Gaming the quantum for control

Faisal Shah Khan
Khalifa University of Science, Technology & Research, Abu Dhabi, UAE.

Abstract

A non-cooperative game theoretic perspective on the dynamics of quantum systems is proposed with the goal of exploring control of these dynamics which is optimal in the sense of Nash equilibrium.

A non-cooperative game

is a function G with domain equal to the product of the *pure strategy* sets S_i of *player i* and image called the *set of outcomes* O such that the players entertain non-identical preferences over these outcomes.

Example (Prisoners' Dilemma):

$$G:\{D, H\}\times\{D, H\}\rightarrow\{(3,3),(0,5),(5,0),(1,1)\}$$

Players' Preferences

$$\text{Player I}:(5,0)\succ(3,3)\succ(1,1)\succ(0,5)$$
$$\text{Player II}:(0,5)\succ(3,3)\succ(1,1)\succ(5,0)$$

The symbol $\succ$ captures "more preferred"

Assuming *rational* players seeking outcomes consistent with their preferences, the optimal outcome here is (3,3).

However, in attempting to satisfy their preferences, the likely outcome for rational players is the *Nash equilibrium* (1,1) where each player gives a best reply to the choice of his opponent's pure strategy.

Nash equilibrium

Prisoners' Dilemma

Pure Strategies — Player II

Player I	D	H
D	(3,3)	(0,5)
H	(5,0)	(1,1)

The outcomes (1,1) with the corresponding pure strategy pair (H, H) is the *Nash Equilibrium* outcome where unilateral deviation by a player from his pure strategy produces an outcome of lesser preference for that player.

A non-cooperative quantum game

is a function Q such that the pure strategy sets S_i are sets admissible in quantum mechanics and set of outcomes O is a Hilbert space of quantum superpositions such that players entertain non-identical preferences over these quantum superpositions.

One way to define players' preferences over quantum superpositions is via observables, that is, elements of an orthogonal basis of Hilbert space of quantum superpositions, as follows.

Let $B=\{b_1, b_2, b_3, b_4\}$ be an orthogonal basis for a four dimensional Hilbert space. Define the preferences of two Player I and Player II over the elements of B to be

$$\text{Player I}: b_1\succ b_2\cong b_3\cong b_4$$
$$\text{Player II}: b_2\succ b_1\cong b_3\cong b_4$$

This symbols captures "indifference between"

A game with players' preferences such as the one's above reflecting the philosophy "one man's meat is another man's poison" is called a strictly competitive (quantum) game.

These preferences over basis elements induce preferences over quantum superpositions via the inner-product (distance notion) of the quantum system.

Player I (II) will prefer a quantum superpositions q over another p if q is closer to $b_1(b_2)$ than p is.

Each rational player will now choose quantum strategies in a way that is consistent with his preferences over quantum superpositions.

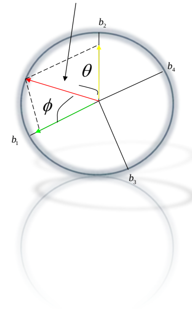
A choice of pure quantum strategies (S,T) in a two player strictly competitive game Q is a Nash equilibrium if Q(S,T) is a quantum superposition that simultaneously minimizes the distance between itself and the most preferred basis element of each player.

Question

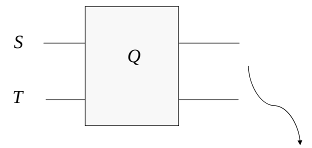
What is the nature of the quantum game Q and the pure strategies S and T of the players such that Q entertains Nash equilibrium?

Strictly competitive quantum game geometry

$v = Q(S,T)$, a Nash equilibrium quantum superposition



A quantum control mechanism at Nash equilibrium



$$\gamma=\gamma_1|00\rangle+\gamma_2|01\rangle+\gamma_3|10\rangle+\gamma_4|11\rangle$$

If this output state satisfies the geometry of a Nash equilibrium, then can input state (S,T) be found and a quantum mechanism (circuit) Q be constructed so that Q(S,T)=r?

Potential applications of gaming the quantum for control

Gaming the quantum allows one to speak of optimal control of quantum mechanisms in the game-theoretic sense of Nash equilibrium.

One potential application of this idea is to quantum circuit design for quantum algorithms. Grover's search algorithm for example exhibits strictly competitive preferences over the outcomes, with the searched item being the most preferred outcome of "Player I". This setting may also provide insights into equilibrium behavior of quantum effects found in biological systems such as photosynthesis.

Another possible application is in the area of quantum neural networks. An ongoing project with Ahmed al Hady at Max-Planck-Institut für Dynamik und Selbstorganisation in Germany attempts to identify optimal learning strategies for quantum mechanical mdels of a simple neural network called the perceptron.

A further application of gaming the quantum could be in the study of relativistic quantum mechanics. The geometry of Nash equilibrium, at least in strictly competitive games, is based on the metric of the Hilbert space of quantum superpositions. Could it be the case that some appropriate functor in the form of a topological quantum field theory be constructed between the category of (projective) Hilbert spaces used in quantum mechanics and the category of differential manifolds used in general relativity? Such a functor would allow to speak meaningfully of relativistic quantum control at Nash equilibrium.

References

[1] Faisal Shah Khan, Simon J.D. Phoenix, *Gaming the Quantum*, Quantum Information & Computation, Vol.13 No.3&4 March 1, 2013.

[2] Faisal Shah Khan, Ahmed el Hady, *Gaming Quantum Neural Networks*, to appear in proceedings of SPIE, Defense, Security + Sensing, 2013.