Quantum-enhanced optical phase tracking. [Science **337**, 1514 (2012)]

H. M. Wiseman[GU], H. Yonezawa[UT], D. Nakane[UT], T. A. Wheatley[UNSW-C], K. Iwasawa[UT], S. Takeda[UT], H. Arao[UT], K. Ohki[KU], K. Tsumura[UT], D. W. Berry[MU], T. C. Ralph[UQ], E. H. Huntington[UNSW-C], **A. Furusawa**[UT]

Centre for Quantum Dynamics

Griffith UNIVERSITY

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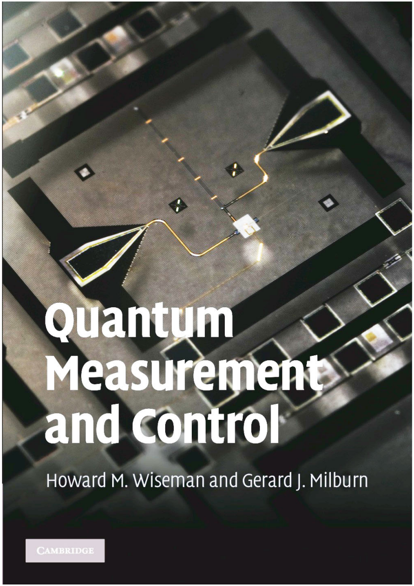
Outline

Outline

Measurement and Control



- The obvious reason to combine measurement and control is feedback, to purposefully **change** the average system evolution.
- Classically non-trivial, even with perfect measurement.
- *cf.* adaptive measurement — controlling future measurements on the basis of the results of past ones, to **obtain better data**, leaving the average system evolution **unchanged**.
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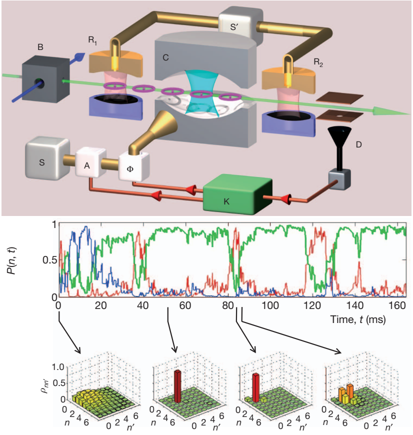
Measurement and Control

LETTER

doi:10.1038/nature10376

Real-time quantum feedback prepares and stabilizes photon number states

Clément Sayrin[1], Igor Dotsenko[1], Xingxing Zhou[1], Bruno Peaudecerf[1], Théo Rybarczyk[1], Sébastien Gleyzes[1], Pierre Rouchon[2], Mazyar Mirrahimi[3], Hadis Amini[2], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,4]



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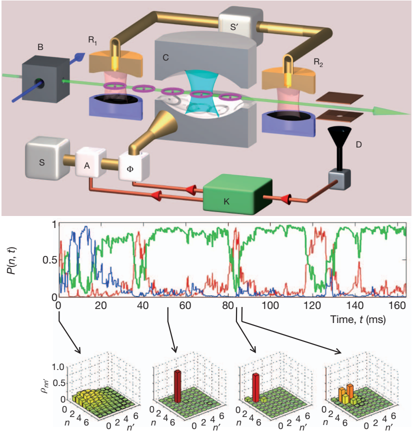

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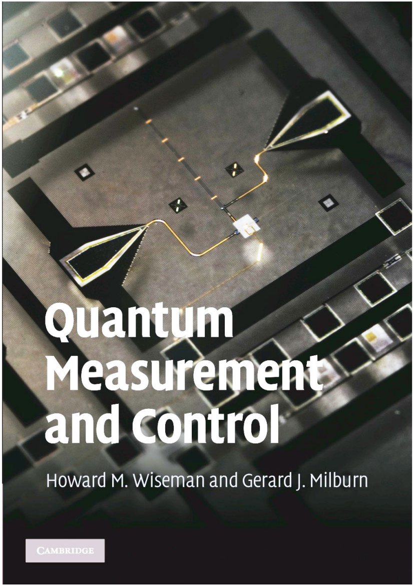

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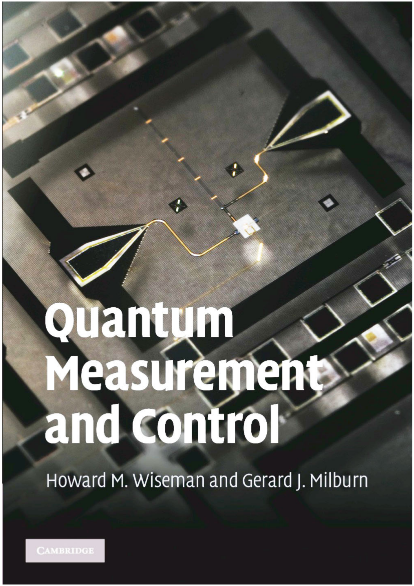

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Adaptive Measurement: Recent Examples

Letter

Nature **445**, 65-69 (4 January 2007) | doi:10.1038/nature05346; Received 6 July 2006; Accepted 11 October 2006

High-speed linear optics quantum computing using active feed-forward

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- Figures and tables

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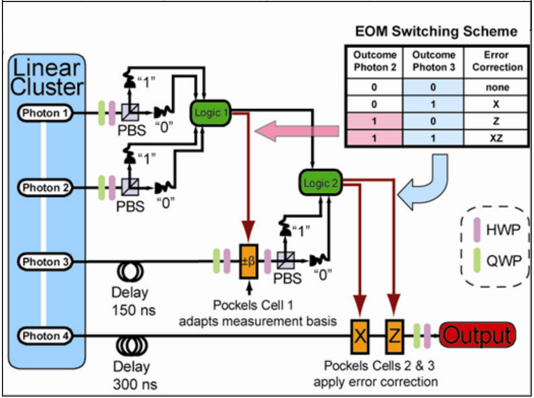


Outcome Photon 2	Outcome Photon 3	Error Correction
0	0	none
0	1	X
1	0	Z
1	1	XZ

0. *Measurement-based Q computation.*
1. Optimal state discrimination of multiple copies.
2. Tracking an open quantum system with a finite-state machine.
3. Phase estimation using quantum states under various conditions.

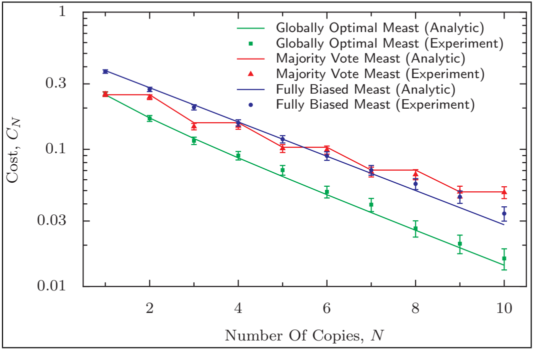
....

Adaptive Measurement: Recent Examples

PRL **103**, 220503 (2009) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2009

Mixed State Discrimination Using Optimal Control

B. L. Higgins,[1] B. M. Booth,[1] A. C. Doherty,[2] S. D. Bartlett,[3] H. M. Wiseman,[1,*] and G. J. Pryde[1,†]
[1]*Centre for Quantum Dynamics, Griffith University, Brisbane, 4111, Australia*



. . . . .

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Adaptive Measurement: Recent Examples

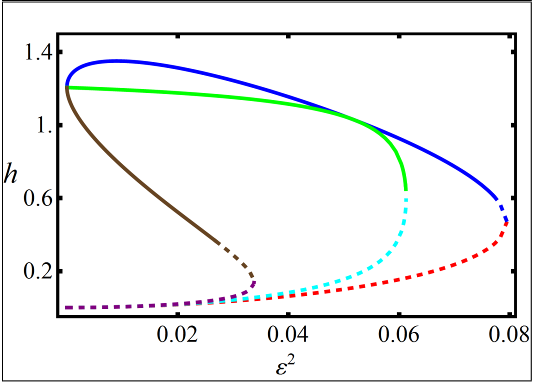
PRL **106,** 020406 (2011) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2011

How Many Bits Does It Take to Track an Open Quantum System?

R. I. Karasik and H. M. Wiseman*

ARC Centre for Quantum Computation and Communication Technology, and Centre for Quantum Dynamics, Griffith University, Brisbane, Queensland 4111, Australia

(Received 1 September 2010; published 14 January 2011)



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Adaptive Measurement: Recent Examples

Vol 450 | 15 November 2007 | doi:10.1038/nature06257 nature

LETTERS

Entanglement-free Heisenberg-limited phase estimation

B. L. Higgins[1], D. W. Berry[2], S. D. Bartlett[3], H. M. Wiseman[1,4] & G. J. Pryde[1]

nature photonics

LETTERS

PUBLISHED ONLINE: 12 DECEMBER 2010 | DOI: 10.1038/NPHOTON.2010.268

Entanglement-enhanced measurement of a completely unknown optical phase

G. Y. Xiang[1,2], B. L. Higgins[1], D. W. Berry[3], H. M. Wiseman[1]* and G. J. Pryde[1]*

21 SEPTEMBER 2012 VOL 337 SCIENCE www.sciencemag.org

Quantum-Enhanced Optical-Phase Tracking

Hidehiro Yonezawa,[1] Daisuke Nakane,[1] Trevor A. Wheatley,[1,2,3] Kohjiro Iwasawa,[1] Shuntaro Takeda,[1] Hajime Arao,[1] Kentaro Ohki,[4] Koji Tsumura,[5] Dominic W. Berry,[6,7] Timothy C. Ralph,[2,8] Howard M. Wiseman,[9]* Elanor H. Huntington,[2,3] Akira Furusawa[1]*

0. *Measurement-based Q computation.*
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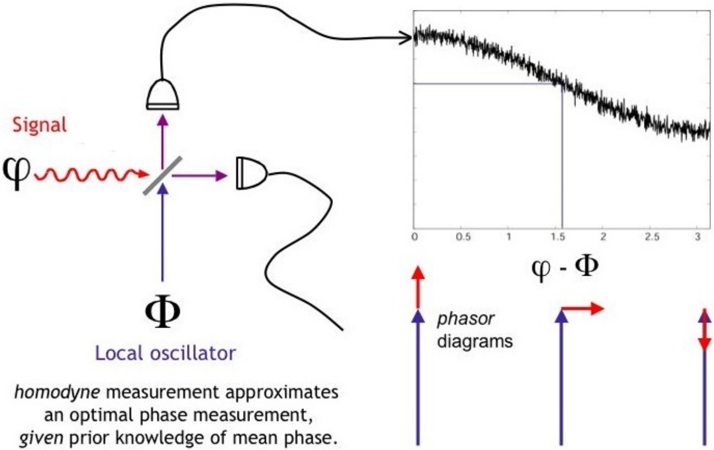
Measurements of Phase with a Local Oscillator



homodyne measurement approximates an optimal phase measurement, *given* prior knowledge of mean phase.

Other schemes: **heterodyne** / dual-homodyne; **adaptive** 'dyne; **optimal**.

Optimal! That sounds good. What is it? I don't know in general. *What about in the simplest case?* Sure it's this POVM. *And how can I do this in the lab?* I haven't a clue.

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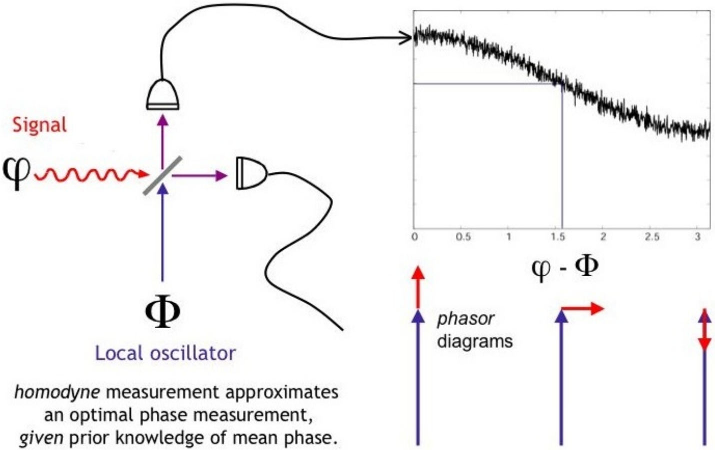

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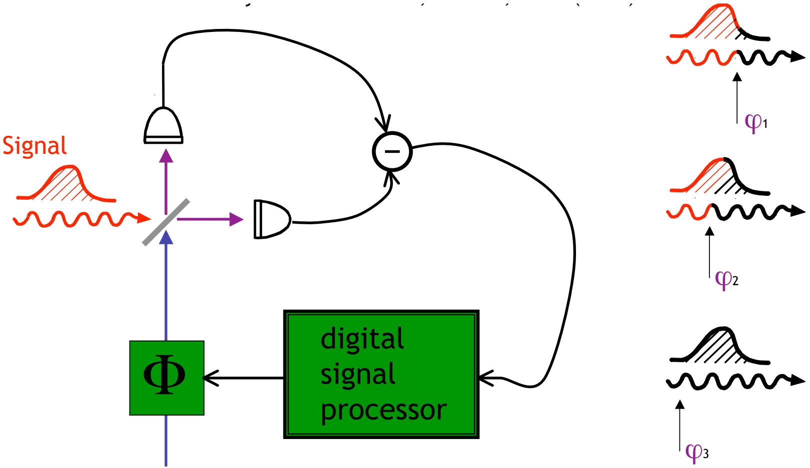

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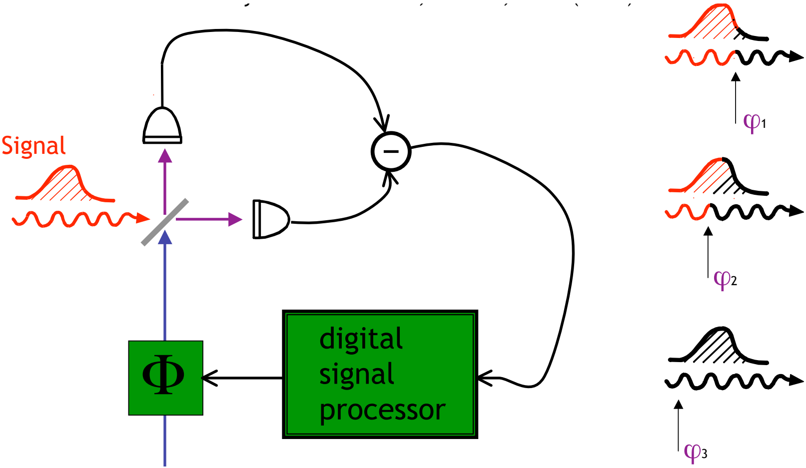

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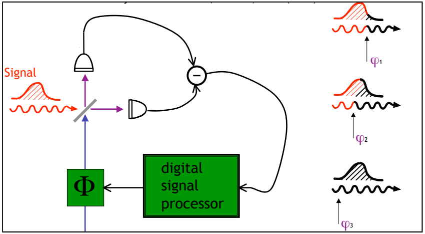
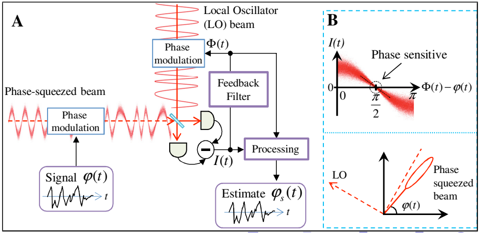
Performance of Schemes

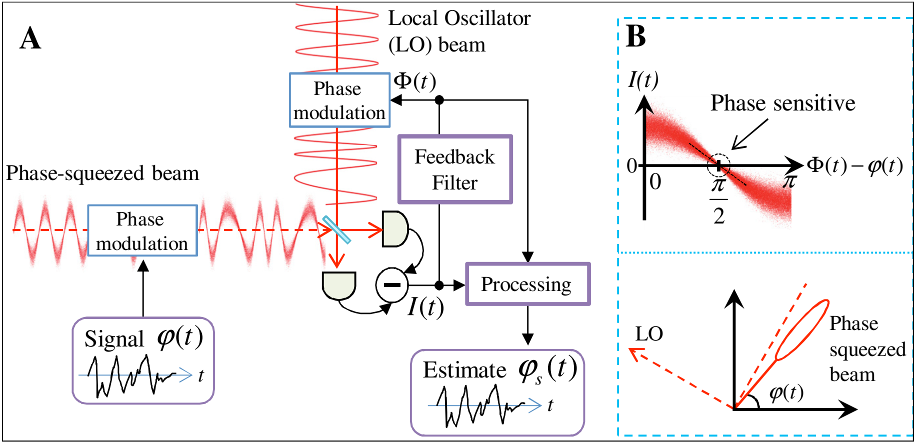
Scenario	Type of Light	Detection	Theory	V (theory)	Experiment	V (expt)
Single pulse, constant phase, n = mean photon number	Coherent	Heterodyne	? 1970s?	$0.50{\cdot}n^{-1}$=SQL	Armen *et al*, PRL '02 ?	$0.62{\cdot}n^{-1}$
		Adaptive	HMW, PRL '95	$0.25{\cdot}n^{-1}$	Armen *et al*, PRL '02	$0.40{\cdot}n_o^{-1}$
		Optimal	? 1950s?	$0.25{\cdot}n^{-1}$=CSL		
	Squeezed	Heterodyne	? (pre '95)	$0.25{\cdot}n^{-1}$	?	
		Adaptive	DWB&HMW, PRA '01	slow(n)·ln(n)·n^{-2}	–	
		Optimal	Collett, PS '93	0.25·ln(n)·n^{-2}		
Continuous beam, Wiener phase, N = n per coherence time.	Coherent	Heterodyne	DWB&HMW, PRA '02	$0.35{\cdot}N^{-1/2}$	± Wheatley *et al*, PRL '10	$0.37{\cdot}N^{-1/2}$
		Adaptive	DWB&HMW, PRA '02	$0.25{\cdot}N^{-1/2}$	± Wheatley *et al*, PRL '10	$0.30{\cdot}N^{-1/2}$
		Optimal	DWB&MJH&HMW up	$0.25{\cdot}N^{-1/2}$=CSL		
	Squeezed (OPO output)	Heterodyne	DWB&HMW, PRA '06	$0.25{\cdot}N^{-1/2}$	–	
		Adaptive	DWB&MJH&HMW up	slow(N) $\cdot N^{-2/3}$	± Yonezawa *et al*, Sci. '12	$0.21{\cdot}N^{-1/2}$
		Optimal	DWB&MJH&HMW up	$\geq 0.21{\cdot}N^{-2/3}$		





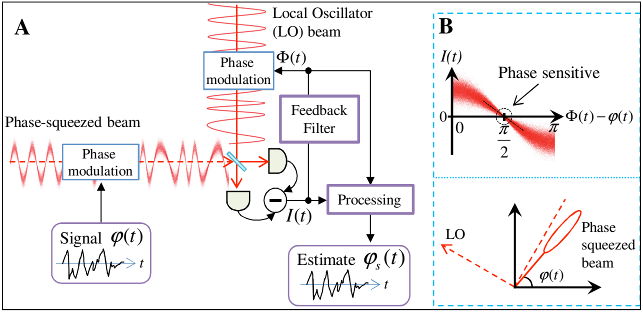
Outline

The Parameter(s) to be Estimated



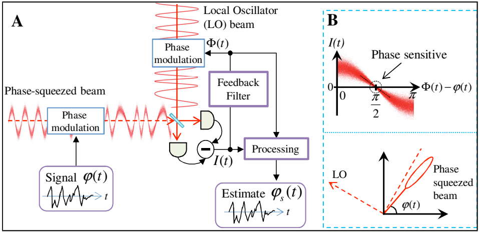
$$\varphi(t) = \sqrt{\kappa}\int_{-\infty}^{t} e^{-\lambda(t-s)}dV(s).\quad \text{For } \lambda = 0,\ \tau_{\rm coh} = \kappa^{-1}$$

The Squeezed Beam



- At centre frequency antisqueezed spectrum $S_p(0) \equiv e^{2r_p}$ $[r_p \geq r_m]$
- …squeezed spectrum $S_m(0) \equiv e^{-2r_m} < 1 \implies$ nonclassical.
- Coherent amplitude α, so that flux = $\mathcal{N} = |\alpha|^2+$squeezed flux.
- Broadband squeezing $\implies$ squeezed flux is “infinite”.
- But we show (numerically) that using narrowband squeezing (with negligible flux) makes little difference so we take $\mathcal{N} = |\alpha|^2$.

The Photocurrent



$$\Phi(t) = \varphi_f(t) + \pi/2 \approx \varphi(t) + \pi/2.$$

$$I(t)dt = 2|\alpha| \sin[\varphi(t) - \varphi_f(t)]\, dt + \sqrt{R_{\rm sq}(t)} dW(t), \quad (1)$$

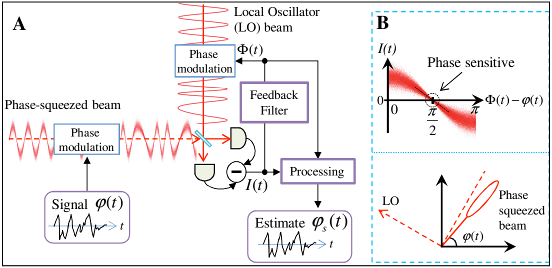
$$R_{\rm sq}(t) = \sin^2[\varphi(t) - \varphi_f(t)]\, e^{2r_p} + \cos^2[\varphi(t) - \varphi_f(t)]\, e^{-2r_m}, \quad (2)$$

For good tracking $\sigma_f^2 \equiv \langle[\varphi(t) - \varphi_f(t)]^2\rangle \ll 1$. We expand $I(t)$ to *second* order in $[\varphi(t) - \varphi_f(t)]$ and approximate $R_{\rm sq}(t)$ by its average:

$$I(t)dt \simeq 2|\alpha| [\varphi(t) - \varphi_f(t)]\, dt + \sqrt{\bar{R}_{\rm sq}} dW(t), \quad (3)$$

$$\bar{R}_{\rm sq} = e^{-2r_m} + \sigma_f^2 \times e^{2r_p}. \quad (4)$$

The Filtered Estimate



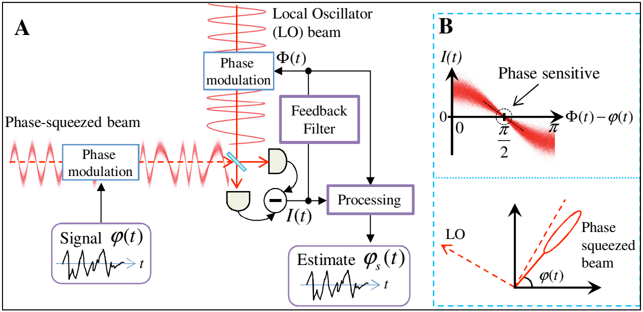
- Under this approx., the optimal (Kalman) *filter* of the current is

$$\varphi_f(t) = \Gamma \int_{-\infty}^{t} e^{-\lambda(t-s)} \frac{I(s)}{2|\alpha|} ds,$$

where $\Gamma = \sqrt{4\,|\alpha|^2\,\kappa/\bar{R}_{\rm sq}}$ must be $\gg \lambda$ to justify the approx.

- Taking $\lambda = 0$ for simplicity gives $\sigma_f^2 = \sqrt{\kappa/\Gamma}$.
- This is still implicit as $\bar{R}_{\rm sq}$ (and hence Γ) depends on σ_f^2.

The Smoothed Estimate



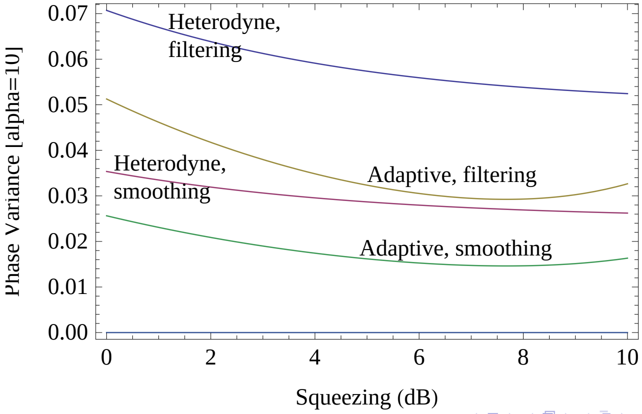
- φ_f is the optimal *causal* estimate, but a better estimate is found by optimally *smoothing* the filter:

$$\varphi_s(t) = (2\lambda + \Gamma) \int_t^{\infty} e^{-(\lambda+\Gamma)(s-t)} \varphi_f(s) ds.$$

- Again taking $\lambda = 0$ for simplicity gives $\sigma_s^2 = \sigma_f^2/2$.

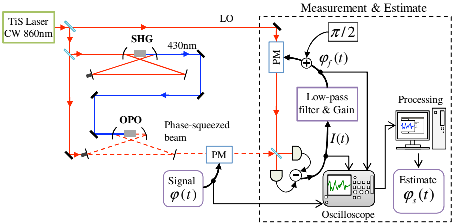
Squeeze till it hurts!

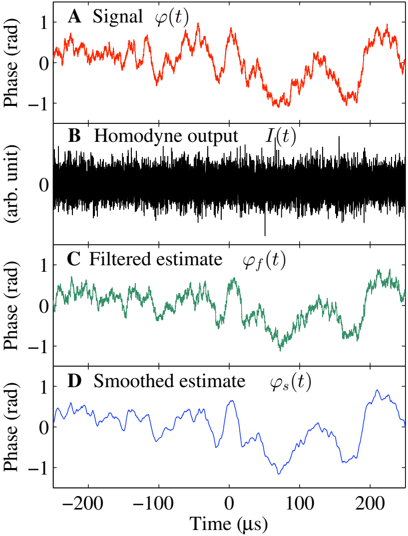
- Even with everything ideal — $\lambda = 0$, $r_m = r_p = r$, $\sigma_f^2 \ll 1$ — too much squeezing hurts the performance of the adaptive scheme because $\bar{R}_{\rm sq} = e^{-2r_m} + \sigma_f^2 \times e^{2r_p}$.
- Shown here for $N = |\alpha|^2/\kappa = 100$, Squeezing (dB) $= 10\log_{10} e^{2r}$.



Outline

The Experiment





$$\kappa \equiv (1.9 \pm 0.1) \times 10^4 \text{ rad/s},$$

$$\lambda \equiv (5.9 \pm 0.5) \times 10^4 \text{ rad/s},$$

$$|\alpha|^2 = (1.00 \pm 0.06) \times 10^6 \text{s}^{-1},$$

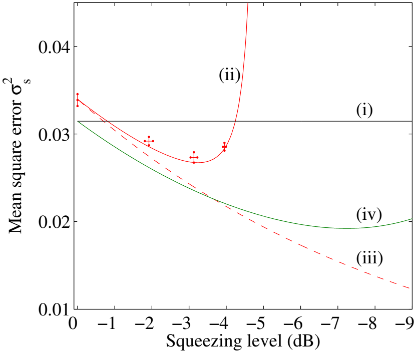
$$\{ \implies \text{Power} = 200 \text{ femtowatts.} \}$$

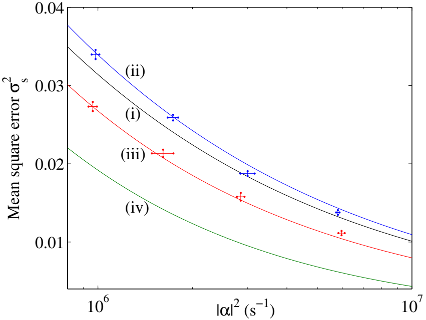
$$r_m = 0.36 \pm 0.01 \text{ (squeezing)},$$

$$r_p = 0.59 \pm 0.01 \text{ (anti-squeezing)}.$$

$$\implies \lambda/\Gamma < 0.2$$

The Results





i) coherent-state limit, $\eta = 1$.
ii) squeezed theory, $\eta = 0.85$.
iii) as per (ii) without $\sigma_f^2 \times e^{2r_p}$.
iv) theory, $\eta = 1$, $r_p = r_m$.

i) coherent-state limit, $\eta = 1$.
ii) coherent-states, $\eta = 0.85$.
iii) squeezing, $\eta = 0.85$, $r_p > r_m$.
iv) squeezing, $\eta = 1$, $r_p = r_m$.

Outline

Quantum Cramér-Rao Bound

Using the theory of Tsang, HMW, & Caves, PRL **106**, 090401 (2011).

$$\langle[\varphi_{\text{est}}(t) - \varphi(t)]^2\rangle \geq F^{-1}_{t,t}, \tag{5}$$

where $F^{-1}_{t,t}$ is the matrix inverse of the continuously indexed Fisher information "matrix" $F_{t,t'} := F^{(Q)}(t,t') + F^{(C)}(t,t')$, where

$$F^{(Q)}(t,t') := \frac{2}{\hbar^2}\left\langle \Delta f(t)\Delta f(t') + \Delta f(t')\Delta f(t)\right\rangle$$
$$F^{(C)}(t,t') := \int D\varphi\, P_{\text{prior}}[\varphi]\frac{\delta \ln P_{\text{prior}}[\varphi]}{\delta\varphi(t)}\frac{\delta \ln P_{\text{prior}}[\varphi]}{\delta\varphi(t')}. \tag{6}$$

where $f(t)$ is the photon-flux operator and $(\lambda,\kappa) \mapsto P_{\text{prior}}[\varphi]$.

In the high-squeezing limit we find

$$F^{(Q)}(t,t') \simeq 4\mathcal{N}\delta(t-t') + 8\mathcal{N}^2 e^{-\gamma_p|t-t'|},$$

and for $\lambda = 0$,

$$\langle[\varphi_{\text{est}}(t) - \varphi(t)]^2\rangle \gtrsim 0.21(\mathcal{N}/\kappa)^{-2/3}.$$

Is this achievable by adaptive measurements?

- DWB & HMW derived a $(\mathcal{N}/\kappa)^{-2/3}$ scaling in 2002 [PRA], but
    - this was for broad-band squeezing, and
    - it ignored the (infinite) flux in the squeezed photons.

- DWB & HMW corrected this short-coming in 2006 [PRA] by
    - considering finite-bandwidth squeezing,
    - taking into account the squeezed photons, and
    - optimizing the bandwidth as well as the degree of squeezing,

  and found a scaling of only $(\mathcal{N}/\kappa)^{-5/8}$ [cf. CSL $\sim (\mathcal{N}/\kappa)^{-1/2}$].

- However, very recently we have [see our 2013 erratum]
    - realized the 2006 paper used the *wrong expression* for the squeezed flux,
    - corrected this in the analytical argument, which now gives a $(\mathcal{N}/\kappa)^{-2/3}$ scaling, and
    - verified this scaling (up to some sub-logarithmic multiplier) numerically for $1 \leq \mathcal{N} \leq 10^{20}$.

Outline

- Squeezed states allow one to beat the coherent-state-limit for phase estimation using a local oscillator.
- However, if the phase is initially completely unknown, or widely varying, to do better than CSL scaling requires an adaptive measurement (at least).
- We have performed the first experiment of this kind that has beaten the CSL (by $15 \pm 4\%$).
- The theory predicts, and we observe experimentally, that there is such a thing as "too much squeezing" — the first time this has been observed for a fundamental task.
- With optimal squeezing we think that, in principle, adaptive measurement could achieve the best possible scaling $\sim \mathcal{N}^{-2/3}$, as determined from the quantum Cramér-Rao bound.
- Future experiments should optimize the bandwidth as well as the degree of squeezing, for a given *total* flux.