Small-x, Diffraction and QCD Results from HERA

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DESY

QCD and String Theory
Santa Barbara
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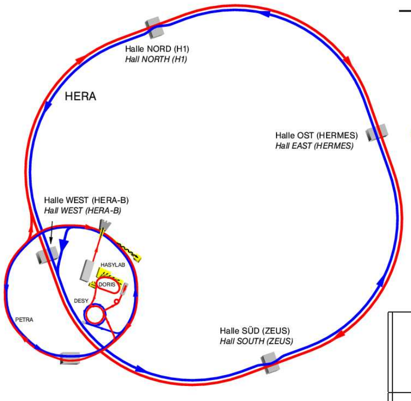
HERA electron-proton collider at DESY



circumference of HERA: 6 km

27.5 GeV e → ← **920 GeV p**

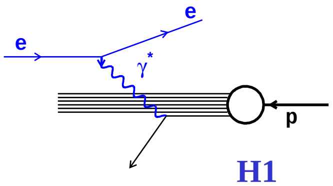
e p c.m. energy $s^{1/2}$ = 318 GeV

equivalent to fixed target experiment with

54000 GeV e → **p at rest**

equivalent to 1000 SLACs in series on fixed target

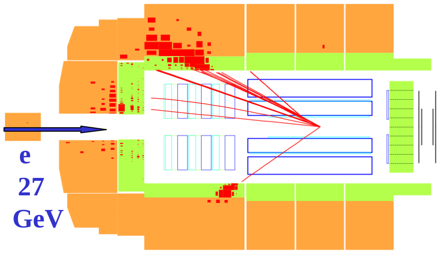
run period	e beam	lumi (pb^{-1}) ZEUS
1994 - 1997	e^+	48
1999 - 2000	e^+	66
1998 - 1999	e^-	17
2003 - 2004	e^+, polarised	45

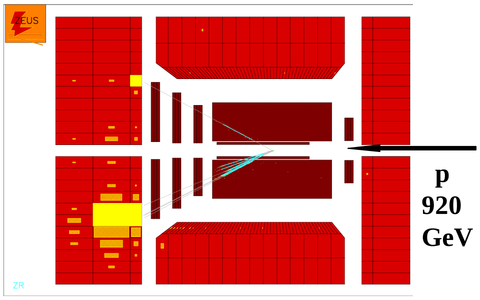


Small-x and Diffraction
Q^2 ~ 2 –100 GeV^2 and 0.05-0.6 GeV^2

H1

ZEUS





Liquid Argon Calorimeter

Uranium-Scintillator Calorimeter

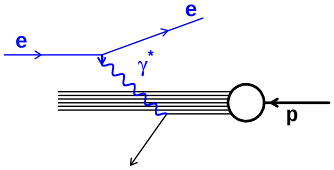
Q^2 - virtuality of the incoming photon
W - CMS energy of the incoming photon-proton system
x - Fraction of the proton momentum carried by the struck quark $x \sim Q^2/W^2$

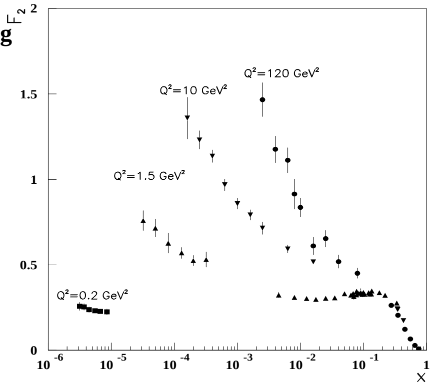
$$\frac{d^2\sigma^{eP}}{dxdQ^2} = \frac{2\pi\alpha_{em}^2}{xQ^4}\cdot[Y\_F_2(x,Q^2) - Y\_xF_3(x,Q^2) - y^2F_L(x,Q^2)]$$
$$Y_\pm = 1 \pm (1-y)^2$$

y – inelasticity
$Q^2 = sxy$

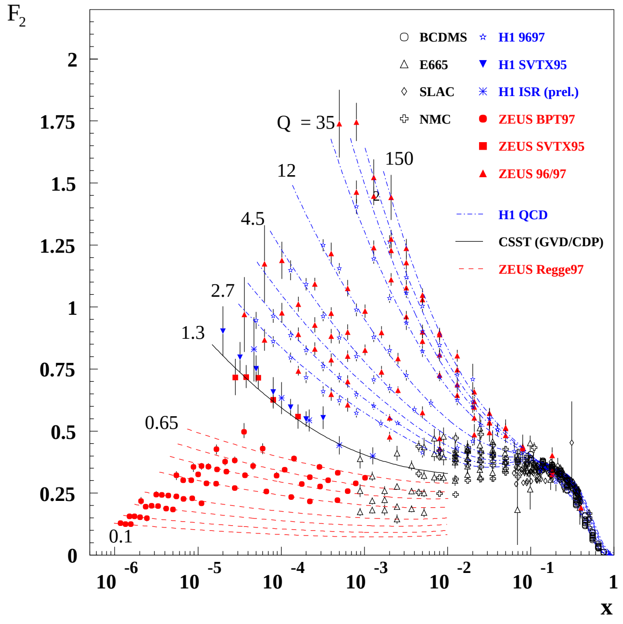
Infinite momentum frame

Proton looks like a cloud of non-interacting quarks and gluons

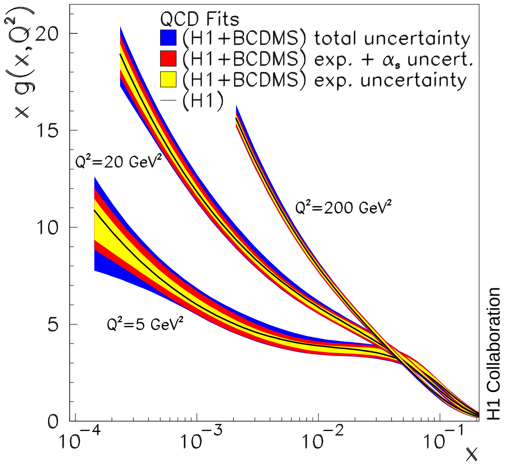




F_2 measures parton density in the proton at a scale Q^2



Gluon density



Gluon density dominates F_2 for *x* < 0.01

Dominant part of evolution eq.

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \int_x^1 \frac{dz}{z} P_{gg}(z,Q^2)\, g\left(\frac{x}{z},Q^2\right)$$

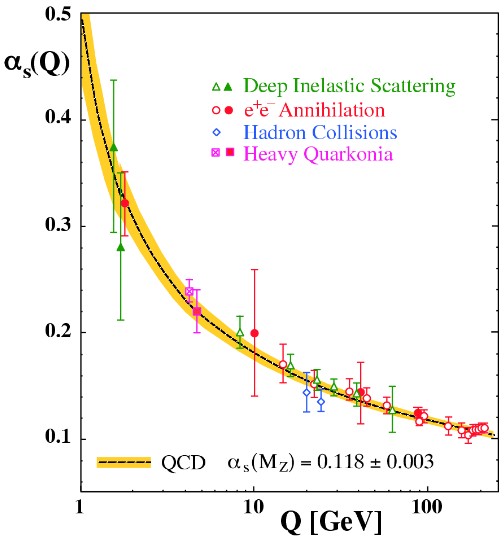
LO Gluon splitting function at low x

$$P_{gg} \approx \frac{\alpha_s(Q^2)}{2\pi}\frac{2C_A}{z}$$

$$C_A = 3$$

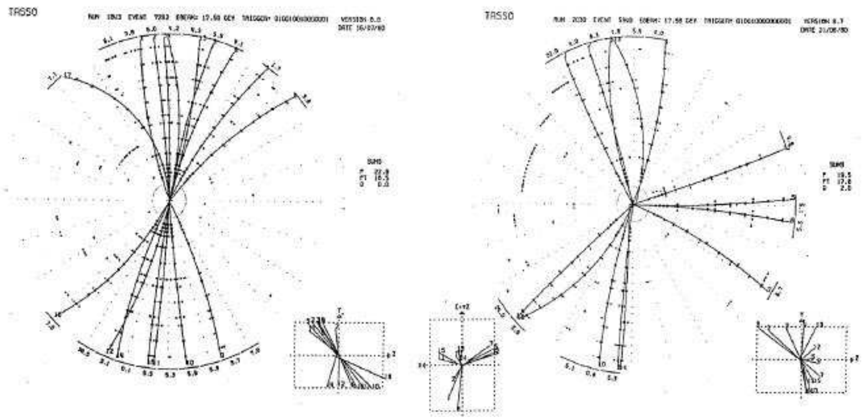
Non-Abelian structure of QCD

Asymptotic Freedom



Nobel Prize for Physics 2004
D. Gross
D. Politzer
F. Wilczek

Experimental Discovery of the Gluon
DESY 1979
TASSO Experiment

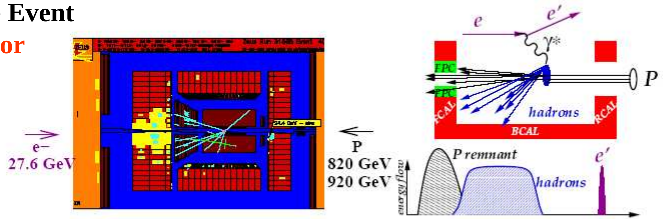


European Physical Society Prize 1995
P. Soeding, B. Wiik, G. Wolf, S.L. Wu

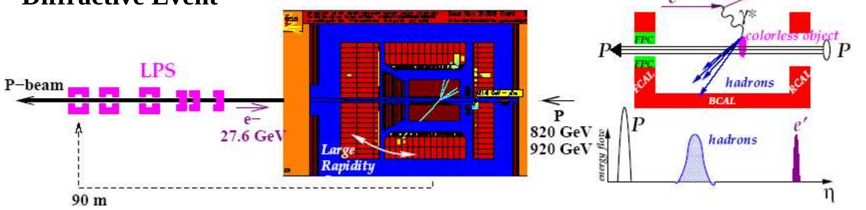
Diffractive Scattering

Non-Diffractive Event

ZEUS detector



Diffractive Event



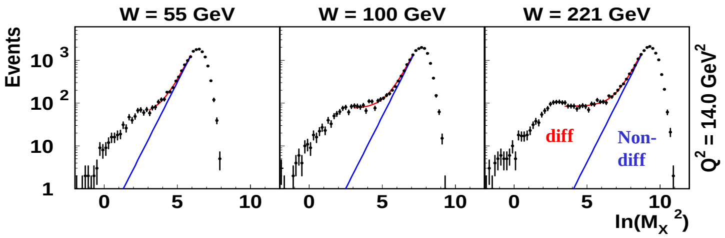
M_X - invariant mass of all particles seen in the central detector

t - momentum transfer to the diffractively scattered proton

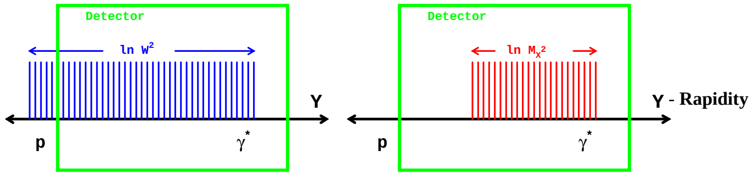
t - conjugate variable to the impact parameter

Diffractive Signature

*



Non-Diffraction **Diffraction**

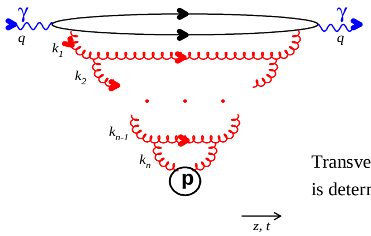


uniform, uncorrelated particle emission along the rapidity axis =>
probability to see a gap ΔY is ~ exp(-<n>ΔY)
<n> - average multiplicity per unit of rapidity

$dN/dM^2_X \sim 1/M^2_X$ =>
$dN/d\log M^2_X \sim$ const
note : $\Delta Y \sim \log(W^2/M^2_X)$

Slow Proton Frame

incoming virtual photon fluctuates into a quark-antiquark pair which in turn emits a cascade-like cloud of gluons



$$\tau_{qq} \approx \frac{1}{\Delta E} \approx \frac{1}{m_p x} \approx 10\text{–}1000 \text{ fm}$$

Transverse size of the quark-antiquark cloud is determined by $r \sim 1/Q \sim 2\ 10^{-14}\text{cm}/\ Q\ (GeV)$

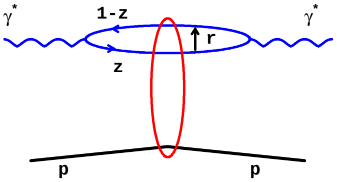
$$\sigma_{tot}^{\gamma p} = \frac{1}{W^2} ImA_{el}(W^2, t=0)$$

$$\sigma_{tot}^{\gamma^* P}(W, Q^2) = \frac{4\pi^2 \alpha_{em}}{Q^2} \cdot F_2(x, Q^2)$$

Rise of $\sigma^{\gamma p}_{tot}$ with *W* is a measure of radiation intensity

Diffraction is similar to the elastic scattering:
replace the outgoing photon by the diffractive final state
ρ, J/Ψ or X = two quarks

Dipole description of DIS



$$\sigma_{tot}^{\gamma^* p} = \int d^2\hat{\vec{r}} \int_0^1 dz \Psi(Q^2, z, \vec{r})^* \sigma_{q\bar{q}}(x, r^2) \Psi(Q^2, z, \vec{r})$$

$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt}\Big|_{t=0} = \frac{1}{16\pi} \left| \int d^2\vec{r} \int_0^1 dz \Psi_{VM}^*(Q^2, z, \vec{r})\ \sigma_{q\bar{q}}(x, r^2) \Psi(Q^2, z, \vec{r}) \right|^2$$

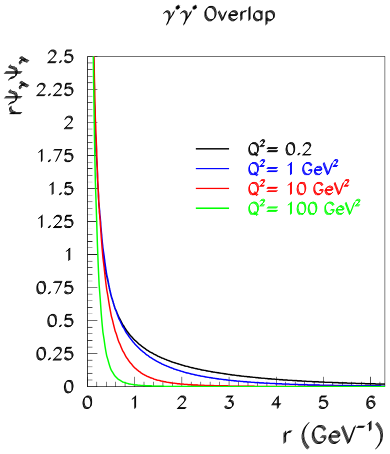
$$\frac{d\sigma_{diff}^{\gamma^* p}}{dt}\Big|_{t=0} = \frac{1}{16\pi} \int d^2\vec{r} \int_0^1 dz\, \Psi^*(Q^2, z, \vec{r}) \sigma_{q\bar{q}}^2(x, r^2) \Psi(Q^2, z, \vec{r})$$

$\Psi(z, \vec{r})$	$\gamma \rightarrow q\bar{q}$ QCD wave function
$\sigma_{q\bar{q}}(x, r^2)$	cross section for scattering of $q\bar{q}$ pair on proton (dipol cross section)

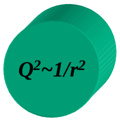
$$\sigma_{T,L}^{\gamma^* P}(x,Q^2)=\int_0^\infty d^2\vec{r}\int_0^1 dz\sum_f\left|\Psi_{T,L}^f(\vec{r},z,Q^2)\right|^2\sigma_{qq}(x,r)$$

$$\left|\Psi_T^f(\vec{r},z,Q^2)\right|^2=\frac{3\alpha_{em}}{2\pi^2}e_q^2\{[z^2+(1-z)^2]\varepsilon^2K_1^2(\varepsilon r)+m_q^2K_0^2(\varepsilon r)\}$$

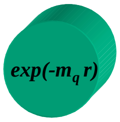
$$\left|\Psi_L^f(\vec{r},z,Q^2)\right|^2=\frac{3\alpha_{em}}{2\pi^2}e_q^2\{4Q^2z^2(1-z)^2K_0^2(\varepsilon r)\}\qquad \varepsilon^2=z(1-z)Q^2+m_q^2$$



$K_1(\varepsilon r)=1/\varepsilon r$ for $\varepsilon r \ll 1$



$K_1(\varepsilon r)=\sqrt{\pi/2x}\,\exp(-\varepsilon r)$ for $\varepsilon r \gg 1$

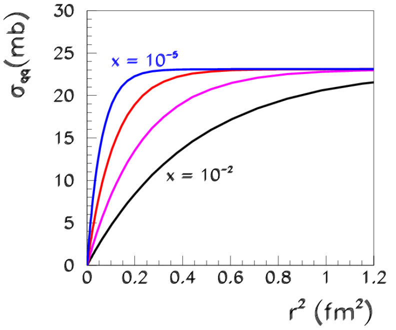


GBW Model

K. Golec-Biernat, M. Wuesthoff

$$\sigma_{qq}(x,r)=\sigma_0[1-\exp(-\frac{r^2}{4R_0^2})]$$

$$R_0^2(x)=\frac{1}{GeV^2}\left(\frac{x}{x_0}\right)^\lambda$$



Scaling in

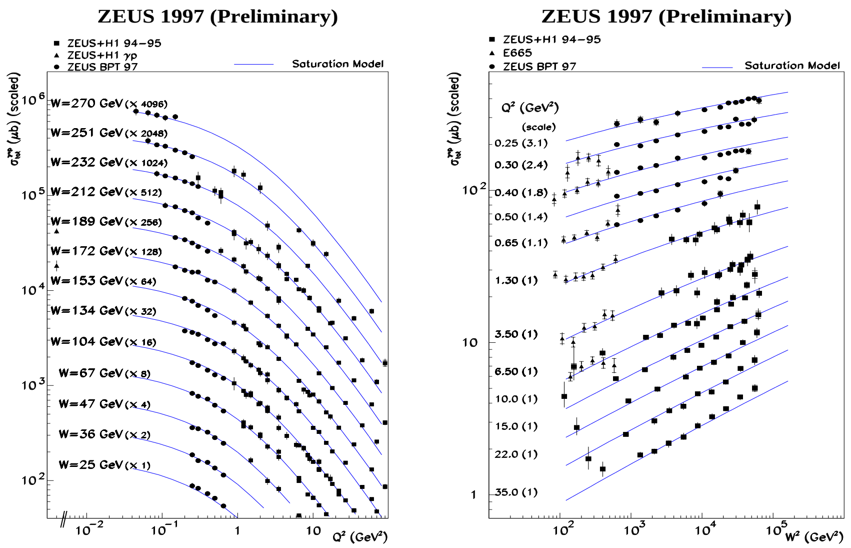
r^2/R_0^2

$\Rightarrow$

Geometrical Scaling

A. Stasto & Golec-Biernat
J. Kwiecinski

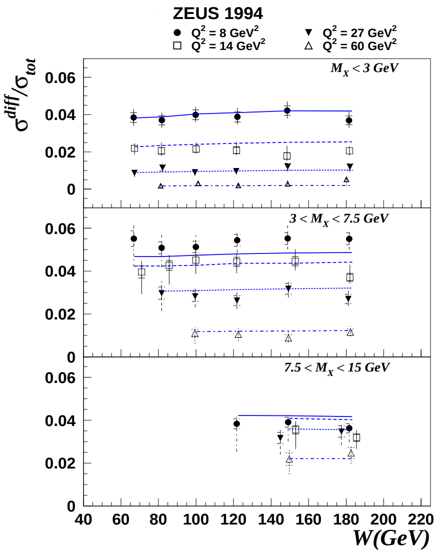
Parameters fitted to DIS F_2 data:
σ_0 = 23 mb λ = 0.29 x_0 = 0.0003

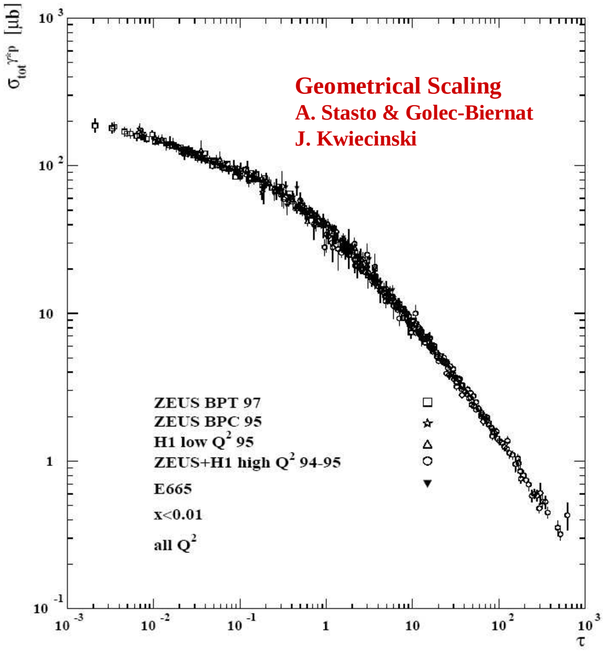


Parameters fitted to HERA DIS data: $\chi^2/N \sim 1$

$\sigma_0 = 23$ mb $\lambda = 0.29$ $x_0 = 0.0003$

Saturation Model
Predictions for Diffraction





Geometrical Scaling

A. Stasto & Golec-Biernat

J. Kwiecinski

$$\tau = Q^2 R_0^2(x)$$

$$R_0^2(x) = \frac{1}{GeV^2}\left(\frac{x}{x_0}\right)^{\lambda}$$

GBW model, in spite of its compelling success has some obvious shortcomings:

The treatment of QCD evolution is only rudimentary

remedy => incorporate DGLAP into dipole cross-section

J. Bartels, K. Golec-Biernat, H. Kowalski

$$\sigma_{qq}(x,r) \approx \sigma_0(1-\exp(-\frac{r^2}{R_0^2})); \qquad R_0^2 = \frac{1}{GeV^2}\left(\frac{x}{x_0}\right)^{\lambda_{GBW}}$$

$$\sigma_{qq}(x,r) \approx \sigma_0(1-\exp(-\frac{\pi^2}{3\sigma_0}r^2\alpha_s xg(x,\mu^2 = C/r^2+\mu_0^2)))$$

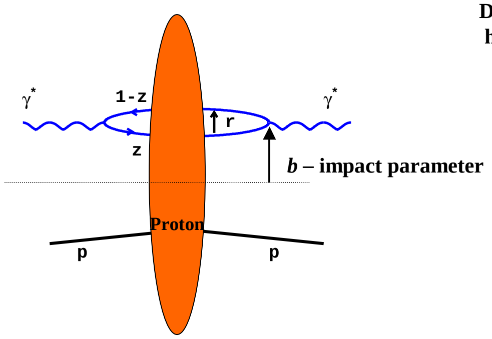
Recently: BFKL motivated Ansatz proposed by

Iancu, Itacura, Munier

The dipole cross section is integrated over the transverse coordinate although the gluon density is expected to be a strongly varying function of the impact parameter.

Impact Parameter Dipole Saturation Model

H. Kowalski
D. Teaney
hep-ph/0304189



well motivated:

Glauber
Mueller
Levin
Capella
Kaidalov

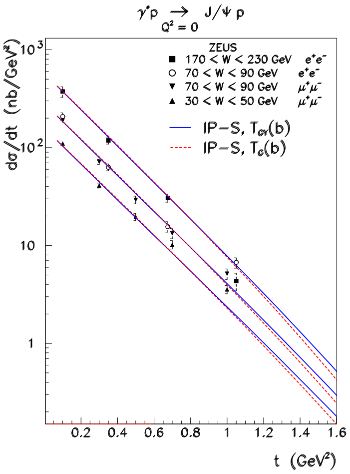
$$\frac{d\sigma_{qq}(x,r)}{d^2b} = 2\cdot\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3}r^2\alpha_s(\mu^2)xg(x,\mu^2)T(b)\right)\right\}$$

***T(b)* - proton shape** $\int_0^\infty T(b)d^2b = 1$

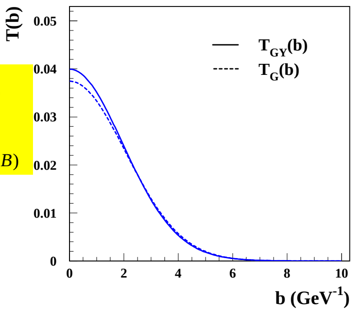
t-dependence of the diffractive cross sections determines the *b* distribution

$t=-\vec{\Delta}^2$ $\vec{\Delta}$ - transv. momentum (2-d) $\vec{b}$ - impact parameter (2-d)

$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt} = \frac{1}{16\pi}\left|\int d^2\vec{r}\int d^2b e^{-i\vec{b}\vec{\Delta}}\int_0^1 dz\Psi_{VM}^*(Q^2,z,\vec{r})\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3}r^2\alpha_s xg(x,\mu^2)T(b)\right)\right\}\Psi(Q^2,z,\vec{r})\right|^2$$



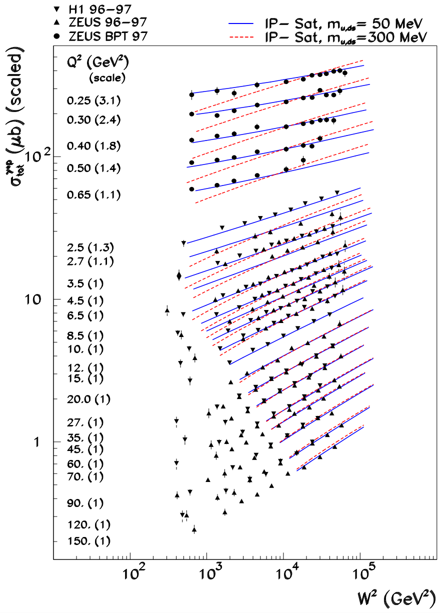
$$\frac{d\sigma^{diff}}{dt}\sim\exp(B\cdot t)$$
$$\Rightarrow$$
$$T(b)\sim\exp(-\vec{b}^2/2B)$$



$T_G(b)\propto\exp(-\vec{b}^2/2w_G)$ $w_G = 4.25\ \text{GeV}^2$

$T_{GY}(b)\propto\int d^2b'\exp(-(\vec{b}-\vec{b}')^2/2w_G)K_0(b'/w_E)$

$$\sigma^{\gamma^* p} = \sum_f \int d^2\vec{r} \int d^2 b \int_0^1 dz \Psi_f^*(Q^2,z,\vec{r}) \left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3} r^2 \alpha_s xg(x,\mu^2) T(b)\right)\right\} \Psi_f(Q^2,z,\vec{r})$$



$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$

$$xg(x,\mu_0^2) = A_g \left(\frac{1}{x}\right)^{\lambda_g} (1-x)^{5.6}$$

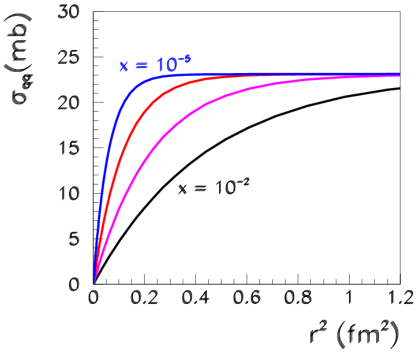
$Q^2 > 0.25$ GeV^2
$m_u = 0.05$ GeV
$m_c = 1.30$ GeV

Fit parameters
$\lambda_g = -0.12$ $C = 4.0$
$Q_0^2 = 0.8$ GeV^2
$\chi^2/N = 0.8$

$x < 10^{-2}$

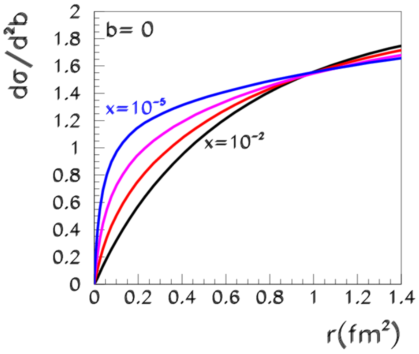
GBW Model

$$\sigma_{qq}(x,r) = \sigma_0[1-\exp(-\frac{r^2}{4R_0^2})]$$



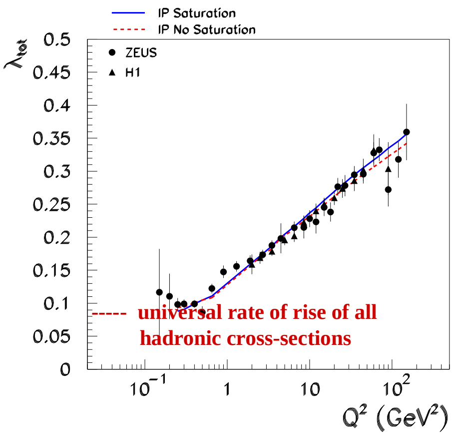
IP Saturation Model

$$\frac{d\sigma_{qq}(x,r)}{d^2 b} = 2\cdot\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3} r^2 \alpha_s xg(x, C/r^2 + Q_0^2) T(b)\right)\right\}$$

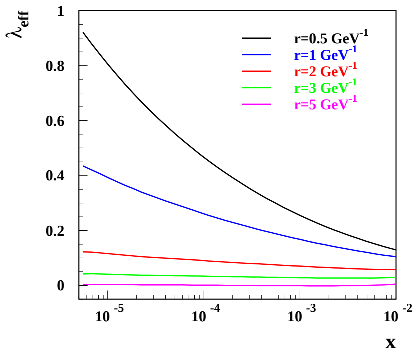


$\sigma^{\gamma^* p} \sim (W^2)^{\lambda_{tot}(Q^2)} \sim (1/x)^{\lambda_{tot}(Q^2)}$

$xg(x,\mu^2) \sim (1/x)^{\lambda_{eff}(\mu^2(r))}$

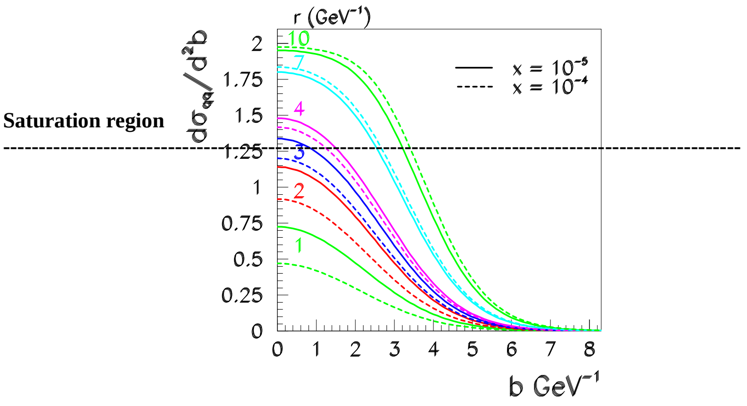


----- universal rate of rise of all hadronic cross-sections



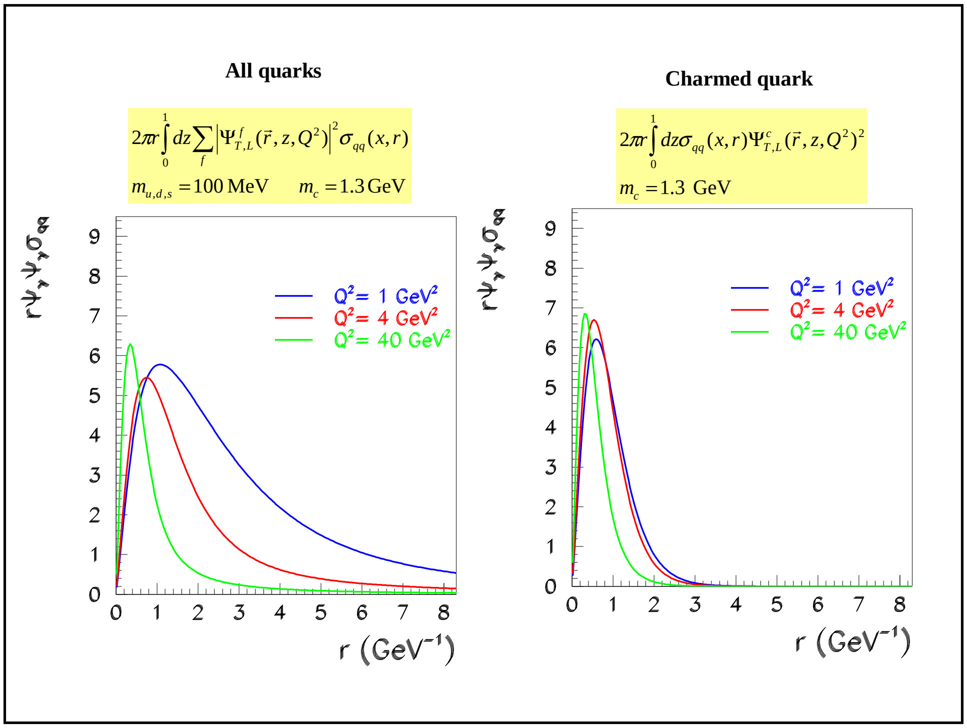
Smaller dipoles ⇔ steeper rise

Large spread of λ_{eff} characteristic for Impact Parameter Dipole Models



Saturation region

$$\frac{d\sigma_{qq}(x,r)}{d^2b} = 2\cdot\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3}r^2\alpha_s xg(x,\mu^2)T(b)\right)\right\}$$



All quarks

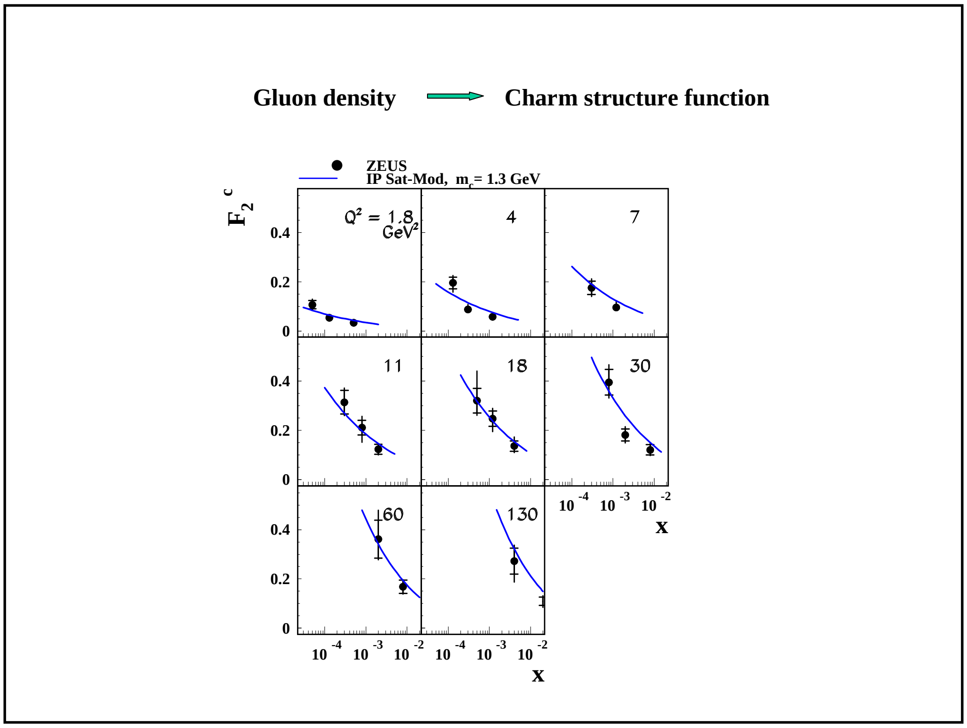
$$2\pi r\int_0^1 dz\sum_f \left|\Psi^f_{T,L}(\vec{r},z,Q^2)\right|^2\sigma_{qq}(x,r)$$

$m_{u,d,s} = 100\,\mathrm{MeV}$ $m_c = 1.3\,\mathrm{GeV}$

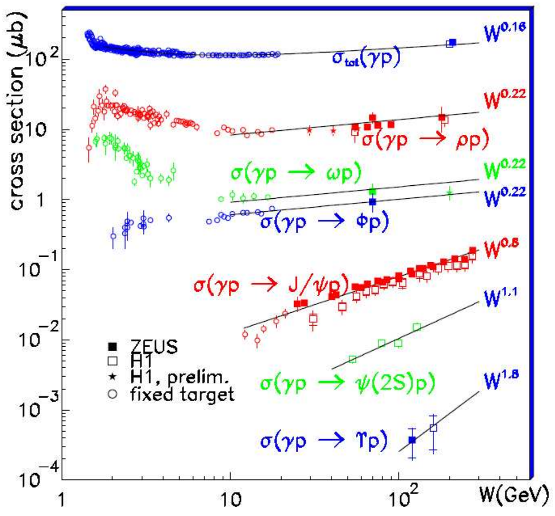
Charmed quark

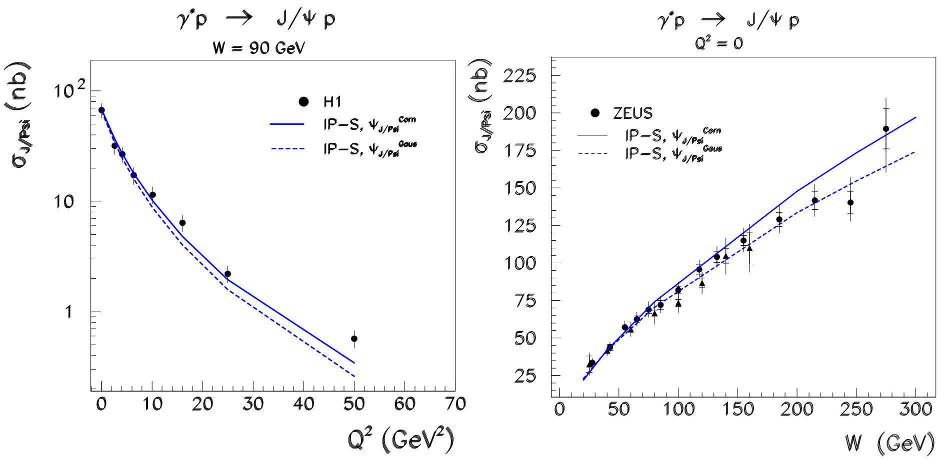
$$2\pi r\int_0^1 dz\sigma_{qq}(x,r)\Psi^c_{T,L}(\vec{r},z,Q^2)^2$$

$m_c = 1.3$ GeV

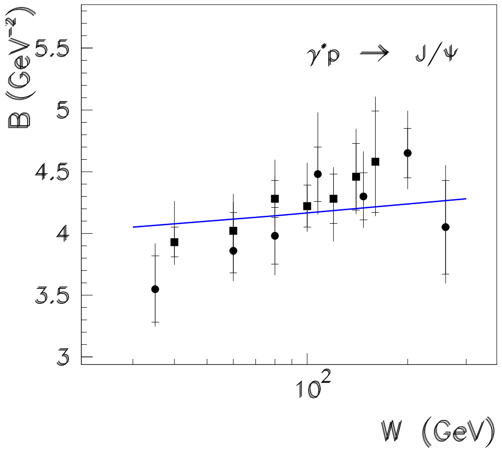


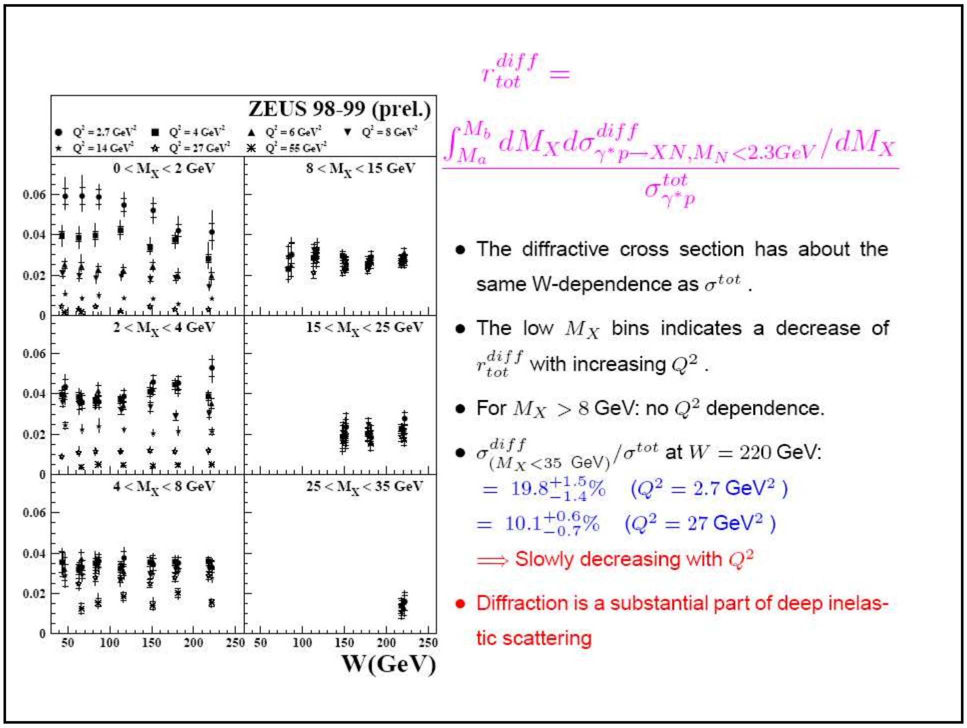
Gluon density → **Charm structure function**





Absolute values of cross sections are strongly dependent on m_c





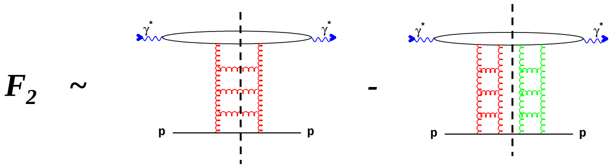
$$r^{diff}_{tot} = \frac{\int_{M_a}^{M_b} dM_X d\sigma^{diff}_{\gamma^* p \to XN, M_N < 2.3GeV} / dM_X}{\sigma^{tot}_{\gamma^* p}}$$

- The diffractive cross section has about the same W-dependence as σ^{tot} .
- The low M_X bins indicates a decrease of r^{diff}_{tot} with increasing Q^2 .
- For $M_X > 8$ GeV: no Q^2 dependence.
- $\sigma^{diff}_{(M_X < 35\ \mathrm{GeV})} / \sigma^{tot}$ at $W = 220$ GeV:
  $= 19.8^{+1.5}_{-1.4}\%$ ($Q^2 = 2.7$ GeV^2)
  $= 10.1^{+0.6}_{-0.7}\%$ ($Q^2 = 27$ GeV^2)
  $\Longrightarrow$ Slowly decreasing with Q^2
- Diffraction is a substantial part of deep inelastic scattering

Absorptive correction to F_2

Example in Dipole Model

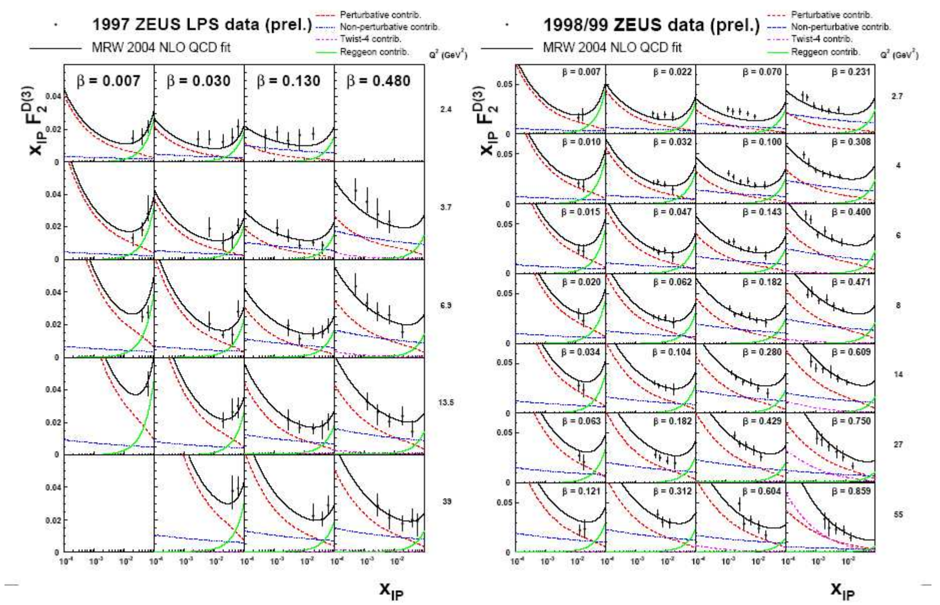
$$\frac{d\sigma}{d^2b} = 2(1-\exp(-\Omega/2)) = \Omega - \Omega^2/4 + \ldots.$$



Single inclusive
pure DGLAP

Diffraction

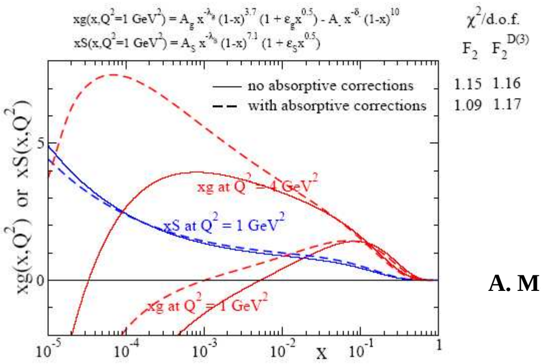
Fit to diffractive data using MRST Structure Functions
A. Martin M. Ryskin G. Watt



$$\boxed{F_2^{\rm data}(x_B,Q^2)=F_2^{\rm DGLAP}(x_B,Q^2)+\Delta F_2^{\rm abs}(x_B,Q^2)}$$

$$\boxed{\Delta F_2^{\rm abs}(x_B,Q^2)\simeq-\int_{x_B}^{0.1}{\rm d}x_{I\!P}\,\left[F_{2,P}^{D(3)}(x_{I\!P},\beta,Q^2)+F_{L,P}^{D(3)}(x_{I\!P},\beta,Q^2)\right]}$$

Gluon and sea quark PDFs



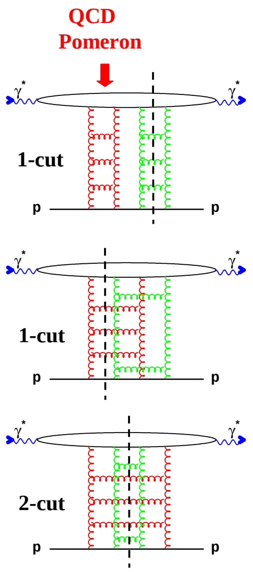
A. Martin M. Ryskin
G. Watt

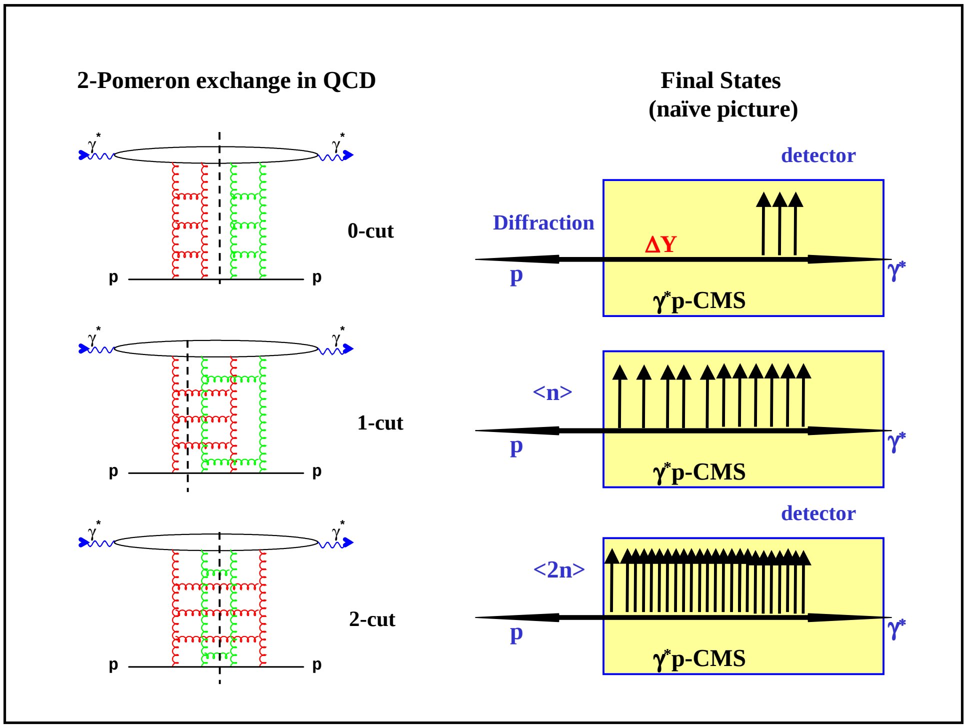
AGK Rules

The cross-section for k-cut pomerons:
Abramovski, Gribov, Kancheli
Sov. ,J., Nucl. Phys. 18, p308 (1974)

$$\sigma_k=\sum_{m=k}^{\infty}(-1)^{m-k}\,2^m\,\frac{m!}{k!(m-k)!}F^{(m)}$$

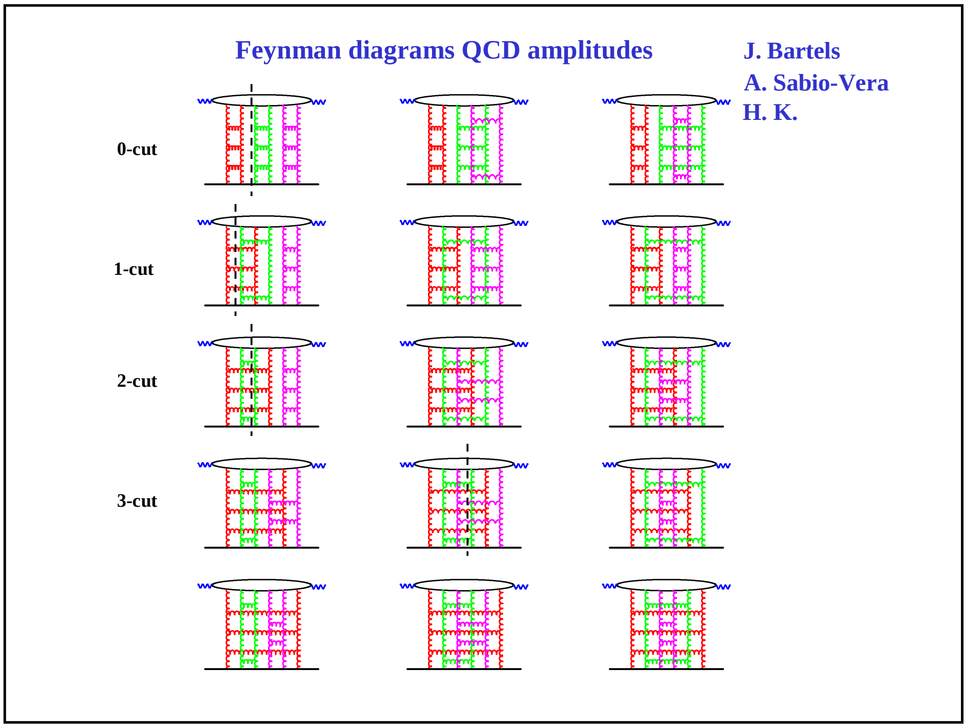
F $^{(m)}$ **– amplitude for the exchange of *m* Pomerons**





2-Pomeron exchange in QCD

Final States
(naïve picture)



Feynman diagrams QCD amplitudes

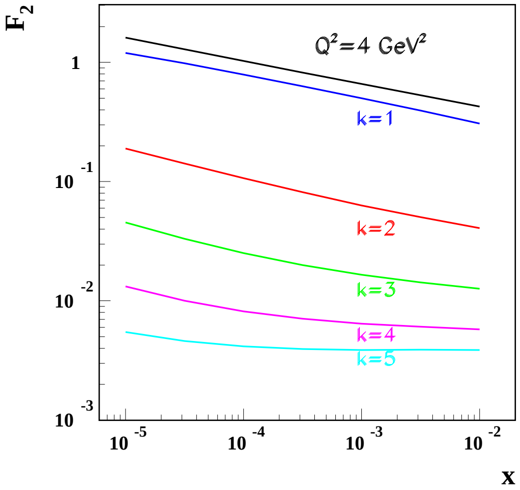
J. Bartels
A. Sabio-Vera
H. K.

Probability of *k*-cut in HERA data

Dipole Model

$$\sigma^{\gamma^* p} = \sum_f \int d^2\vec{r} \int d^2 b \int_0^1 dz \Psi_f^*(Q^2, z, \vec{r})\ 2\left\{1 - \exp(-\frac{\Omega}{2})\right\}\Psi_f(Q^2, z, \vec{r})$$

$$\sigma_k^{\gamma^* p} = \sum_f \int d^2\vec{r} \int d^2 b \int_0^1 dz \Psi_f^*(Q^2, z, \vec{r})\ \frac{\Omega^k}{k!}\exp(-\Omega)\Psi_f(Q^2, z, \vec{r})$$



Problem of DGLAP QCD fits to F_2

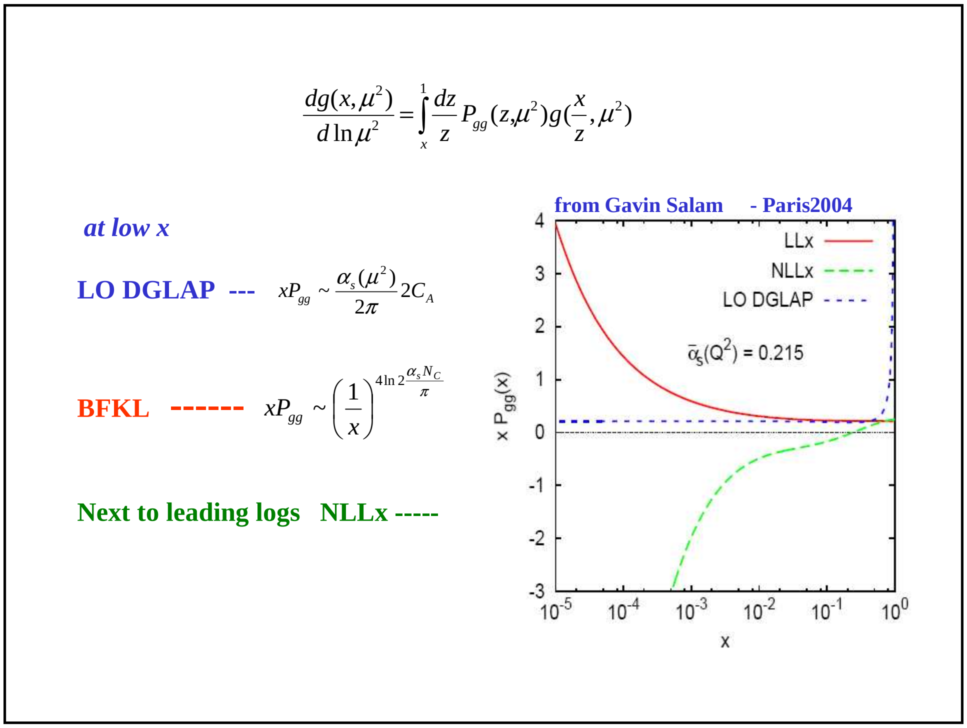
CTEQ, MRST, …., IP-Dipole Model

$xg(x, \mu_0^2) \sim x^\lambda, \quad \lambda > 0$ **at small *x***

valence like gluon structure function ?

Remedy: **Absorptive corrections?** **MRW**

Different evolution? **BFKL, CCSS, ABFT**



$$\frac{dg(x,\mu^2)}{d\ln\mu^2} = \int_x^1 \frac{dz}{z} P_{gg}(z,\mu^2)\, g(\frac{x}{z},\mu^2)$$

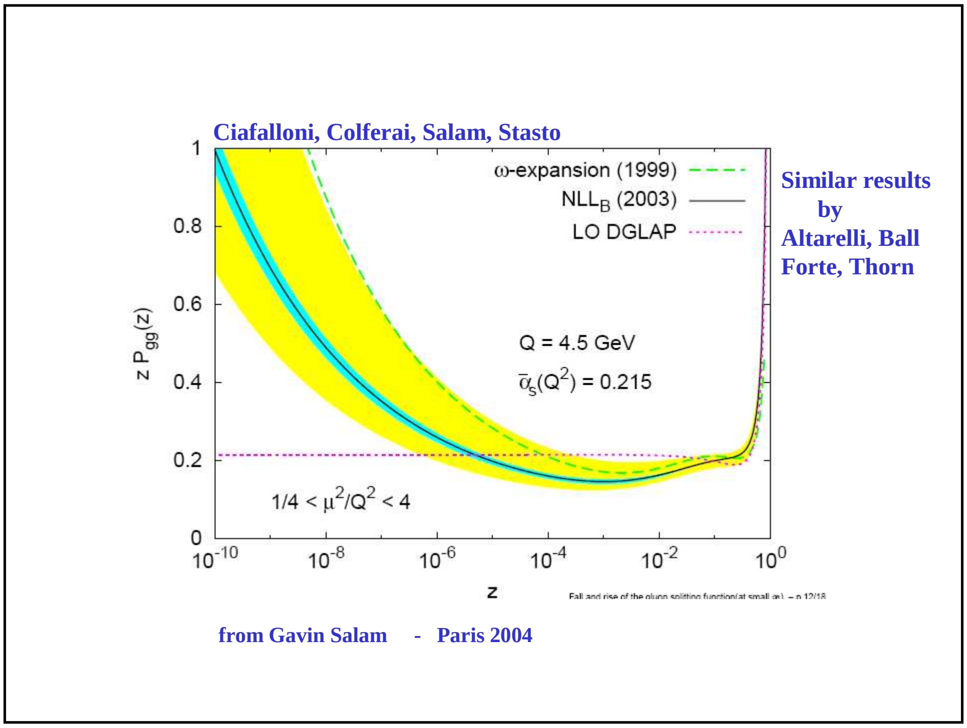
at low x

LO DGLAP --- $xP_{gg} \sim \frac{\alpha_s(\mu^2)}{2\pi} 2C_A$

BFKL ------ $xP_{gg} \sim \left(\frac{1}{x}\right)^{4\ln 2\frac{\alpha_s N_C}{\pi}}$

Next to leading logs NLLx -----

from Gavin Salam - Paris2004



Ciafalloni, Colferai, Salam, Stasto

Similar results by Altarelli, Ball Forte, Thorn

from Gavin Salam - Paris 2004

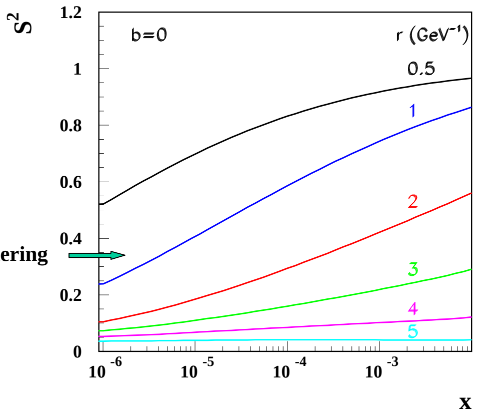
Density profile $D = \frac{2\pi^2}{3}\alpha_s(\mu^2(r))xg(x,\mu^2(r))T(b)$ *grows with diminishing x and r*

approaches a constant value

Saturated State - Color Glass Condensate

S – Matrix => interaction probability
Saturated state = high interaction probability
S^2 => 0

$$S^2 = \exp\left(-D\cdot\frac{r^2}{2}\right)$$



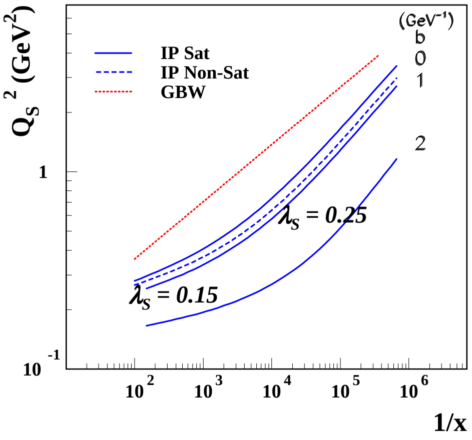
multiple scattering

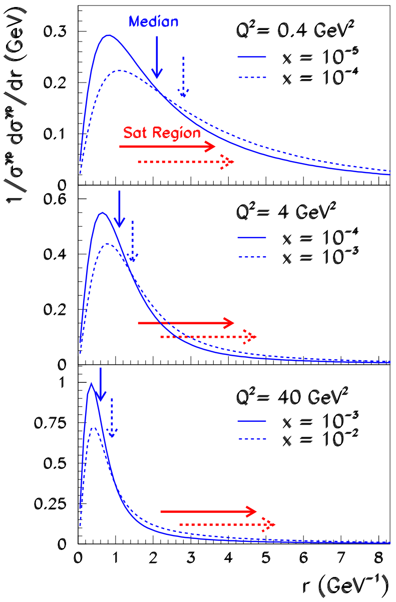
$$S^2 = e^{-1}$$

r_S - dipole size for which proton consists of one int. length

Q_S^2 – **Saturation scale** **= Density profile at the saturation radius r_S**

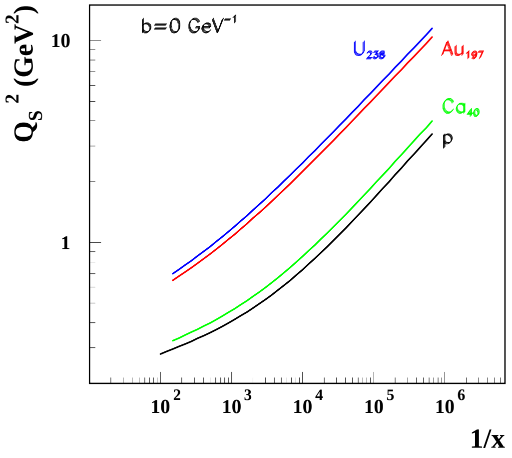
$$Q_S^2 = \frac{2}{r_S^2}$$





Saturated state
is partially perturbative

cross-section
exhibits the universal rate of growth



$$(Q_S^2)_g = \frac{N_C}{C_F}(Q_S^2)_q = \frac{9}{4}(Q_S^2)_q$$

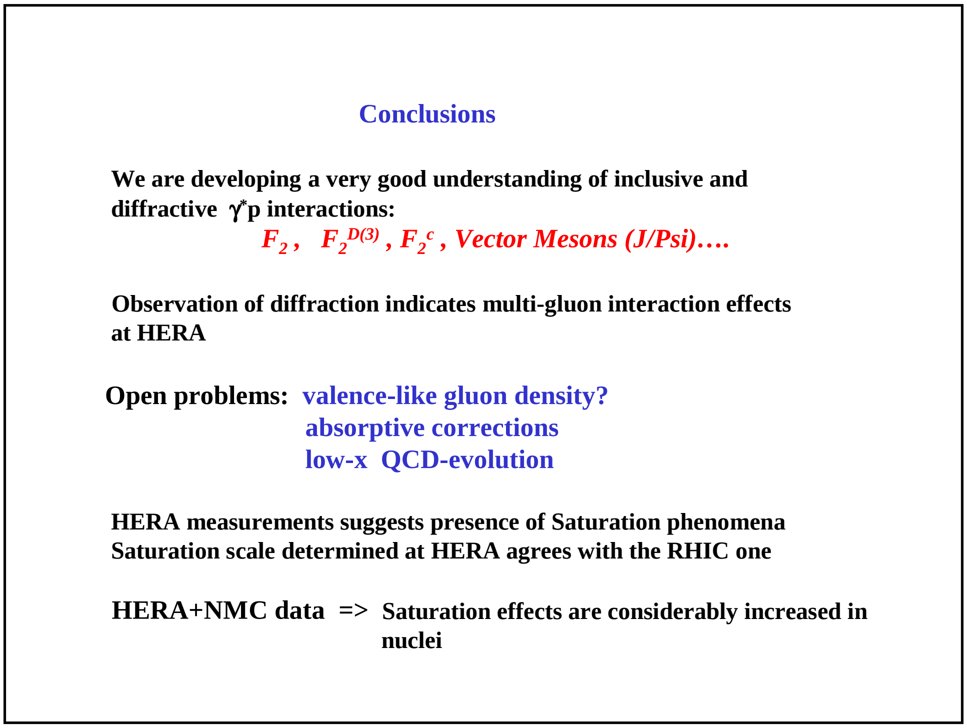
RHIC

$$Q_S^2 = \frac{4\pi^2\alpha_s N_C}{N_C^2-1}\frac{1}{\pi R^2}\frac{dN}{dy}$$

$$\frac{dN}{dy} \approx 1000 \qquad R \approx 7\ \mathrm{fm}$$

$$Q_S^2 \approx 1$$

$$(Q_S^2)_{RHIC} \cong (Q_S^2)_{HERA}$$



Conclusions

We are developing a very good understanding of inclusive and diffractive $\gamma^{*}p$ interactions:

F_2 , $F_2^{D(3)}$, F_2^{c} , *Vector Mesons (J/Psi)….*

Observation of diffraction indicates multi-gluon interaction effects at HERA

Open problems: **valence-like gluon density?**
absorptive corrections
low-x QCD-evolution

HERA measurements suggests presence of Saturation phenomena
Saturation scale determined at HERA agrees with the RHIC one

HERA+NMC data => **Saturation effects are considerably increased in nuclei**