Confinement without a mass gap from strings on the deformed conifold

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Contents

Based primarily on work with I. Klebanov and C. Herzog, hep-th/0405282.

Section 5 based on work with J. Heckman, hep-th/0411001.

1. Historical introduction to AdS/CFT

(Encoding, no doubt, my personal prejudices...)

- Genus expansion for large N gauge theories suggests a string theory *['t Hooft 74]*
- D-branes carry a gauge field *[Polchinski 95]*
- Non-abelian gauge dynamics from coincident D-branes *[Witten 95]*
- Black hole microstate counting via D-branes *[Strominger-Vafa 96, Callan-Maldacena 96]*
- D3-brane entropy from N=4 super-Yang-Mills *[Gubser-Klebanov-Peet 96]*
- D3-brane absorption cross-sections from N=4 SYM *[Klebanov 97]*
- Cross-sections related to protected 2pt fcts in super-Yang-Mills *[Gubser-Klebanov 97]*
- Non-critical strings in a warped 5-dim geometry can be dual to a gauge theory in 4-dim *[Polyakov 97]*
- Near-horizon geometry ($AdS_5 \times S^5$) entirely encodes strongly coupled gauge dynamics *[Maldacena 97]*
- All correlators for gauge theory on the boundary of $AdS_5 \times S^5$ calculable from bulk amplitudes *[Gubser-Klebanov-Polyakov 98, Witten 98a]*

- Wilson lines in gauge theory correspond to strings hanging into AdS_5 *[Maldacena 98, Rey-Yee 98]*
- Confinement can be described by a deformation of AdS *[Witten 98b]*

AdS/CFT generalizes beautifully from its original setting to a duality

- between gauge theory and gravity
- between open and closed strings
- in opposite limits of coupling—a bit like S-duality.

It encodes and illuminates

- RG flow, confinement, χ_{SB}
- black hole entropy, holographic principle
- a version of the c-theorem
- ... and much much more

A review (by now out of date) may still be of some use: *[Aharony-Gubser-Maldacena-Ooguri-Oz 99]*

Flies in the ointment

We can't yet fully realize the old dream of casting 4-dim large N QCD as a string theory.

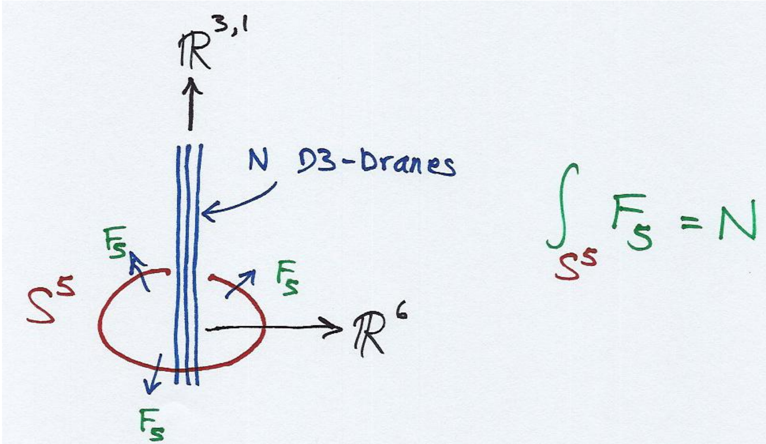
- AdS/CFT gives us a handle on large $g_{YM}^2 N$ calculations, but small $g_{YM}^2 N$ is hard work at best, because...
- It's hard to quantize strings in backgrounds with Ramond-Ramond flux.
- Open strings give gauge theory plus junk. The junk is important and sometimes even interesting, as we'll see.

Nevertheless, it's well worth studying string theory backgrounds exhibiting confinement.

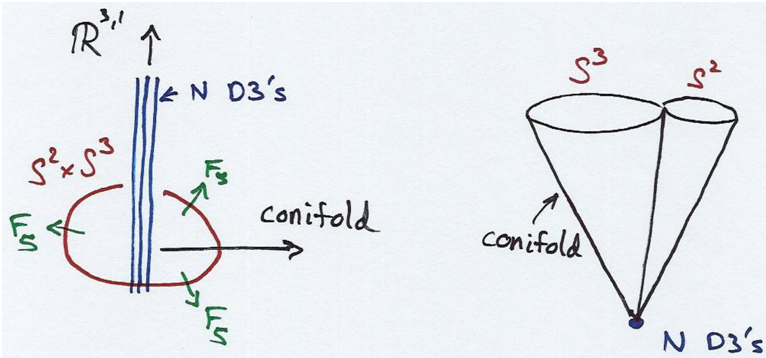
A favorite: strings on the warped deformed conifold *[Klebanov-Strassler 00]*

2. The warped deformed conifold

$\mathcal{N} = 4\ SU(N)$ SYM describes dynamics of N coincident D3-branes in otherwise flat empty $\mathbf{R}^{9,1}$:

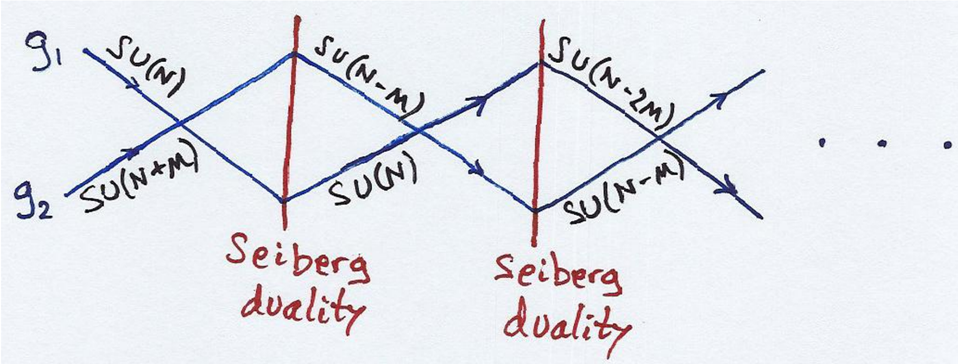


Put N D3-branes on an isolated singularity to get gauge theories with less SUSY: e.g. $\mathcal{N} = 1$:

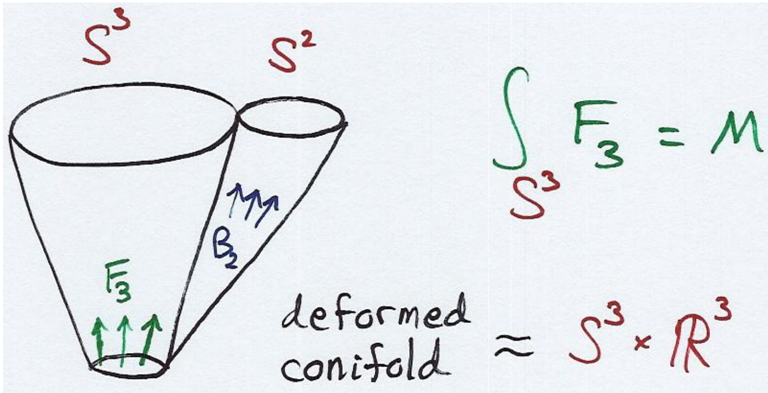


Nothing sets a scale in this geometries. So gauge theory is a CFT: $SU(N) \times SU(N)$ with certain $(N, \bar{N})$-type matter.

A "fractional D3-brane" leads to $SU(N+M) \times SU(N)$. Now it can't be a CFT: $SU(N+M)$ flows to strong coupling and undergoes a Seiberg duality.



String theory description: add some F_3 so that S^3 stays of finite size. Need some B_2 to maintain $\mathcal{N}=1$ SUSY.



Certain Bianchi identities force

$$\int_{S^2 \times S^3} F_5 = N_{\text{eff}}(r) = N + \frac{3}{2\pi} g_s M^2 \log(r/r_0) \,,$$

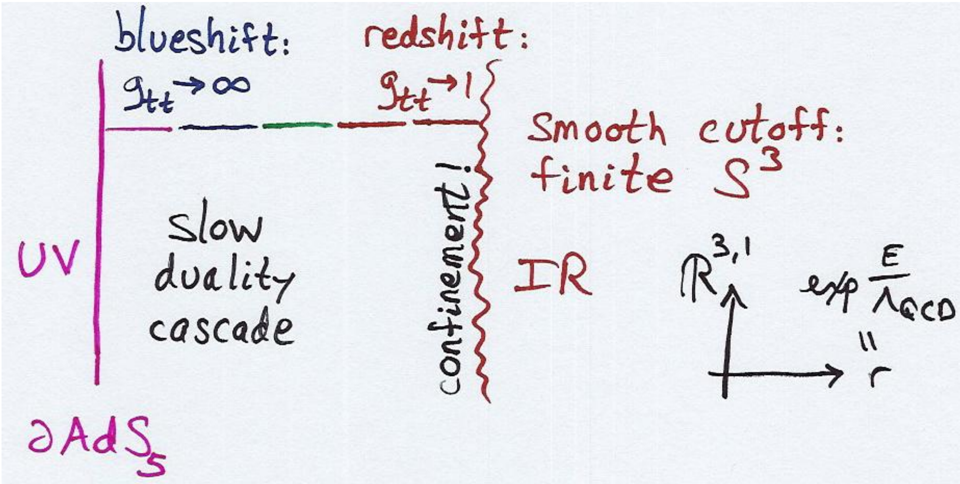
an expression in supergravity of the Seiberg cascade.

What's the infrared physics?

Duality cascades terminates...

$$\ldots \to SU(9M) \times SU(8M) \to SU(8M) \times SU(7M) \to \ldots$$
$$\to SU(2M) \times SU(M) \to SU(M)$$

at a confining gauge theory: $\mathcal{N} = 1$ $SU(M)$ pure glue. A cartoon summary:



3. A Goldstone boson

So there's a mass gap, right?

WRONG. There's a Goldstone boson, anticipated in *[Aharony 01]*.

To see it, we have to understand the chiral superfields better in the penultimate theory, $SU(2M) \times SU(M)$:

	$SU(2)_A$	$SU(2)_B$	$SU(2M)$	$SU(M)$
A	2	1	$2M$	$\overline{M}$
B	1	2	$\overline{2M}$	M

$SU(2)_A \times SU(2)_B$ is the expected $SO(4)$ symmetry of our S^3. A and B go away in the pure $SU(M)$ theory. A chiral baryon-like operator can be formed from A_1, A_2—both in the $(2M, \overline{M})$:

$$\mathcal{B} \sim \epsilon_{\alpha_1 \alpha_2 \ldots \alpha_{2M}} (A_1)_1^{\alpha_1} (A_1)_2^{\alpha_2} \ldots (A_1)_M^{\alpha_M} (A_2)_1^{\alpha_{M+1}} (A_2)_2^{\alpha_{M+2}} \ldots (A_2)_M^{\alpha_{2M}}$$

$$\overline{\mathcal{B}} \sim \epsilon^{\alpha_1 \alpha_2 \ldots \alpha_{2M}} (B_1)^1_{\alpha_1} (B_1)^2_{\alpha_2} \ldots (B_1)^M_{\alpha_M} (B_2)^1_{\alpha_{M+1}} (B_2)^2_{\alpha_{M+2}} \ldots (B_2)^M_{\alpha_{2M}} \, .$$

The non-perturbative superpotential for A and B has among its flat directions the baryonic branch:

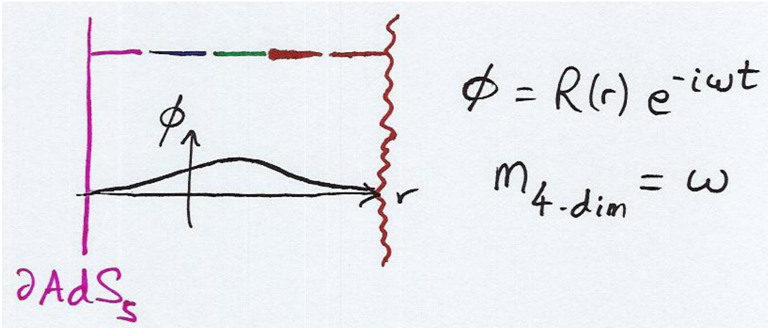
$$\mathcal{B}\overline{\mathcal{B}} = -\Lambda_{2M}^{4M}$$

where Λ_{2M} is the scale for strong dynamics in $SU(2M)$. Any given vacuum clearly breaks the global symmetry

$$U(1)_B : \begin{cases} \mathcal{B} \to e^{i\theta}\mathcal{B} \\ \overline{\mathcal{B}} \to e^{-i\theta}\overline{\mathcal{B}} \end{cases}$$

So there's got to be an associated Goldstone boson.

Recall the usual story about glueballs in AdS/CFT: massless supergravity fields ϕ in deformed AdS_5 have normalizable energy eigenstates:



So our Goldstone boson should come from some radial zero-mode.

Goldstone is a pseudo-scalar, $a = a(t, \vec{x})$ with $\Box_4 a = 0$. Say $da = *_4 f_3$. Then $df_3 = 0$. Maybe $\delta F_3 = f_3 = *_4 da$ is our Goldstone boson. Almost right...

$$\delta F_3 = *_4 da + f_2(\tau) da \wedge dg^5 + f_2' da \wedge d\tau \wedge g^5$$
$$\delta F_5 = (1 + *) \delta F_3 \wedge B_2 = \left(*_4 da - \frac{\epsilon^{4/3}}{6K(\tau)^2} h(\tau) da \wedge d\tau \wedge g^5 \right) \wedge B_2 \,,$$

The extra junk is whatever we needed to satisfy all linearized eom's. And

$$f_2(\tau) \propto -\frac{2}{K(\tau)^2 \sinh^2 \tau} \int_0^\tau dx\, h(x) \sinh^2 x \,,$$

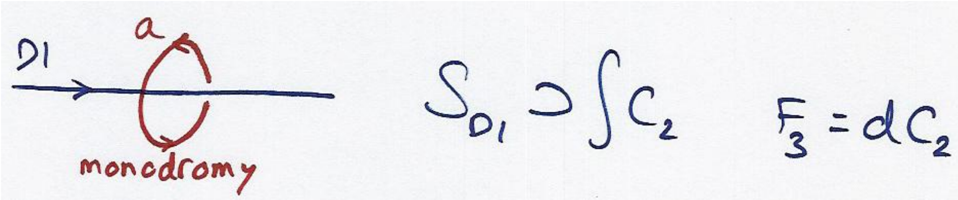
which vanishes both for $\tau \to 0$ (IR) and $\tau \to \infty$ (UV). Normalizability follows.

We guessed $\delta F_3 = *_4 da$ for a reason: wanted to understand D1-brane in this geometry. NOT a Wilson line, so what is it?

$D1$ carries electric charge of F_3, hence magnetic charge of $*_4 F_3 = da$.

D1-brane is a solitonic string

This makes sense because D1 sources F_3. This ANO vortex-string is a stable, non-BPS object.



A natural question: What is the scalar superpartner of our Goldstone boson?

Must be a modulus: a normalizable deformation of the warped deformed conifold. It should also

- Preserve $SO(4)$ symmetry
- Comprise perturbations of NS-NS fields (cf. $\chi + ie^{-\phi}$)
- Break a certain $\mathbf{Z}_2$ symmetry ($A \leftrightarrow B$)

An ansatz which does all this is

$$\delta B_2 = \chi(\tau) dg^5 \qquad \delta(ds^2_{10}) = m(\tau)(g^{(1}g^{3)} + g^{(2}g^{4)})$$

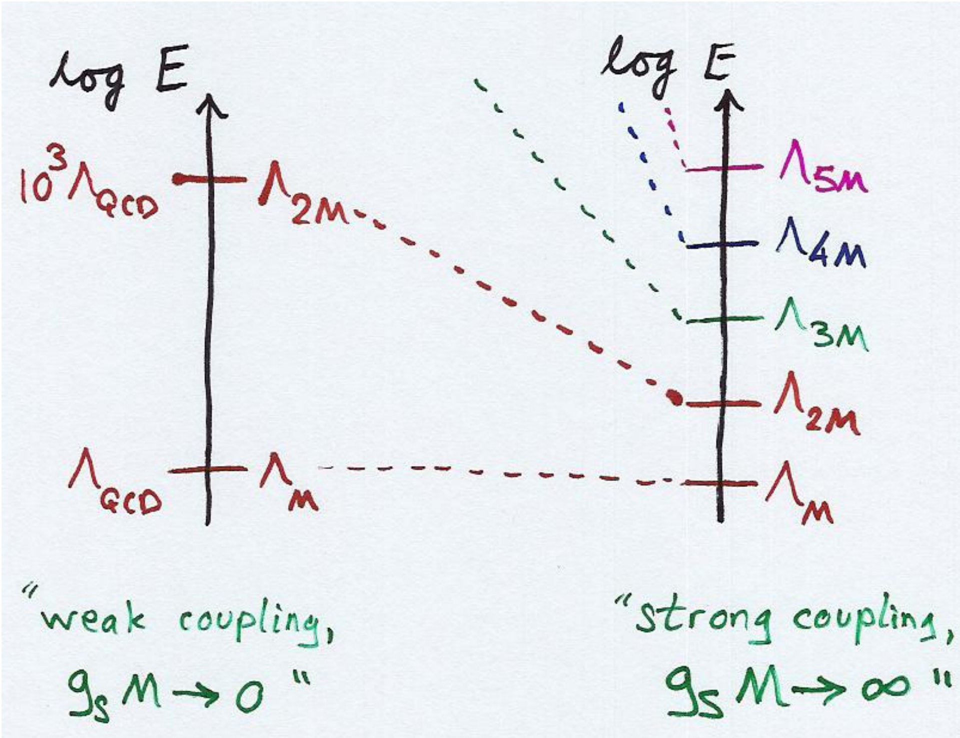
Suitable $z(\tau)$ and $m(\tau)$ can be found which are normalizable.

Turning on $m(\tau)$ is what one does to resolve the conifold. *After* turning on $m(\tau)$, 6-manifold is still Ricci-flat (SUSY, but not CY): resolved warped deformed conifolds.

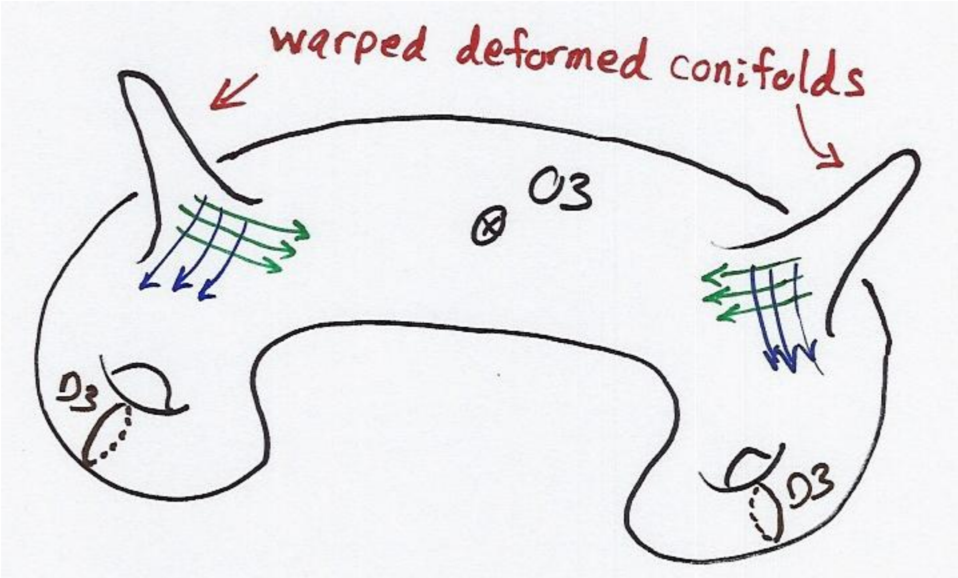
4. Some loose ends

- Haven't shown that our NS-NS perturbation is really a Goldstone superpartner.
- Haven't obtained R-W-D conifolds past first order pertubation theory.
- Didn't explain how earlier intuitions about mass gap went wrong.

Couplings of Goldstone mode are suppressed by $1/\Lambda_{2M}$. In "weak coupling" limit, where scales separate, this amounts to complete decoupling from IR physics.



- Should explain what happens to new modulus after compactification.

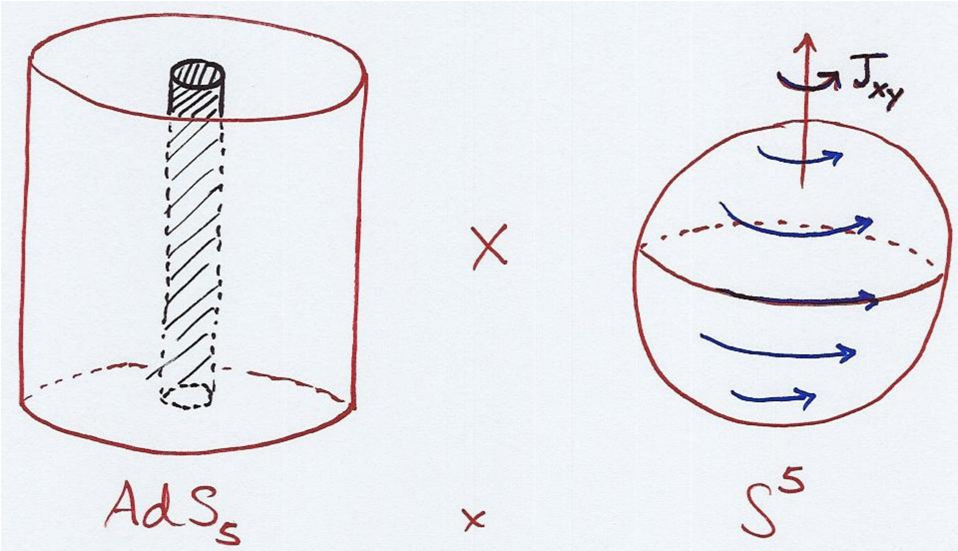


$U(1)_B$ is presumably gauged, so Goldstone boson gets eaten via abelian Higgs mechanism. Modulus acquires a mass m_H because of SUSY.

Preliminary estimates suggest m_H/Λ_{QCD} has power-law behavior in $g_s M$ and $K = \int H_3$.

5. Spinning black holes in AdS and a dimension gap

[Gubser-Heckman 04]
Consider black holes in $AdS_5 \times S^5$ with angular momentum in S^5 directions:

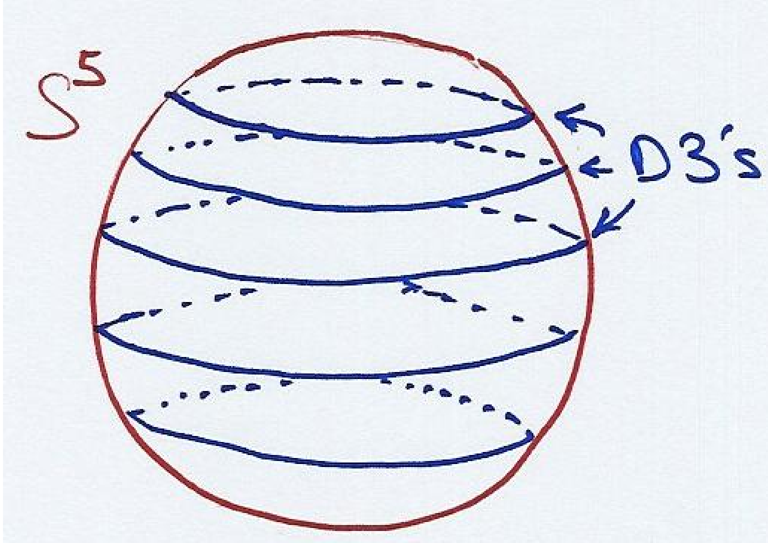


Three independent spins correspond to $U(1)^3 \subset SO(6)$.

The three-charge BH's are an AdS version of the extensively studied D1-D5-KK systems [Strominger-Vafa 96, Callan-Maldacena 96].

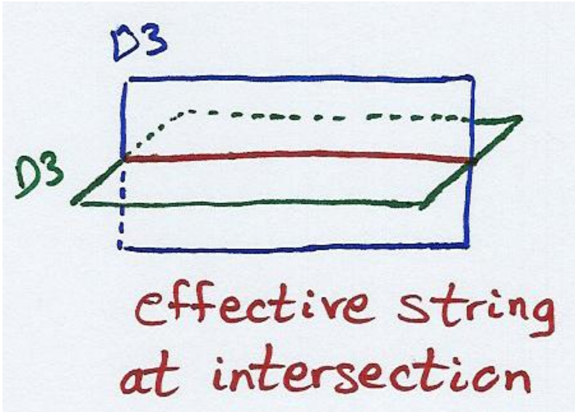
Their salient thermodynamic properties may be roughly understood in terms of effective strings.

The angular momentum is carried by rotating distributions of giant gravitons *[Myers-Tafjord 01]*.



Three angular momenta are carried by three partially orthogonal distributions.

The effective string arises from the intersection of two partially orthogonal D3's:



This picture on a T^5 is literally the Strominger-Vafa story. On an S^5 it's not too different.

But there's something VERY different about these BH's if there's more than one non-zero charge...

Forming a horizon

Horizon radius is determined by largest zero of

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2} \prod_{i=1}^{3} \left(1 + \frac{q_i}{r^2}\right),$$

and translating into CFT quantities gives (for $q_i \ll L^2$)

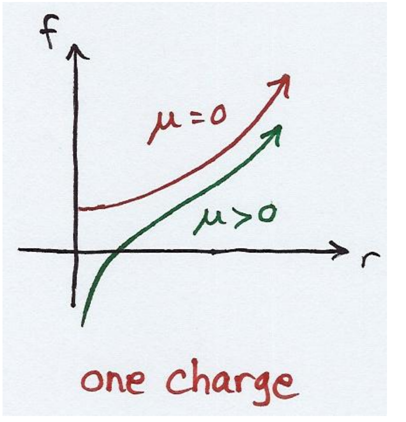
$$\frac{J_i}{N^2} = \frac{1}{2} \frac{q_i}{L^2} \qquad \frac{\Delta - \sum_i J_i}{N^2} = \frac{3}{4} \frac{\mu}{L^2}.$$

If only $q_1 \neq 0$, then for small μ we're discussing operators with low "R-twist" $\Delta - J_1$.

Example: acting with $\operatorname{tr} Z_1^{J_1}$ preserves $1/2$ of SUSY. Here $\Delta = J_1$.

Finite $\Delta - J_1$ leads to $\sim e^{(\Delta - J_1)/TL}$ operators, corresponding to a black hole horizon forming in AdS_5.

T is the Hagedorn temperature of the effective string—roughly calculable in giant graviton picture.



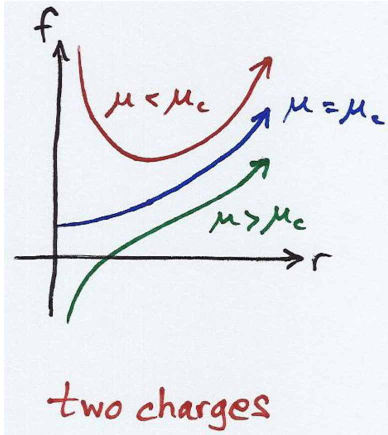
And $T_{\text{Hagedorn}} = T_{\text{Hawking}}$.

The dimension gap

But if q_1 and q_2 are non-zero, then only if $\mu > \mu_c = q_1 q_2 / L^2$ is there a horizon *[Berhndt-Chamseddine-Sabra 98]*.

$\mu = 0$ is again SUSY.

Similar behavior arises for three-charge case.



The dimension gap is

$$\frac{\Delta_c - J_1 - J_2}{N^2} = \frac{3}{4}\frac{\mu_c}{L^2} = \frac{3}{4}\frac{q_1}{L^2}\frac{q_2}{L^2} = 3\frac{J_1}{N^2}\frac{J_2}{N^2} \, .$$

For $J_1 + J_2 < \Delta < \Delta_c$ there are insufficiently many operators to correspond to BH entropy.

For $\Delta > \Delta_c$ there are qualitatively more operators.

6. Conclusions

- AdS/CFT exhibits many if not all of the distinctive dynamical features of 4d gauge theories.

- Confinement without a mass gap from warped deformed conifold is a good example: strong gauge dynamics meets Goldstone's theorem.

- But supergravity has various ways of telling us that a clean analytic description of large N confinement requires us to better understand string theory.

- Example: compression of scales prevents Goldstone from decoupling.

- The dimension gap for R-charged black holes shows we still have much to learn even about $\mathcal{N} = 4$ super-Yang-Mills theory.

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