# Open-loop control and reservoir engineering

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# Conference Announcement — QuAMP 2013

If you would like an excuse to visit Swansea, **QuAMP**, the première UK conference devoted to quantum, atomic, molecular and plasma physics, will take place **8-12 September 2013** hosted by **Swansea University**.

**Conference Topics:**

1. Atomic and molecular systems - interactions and physics,
2. Ultra-fast phenomena, Metrology, Antimatter physics
3. Quantum optics, Quantum information and computing
4. Plasma physics
5. Ultra-cold matter
6. and hopefully some Quantum Control!

**Details to follow** – drop me a line at `sgs29@swan.ac.uk` if you would like to be included in the conference mailing list.

# Overview: Control Paradigms & Applications

1. Model-based Open-loop Coherent Control
2. Model-free Adaptive/Learning Coherent Control
3. Dissipation-Assisted Coherent Control
4. Static Reservoir Engineering
5. Dynamic Reservoir Engineering using Direct Feedback
6. . . .

# Model-based Open-loop Coherent Control [OLCC]

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Many approaches to solve OLCC problem:

1. Initiuitive or physic-based designs
   Pump-dump sequences, adiabatic passage (STIRAP), etc
2. Constructive geometric techniques
   Design principle for conventional NMR pulse sequences
3. Optimal control based on optimization of objective

# Optimal OLCC

Model-based formulation straight-forward in principle:

**Problem**

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But have to make many choices

1. Different ways to formulate objectives and constraints
2. Many control parametrizations – piecewise constant, splines, wavelets, harmonic functions
3. Many optimization algorithms

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**Hard problems** do exist though & then smart choices matter!

- Large systems, many qubits (huge Hilbert space)
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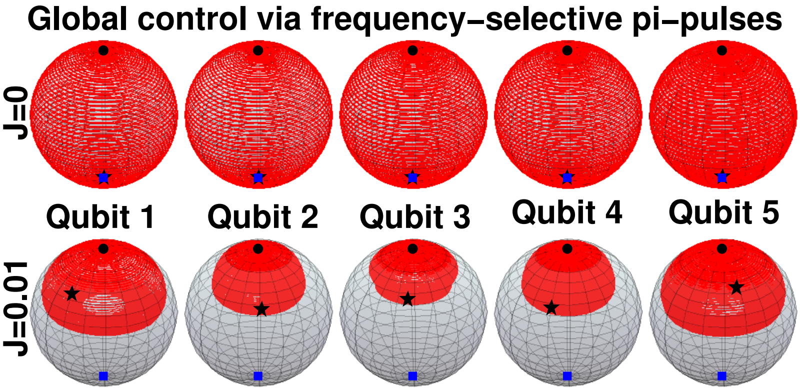
**Example:** Implementation of Encoded Logic Gates
Find ways to realize logic gates (esp. non-transversal gates) for five-qubit code with highly constrained control

NJP **11**, 105003 (2009)

# Implementation of Encoded Logic Gates

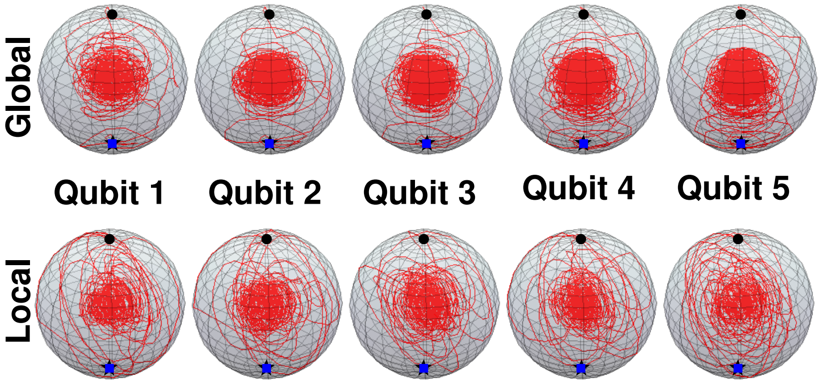



…fails even for modest fixed couplings

Competition of maximizing frequency selectivity (favors long pulses) and minimizing coupling effect (favors short pulses)

# Implementation of Encoded Logic Gates II

Optimal control well suited to solve this type of problem




Found solutions for all single-qubit logic gates (5-qubit gates) including non-transversal ones for different models even for large coupling (shown: simulataneous X-gate with $J = 1$)

# Challenges Encountered and Solutions

1. Choice of control objective: logic gate is 5-qubit unitary
   - must perform correct operation on 2-D logic subspace and
   - fault-tolerance requires correct mapping of error subspaces
   - but many unitaries in $\mathbf{SU}(32)$ satisfy these requirements

   Smart objective functional should reflect this freedom.
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2. Optimization algorithms: tried different approaches
   - 0th order (function values only) Simplex
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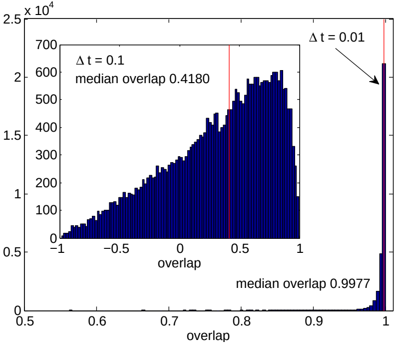
   Observed good initial convergence for various algorithms
   - all struggled near the top
   - most efficient quasi-Newton method struggled the most
   - bizarre randomized sequential update performed best!?

# Problem: Gradient accuracy

- Gradient accuracy important crucial to get to the top
- Signal-to-noise ratio decreases as gradient becomes small
- Linear and finite difference approximations problematic




[NJP13, 073029 (2011)]

- Doubly so for quasi-Newton methods which construct hessian from gradient record [JMR212, 412 (2011)]

# Solution: Accurate Gradients

- Parametrization of controls $\Rightarrow$ gradient functional becomes vector of partial derivatives. PCC: $f_m(t) = \sum_k f_{mk} \chi_{[t_{k-1},t_k]}$

$$\frac{d\mathfrak{F}}{df_{mk}} = \Re \operatorname{Tr}\left[ V^\dagger \Lambda(T, t_k) I_{mk} \Lambda(t_{k-1}, 0) \right]$$

Setting $\Lambda_f(t, t_{k-1}) = e^{(t-t_{k-1})L^{(k)}}$, $L^{(k)} = L_0 + \sum_m f_{mk} L_m$

$$I_{mk} = \int_{t_{k-1}}^{t_k} \Lambda_f(t_k, t) L_m \Lambda_f(t, t_{k-1})\, dt$$

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- Standard approximation $I_{mk} \approx L_m \Lambda(t_k,t_{k-1})\Delta t$ inaccurate but $I_{mk}$ can be evaluated exactly by spectral decomposition [SD] or augmented matrix exponential [NJP14, 073023]:

$$\exp\begin{pmatrix} A & B \\ 0 & C \end{pmatrix} = \begin{pmatrix} e^A & \int_0^1 e^{A(1-s)} B e^{Cs}\, ds \\ 0 & e^C \end{pmatrix}$$

  setting $A = C = L(f_{mp})\Delta t$ and $B = L_m \Delta t$.

# Convergence Behaviour

1. With accurate gradient formulas we now observe good convergence behaviour using quasi Newton methods for
   - closed, markovian and non-markovian systems
   - pure-state, mixed state and gate control problems.
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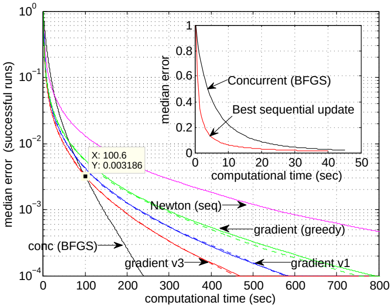
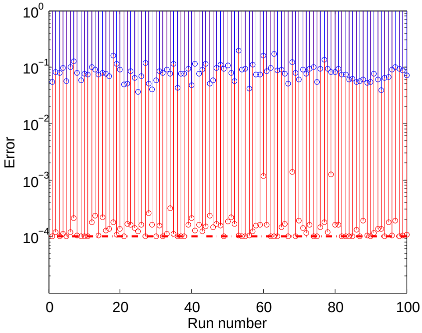
2. SD-method incorporated in Shai's Optimal Control toolbox AME now implemented in Spinach – significant speedup!
3. Sequential update ('Krotov') still better at lower fidelities




All sequential update algorithms initially faster, concurrent quasi-Newton always wins near top, crossover around $10^{-2}$ to $10^{-3}$ error.

[NJP13, 073029 (2011)]

# Markovian vs Non-Markovian Control

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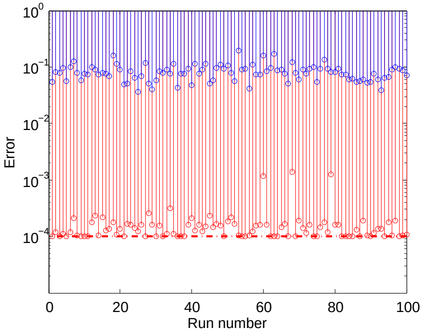
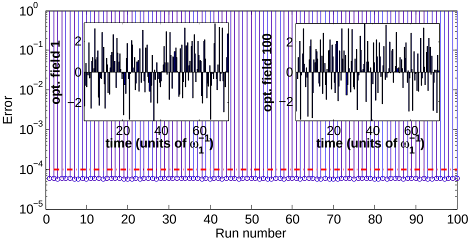
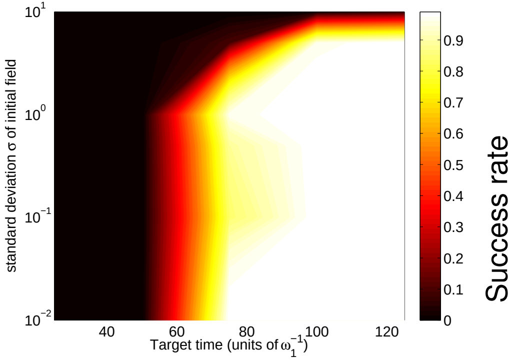



**Non-markovian:**
control performance hugely improves if noise qubits taken into account

**Markovian:**
save time by optimizing neglecting environment, often just as good



# Speed Limits?



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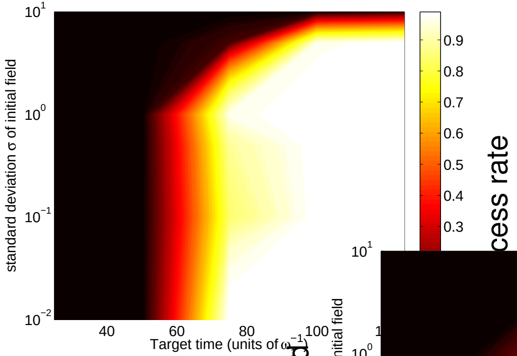
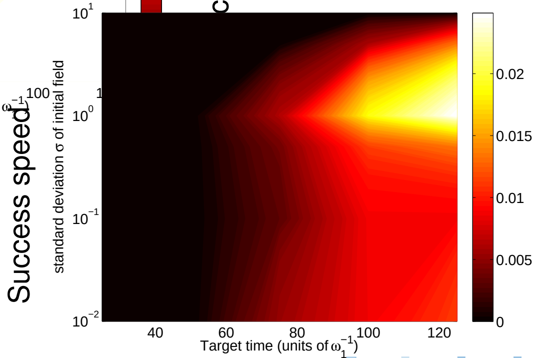
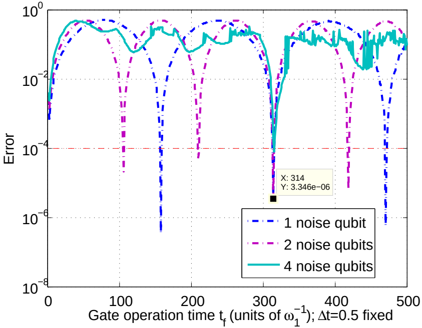


# Limits of Robustness

- Controls generally robust with regard to many errors (control noise, slight system imperfections)
- Very sensitive to **control leakage** i.e., inadvertent excitation of noise qubits/environment highly deleterious
- Can be mitigated by incorporating leakage effect into optimization, but resulting control problem hard



# Dealing with model uncertainty or ignorance

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- Methods based on higher-order local models generally more efficient than simplex or direct search strategies
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**Problem:** Noisy data can be a killer — especially when trying to compute accurate gradients or Hessians — but can be dealt with — e.g., using 'digital filtering' techniques.

# Comparison of optimisation algorithms

**Benchmark problem:** [PRA 80, 030301 (2009)]

- Information transfer in Heisenberg spin chain ($N = 10$)
- Same 100 randomly generated initial $x_0$ used for each run
- Success if transfer fidelity $> 99.99\%$

| | Success % | F.evals | E.time | Best T |
|---|---|---|---|---|
| genetic | 0 | 12,300 | 35.4 | 15.7071 |
| simplex | 75 | 8,700 | 21.8 | 94.9778 |
| BFGS | 95 | 2,900 | 7.3 | 86.0347 |
| qNewton | 100 | 1,400 | 3.6 | 74.6144 |

$\Rightarrow$ constrainted optimization using quasi-Newton method with digital filtering: high success rate and efficient computation

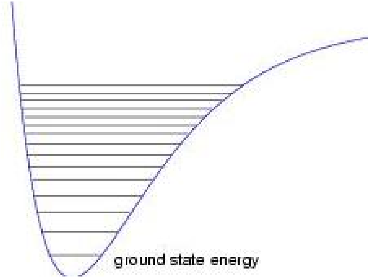
Similar results for many other problems studied — including frequency-domain controls.

# Dissipation-assisted control

Coherent control has fundamental limits: many properties are conserved under unitary evolution, i.e., no entropy reduction

Can't solve problems that require entropy reduction

- preparing known state from fully or partially unknown state
- cooling to ground state, e.g., of vibrational modes




- Molecule with many vibrational modes
- Initial ensemble, many modes populated, populations may be unknown
- Coherent control can swap populations around but not increase purity

Dissipation to the rescue?

PRA63, 013407 (2001)

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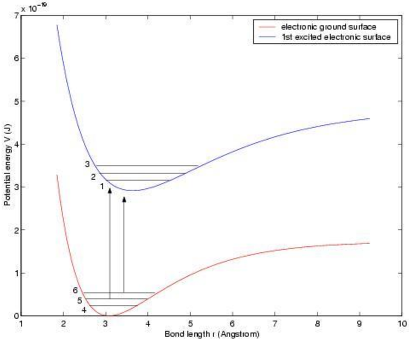


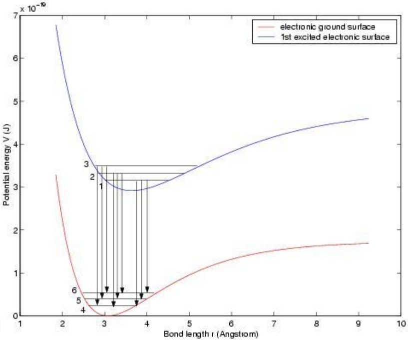

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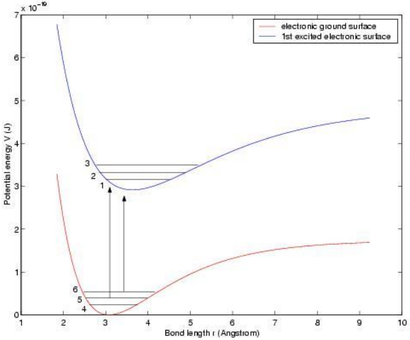
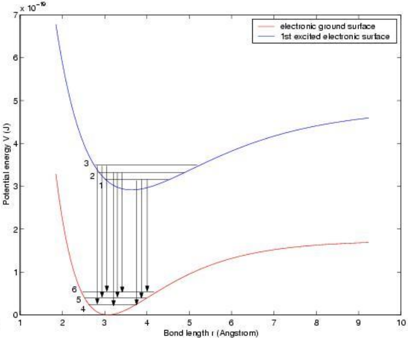
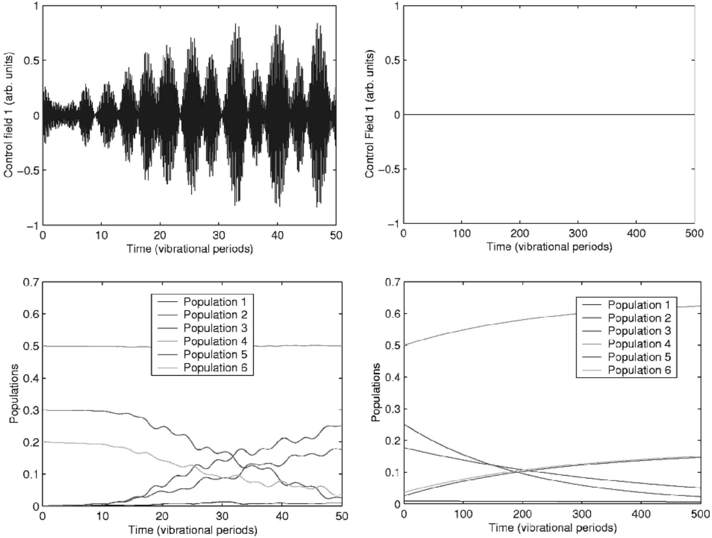



3. Repeat.

# Vibrational Cooling Example

Works at least in theory for toy model: PRA63, 013407 (2001)



# Reservoir Engineering

**Hamiltonian engineering** (coherent control) not sufficient for

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- stabilization of quantum states

These tasks can be accomplished by **reservoir engineering.**

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**Motivation:** Direct-feedback master equation [Wiseman 94]

$$\dot{\rho}(t) = -i[H_{tot}, \rho(t)] + D[M - iF]\rho(t) + \frac{1-\eta}{\eta} D[F]\rho(t)$$
$$H_{tot} = H_0 + H_c + \tfrac{1}{2}(M^\dagger F + FM),$$

**Basic observation:** Any state — pure or mixed — can be stabilized for any system for suitable static choice of hermitian operators $H$, $F$, $M$. Even with limited resources many states are still stabilizable. See Lorenza' talk last week.

# Protecting Entanglement from Markovian dephasing

**Example:** two qubits, Heisenberg coupling with local control (Z-type) subject to Markovian dephasing

- If system is prepared in Bell state $|00\rangle + e^{i\phi}|11\rangle$, Markovian dephasing leads to exponential decay of entanglement in almost all cases.

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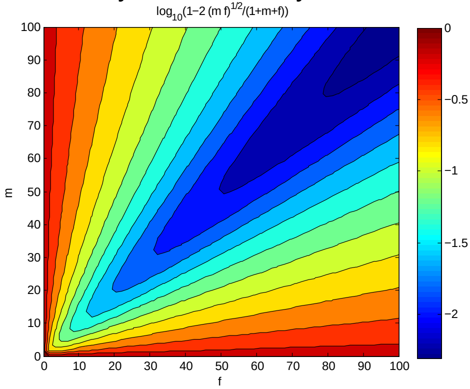
$$\dot{\rho} = [H_0 + f(\rho(t))H_c, \rho] + D[Z]\rho + D[M]\rho \qquad (1)$$

- Bell state approximately stabilizable using direct feedback, e.g. choose $H = 0$, $M = \sqrt{m}ZI$, $F = \sqrt{f}XX$

# Direct Feedback Stabilization

Although we can't stabilize pure state, purity and concurrence can be made arbitrarily close to unity:



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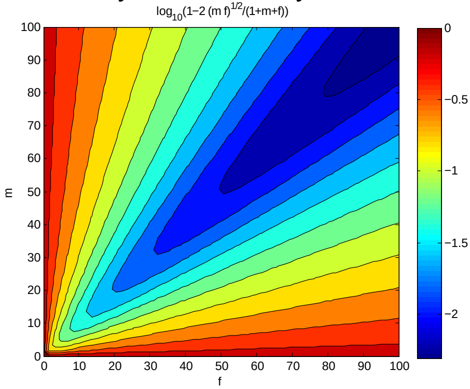


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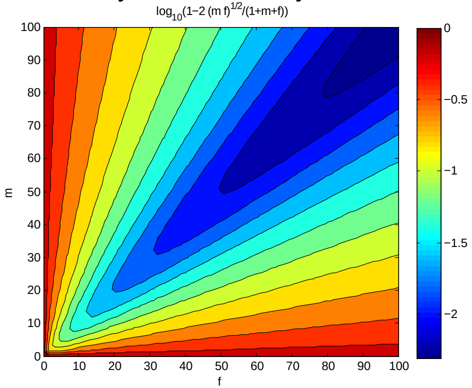


Even better, we can prepare entanglement too! Show movies. Feedback not trivial to implement, realizeable using quantum circuits? Non-uniqueness of $M$ and $F$ may help.

[J. Russian Laser Research 32, 502 (2011)]