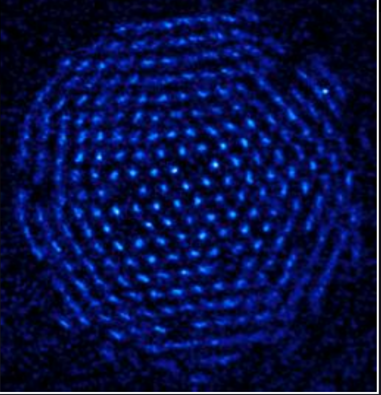

Filtering Classical Noise by Quantum Control

KITP 2013

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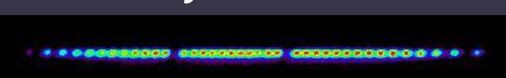
Formerly



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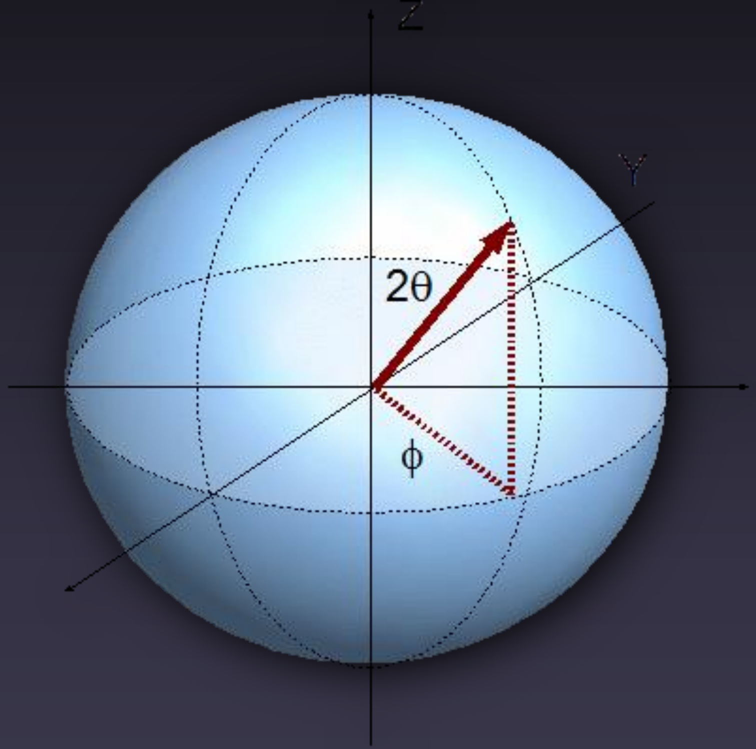
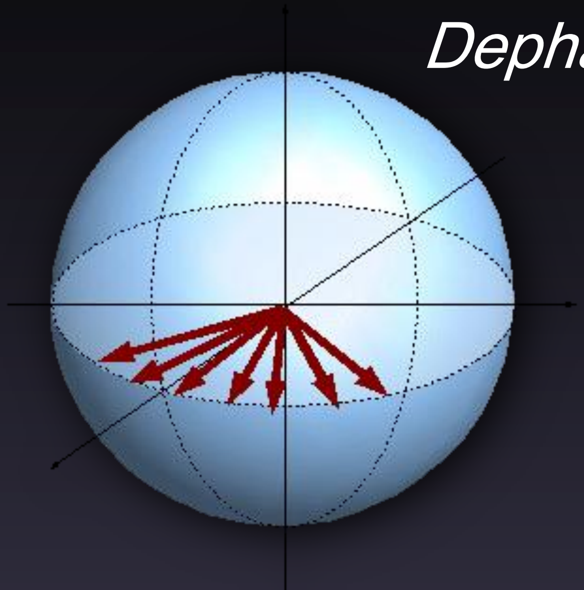
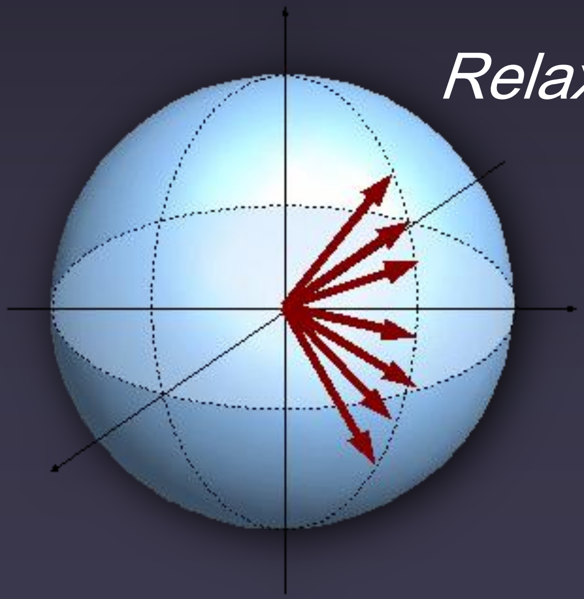
Errors: A challenge for experimentalists



Dephasing



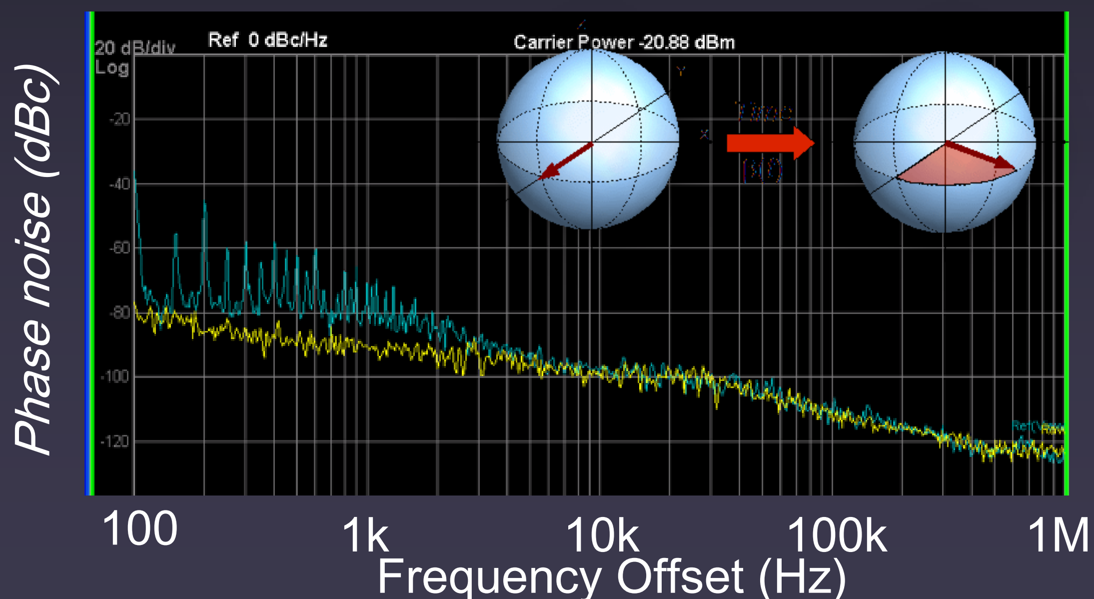
Relaxation



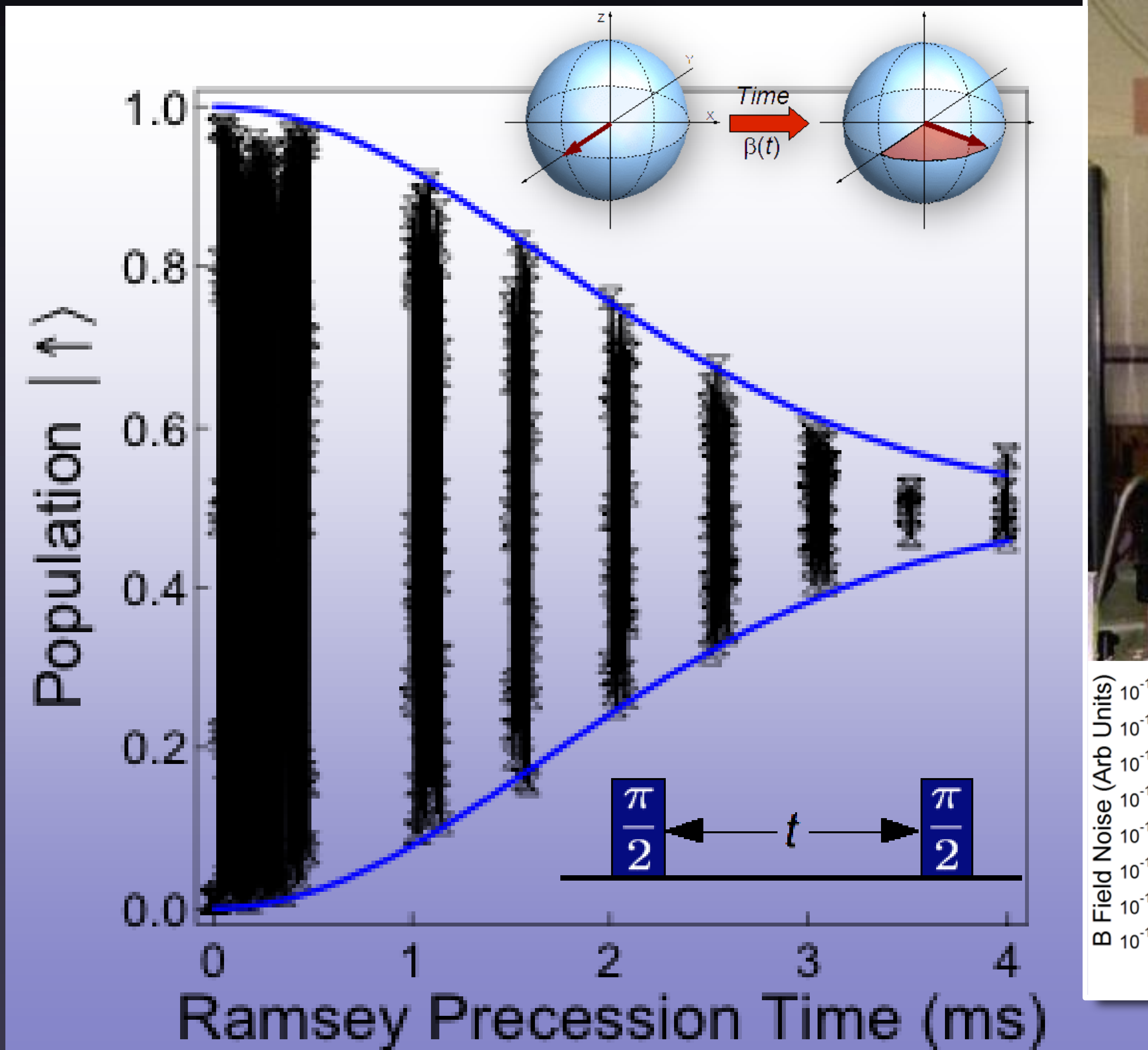
What error model should we pursue?

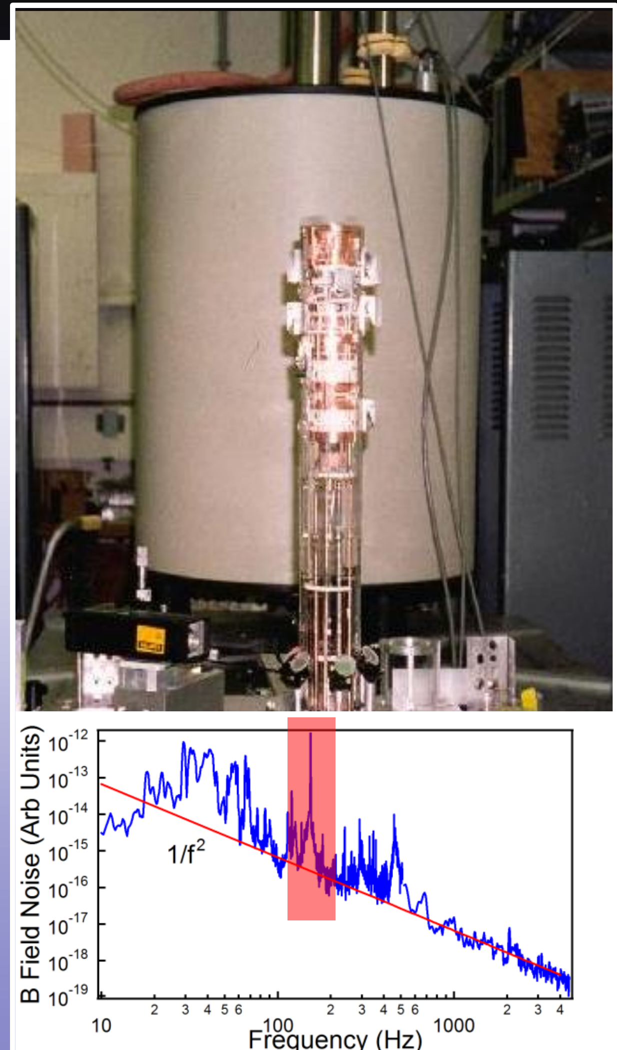
- Independent stochastic errors
- Full quantum mechanical bath
- Classical colored noise
  - Ambient fields
  - Local Oscillator instabilities

12.6 GHz carrier



Dephasing due to classical fields





$^{9}Be^{+}$ @4.5T

Ω_0~124 GHz

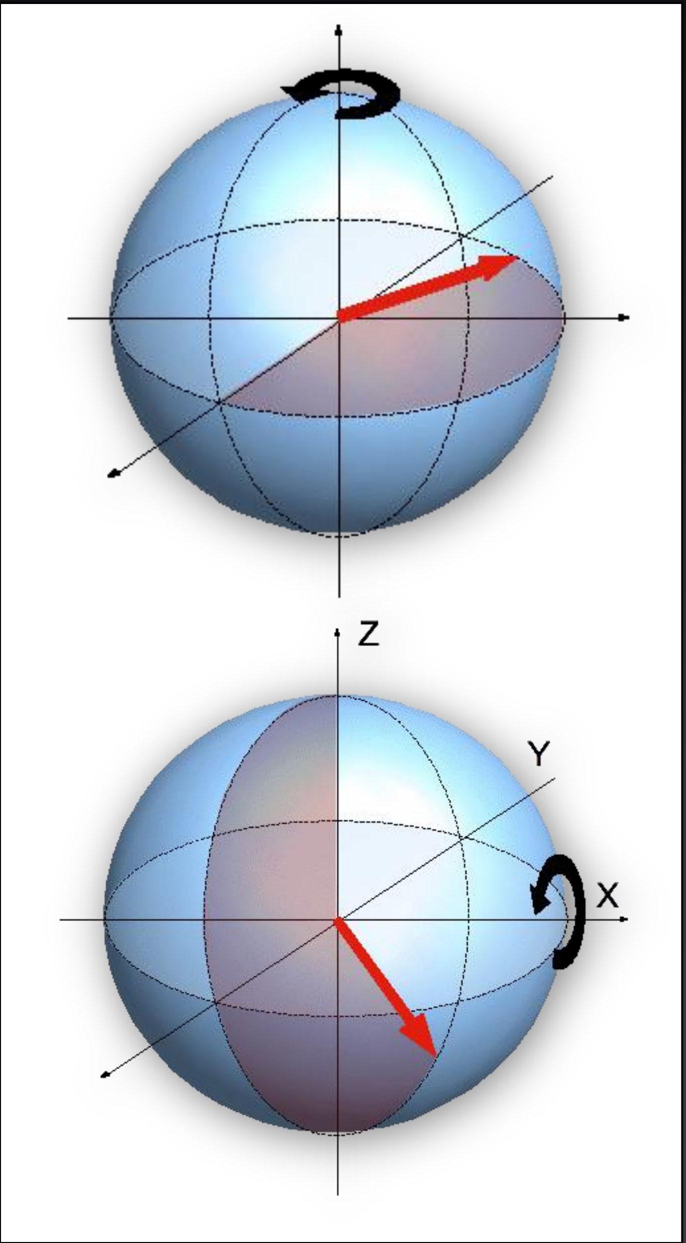
Biercuk *et al.*, *Quant. Info. Comp.* **9**, 920 (2009).

How do we deal with resulting errors?

- Closed-loop feedback control
  $\Rightarrow$ Quantum Error Correction

- Open-loop control
  $\Rightarrow$Dynamic Error Suppression
  (Unitary Quantum control)

What tools do we have?



Dissipative State Prep.

"Single Qubit Gates"

$$I,\ X,\ Y,\ Z,\ H,\ S,\ T,$$

Telecom-style Modulation

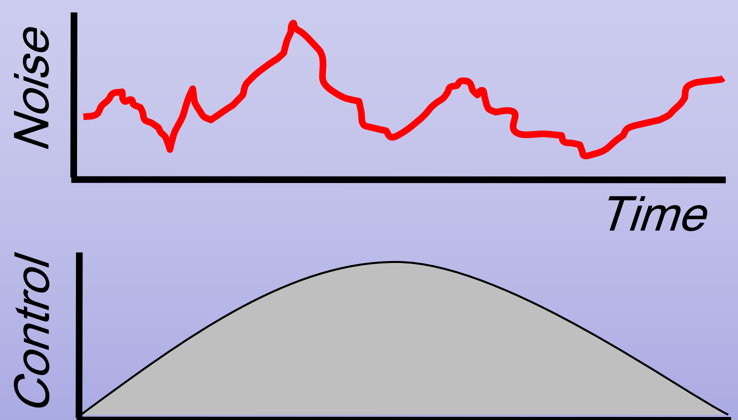
$$I(t),\ \ Q(t)$$

Questions: What is the influence of these techniques on control fidelity in a noisy environment? How well can we do?

Quantum Control as Noise Filtering

Requirement: Evaluate the performance of quantum control operations in the presence of *time-dependent* noise

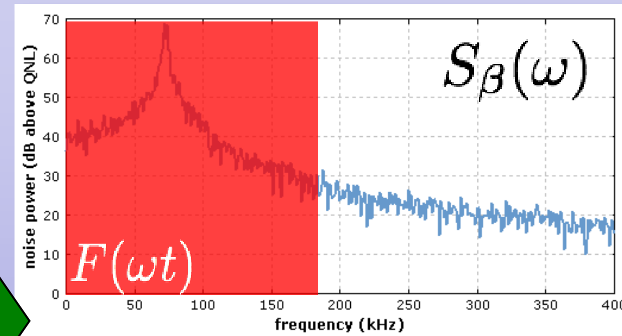
Time Domain



$$|\psi(0)\rangle \rightarrow |\psi(t)\rangle$$

Time-domain *convolution*

Fourier Domain



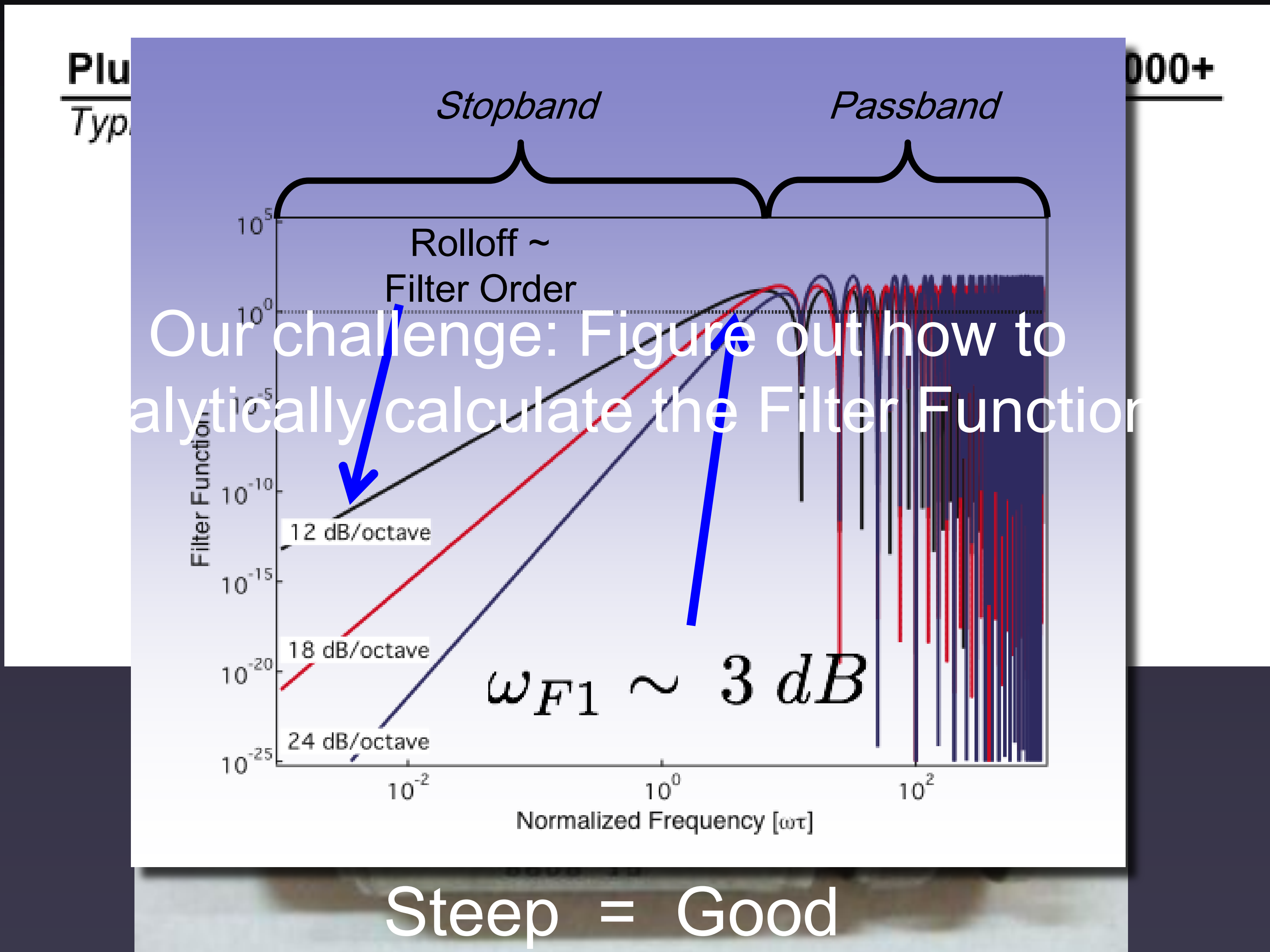
Coherence: $W = e^{-\chi(t)}$

$$\chi(t) = \frac{1}{\pi}\int_0^\infty S(\omega)\frac{F(\omega t)}{\omega^2}d\omega$$

Fourier-domain *product*

Kurizki *et al.*, *PRL* 87, 270405 (2001). Martinis *et al.*, *PRB* 64 094510
Uhrig *et al.*, *PRL* 98, 100504 (2007); Cywinski et al., *PRB* 77, 174509 (2008).
MJB et al., *J. Phys. B* 44, 154002 (2011). Green, Uys, MJB, *PRL* 109 020501 (2012)

The Filter Function

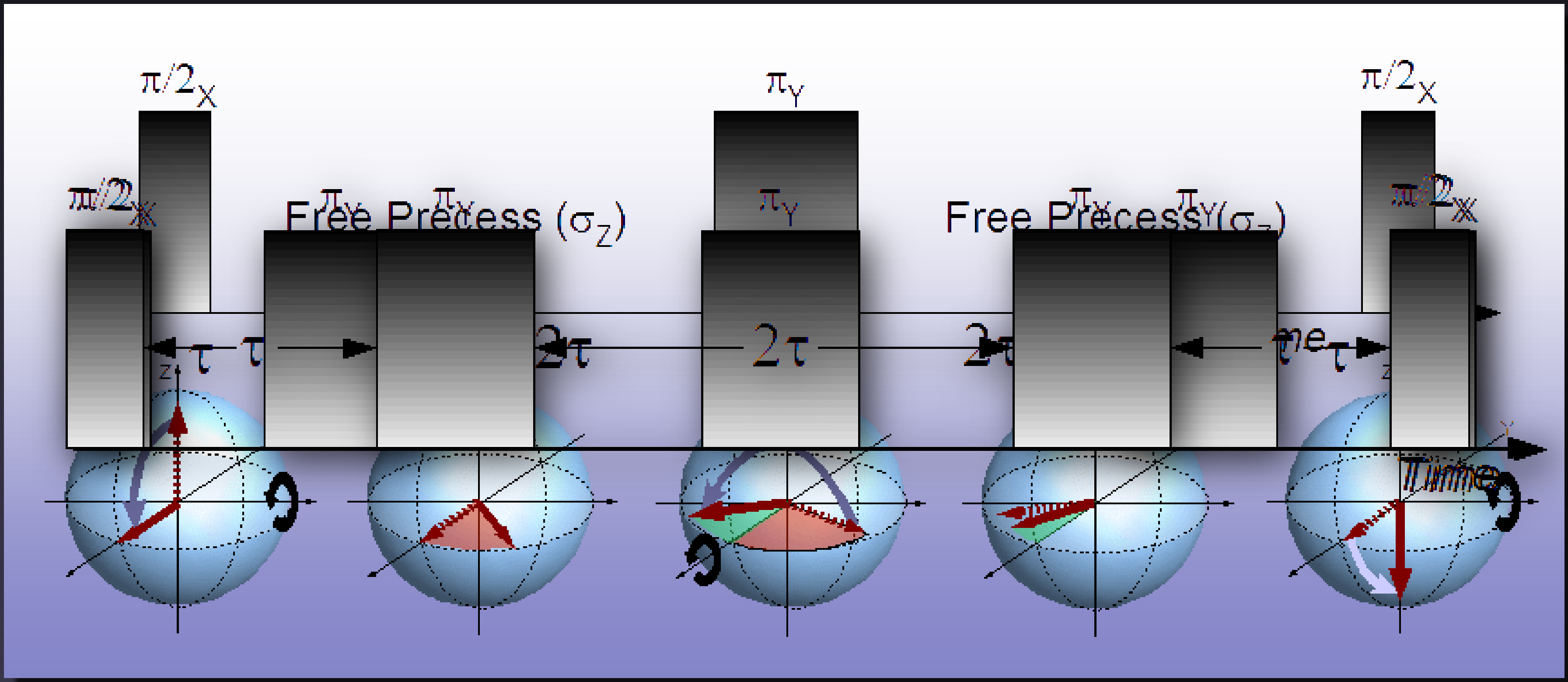


Our challenge: Figure out how to
alytically calculate the Filter Functior

Steep = Good

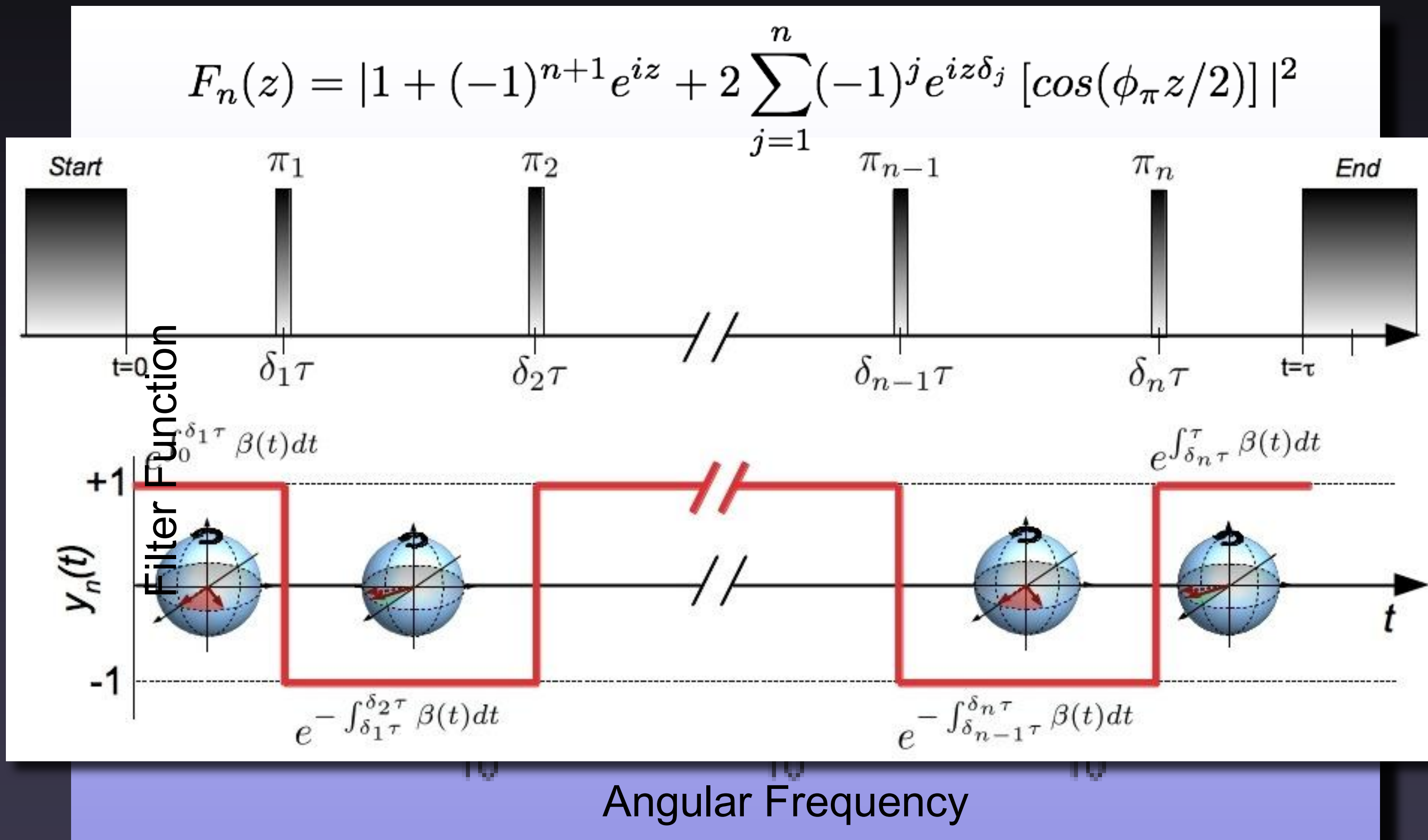
Biercuk et al., *J. Phys. B* 44, 154002 (2011).

Simplest Example: Dynamical Decoupling



Hahn 1950, NMR

Dynamical Decoupling as Noise Filtering

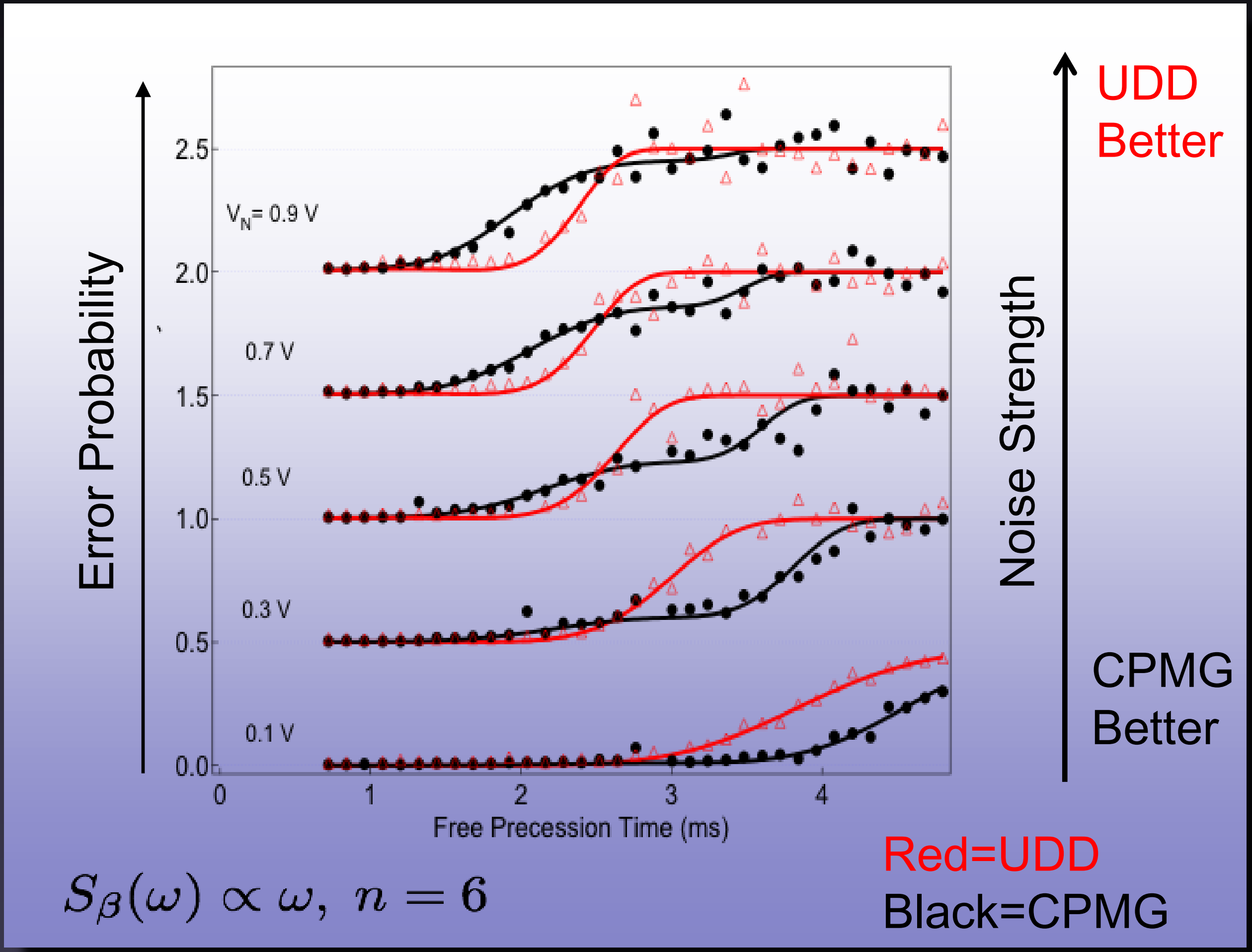


Adjust pulse timing to modify filter

Uhrig *et al.*, *PRL* 98, 100504 (2007); Cywinski et al., *PRB* 77, 174509 (2008).
MJB et al., *J. Phys. B* 44, 154002 (2011).

$H_c \rightarrow I$

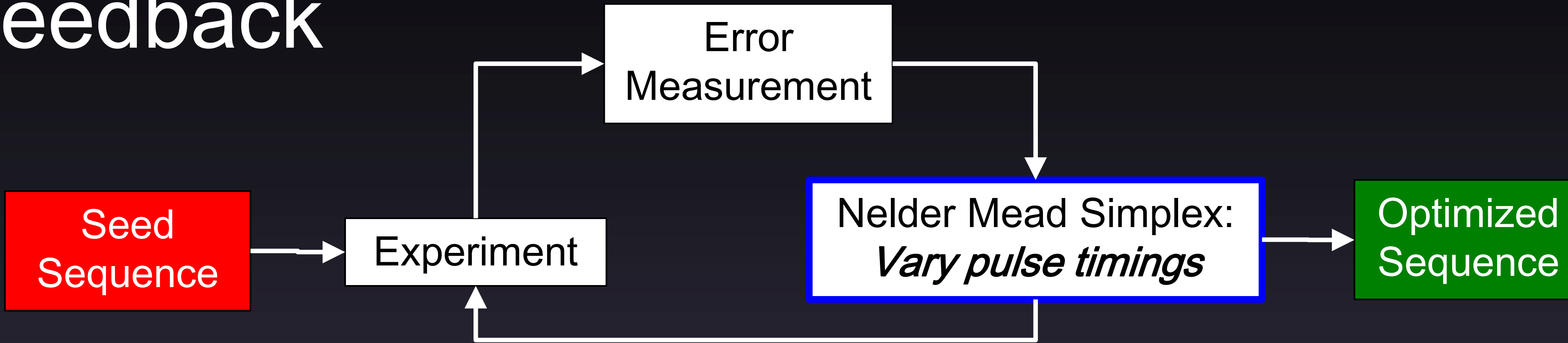
FF approach is *experimentally verified*

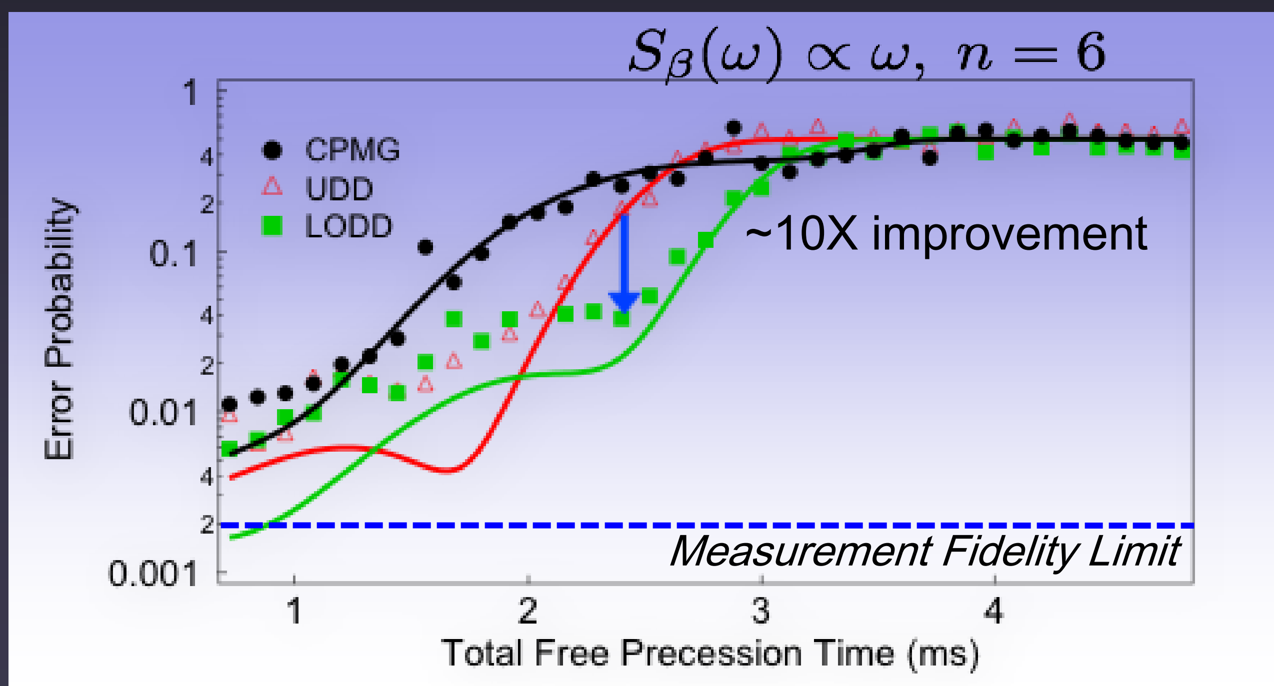


Experiments performed with trapped ions and *Engineered Noise*

MJB *et al.*, *Nature* **458**, 996 (2009). MJB *et al.*, *Phys. Rev. A* **79** 062324 (2009)., MJB et al QIC 9, 920 (2009)

Filter optimization by autonomous feedback





Biercuk *et al.*, *Nature* **458**, 996 (2009)

Noise filtering beyond Memory

$I, X, Y, Z, H, S, T, CNOT/CPHASE$

Challenge: Noncommuting control/ noise

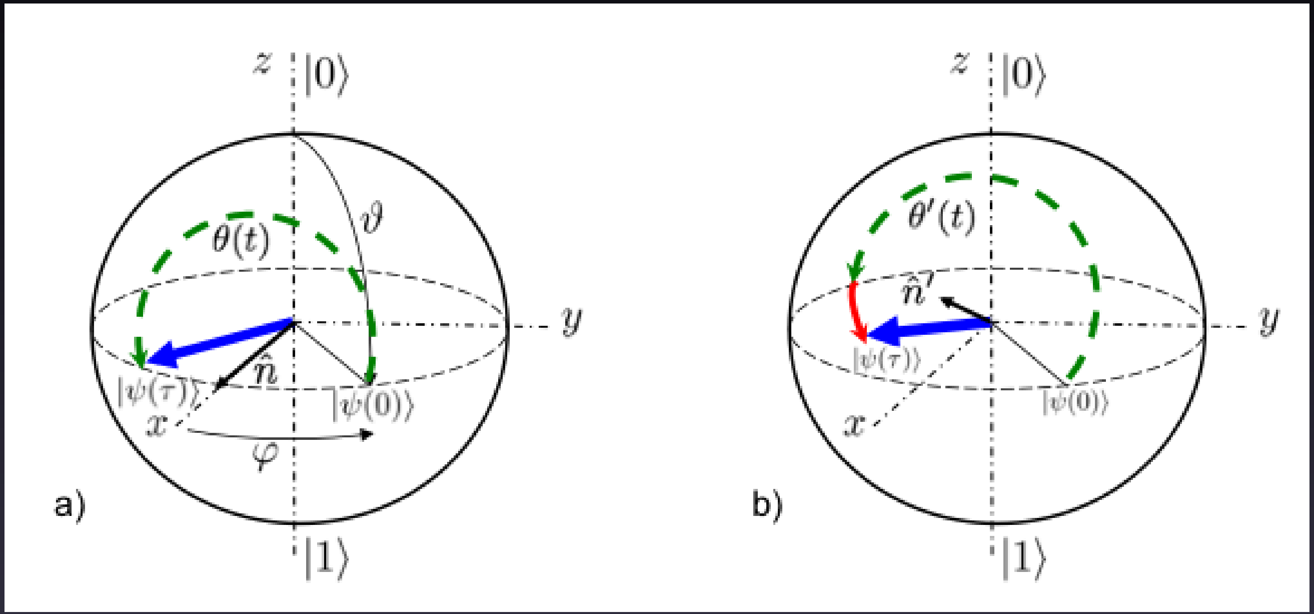
$$H_0 \propto \beta(t)\sigma_z \quad H_c \propto \sigma_x \quad U(t) = T exp\left(-i\int_0^\infty H(t')dt'\right)$$

$$\mathcal{F}_{av} = 1 - \sum_{n=2}^{\infty}\left\{\frac{1}{(2\pi)^n}\sum_{i_1...i_n}\int d\omega_1 ... \int d\omega_n \mathcal{S}_{i_1...i_n}(\omega_1, ..., \omega_n)\mathcal{R}i_1...i_n(\omega_1, ..., \omega_n)\right\}$$

- Treat any piecewise-constant single-qubit control protocol
- Treat the effects of universal decoherence
- Reduces error estimation from hours to milliseconds

Green, Uys, & MJB *PRL* **109** 020501 (2012), Green, Sastrawan, Uys & MJB, arxiv:1211.1163

Example: FF's for simple Pi pulses



$$\mathcal{F}_{av} = 1 - \sum_{n=2}^{\infty} \left\{ \frac{1}{(2\pi)^n} \sum_{i_1 \ldots i_n} \int d\omega_1 \ldots \int d\omega_n \mathcal{S}_{i_1 \ldots i_n}(\omega_1, \ldots, \omega_n) \mathcal{R} i_1 \ldots i_n(\omega_1, \ldots, \omega_n) \right\}$$

Multi-axis errors from single-axis control

Fully generalized Filter Functions

$$R_{zz}^{(Prim)}(\omega) = \frac{\omega^2}{\omega^2 - \Omega^2} \left(e^{i\omega\tau_\pi} + 1\right) \qquad R_{zy}^{(Prim)}(\omega) = \frac{i\omega\Omega}{\omega^2 - \Omega^2} \left(e^{i\omega\tau_\pi} + 1\right)$$

Green, Uys, MJB, *PRL* **109** 020501 (2012)

$$\langle\mathcal{F}\rangle \simeq 1-\sum_i \langle a_{1,i}^2\rangle - \left\{\sum_i \left(\langle a_{2,i}^2\rangle + 2\langle a_{1,i}a_{3,i}\rangle\right) - \frac{1}{3}\sum_{i,j}\langle a_{1,i}^2 a_{1,j}^2\rangle\right\} \tag{38}$$

In the frequency domain

$$\langle\mathcal{F}\rangle = 1-\frac{1}{4\pi}\int_0^\infty \frac{d\omega}{\omega^2}S(\omega)F_1(\omega,\tau) - \frac{1}{(4\pi)^2}\int_0^\infty \frac{d\omega}{\omega^2}S(\omega)\int_0^\infty \frac{d\omega'}{\omega'^2}S(\omega')F_2(\omega,\omega'\tau)+\ldots \tag{39}$$

where

$$F_1(\omega,\tau) \equiv \sum_i |y_{1,i}(\omega,\tau)|^2 \tag{40}$$

$$F_2(\omega,\omega'\tau) \equiv \sum_i \left(F_{2a,i}(\omega,\omega',\tau)+2F_{2b,i}(\omega,\omega',\tau)\right) - \frac{1}{3}\sum_{i,j}F_{2,c,i,j}(\omega,\omega',\tau). \tag{41}$$

The terms making up $F_2(\omega,\omega'\tau)$, and may be written

$$F_{2,a,i}(\omega,\omega',\tau) \equiv \{y_{2,i}(\omega,\omega',\tau)+y_{2,i}(\omega,-\omega',\tau)+y_{2,i}(-\omega,\omega',\tau)+y_{2,i}(-\omega,-\omega',\tau)\} \tag{42}$$

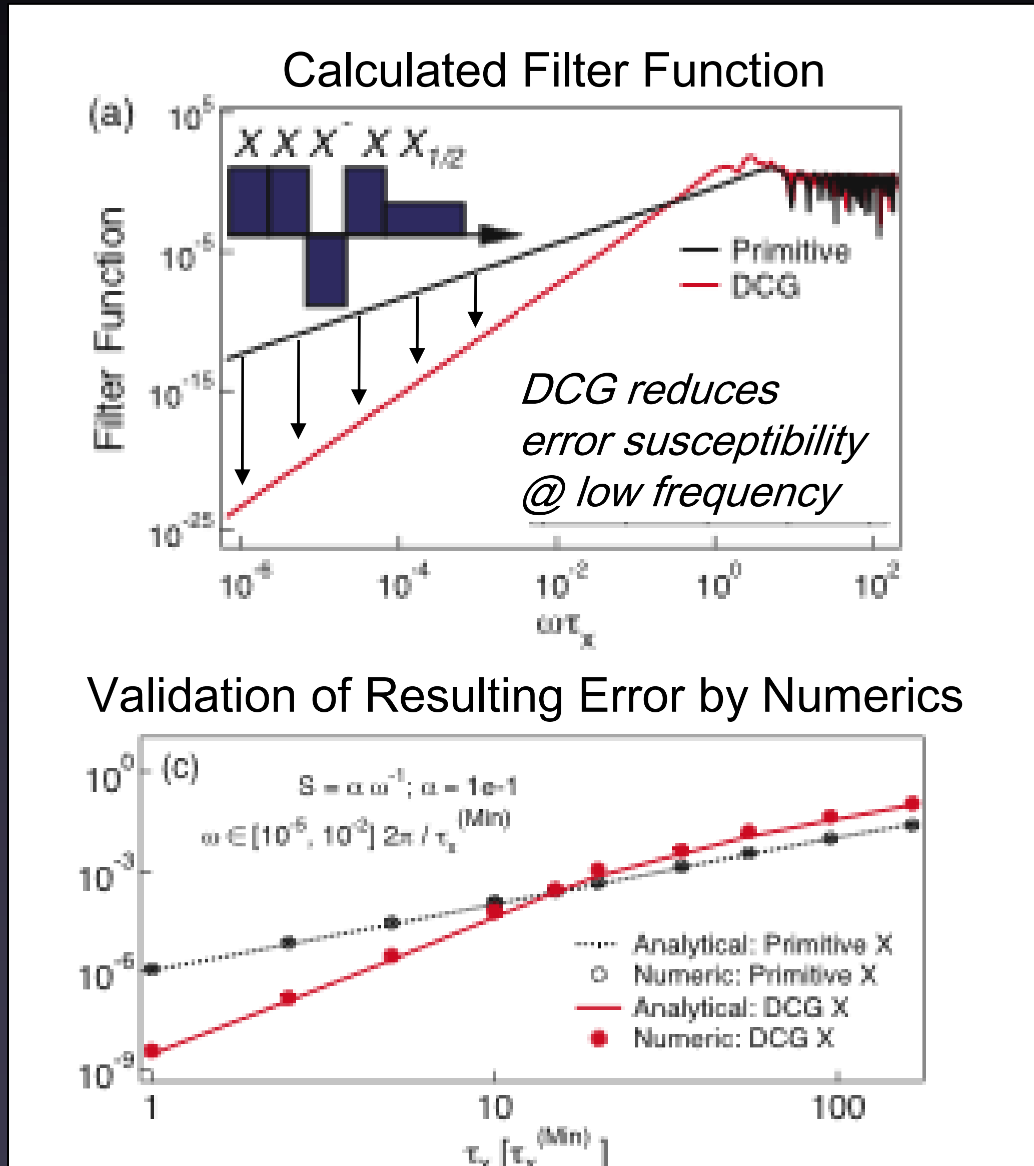
$$\begin{aligned} y_{2,i}(\omega,\omega',\tau) = \frac{\omega^2\omega'^2}{4}\Bigg\{&\int_0^\tau dt_4 e^{i\omega' t_4}\int_0^{t_4} dt_3 e^{-i\omega' t_3}\int_0^\tau dt_2 e^{i\omega t_2}\int_0^{t_2} dt_1 e^{-i\omega t_1} s_{2,i}(t_1,t_2)s_{2,i}(t_3.t_4)\\ &+\int_0^\tau dt_4 e^{i\omega' t_4}\int_0^{t_4} dt_3 e^{i\omega t_3}\int_0^\tau dt_2 e^{-i\omega' t_2}\int_0^{t_2} dt_1 e^{-i\omega t_1} s_{2,i}(t_1,t_2)s_{2,i}(t_3.t_4)\\ &+\int_0^\tau dt_4 e^{i\omega t_4}\int_0^{t_4} dt_3 e^{i\omega' t_3}\int_0^\tau dt_2 e^{-i\omega' t_2}\int_0^{t_2} dt_1 e^{-i\omega t_1} s_{2,i}(t_1,t_2)s_{2,i}(t_3.t_4)\Bigg\} \end{aligned} \tag{43}$$

$$F_{2,b,i}(\omega,\omega',\tau) \equiv \{y_{3,i}(\omega,\omega',\tau)+y_{3,i}(\omega,-\omega',\tau)+y_{3,i}(-\omega,\omega',\tau)+y_{3,i}(-\omega,-\omega',\tau)\} \tag{44}$$

$$\begin{aligned} y_{3,i}(\omega,\omega',\tau) = \frac{\omega^2\omega'^2}{4}\Bigg\{&\int_0^\tau dt_4 e^{i\omega' t_4}\int_0^\tau dt_3 e^{-i\omega' t_3}\int_0^{t_3} dt_2 e^{i\omega t_2}\int_0^{t_2} dt_1 e^{-i\omega t_1} s_{1,i}(t_4)s_{3,i}(t_1,t_2,t_3)\\ &+\int_0^\tau dt_4 e^{i\omega' t_4}\int_0^\tau dt_3 e^{i\omega t_3}\int_0^{t_3} dt_2 e^{-i\omega' t_2}\int_0^{t_2} dt_1 e^{-i\omega t_1} s_{1,i}(t_4)s_{3,i}(t_1,t_2,t_3)\\ &+\int^\tau dt_4 e^{i\omega t_4}\int^\tau dt_3 e^{i\omega' t_3}\int^{t_3} dt_2 e^{-i\omega' t_2}\int^{t_2} dt_1 e^{-i\omega t_1} s_{1,i}(t_4)s_{3,i}(t_1,t_2,t_3)\Bigg\} \end{aligned}$$

$$F_{2,c,i,j}(\omega,\omega',\tau) \equiv |y_{1,i}(\omega,\tau)|^2\,|y_{1,j}(\omega',\tau)|^2 + 2Re\left[y_{1,i}(\omega,\tau)y_{1,j}(\omega,\tau)^*\right] Re\left[y_{1,i}(\omega',\tau)y_{1,j}(\omega',\tau)^*\right]$$

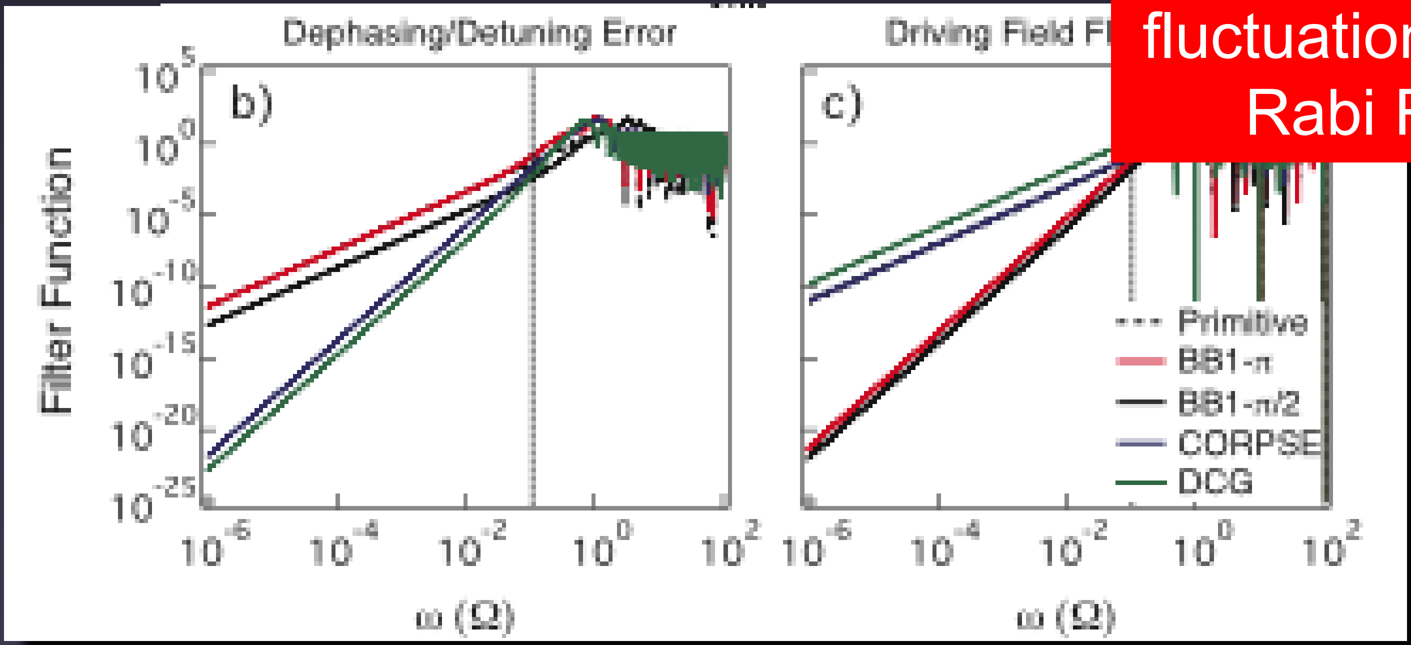
Example FF's for Corrected Gates



Green, Uys, MJB, *PRL* **109** 020501 (2012)

Filter Functions for Composite Pulses

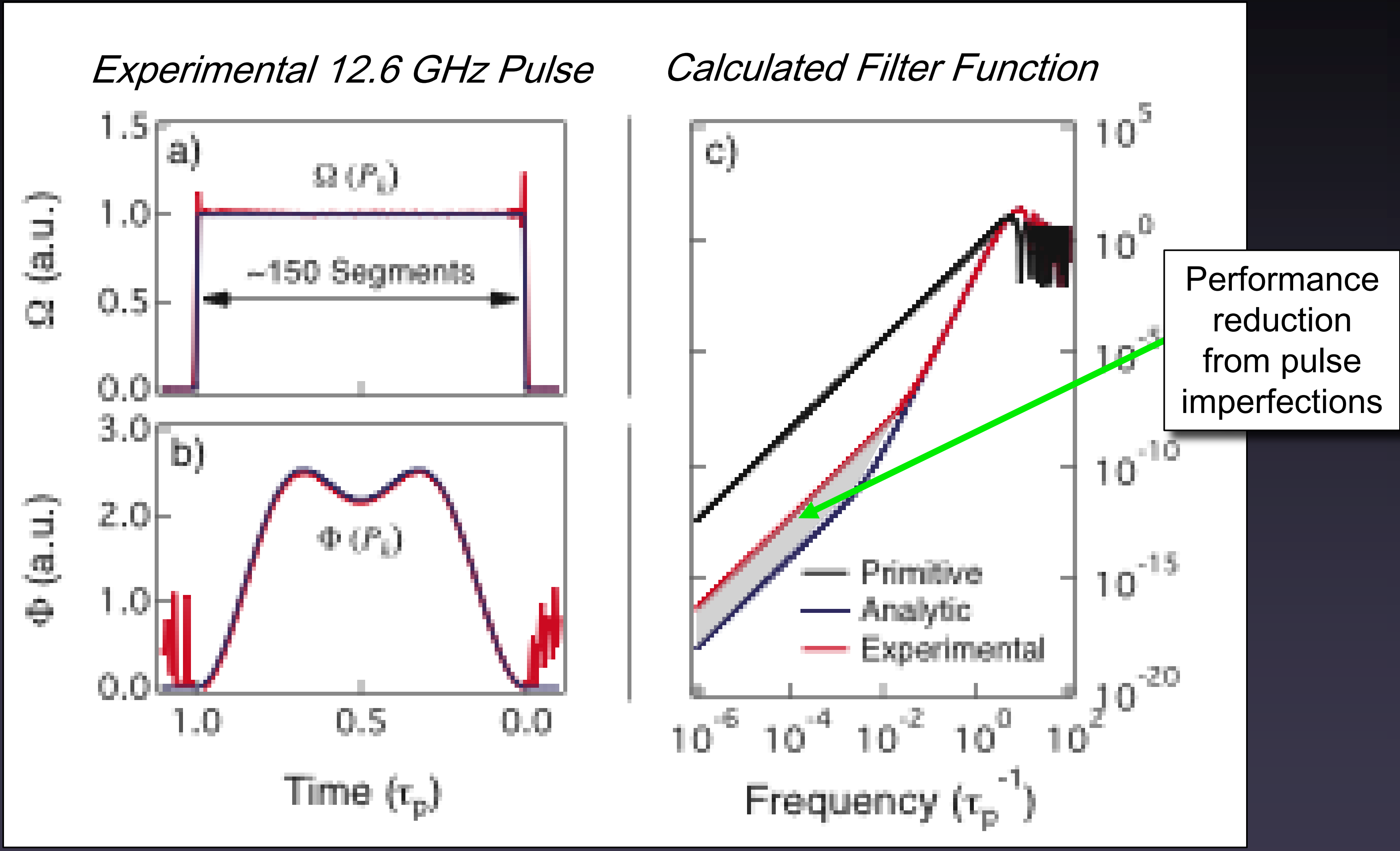
- BB1: $$U(\pi,\beta)U(2\pi,3\beta)U(\pi,\beta)$$ $$\beta = \cos^{-1}(-1/4)$$
- CORPSE: $$U(\pi/3,0)U(5\pi/3,\pi)U(7\pi/3,0)$$
- DCG: $$U(\pi,0)U^{(1/2)}(\pi,0)U$$



Robust to fast fluctuations~10% Rabi Rate

Katatyev, Green, Soare, Viola, Brown, MJB, In preparation

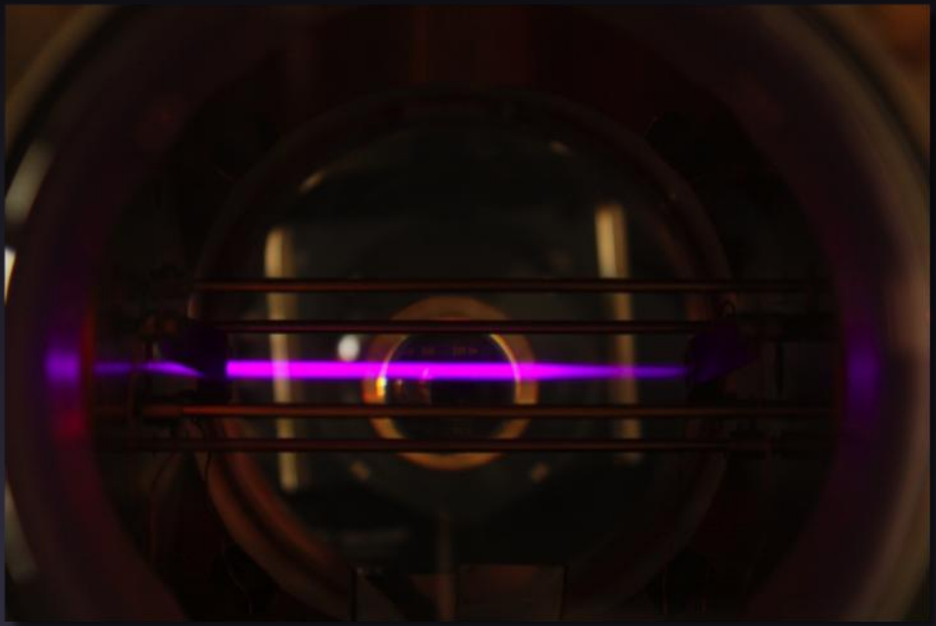
Experimental control imperfections



Fasuweh et al., PRA 85 022310 (2012)

Green, Sastrawan, MJB, arXiv:1211.1163 (2012). Kabytaev, Soare, Green, …MJB, *In preparation*

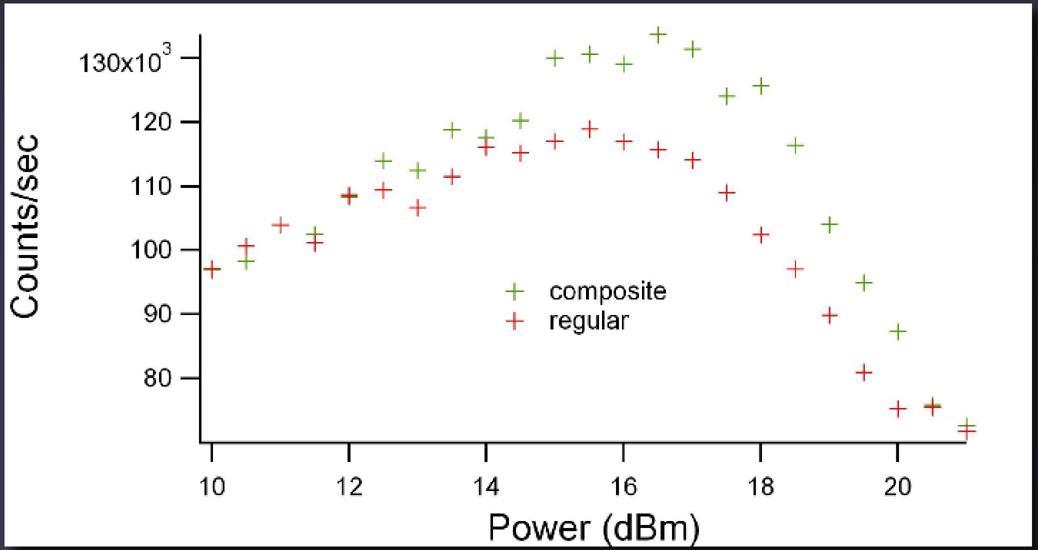
Currently studying in the lab…



- 12.6 GHz 171Yb+ qubit
- Vector source
- Engineered noise bath

Simple composite pulse

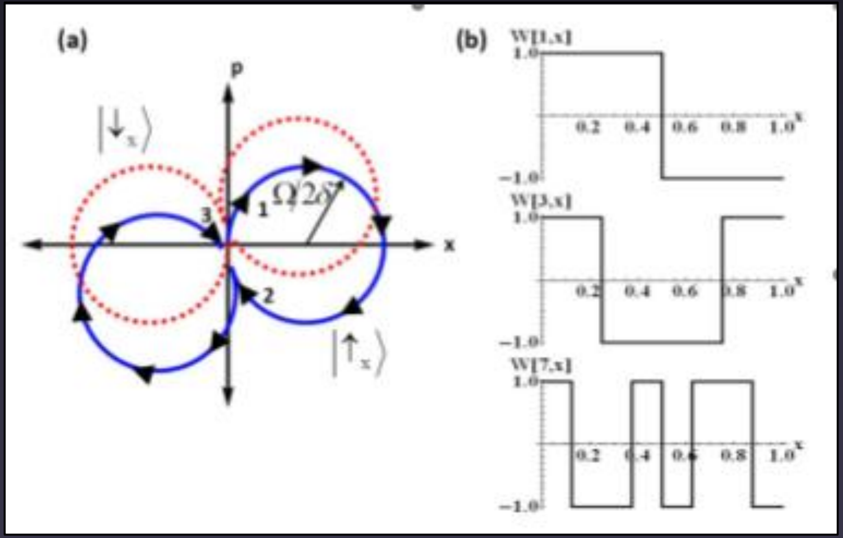
$$\frac{\pi}{2}_x - \pi_y - \frac{\pi}{2}_x$$

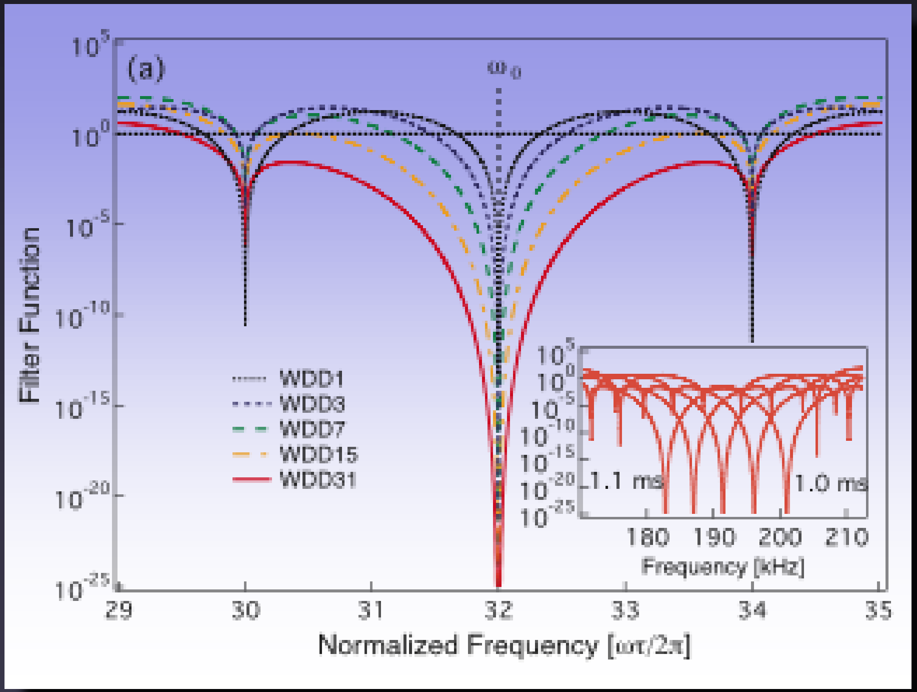


Moving to multiqubit gates

$I,\ X,\ Y,\ Z,\ H,\ S,\ T,$ $CNOT/CPHASE$

MS Gates with Laser Detuning & Amplitude Errors

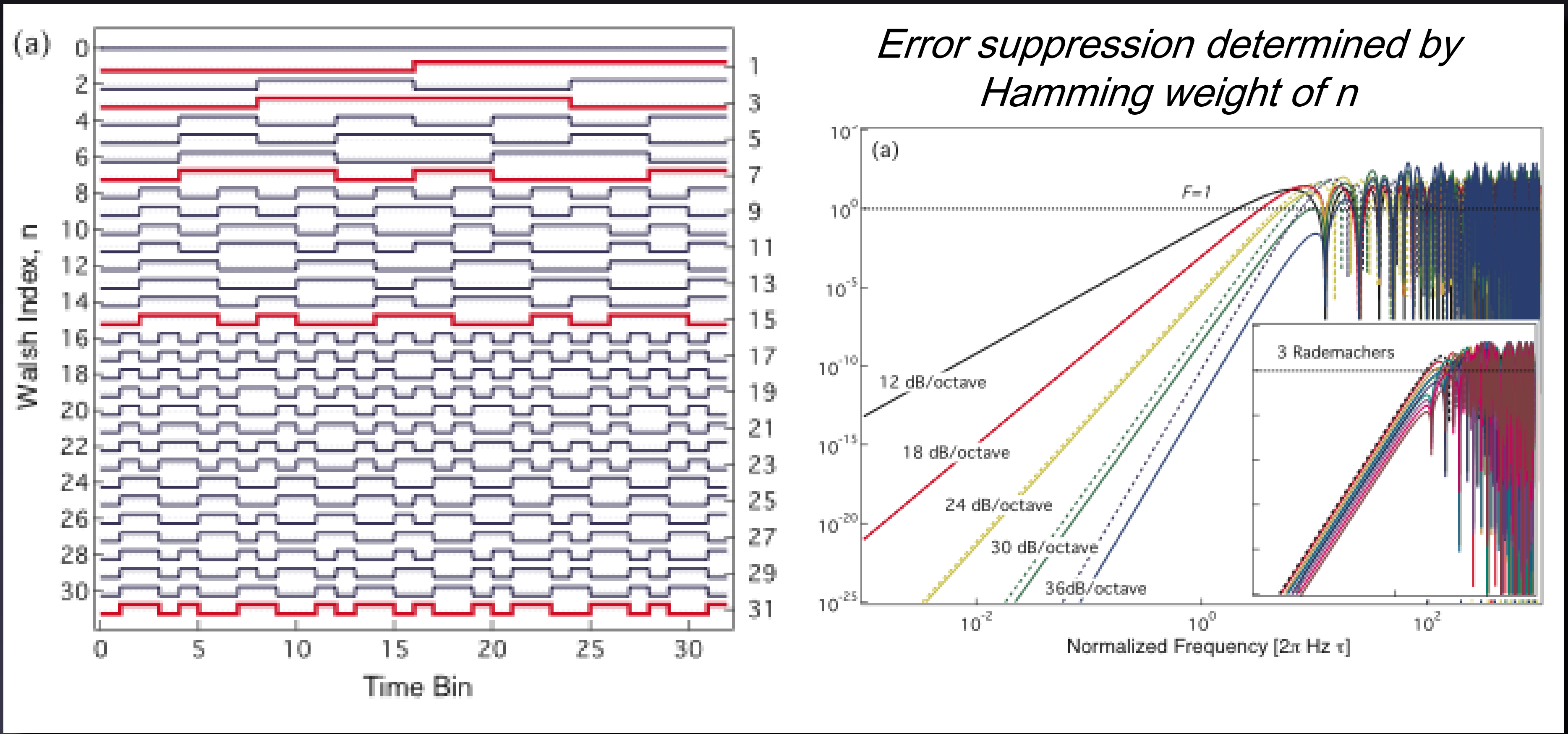




$$\overline{|\alpha(\tau)|^2} = \frac{\Omega^2}{4}\int_{-\infty}^{\infty}\frac{F_p\left((\delta+\Delta)\tau\right)}{(\delta+\Delta)^2}P(\Delta)d\Delta$$

Hayes et al., *PRL*; Hayes, Khodjasteh, Viola, MJB, *PRA* 84, 062323 (2011).

Our basis: Walsh functions



Structure gives interesting implications at the system level

Hayes, Khodjasteh, Viola, MJB, *PRA* 84, 062323 (2011).

Noise filtering at the system level

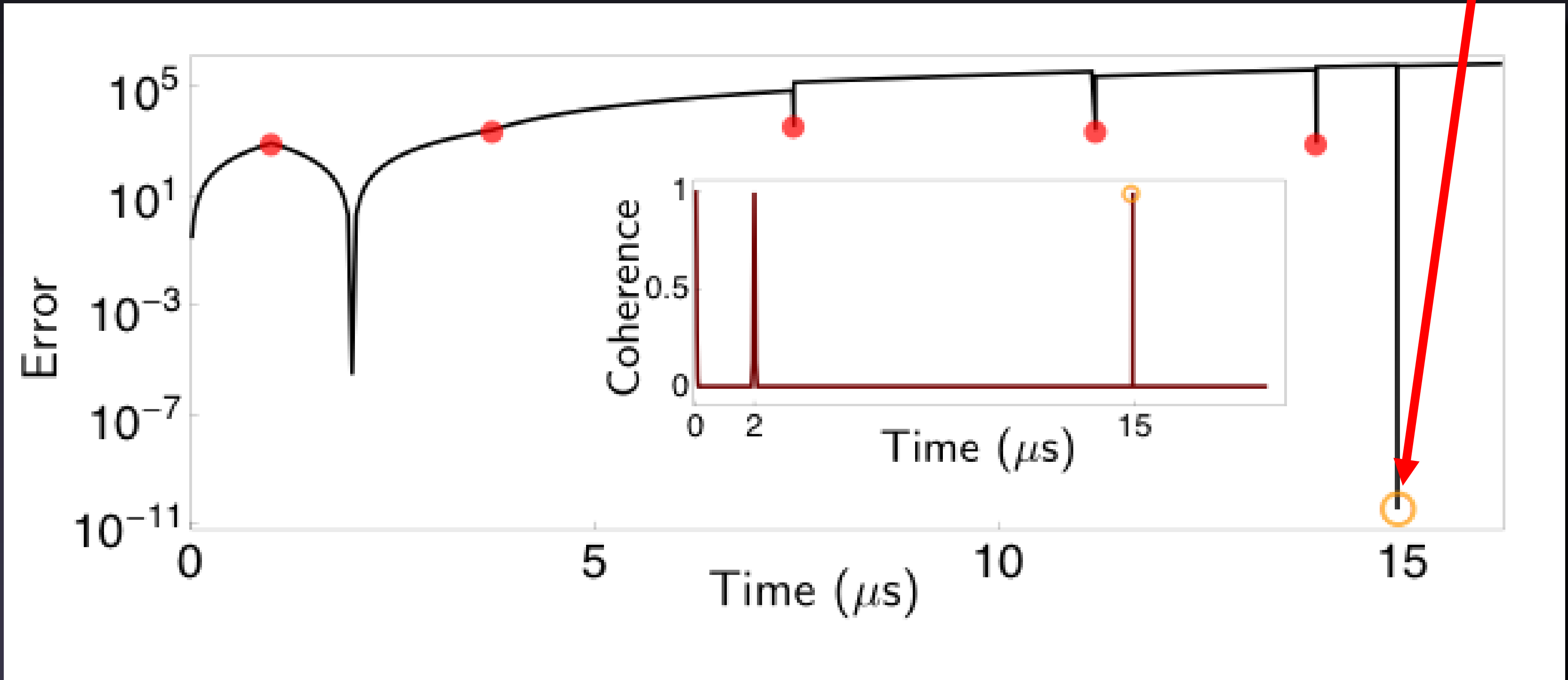
Designing a practical long-time quantum memory – considerations:

1. Perturbative DD (increasing *N*) fails for long-time storage, by requiring the unphysical constraint τ_{min} -> 0
2. Numerical optimization becomes impractical in long time (large-*N*) limit
3. Access to quantum state required throughout memory time, with minimum latency

Khodjasteh, Sastrawan, Hayes, Green, MJB, Viola, arXiv:1206.6087

Latency is a killer

Error low ONLY here

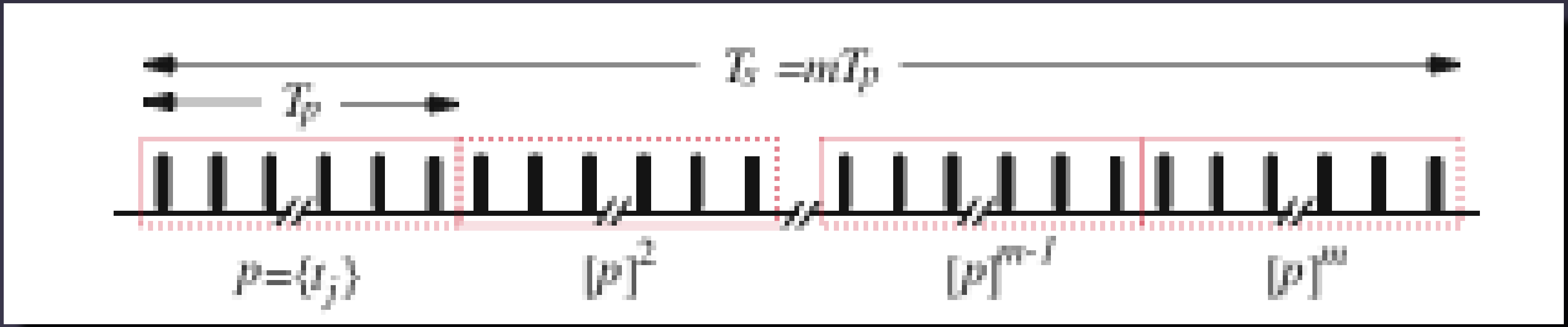


Optimizing the filter directly fails to address third consideration

Error calculated for UDD and assuming noise common for singlet-triplet qubit

Systematic approach: Repetition

- Take a short, high-order “base sequence” and repeat as needed.
- Interrupt latency is limited to integer multiples of the base sequence length, not the total storage time
- Analytically evaluate the effect of repetition in the long-storage-time limit



Khodjasteh, Sastrawan, Hayes, Green, MJB, Viola, arXiv:1206.6087

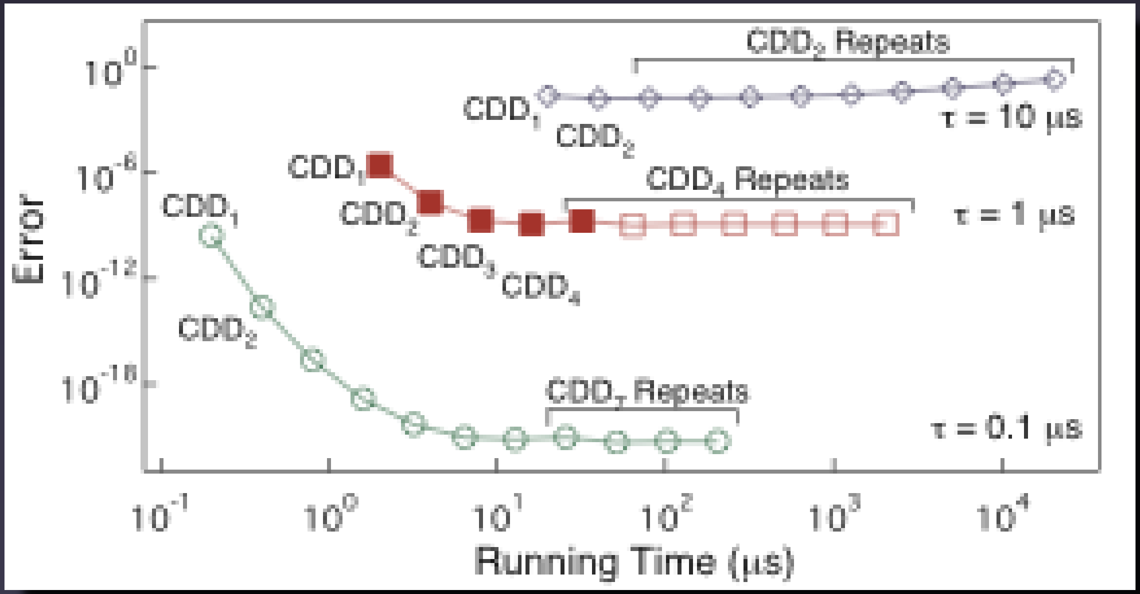
Repeated application of noise filter

$$\chi_p = \int_0^\infty \frac{S(\omega)}{2\pi\omega^2} F_p(\omega) d\omega \xrightarrow[m \text{ times}]{\text{Repeat}} \chi_{[p]^m} = \int_0^\infty \frac{S(\omega)}{2\pi\omega^2} \frac{\sin^2(m\omega T_p/2)}{\sin^2(\omega T_p/2)} F_p(\omega)\, d\omega$$

$$\lim_{m\to\infty} \chi_{[p]^m} \equiv \chi_{[p]^\infty} = \int_0^{\omega_c} \frac{S(\omega)}{4\pi\omega^2} \frac{F_p(\omega)}{\sin^2(\omega T_p/2)} d\omega$$

$$\boxed{\chi_{[p]^m} \leq 2\chi_{[p]^\infty}}$$

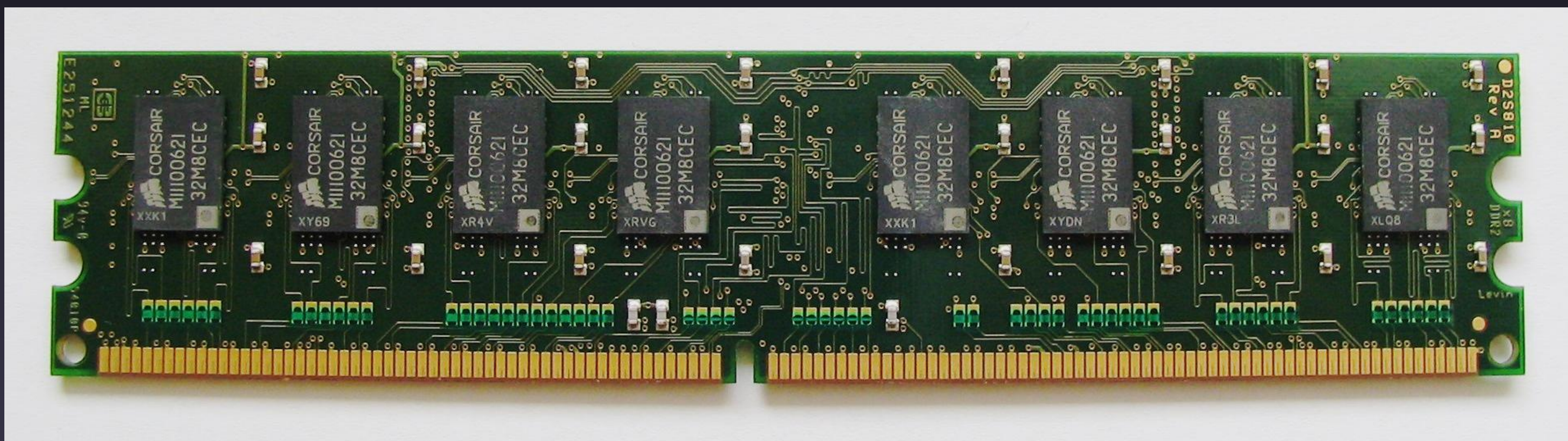
If integral converges, coherence bounded at long times



Khodjasteh, Sastrawan, Hayes, Green, MJB, Viola, arXiv:1206.6087

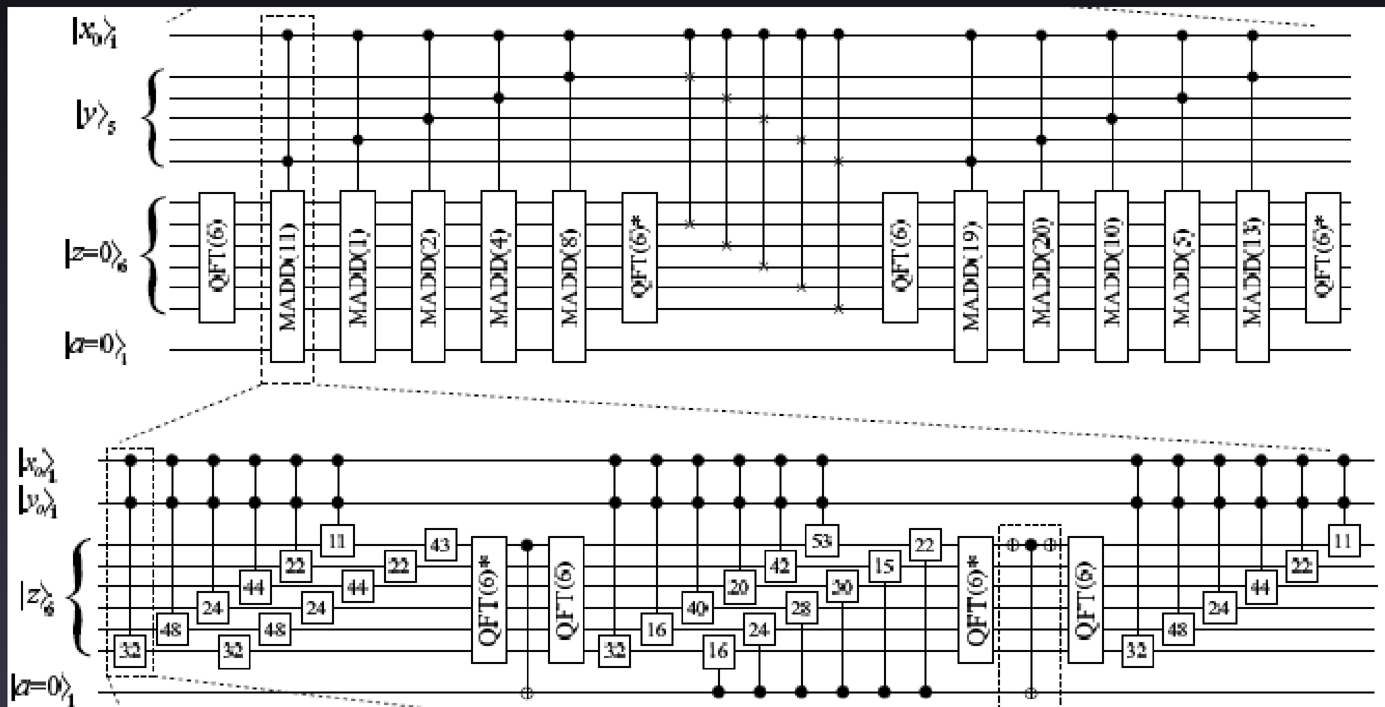
Search over Walsh basis

Similar effects already in use



DRAM, Corsair

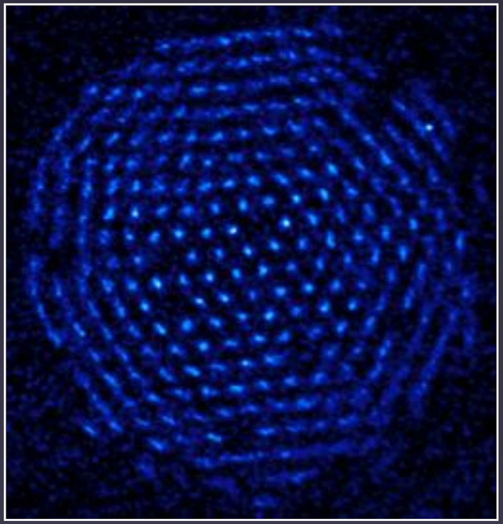
Future Possibilities: Algorithmic Design(?)



Exploit echo-like effects in algorithmic design
Can produce filter functions for algorithms, blocks, etc.

Summary

- Quantum control as noise filtering
  - New analytical approach based on noise filters
  - Filtering *during* gate operations
  - Approaches compatible with large-scale systems
- We're bringing a "30,000 foot" viewpoint to these analyses.



Other stuff:
Quantum Simulation of the variable-range 2D Ising model on a triangular lattice with N ~300 qubits

Acknowledgements & Collaborators

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Jarrah Sastrawan
Alex Soare

DARTMOUTH 1769

Lorenza Viola
Kaveh Khodjasteh

CSIR our future through science

Hermann Uys

NIST

John Bollinger
Joe Britton
Brian Sawyer
David Wineland

PhD opportunities and postdoctoral fellowships available in my Group

Visitors Welcome!

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Photo: Tourism Australia