# Theoretical and experimental error correction of programmable quantum annealing

New Directions in the Quantum Control Landscape
KITP, Santa Barbara, March 1, 2013

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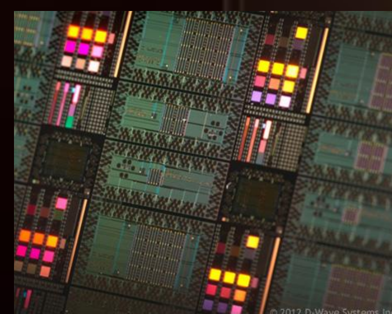
University of Southern California

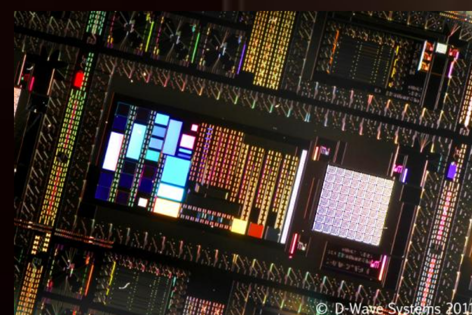
# We'd like to error-correct this machine

D-Wave 1, 128-qubit "Rainier" processor
Purchased by Lockheed Martin Corp.
Installed at USC's Information Sciences Institute (ISI)
Operational 12/23/11-12/31/12



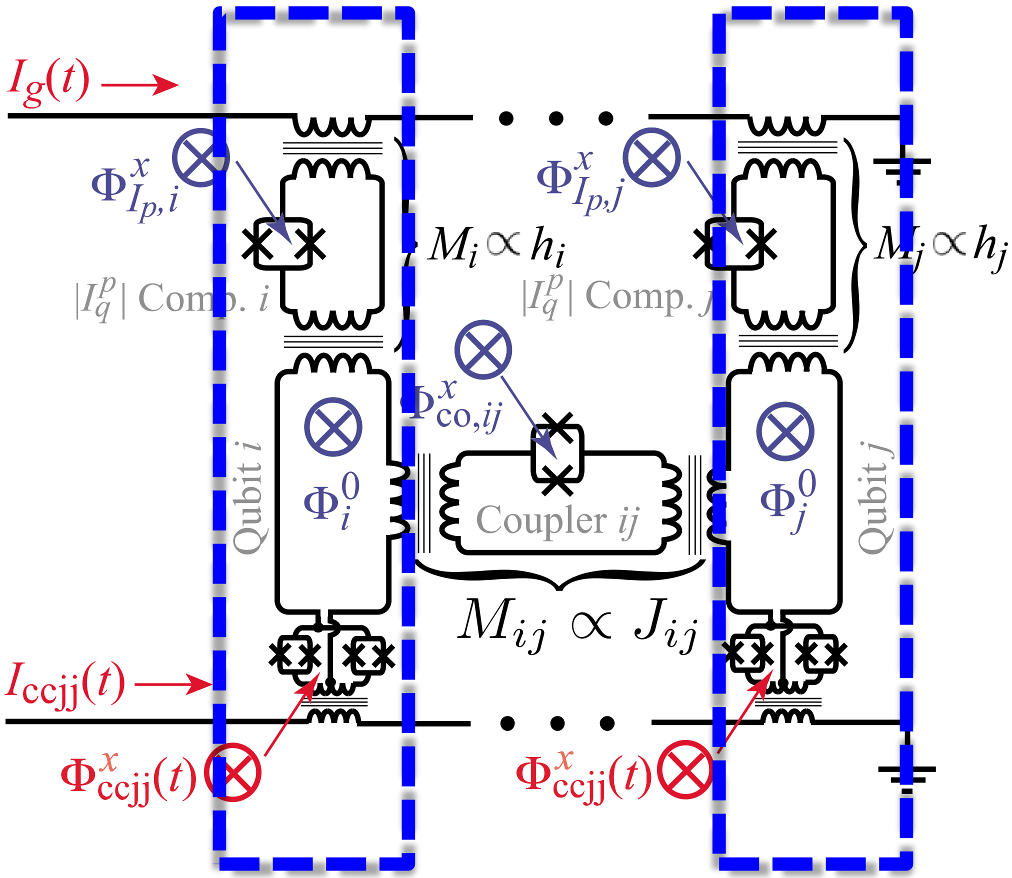
# We'd like to error-correct this machine

D-Wave 2, 512-qubit "Vesuvius" processor
Purchased by Lockheed Martin Corp.
Being installed at USC's Information Sciences Institute (ISI)
Not yet operational.



# Flux Qubits

niobium based compound-compound Josephson junction (CCJJ) rf SQUIDs



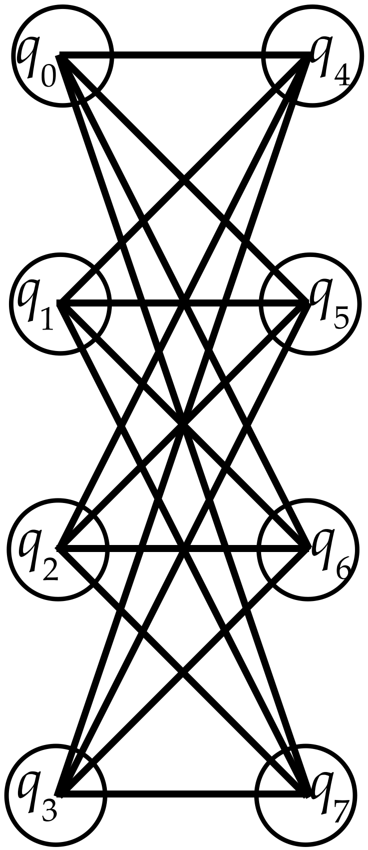

$T_1 \sim 10 - 100\mathrm{ns}$
relaxation time

error correction is important

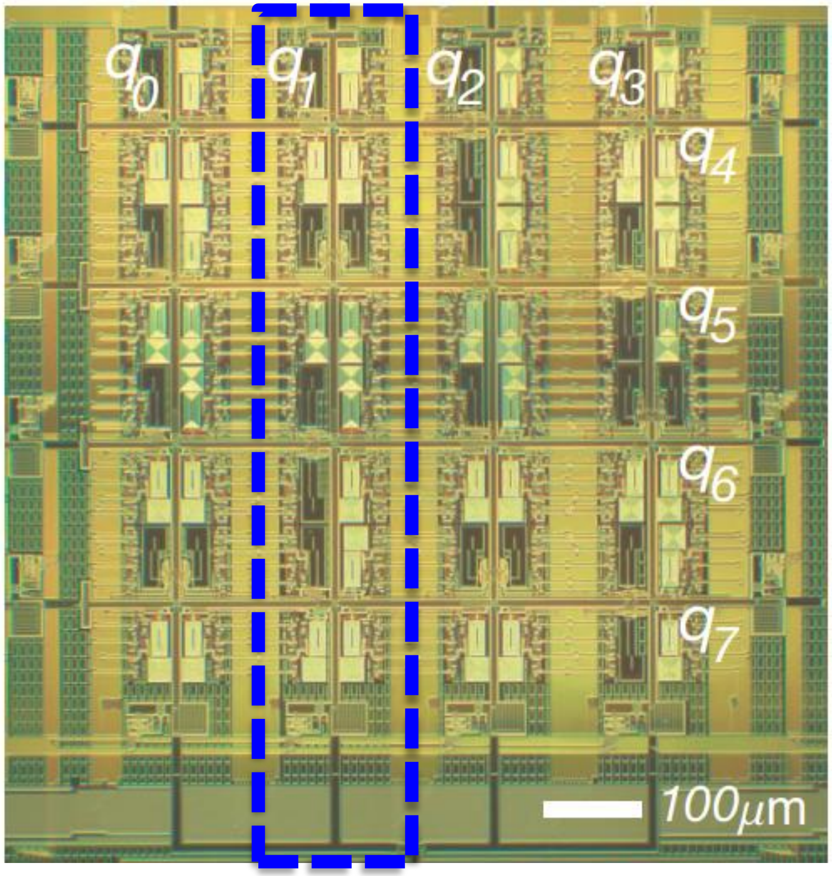
Described in detail in Harris *et. al*, PRB 2010

# Eight-Qubit Unit Cell

unit cell coupling graph



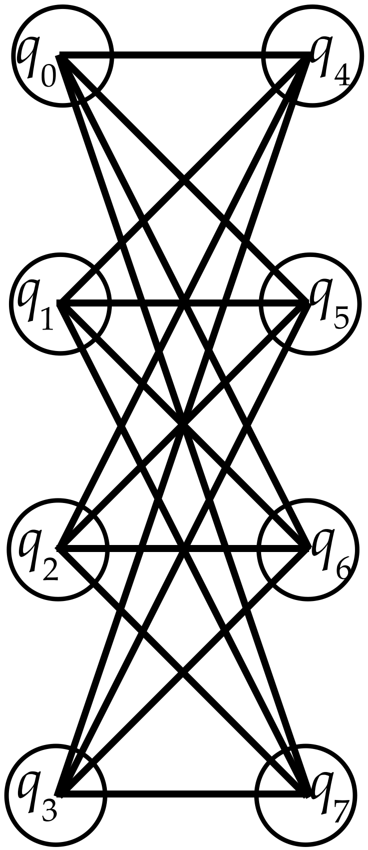

actual unit cell



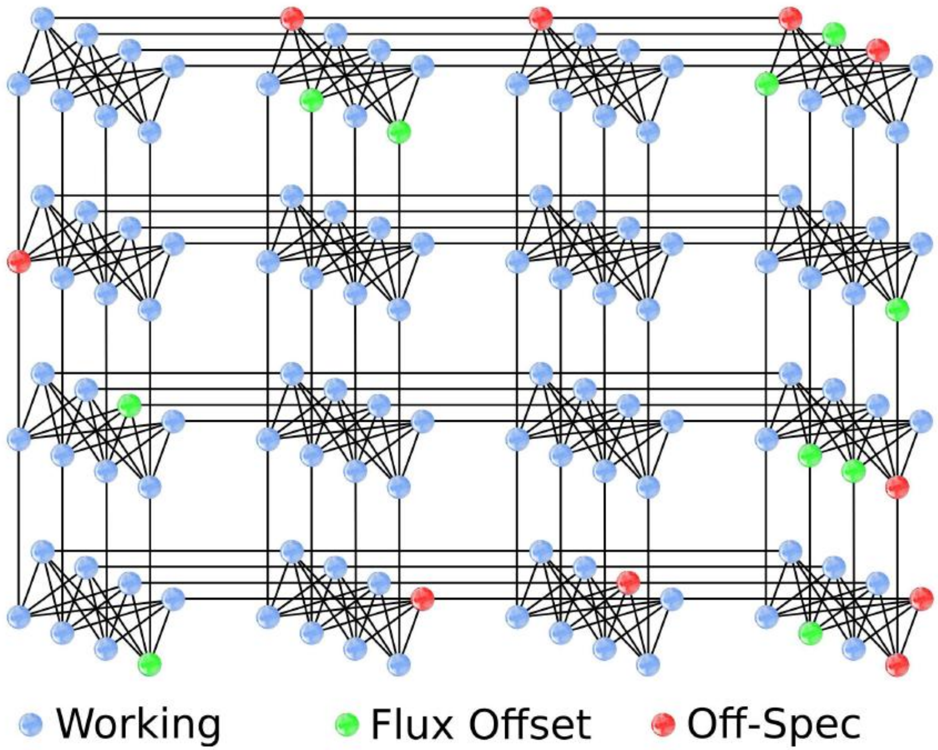

qubit 1

# Eight-Qubit Unit Cells and Tiling into 4 x 4 array

coupling graph




“Chimera” coupling graph of entire chip




108 functional qubits

# What does it do?

# D-Wave processor is not a universal computer

Special-purpose optimizer designed to find the ground state of classical Ising spin models:

$$H_{\text{Ising}} = \sum_{j\in V} h_j \sigma_j^z + \sum_{(i,j)\in E} J_{ij} \sigma_i^z \sigma_j^z$$

User can program $\{h_i\},\{J_{ij}\}$

(subject to Chimera graph connectivity)

NP-hard already for $J_{ij} = \pm$ (Barahona, 1982)

"Chimera" graph retains NP-hardness.

For error correction, in this talk, we'll consider only open antiferromagnetic chains, whose ground state is trivial.

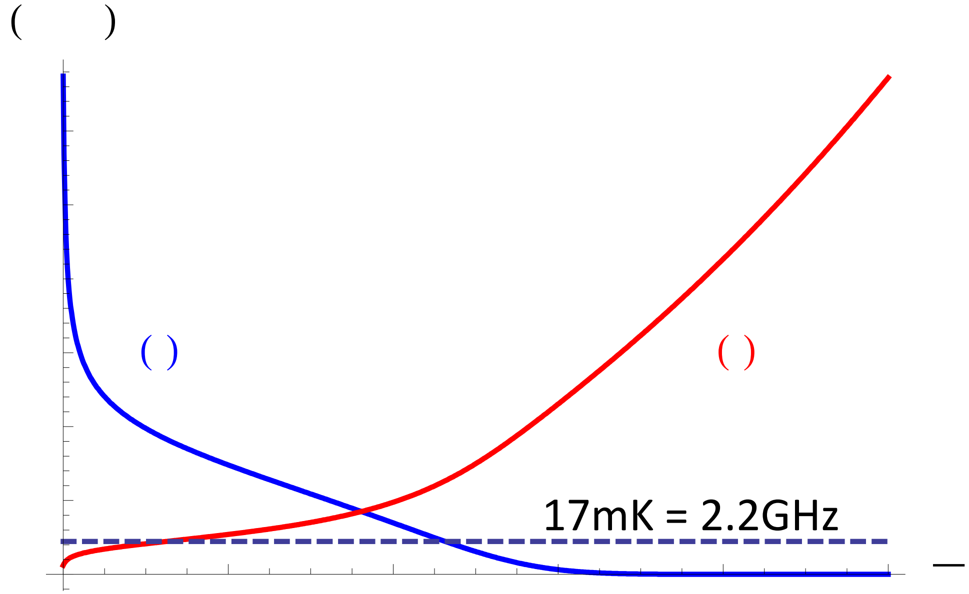
# How does it find ground states?

# Quantum Annealing / Adiabatic Quantum Optimization

(special case of adiabatic quantum computation)

1. Cool into ground state of transverse field $\sum_j \sigma_j^x$

2. Evolve via $H_S(t) = A(t)\sum_j \sigma_j^x + B(t) H_{\text{Ising}}$ $\quad t \in [0, t_f]$




$t_f \in [5\mu s, 20ms]$

3. Measure $\sigma^z$ on each qubit

Ideal output: ground state of $H_{\text{Ising}}$

# Fundamental Challenge: That Pesky Bath

Most common objection to D-Wave qubits:

Single qubit relaxation time 10-100 nsec, dephasing even less;
adiabatic evolution times are μsec-msec,
so surely decoherence will kill!

Not so fast:

Quantum annealing is about staying in the ground state (GS);

- When system Hamiltonian is dominant energy scale, dephasing is between energy eigenstates, not computational basis states
- Thermal relaxation into GS is helpful
- "Raw" dephasing/excitation rate (FT of bath correlation func.) is multiplied by Boltzmann factor, hence suppressed by finite gap (of course gap shrinks as problems get harder for more spins)

Formal treatment and details:

T. Albash, S. Boixo, DAL, P. Zanardi, New J. of Physics **14**, 123016 (2012)

# Summary of Error Sources in QA

- Closed system non-adiabatic transitions:

  Adiabatic theorem: transition rate $\propto 1/(t_f \Delta)^k,\ k \geq 1$

  Gap decreases with problem size for hard/interesting problems.

- Open system thermal excitation:

  Thermal excitations happen at any finite temperature, but

  $$\text{excitation rate} \sim \gamma(\Delta) e^{-\Delta/kT} \to \gamma(0) \text{ as } \Delta \to 0$$

  Therefore non-adiabatic transitions dominate in this limit, forcing longer evolution time.

  Either way, the larger the gap the more errors are suppressed.
  ➔ Can we engineer larger gaps to suppress errors, and correct errors after they've occurred?

# Error suppression & correction of QA using stabilizer encoding and gap enhancement

Inspiration: Jordan, Farhi, Shor PRA **74**, 052322 (2006)

- Pick a stabilizer code. Take $H_S$ and replace every Pauli operator by the corresponding *encoded* Pauli operator. The ground state of the encoded Hamiltonian is the encoded version of the original ground state.

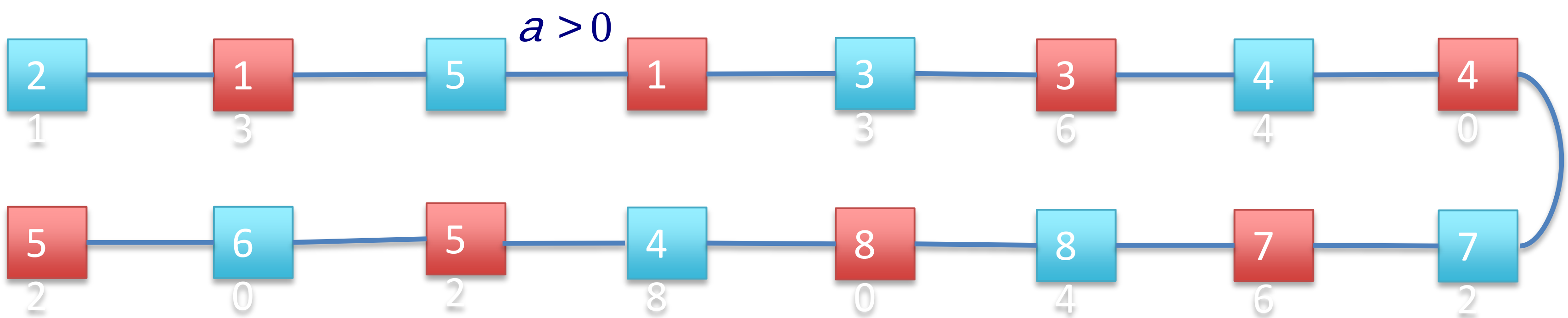
- Add an *energy penalty* term: sum over the stabilizer generators of the code.

- Each error detected by the code anticommutes with at least one generator so pays a penalty of at least two energy units: *errors are energetically suppressed*.

- Implementation problems:
  - General: Requires at least 4-body interactions to penalize arbitrary single-qubit errors. We need at most 2-body.
  - Us: We can't encode the initial Hamiltonian, only the final (recall we can only program $\{h_i\},\{J_{ij}\}$). Thus penalty won't commute with initial Hamiltonian and there will be an optimal penalty value.

- In practice: We implement the classical repetition code and error-correct by majority voting

# Antiferromagnetic Chain

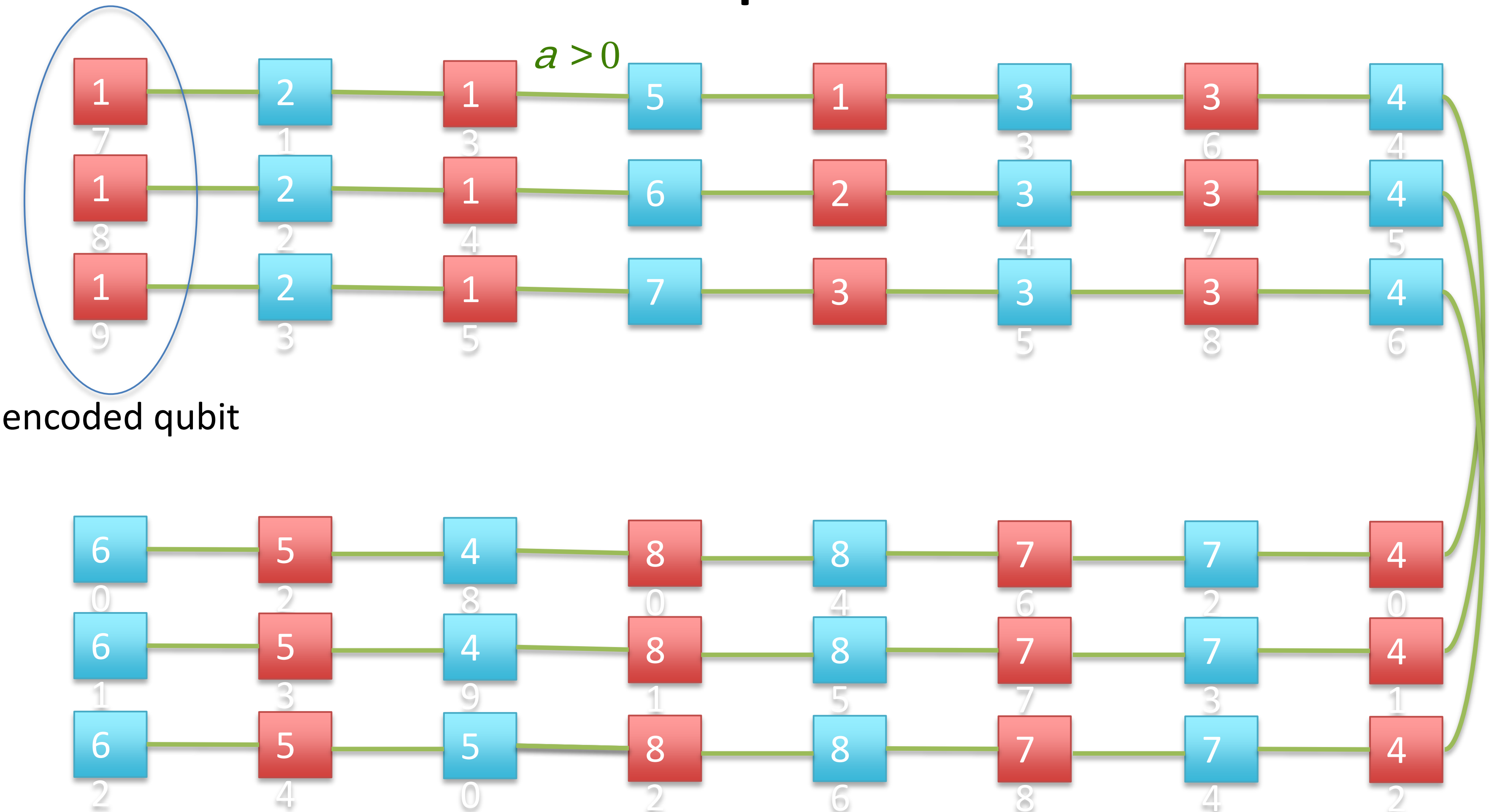



unencoded problem embedding,
16 qubit example

Ground state is doubly degenerate: spins alternate up/down

$$\left|0101\cdots01\right\rangle \text{ or } \left|1010\cdots10\right\rangle$$

# Antiferromagnetic Chain with 3-bit Repetition Code




Decode by majority voting in each three-bit block.
Flip bits accordingly in a post-processing step.

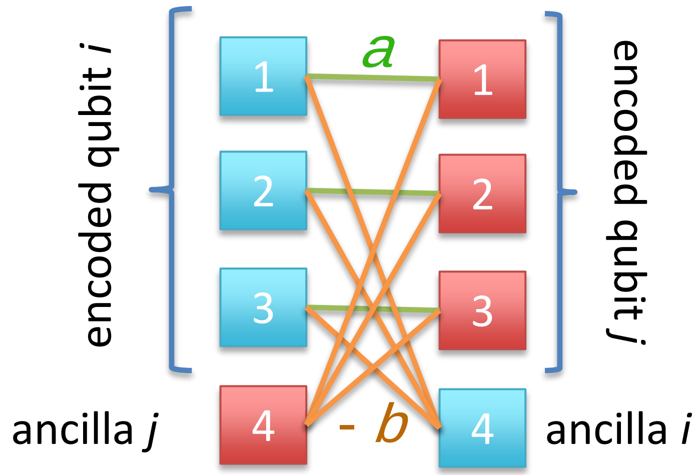
# Encoding & Penalty
# Example: two anti-FM coupled qubits

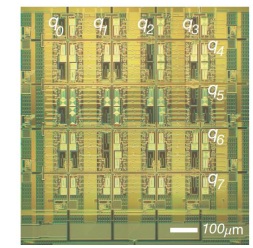
$$H_{\text{Ising}} = \alpha Z_i Z_j$$

unencoded embedding


$$\overline{H}_{\text{Ising}} = \alpha \sum_{i,j} \overline{Z_i Z_j} + b H_{\text{penalty}}$$

encoded embedding




$a > 0$ : problem scale

$b > 0$ : penalty scale

Boost the energy scale by a factor of 3:

$$\overline{Z}_i = \sum_{k=1}^{3} Z_{i,k}, \quad i = 1,2$$

$$\overline{Z_i Z_j} = \sum_{k=1}^{3} Z_{i,k} Z_{j,k}$$

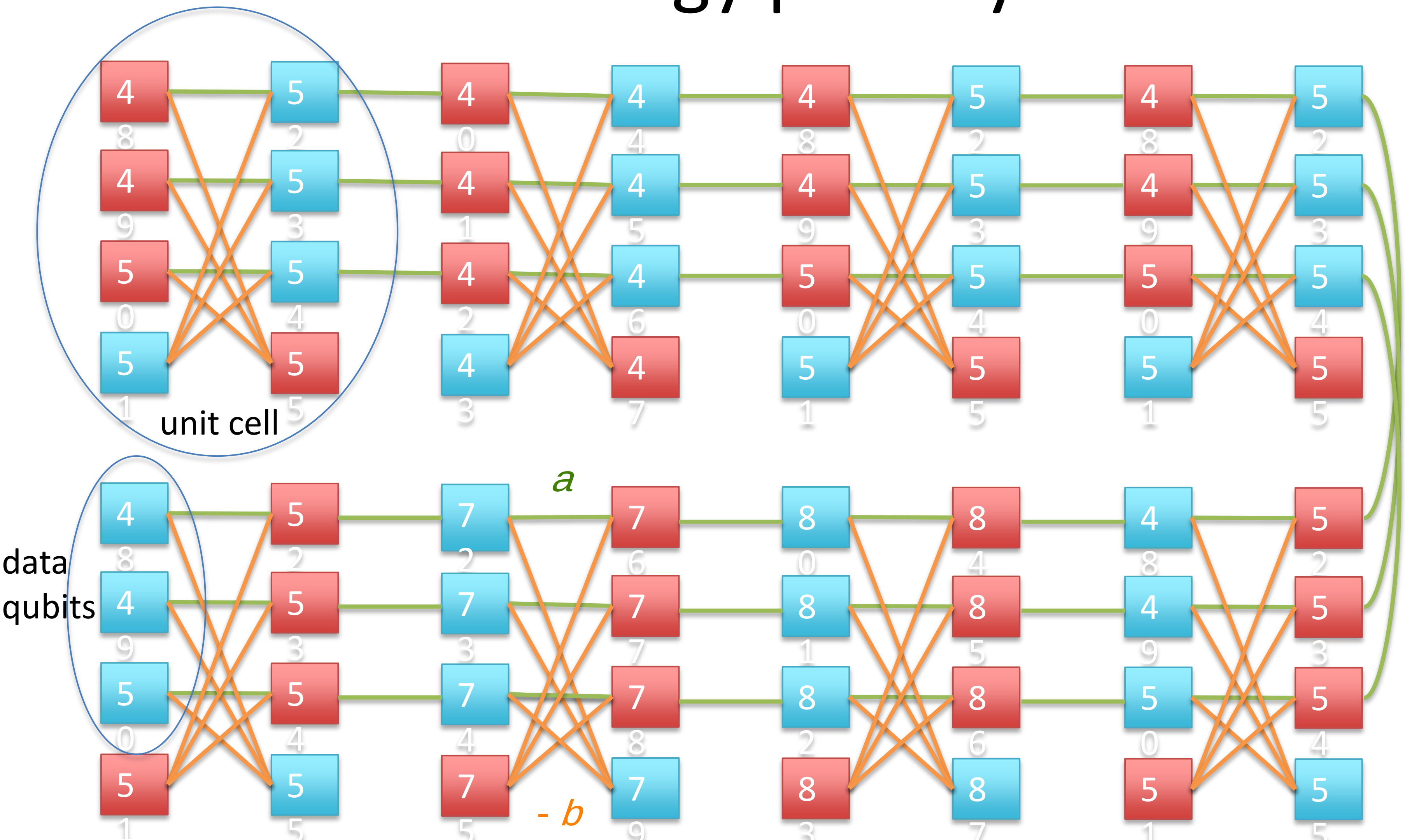
logical ops

stabilizer generators

penalize single-bit-flip errors:

$$H_{\text{penalty}} = \sum_{k=1}^{3} Z_{1,k} Z_{1,4} + Z_{2,k} Z_{2,4}$$

# Encoded antiferromagnetic chain with energy penalty

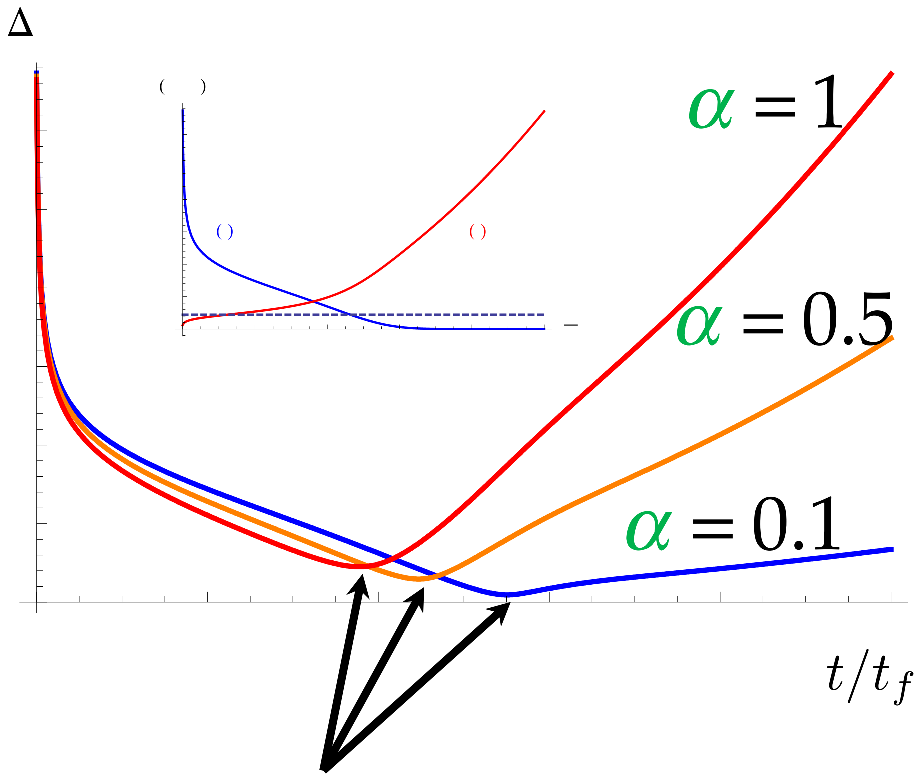



16 encoded qubits (groups of red or blue)

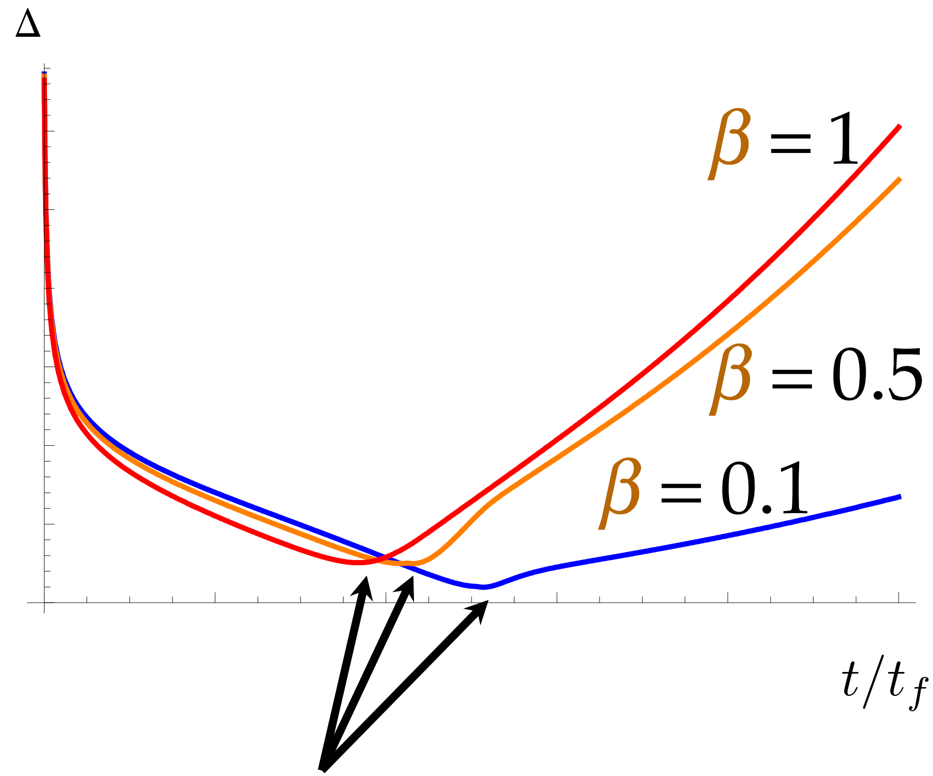
Decode by majority vote on data qubits, flip accordingly in post-processing step

# Roles of problem scale and penalty scale

8 qubit chain




2 encoded-qubit chain




Min gap **increases** and shifts to earlier times;
Earlier is also better since excitations are most damaging while transverse field is on.

# Gap doesn't tell the whole story: state identity matters too

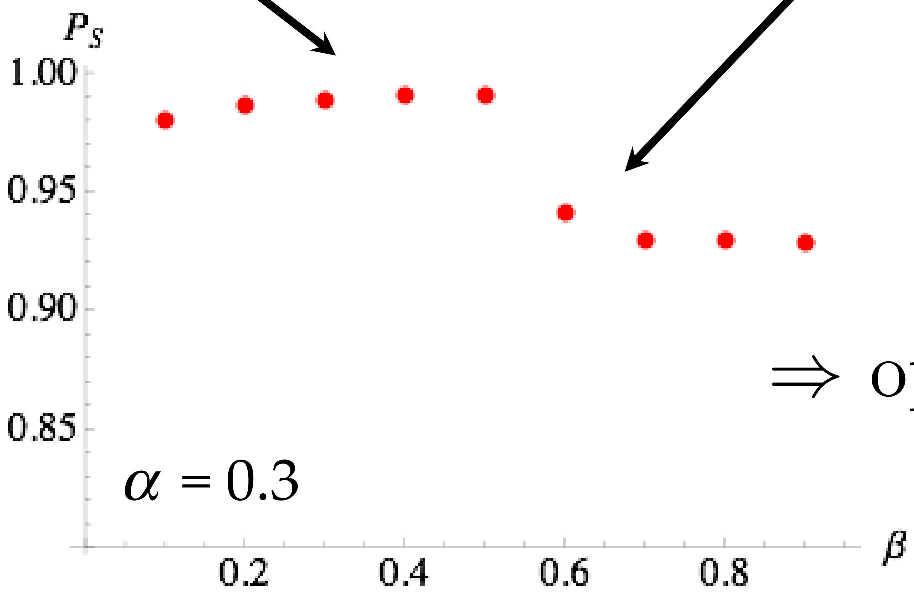
Excited spectrum labeling changes with $\beta$.

This affects the decoding.

1st excited states
are correctly decoded

1st excited states
are incorrectly decoded

probability of finding correct ground state after decoding; 2 encoded-qubit chain

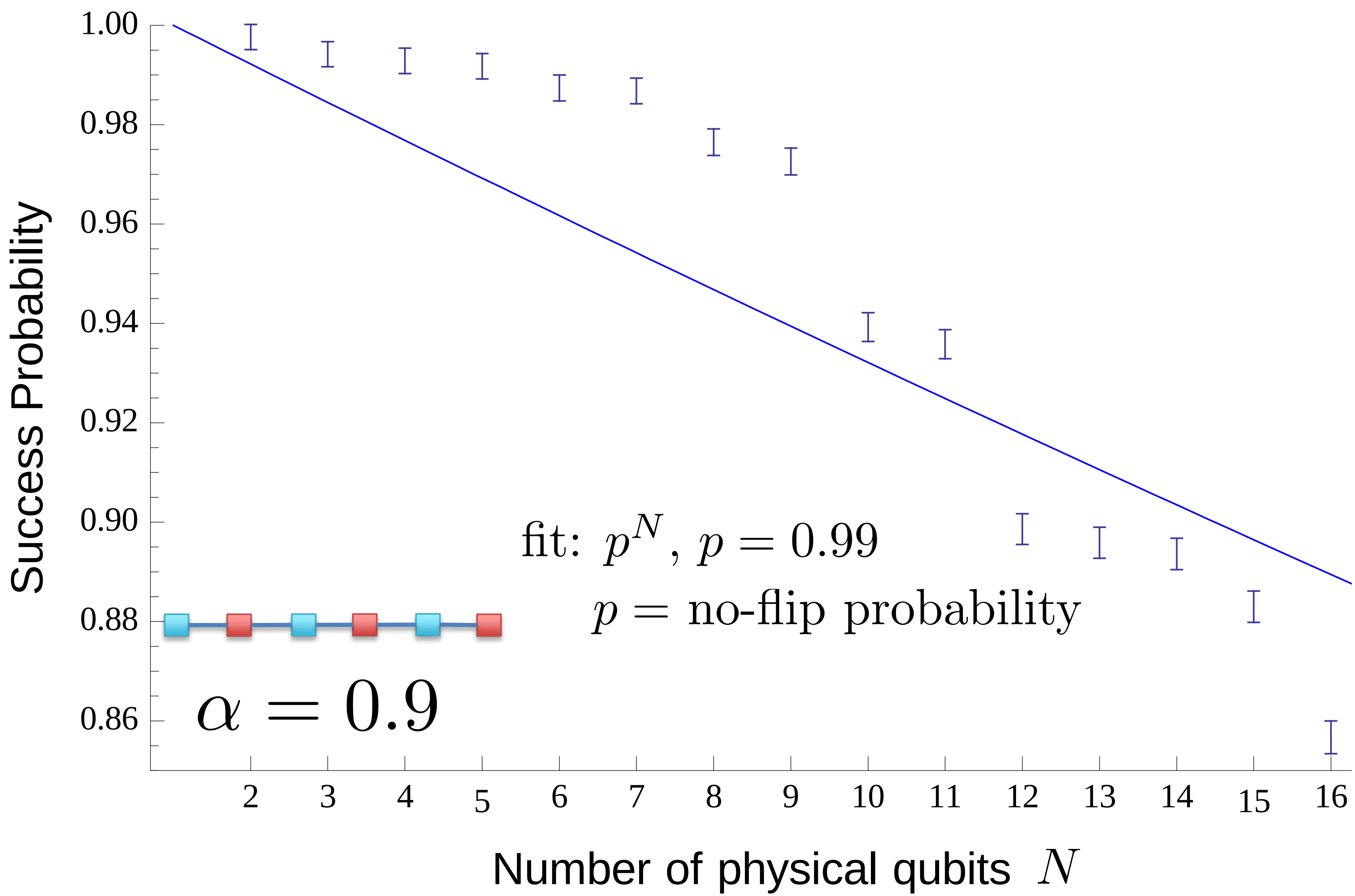



$\Rightarrow$ optimal $\beta$ for each $\alpha$

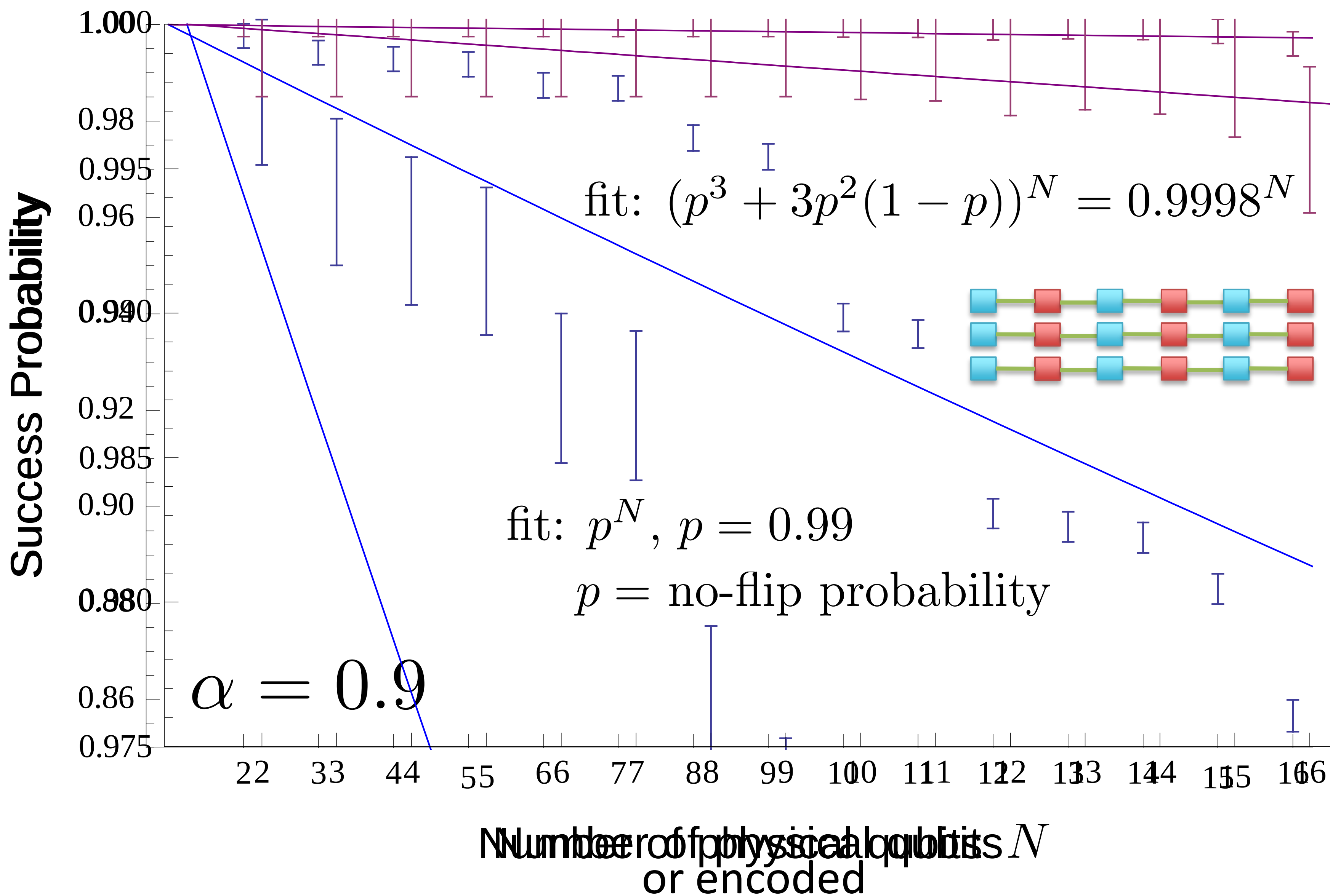
# Experimental Results

using empirical optimal $\beta$

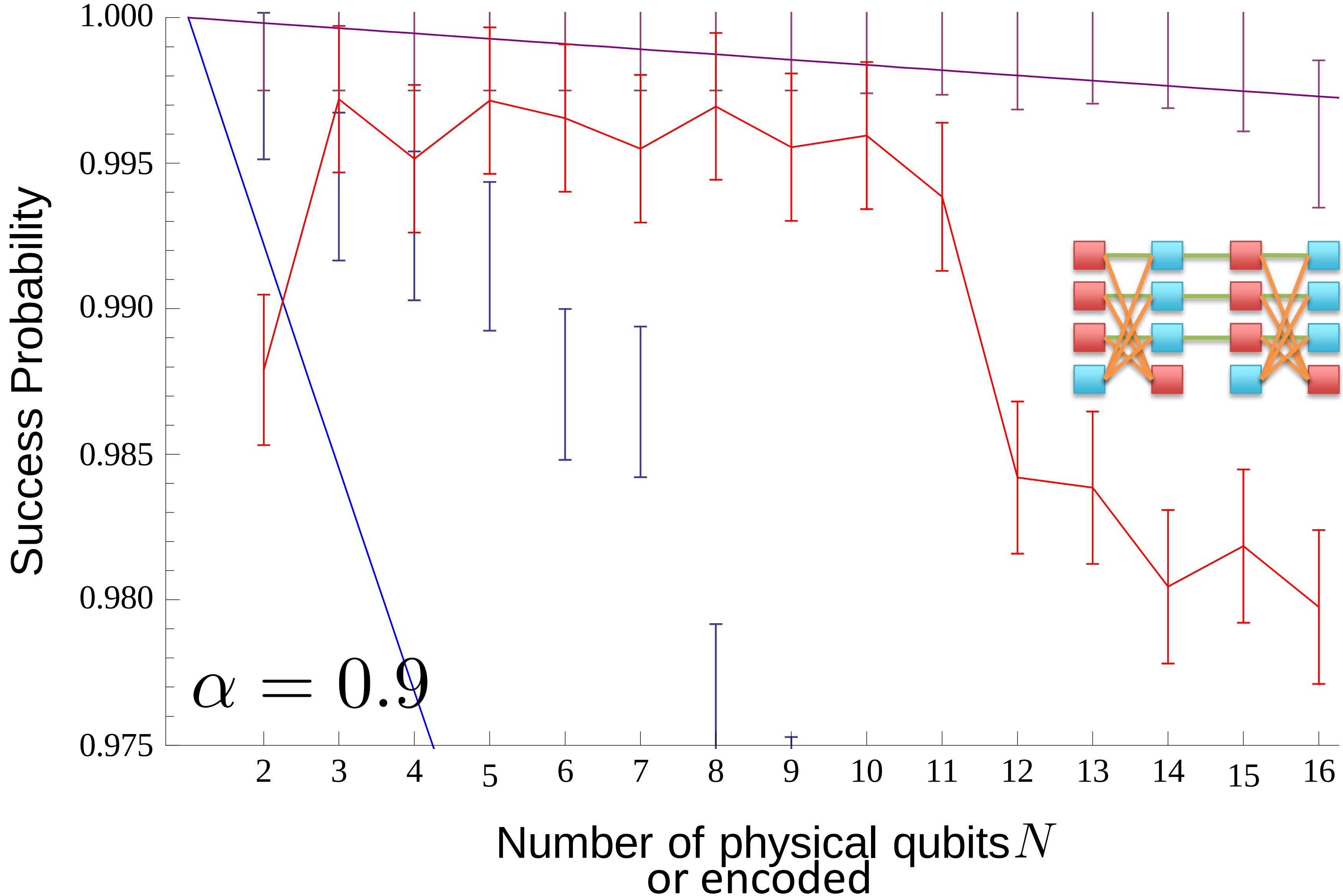
# Antiferromagnetic chain experimental results unprotected



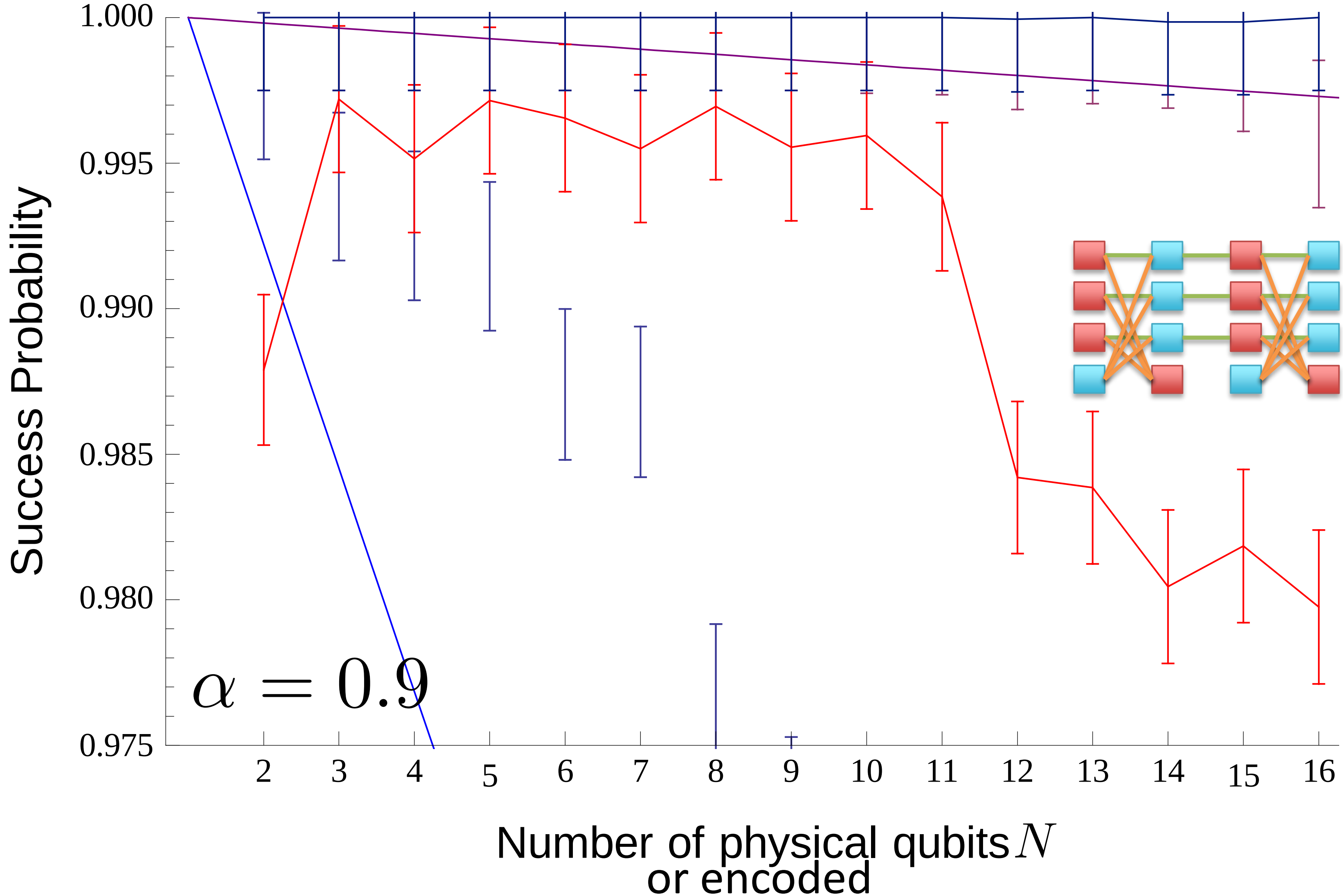
# Antiferromagnetic chain experimental results majority vote on 3 unprotected copies



# Antiferromagnetic chain experimental results
# Repetition code, energy penalty, undecoded



# Antiferromagnetic chain experimental results
# Repetition code, energy penalty, decoded

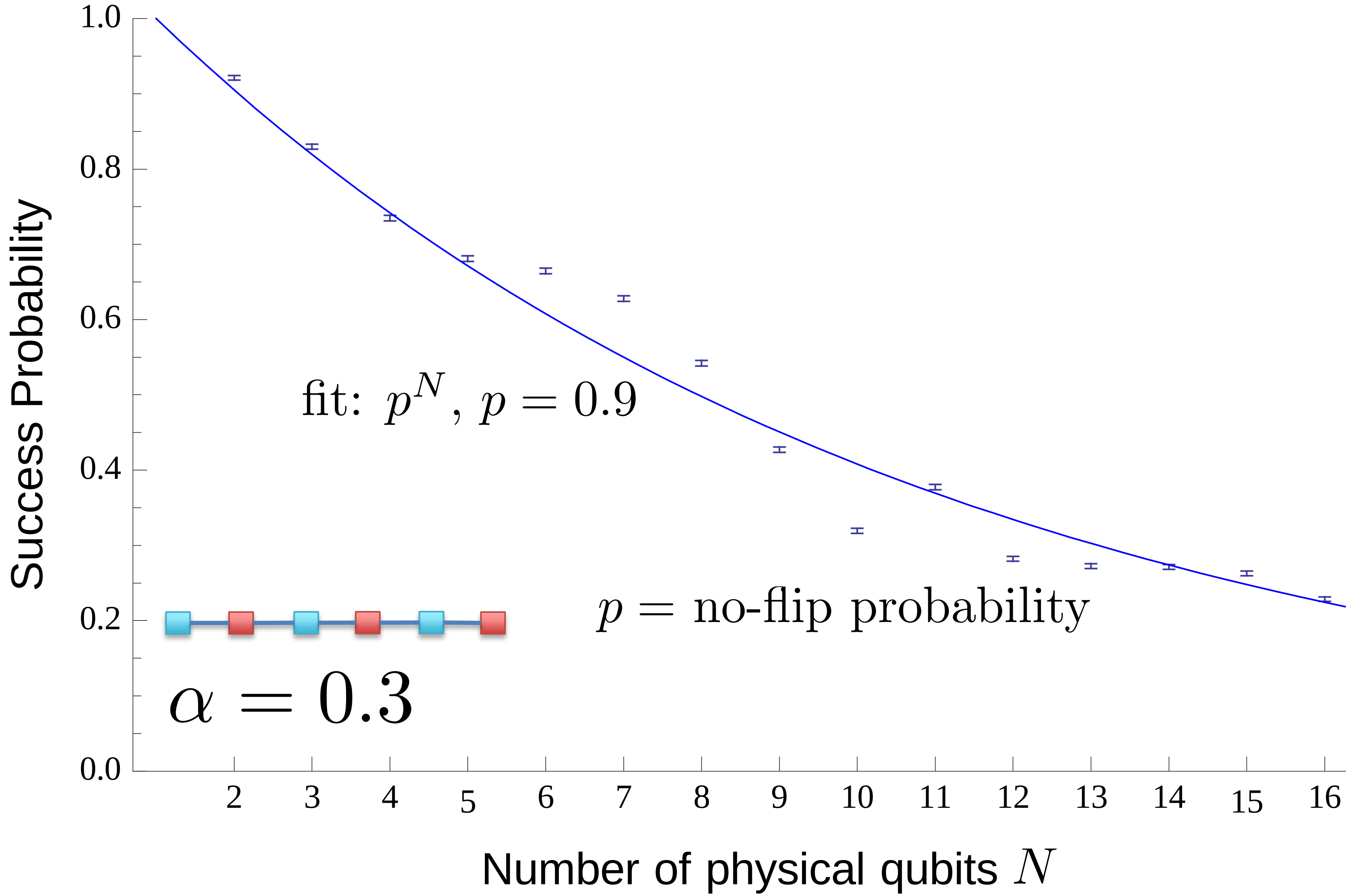


# What happens when we lower the problem scale?

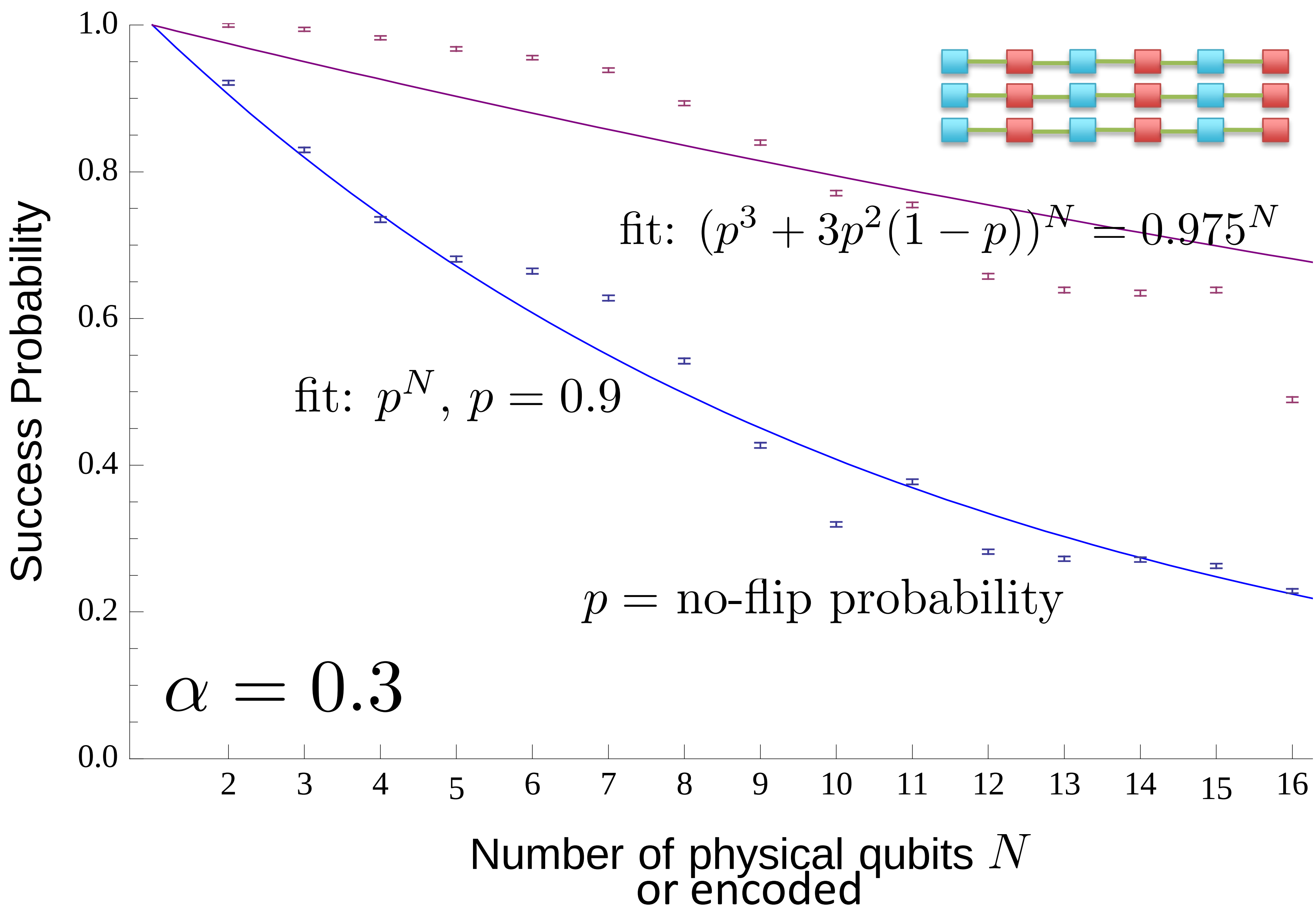
Lowering $\alpha \equiv$ raising temperature

Does this result in a relative advantage for the energy penalty?

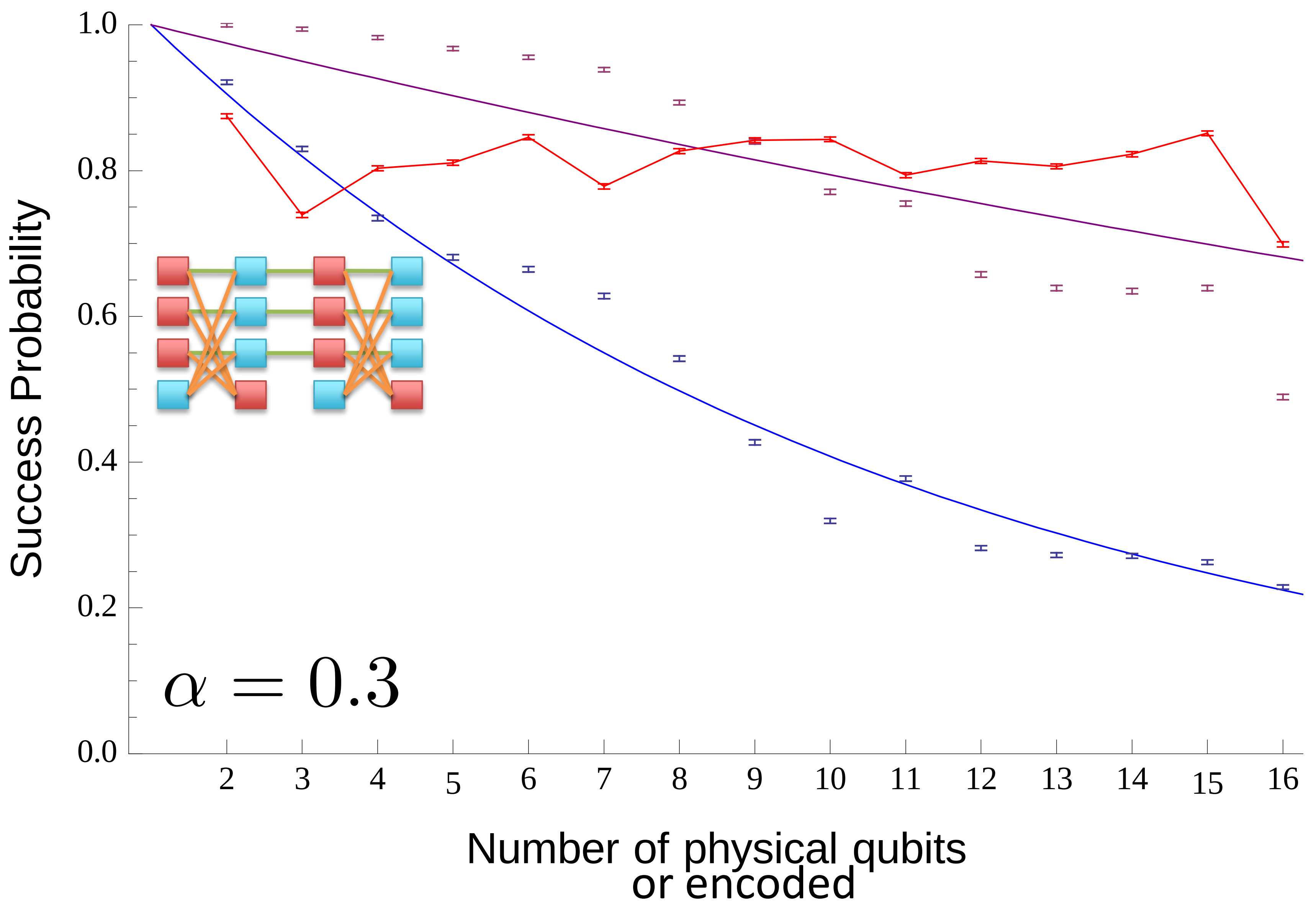
# Antiferromagnetic chain experimental results unprotected



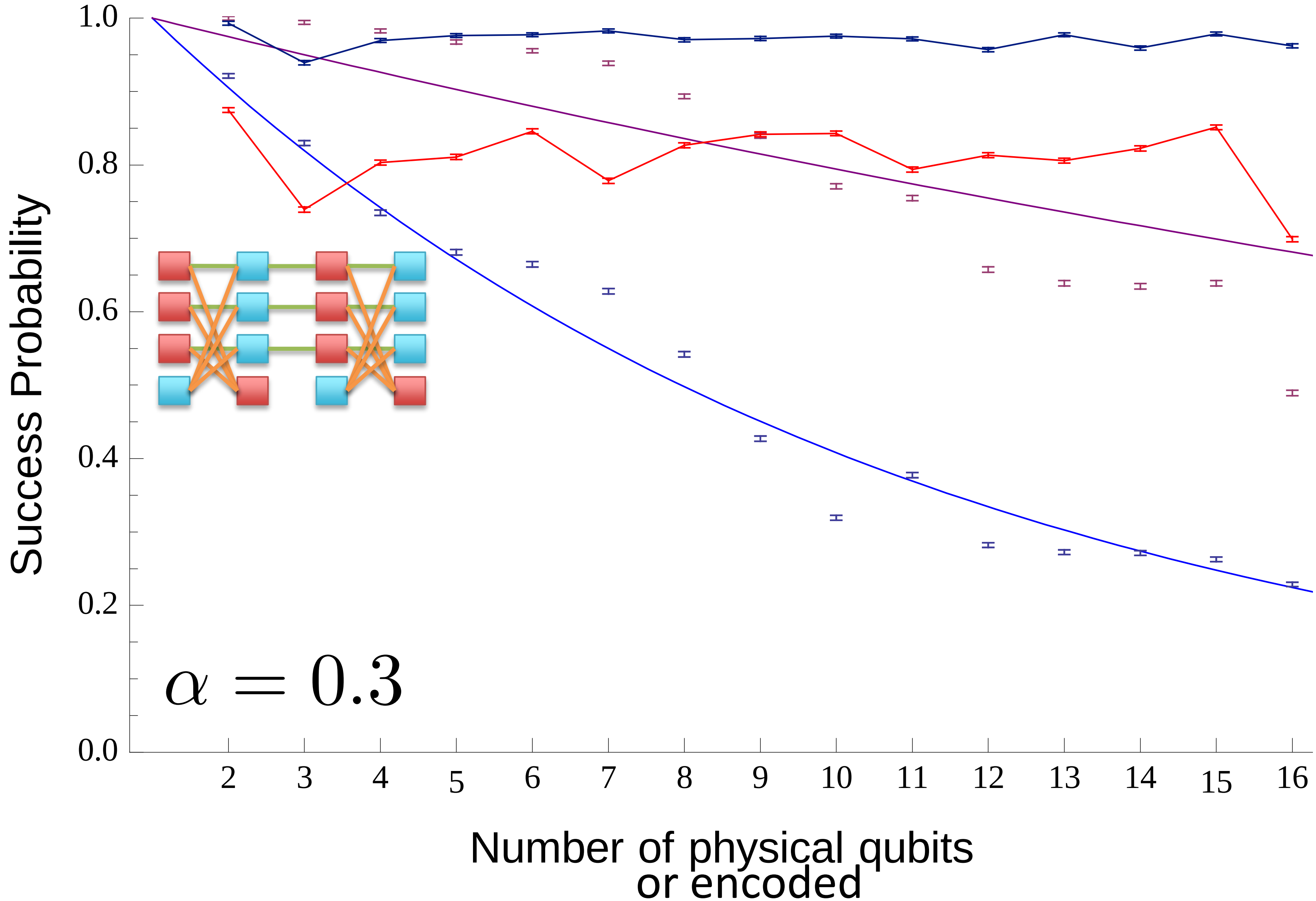
# Antiferromagnetic chain experimental results decoded (majority vote) on 3 unprotected copies



# Antiferromagnetic chain experimental results
# Repetition code, energy penalty, undecoded



# Antiferromagnetic chain experimental results
# Repetition code, energy penalty, decoded



# Conclusions

We've introduced and implemented a strategy for error suppression and correction of open system quantum annealing.

The improvement is due to:

- The addition of an energy penalty (gap enhancement)
- Access to a higher energy scale through the use of more couplers (gap enhancement again)
- Majority vote decoding in a postprocessing step

While the strategy is general, it remains to be seen whether it will be as useful for problems with "hard" ground states.

# Collaborators

**Tameem Albash**

**Kristen Pudenz**

Error Correction
(paper soon)

Master Equation
New J. of Physics **14**, 123016 (2012)

**Sergio Boixo**

**Paolo Zanardi**