# Regge Scattering in Gauge / Gravity Duality

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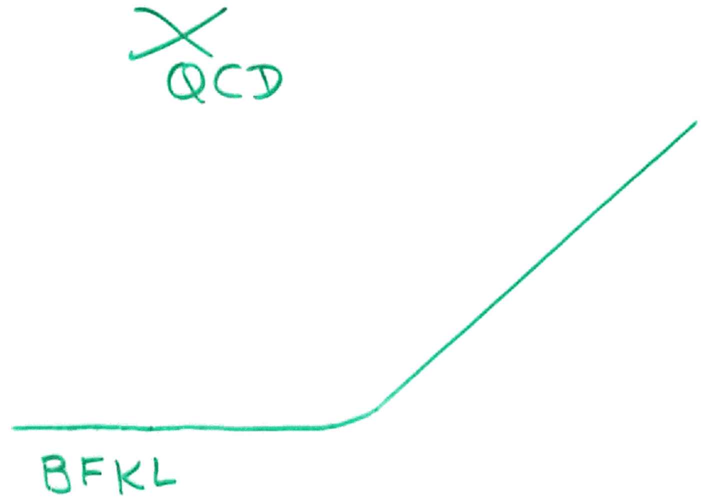
cf. O. Andreev
Andreev + Siegel
R. Janik



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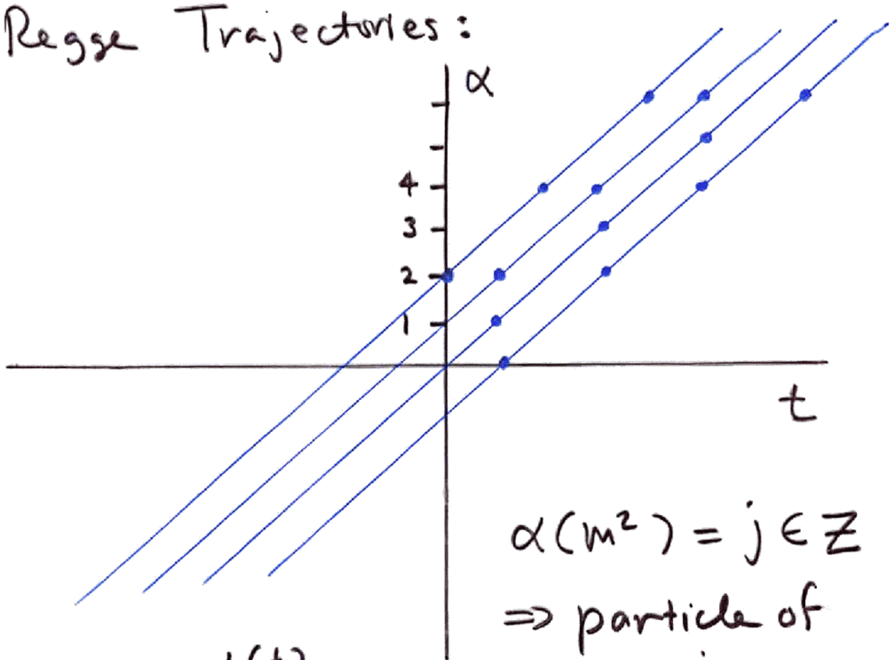
## What String Theorists Know About

Regge Trajectories:




$\alpha(m^2) = j \in \mathbb{Z}$

$\Rightarrow$ particle of spin $j$

$A \sim s^{\alpha(t)}$

$(s \to \infty,\ t$ fixed$)$

e.g. Veneziano, Virasoro-Shapiro



## Gauge/gravity duality w/ confinement:

$$ds^2 \sim \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2$$

AdS radius $\quad R^2 = \sqrt{4\pi g N}\,\alpha' = \sqrt{\lambda}\,\alpha'$

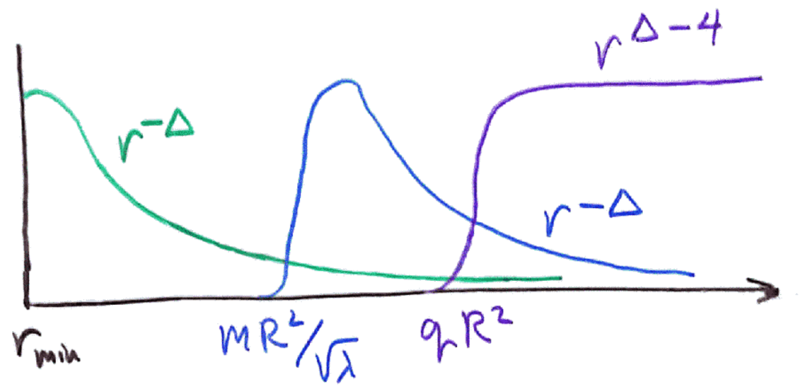
't Hooft param.

$r > r_{min} \Rightarrow$ mass gap, confinement

will consider only leading $1/N$

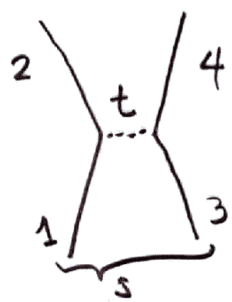
$\Rightarrow$ one-particle exchange

External states




light hadron. $\Delta$ = conformal twist (= # of partons in p.t.)

- Onium, constituents of mass $m$.
- off-shell current, momentum $q$.

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$R = \lambda^{1/4}\sqrt{\alpha'} \gg \sqrt{\alpha'}$ : geometry flat on string scale. For scattering processes localized on scale $\sqrt{\alpha'}$, can fold flat-spacetime amplitude $A_{10}$ into external states:

$$\mathcal{A} = \int dr\,\sqrt{-g}\,\phi_1(r)\phi_2^*(r)A_{10}(\tilde{p})\phi_3(r)\phi_4^*(r)$$

$$\tilde{p} = \frac{R}{r}\,p$$

$p$: 4-d (Noether) momentum

$\tilde{p}$: momentum seen by local 10-d observer

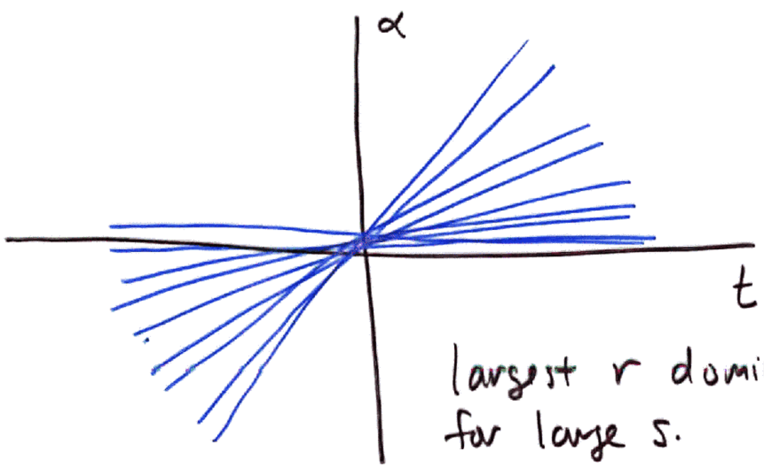
Regge:

$$A_{10} \sim (\alpha' \tilde{s})^{2+\alpha'\tilde{t}}$$

$$2+\alpha'\tilde{t} = 2+\alpha'(t R^2/r^2)$$
$$= 2+\left(\alpha' \frac{R^2}{r^2}\right) t$$

(n.b. $t<0$)

Effective Regge slope depends on $r$ at which scattering occurs.




largest $r$ dominates for large $s$.

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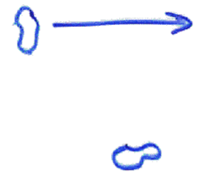
Important fact (Susskind): strings grow with boost due to time dilation



$$s^{\alpha' t/2} = s^{-\alpha' k^2/2} = e^{-\alpha' k^2 \ln s/2}$$
$$\sim e^{-x^2/2\alpha' \ln s}$$

(gaussian form factor, spreading as $\sqrt{\ln s}$ )

Asymptotically in $s$, string size is greater than $R$ of AdS.

$$A \sim \int dr \sqrt{g}\, \phi_1 \phi_2^* \tilde{s}^2 e^{(\ln \tilde{s})\frac{\alpha'}{2} D^2} \phi_3 \phi_4^*$$

diffusion operator

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Regge behavior in flat spacetime

V-S: $\mathcal{A} = \int d^2z\, |z|^{-4-\alpha' t/2}\, |1-z|^{-4-\alpha' s/2}$


saddle point at $z = \dfrac{t}{t+s} = O(s^{-1})$

$$\mathcal{A} \sim \int d^2z\, |z|^{-4-\alpha' t/2}\, e^{\alpha' s(z+\bar{z})/4}$$

$$\sim \frac{\Gamma(-1-\frac{\alpha'}{2}t)}{\Gamma(2+\frac{\alpha'}{2}t)}\, (\alpha' s)^{2+\alpha' t/2}$$

$z \ll 1 \Rightarrow$ OPE

$$:e^{ip_1\cdot X(z)}:\, :e^{ip_2\cdot X(0)}: \sim |z|^{-4-\frac{\alpha'}{2}t} :e^{i(p_1+p_2)\cdot X(0)}$$
$$\times\, e^{ip_1\cdot(z\partial_z + \bar{z}\partial_{\bar{z}})X(0)}:$$

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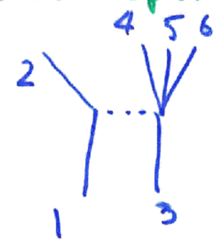
$$\int d^2z \,\langle V_1(0)\, V_2(z)\, V_3(1)\, V_4(\infty)\rangle$$

$$= \frac{\Gamma(-1-\frac{\alpha' t}{2})}{\Gamma(2+\frac{\alpha'}{2}t)}\, s^{2+\frac{\alpha'}{2}t} \langle V_p(0)\, V_3(1)\, V_4(\infty)\rangle$$

$$V_p = e^{i(p_1+p_2)\cdot X}\, (\partial X^+ \bar{\partial} X^+)^{1+\frac{\alpha'}{4}t}$$

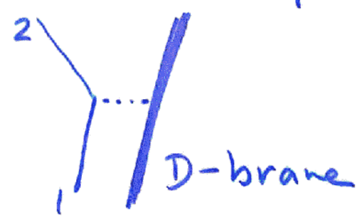
"Pomeron vertex operator" (on-shell)

Extends to




$\langle V_p V_3 V_4 V_5 V_6\rangle$

+ to




$\langle V_p \rangle_{disk}$

More generally

$$\langle \cdots\, V_p\rangle \langle V_p \,\cdots\rangle$$

large relative boost

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Extends to curved space, keeping complete set in radial direction

$$K = \sum_i |i\rangle\, s^{j_i} \langle i|$$

$$\left(-\frac{\alpha'}{2} D^2(j_i) + j_i - 2\right)|i\rangle = 0$$

(phys. st.)

$L_0 + \bar{L}_0 - 2$

$$D^2(j_i) \approx D^2(2)$$

$$= \frac{1}{R^2}\left[\partial^2_{\ln r} - 4\right] + \frac{R^2}{r^2} t$$

$$= \frac{1}{R^2}\left[(\underset{\uparrow\ \text{dimension}}{\Delta} - 2)^2 - 4\right] + \frac{R^2}{r^2} t$$

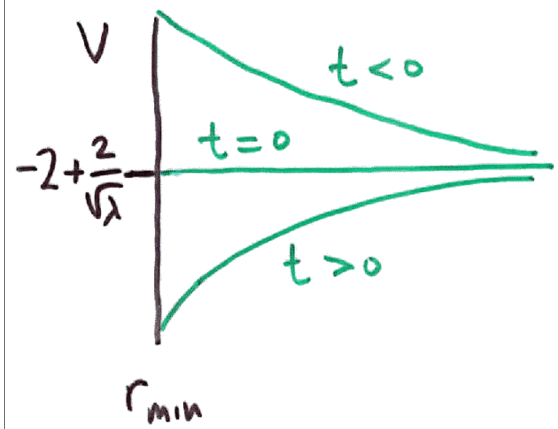
$$K \approx s^{+\frac{1}{2\sqrt{\lambda}}\left(\partial^2_{\ln r} + \frac{R^4}{r^2} t\right) + 2 - \frac{2}{\sqrt{\lambda}}}$$

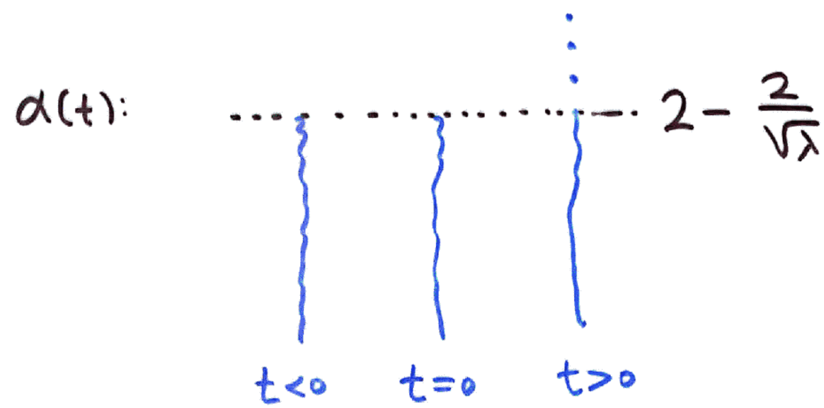


Regge trajectories:

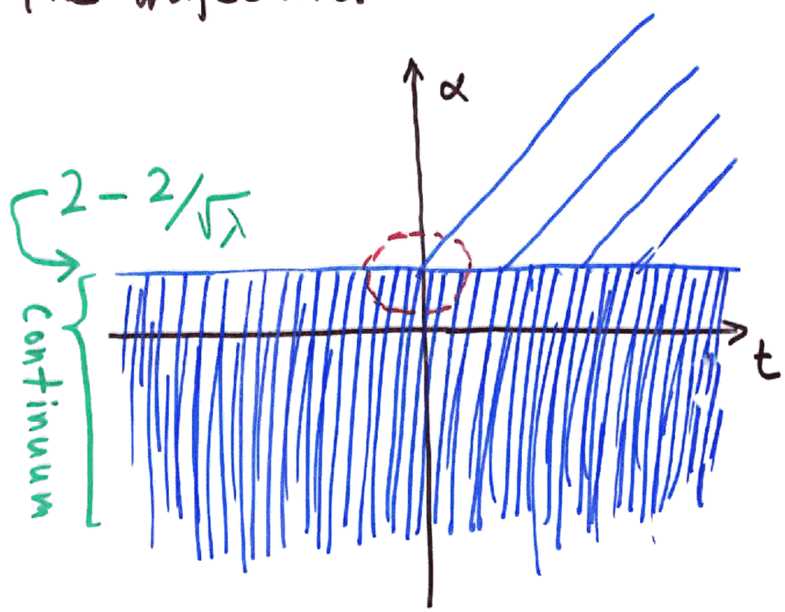
$-\alpha(t)$ given by spectrum of

$$-2 + \frac{2}{\sqrt{\lambda}} + \frac{1}{2\sqrt{\lambda}}\left(-\partial^2_{\ln r} - \frac{R^4}{r^2} t\right)$$



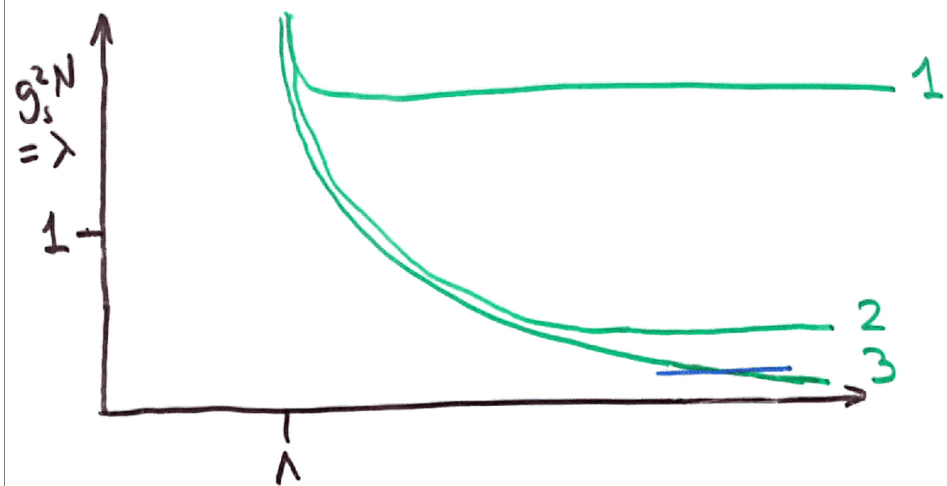
The trajectories:




model-dependent



$\Rightarrow$ weak coupling, BFKL

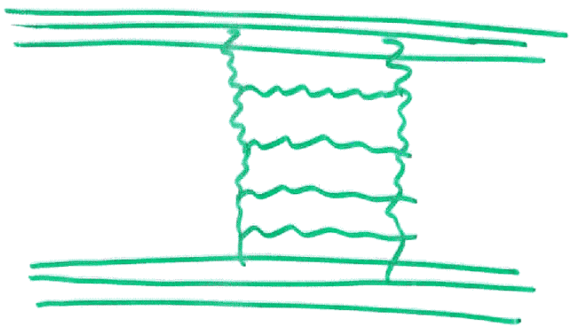



1. $N = 1^*$ (deformed, confining AdS/CFT at large UV $\lambda$): (the story thus far)

Can apply BFKL to

2. $N = 1^*$ at small UV $\lambda$
3. QCD

BFKL: treat $\alpha_s$ as small, but not $\alpha_s \ln s$. Keep and sum $(\alpha_s \ln s)^n$ for all $n$.



$\approx$ gluon ladders (highly nontrivial)

Result ($N=1^*$)

- diffusion in $\ln b$ — size of 2-gluon wavefunction
- continuum below $\alpha = 1 + \frac{4 \ln 2}{\pi} \lambda$
- discrete states for positive $t$.



$1 + \frac{4 \ln 2}{\pi} \lambda$

BFKL

# BFKL vs. DGLAP

$$F_{\mu+}(D_+)^{j-2}F_{\mu+}$$

has dimension $\Delta(j) = \gamma(j) + j + 2$.

- $\Delta(j)$ governs OPE, parton moments
- can be extended to real, complex $j$.
- solving $\Delta(j) = 2$ gives BFKL intercept (for any $\lambda$*).

"principal continuous series" of $SO(4,2)$: $\Delta = 2 + 2i\nu$

* We think...



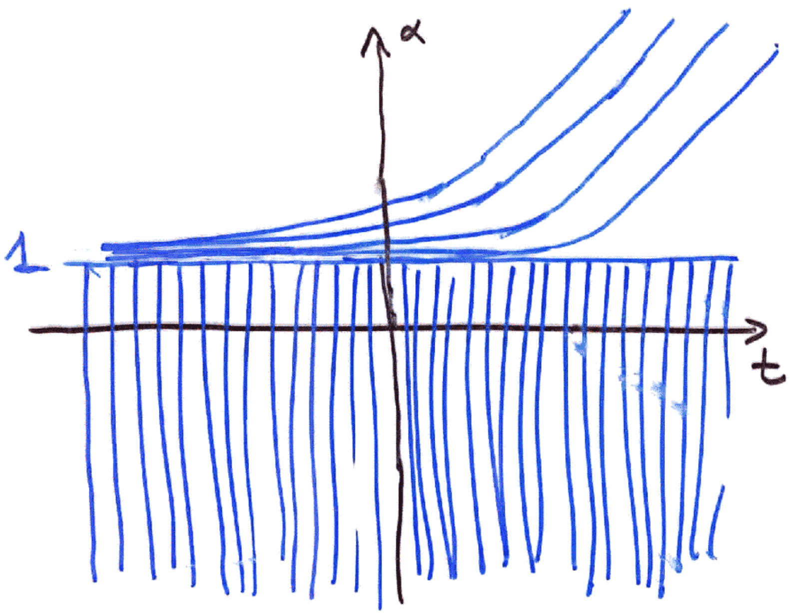
# Large-N QCD

$$V \sim -1 - \frac{4\ln 2}{\pi} N\alpha_s(R^2/r) - \frac{R^4 t}{r^2}$$


$\Rightarrow$ poles even for $t<0$.

- poles starting around $1 + \frac{4\ln 2}{\pi} N\alpha_s(t)$
- continuum below 1

Large-N QCD trajectories




How can planar graphs in a confining theory give a continuum (cut)?

Because QCD is really a five-dimensional theory.

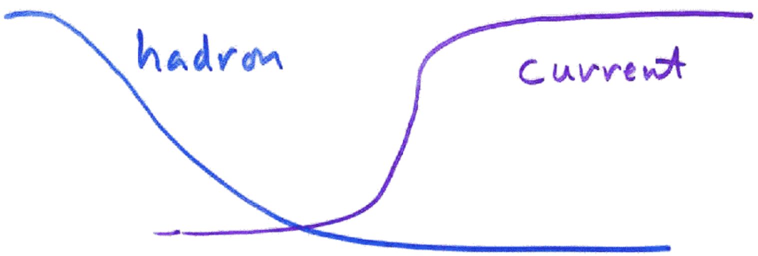


$e^{\frac{1}{\sqrt{\lambda}} \ln s \, \partial^2_{\ln r}}$ :

- diffusion significant only for $s \gtrsim e^{O(\sqrt{\lambda})}$
- large N approx breaks down (black hole production, Giddings) unless $N^2 s \lesssim 1 \Rightarrow N \lesssim e^{-O(\sqrt{\lambda})}$
- both inequalities are much weaker in QCD.

One manifestation of diffusion:

In deep inelastic scattering, external states have very different scales, small overlap:




Diffusion initially enhances overlap (this is not asymptotic Regge behavior).