QCD and Strings, KITP

The Elusive Pentaquark

Leonardo Rastelli

Renaissance of hadron spectroscopy

New mesons (η_c', D_{sJ}, h_c, X)

Exotic baryons?!

Of course, QCD is correct. But we are still far from deriving observed spectroscopy from first principles.

These new discoveries are a powerful reminder of how little we know.

(Absence of) Exotics

A mysterious aspect of QCD is the absence of "exotic" hadrons

Configurations with valence (=minimum) quark structure $q\bar{q}$ and qqq account for all observed states. Gell-Mann 1964

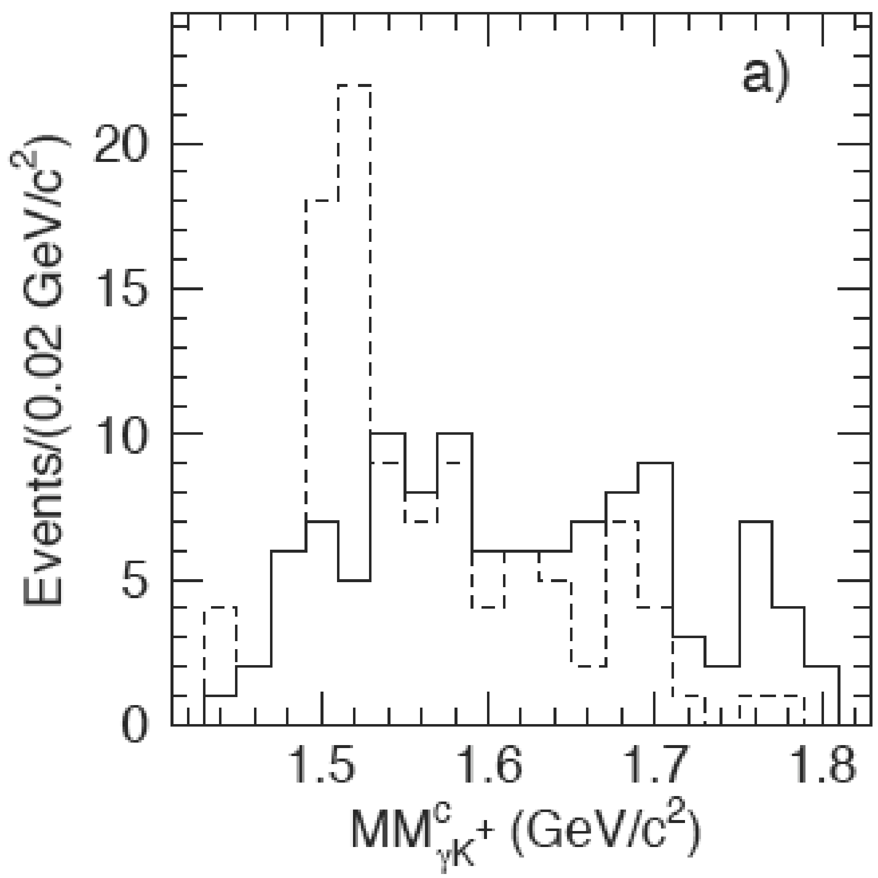
$p = uud$, $n = udd$, $K^+ = u\bar{s}$, $K^- = \bar{u}s$, $K^0 = d\bar{s}$, $\bar{K}_0 = \bar{d}s$ ...

Or they did till January 2003, when LEPS announced the discovery of an $S = +1$ baryon resonance Z^* (now called Θ^+), surprisingly light and narrow.

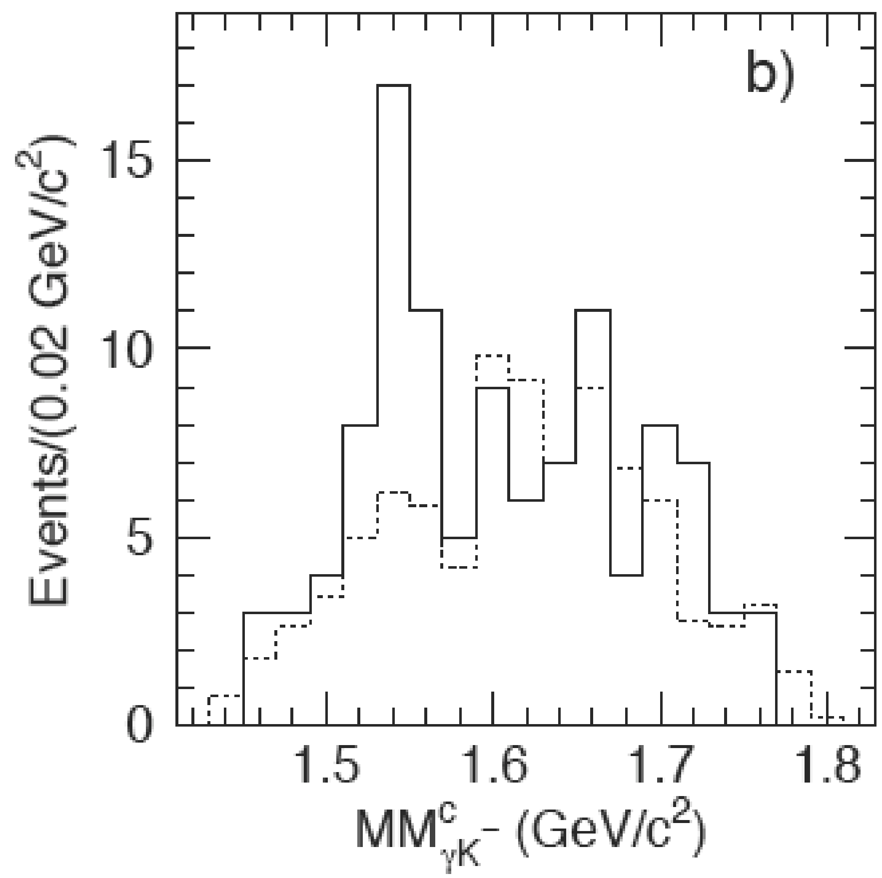
Minimal quark content of $\Theta^+ = uudd\bar{s}$.

Original LEPS experiment

$\gamma n \to n K^+ K^-$ with carbon as the neutron target



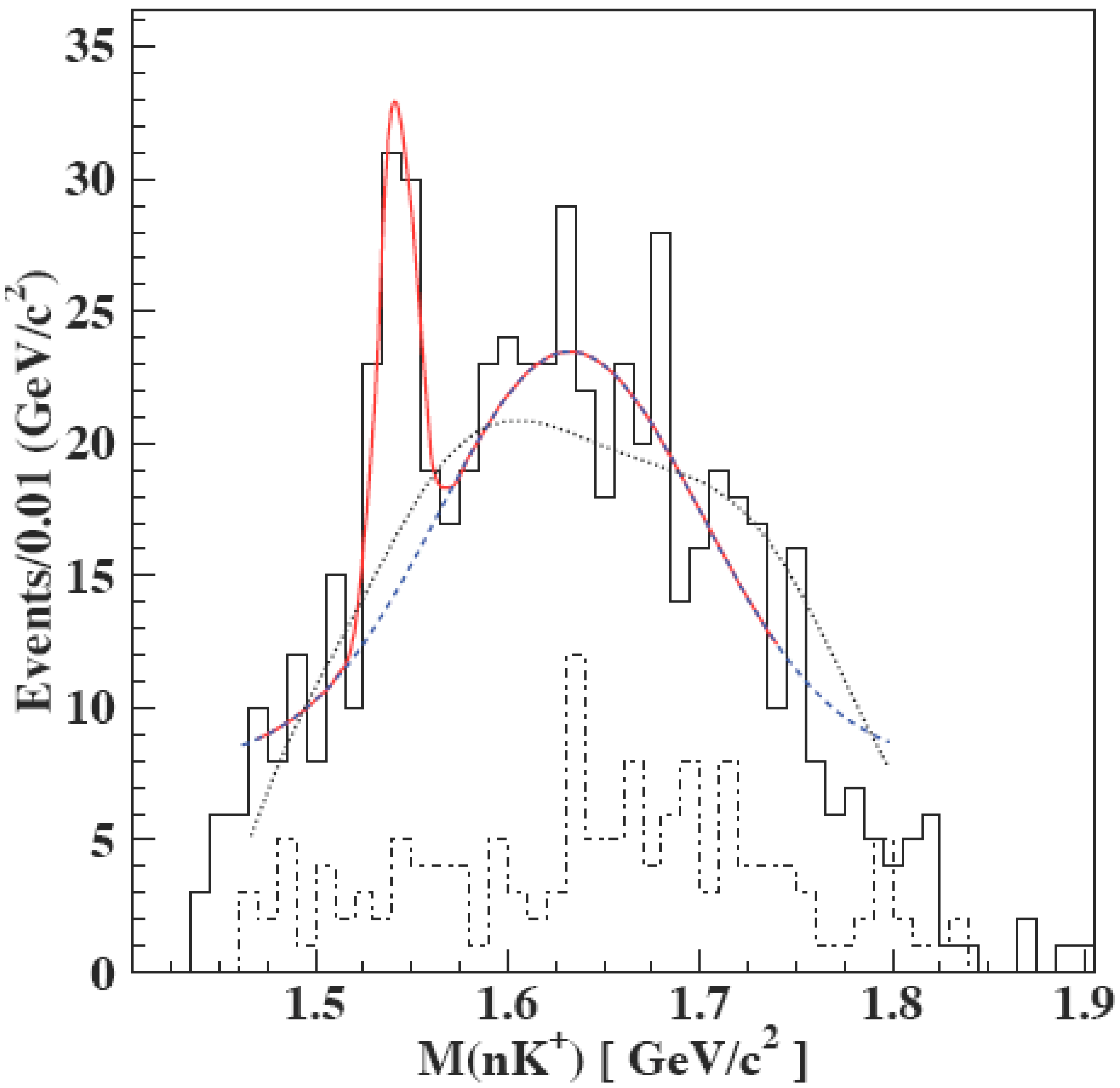
$\Lambda(1520)$ peak in $K^- n$ invariant mass



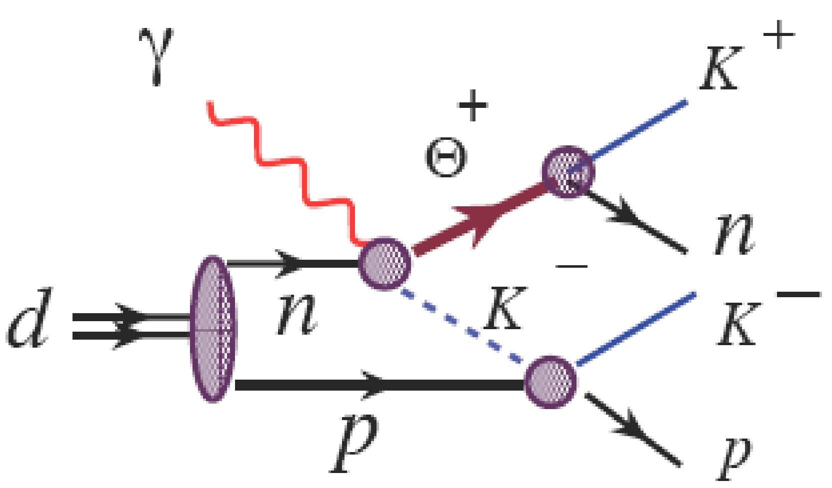
$\Theta(1540)$ peak in $K^+ n$ invariant mass

19 events
over background of 17

Clas I



$\gamma d \rightarrow npK^+K^-$

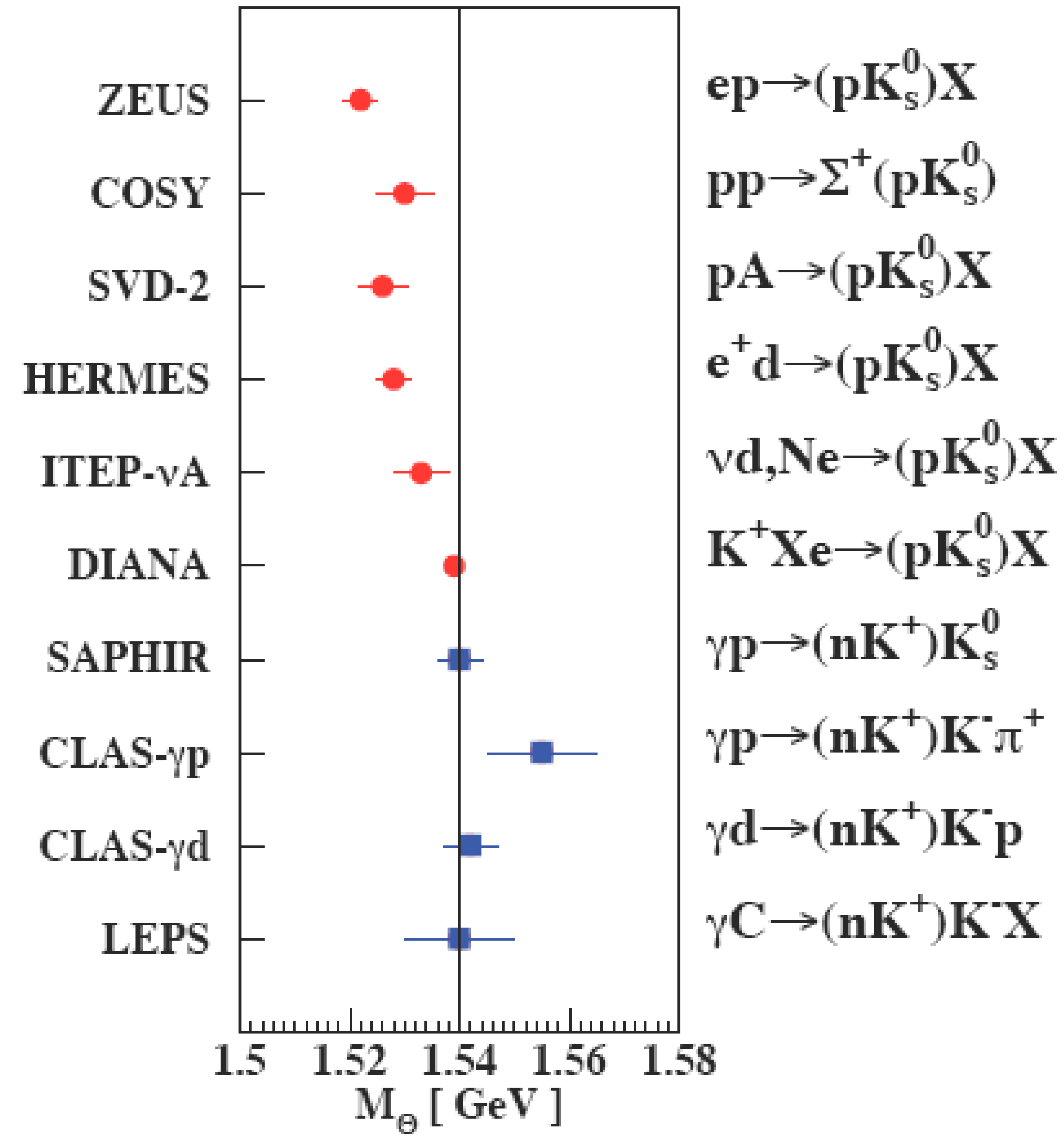


43 events over background of 54

Sightings of Θ^+

Experiment	Reaction	Mass	Width	σ_{std}
LEPS	$\gamma C \to K^+K^-\ X$	1540 $\pm$ 10	$<$ 25	4.6
Diana	$K^+Xe \to K^0pX$	1539 $\pm$ 2	$<$ 9	4.4
CLAS	$\gamma d \to K^+K^-p(n)$	1542 $\pm$ 5	$<$ 21	5.2
SAPHIR	$\gamma p \to K^+K^0(n)$	1540 $\pm$ 6	$<$ 25	4.8
ITEP	$\upsilon A \to K^0pX$	1533 $\pm$5	$<$ 20	6.7
CLAS	$\gamma p \to \pi^+K^-K^+(n)$	1555 $\pm$ 10	$<$ 26	7.8
HERMES	$e^+d \to K^0pX$	1528 $\pm$ 3	13 $\pm$ 9	$\sim$5
ZEUS	$e^+p \to e'K^0pX$	1522 $\pm$ 3	8 $\pm$ 4	$\sim$5
COSY	$pp \to K^0p\sum^+$	1530 $\pm$ 5	$<$ 18	4 – 6

(from Jaffe's review)

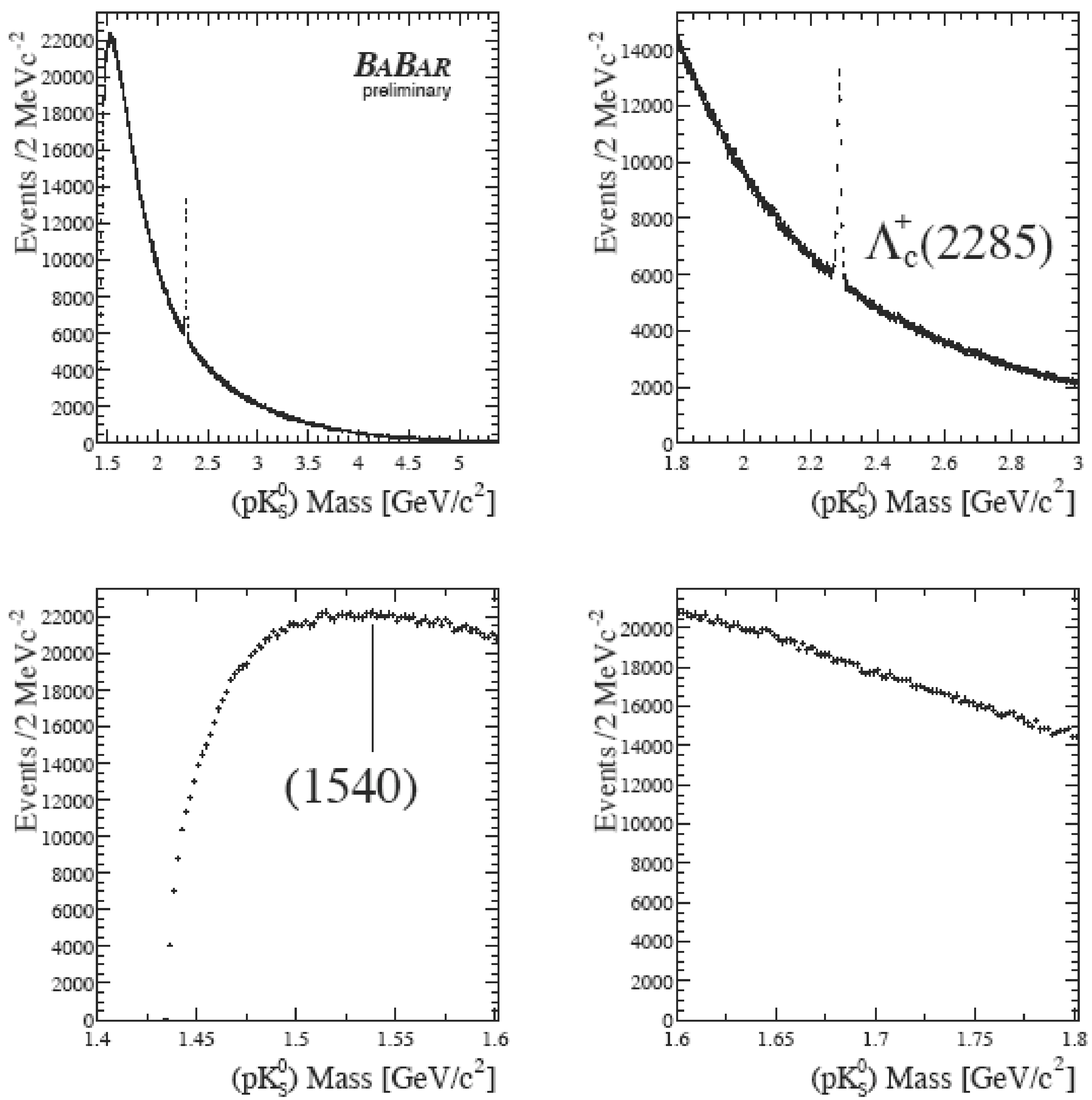


On the other hand, old kaon scattering data do not show any bump.

This gives an upper bound on the width of Θ^+ $\Gamma < 1$ Mev !

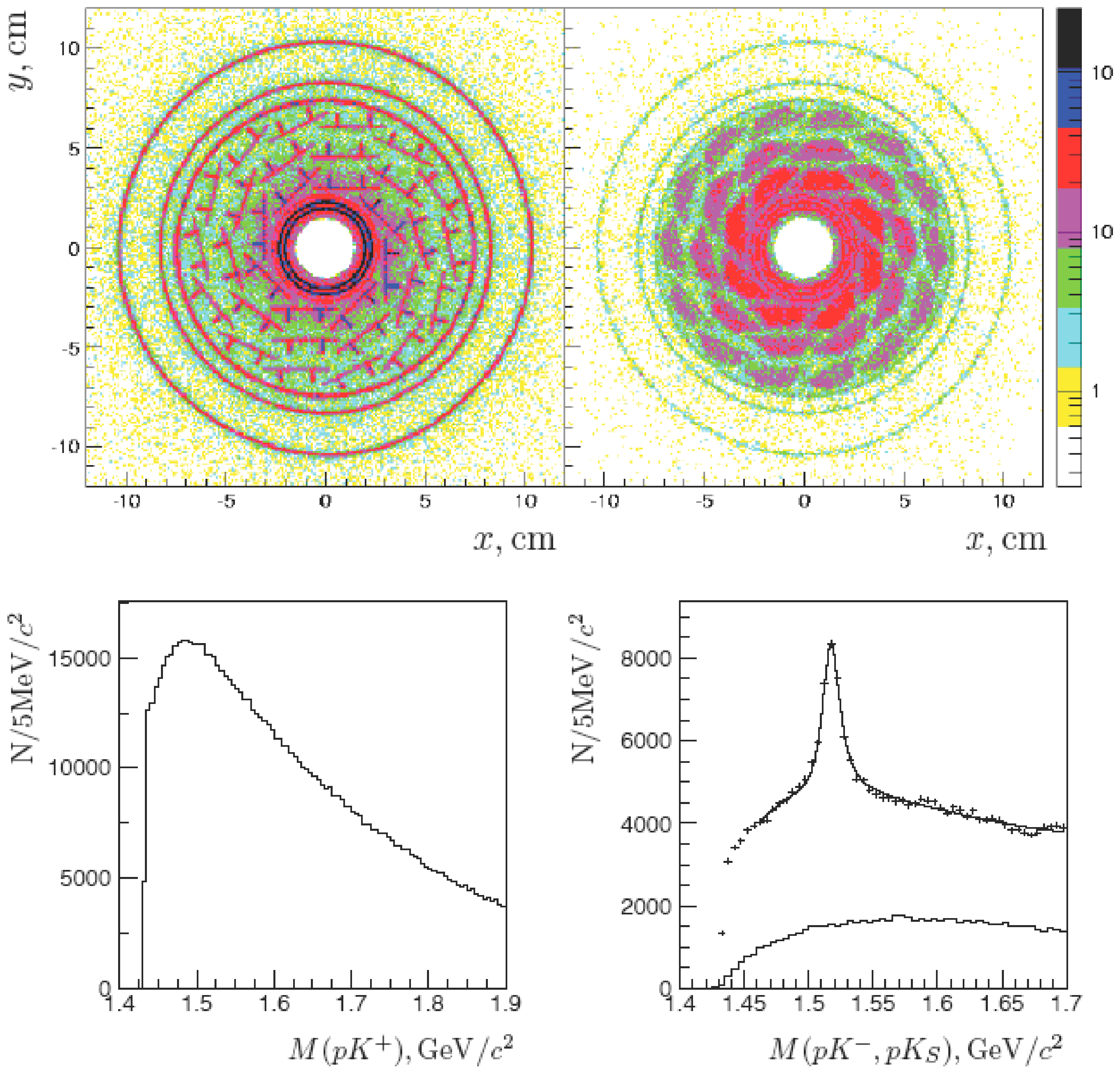
Recent searches in e^+e^- data with extremely high statistics came empty-handed

BaBar



BABAR
preliminary
Events /2 MeVc-2
(pK^0_S) Mass [GeV/c^2]
Λ_c^+(2285)
(1540)

Belle

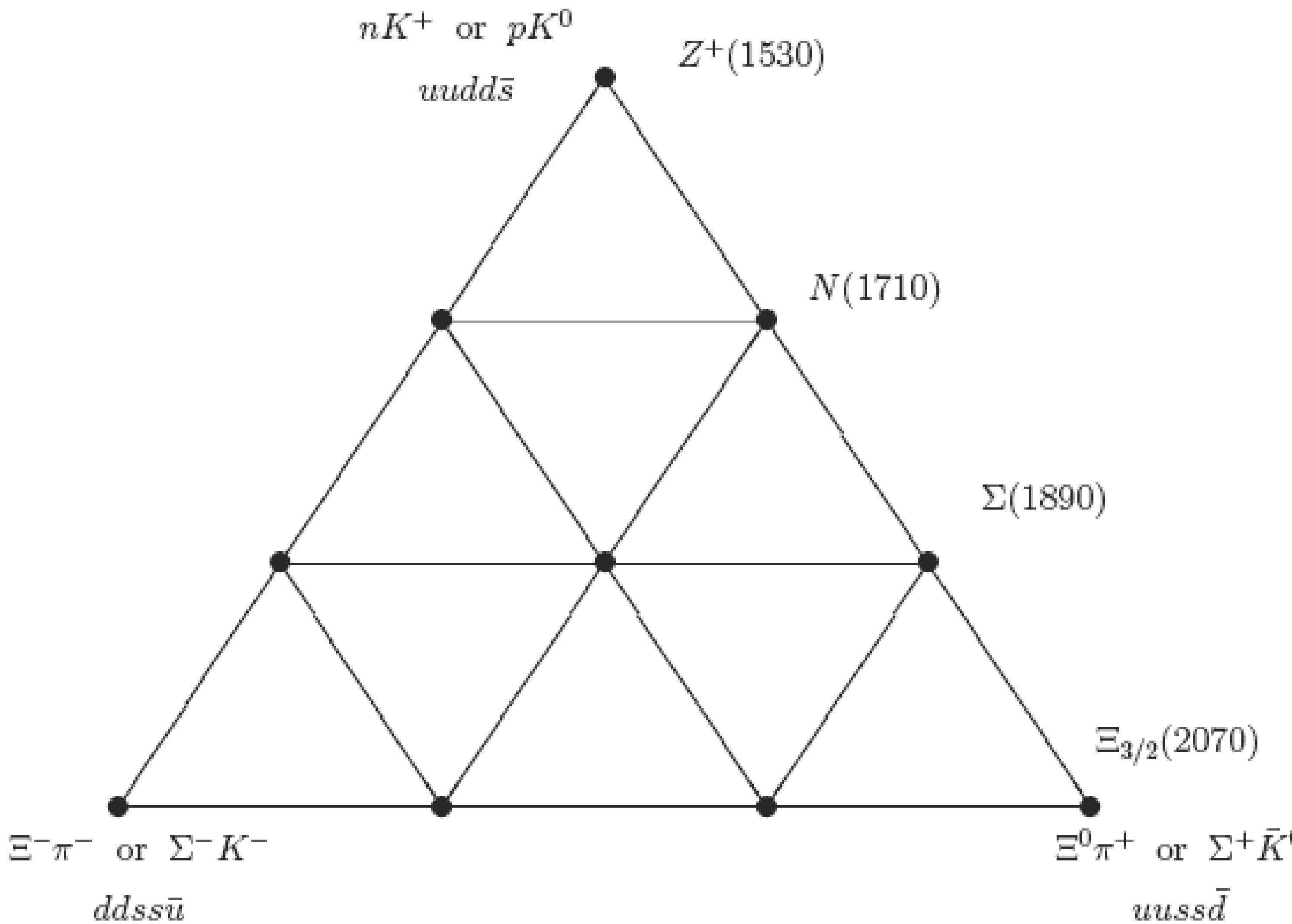


LEPS experiment motivated by 1997 paper by Diakonov, Petrov, Polyakov, based on collective quantization of $SU(3)_f$ chiral soliton model.

Remarkable prediction of Θ^+ resonance:

$I = 0$, $J^P = \frac{1}{2}^+$, $M = 1530$ Mev and $\Gamma < 15$ Mev
belonging to a $\bar{\mathbf{10}}$ $SU(3)_f$ multiplet.

Diakonov Petrov Polyakov 1997



(Early prediction of mass by Praszalowicz)

Baryons as solitons

Baryons interpreted as solitons of effective $SU(3)_f$ chiral Lagrangian Skyrme, Witten

$$L_\chi = \frac{f_\pi^2}{16}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2) + \mathrm{Tr}(M(U+U^\dagger-2)) + S_{WZ} + \dots$$

$$S_{WZ} = -\frac{iN}{240\pi^2}\int d^5x \epsilon^{\mu\nu\alpha\beta\gamma}\mathrm{Tr}(\partial_\mu U U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \partial_\gamma U U^\dagger)$$

Hedgehog solitons living purely in isospin sector, topologically stable

$$U_0 = U_{\pi,0} = \begin{pmatrix} e^{i\tau\cdot\hat{r}F(r)} & 0 \\ 0 & 1 \end{pmatrix} \quad F(0) = \pi,\ F(\infty) = 0$$

$SU(2)_I$ sector

Non-strange low-lying excitations given by rigid rotations

$$U(x,t) = A(t)U_0A^{-1}(t).$$

with $A(t)$ lying in $SU(2)_I$. Canonically quantizing the rotation,

$$H = M_{cl} + \frac{1}{8\Omega}\sum_{j=0}^{3}\pi_j^2 \rightarrow H = M_{cl} + \frac{1}{2\Omega}J(J+1)\,.$$

States must have $I = J$. If integer, bosonic; if half-integer, fermionic.

$I = J = 1/2$: nucleons; $I = J = 3/2$, Δ's.

$f_\pi \sim \sqrt{N}$, $e \sim 1/\sqrt{N} \rightarrow M_{cl} \sim \Omega \sim N$.

Motion is slow at large N, excitations are $\sim 1/N$. This analysis is self-consistent for $I = J = 1/2, 3/2, \ldots O(N)$.

$SU(3)_f$ rigid rotators

$$H = \frac{1}{2\Omega}\sum_{j=1}^{3}\pi_j^2 + \frac{1}{2\Phi}\sum_{a=4}^{7}\pi_a^2$$

WZ term gives constraint on wavefunction

$$\pi_8\psi(\mathcal{A}) = \frac{N}{2\sqrt{3}}\psi(\mathcal{A})$$

Allowed $SU(3)_f$ multiples must contain states with $S = 0$.

Witten, Guadagnini,Chemtob, Manohar

Multiplets depend on $N \equiv 2n + 1$ and become large as $N \to \infty$.

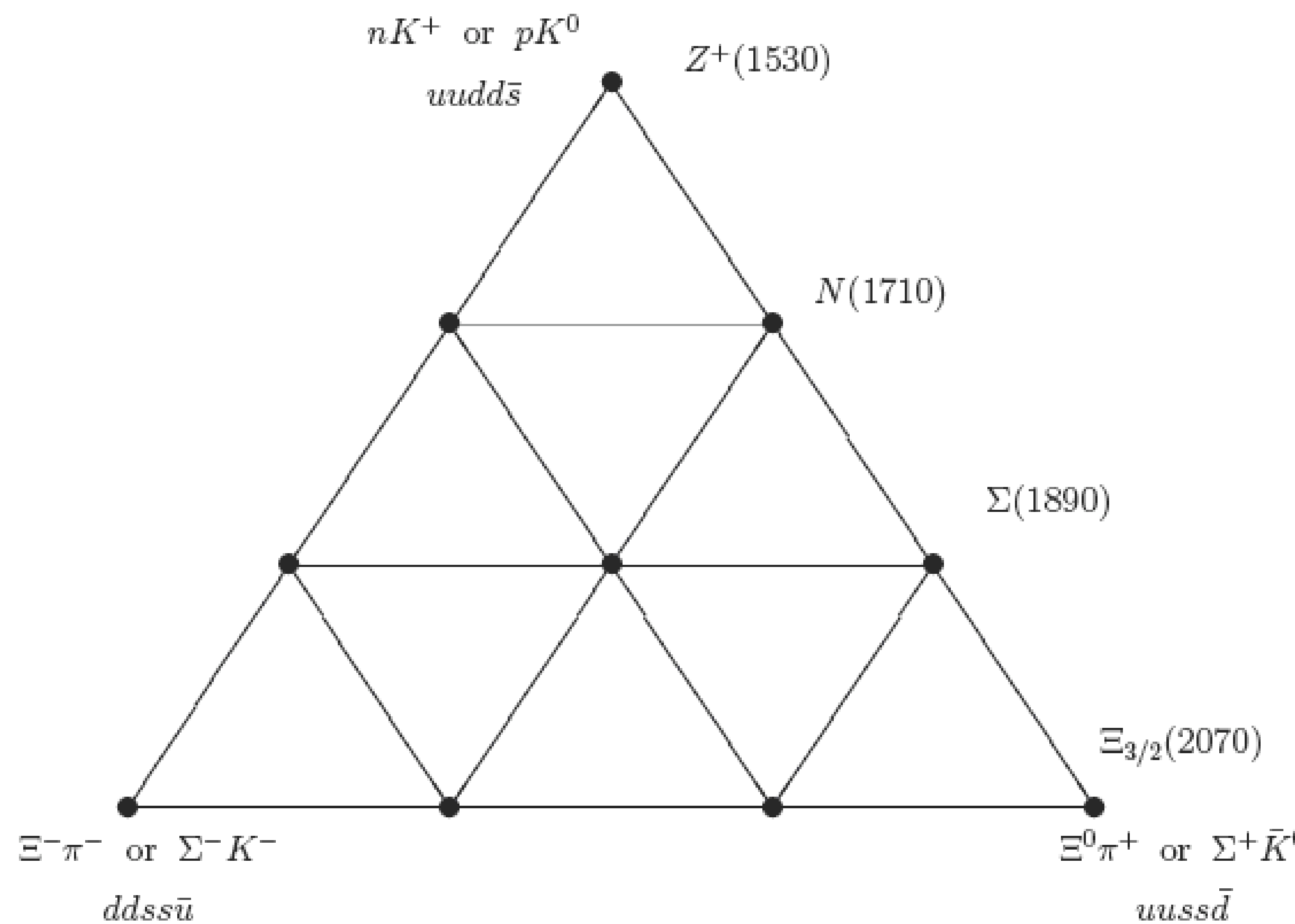
$(1, n)$ with $J = 1/2$ (the "octect")

$(3, n - 1)$ with $J = 3/2$ (the "decuplet")

$(0, n + 2)$ with $J = 1/2$ (the "antidecuplet")

. . . . . .

Diakonov Petrov Polyakov 1997



(Early prediction of mass by Praszalowicz)

$$M^{(p,q)} = M_{cl} + \frac{1}{2\Omega} J(J+1) + \frac{1}{2\Phi} \left(C^{(p,q)} - J(J+1) - \frac{N^2}{12} \right) ,$$

$$C^{(p,q)} = \frac{1}{3} [p^2 + q^2 + 3(p+q) + pq]$$

For "non-exotic" multiplets $(2J, n + 1/2 - J)$ of spin $J = 1/2, 3/2, \ldots$

$$M(J) = M_{cl} + \frac{N}{4\Phi} + \frac{1}{2\Omega} J(J+1)$$

splittings are $O(1/N)$. These are the multiplets that appear in constructing ordinary baryons with N quarks. Manohar

However, for "exotic" multiplets like the "antidecuplet" $(0, n+2)$, splitting is

$$\frac{N}{4\Omega} = O(1)$$

This $O(1)$ splitting invalidates the semiclassical quantization of collective coordinates in the exotic sector Cohen, Itzhaki Klebanov Ouyang LR

Many ways to see this: Born-Oppenheimer separation of scales breaks down, width is $O(1)$, bound state approach gives no state in the chiral limit, ...

Large N cannot be used to predict exotics.
Their existence is a dynamical question.

If exotics do exist, large N can be used to constrain their spin-flavor properties.
Jenkins Manohar

Bound State Approach

The dynamical problem can be studied in the Skyrme model, the simplest truncation of the chiral Lagrangian supporting solitons.

BS approach Callan Klebanov: expand the action to quadratic order in kaon fluctuations around the classical hedgehog; study the Schoedinger problem for the kaon partial waves.

$$-f(r)\ddot{k} + 2i\lambda(r)\dot{k} + \mathcal{O}k = 0 \;\; \mathcal{O} \equiv \frac{1}{r^2}\partial_r h(r) r^2 \partial_r - m_K^2 - V_{eff}(r)\,.$$

Expansion of k in eigenmodes:

$$k(r,t) = \sum_{n>0} (\tilde{k}_n(r) e^{i\tilde{\omega}_n t} b_n^\dagger + k_n(r) e^{i\omega_n t} a_n)$$

with $\omega_n, \tilde{\omega}_n > 0$. Eigenvalue equations:

$$(f(r)\omega_n^2 + 2\lambda(r)\omega_n + \mathcal{O})k_n = 0 \qquad (S = -1)\,,$$

$$(f(r)\tilde{\omega}_n^2 - 2\lambda(r)\tilde{\omega}_n + \mathcal{O})\tilde{k}_n = 0 \qquad (S = +1)\,.$$

For $m_K \to 0$, find exact solution of $S = -1$ equation with $\omega = 0$, but no solution to $S = +1$ equation. Hence the rigid rotor mode with $\tilde{\omega} = N/(4\Omega)$ is not reproduced.

Particle	J	I	L	Mass (expt)	Mass (a)	Mass (b)	Mass (c)
Λ	$\frac{1}{2}$	0	1	1115	1048	1059	1121
Σ	$\frac{1}{2}$	1	1	1190	1122	1143	1289
Σ	$\frac{3}{2}$	1	1	1385	1303	1309	1330
Λ	$\frac{1}{2}$	0	0	1405	1281	1346	1366

Table 1: Masses (in MeV) of the light $S = -1$ hyperons as calculated from the bound state approach, with (a) $e = 5.45$, $f_\pi = f_K = 129$ MeV, (b) $e = 4.82$, $f_\pi = f_K = 186$ MeV, with an overall constant added to fit the N and Δ masses, and (c) the same parameters as (a) but with the WZ term artificially decreased by a factor of 0.4. In all cases $m_\pi = 0$.

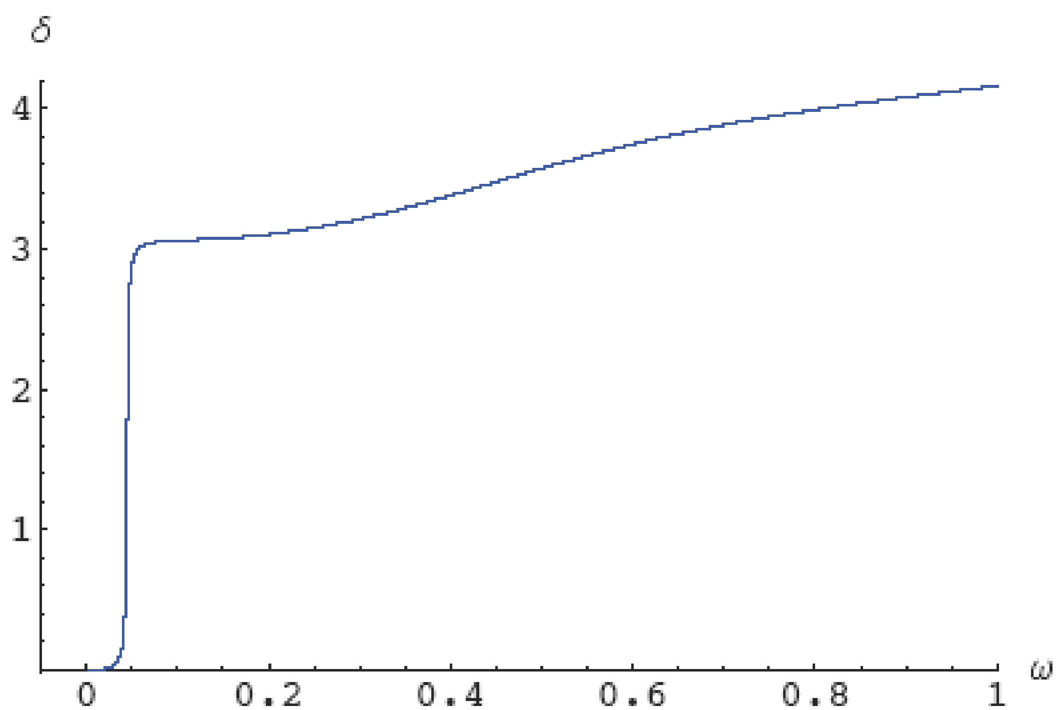


Figure 1: Phase shift as a function of energy in the $L = 2$, $T = \frac{3}{2}$, $S = -1$ channel. The energy ω is measured in units of ef_π (with the kaon mass subtracted, so that $\omega = 0$ at threshold), and the phase shift δ is measured in radians. Here $e = 5.45$ and $f_\pi = 129$ MeV.

Particle	J	I	L	Mass (expt)	Mass (th)
Λ (D_{03})	$\frac{3}{2}$	0	2	1520	1462
Σ (D_{13})	$\frac{3}{2}$	1	2	1670	1613
Σ (D_{15})	$\frac{5}{2}$	1	2	1775	1723

Table 2: Masses (in MeV) of the $S = -1$ D-wave resonances calculated from the bound state approach, with $f_\pi = 129$ MeV, $e = 5.45$.

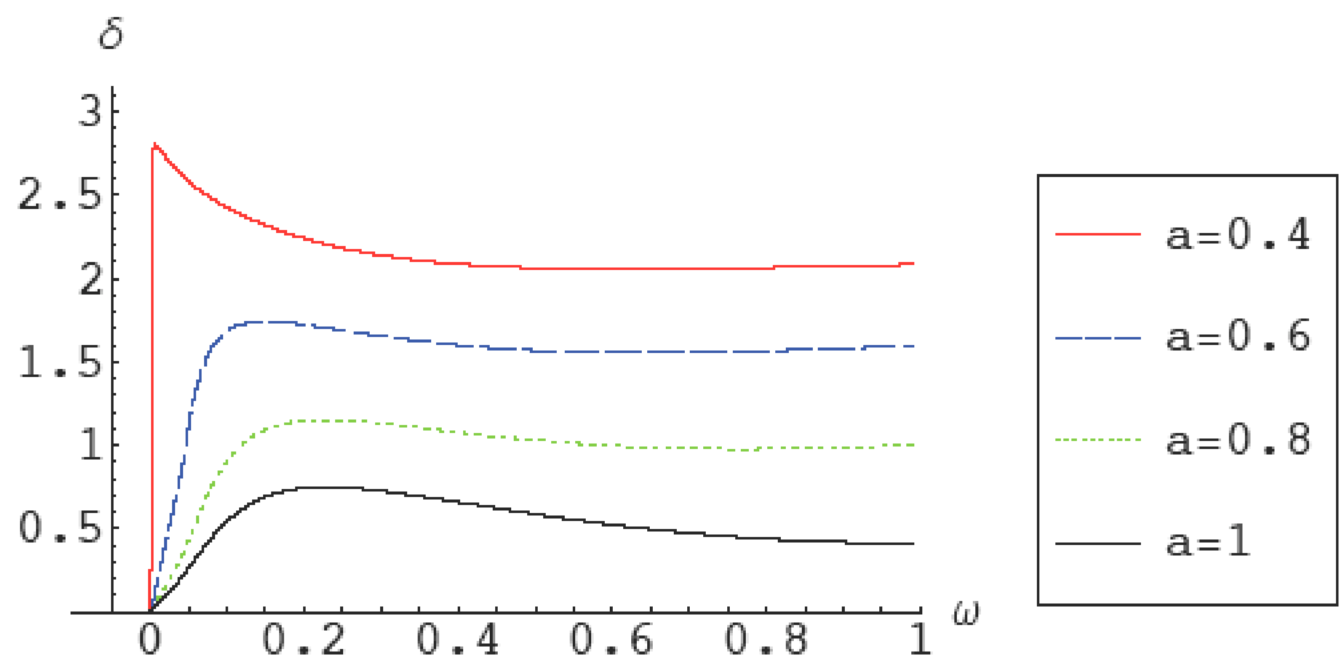


Figure 2: Phase shifts δ as a function of energy in the $S = +1$, $L = 1$, $T = \frac{1}{2}$ channel, for various choices of the parameter a (strength of the WZ term). The energy ω is measured in units of ef_π ($e = 5.45$, $f_\pi = f_K = 129$ MeV) and the phase shift δ is measured in radians. $\omega = 0$ corresponds to the $K - N$ threshold.

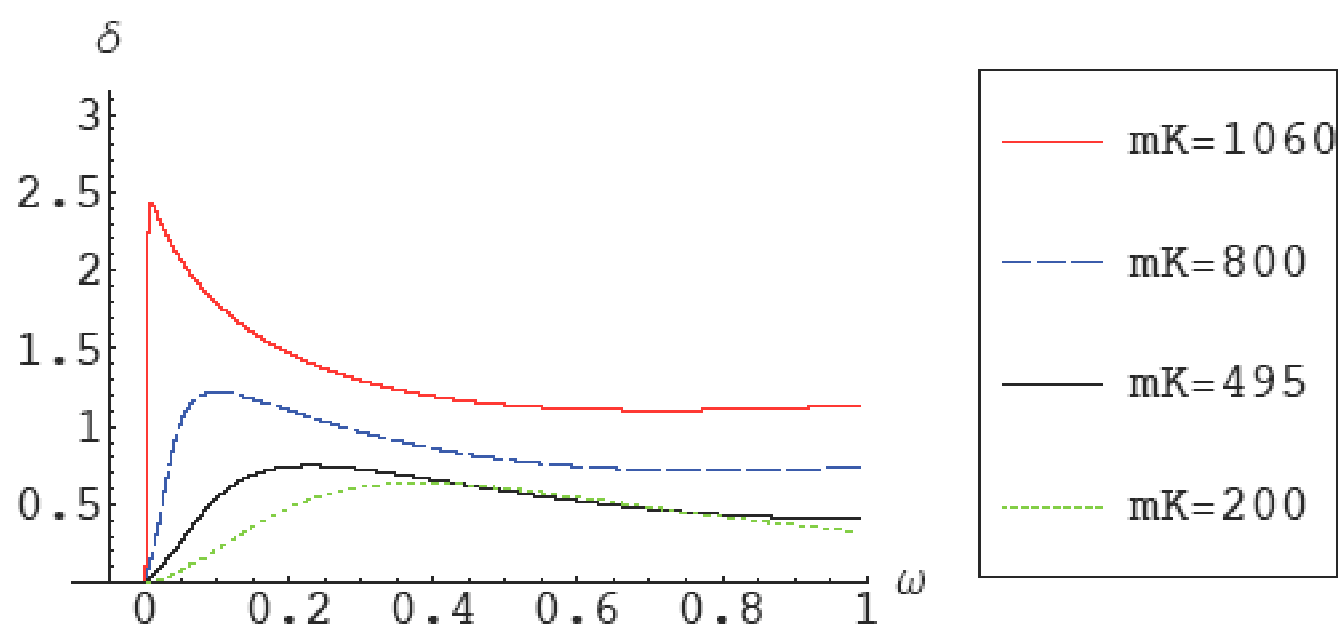


Figure 3: Phase shifts δ as a function of energy in the $S = +1$, $L = 1$, $T = \frac{1}{2}$ channel, for various values of m_K. Here $e = 5.45$ and $f_\pi = f_K = 129$ MeV.

The BS approach reproduces the spectrum of non-exotic baryons, both bound states and resonances. Mass-splittings are in good agreement with experiment.

However no $S = +1$ baryon is found unless one makes rather drastic adjustments of the parameters (reducing the WZ to 40%, or increasing m_K to 1 GeV).

Model independent relations

If Θ^+ does exist, it comes with $SU(2)$ rotor excitations with $I = 1,\ J^P = \frac{3}{2}^+$ and $I = 1,\ J^P = \frac{1}{2}^+$ (they would lie in the **27** of $SU(3)_f$).

Certain model-independent relations hold:

$$2M(1, \frac{3}{2}) + M(1, \frac{1}{2}) - 3M(0, \frac{1}{2}) = 2(M_\Delta - M_N) = 586 \text{ MeV} \ ,$$

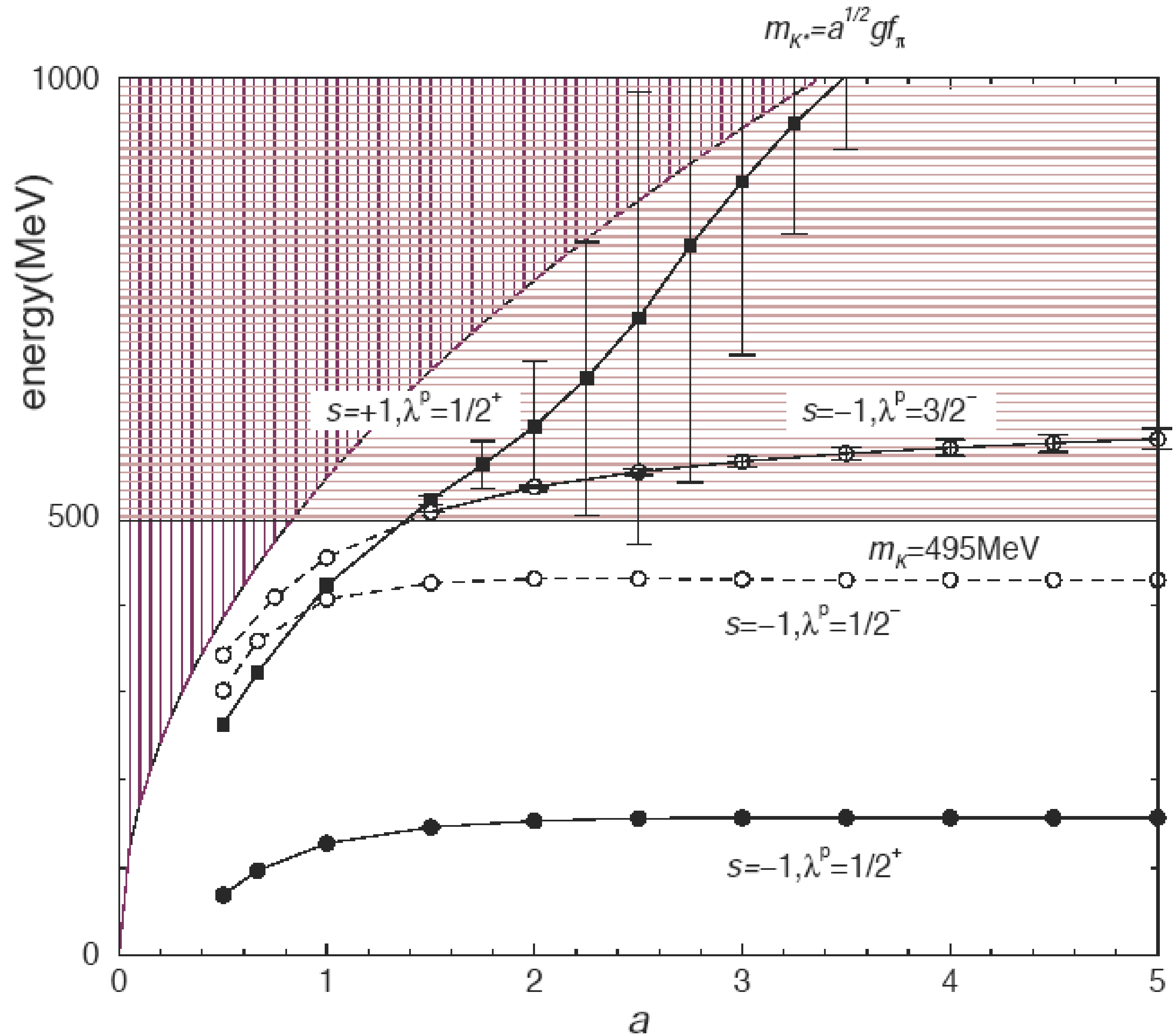
$$\frac{3}{2}M(2, \frac{5}{2}) + M(2, \frac{3}{2}) - \frac{5}{2}M(0, \frac{1}{2}) = 5(M_\Delta - M_N) = 1465 \text{ MeV} \ ,$$

$$M(2, \frac{3}{2}) - M(2, \frac{5}{2}) = \frac{5}{3}\left(M(1, \frac{1}{2}) - M(1, \frac{3}{2})\right),$$

Special cases of large N relations following from contracted $SU(6)$ symmetry

Jenkins Manohar

Park Rho Min, hep-ph 0405246:



Results for a model with explicit vector mesons K^*

Quark Models

"Uncorrelated" quark models doomed from the start. Simplest wavefunction (S-wave, P = -1), would "fall-apart" in Kn.

Need to place the quarks in a higher spatial wavefunction.

Neat way to do it motivated by "diquark" correlations Jaffe Wilczek

Such a model can accomodate the Θ^+, but it probably could not have been used to unambiguosly predict it.

Lattice Results

Most groups see a $P = -1$ state.

Summary

The original chiral soliton model prediction of Θ^+ was largely accidental.

No model sharply predicts the Θ^+. Rather the opposite.

If it exists, there are ways to contrive it, both in quark models and in soliton models, but no good idea of why it should be so narrow.

- It it goes away, we may even take this as a success of large N ideas combined with a simple dynamical model (the Skyrme Lagrangian).

- If it is confirmed, major theoretical challenge.

Clues for/from gauge/string duality?

No Conclusions