Witten's superconducting strings and other topological defects in high density QCD*

Ariel Zhitnitsky
University of British Columbia
Vancouver

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* *(Based on works done with M.Metlitski, M.Stephanov, D.T.Son.)*

I. Introduction. General remarks.

1. We work in the limit:

a) $\mathbf{d}$ $(dimensionality)$ =3+1
b) $\mathcal{N}$ ($number\ of\ supercharges$)=0
c) N_f $(number\ of\ light\ flavors$) =3

2. It is well-known that there are NO topological solitons in the Standard model.
(like monopoles, domain walls, strings...)

$$SU(3)_c \times SU(2) \times U(1) \times (global) \Rightarrow SU(3)_c \times U(1)_{EM}$$

3. However! In large N_c limit and in dense matter (large chemical potential $\mu \gg \Lambda_{QCD}$) the situation is very different.

4.

Q: What are the common features between these theories ? (large N_c limit vs. large $\mu \gg \Lambda_{QCD}$ limit)

A: In both cases there is a light singlet $U(1)_A$ field (η'-meson).

Large N_c	Large μ (dense matter)
η'-meson is light, $m^2_{\eta'} \sim 1/N_c$ (Witten)	η' is light, $m^2_{\eta'} \sim (\frac{\Lambda_{QCD}}{\mu})^b$ due to the suppressed instantons $n(\rho) \sim e^{-\mu^2 \rho^2 N_f}$
$\theta \rightarrow \theta + 2\pi$ periodicity implies there existence of the η' domain walls	$U(1)_A$ is explicitly broken. η' is a periodic Goldstone field: there is η' domain wall
$U(1)_A$ is spontaneously broken; therefore η' strings with attached η' domain wall must exist	$U(1)_A$ is spontaneously broken; therefore η' strings with attached η' domain wall must exist

II. Color Superconductivity

($\mu\bar{\Psi}\gamma_0\Psi$ - term in the Lagrangian)

1. If there is a channel in which the quark-quark interaction is attractive, than the true ground state of the system will be a complicated coherent state of Cooper pairs like in **BCS** theory (ordinary superconductor).

2. Diquark condensates break color symmetry (CFL phase, $N_c = N_f = 3$):

$$\langle\psi^{ia}_{L\alpha}\psi^{jb}_{L\beta}\rangle^* \sim \epsilon_{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{abc}X^{\gamma}_{c},$$
$$\langle\psi^{ia}_{R\alpha}\psi^{jb}_{R\beta}\rangle^* \sim \epsilon_{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{abc}Y^{\gamma}_{c}$$

3. $SU(3)_c \times U(1)_{EM} \times SU(3)_L \times SU(3)_R \times U(1)_B$

$$\Downarrow$$

$$SU(3)_{c+L+R} \times U(1)^*_{EM}$$

a) Color gauge group is completely broken;
b) $U(1)_B$ is spontaneously broken;
c) $U(1)_{EM}$ is not broken;
d) $U(1)_A$ is broken spontaneously and explicitly (by instantons)

4. Goldstone fields are the phases of the condensate

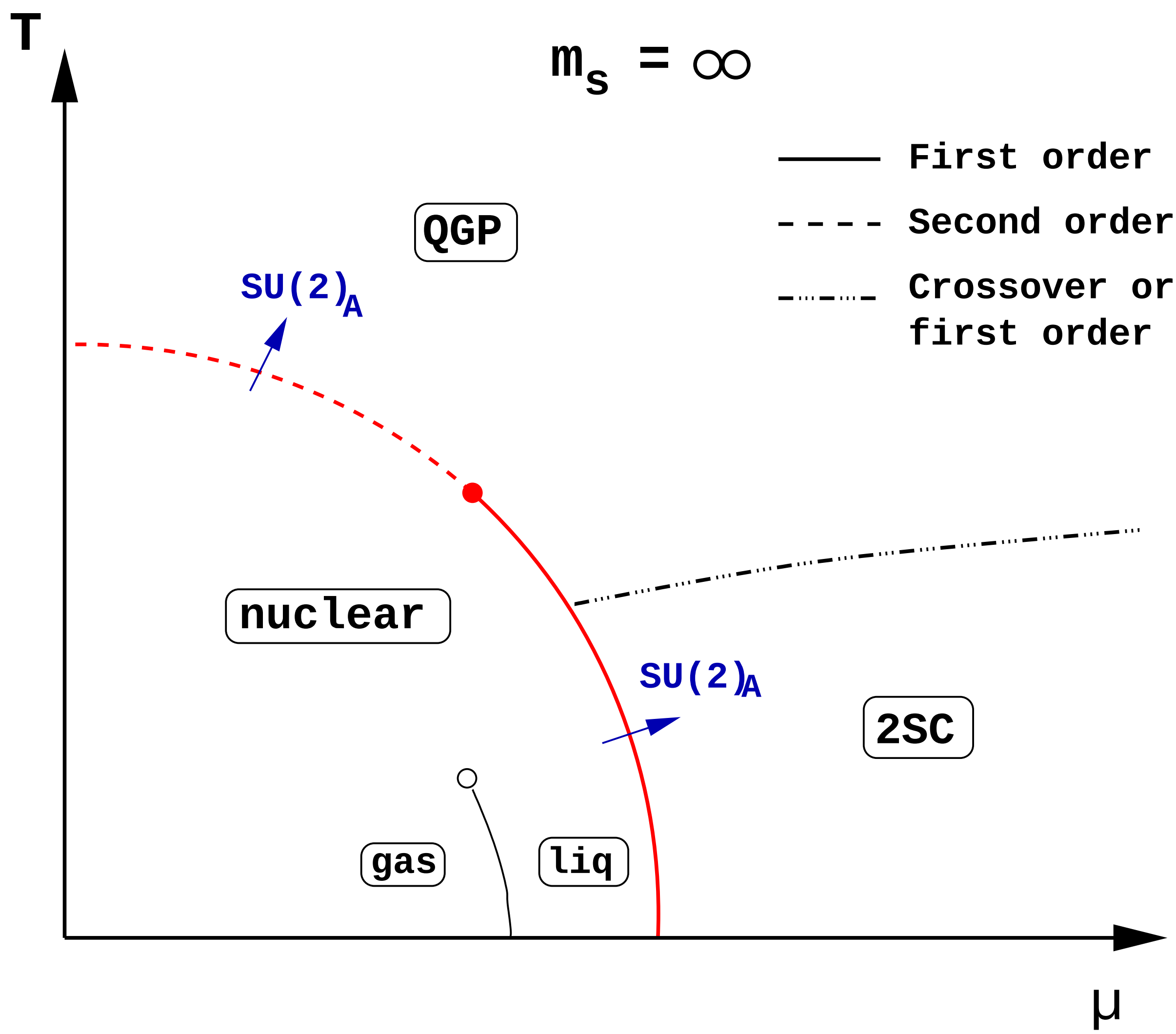
$$\Sigma_\gamma^\beta = \sum_c X_c^\beta Y_\gamma^{c*} \sim e^{\lambda^a \pi^a} e^{i\eta'}.$$

5. Superfluid phonon field is the phase of the condensate

$$detX \sim detY \sim e^{i\varphi_B}.$$

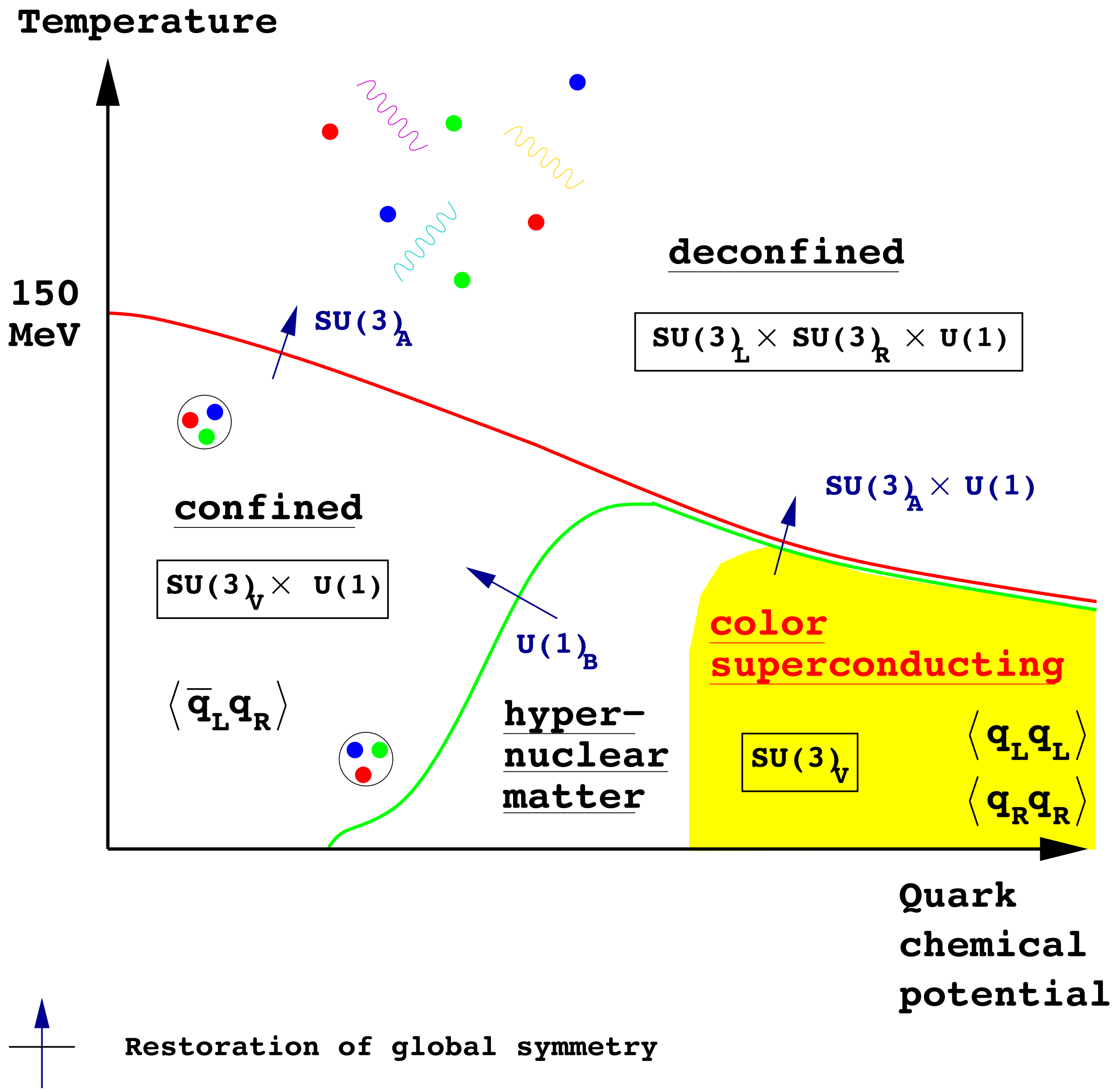
6. The gap $\Delta \simeq 100 MeV$ is large.
The critical temperature $T_c \simeq 0.6\Delta$ is also large.

7. The phase structure in QCD (with parameters realized in nature) could be much more complicated. In particular, condensates of different fields: $\langle K^0 \rangle, \langle K^+ \rangle, \langle \eta \rangle$ along with diquark condensate $\langle \psi_{L\alpha}^{ia} \psi_{L\beta}^{jb} \rangle$ and $\langle \psi_{R\alpha}^{ia} \psi_{R\beta}^{jb} \rangle$, could develop.



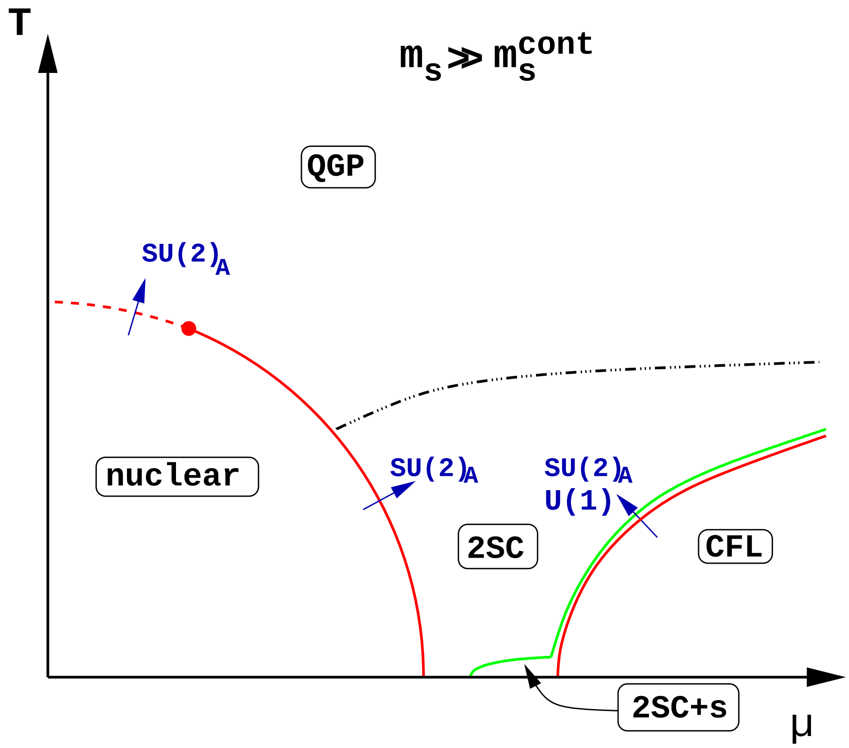
(From Mark Alford's review paper).

Conjectured phase diagram for QCD with $N_f = 2$, $N_c = 3$. ($2SC \equiv$ two flavor superconducting phase.)



(From Mark Alford's review paper).

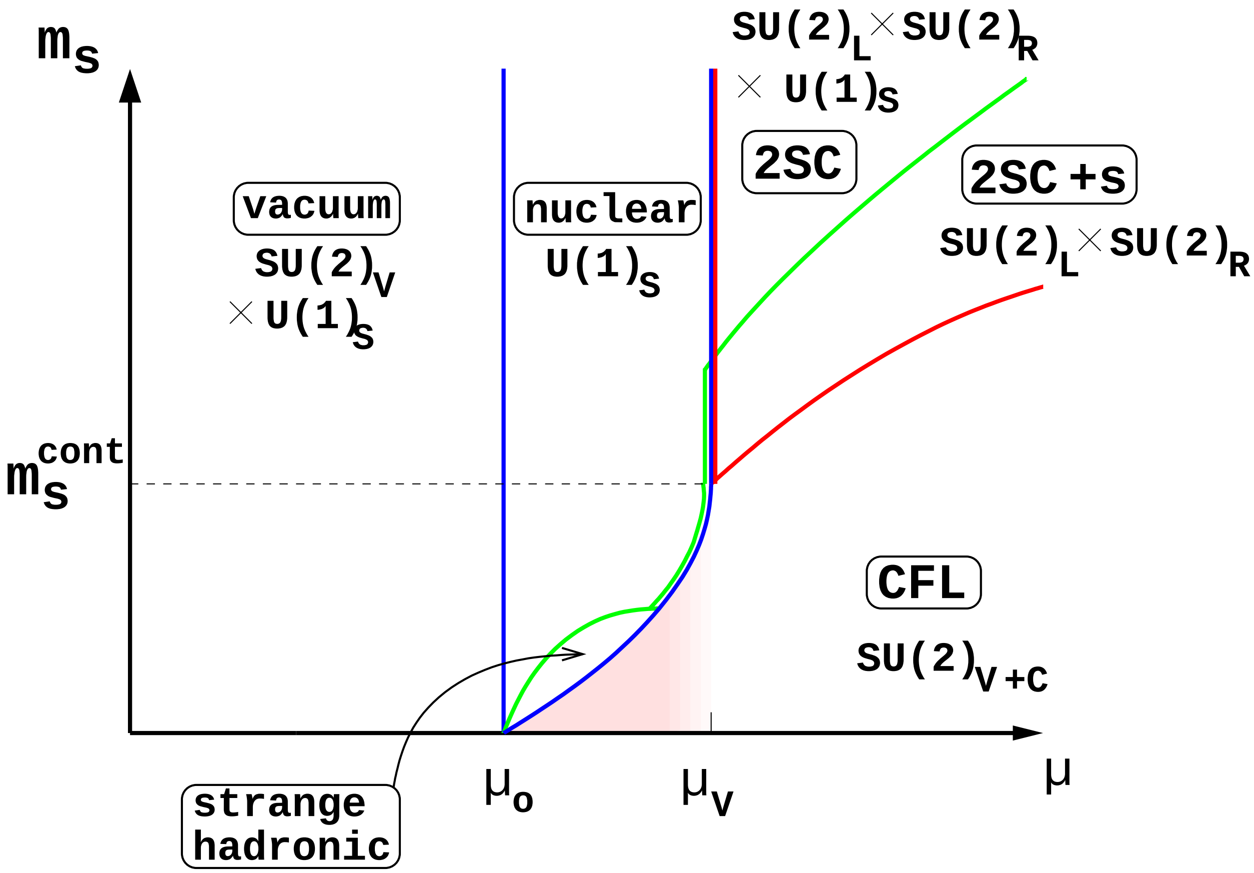
Conjectured phase diagram for QCD with $m_q = 0$, $N_f = 3$, $N_c = 3$.



(From Mark Alford's review paper).

Conjectured phase diagram for QCD with $N_f = 2+1$ ($m_s \neq 0$ is large) , $N_c = 3$.

It is interesting to note that the phase diagram for high temperature superconductors has similar structure with replacements :
chemical potential $\mu \rightarrow x$ (doping);
hadronic/nuclear phase $\rightarrow$ antiferromagnetic phase;
CFL phase $\rightarrow$ high T_c superconducting phase;
Some mess in the transition region in both cases.



(From Mark Alford's review paper).

This diagram shows a huge sensitivity to the strange quark mass, m_s .

III. Topological Defects in Dense Matter

1. $U(1)_B$ is spontaneously broken $\Rightarrow$ there are global vortices, similar to what is observed in He^4. Effective lagrangian is

$$L_B \sim f_B^2[(\partial_0\varphi_B)^2 - u^2(\partial_i\varphi_B)^2]$$

2. $U(1)_A$ is spontaneously broken (by the condensate $\langle\Sigma_\gamma^\beta\rangle \neq 0$) $\Rightarrow$ there are axial global vortices.

3. The symmetry is broken also explicitly by the instantons $\Rightarrow$ there are axial domain walls. Effective lagrangian is

$$L_A \sim f_A^2[(\partial_0\varphi_A)^2 - u^2(\partial_i\varphi_A)^2] + a\cos(\varphi_A - \theta),$$

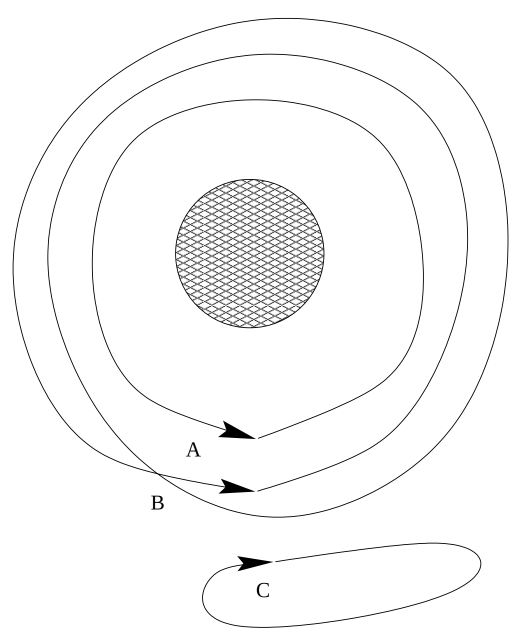
where $a \sim \int d\rho n(\rho) \sim (\frac{\Lambda_{QCD}}{\mu})^b \ll 1$ with $n(\rho) \sim e^{-\mu^2\rho^2 N_f}$ such that instanton calculations are under complete theoretical control.

4. The η' is light: $m_{\eta'}^2 \sim \frac{a}{f^2} \sim (\frac{\Lambda_{QCD}}{\mu})^b \to 0$.

5. The η' domain wall solutions correspond to the transitions between $\varphi_A = 0, 2\pi, 4\pi...$, which are the same physical points.

6. Analytical solution: $\varphi_A = 4 \tan^{-1}[\exp(\frac{m_{\eta'}x}{4})]$. Domain wall tension $\sigma \sim \sqrt{a} f \sim f^2 m_{\eta'}$.

7. Few remarks on domain walls at large μ:
a). If states $\varphi_A = 0, 2\pi, 4\pi...$, were physically different states, these domain walls would be absolutely stable (ferromagnetic domain walls);



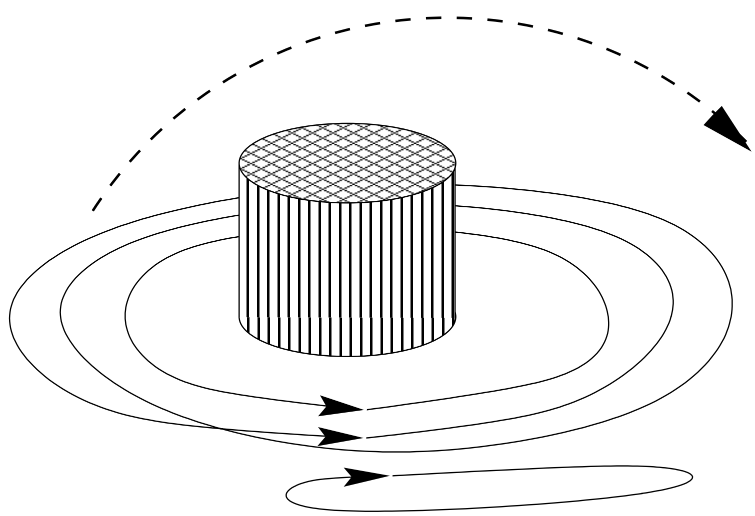
b). If φ_A is the only degree of freedom of the theory, the soliton would be absolutely stable, like in $2d$ Sine Gordon model;

c).If the heavy degrees of freedom $m \sim \Delta$ are taken into account, the domain walls become metastable (tunneling with the excitations of heavy degrees of freedom is possible);

d). The life time is expected to be very large for large μ when the instanton density is small and η' is light.

Quantum mechanical tunneling :

$\tau(a \to 0) \sim \exp\left(\frac{\pi^2}{24}\frac{f^6 u^2}{a\Delta^2}\ln^3\frac{1}{\sqrt{a}}\right) \to \infty$



IV. Instanton interactions in dense QCD

1. Instanton calculations are under control due to the $e^{-N_f\mu^2\rho^2}$ suppression,

$$n_0(\rho) = const.\left(\frac{8\pi^2}{g^2}\right)^{2N_c}\rho^{-5}\exp\left(-\frac{8\pi^2}{g^2(\rho)}\right)e^{-N_f\mu^2\rho^2} .$$

2. Partition function for η'

$$Z = \int \mathcal{D}\varphi_A \; e^{-f^2 u\int d^4x(\partial\varphi_A)^2} \, e^{a\int d^4x \cos(\varphi_A(x)-\theta)} ,$$

3. Different representation for η'

$$\mathrm{e}^{a\int d^4x \cos(\varphi_A(x)-\theta)} \equiv$$

$$\sum_{M_\pm=0}^{\infty} \frac{(a/2)^M}{M_+!M_-!}\int d^4x_1 \ldots \int d^4x_M \; e^{i\sum_{a=0}^{M} Q_a(\varphi_A(x_a)-\theta)} .$$

$$Z = \sum_{M_\pm=0}^{\infty} \frac{(a/2)^M}{M_+!M_-!} \int d^4x_1 \ldots \int d^4x_M \; e^{-i\theta \sum_{a=0}^{M} Q_a} \cdot$$

$$e^{-\frac{1}{2f^2u}\sum_{a>b=0}^{M} Q_a Q_b G(x_a - x_b)} \;, \quad G(x_a - x_b) = \frac{1}{4\pi^2 (x_a - x_b)^2} \,.$$

The two representations of the partition function are equivalent.

4. Physical Interpretation.

a) Since $Q_{\rm net} \equiv \sum_a Q_a$ is the total charge and it appears in the action multiplied be the parameter θ, one concludes that $Q_{\rm net}$ *is* the total topological charge of a given configuration.

b) Each charge Q_a in a given configuration should be identified with an integer topological charge well localized at the point x_a. This, by definition, corresponds to a small instanton positioned at x_a.

c) Further saupport for the identification: every particle with charge Q_a brings along a factor of fugacity $\sim a$ which contains the classical one-instanton suppression factor $\exp(-8\pi^2/g^2(\rho))$ in the density of instantons.

5. The following hierarchy of scales exists: The typical size of the instantons $\bar{\rho} \sim \mu^{-1}$ is much smaller than the short-distance cutoff of our effective low-energy theory, Δ^{-1},

(size)	$\ll$	(cutoff)	$\ll$	(II distance)	$\ll$	(Debye)
μ^{-1}	$\ll$	Δ^{-1}	$\ll$	$(\sqrt{a}\mu\Delta)^{-1/2}$	$\ll$	$(\sqrt{a}\Delta)^{-1}$

Due to this hierarchy, ensured by large $\mu/\Lambda_{\rm QCD}$, we acquire analytical control.

6. The starting low-energy effective Lagrangian contains only a colorless field φ, we have ended up with a representation of the partition function in which objects carrying color (the instantons) can be studied.

7. In particular, II and $I\bar{I}$ interactions (at very large distances) are exactly the same up to a sign and are Coulomb-like.

8. Very complicated picture of the **bare** II and $I\bar{I}$ interactions becomes very simple for **dressed** instantons/anti-instantons when all integrations over all possible sizes, color orientations and interactions with background fields are properly accounted for!

V. Domain Walls in Large N_c QCD

(In large N_c limit the domain wall structure appears to be very similar to what was discussed for dense matter).

1. In large N_c limit η' is light, $m^2_{\eta'} \sim 1/N_c$

2. Tilted Mexican potential has a form when the points $\varphi_A = 0, 2\pi, 4\pi ...$ are identified (similar to dense matter case).

3. The η' domain wall solutions correspond to the transitions between identical physical points $\varphi_A = 0, 2\pi, 4\pi ....$ Therefore, the domain wall solution is metastable rather than absolutely stable.

4. For the specific model for the η' potential, $V \sim N_c^2[1-\cos(\frac{\theta-\eta'}{N_c})]$,

a) the peak of the potential is very high, $\Delta V_1 \sim N_c^2$– this corresponds to the excitation of the heavy degrees of freedom;

b) the part related to the η' meson is relatively small, and proportional to $\Delta V_2 \sim f_{\eta'}^2 m_{\eta'}^2 \sim 1$.

5. The life time is expected to be very large in large N_c limit, $\tau \sim e^{N_c^2}$

6. Long-lived η' domain walls ($N_c = 3$ is qualitatively similar to $N_c = \infty$ case) in principle can be studied at RHIC (Shuryak, AZ).

VI. Quantum Anomalies in Dense Matter

1. Few general remarks:

a). It is known, that the **quantum anomalies** in QFT play very important role in theory and phenomenology.

b). Our goal is to derive a new **anomalous effective lagrangian** describing the interaction of three light fields: the electromagnetic photons A_μ, neutral light Nambu-Goldstone bosons (π, η, η'), and the **superfluid phonon** φ_B .

c). Anticipating the event.
The main result of the calculations– **new anomalous** terms which include superfluid phonon φ_B identically **vanish** unless background contains topological defects such as **domain walls or/and vortices.**
(it was the main motivation to introduce vortices and domain walls in the previous sections)

d). If the anomaly induced interaction does not vanish (in case of nontrivial background, e.g. vortices) it leads to a number of interesting phenomena, such as:

i) *superconducting strings* (similar to the Witten' cosmic superconducting strings),

ii) *classical weak neutral* currents flowing on superfluid vortices;

iii) *electric currents* flowing on $U(1)_A$ vortices;

iv) *magnetization* of the η' domain wall, and many other macroscopically *large coherent phenomena*.

2. Main Idea

a) Consider QCD in the background of two U(1) fields: the electromagnetic field A_μ and a fictitious (spurion) B_μ field which couples to the baryon current. At the end of the calculations we will put $B_\mu = \mu n_\mu$, $n_\mu = (1, \vec{0})$.

b) The fundamental Lagrangian describing the coupling of quarks with B_ν is

$$L_B = \bar{\psi}\gamma^\nu \left(i\partial_\nu + \frac{1}{3}B_\nu\right)\psi,$$

It is invariant under $U(1)_B$ local gauge transformations $q \to e^{i\beta(x)/3}q, \quad B_\mu(x) \to B_\nu(x) + \partial_\mu\beta(x)$,similar to the conventional electrodynamics, $U(1)_{EM}$.

c)The effective low-energy description must respect the $U(1)_B$ gauge symmetry. Therefore, in the effective Lagrangian the covariant derivative $D_\mu\varphi_B = \partial_\mu\varphi_B - 2B_\mu$ (similar to the conventional $E\&M$) must appear. The effective Lagrangian should be relativistically invariant before the replacement $B_\mu \to (\mu, \vec{0})$ is made.

3. Anomalous effective lagrangian

a). Consider the transformation properties of the path integral under the $U(1)_A$ chiral transformation. As is known, the measure is not invariant under these transformations due to the chiral anomaly: it receives an additional contribution $\delta S = -\int d^4x\, \partial^\mu \alpha\, j^{\rm A}_\mu = \int d^4x\, \alpha\, \partial^\mu j^{\rm A}_\mu$.

b). The problem is reduced to the calculation of the divergenece of the axial current in the presence of the electromagnetic A_μ background field as well as in the presence of fictitious B_μ background field,

$$\partial^\mu j^{\rm A}_\mu = -\frac{1}{16\pi^2}\big(e^2 C_{\rm A\gamma\gamma} F^{\mu\nu}\tilde{F}_{\mu\nu} - 2e C_{\rm AB\gamma} B^{\mu\nu}\tilde{F}_{\mu\nu} + C_{\rm ABB} B^{\mu\nu}\tilde{B}_{\mu\nu}\big), \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

c). Coefficients $C_{\rm A\gamma\gamma}$ are well-known ($\pi^0 \to 2\gamma$); Coefficients $C_{\rm AB\gamma}, C_{\rm ABB}$ (when fictitious B_μ field is included) can be easily calculated in similar way.

d) At first sight, these new terms vanish identically ($B_{\mu\nu} \equiv 0$ for pure gauge field $B_\mu \sim \partial_\mu \varphi_B$) and cannot have any physical effect. This is true when the NG fields $\varphi_{\rm A}$ and $\varphi_{\rm B}$ are small quantum fluctuations.

e) However, as $\varphi_{\mathrm{A,B}}$ **are periodic variables,** the action can be nonzero. This occurs in the presence of topological defects like vortices or domain walls. For example, for vortex configuration,

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu = (\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\varphi = 2\pi\delta^2(x_\perp).$$

f) In terms of physical fields (A_μ-electromagnetic, φ_B -baryon phase of the diquark condensate , φ_A–NG fields, π, η'), the effective lagrangian takes the form,

$$\mathcal{L}_{\mathrm{anom}} = \frac{1}{8\pi^2 q_{\mathrm{A}}}\partial_\mu\varphi_{\mathrm{A}}\Big[e^2 C_{\mathrm{A}\gamma\gamma}A_\nu\tilde{F}^{\mu\nu} - eC_{\mathrm{AB}\gamma}\epsilon^{\mu\nu\alpha\beta}\left(\mu n_\nu - \tfrac{1}{2}\partial_\nu\varphi_{\mathrm{B}}\right)F^{\alpha\beta} - \tfrac{1}{2}C_{\mathrm{ABB}}\,\epsilon^{\mu\nu\alpha\beta}\left(\mu n_\nu - \tfrac{1}{2}\partial_\nu\varphi_{\mathrm{B}}\right)\partial_\alpha\partial_\beta\varphi_{\mathrm{B}}\Big].$$

VII. Applications

1. Magnetization of axial η' domain walls.

a) Consider an axial domain wall in an external magnetic field. The baryon field $B_\nu = (1, \vec{0})$ is considered as a background which is at rest. The following term is present in the anomaly Lagrangian:

$$\mathcal{L}_{\mathrm{AB}\gamma} = -\frac{eC_{\mathrm{AB}\gamma}\mu}{8\pi^2}\partial_\mu \varphi_{\mathrm{A}} n_\nu \tilde{F}_{\mu\nu} = \frac{eC_{\mathrm{AB}\gamma}\mu}{8\pi^2}\vec{B}\cdot\vec{\nabla}\varphi_{\mathrm{A}}$$

b). For a U(1)$_{\mathrm{A}}$ domain wall streched along xy directions, the phase φ_{A} has a jump by 2π. External magnetic field $B_z \neq 0$.

c) Anomaly lagrangian $\mathcal{L}_{\mathrm{AB}\gamma}$ implies that the energy is changed by a quantity proportional to BS, where S is the area of the domain wall. This means that the **domain wall is magnetized**, with a finite magnetic moment per unit area equal to $eC_{\mathrm{AB}\gamma}\mu/(4\pi)$.

d)The magnetic moment is directed perpendicularly to the domain wall. For the 2SC and CFL phases the magnetic moment per unit area is $e\mu/(12\pi)$ and $e\mu/(6\pi)$, respectively.

2. Currents on axial η' vortices.

a) The same effect can be looked at from a different perspective. One rewrites lagrangian $\mathcal{L}_{\mathrm{AB}\gamma}$ into the following form, $\mathcal{L}_{\mathrm{AB}\gamma} = \frac{eC_{\mathrm{AB}\gamma}\mu}{4\pi^2}\epsilon_{ijk}A_i\,\partial_j\partial_k\varphi_{\mathrm{A}}$.

b). Since $\epsilon_{ijk}\partial_j\partial_k\varphi_{\mathrm{A}} \sim 2\pi\delta^2(x_\perp)$ on the vortex core, the action can be written as a line integral along the vortex,

$$\mathcal{S}_{\mathrm{anom}} = \frac{eC_{\mathrm{AB}\gamma}\mu}{2\pi}\int d\vec{\ell}\cdot\vec{A} \qquad (2)$$

which means that **there is an electric current running along the core of the axial vortex**. The magnitude of the current is $j^{\mathrm{em}} = \frac{eC_{\mathrm{AB}\gamma}\mu}{2\pi}$

c). The electromagnetic current running along a closed vortex loop generates a **magnetic moment equal to $\frac{1}{2}jS$,** where S is the area of the surface enclosed by the loop.

d). A large vortex loop has a magnetic moment that can be interpreted as created by the current running along the loop, *or* as the total magnetization of the domain wall stretched on the loop.

3. Axial current on a superfluid vortex.

a) The following term is present in the anomaly Lagrangian: $\mathcal{L}_{\rm ABB} \sim \mu\epsilon^{\mu\nu\alpha\beta}\partial_\mu\varphi_A n_\nu\partial_\alpha\partial_\beta\varphi_{\rm B}$.

b). Consider a superfluid vortex. At the vortex core, as usual, $(\partial_x\partial_y - \partial_y\partial_x)\varphi_B = 2\pi\delta^2(x_\perp)$. The action becomes

$$\mathcal{S}_{\rm ABB} = \frac{\mu}{2\pi}\int dtdz\partial_z\varphi_A \qquad (3)$$

where the integral is a linear integral along the vortex line. As usual, it is a total derivative, but leads to the physically observable phenomena.

c). There is an axial current running on the superfluid vortex. The magnitude of the current is $\mu/(2\pi)$.

d). If one now turns on weak interactions, this current is proportional to the weak neutral current which is coupled to the Z boson. This implies that the system induces a strong classical weak (rather than electromagnetic) field along the vortex.

4. Superconducting strings in dense matter.

a) Everything what is said above about the neutral vortex is also applicable to the case when the Cooper pairs carry the electric charge (2SC phase). The phenomenon in this case becomes even more interesting.

b) The anomalous interaction with electromagnetic field $\sim \epsilon^{\mu\nu\alpha\beta}(\partial_\mu\varphi_A)\partial_\nu\varphi_{\rm B}\partial_\alpha {\rm A}_\beta \sim \epsilon^{\mu\nu\alpha\beta}(\partial_\mu\varphi_A)\cdot$ $(\partial_\alpha\partial_\nu\varphi_{\rm B}){\rm A}_\beta$ does not vanish, and the action includes additional terms ($\epsilon^{\alpha\nu}\partial_\alpha\partial_\nu\varphi_{\rm B} \rightarrow 2\pi\delta^2(x_\perp)$ for vortex),

$$\mathcal{S}_{\rm anom} = \int dtdz \Big[\frac{\mu}{2\pi}\partial_z\varphi_A + eA_0\partial_z\varphi_A - eA_z\partial_0\varphi_A\Big].$$

c). This equation implies that $\partial_z\varphi_A$ can be interpreted as the density of electrical (not axial) charge, while $\partial_0\varphi_A$ can be interpreted as the electromagnetic current along the string.

d). The effective lagrangian $eA_a\epsilon^{ab}\partial_b\varphi_A$ is very similar to the one introduced by Witten, 1984 to describe the superconducting strings.

5. Moral.

It is quite remarkable that a very nontrivial construction invented by Witten for cosmic strings is automatically realized in dense matter systems. Therefore, many consequences of the Witten's construction, such as there existence of the closed loops of superconducting strings (vortons), may be realized in dense matter.

VIII. Possible "contact points" with the string theory

1. *QFT -side .* Physics of the light η' meson, η' domain walls and strings, "dressed" instantons generating η' potential , etc

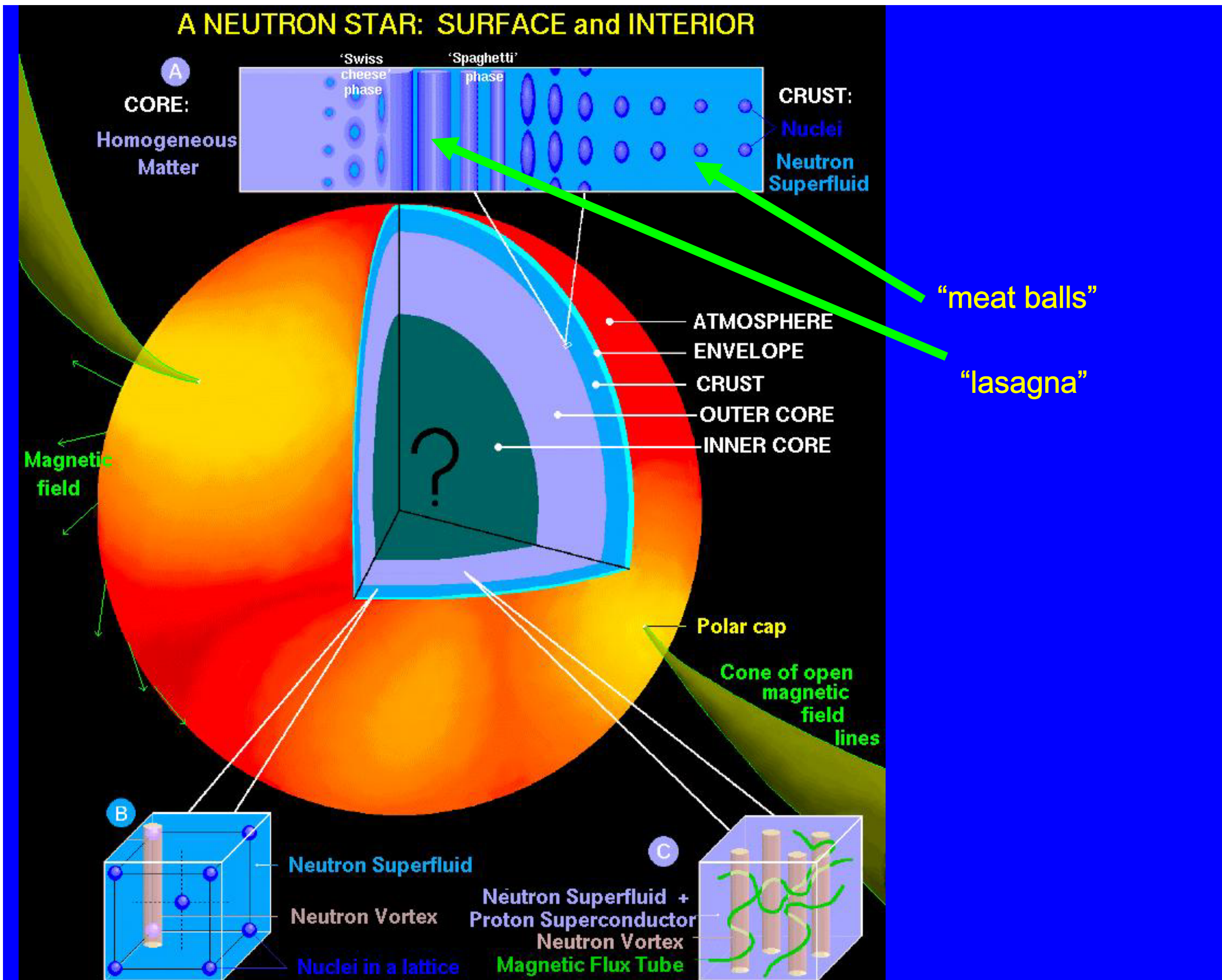
$$\Updownarrow$$

Strings- side. The holographic life of the η' M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, JHEP **0405**, 041 (2004); A. Armoni, JHEP **0406**, 019 (2004)

2. Close analogy between Witten's cosmic superconducting strings (*QFT -side*) and D, F strings in D -brane framework (*Strings- side*) has been recently pointed out (Gia Dvali, Alex Vilenkin, 2003). QCD can serve as the underling fundamental theory to describe the Witten's superconducting strings.

Dense Matter: where it can be realized in Nature?

1. neutron stars



Borrowed from http://www.lsw.uni-heidelberg.de/~mcamenzi/NS_Mass.html

2. Dark matter in our Universe?

a) Strangelets (nuggets): E. Witten, Phys. Rev.**D 30** (1984) 279.

b) Recent Development: Witten's nuggets could be the chunks of matter in the color superconducting phase, A. Zhitnitsky "'Nonbaryonic' dark matter as baryonic color superdonductor," JCAP **0310**, 010 (2003)

c) D. P. Anderson, E. T. Herrin, V. L. Teplitz and I. M. Tibuleac, "Unexplained Sets of seismographic Station reports and a set consistent with a quark nugget passage" arXiv:astro-ph/0205089.

Unexplained Sets of Seismographic Station Reports and A Set Consistent with a Quark Nugget Passage

David P. Anderson
Department of Geological Sciences
Southern Methodist University
dpa@io.isem.smu.edu

Eugene T. Herrin
Department of Geological Sciences
Southern Methodist University
herrin@passion.isem.smu.edu

Vigdor L. Teplitz
Department of Physics
Southern Methodist University
teplitz@mail.physics.smu.edu

Ileana M. Tibuleac
Weston Geophysical
Boston, MA
ileana@westongeophysical.com

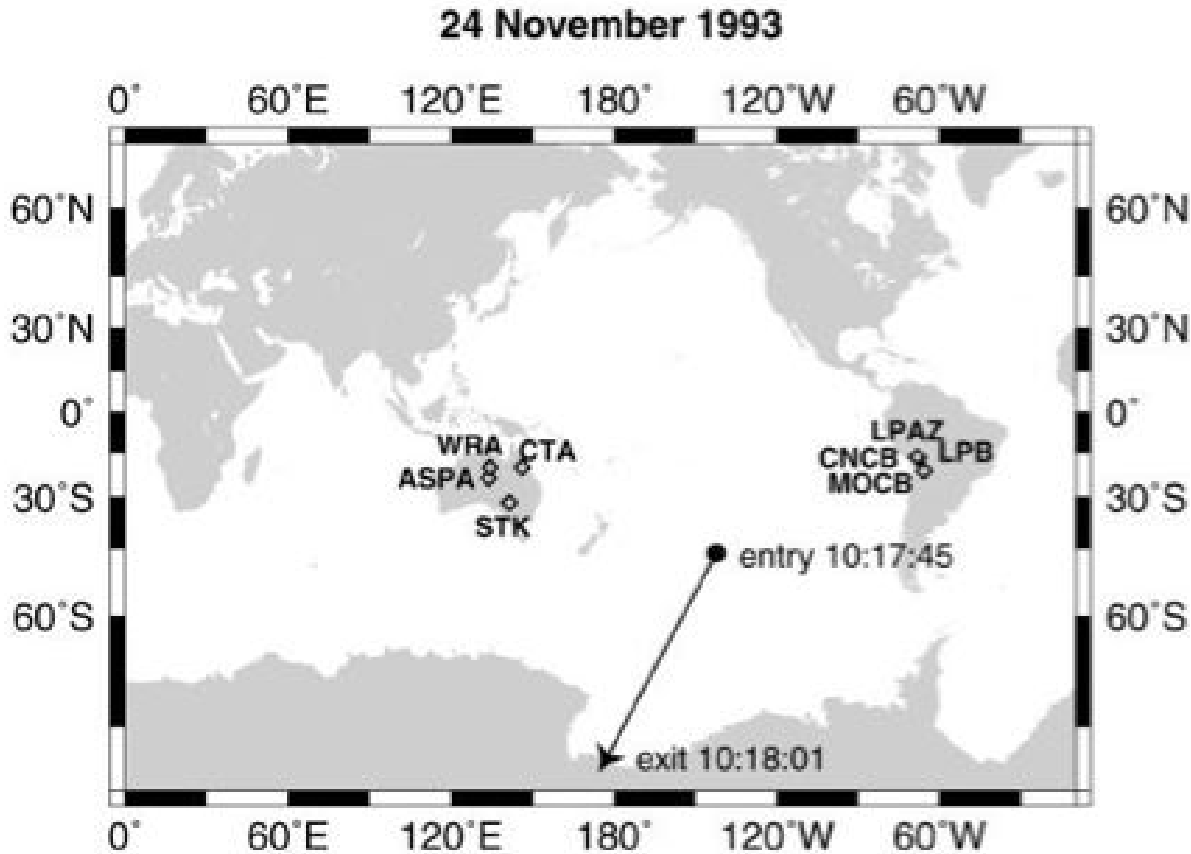


Figure 4. Surface trace for November 1993 linear event.

VI. The November 1993 Event

A. Fit to an epilinear source

Figure 4 plots the surface trace for a linear source fit of the 9 unassociated station reports from November 24, 1993. The source has an entry time of 10:17:45 in the South Pacific and an exit point 16 seconds later in the Ross Ice Shelf near the South Pole. The model and residuals are given in Table 3. All residuals are less than ± 0.4 sec., an excellent fit of data to the model. The root-mean-square (RMS) residual is 0.23 sec. This result is better than many obtained for well-located earthquakes.