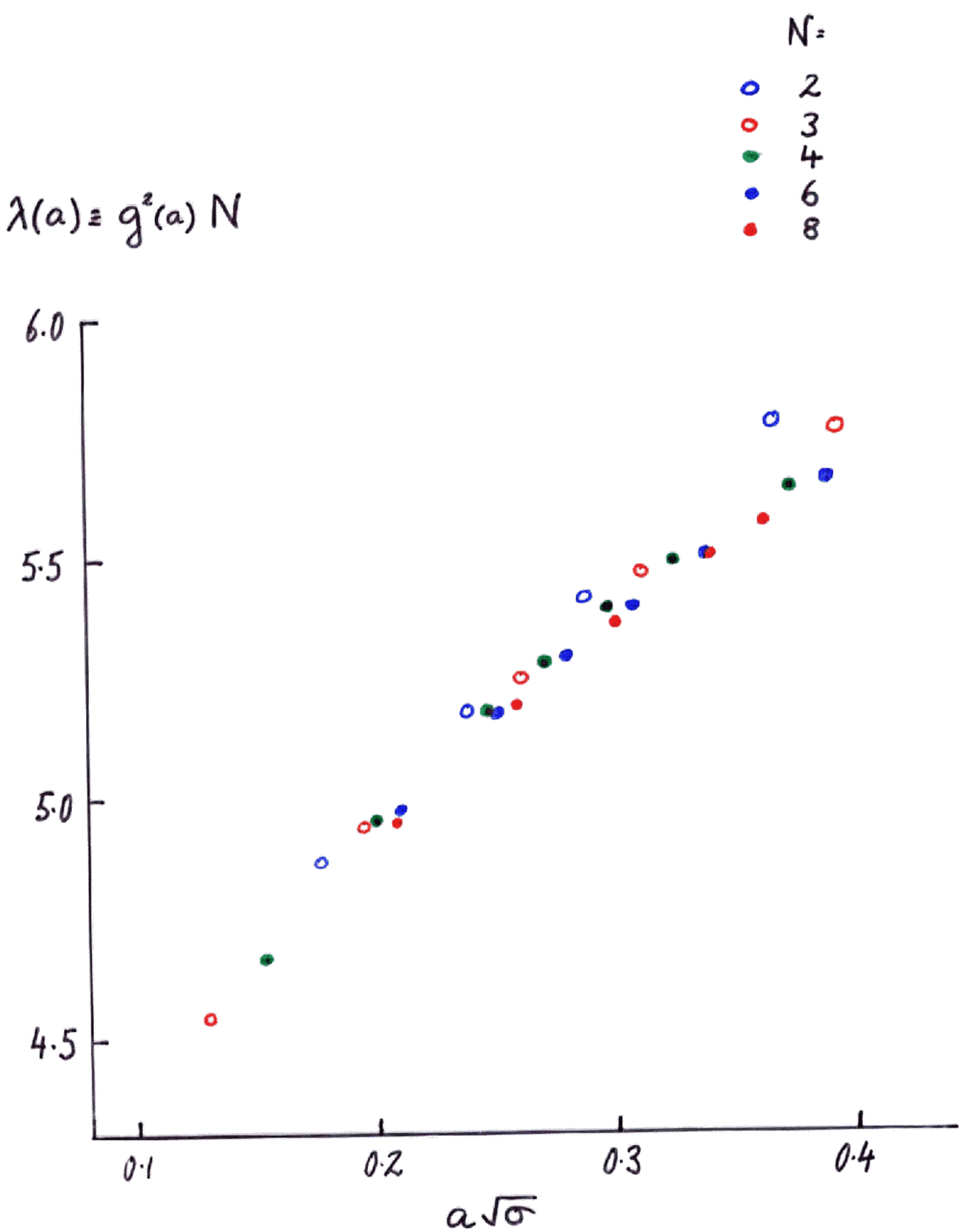
on the lattice $\beta \xrightarrow[a \to 0]{} \frac{2N}{g^2}$

so we can define

$$\lambda(a) = g^2(a)\,N = \frac{2N^2}{\beta}\frac{1}{\langle \frac{1}{N}\mathrm{ReTr}U_p \rangle}$$

↑
"mean field" improvement
$\langle \cdots \rangle = 1 - O(g^2)$

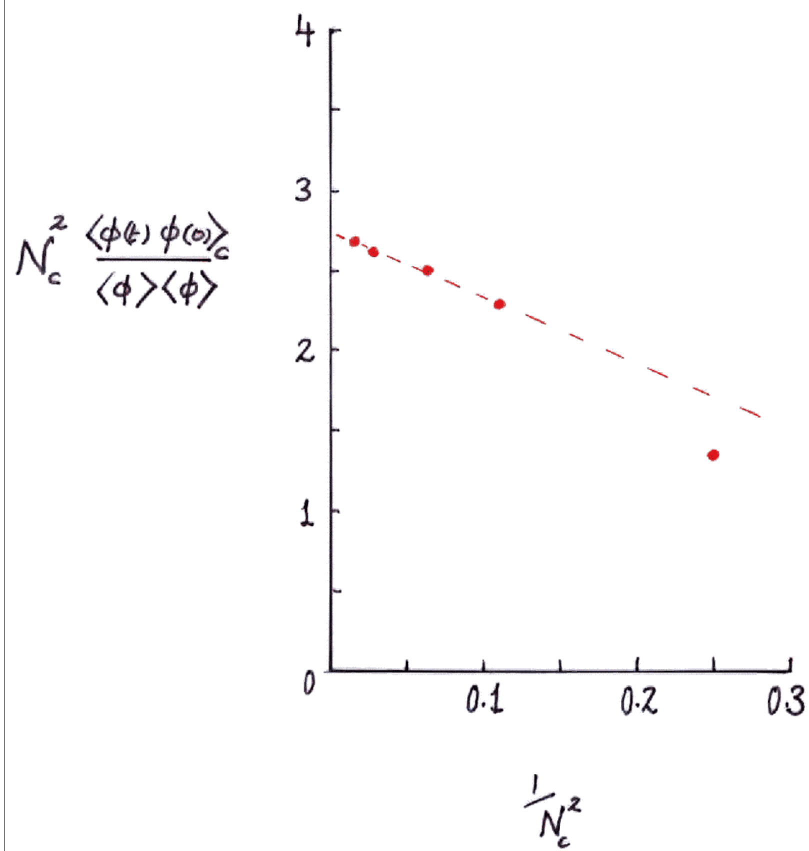
$\Longrightarrow$



factorisation

$$\langle \phi(t)\,\phi(0)\rangle = \langle \phi\rangle\langle \phi\rangle\left\{1 + O\left(\tfrac{1}{N^2}\right)\right\} \quad ?$$

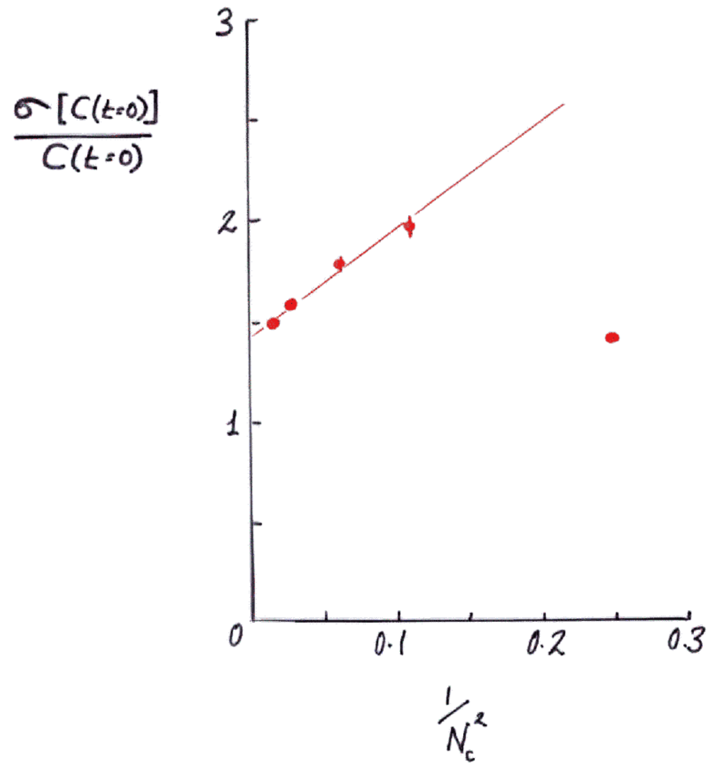
connected correlator → masses



$\phi_v = \phi - \langle \phi \rangle$

fluctuations on $C(t) = \langle \phi_v(t)\, \phi_v(0) \rangle$

$$\sigma^2 [C(t)] = \langle \{ \phi_v(t)\, \phi_v(0) - \langle \phi_v(t)\, \phi_v(0) \rangle \}^2 \rangle$$

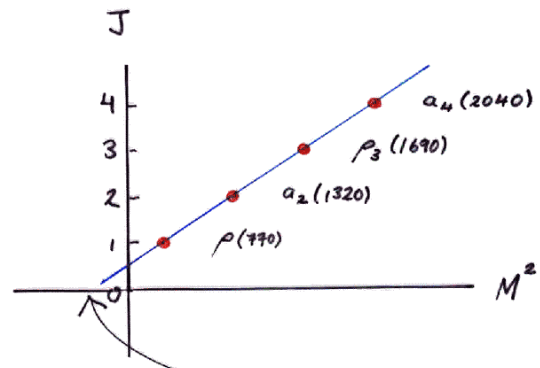


† Harvey Meyer

- Regge trajectories

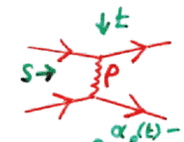
$J = \alpha_0 + \alpha' M^2$

$\alpha'_R \approx 0.87\ \mathrm{GeV}^{-2}$



for $t = M^2 \le 0$: $s \rightarrow$

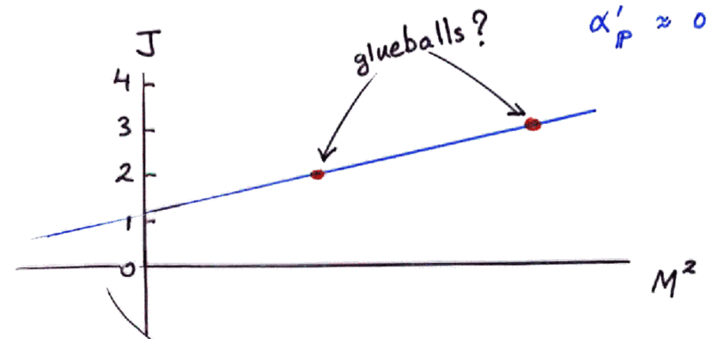
$J = \alpha_\rho(t)$

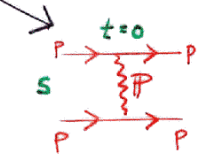


$\sim s^{\alpha_\rho(t)-1}$

- Pomeron

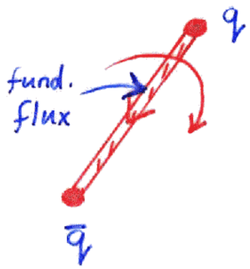
$\alpha'_{\mathbb{P}} \approx 0.25\ \mathrm{GeV}^{-2}$





optical theorem

$s \cdot \sigma_{pp}^{tot}$ slowly rising a simple model for $\mathbb{R}$

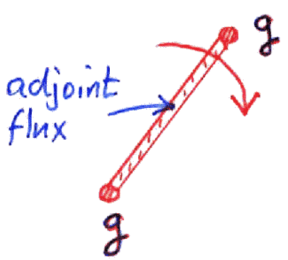


$$J = \alpha'_{\mathbb{R}} M^2$$

with

$$\alpha'_{\mathbb{R}} = \frac{1}{2\pi\sigma_f}$$

a simple model for $\mathbb{P}$



$$J = \alpha'_{\mathbb{P}} M^2$$

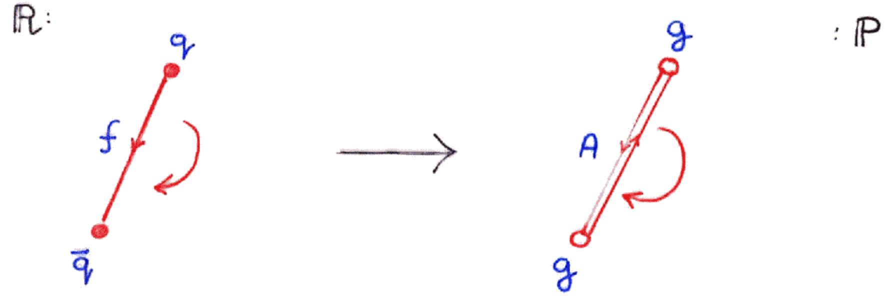
with

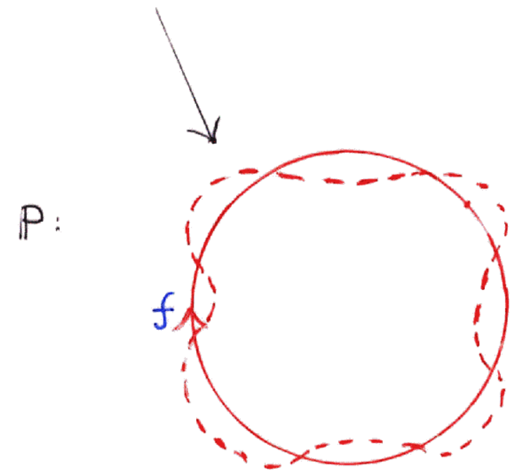
$$\alpha'_{\mathbb{P}} = \frac{1}{2\pi\sigma_A} = \alpha'_{\mathbb{R}} \cdot \frac{\sigma_f}{\sigma_A} \approx 0.45\,\alpha'_{\mathbb{R}}$$

not too bad!

& same in pure gauge theory

(two?) Pomerons ?





closed loop of fund. flux
+
oscillations
phonons

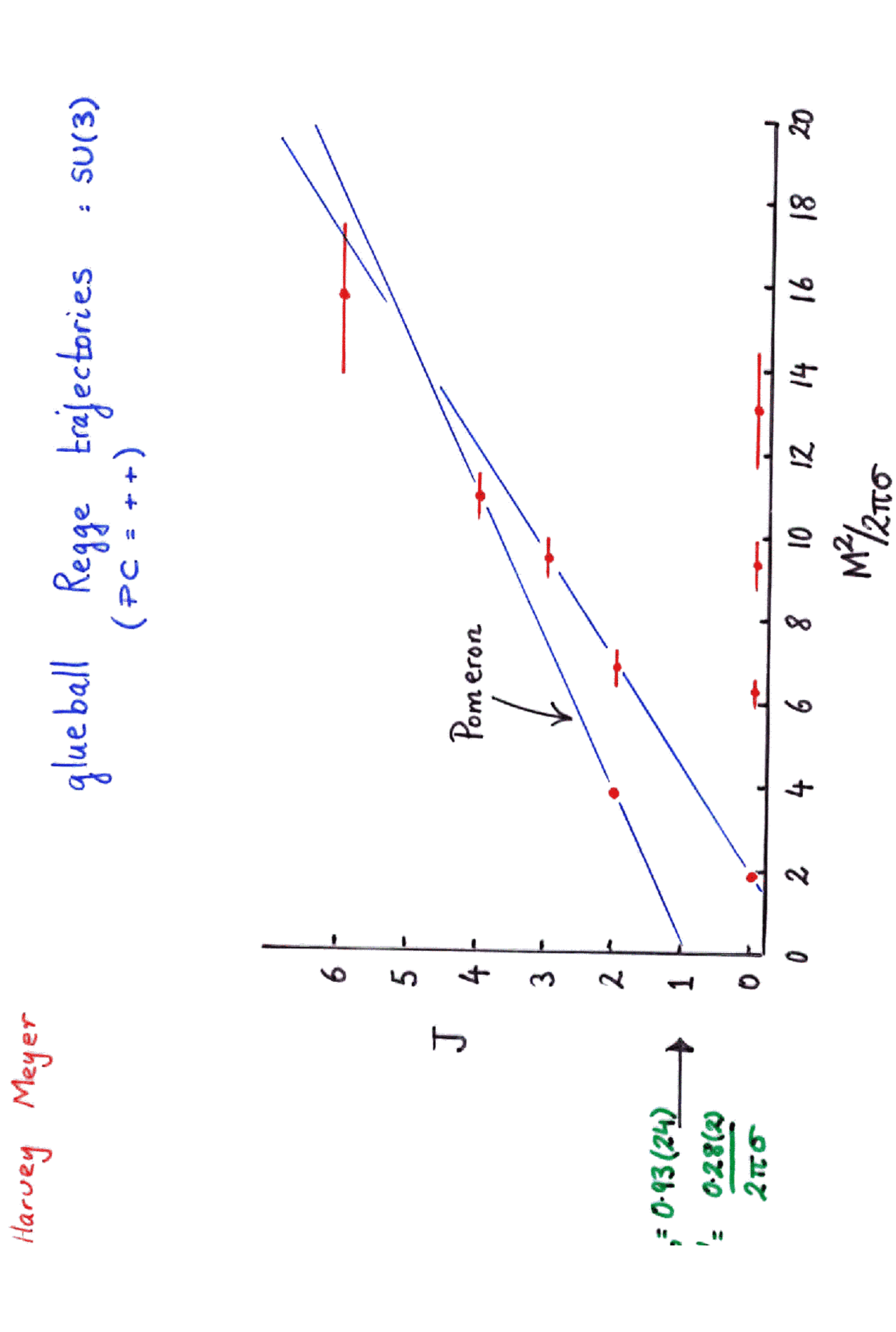
Isgur, Paton: '85

radius ρ : $E_{min}(\rho; J) \underset{phonons}{\approx} 2\pi\rho\sigma_f + \frac{J-c}{\rho}$

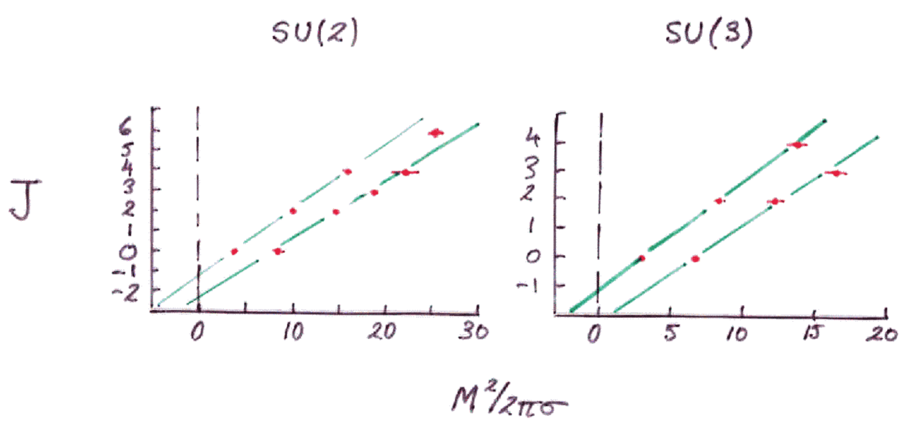
using $\frac{dE}{d\rho} = 0$

$\Longrightarrow$ $J = \alpha' M^2$ for $J \gg 1$

with $\alpha' = \ldots$



Pomeron in D=2+1



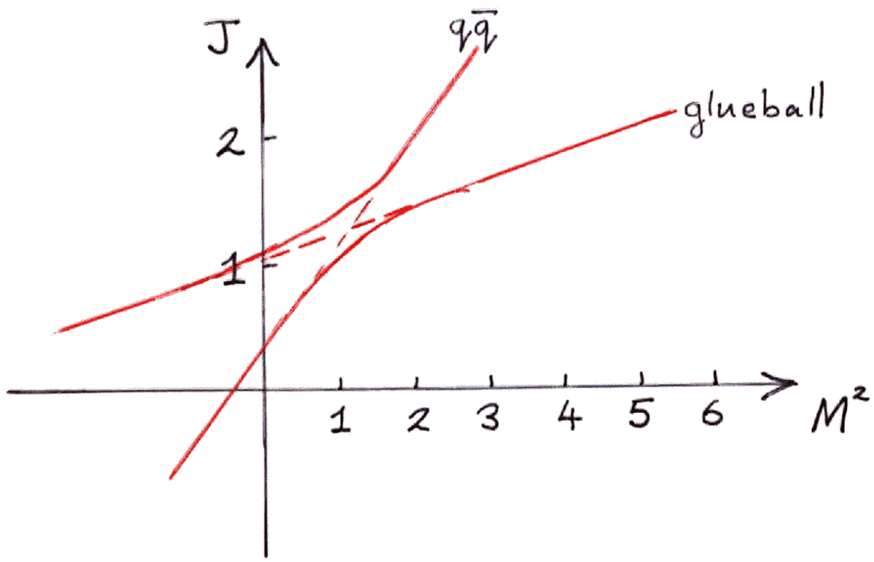
evidence for good linearity:

$$J = \alpha_0 + \alpha' \frac{M^2}{2\pi\sigma}$$

$\alpha_0 \approx -1$ $\quad$ $\alpha' \approx \frac{1}{3}$ $\Longrightarrow$ not important at high energy!

in reality there will be mixing between flavour-singlet $q\bar{q}$ and glueballs

→



'crossing' near $M^2 \sim 1\ GeV^2$

⇒ small but significant shifts in $\alpha_P(0)$ and $\alpha'_P(0)$

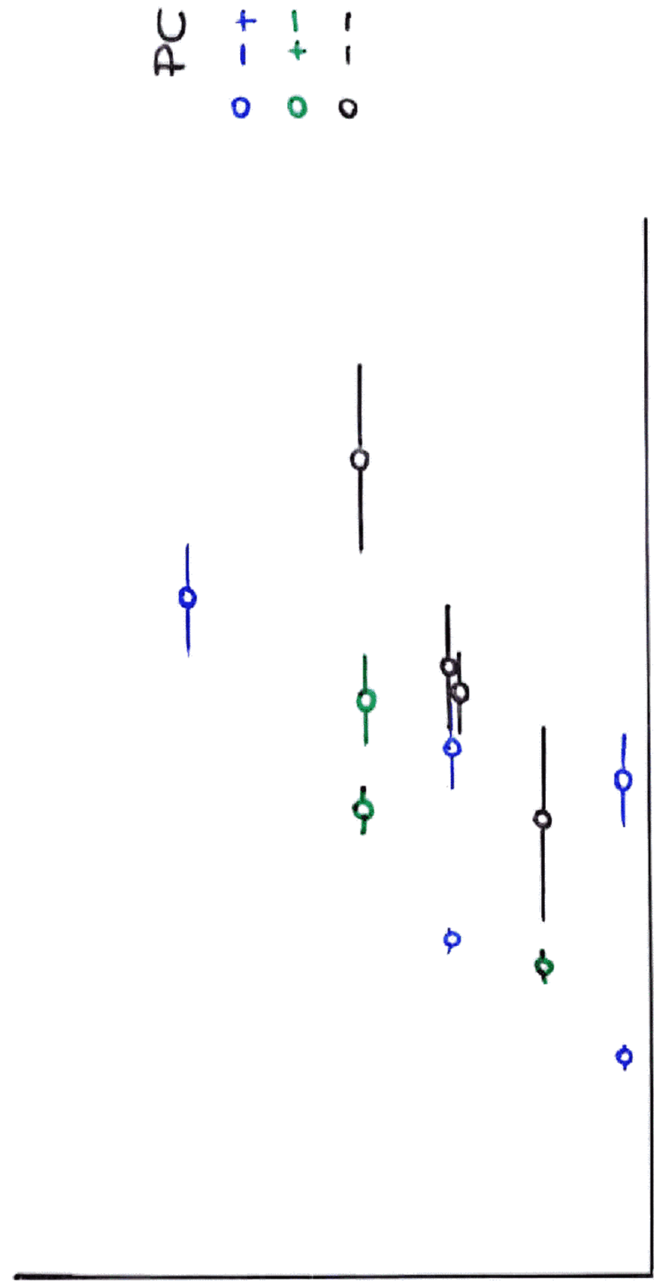
but 2 $q\bar{q}$ trajec.

with

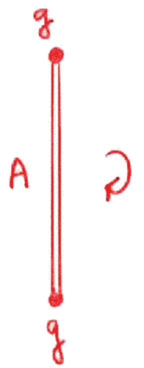
$$1{\cdot}09 \approx \alpha_P(0) > \alpha^G_P(0) \overset{??}{\approx} 1$$

$$0{\cdot}25 \approx \alpha'_P(0) > \alpha^{G'}_P(0) \overset{?}{\approx} 0{\cdot}2$$

- $N_c \to \infty$ things become simple



observed pattern of states suggests:



J = even

P, C = +,+

— leading (Pomeron) trajectory

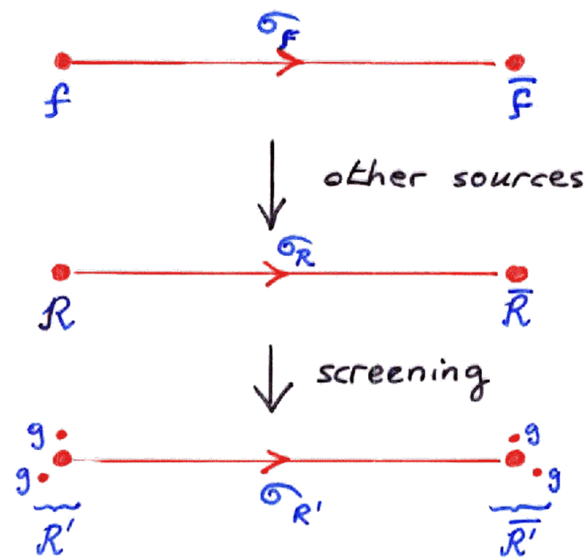
no light $J=1$; all other J

parity doubling

$C=+/-$ degeneracy

— sub-leading trajectory

k-strings



e.g. $f = q$

k-string: $R' \sim f^k \sim k$ quarks $\rightarrow \sigma_k$

if $\sigma_{R'} < \sigma_R$

So: stable strings will be 'lightest' strings in classes of R
— each class contains all R that can be reached from each other by gluon screening

How do we label these classes?

Label sources by how they transform under the centre

$$Z_N = \{ z_j = e^{i\frac{2\pi j}{N}} \cdot 1\!\!1 \} \subset SU(N)$$

Since the adjoint gauge fields transform trivially $\quad A_\mu \to z A_\mu z^+ = A_\mu \qquad z \in Z_N$

if $\quad \phi_R \to z^k \phi_R \qquad R' \in R \otimes A \otimes A + \ldots$

then $\quad \phi_{R'} \to z^k \phi_{R'}$

So

the stable strings can be labelled by this k: $\quad \phi_R \to z^k \phi_R$

subject to constraints

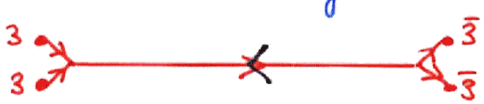
$k \leftrightarrow -k \qquad$ Charge conjugation

$k \leftrightarrow N-k \qquad$ as $z^N = 1$

e.g.

k quarks $\sim z^k$

Note: no new strings in SU(3):



$k \to N-k$

$k=2$ need $SU(N\geq 4)$

$k=3$ need $SU(N\geq 6)$

The values of $\sigma_k/\sigma \big|_N$ should tell us something about the dynamics of confinement ...

some possibilities:

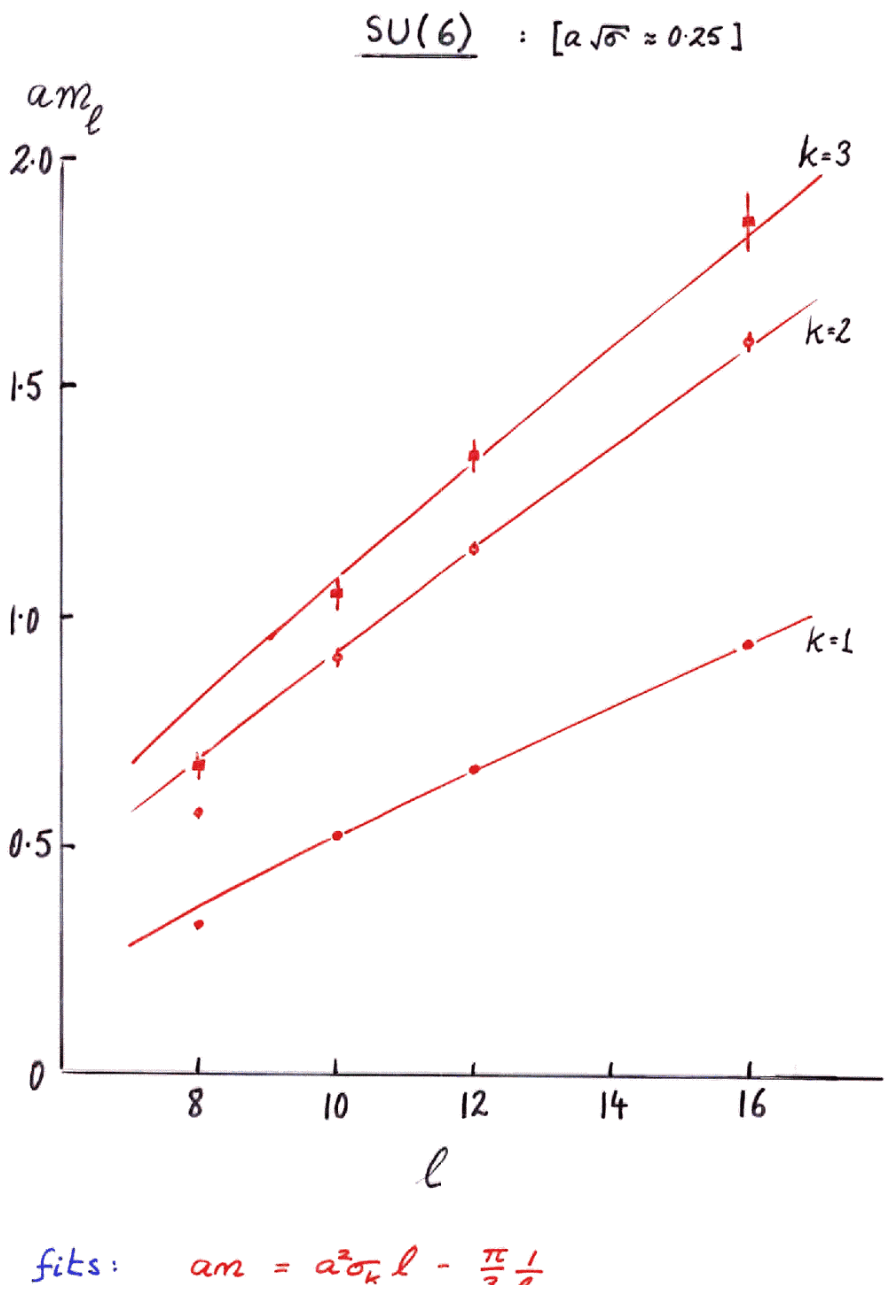
no binding "flux counting": $\quad \sigma_k/\sigma = k \quad$ for $k<N-k$

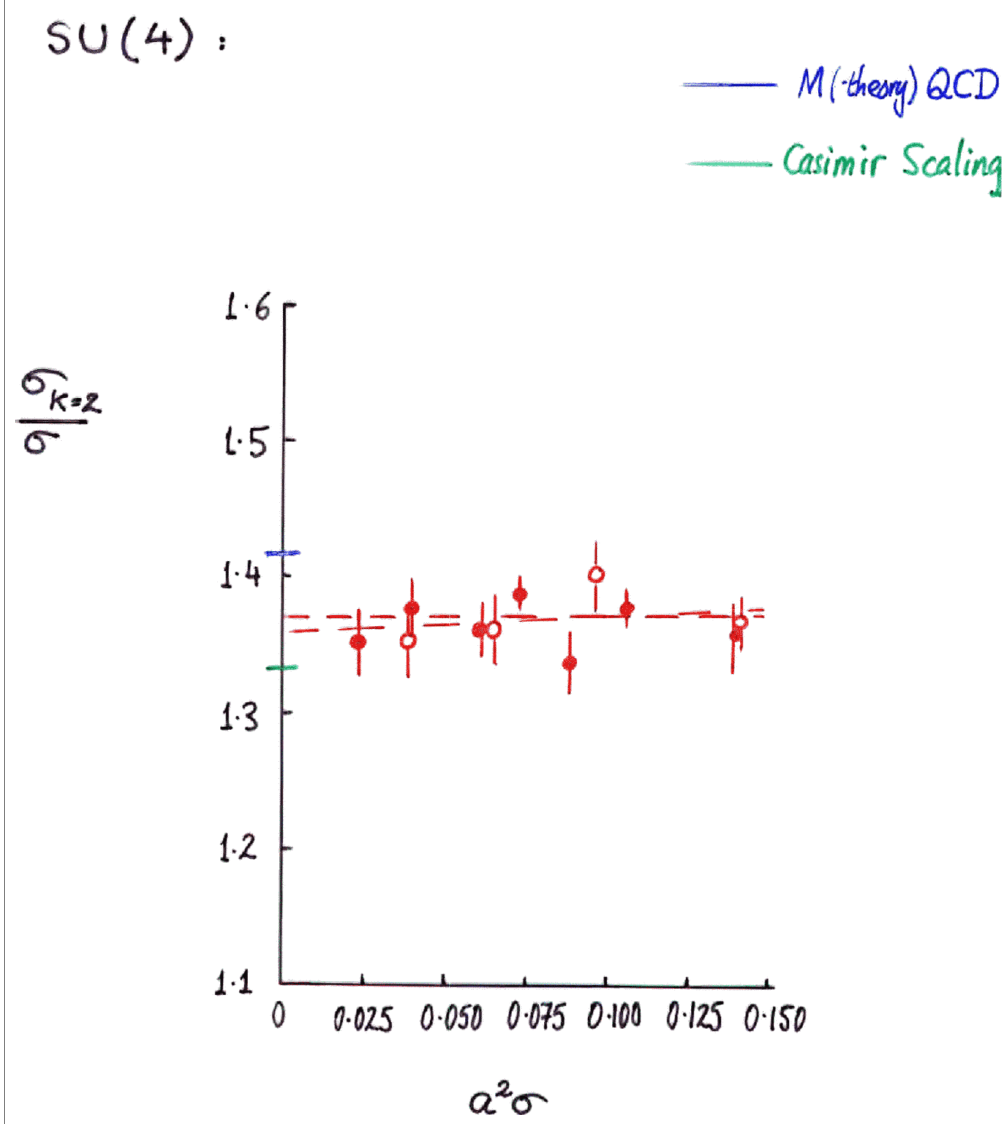
MQCD (Aharony et al: hep-th/9707244): $\quad \sigma_k/\sigma = \dfrac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}}$

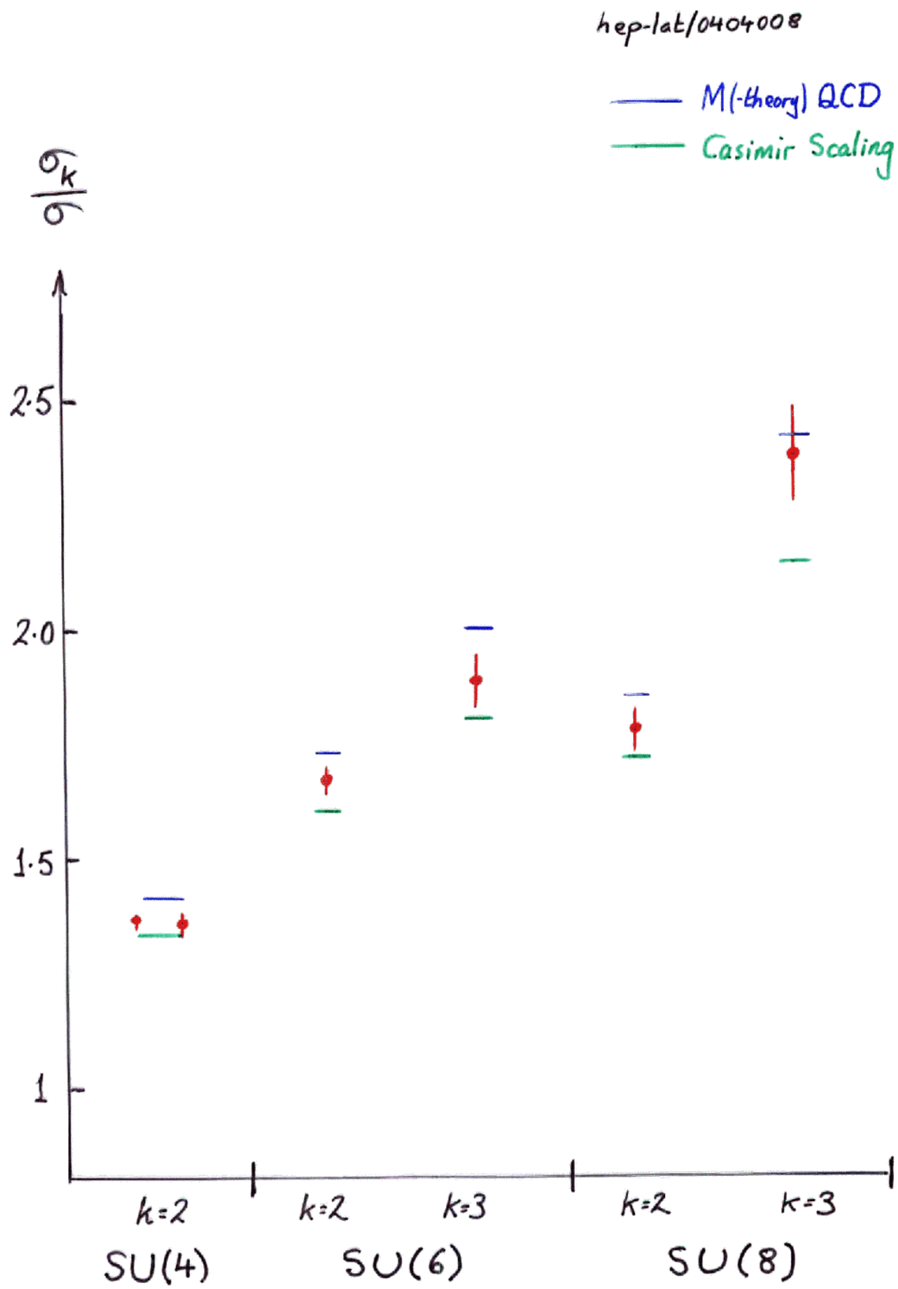
Casimir scaling $\propto T_R^a T^a$: $\quad \sigma_k/\sigma = \dfrac{k(N-k)}{N-1}$

Coulomb; Hamiltonian sc; random vacuum.

(old) Bag model: $\quad \sigma_k/\sigma = \left\{\dfrac{k(N-k)}{N-1}\right\}^{1/2}$

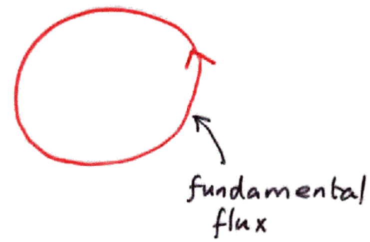

SU(6) : [a √σ ≈ 0·25]
am_ℓ
k=3
k=2
k=1
2·0
1·5
1·0
0·5
0
8
10
12
14
16
ℓ
fits: am = a²σ_k ℓ - π/3 1/ℓ


SU(4) :
M(-theory) QCD
Casimir Scaling
σ_{k=2}/σ
1·6
1·5
1·4
1·3
1·2
1·1
0
0·025
0·050
0·075
0·100
0·125
0·150
a²σ



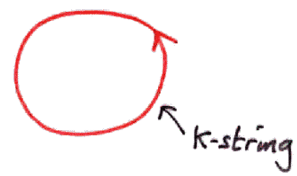
implications for spectrum of 'bound' k-strings

glueball model



$\Rightarrow \quad m_i^f = c_i \sqrt{\sigma}$

SU(N≥4) ↓



$\Rightarrow \quad m_i^k = c_i \sqrt{\sigma_k}$

$= m_i^f \times \sqrt{\frac{\sigma_k}{\sigma}}$

$\Longrightarrow$

As N ↑ get extra "towers" of glueball states scaled up factors of k-string tension ratios

... perhaps only for higher excitations

an interesting possibility

Casimir scaling (if true)

$$\Longrightarrow \quad \frac{m_i^k}{m_i^f} = \sqrt{\frac{\sigma_k}{\sigma}} = \sqrt{\frac{k(N-k)}{N-1}}$$

$$= k^{1/2}\left\{1 + O\left(\frac{1}{N}\right)\right\}$$

k fixed
$N \to \infty$

not $O\left(\frac{1}{N^2}\right)$

Are fundamental flux loops introducing corrections as though the theory contained fund. rep. fields?

topological susceptibility

$$\chi_t = \frac{\langle Q^2 \rangle}{\text{space-time volume}}$$

is expected to be related to $m_{\eta'}$:

't Hooft
Witten
Veneziano

$$\chi_t \simeq \frac{m_{\eta'}^2 f_{\eta'}^2}{2N_f} \sim (180\ \text{MeV})^4$$

χ_t: of SU(N) pure gauge theory

$\simeq$: N large

$\sim$: if N=3 is large

$\Downarrow$

- $m_{\eta'}^2 = \frac{c}{N}$ for $N \geq 3$ — we cannot check this!
- $\chi_t \big|_{SU(3)} \approx \chi_t \big|_{SU(\infty)}$ — we can try to check this

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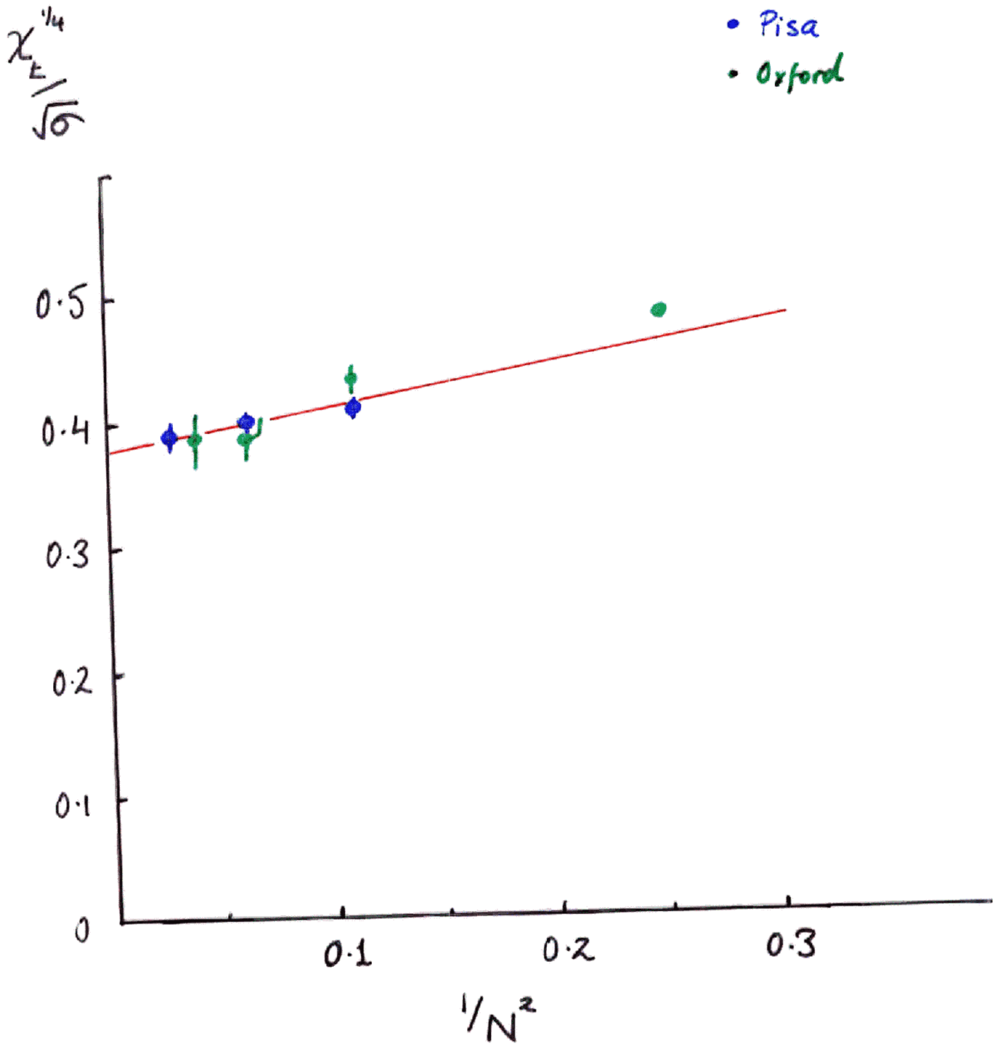
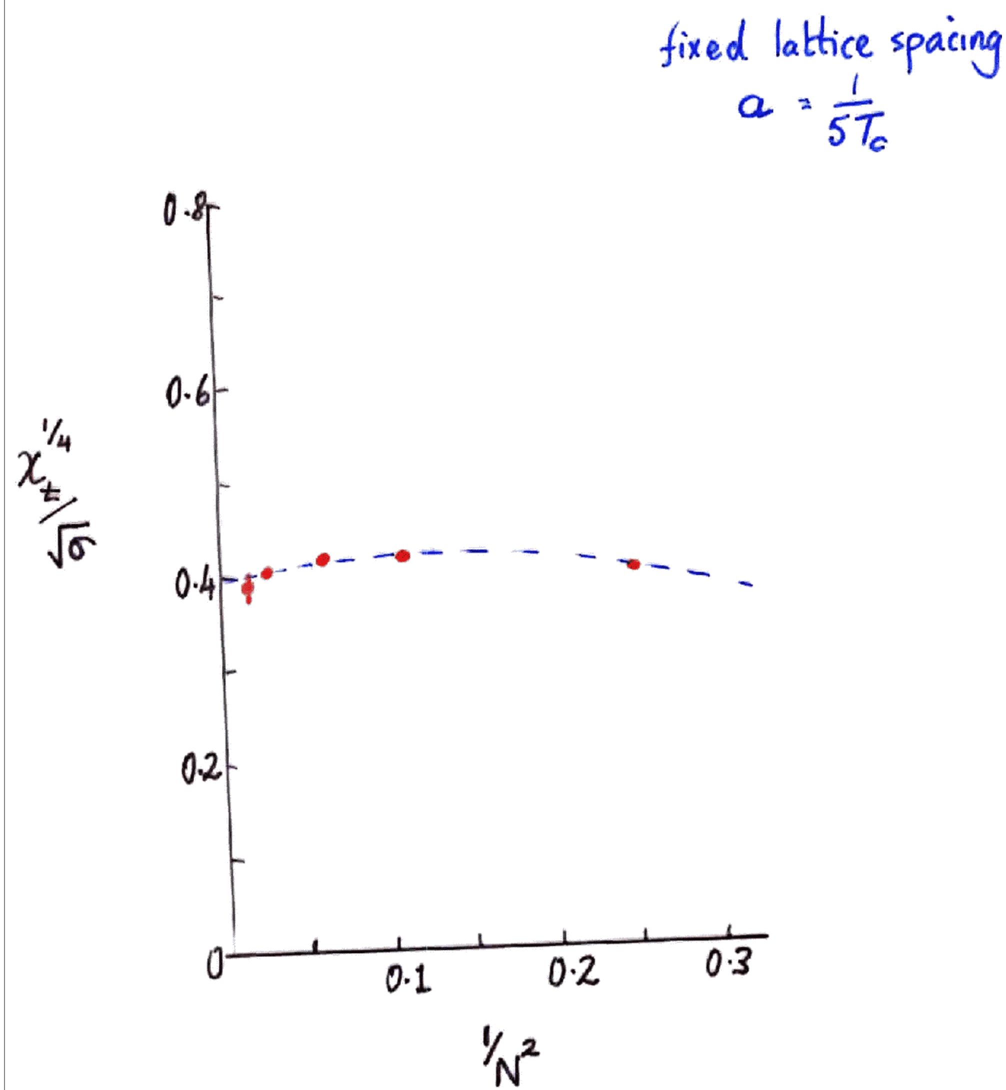
B. Lucini, MT : hep-lat/0103027

BL, MT + U. Wenger: hep-lat/0401028

L. Del Debbio, H. Panagopoulos, E. Vicari : hep-th/0204125



$$\chi_t^{1/4} \approx 0.375\sqrt{\sigma} \approx 165\ \text{'MeV'} \quad \left\{ \begin{array}{l} N \to \infty \\ \sqrt{\sigma} = 0.44\ \text{'GeV'} \end{array} \right.$$



small instantons

$$D(\rho)\,d\rho \underset{\rho \ll 1\text{fm}}{\approx} \frac{d\rho}{\rho} \cdot \frac{1}{\rho^4}\, e^{-\frac{8\pi^2}{g^2(\rho)}} \cdot c(g^2;N)$$

↑"negligible" as $\rho \to 0$

$$g^2(\rho) = \frac{24\pi^2}{11N \ln\left(\frac{1}{\rho\Lambda}\right)}$$

$$\underset{\rho \to 0}{\propto} \rho^{\frac{11N}{3} - 5}\, d\rho$$

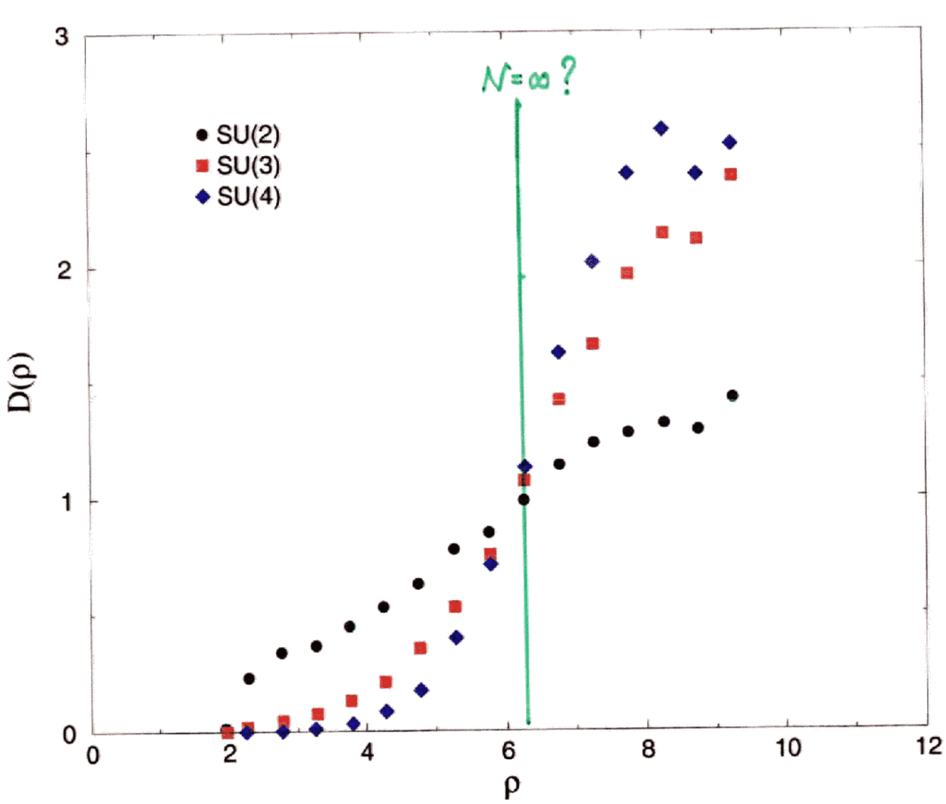
$\Longrightarrow$

for small enough ρ

$$\text{prob}(I) \propto e^{-cN}$$

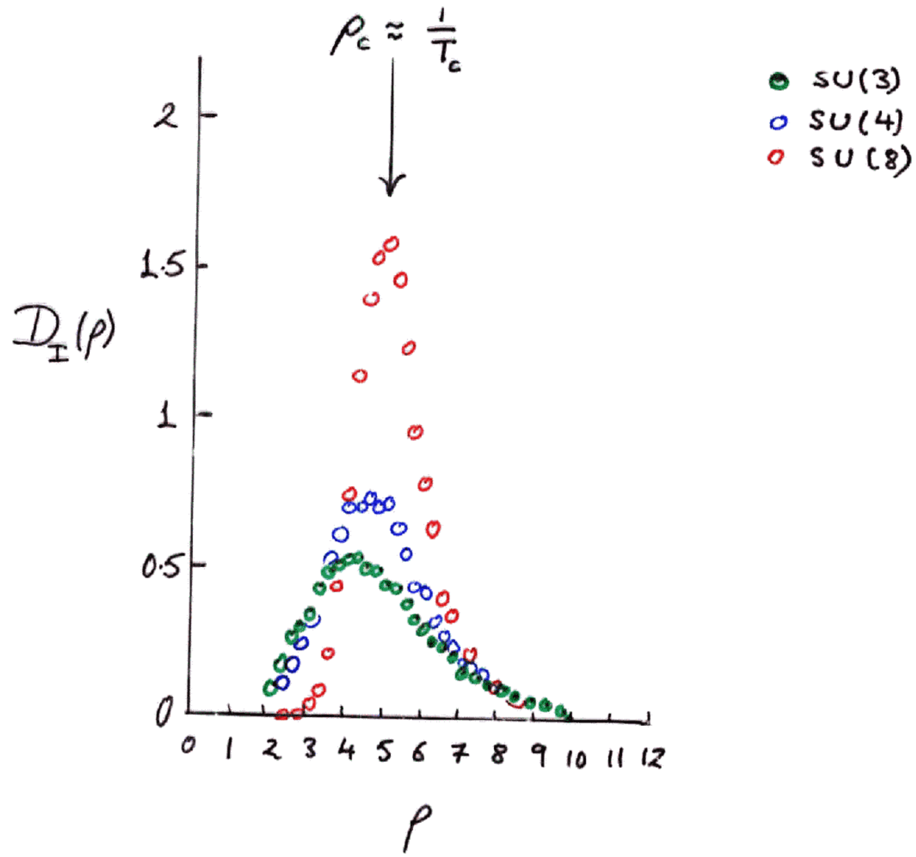
i.e. no contribution to any finite order in the $1/N$ expansion

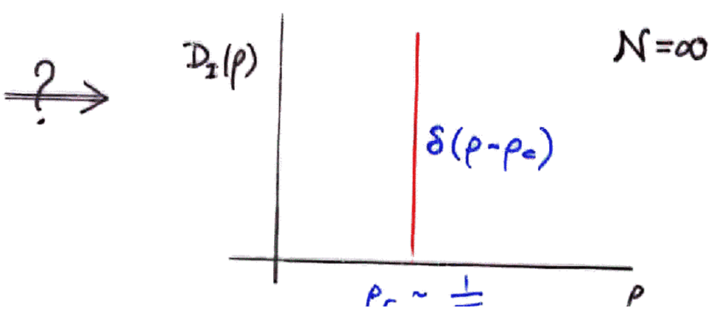
20^4 for $a\sqrt{\sigma} \approx 0.16$



$\Rightarrow$ overlap fermions on SU(N) fields

Urs Wenger
Nigel Cundy
MT.





does topology drive chiral ssb ?

$$\langle \bar{\psi}\psi \rangle \sim \left\langle Tr \frac{1}{i\not{D}+m} \right\rangle \sim \sum_{\lambda} \frac{1}{i\lambda + m} \sim \int_0^\infty \frac{m\,\rho(\lambda)\, d\lambda}{\lambda^2+m^2} \propto \rho(\lambda=0)$$

are small non-zero modes of $\not{D}$, topological?

measure topological content of $\psi(x)$ by

$$C^5 = \int d^4x \, |\psi^\dagger \gamma_5 \psi|^{1/2} \, |Q(x)|^{1/2} \, \mathrm{sign}(\psi^\dagger \gamma_5 \psi)\, \mathrm{sign}(Q(x))$$

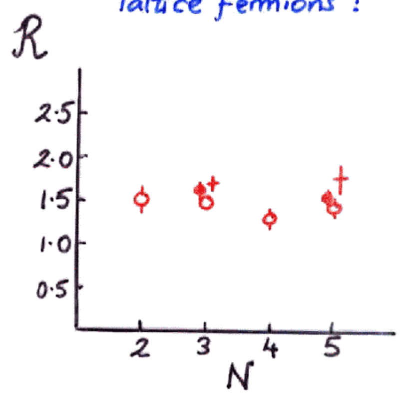
and then look at

$$R \equiv \frac{C^5(\lambda>0)}{C^5(\lambda=0)}$$

← lowest few modes

← know zero modes entirely topological

using good (overlap) lattice fermions:



⇒

- small modes topological
- ind. of a & V
- ind. of N

⇓ some evidence

topology drives chiral ssb at all N

N. Cundy, MT, U. Wenger hep-lat/0309011

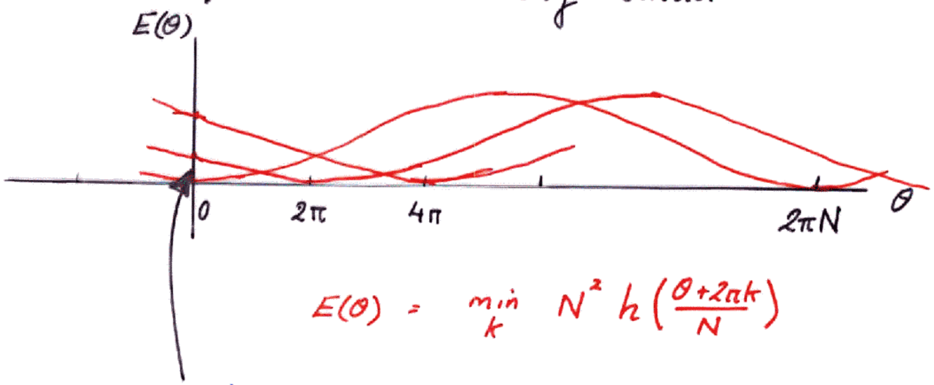
multiple vacua as $N \to \infty$ — Witten, Shifman

$$e^{-E(\theta)T} = \int \cdots e^{i\theta Q}$$

N counting: $E(\theta) = N^2 h\left(\frac{\theta}{N}\right)$

Q integer: $E(\theta) = E(\theta + 2\pi)$

$\Longrightarrow$ intertwining vacua



$$E(\theta) = \min_k N^2 h\left(\frac{\theta + 2\pi k}{N}\right)$$

multiple vacua at $\theta = 0$

– tunnelling $\propto e^{-cN}$ as $N \to \infty$

$\underset{\theta \approx 0}{\Longrightarrow}$ $E(\theta) = b_0 \theta^2 + b_1 \theta^4 + \ldots$

$b_0 \to O(N^0)$, $b_1 \to O\left(\frac{1}{N^2}\right)$

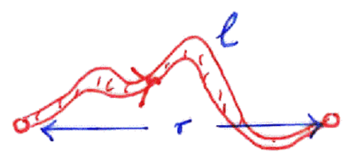
$\frac{d}{d\theta} \int \cdots e^{i\theta Q} \sim \int \cdots Q e^{i\theta Q} \Rightarrow b_1 \leftrightarrow \begin{cases} \langle Q^4 \rangle_{\theta=0} \\ \langle Q^2 \rangle_{\theta=0} \end{cases}$

$\Longrightarrow$ $b_1(N=3) : b_1(N=4) : b_1(N=6) \propto$ "$\frac{1}{N^2}$"

Pisa $\approx$

deconfinement

Polyakov



costs: $e^{-E(l)/T}$, $E(l) = \sigma l$

$n(l)$ — number of paths $\propto e^{cl}$

as $n(l) \underset{l \gg r}{\approx} n\left(\frac{l}{2}\right) n\left(\frac{l}{2}\right)$

i.e. $e^{-\frac{\sigma l}{T}} \cdot e^{+cl}$

$\Rightarrow$ cost of ∞ flux tube drops to zero at $T_c = \frac{\sigma}{c}$ $\to$ deconfinement

— — — — — — —

$\Rightarrow$ partition function $\sum_{\text{states}} e^{-E_s/T}$

diverges as $T \to T_c \approx 0.6\sqrt{\sigma} \approx 280\,\text{MeV}$

but lowest energy states have

$$\frac{E_{0^{++}}}{T_c} \approx \frac{3.5\sqrt{\sigma}}{0.6\sqrt{\sigma}} \approx 6 \quad !?$$

but

$$\sum_{\text{states}} e^{-E_s/T} = \sum_E n(E)\, e^{-E/T}$$

↑ density of states

$\Rightarrow$ what is important is

$\lim_{E\to\infty} n(E)$

and not $E_0, E_1, \ldots$

e.g. flux loop model of glueballs

ground state

$m = \sigma\ell = 2\pi\rho\sigma$

excited states

$m = \sigma\ell$

but $n(\ell) \propto e^{c\ell}$

quantum → classical
for large quantum numbers

generically:

$$e^{-\frac{\sigma_{eff}(T)\,\ell}{T}} = e^{-\frac{\sigma\ell}{T}}\, e^{+c\ell}$$

$\Rightarrow$

$$\sigma_{eff}(T) = \sigma(T=0) - cT \xrightarrow[T\to T_c]{} 0$$

$\Rightarrow$ 2nd order transition

Note:

deconfinement entirely within confined, stringy theory

— no reference to microscopic df such as gluons neglects

stringy states ← $O(N_c^0)$ entropy

'gluon plasma' states ← $O(N_c^2)$ entropy

→ phase transition, $T = T_c$,

$$e^{-\frac{F_{conf}}{T_c}} = e^{-\frac{F_{decon}}{T_c}}$$

⇒ $F = E - TS$

$$E_{conf} - T_c S_{conf} = E_{decon} - T_c S_{decon}$$

E_{conf}: vacuum + $O(N_c^0)$ ~ $-O(N_c^2)$; S_{conf}: $\sim N_c^0$; E_{decon}: $\sim N_c^2$ (probably); S_{decon}: $\sim N_c^2$

⇛ $N_c \to \infty$

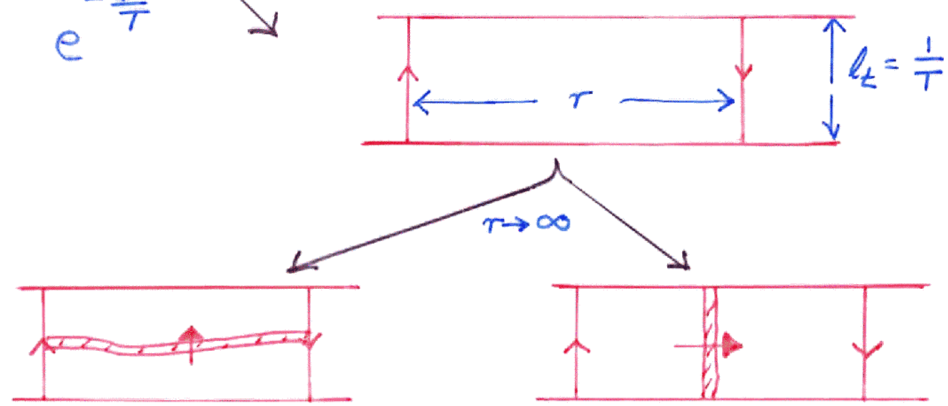
- $L_h = \langle E \rangle_{conf} + \cdots \simeq$ gluon condensate (extrap E_{dec} to $T \to 0$)
- without $\langle E \rangle_{conf} \propto -N_c^2$ we would expect $T_c \to 0$ BUT we find $T_c \neq 0$ as $N_c \to \infty$ $\Rightarrow \langle E \rangle_{conf} \overset{\infty}{\propto} -N_c^2$

not sure!

$\sigma(T)$?

free energy static sources r apart $= \cdots + \sigma(T)\, r$

$e^{-\frac{E}{T}}$



(excited) strings length r

ground state string length $l_t = 1/T$

$$\sigma(T) = \frac{E_0(l_t)}{l_t} = \sigma - \frac{\pi}{3}\frac{1}{l_t^2} + O\left(\frac{1}{l_t^4}\right)$$

$$= \sigma - \frac{\pi}{3} T^2 + O(T^4)$$

universal (bosonic $c_s = 1$) string correction

T_c from $\sigma(T=T_c)=0$

2nd order string condensation transition

Nambu - Gotto string
Arvis '83
Luscher, Weisz '04

$$\Longrightarrow \quad E_0 = \sigma l_t \left\{1 - \frac{8\pi}{\sigma l_t^2}\frac{d-2}{24}\right\}^{1/2}$$

$$\overset{l_t = \frac{1}{T}}{=} \quad \frac{\sigma}{T}\left\{1 - \frac{T^2}{\sigma}\frac{\pi(d-2)}{3}\right\}^{1/2}$$

$$= 0 \quad \text{for} \quad \frac{T_c}{\sqrt{\sigma}} = \begin{cases} 0.977 & d=3 \\ 0.691 & d=4 \end{cases}$$

candidates : observed 2nd order transitions

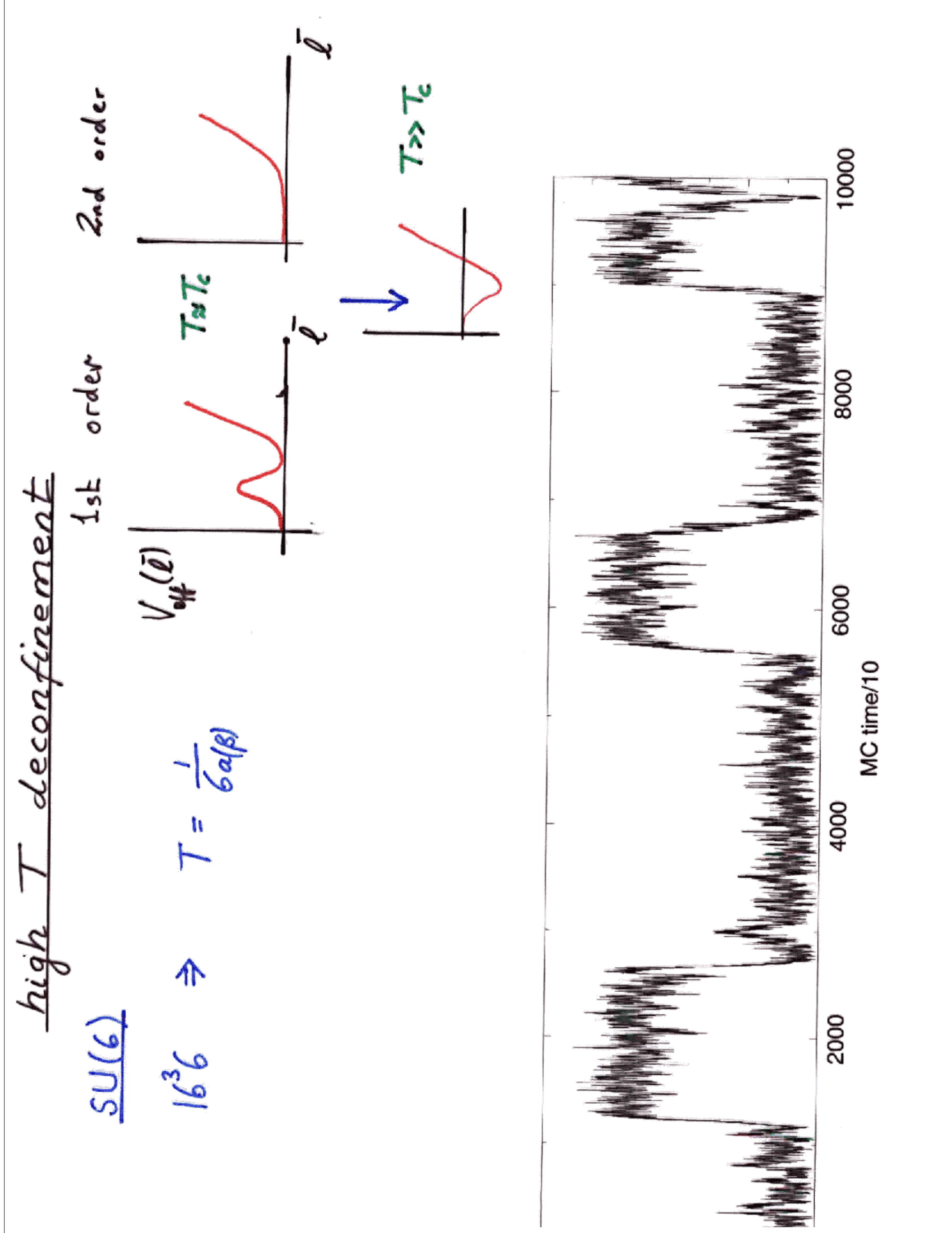
$d=3$:

$$\frac{T_c}{\sqrt{\sigma}} = \begin{cases} 1.121(8) & SU(2) \\ 0.985(12) & SU(3) \end{cases}$$

$d=4$:

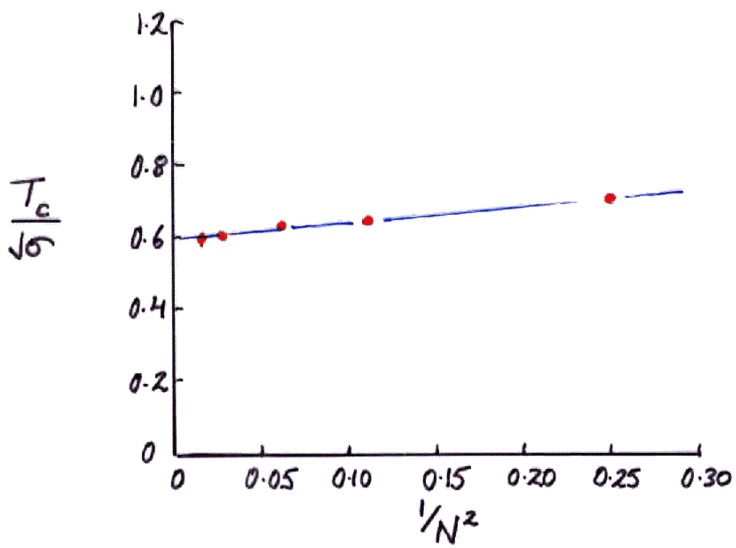
$$\frac{T_c}{\sqrt{\sigma}} = 0.709(4) \qquad SU(2)$$

remarkably close!



(BL, MT, UW)

1st order ⟵|⟶ 2nd order



$$\frac{T_c}{\sqrt{\sigma}} = 0.596(4) + \frac{0.453(30)}{N^2}$$

↑ $N=\infty$

multiple deconfinement?

e.g. SU(4)

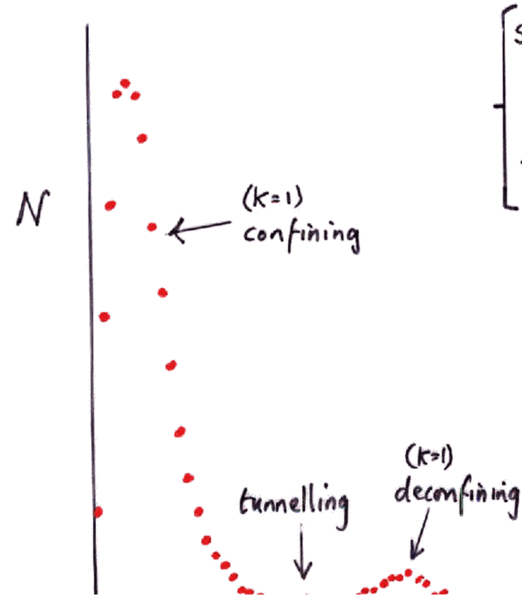
$\langle l \rangle = 0$, $\langle l^2 \rangle = 0$ — $Z_4 \to Z_2$ at $T_c(k=2)$ — $\langle l \rangle = 0$, $\langle l^2 \rangle \neq 0$ — $Z_2 \to 1$ at $T_c(k=1)$ — $\langle l \rangle \neq 0$, $\langle l^2 \rangle \neq 0$ → T

simple energetics / entropy argument suggests that for string condensation

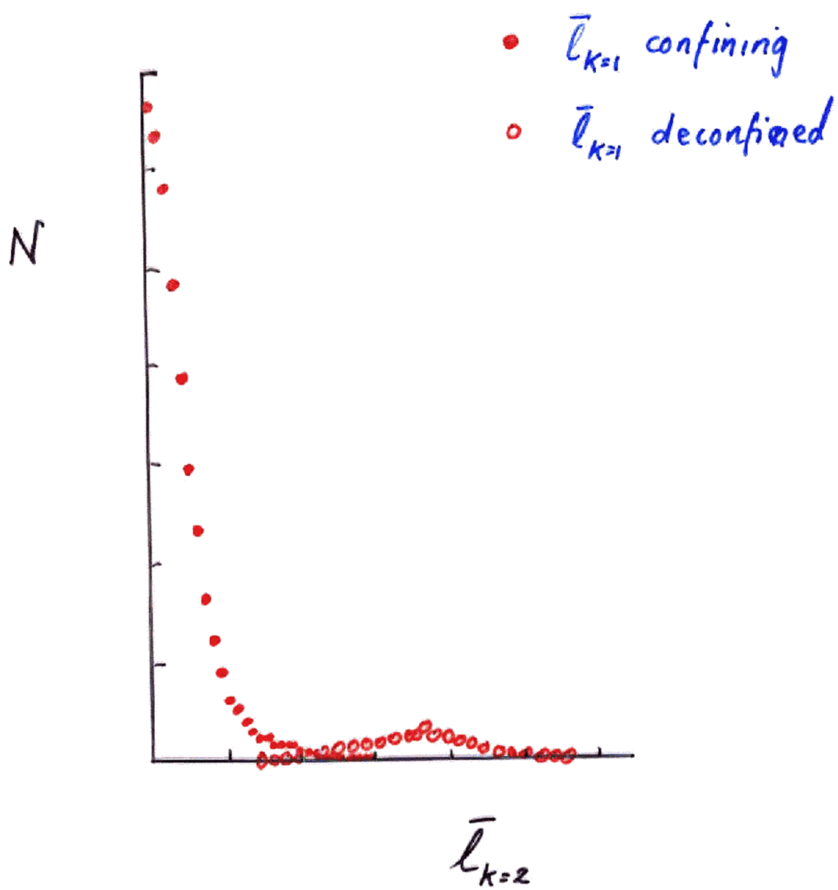
$$T_c^{sc}(k=2) \gtrsim T_c^{sc}(k=1)$$

so multiple deconfinement looks unlikely

SU(4)
$a \approx \frac{1}{5T_c}$
$T \simeq 0.996\, T_c$



$\Rightarrow$



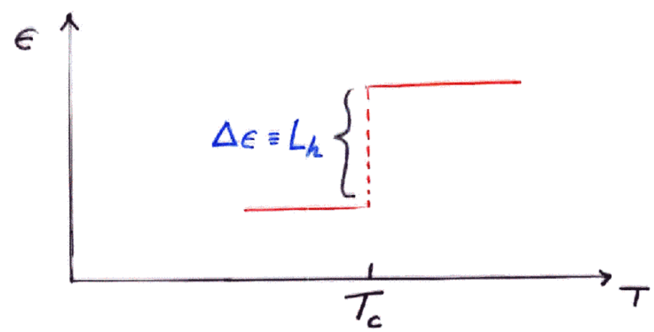
$\Rightarrow$ no multiple transitions

latent heat:

$$Z = \sum_{\text{states}} e^{-E/T} \longleftrightarrow \text{EFPI}\big|_{L_t = \frac{1}{aT}}$$

$\rightarrow$

$$\epsilon = \frac{\bar{E}}{V} = \frac{T^2}{V} \frac{\partial}{\partial T} \ln Z$$



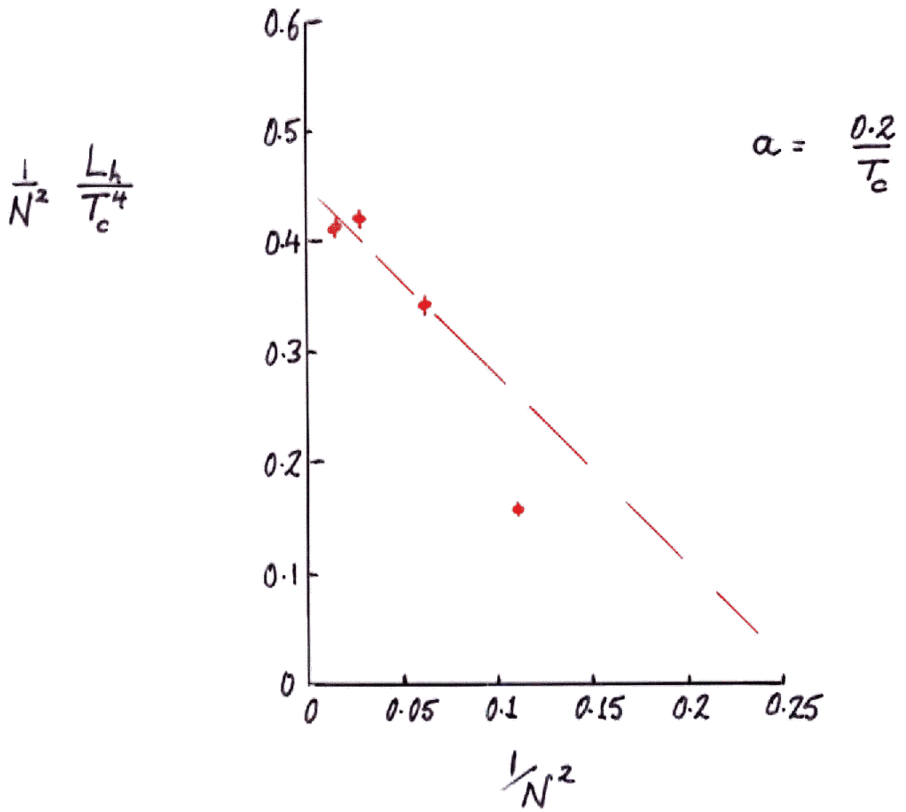
$\longrightarrow$

$$\frac{\partial}{\partial T} \to \frac{\partial}{\partial a} \to \frac{\partial \beta}{\partial a} \frac{\partial}{\partial \beta}$$

$$\frac{\Delta \epsilon}{T_c^4} = - L_t^4 \underbrace{a \frac{\partial \beta}{\partial a}\Big|_{T=T_c}} 6 \Delta \langle u_p \rangle$$

interpolate $a\sqrt{\sigma}(\beta)$

$$\rightarrow \frac{1}{a\sqrt{\sigma}} \frac{\partial (a\sqrt{\sigma})}{\partial \beta} = \frac{1}{a} \frac{\partial a}{\partial \beta}$$



$\Longrightarrow$

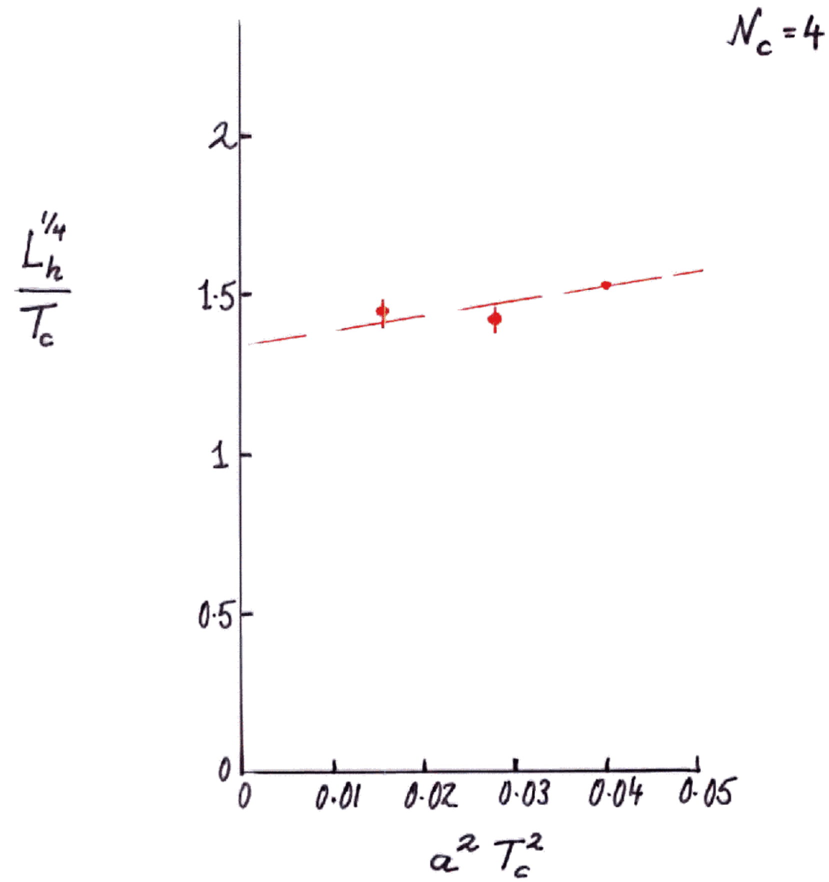
1'st order $\forall N_c \geq 3$

SU(3) is "weakly" first order

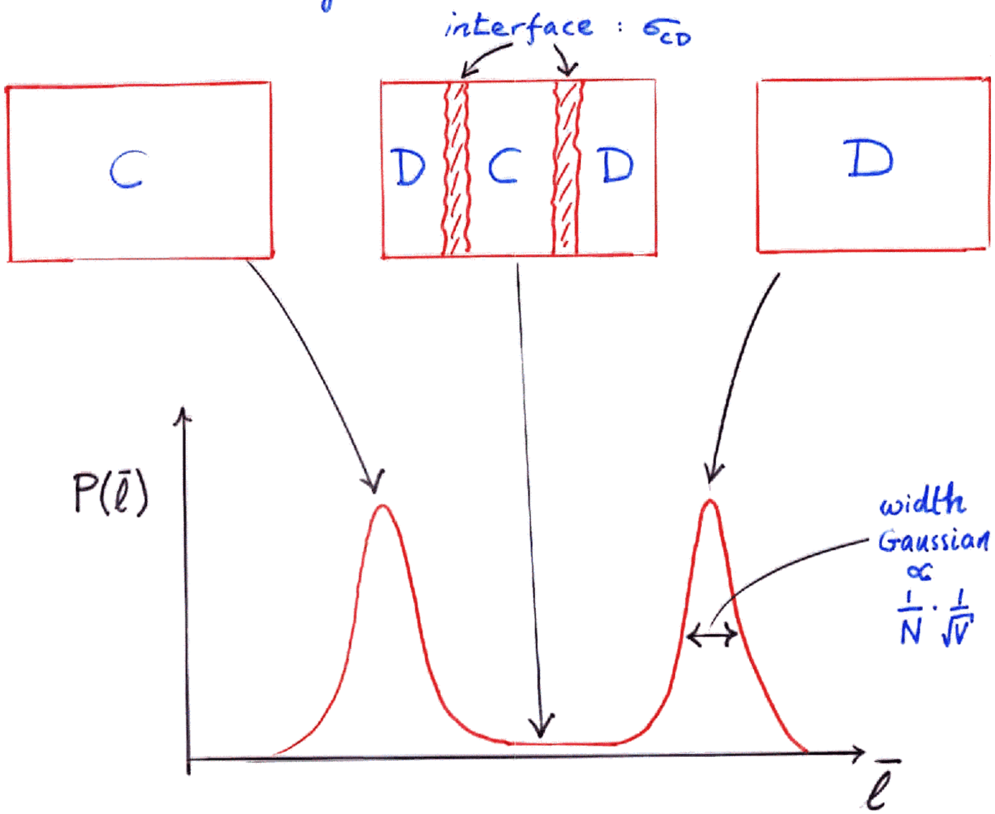
$L_h \propto N_c^2$ as $N_c \to \infty$

$\searrow$ finite-V smearing of $T_c \longrightarrow 0$

... continuum limit



tunnelling ↔ interface tension



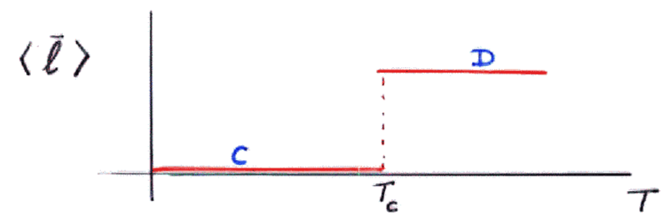
$$\Rightarrow \quad \frac{P_{min}}{P_{max}} \propto e^{-2\sigma_{CD}\, area/T}$$

$$\propto e^{-2a^3\sigma_{CD}\, l_s^2 l_t}$$

$\Rightarrow$ $L_h \propto N^2$

two neat possibilities :

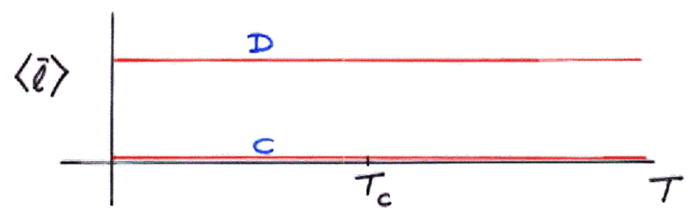
- $\sigma_{CD} \propto N$



for any V, however small or large

~EK reduction ?

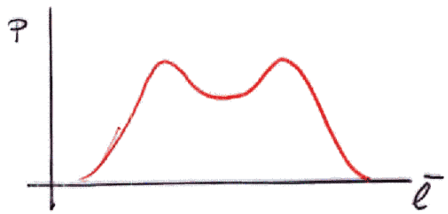
- $\sigma_{CD} \propto N^2$



"infinite hysteresis"

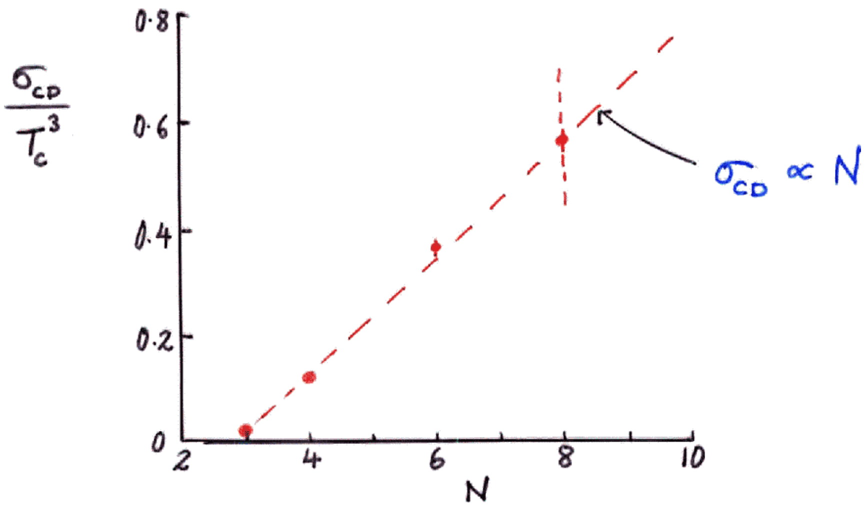
Hagedorn will intervene at some $T > T_c$?

in practice:



$\Rightarrow$ need finite-V extrapolation

preliminary



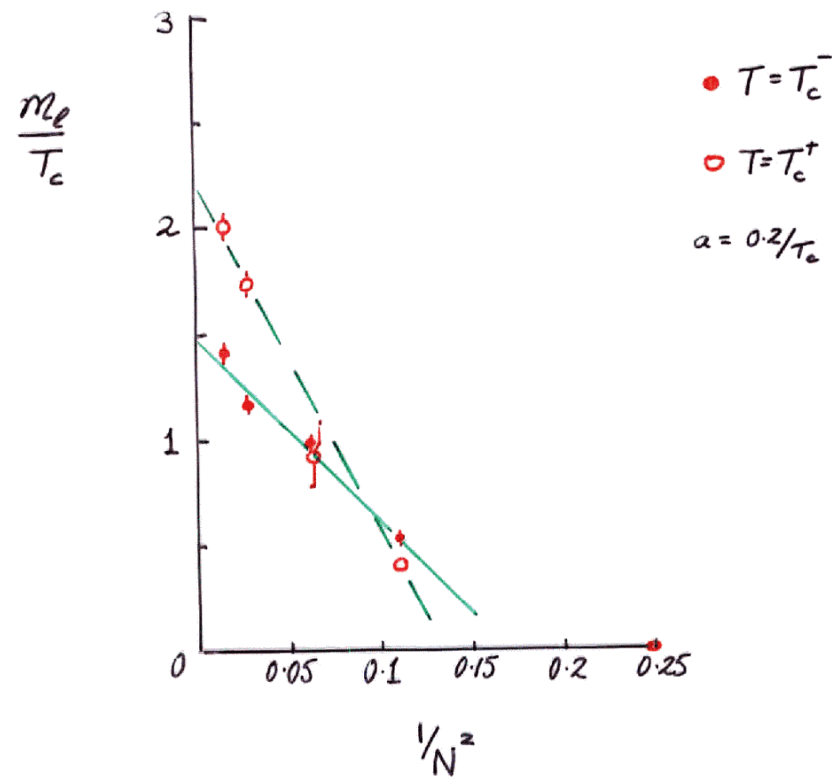
(theoretically I prefer $\propto N^2$)

lightest mass coupling to Polyakov loop at $T = T_c$:

confined phase : T_c^-

deconfined phase : T_c^+

$\xi = \frac{1}{m_\ell} \downarrow$ as $N \uparrow$ $\Rightarrow$ more strongly 1st order as $N \uparrow$



at high T

$$P\, e^{i\int dx_0 A_0} \sim 1 + c A_0^2 + \cdots$$

couples to 2 electric screened gluons

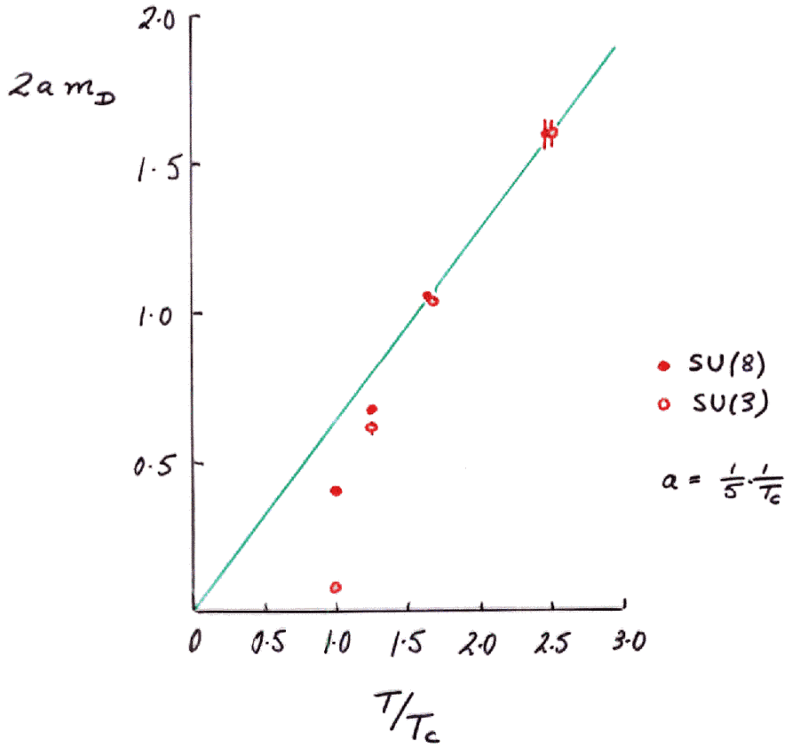
$\Longrightarrow$ lightest mass coupling to $\ell - \langle \ell \rangle$

is $am_\ell = 2am_D$

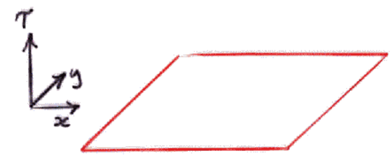
and at $T \to \infty$

$$m_D^2 = c\, g^2(T) \cdot T^2$$

↑ lowest order ↖ dimensions



'spatial' string tensions : ← ~ $e^{-\sigma \cdot \text{area}}$



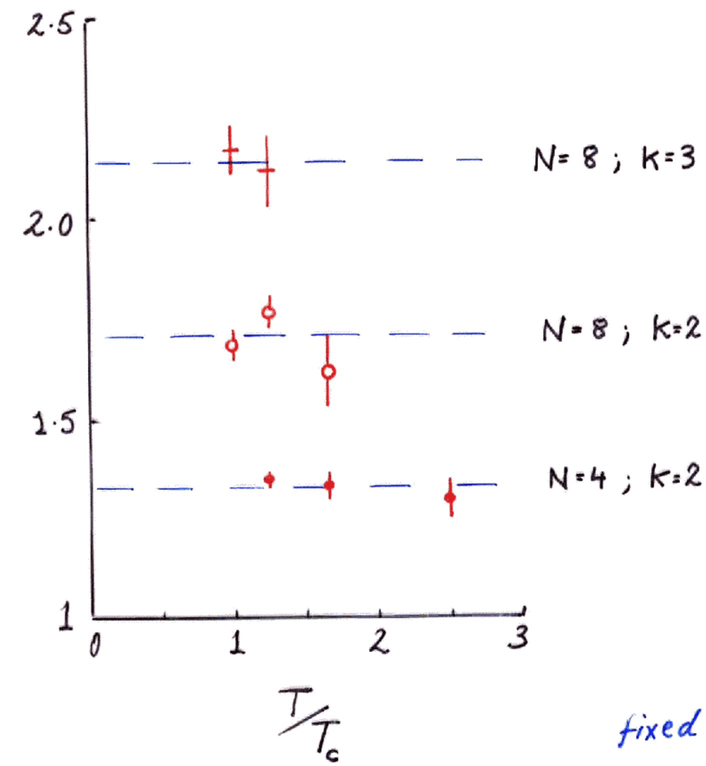
$\sim e^{-\sigma \cdot \text{area}}$

$\Longrightarrow$

---- Casimir scaling $\propto Tr_{\mathcal{R}}\, T^a T^a$

$\to \dfrac{k(N-k)}{N-1}$

$\dfrac{m_\ell(k)}{m_\ell(1)}$

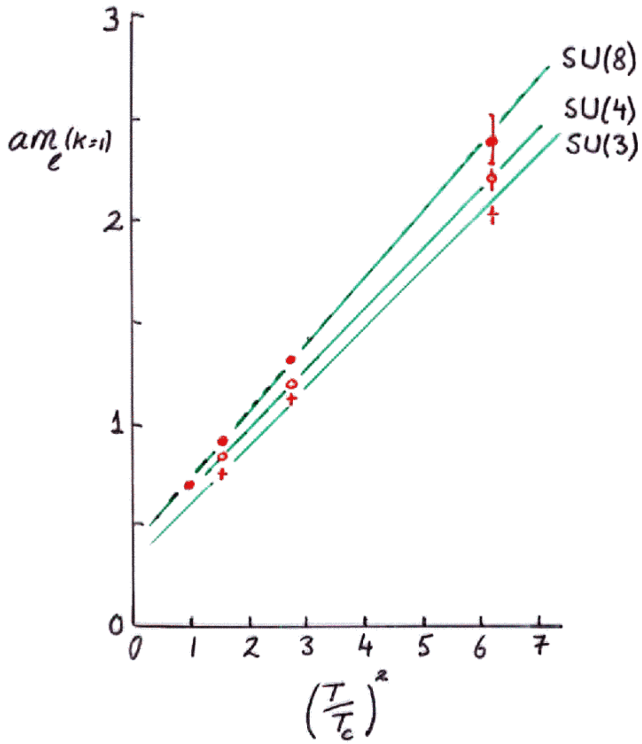


fixed $a \simeq \frac{1}{5}\cdot\frac{1}{T_c}$

at high T, only scale

$\Longrightarrow$

$\sigma \propto T^2$



$\Longrightarrow$ early onset asymptotic behaviour

instanton density

$\rho \to 0$ any T :
$$D(\rho)\,d\rho \propto \frac{d\rho}{\rho^5} e^{-\frac{8\pi^2}{g^2(\rho)}}$$
$$\propto \rho^{\frac{11N}{3}-5}$$
$$\xrightarrow[\rho\to 0]{} 0 \quad \text{for large } N$$

$T \gg T_c$:
$$m_{el}^2(T) \propto \lambda(T)\,T^2$$

decoheres E-fields on scale $\rho \sim \frac{1}{T}$

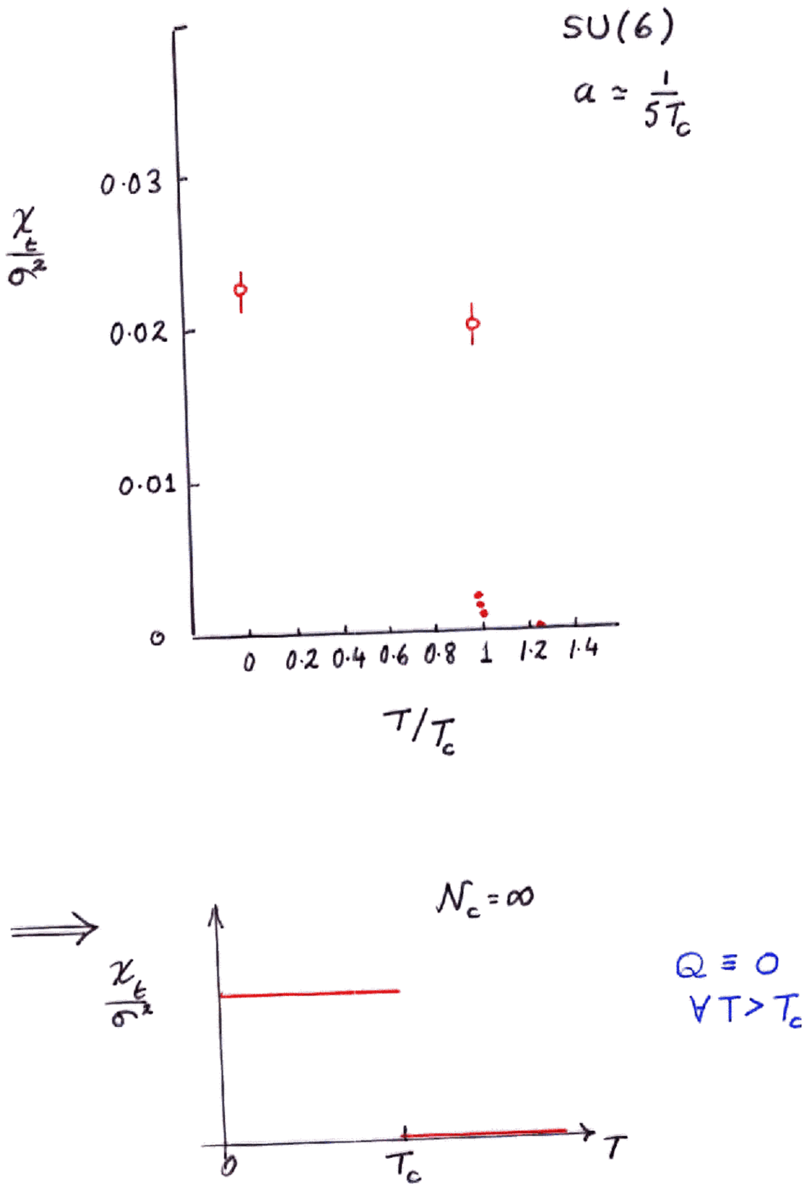
$\underset{GPY}{\Longrightarrow}$
$$D(\rho,T) = D(\rho,0)\, e^{-\frac{2N}{3}(\pi\rho T)^2 + \cdots}$$
$$\longrightarrow 0 \quad \text{for large } T$$
so PT valid

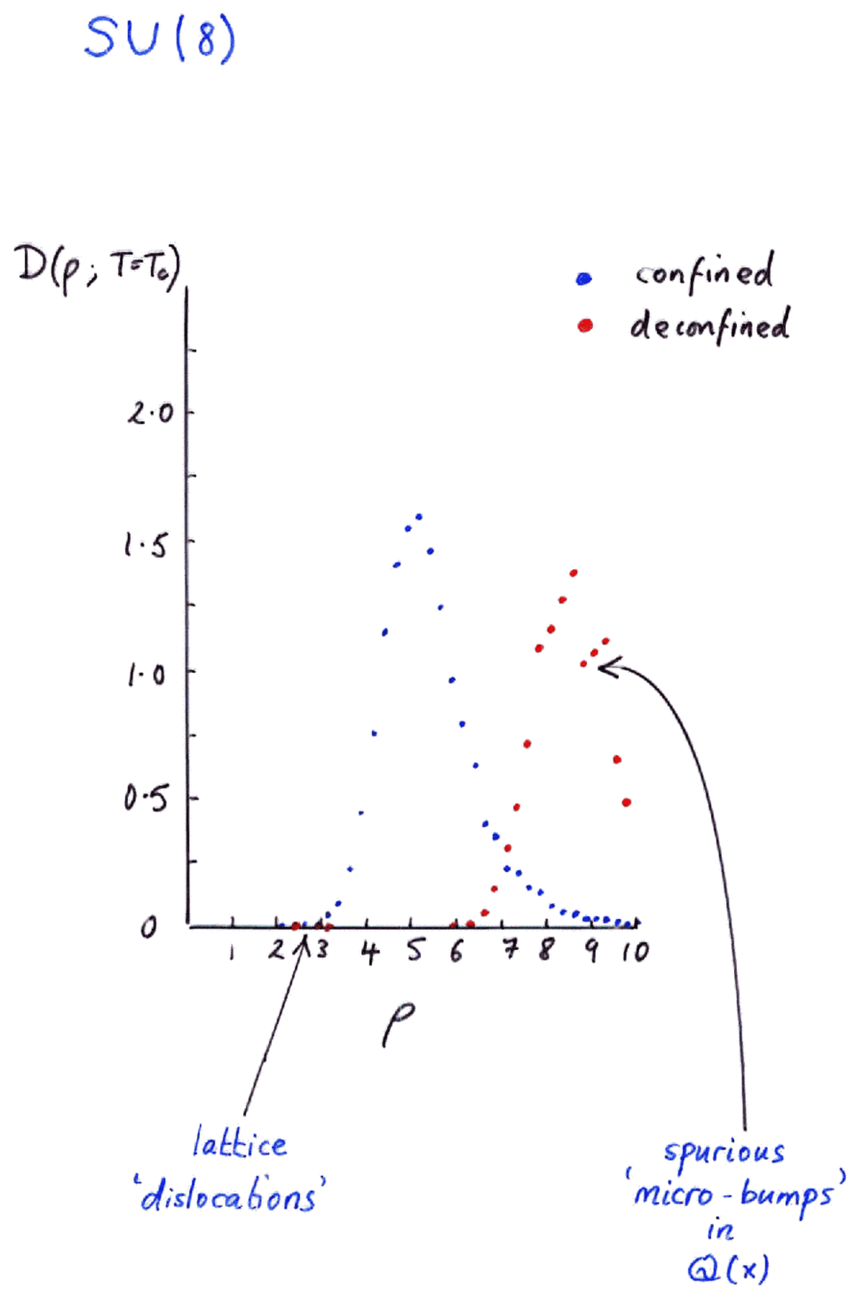
do these suppressions overlap as $N \to \infty$

so that

$$D(\rho,T) = 0 \qquad \forall\rho, \forall T, \; N=\infty$$

in the deconfined phase


SU(6)
a ≃ 1/5T_c
0·03
0·02
0·01
0
χ_t/σ²
0 0·2 0·4 0·6 0·8 1 1·2 1·4
T/T_c
⟹
N_c = ∞
χ_t/σ²
0
T_c
T
Q ≡ 0
∀ T > T_c


SU(8)
D(ρ; T=T_c)
confined
deconfined
2·0
1·5
1·0
0·5
0
1 2 3 4 5 6 7 8 9 10
ρ
lattice
'dislocations'
spurious
'micro-bumps'
in
Q(x)

- disconnected pieces dominate
  e.g. $\langle \frac{1}{N}\mathrm{Tr}F^2(x)\cdot\frac{1}{N}\mathrm{Tr}F^2(0)\rangle$

i.e

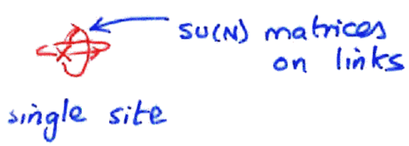
$\Longrightarrow$ factorisation

$$\langle \mathrm{Tr}G\,.\,\mathrm{Tr}G'\,.\,\mathrm{Tr}G''\ldots\rangle \underset{N=\infty}{=} \langle\mathrm{Tr}G\rangle\langle\mathrm{Tr}G'\rangle\langle\mathrm{Tr}G''\rangle\ldots$$

$\Longrightarrow$ $\int [dA]\, \mathrm{Tr}G\,\mathrm{Tr}G'\ldots\, e^{\beta S[A]}$
is dominated by
a single Master Field (Orbit) written
as $N\to\infty$

trans. inv. ?

- space-time reduction Eguchi-Kawai



: planar limit
on $L^4 = N^2$ lattice

$N\uparrow \longrightarrow$ transition sharper on smaller volumes

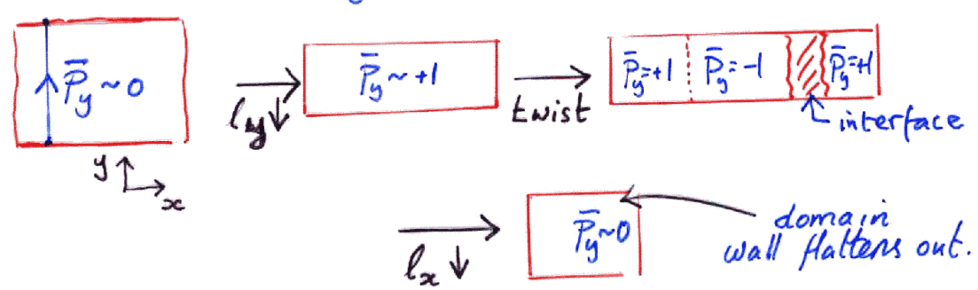
e.g. at $aT_c = \frac{1}{l_t} = \frac{1}{5}$
we use:

$32^3 5$	-	$64^3 5$	SU(3)
$14^3 5$	-	$20^3 5$	SU(4)
$10^3 5$	-	$14^3 5$	SU(6)
$6^3 5$	-	$10^3 5$	SU(8)
⋮			

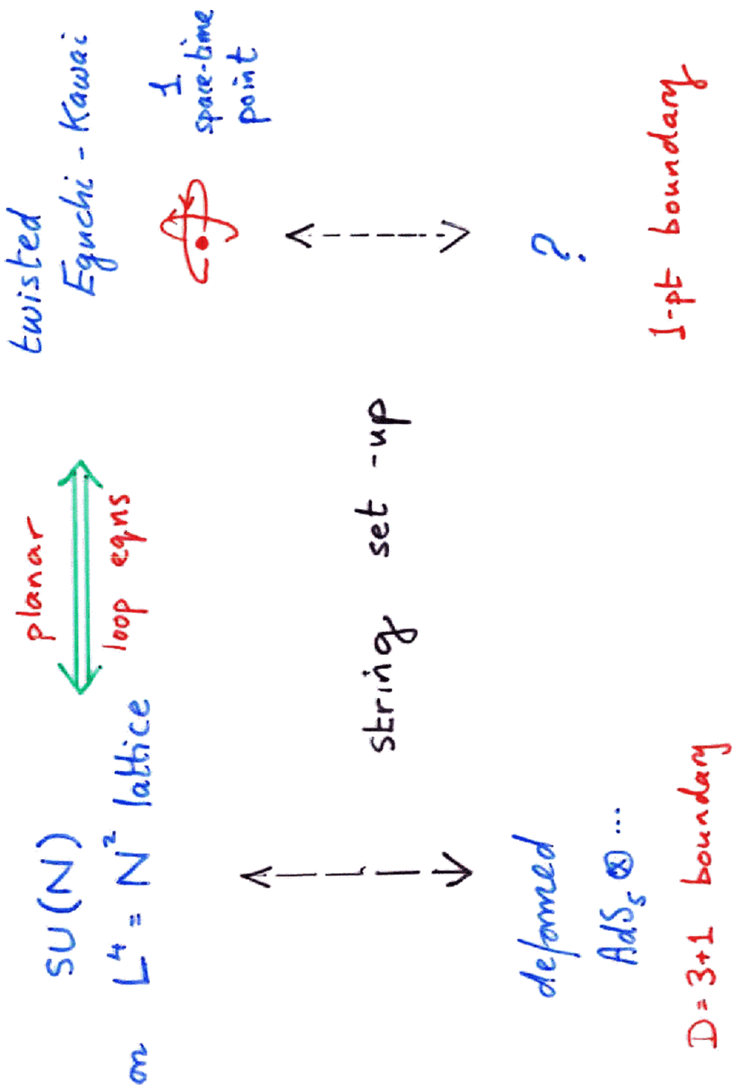
$\Longrightarrow$ $l_s^3\, l_t$ with $l_s \to 0$ as $N\to\infty$
~ Eguchi-Kawai reduction ?

BUT: when $l_s \lesssim \frac{1}{aT_c}$ 'spatial' Wilson loops deconfine $\to$ centre symmetry breaking

$\Longrightarrow$ remedy with space-space twist:



$\Longrightarrow$ dynamical route to twisted Eguchi-Kawai ?


SU(N)
on $L^4 = N^2$ lattice
planar
loop eqns
twisted
Eguchi - Kawai
1 space-time point
string set-up
deformed
$AdS_5 \otimes \dots$
D=3+1 boundary
?
1-pt boundary