Boundary Conditions in AdS
Life Without a Higgs

Csáki, Grojean, Murayama, Pilo, JT hep-ph/0305237

Csáki, Grojean, Pilo, JT hep-ph/0308038

Csáki, Grojean, Hubisz, Shirman, JT hep-ph/0310355

Cacciapaglia, Csáki, Grojean, JT hep-ph/0401160,
hep-ph/0409126

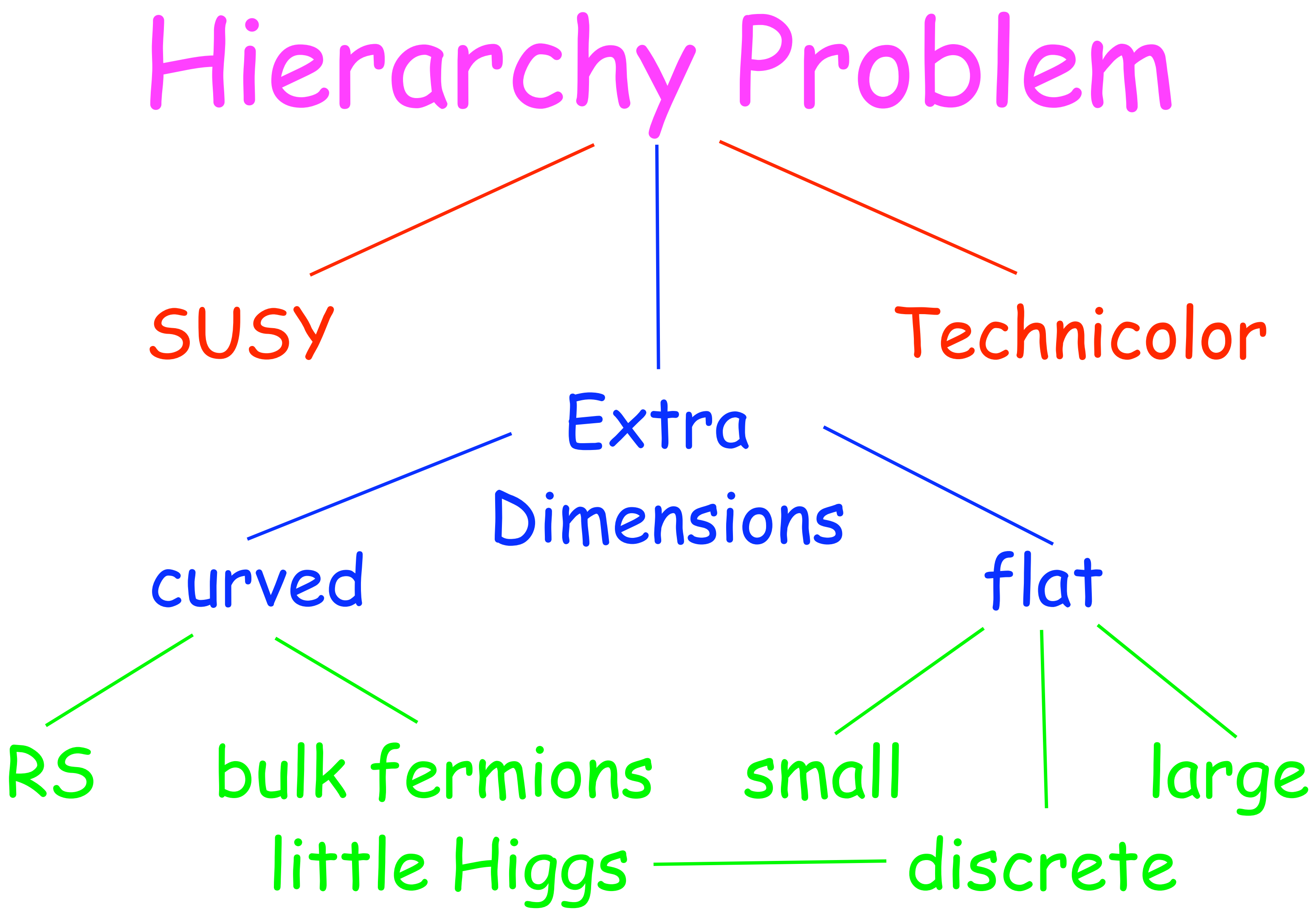
Outline

- Motivation
- Gauge Theory on an Slice of AdS
- Unitarity of WW Scattering
- Life without a Higgs
- Models
- Conclusions

Hierarchy Problem

SUSY

Technicolor


Hierarchy Problem
SUSY
Technicolor
Extra Dimensions
curved
flat
RS
bulk fermions
small
large
little Higgs
discrete

Klebanov-Strassler

Klebanov, Strassler hep-th/0007191

with a single wrapped D5 brane

at the bottom of the duality cascade:

	$SU(2)$	$SU(2)_L$	$SU(2)_R$	$U(1)$
A	$\square$	$\square$	$\mathbf{1}$	1
B	$\overline{\square}$	$\mathbf{1}$	$\square$	-1

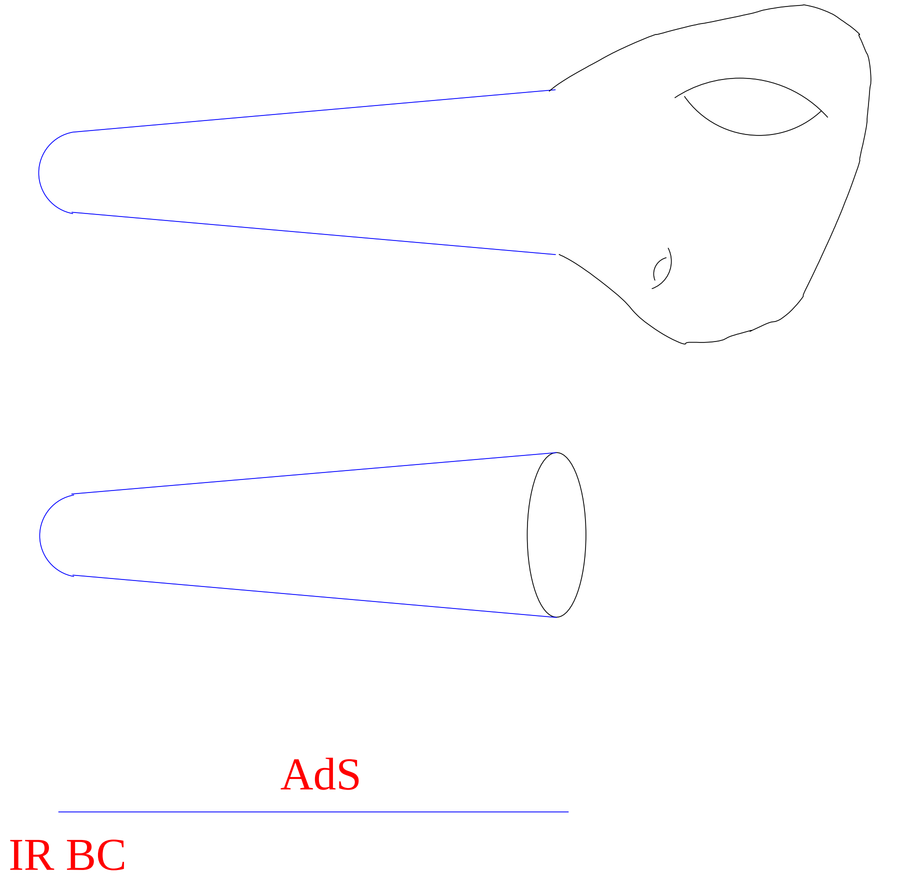
on part of moduli space

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)$$

electroweak symmetry breaking!

Luty, Grant, JT hep-ph/0006224

Klebanov-Strassler Cartoon



Can we break Electroweak Symmetry with AdS Boundary Conditions?

- how do we reduce the rank of the gauge group?
- is WW scattering unitary?
- why does $M_W^2 = \cos\theta_W M_Z^2$?
- how do we get quark and lepton masses??
- precision electroweak measurements???

Ground Rules

extra dimensions
branes
AdS/CFT correspondence

low-scale SUSY

Ground Rules

extra dimensions
branes
AdS/CFT correspondence

low-scale SUSY
strong coupling

Ground Rules

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anthropic incantations

Gauge Theory on an Slice of AdS

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$R \leq z \leq R'$$

Mixed Boundary Conditions

$$\partial_z A_\mu(x,z) = -\frac{g_5^2 v^2}{2} A_\mu(x,z)$$

Dirichlet and Neumann are special cases

KK Modes

$$A^a_\mu(x,z) = \sum_n \psi^a_n(z) a_\mu(x) e^{ip_n x}, \text{ where } p_n^2 = M_n^2$$

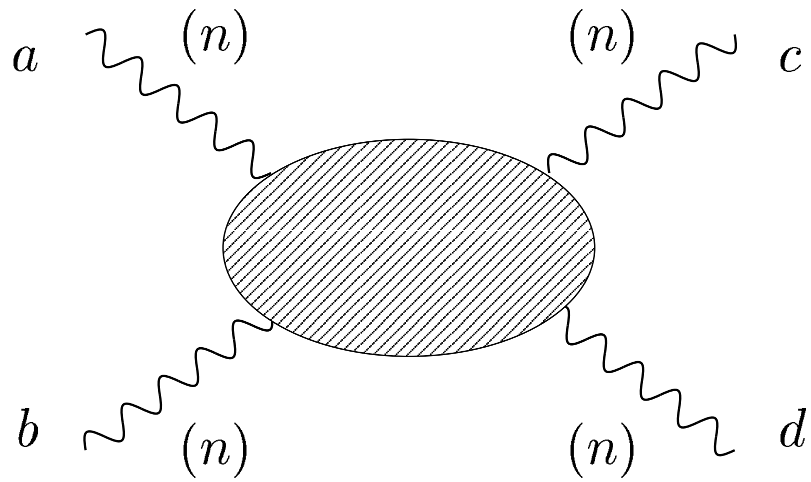
$$\left(\partial_z^2 - \tfrac{1}{z}\partial_z + M_n^2\right)\psi^a_n(z) = 0, \quad \psi^{a\,\prime}_n = V^a_b \psi^b$$

$$g_{cubic} \quad \longrightarrow \quad g^{abc}_{mnk} = g_5 \int dz \left(\tfrac{R}{z}\right) \psi^a_m(y)\psi^b_n(y)\psi^c_k(y)$$

$$g^2_{quartic} \quad \longrightarrow \quad g^{2\,abcd}_{mnkl} = g_5^2 \int dz \left(\tfrac{R}{z}\right) \psi^a_m(y)\psi^b_n(y)\psi^c_k(y)\psi^d_l(y)$$

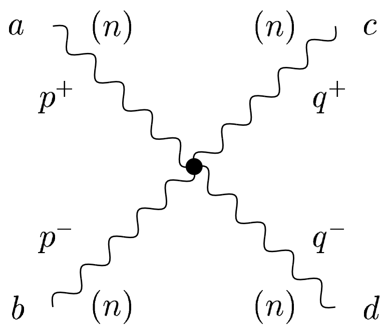
Scattering Amplitude

incoming: $p_\mu = (E, 0, 0, \pm\sqrt{E^2 - M_n^2})$
outgoing: $k_\mu = (E, \pm\sqrt{E^2 - M_n^2}\sin\theta, 0, \pm\sqrt{E^2 - M_n^2}\cos\theta)$
longitudinal polarization: $\epsilon_\mu = (\frac{|\vec{p}|}{M}, \frac{E}{M}\frac{\vec{p}}{|\vec{p}|})$

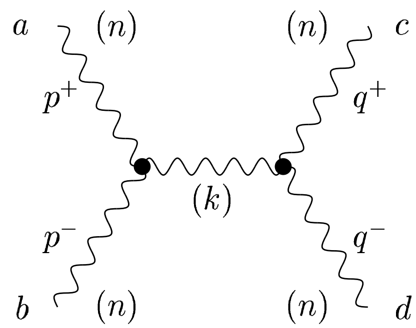


$$\mathcal{A} = A^{(4)}\frac{E^4}{M_n^4} + A^{(2)}\frac{E^2}{M_n^2} + A^{(0)} + \ldots$$

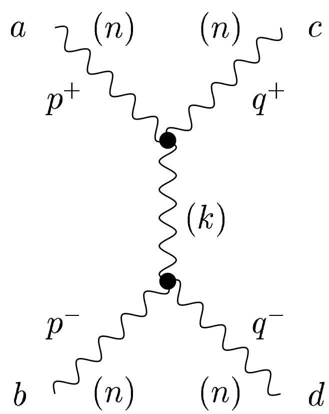
WW Scattering via KK bosons



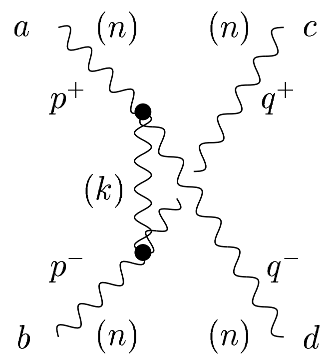
contact interaction



s channel exchange



t channel exchange



u channel exchange

Cancellation

E^4 term: $g_{nnnn}^2 - \sum_k g_{nnk}^2$

$$\int dy\, \psi_n^4(y) = \sum_k dy \int dz\ \psi_n^2(y)\psi_n^2(z)\psi_k(y)\psi_k(z)$$

completeness of hermitian operator:

$$\sum_k \psi_k(y)\psi_k(z) = \delta(y-z)$$

E^2 term: $4g_{nnnn}^2 M_n^2 - 3\sum_k g_{nnk}^2 M_k^2$

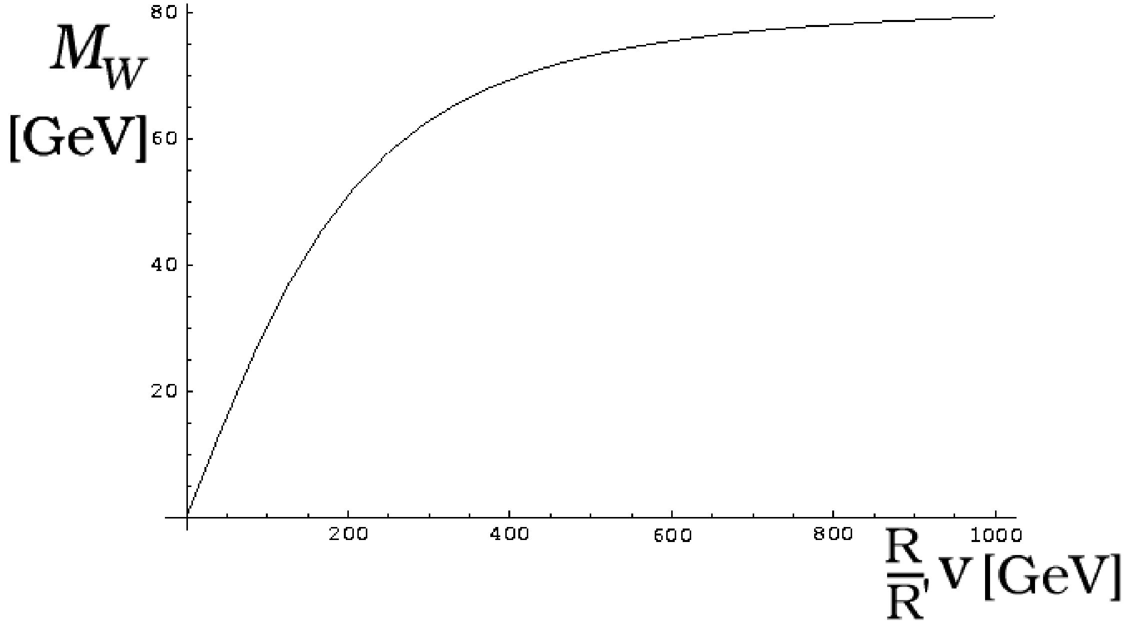
$$\begin{aligned}\sum_k\ M_k^2\ \left(\int dy\, \psi_n^2(y)\psi_k(y)\right)^2 \quad = \quad & \tfrac{4}{3} M_n^2 \int dy\, \psi_n^4(y) - \tfrac{2}{3}[\psi_n^3\psi_n'] \\ & +2\sum_k [\psi_n\psi_n'\psi_k] \int dy\, \psi_n^2(y)\psi_k(y) \\ & -\sum_k [\psi_n^2\psi_k'] \int dy\, \psi_n^2(y)\psi_k(y)\end{aligned}$$

for Dirichlet or Neumann BC's the E^2 terms cancel

Finite VEV

$$\partial_z \psi(z) = -\frac{g_5^2 v^2}{2} \psi(z)$$

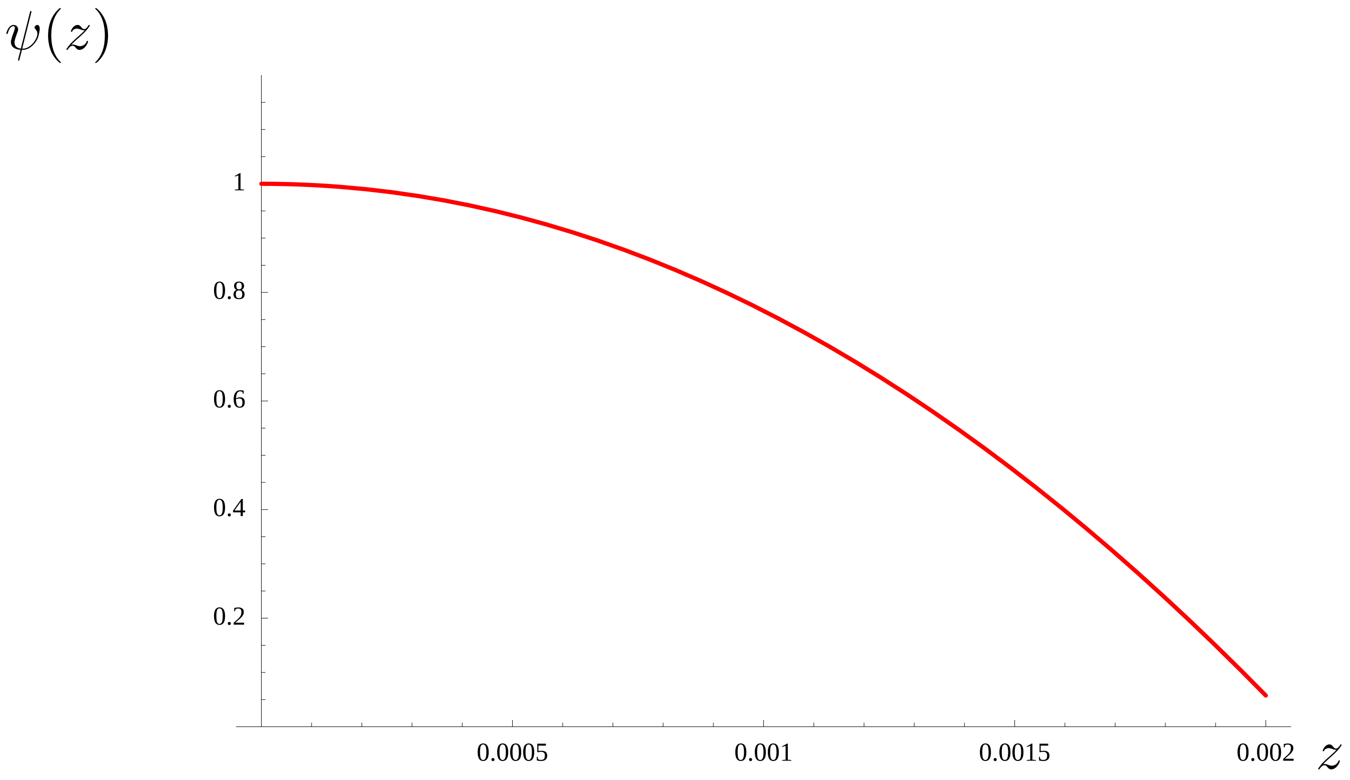
for small v: $M_W^2 = \frac{g^2 v^2}{4} \frac{R^2}{R'^2}$



for $R' = 2 \cdot 10^{-3}$ GeV^{-1}, $R = 10^{-19}$ GeV^{-1}

Decoupling the Higgs

for $v = 1$ TeV



Higgs decouples from scattering as $v \to \infty$

Towards a Realistic Model

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

BC's:

at $z = R$:
$SU(2)_R \times U(1)_{B-L}$
$\downarrow$
$U(1)_Y$

$$\begin{cases} \partial_z A_\mu^{L\,a} = 0, \; A_\mu^{R\,1,2} = 0 \\ \partial_z \left(g_5 B_\mu + \tilde{g}_5 A_\mu^{R\,3}\right) = 0, \; \tilde{g}_5 B_\mu - g_5 A_\mu^{R\,3} = 0 \\ A_5^{L\,a} = 0, \; A_5^{R\,a} = 0, \; B_5 = 0 \end{cases}$$

at $z = R'$:
$SU(2)_L \times SU(2)_R$
$\downarrow$
$SU(2)$

$$\begin{cases} \partial_z \left(A_\mu^{L\,a} + A_\mu^{R\,a}\right) = 0, \; \partial_z B_\mu = 0 \\ A_\mu^{L\,a} - A_\mu^{R\,a} = 0, \\ A_5^{+\,a} = 0, \; \partial_z A_5^{-\,a} = 0, \; B_5 = 0 \end{cases}$$

at $z = R'$, $F_{\nu 5}^{La} + F_{\nu 5}^{Ra} = 0$

KK Modes

$$
\begin{aligned}
B_\mu(x,z) &= g_5\, a_0 A_\mu(x) + \textstyle\sum_{k=1}^\infty \psi_k^{(B)}(z)\, Z_\mu^{(k)}(x)\,, \\
A_\mu^{L\,3}(x,z) &= \tilde{g}_5\, a_0 A_\mu(x) + \textstyle\sum_{k=1}^\infty \psi_k^{(L3)}(z)\, Z_\mu^{(k)}(x)\,, \\
A_\mu^{R\,3}(x,z) &= \tilde{g}_5\, a_0 A_\mu(x) + \textstyle\sum_{k=1}^\infty \psi_k^{(R3)}(z)\, Z_\mu^{(k)}(x)\,, \\
A_\mu^{L\,\pm}(x,z) &= \textstyle\sum_{k=1}^\infty \psi_k^{(L\pm)}(z)\, W_\mu^{(k)\,\pm}(x)\,, \\
A_\mu^{R\,\pm}(x,z) &= \textstyle\sum_{k=1}^\infty \psi_k^{(R\pm)}(z)\, W_\mu^{(k)\,\pm}(x)\,.
\end{aligned}
$$

$$
\psi_k^{(A)}(z) = z\left(a_k^{(A)} J_1(M_k z) + b_k^{(A)} Y_1(M_k z)\right)
$$

$$
M_W^2 = \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}
$$

$$
M_Z^2 = \frac{g_5^2 + 2\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2}\,\frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}
$$

SM Gauge Couplings

$$\tilde{g}_5 g_5 a_0 Q\gamma_\mu + g_5 \psi_1^{L\pm}(R) T_\pm W_\mu^\pm + \left(g_5 \psi_1^{(L3)}(R) T_3 + \tilde{g}_5 \psi_1^{(B)}(R) \frac{Y}{2}\right) Z_\mu$$

$$\begin{aligned} g^2 &= \frac{g_5^2 \psi_1^{(L\pm)}(R)^2}{\int_R^{R'} dz \frac{R}{z} (\psi_1^{(L\pm)}(R)^2 + \psi_1^{(R\pm)}(R)^2)} = \frac{g_5^2}{R\log(R'/R)} \\ e^2 &= \frac{\tilde{g}_5^2 g_5^2 a_0^2}{\int_R^{R'} dz \left(\frac{R}{z}\right)(2\tilde{g}_5^2 + g_5^2) a_0^2} = \frac{g_5^2 \tilde{g}_5^2}{(g_5^2 + 2\tilde{g}_5^2) R\log(R'/R)} \end{aligned}$$

the Z couplings are also reproduced:

$$\begin{aligned} g^2 \cos\theta_W^2 &= \frac{g_5^2 \psi^{(L3)}(R)^2}{\int_R^{R'} dz \left(\frac{R}{z}\right)(\psi^{(L3)}(R)^2 + \psi^{(R3)}(R)^2 + \psi^{(B)}(R)^2)} \\ &= \frac{g_5^2}{R\log(R'/R)} \frac{g_5^2 + \tilde{g}_5^2}{g_5^2 + 2\tilde{g}_5^2}, \\ g'^2 &= \frac{g_5^2 \tilde{g}_5^2}{(g_5^2 + \tilde{g}_5^2) R\log(R'/R)}, \\ \sin\theta_W &= \frac{\tilde{g}_5}{\sqrt{g_5^2 + 2\tilde{g}_5^2}} = \frac{g'}{\sqrt{g^2 + g'^2}} \end{aligned}$$

Custodial Symmetry

$$\cos\theta_W^2 \quad = \quad \frac{g_5^2 + \tilde{g}_5^2}{g_5^2 + 2\tilde{g}_5^2},$$

$$M_W^2 = \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}$$

$$M_Z^2 = \frac{g_5^2 + 2\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2} \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}$$

Hence $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$

Anthropic Misunderstandings

The universe must be such that life can be advanced enough to contemplate the universe and be primitive enough to contemplate the anthropic principle.

what varies between different universes?

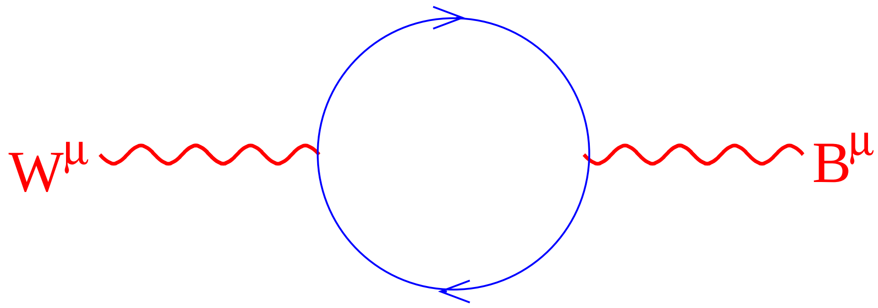
Anthropic Chestnut: The Higgs VEV must be in a narrow range for complex chemistry to arise

. . .

unless we make other modifications in the theory,

with a warped extra dimension an infinite range is allowed!

Precision Electroweak



$$\mathcal{L}_{\rm eff} = -\frac{S}{16\pi} F_L^{3\mu\nu} F_{\mu\nu}$$

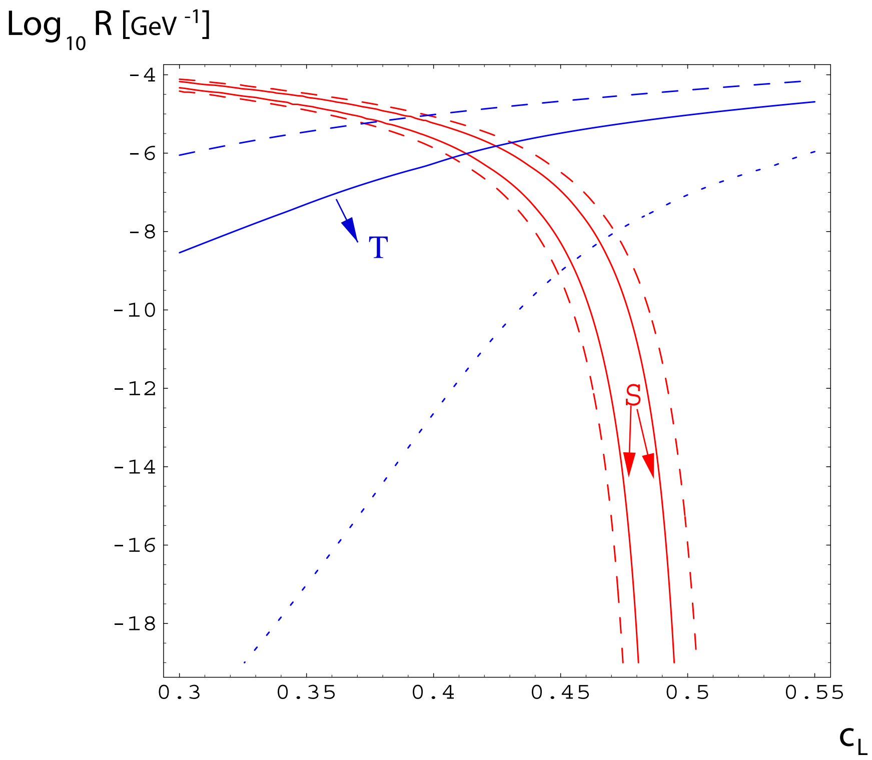
QCD, technicolor $\rightarrow S = \mathcal{O}(1)$

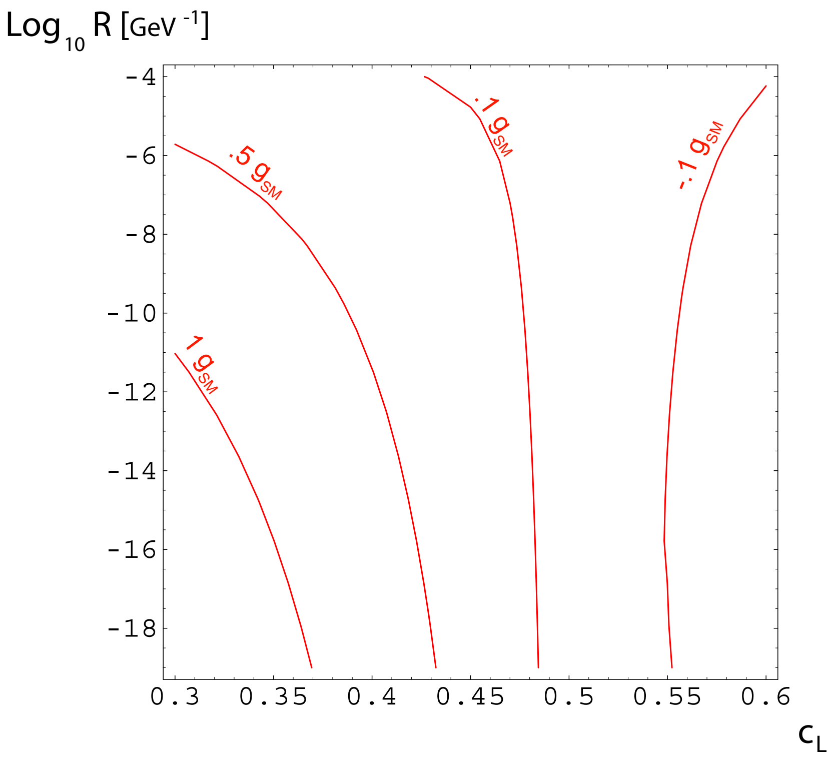
RS with TeV fermions $\rightarrow S = \mathcal{O}(-30)$

Golden, Randall; Holdom, JT; Peskin, Takeuchi

Csaki, Erlich JT, hep-ph/0203034

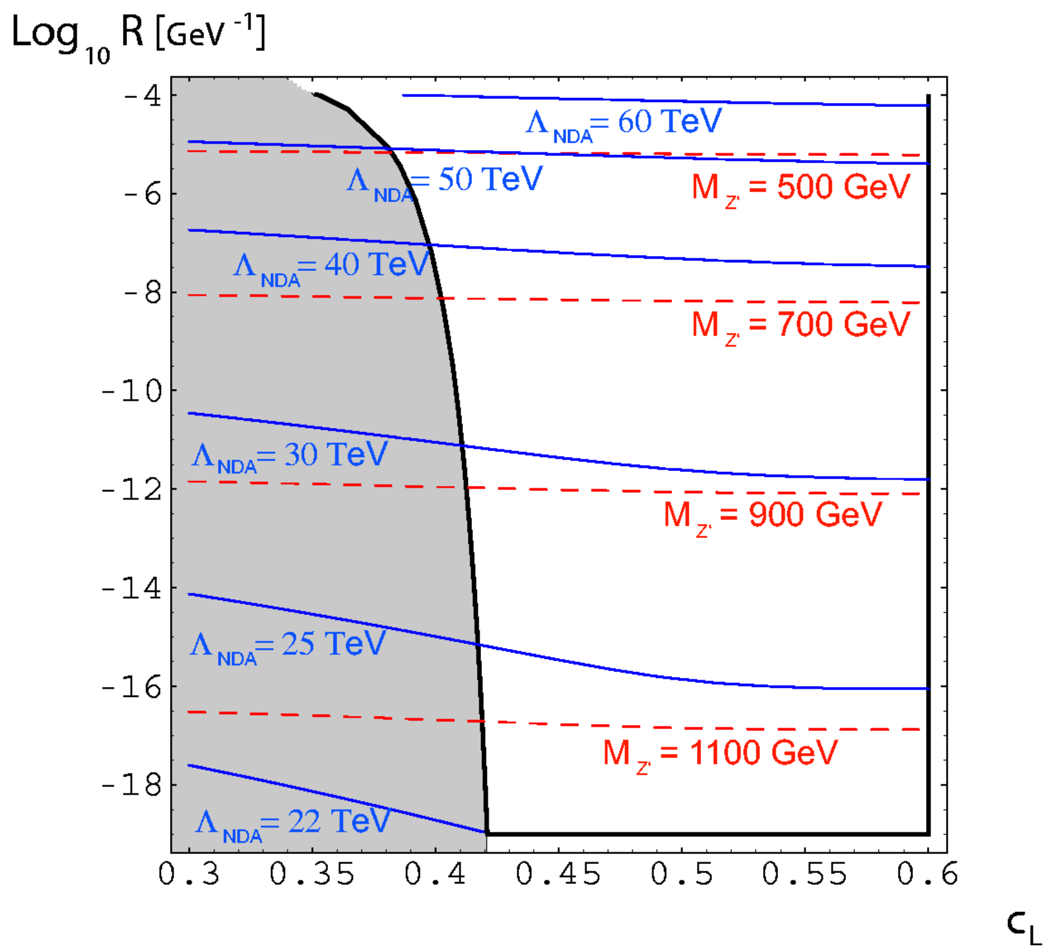
Light Resonances, Small S





Perturbative Unitarity

$$\Lambda_{\rm NDA} \sim \frac{24\pi^3}{g_5^2}\frac{R}{R'} \sim \frac{12\pi^4 M_W^2}{g^2 M_{W'}} = \mathcal{O}\left(\frac{12\pi^4 R'}{\log(R'/R)}\right)$$



Fermion Boundary Conditions

$$\chi_{L,R} = z^{\frac{5}{2}} \left[A_{L,R} J_{\frac{1}{2}+c_{L,R}}(m_n z) + B_{L,R} J_{-\frac{1}{2}-c_{L,R}}(m_n z) \right]$$

$$\psi_{L,R} = z^{\frac{5}{2}} \left[A_{L,R} J_{\frac{1}{2}-c_{L,R}}(m_n z) + B_{L,R} J_{-\frac{1}{2}+c_{L,R}}(m_n z) \right]$$

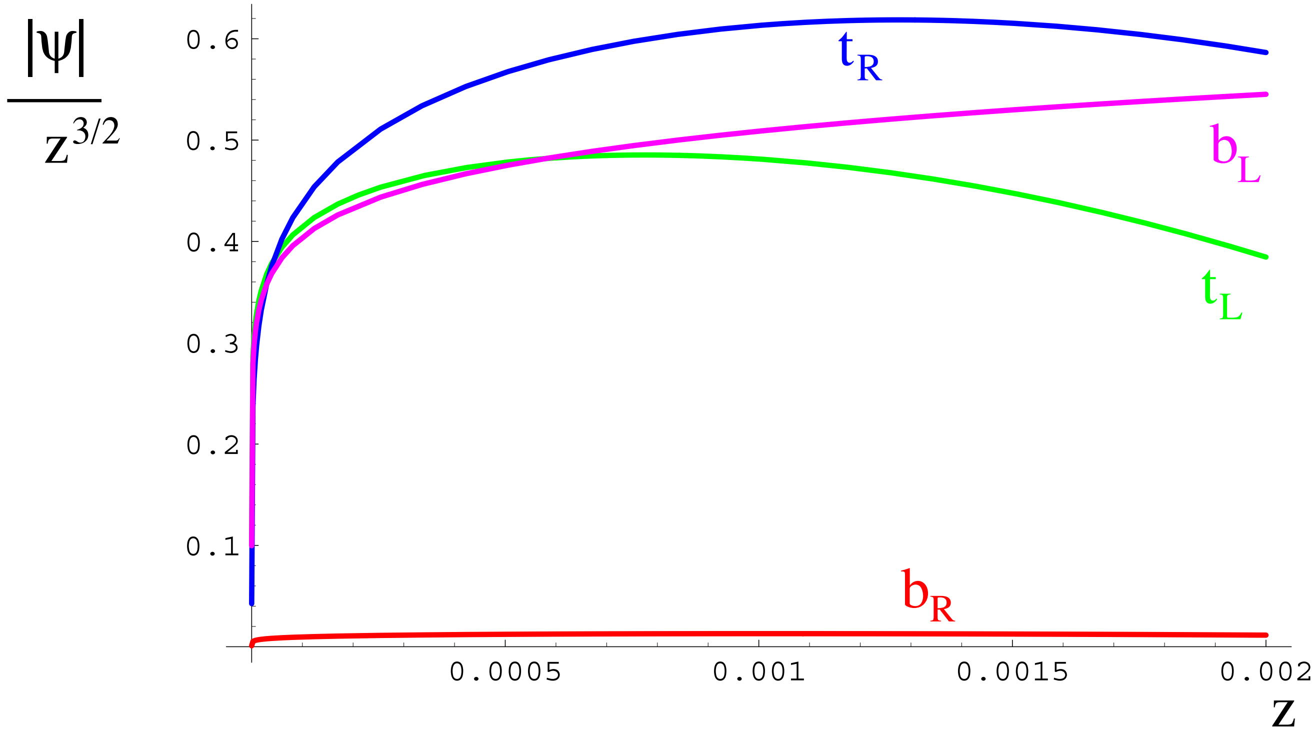
$$S_{TeV} = \int d^4x \left(\frac{R}{z}\right)^4 M_D R' \left[\psi_R \chi_L + \bar{\chi}_L \bar{\psi}_R + \psi_L \chi_R + \bar{\chi}_R \bar{\psi}_L \right]$$

$$S_{Pl} = \int d^4x - i\bar{\xi}\bar{\sigma}^\mu \partial_\mu \xi - i\eta\sigma^\mu \partial_\mu \bar{\eta} + f\left(\eta\xi + \bar{\xi}\bar{\eta}\right) + M\sqrt{R}\left(\psi_R \xi + \bar{\xi}\bar{\psi}_R\right)$$

Taking $c_L \approx 0.4$, $c_R \approx -\frac{1}{3}$, $M_D = 900$, $M_t = 0$, $M_b = 10^{18}$ and $f = 3 \cdot 10^{13}$ GeV gives

$$m_t \approx 170\,\text{GeV}$$
$$m_b \approx 4.5\,\text{GeV}$$

Fermion Wavefunctions



Delocalized Higgs?

AdS/CFT Correspondence:

a localized Higgs $\rightarrow \mathcal{O}$ with $d[\mathcal{O}] = \infty$

more realistic: $d[\mathcal{O}]$ to be finite $\rightarrow$ Higgs profile in bulk, finite VEV

walking technicolor limit $d[\mathcal{O}] = 2$

Two Branes are Better than One?

Two AdS_5 with a shared Planck brane and two TeV branes

Dimopoulos, Kachru, Kaloper Lawrence, Silverstein hep-th/0104239

different R' and R for third generation

third generation couples to a different CFT sector

Conclusions/Questions

- BC's can be used to break electroweak symmetry
- WW scattering can be unitarized without a Higgs
- models with custodial symmetry exist
- oblique corrections can be small
- can m_t vs $Zb\bar{b}$ problem be fixed?
- can we do this analysis for Klebanov-Strassler?