

Design of a Superconducting Quantum Computer: Surface Code & Fidelity

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1. Why surface code design?
2. Surface code review
3. Cell design methodology
4. χ matrix
5. Design comments

Superconducting Qubits and Surface Code

3D transmons:

- $T_1, T_2 \sim 60 \mu\text{s}$
- $t_{\text{gate1}} = 15 \text{ ns}$:
fidelity = 99.8%
- $t_{\text{gate2}} = 200 \text{ ns}$:
fidelity = 99%
- $P_{\text{meas}} = 99.4\%$;
1000's meas./ T_1
- Scalable?

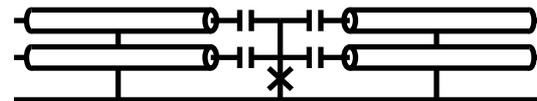
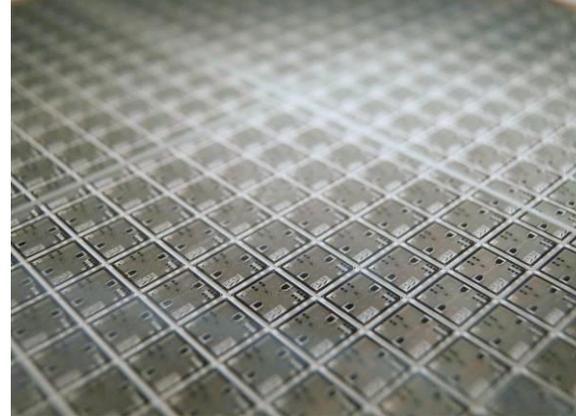
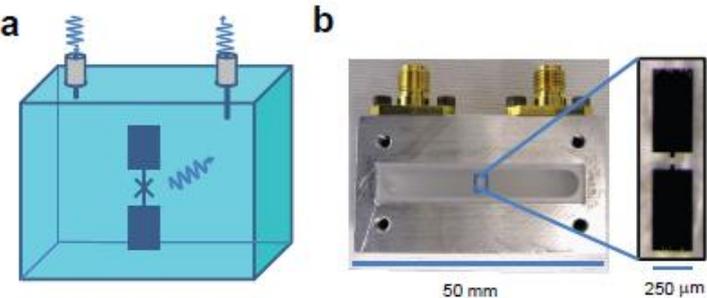
qulC's, UCSB strategy:

- Interconnectivity
- Theory: $t_{\text{CZ}} = 25 \text{ ns}$,
intrinsic fidelity = 99.99%
- UCSB Xmon: $T_1 \sim 42 \mu\text{s}$

$t_{\text{th}} \sim 2.5 \mu\text{s}$

x100

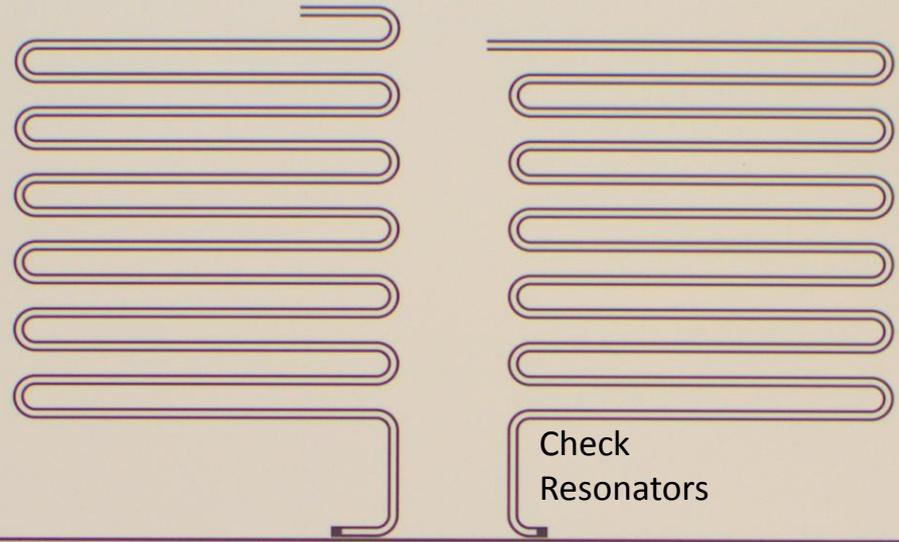
$t_{\text{gate}} \sim 25 \text{ ns}$



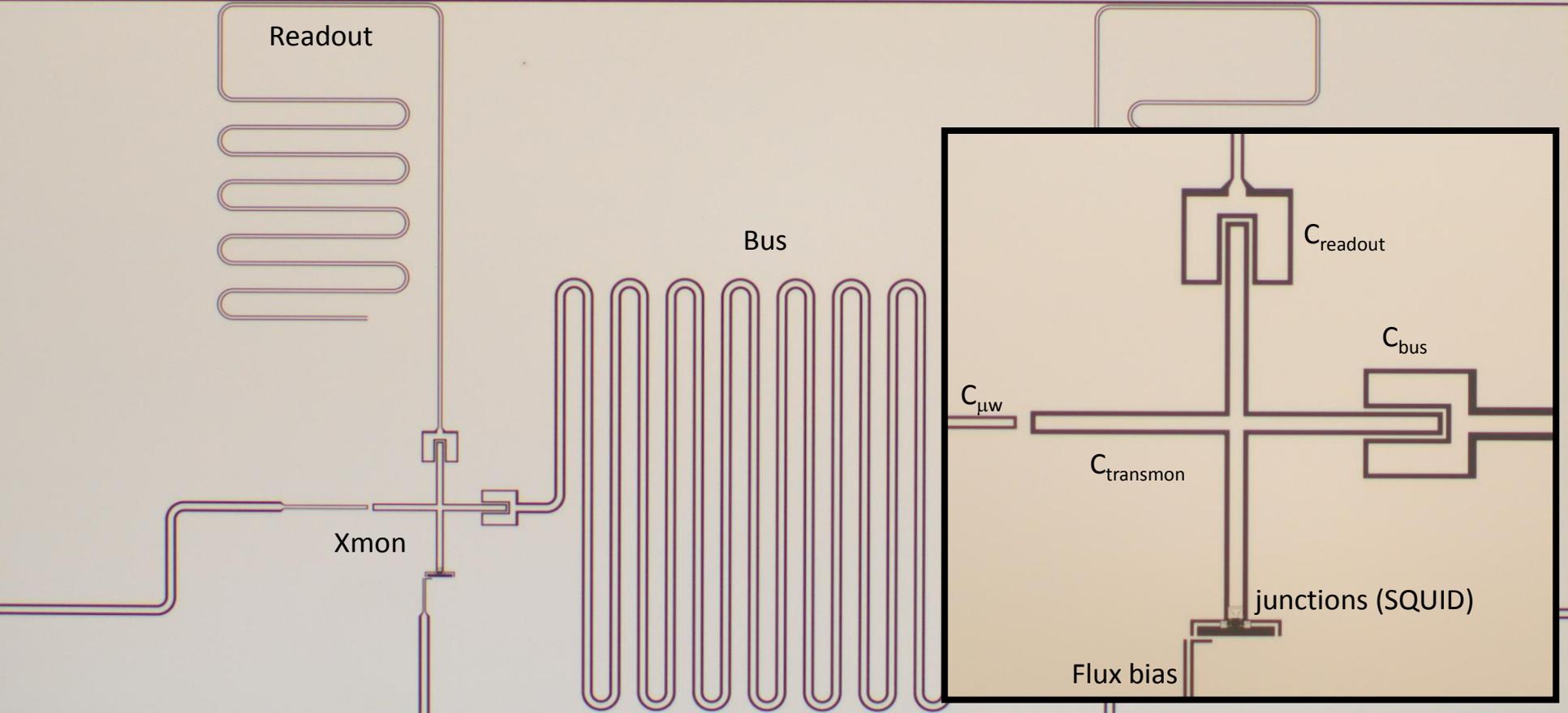
UCSB Xmons

200 μ m

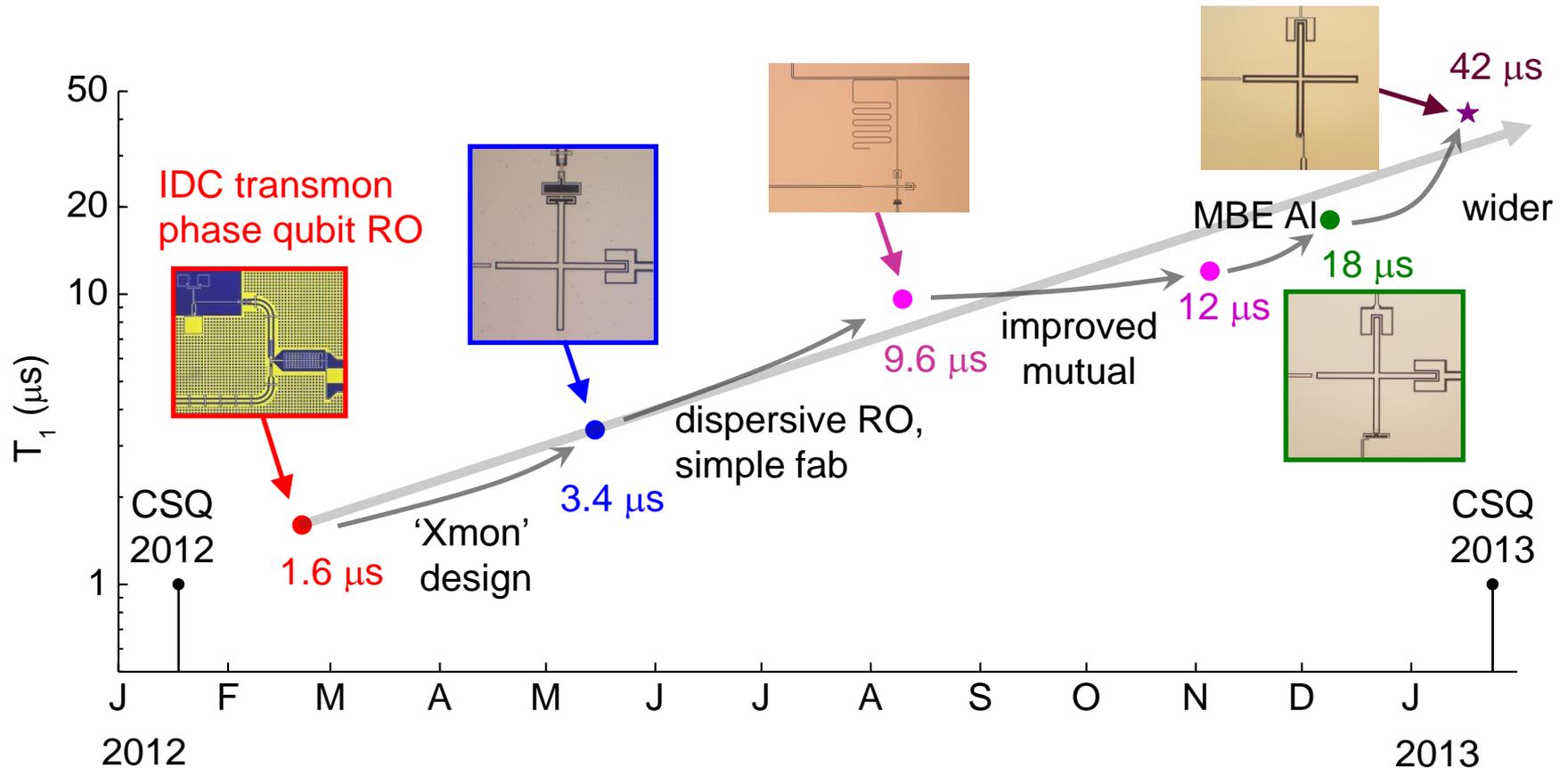
Multiplexed Measurement



- Broken "design rules":
- Transmon outside of resonator
 - Direct μ wave drive
 - Single ended: galvanically connected (SQUID with high M)
- (No IDC, 3 μ m gap)
(C not double-evap.)



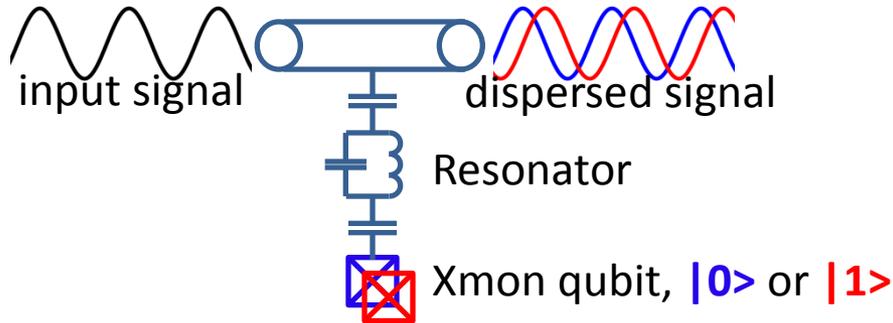
Timeline of T1 coherence



Xmon yield: 26/26
Qubit freq. to 100 MHz

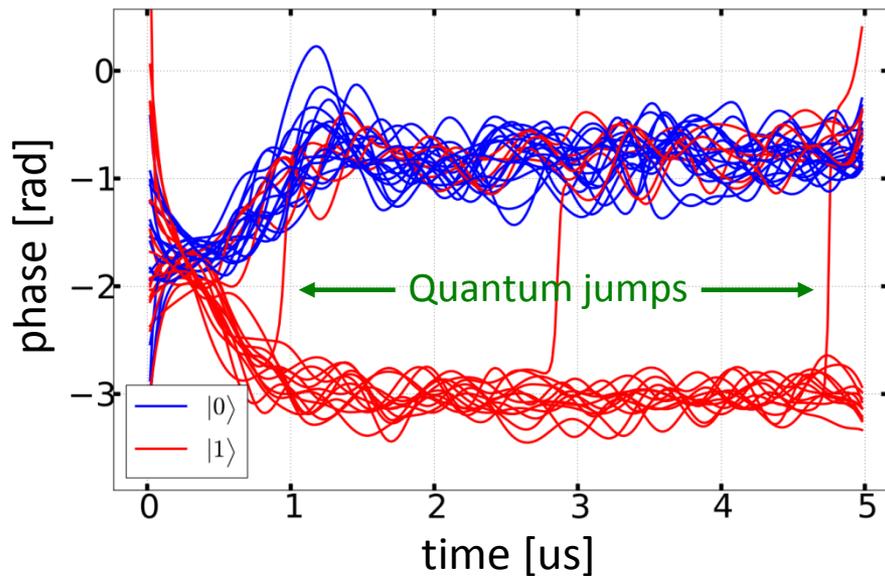
For next run, expect:
70 μs best, 40 μs avg.

UCSB single shot readout with in-house paramp



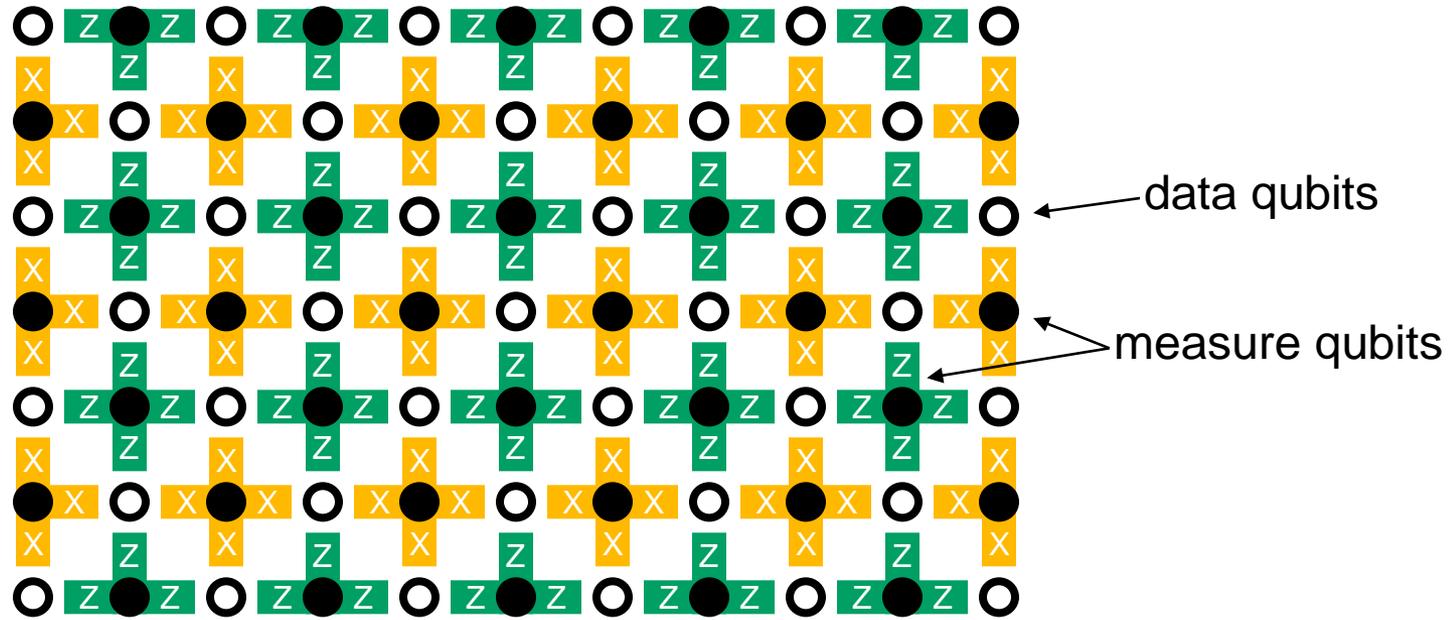
Parameter	Value
$g/2\pi$	30 MHz
κ	1/500 ns
$\Delta/2\pi$	1 GHz
$\langle n \rangle$	160

Phase of single-shot time traces



- Measurement time: $T_{\text{meas}} = 800$ ns
- High separation gives 100% intrinsic fidelity
- Errors from T_1 only: $T_{\text{meas}}/T_1 = 0.05$
- Surface code: $\kappa = 1/50$ ns possible

Surface Code Hardware



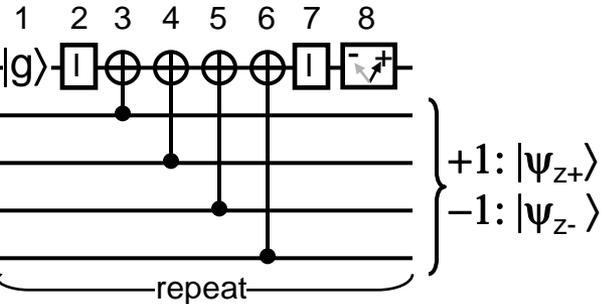
Measurement

Symbol

Physical Logic Sequence

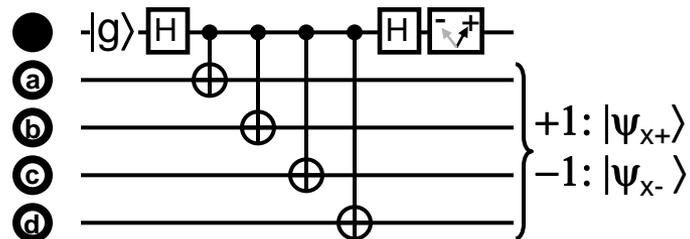
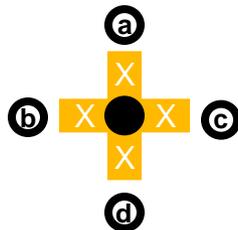
4-bit parity

$$Z_{abcd} = Z_a Z_b Z_c Z_d$$

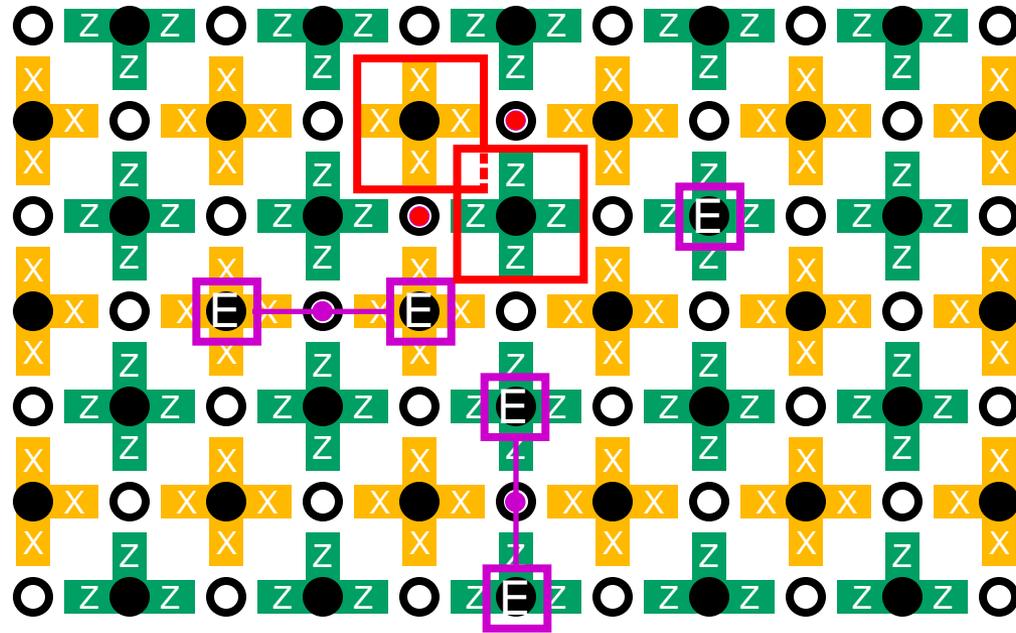


4-phase parity

$$X_{abcd} = X_a X_b X_c X_d$$



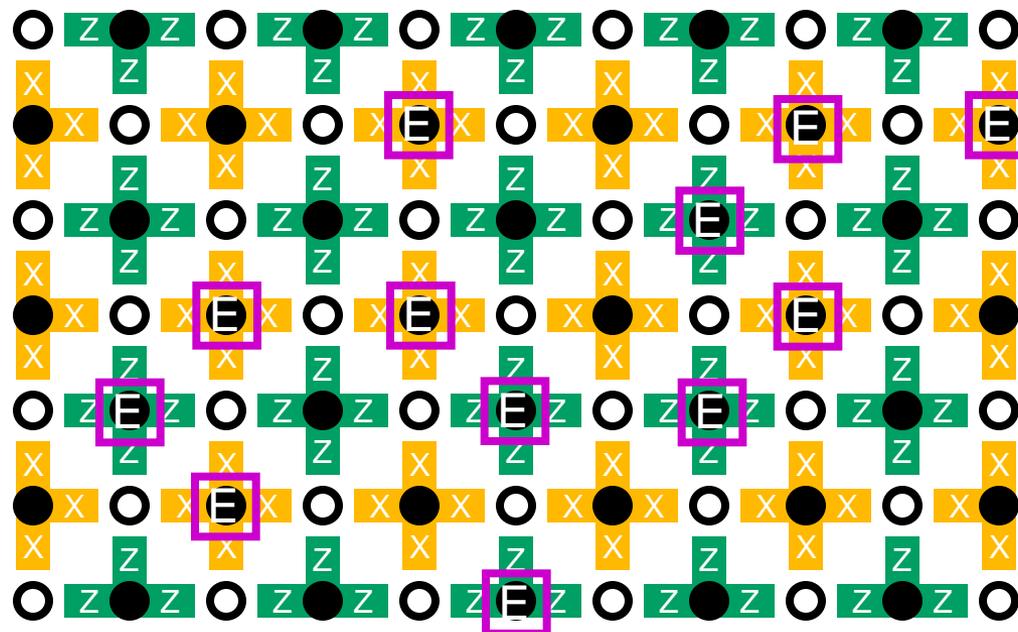
Stabilized State and Identifying Qubit Errors



All measurements X and Z commute:
Measurement outcomes unchanging

When errors:
Data qubit errors – pairs in space
Measure errors – pairs in time

Stabilized State and Identifying Qubit Errors

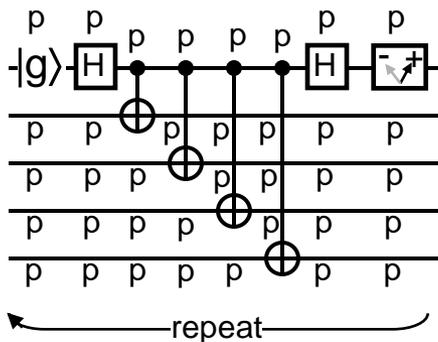


When large density -
Backing out errors not unique

Logical Error Probability

Model: total error p each step & qubit

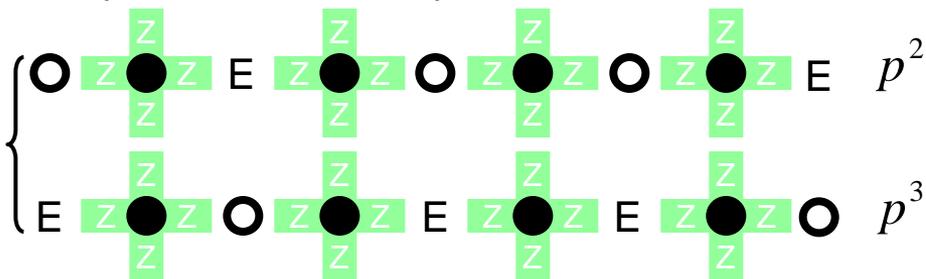
- 1: $p/3$ for X, Y, Z
- 2: $p/15$ for XX, YY, IX, ZI ...



measured error chain:

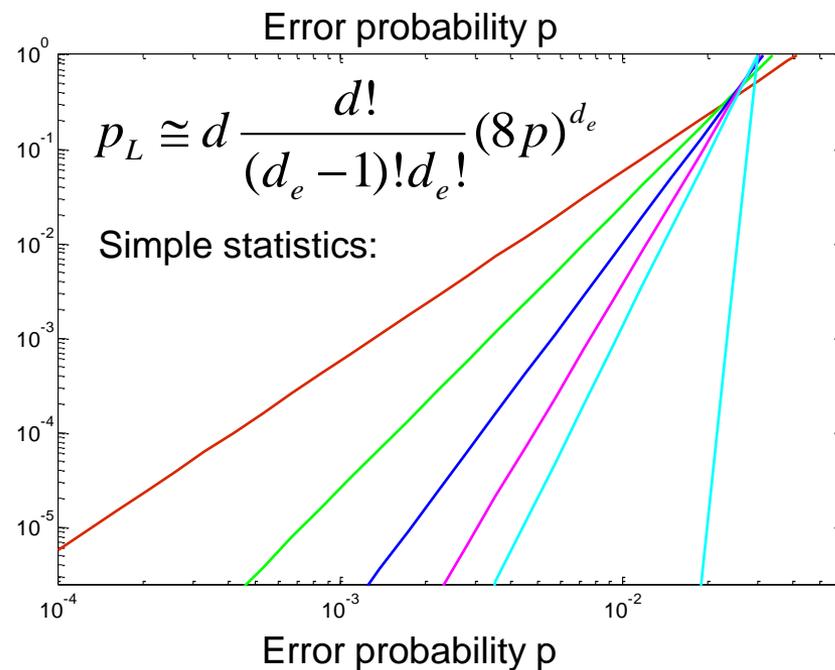
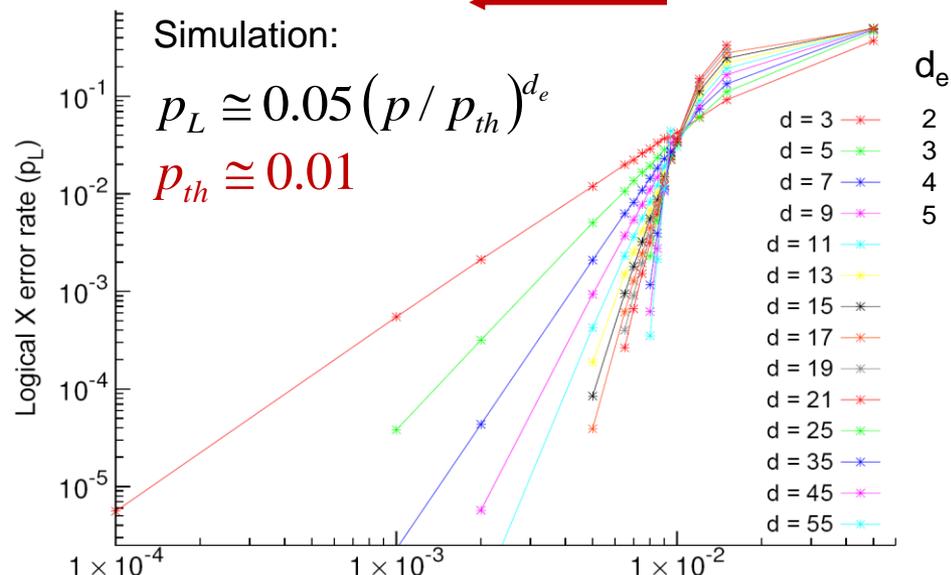


computed error of data qubits:



$$P_L \cong \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} (8p)^3$$

logical error improves with size



The Problem: Characterizing Errors

Quantum
Process Tomography

Complex
Not scalable
Sensitive to calibration
 χ matrix is overly complex

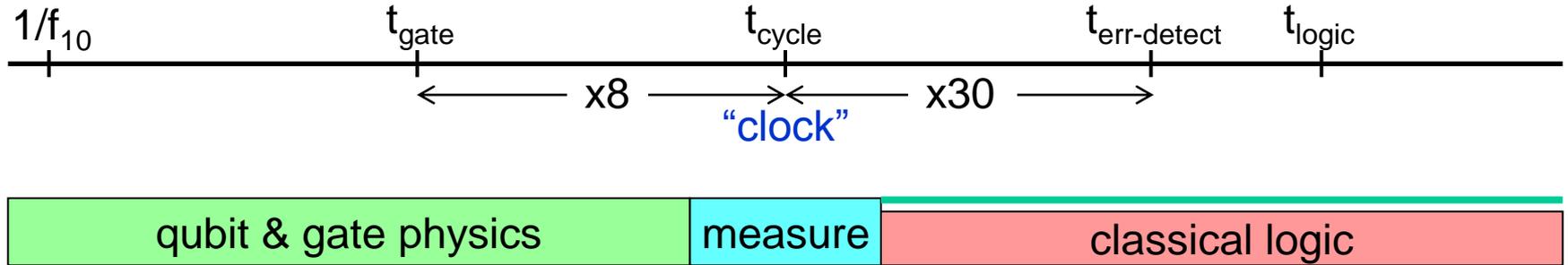
Pauli Gate Errors

Pauli error after every gate
One & two qubit errors
Need 3 to 15 probabilities
How to measure?

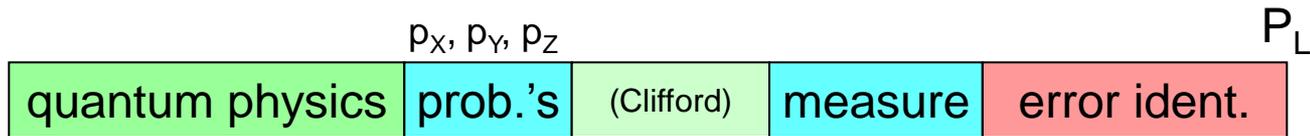
Randomized Benchmark

Scalable
One average fidelity
Overly simple

Designing for the Surface Code



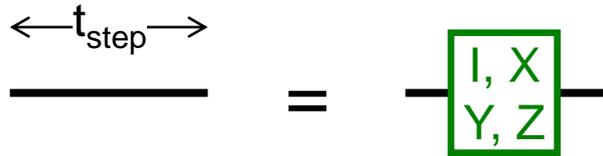
surface code design:



p 's are diagonal matrix elements of
computed or measured QPT

Theory Example: T_1, T_2 errors

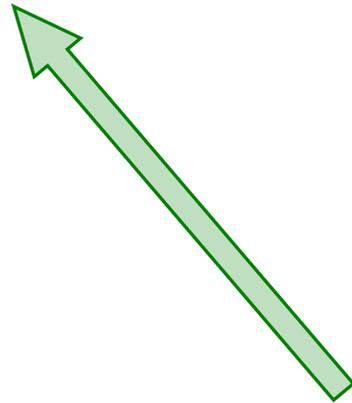
$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$



$$\rho = \begin{pmatrix} 1 - \rho_{11} e^{-t/T_1} & \rho_{01} e^{-t/2T_1} e^{-(t/T_\phi)^{1+\alpha}} \\ \rho_{01}^* e^{-t/2T_1} e^{-(t/T_\phi)^{1+\alpha}} & \rho_{11} e^{-t/T_1} \end{pmatrix}$$

3 possible probability errors:

p_X, p_Y, p_Z



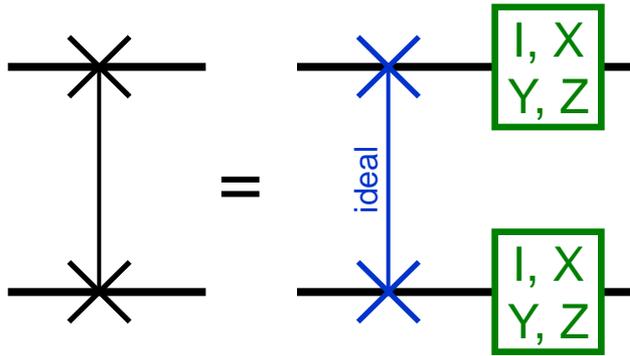
$$= (1 - p_X - p_Y - p_Z) \rho + p_X X \rho X + p_Y Y \rho Y + p_Z Z \rho Z$$

$$p_X = p_Y = \frac{t_{\text{step}}}{4T_1}$$

$$p_Z = \frac{1}{2} \left(\frac{t_{\text{step}}}{T_\phi} \right)^{1+\alpha}$$

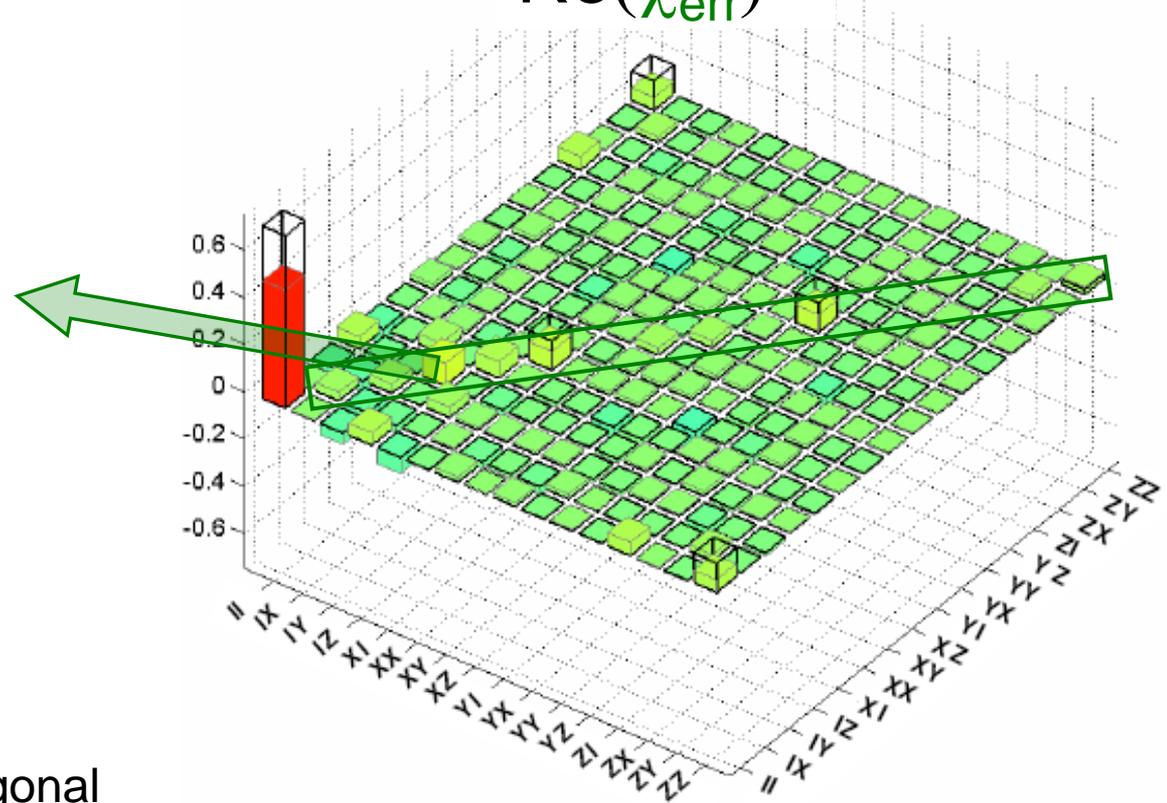
$\alpha \neq 0$ is for correlated phase errors

Experiment Example: χ_{err} of $\text{iswap}^{1/2}$ gate



$$\chi_{\text{meas}} = \chi_{\text{ideal}}^+ \chi_{\text{err}} \chi_{\text{ideal}}$$

$\text{Re}(\chi_{\text{err}})$



15 possible probability errors:

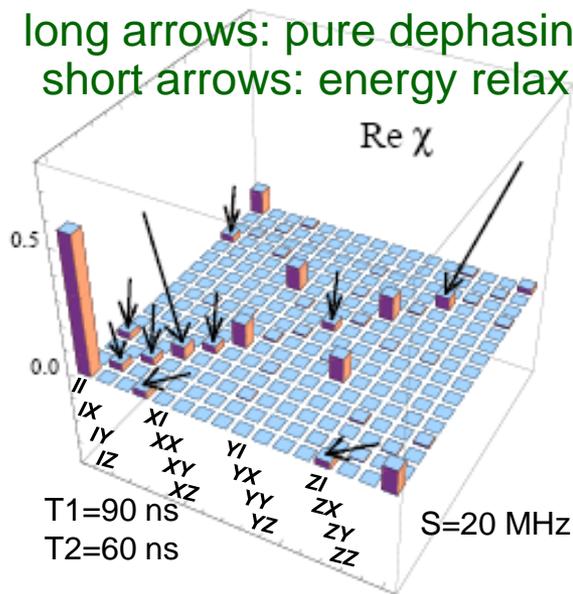
- $\rho_{XI}, \rho_{YI}, \rho_{ZI}$
- $\rho_{IX}, \rho_{IY}, \rho_{IZ}$
- $\rho_{XX}, \rho_{XY}, \rho_{XZ}$
- $\rho_{YX}, \rho_{YY}, \rho_{YZ}$
- $\rho_{ZX}, \rho_{ZY}, \rho_{ZZ}$

Simple: Fidelity = $\chi_{II,II}$
 Modeling: Pauli errors = diagonal

Using χ_{err} Matrix to Test for Errors

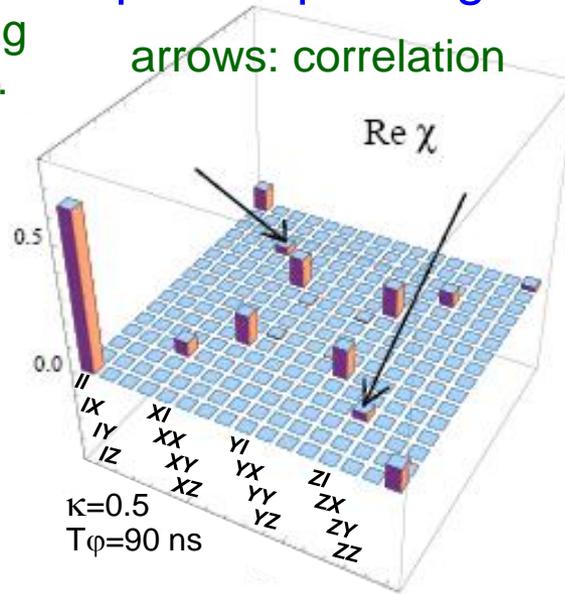
Local T_1 & T_2

long arrows: pure dephasing
short arrows: energy relax.

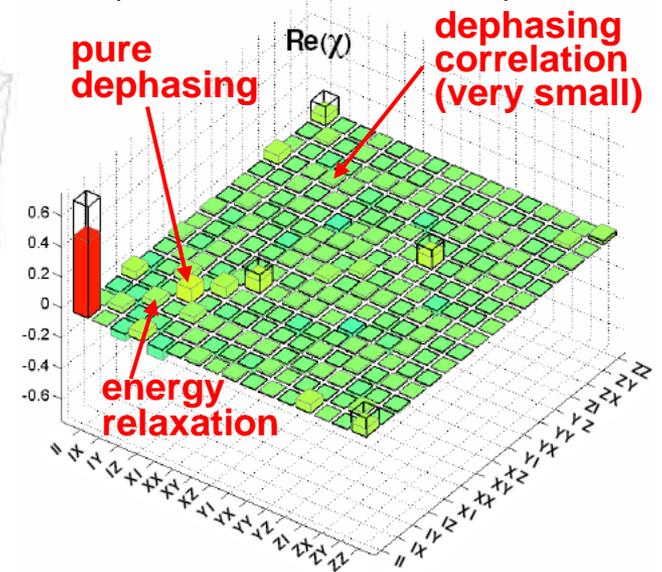


Correlated ($\kappa=0.5$) pure dephasing

arrows: correlation



Actual experiment (modified Pauli basis)

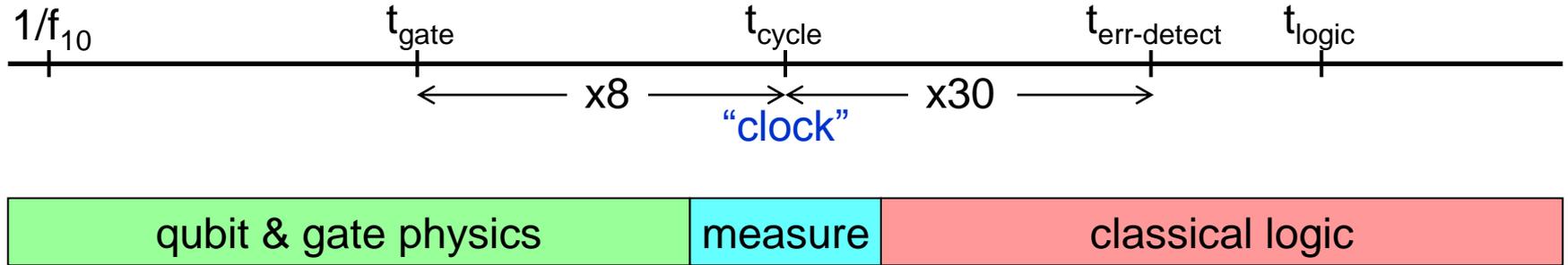


Kofman & Korotkov, arXiv:0903.0671

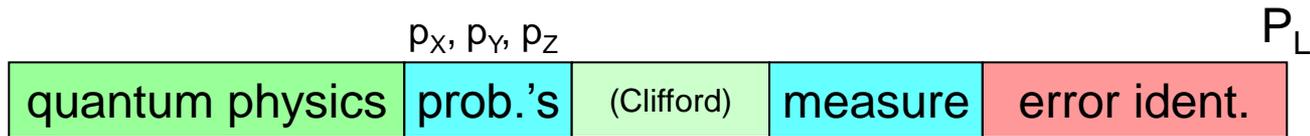
Bialczak et al.

T_ϕ extracted from QPT is close to independently measured value

Designing for the Surface Code



surface code design:



P 's are diagonal matrix elements of computed or measured QPT

full model:



← incoherent avg.
(twirling approx.)

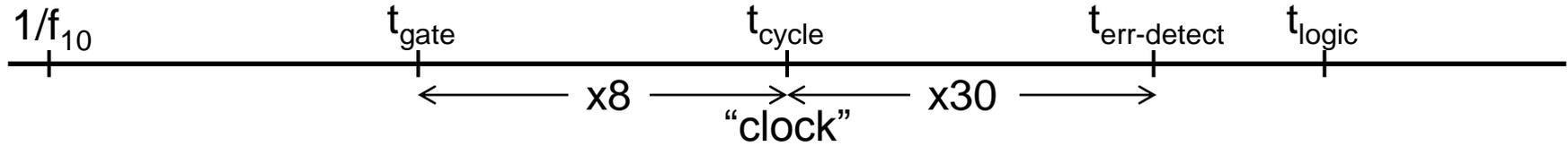
χ_{err} expansion:

$$p_{\text{err}} = p_{X_1} + p_{X_1 Y_2} + \dots \quad (\text{order } 10^{-3})$$

$$+ a p_{X_1} p_{X_2} + \dots \quad (\text{order } 10^{-6})$$

↪ need χ to compute interferences, $\langle a \rangle = 1$

Surface Code Output

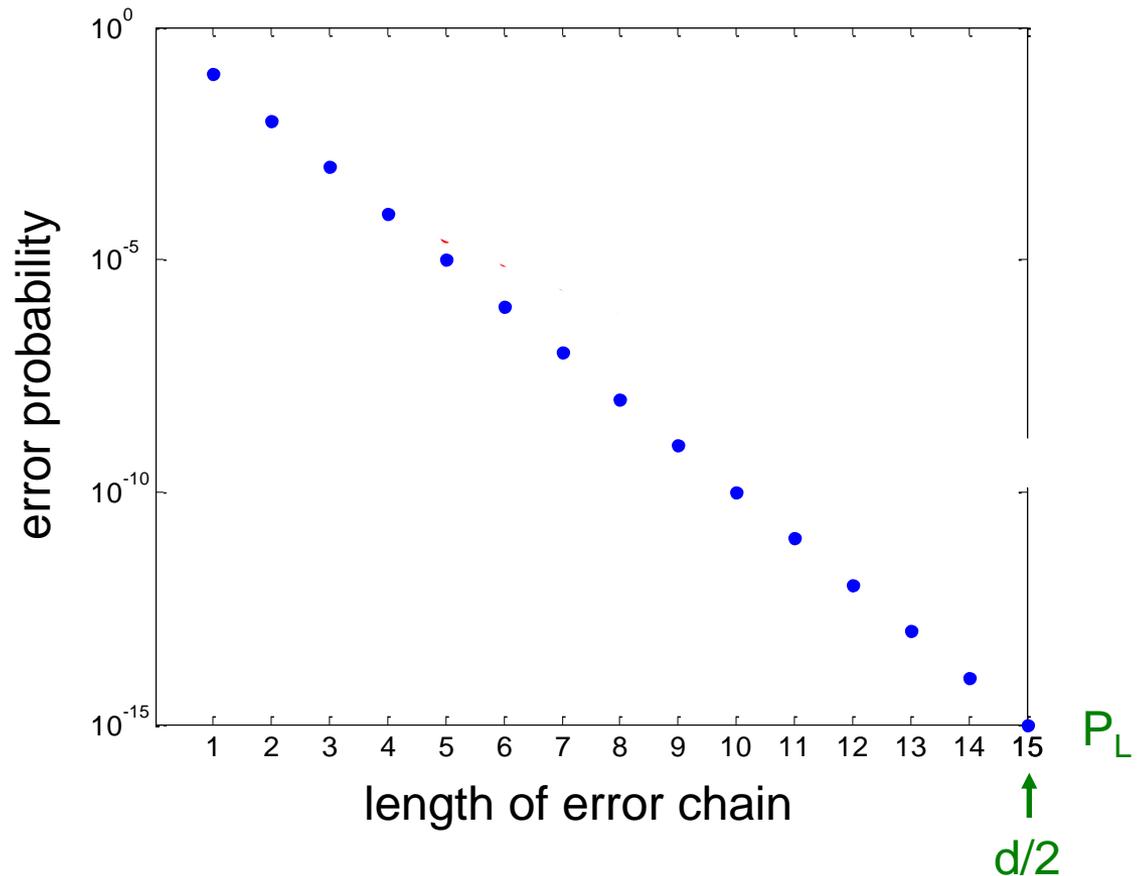


(1) Statistics of errors indicates where and type of errors

p_X, p_{XY}, \dots

(2) Higher probability from correlated errors

How identify correlated errors?



Example: CZ gate

Natural, fast, easy to characterize

$$CZ = \begin{matrix} & \begin{matrix} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{matrix} \\ \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix} & \begin{pmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & -1 \end{pmatrix} \end{matrix}$$

Do not care about phases of **single** and **2-qubit** errors:

Just measure (small) prob's to put in SC errors, e.g.

$P_{IX}, P_{YI}, P_{XY}, P_{YY} \dots$

Measure accurately phases & coherences of diagonal elements

For accuracy & get around single qubit fidelity errors,
is it possible to measure reduced process tomography?

(Still do full QPT to ensure huge error not obscured)

Architecture Design: Surface Code Cells

1) Figure of merit: size $\sim t_{\text{cycle}} (t_{\text{cycle}}/t_{\text{coh}})$ fast is important

Quantum Hardware: Shor Algorithm

Limited by modular exponentiation
 Need $\sim 40N^3$ Toffoli (qu-AND) gates

2048 factoring size N
~~x3600~~ logical overhead
~~x30~~ A states for Toffoli
 220M physical qubits (1day)

2048 bit number, for 0.1 of threshold:

Number of computational logical qubits		Sequential Toffoli gates	Total Toffoli gates	physical qubits		run time			
						10ns	1us	100us	gate cycle
serial	$2N$	$40N^3$	$40N^3$	15M	200M	100ns	10us	1ms	gate cycle
↓	$5N$	$600N^2$	$\mathcal{O}(N^3 \log N)$	37M	20B	1d	100d	30y	
	$2N^2$	$15N(\log_2 N)^2$	$\mathcal{O}(N^3 \log^2 N)$	31B	20T	10m	20h	70d	
parallel	$\mathcal{O}(N^3)$	$\mathcal{O}(\log^3 N)$	$\mathcal{O}(N^3 \log^3 N)$			1s	2m	3h	

\sim constant

Size $\sim (p/p_{th}) t_{cycle}$

$\sim t_{cycle}^2 / t_{coh}$

Interesting example – quantum memory :

100x coherence time, 10x cycle time

Relative size = 1

Error detection: once below threshold, want to extract information quickly

Architecture Design: Surface Code Cells

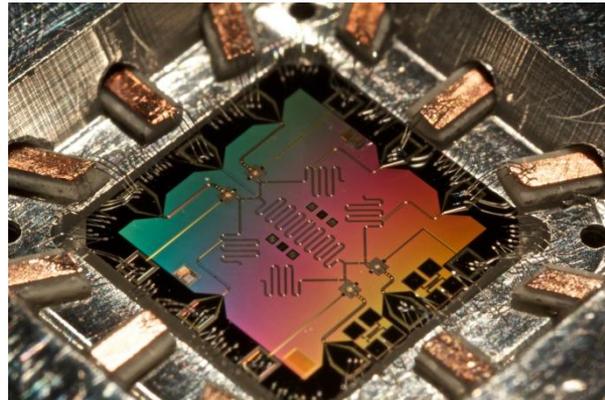
- 1) Figure of merit: size $\sim t_{\text{cycle}} (t_{\text{cycle}}/t_{\text{coh}})$ fast is important
- 2) Global optimization: data qubit with NO decoherence, $p_{\text{th}} = 99\% \rightarrow 97\%$
- 3) Performance does not add trivially: 1 qubit exp. + 2 qubit exp. NOT scalable
Need interconnectivity
- 4) Quantum bus:



The quantum bus is the smart car



- 5) Use ideas from the RezQu architecture



Conclusions

- 1) Compute χ_{err}
- 2) Need error probabilities, diagonal elements of χ_{err}
- 3) Full χ_{err} matrix not very useful; maybe for diagnostics
- 4) Are there alternative forms to quantify useful fidelity based on **direct measure of error probabilities**?
- 5) Ready to test good qubits

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Ted White
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Aaron O'Connell
Matthew Neeley

Peter O'Malley
James Wenner
Jian Zhang
Michael Lenander
Erik Lucero

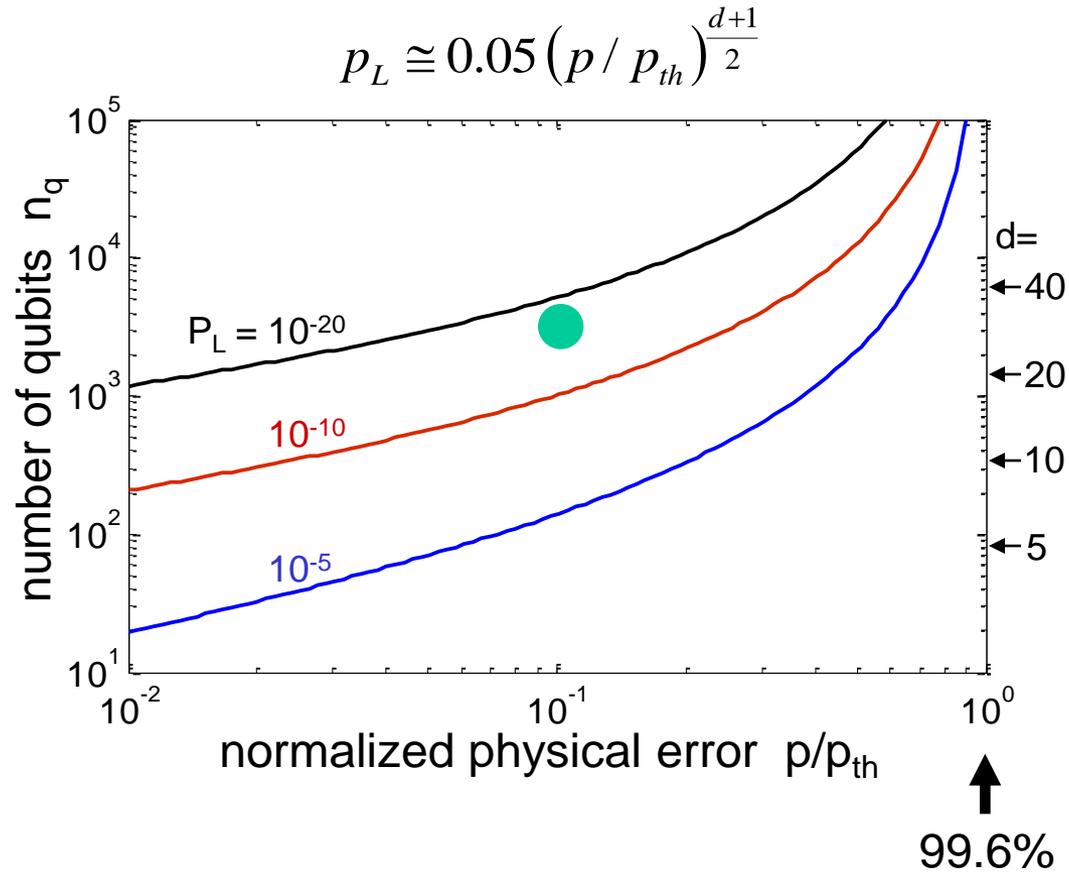
Martin Weides
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Radek Bialczak

John Martinis
Julian Kelley
(Daniel Sank)



(Yu Chen) (Josh Mutus) (Shunobu Ohya) (Andrew Dunsworth)
(Rami Barends) (Evan Jeffrey) (Jimmy Chen) (Ben Chiaro)
(Charles Neill) (Anthony Megrant)

Size of Logical Qubits



$p=99.9\%$, $n_q \sim 3000$
 $p=99.99\%$, $n_q \sim 600$

physical qubits per logical qubit