Atomic Quantum Simulation of (Non-)Abelian Lattice Gauge Theories: Quantum Link Models

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M. Dalmonte
E. Rico → Ulm
M. Müller → Madrid
D. Marcos
K. Stannigel

(quantum optics)

ITP Bern

D. Banerjee
M. Bögli
P. Stebler
U. J. Wiese

(high-energy physics)

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UQUAM
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FWF Der Wissenschaftsfonds.

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… possible future developments at the interface between AMO and cond mat & particle physics

- Abelian U(1) Schwinger model, D. Banerjee et al., PRL 2012
- U(N), SU(N) Non-Abelian QLM, D. Banerjee et al., PRL in print
- Other
  - dissipative techniques K. Stannigel et al.
  - other implementations: superconducting qubits, D. Marcos et al.

Related work: MPQ-Tel Aviv, ICFO, Cornell, ...

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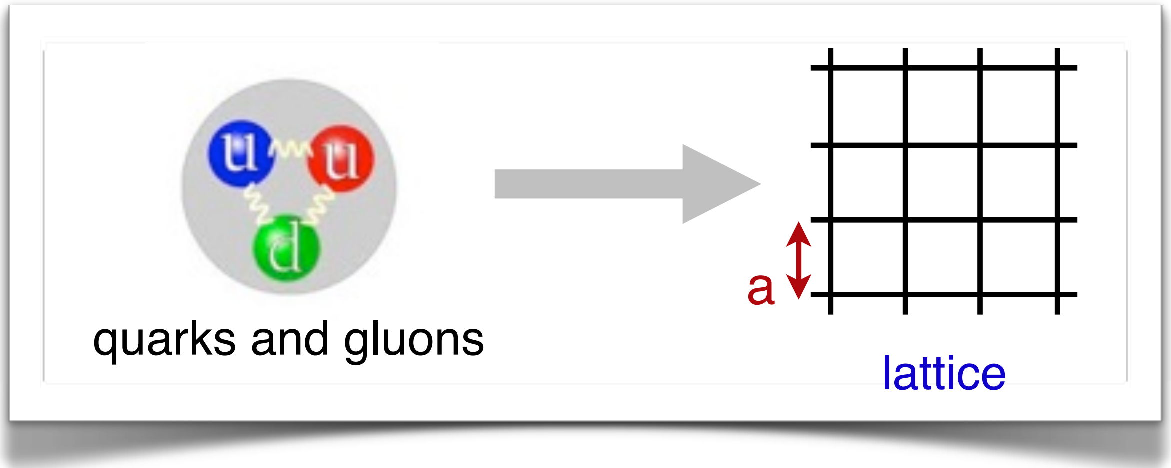
FWF Der Wissenschaftsfonds.

… lattice gauge theories [in particle physics]

- Gauge theories on a discrete lattice structure K. Wilson, Phys. Rev. D (1974).
- **Fundamental gauge symmetries:** standard model (every force has a gauge boson)



non-perturbative approach to fundamental theories of matter (e.g. QCD)
→ classical statistical mechanics

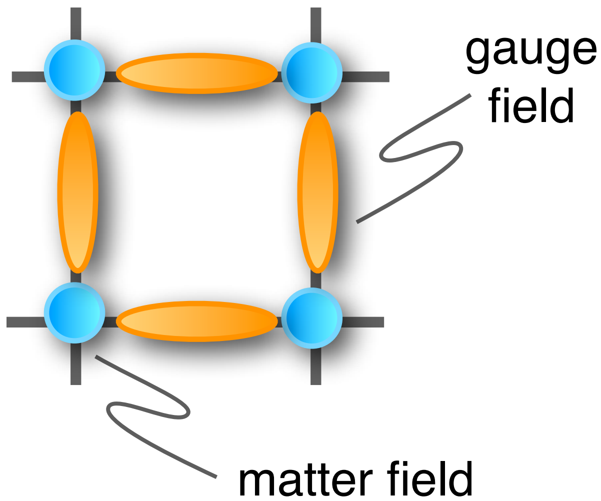
Classical Monte Carlo simulations:

achievements	issues
• first evidence of quark-gluon plasma • ab initio estimate of proton mass • entire hadronic spectrum	Sign problem in its various flavors: • finite density QCD (=fermions) • real time evolution

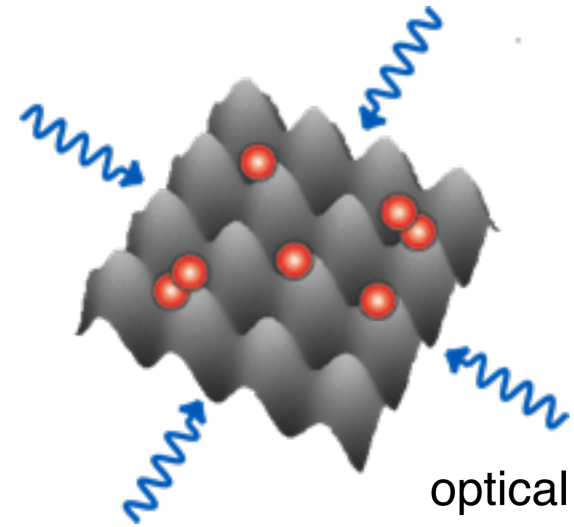
Quantum simulation (with atoms)? … toy models & simple phenomena

Quantum Simulation of Lattice Gauge Theories with Atoms

Lattice Gauge Theories



Atomic Quantum Simulation



Hamiltonian formulation

Kogut & Susskind

(Non-)Abelian LGT:

✓QED U(1)

✓QCD SU(N), U(N)

gauge symmetry as
local symmetry

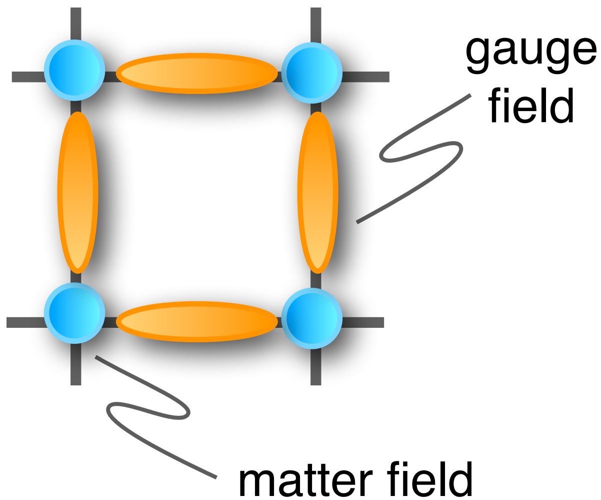
?

Hubbard models with bosonic and fermionic atoms in optical lattices

"emergent lattice gauge theory"

Quantum Simulation of Lattice Gauge Theories with Atoms

Lattice Gauge Theories



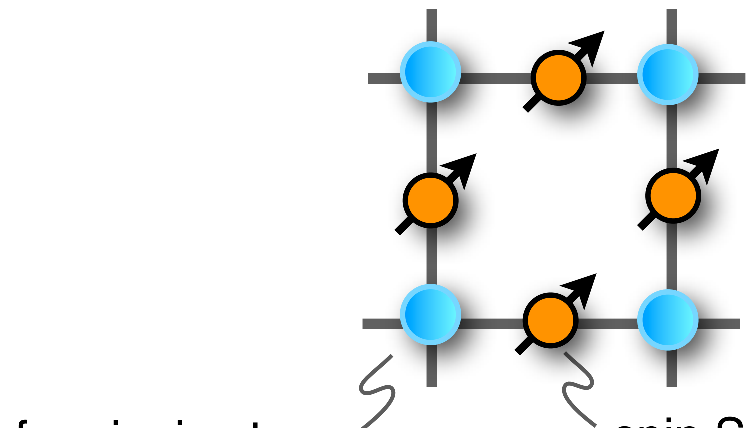
(Non-)Abelian LGT:

✓QED U(1)

✓QCD SU(N), U(N)

Quantum Link Models (QLM)

Example:
U(1) / QED



QLM

✓gauge fields in *finite dim* spaces

✓Abelian & Non-Abelian

particle physics: U.J. Wiese et al., Horn, Orland & Rohrlich, …
cond mat: … [spin ice, quantum spin liquids]

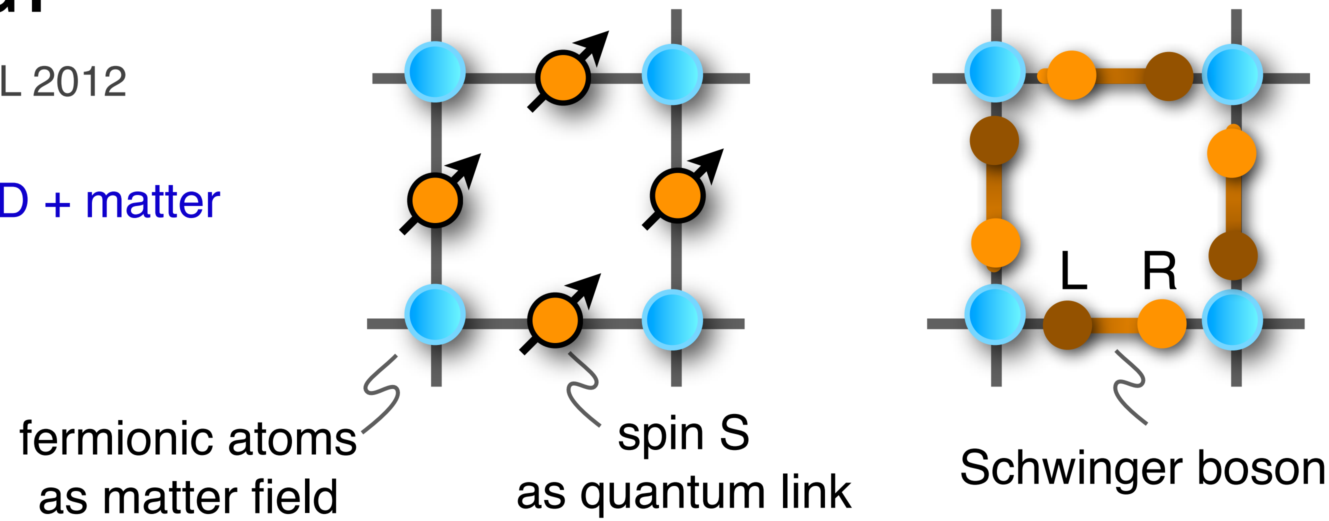
QLM have "natural" implementations
with cold atoms in optical lattices

- **Quantum Link Models (as Hubbard Models for Atoms)**

U(1) Abelian LGT

D. Banerjee et al., PRL 2012

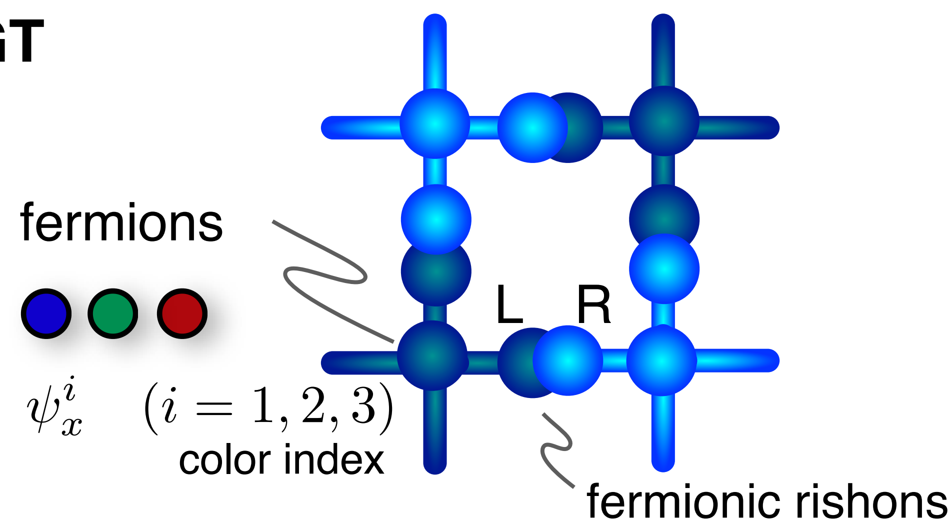
“spin ice” QED + matter



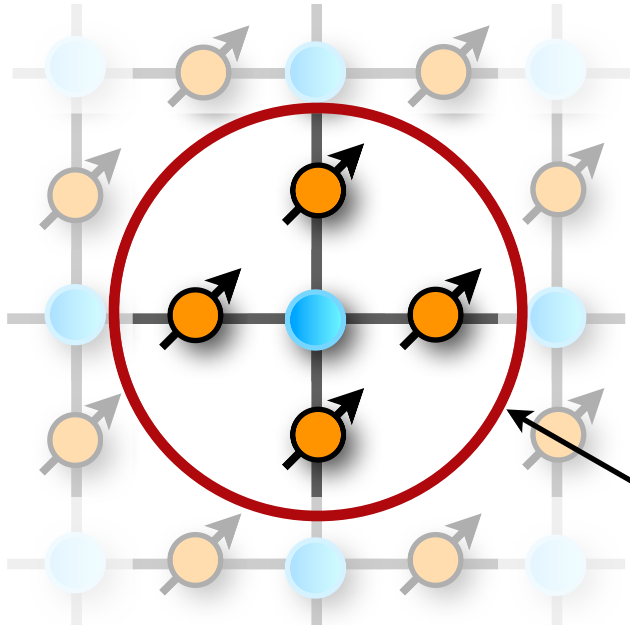
atomic boson-fermi mixtures in optical lattices

U(N),SU(N) Non-Abelian LGT

D. Banerjee et al., PRL in print



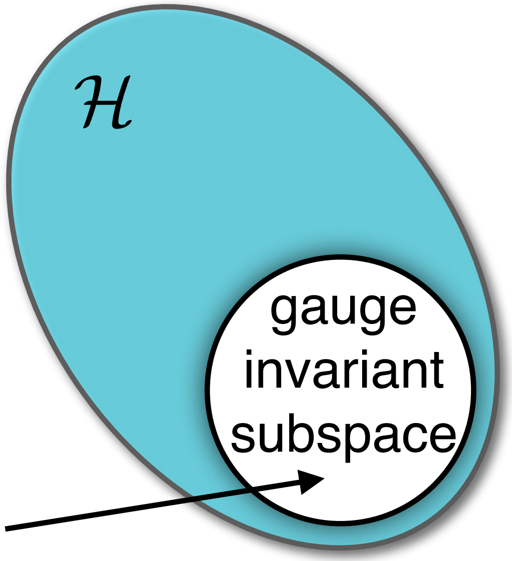
multi-species fermi gases



“spin ice” QED + matter

$$\rho - \nabla \cdot E = 0$$

Gauss law as a constraint



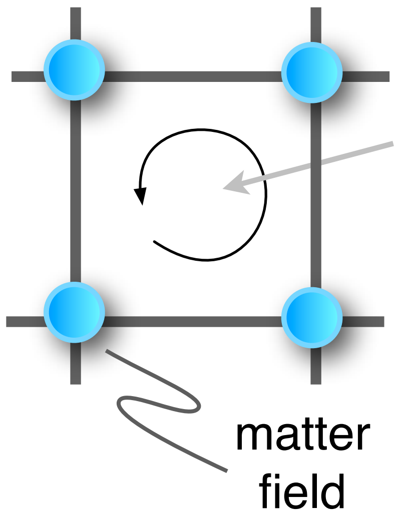
Hamiltonians, (local) gauge symmetries, Gauss law etc.

an AMO perspective

Static vs. Dynamical Gauge Fields on Lattices: U(1)

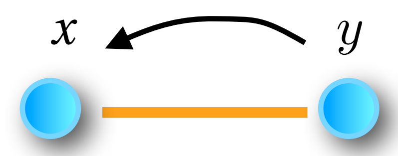
- **c-number / static gauge fields**

particles hopping around a plaquette acquire a finite phase



$$\Phi = \int d^2\vec{f} \cdot \vec{\nabla} \times \vec{A}$$

flux



$$H = -t\psi_x^\dagger e^{i\varphi_{xy}}\psi_y + \text{h.c.}$$

phase $\varphi_{xy} = \int_x^y d\vec{l} \cdot \vec{A}$

U(1) (abelian)

theory & AMO experiments
on “synthetic gauge fields”

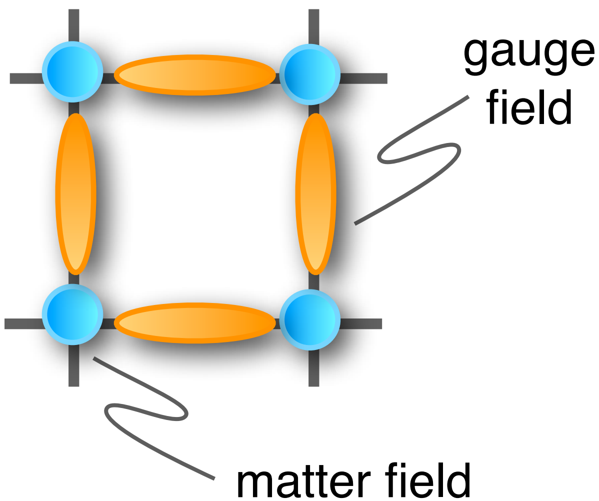
- Hofstadter butterfly
- Fractional Quantum Hall,
  Fractional Chern insulators

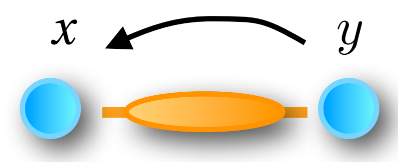
Review: J. Dalibard et al. Rev. Mod. Phys. (2011)

Static vs. Dynamical Gauge Fields on Lattices: U(1)

- **dynamical gauge fields**

  particles hopping around a plaquette assisted by link degrees of freedom





$$H = -t\psi_x^\dagger U_{xy}\psi_y + \text{h.c.} + \ldots$$

link operator

Static vs. Dynamical Gauge Fields on Lattices: U(1)

- **dynamical gauge fields**

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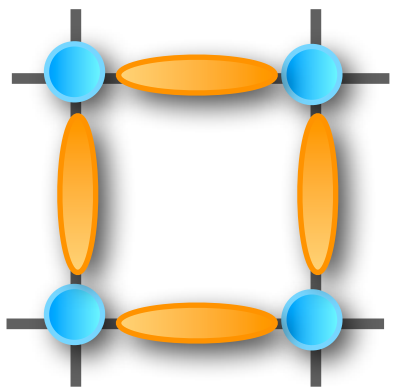
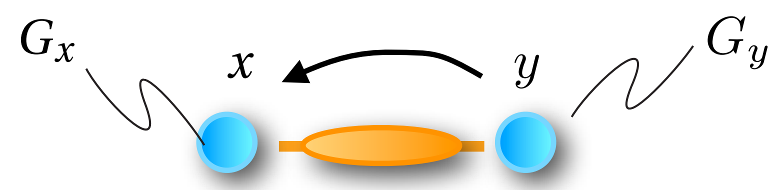




$$H = -t\psi_x^\dagger U_{xy}\psi_y + \mathrm{h.c.} + \dots$$

gauge *(local)* symmetry U(1)

gauge bosons $\quad U_{xy} \xrightarrow{V} e^{i\alpha_x} U_{xy} e^{-i\alpha_y},$

fermions $\quad \psi_x \xrightarrow{V} e^{i\alpha_x}\psi_x$

unitary trafo: $\quad V = \prod_x e^{i\alpha_x G_x}$

generator

***local* conserved quantity**

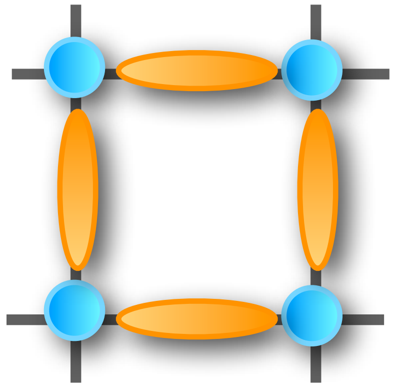
$$[H, G_x] = 0 \;\forall x$$

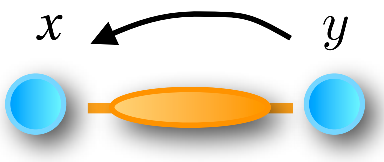
generator of gauge transformation

Static vs. Dynamical Gauge Fields on Lattices: U(1)

- **dynamical gauge fields**

particles hopping around a plaquette assisted by link degrees of freedom





$$H = -t\psi_x^\dagger U_{xy}\psi_y + \text{h.c.} + \ldots$$

Gauss Law

$$G_x = \psi_x^\dagger \psi_x - \sum_i \left(E_{x,x+\hat{i}} - E_{x-\hat{i},x} \right)$$

matter — electric field operator

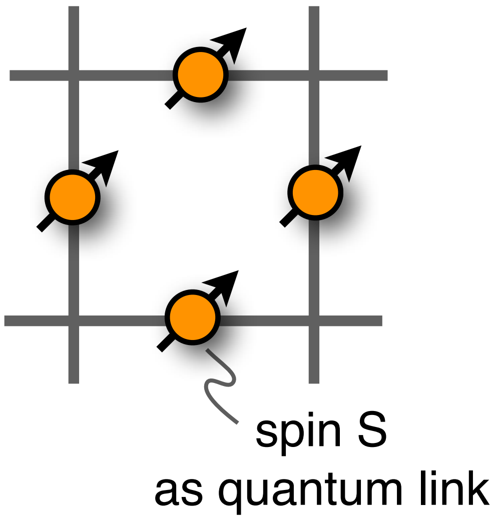
$$\rho - \nabla \cdot E = 0$$

***local* conserved quantity**

$$[H, G_x] = 0 \;\forall x$$

generator of gauge transformation

QED as "Quantum Spin Ice" [Quantum Link Model]

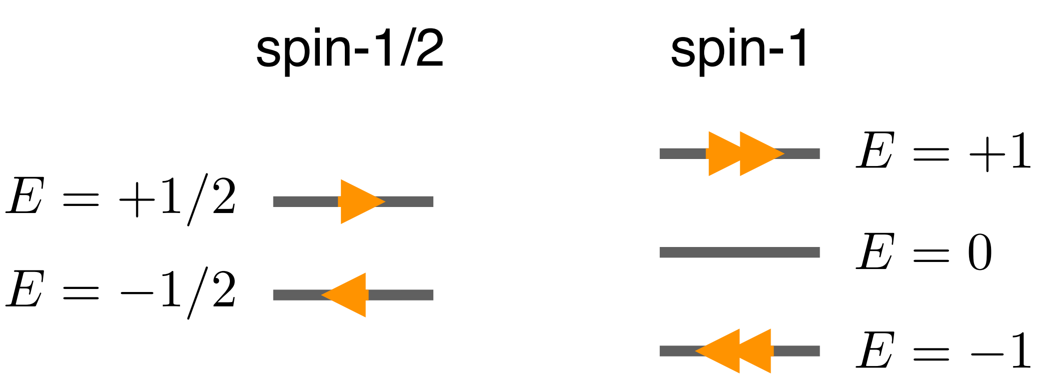


$$U_{x,x+1} \to S^{+}_{x,x+1} \qquad E_{x,x+1} \to S^{z}_{x,x+1}$$

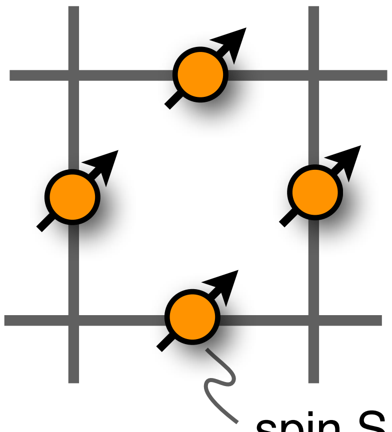
electric flux

Spin S=½,1,...

quantum link carrying an electric flux



QED as "Quantum Spin Ice" [Quantum Link Model]

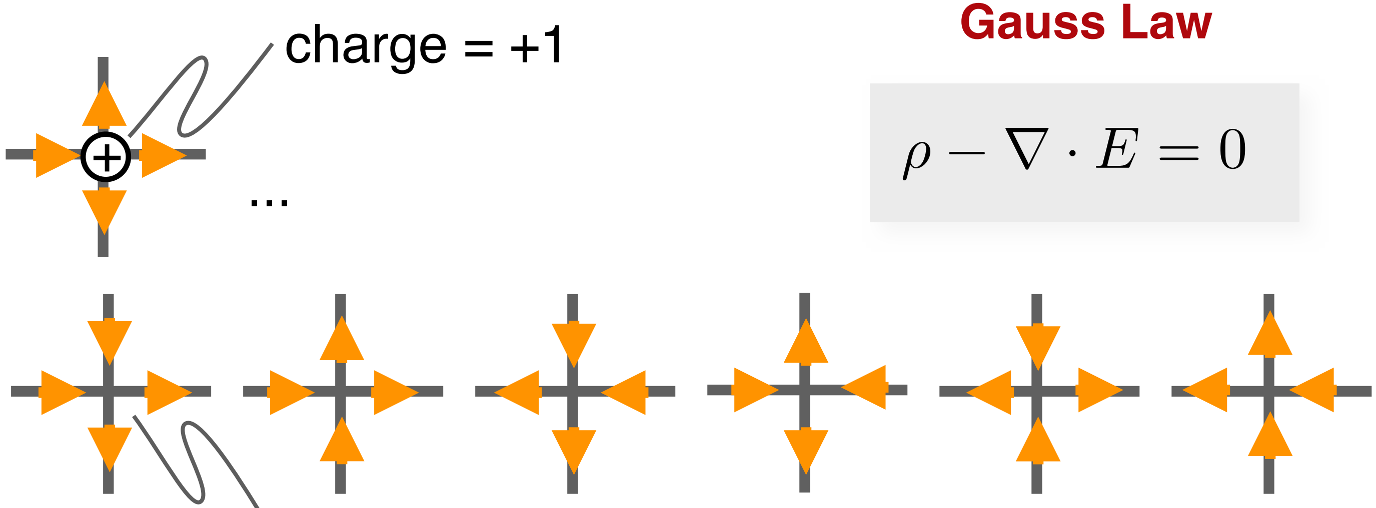


$$U_{x,x+1} \rightarrow S^{+}_{x,x+1} \qquad E_{x,x+1} \rightarrow S^{z}_{x,x+1}$$

electric flux

Spin S=½,1,...

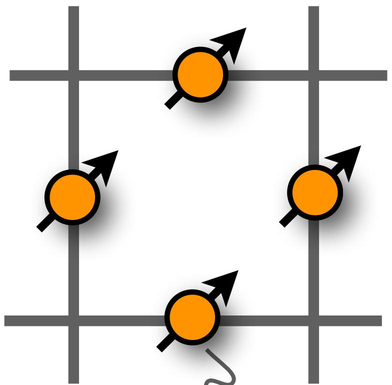
configurations: spin-1/2



Gauss Law

$$\rho - \nabla \cdot E = 0$$

QED as "Quantum Spin Ice" [Quantum Link Model]



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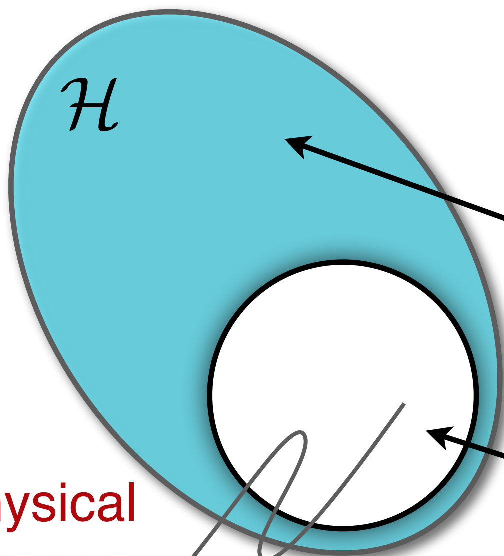
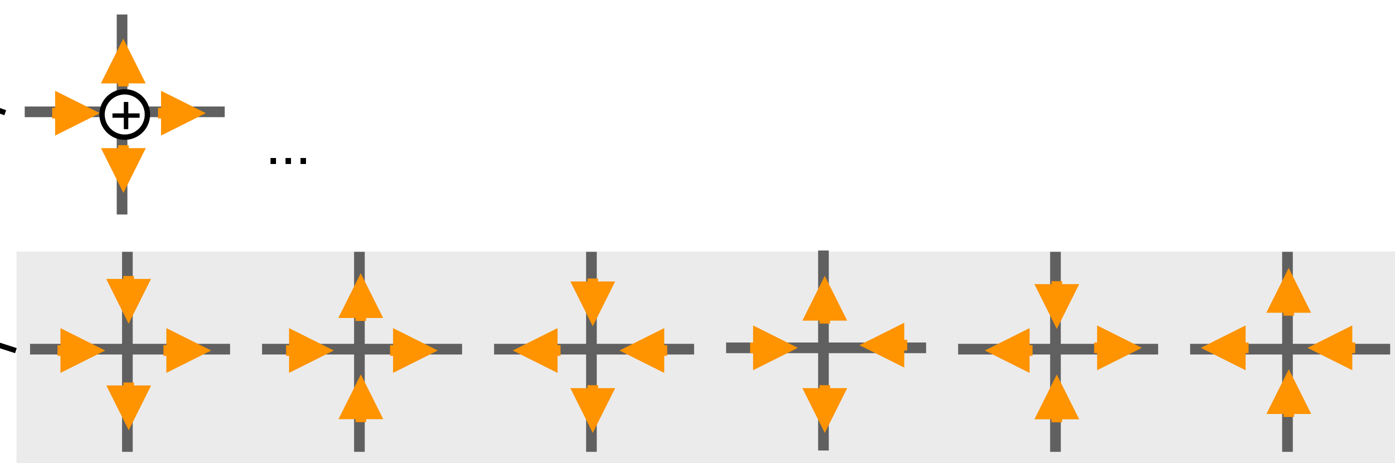


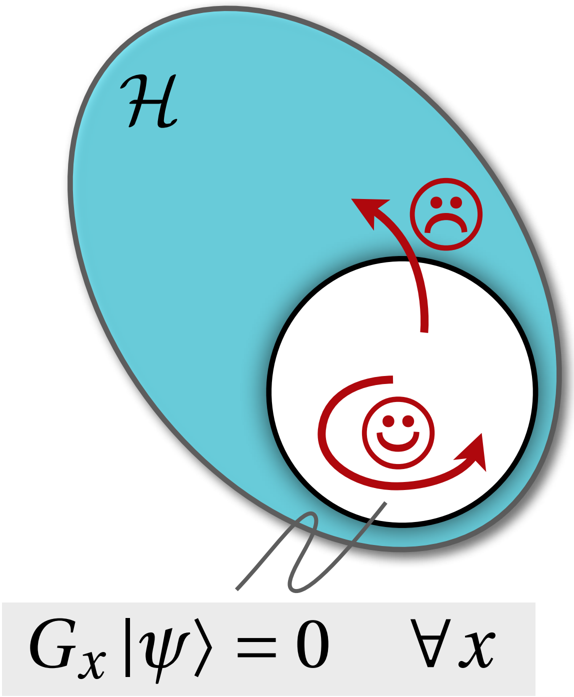
$$G_x\,|\psi\rangle = 0 \quad \forall x$$



"ice rule" $G_x = \Sigma_\mu \left(S^3_{x-\hat{\mu},\mu} - S^3_{x,\mu} \right) \leftrightarrow \nabla \cdot E = 0$

A first remark on implementation: enforcing "Gauss law"

- **Lattice Gauge Theory:** gauge symmetry fundamental
- **Implementation:** gauge symmetry approximate → protect against errors



physical gauge invariant subspace

<u>Strategies for Microscopic Implementation</u>

Hamiltonian

+ constraints $G_x\,|\psi\rangle = 0 \quad \forall x$

A first remark on implementation: enforcing "Gauss law"

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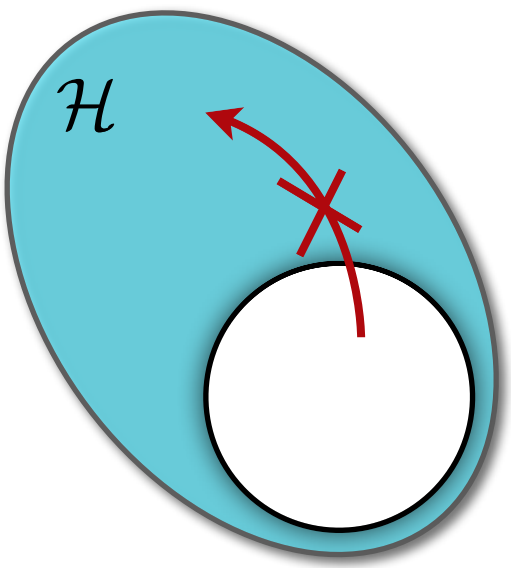
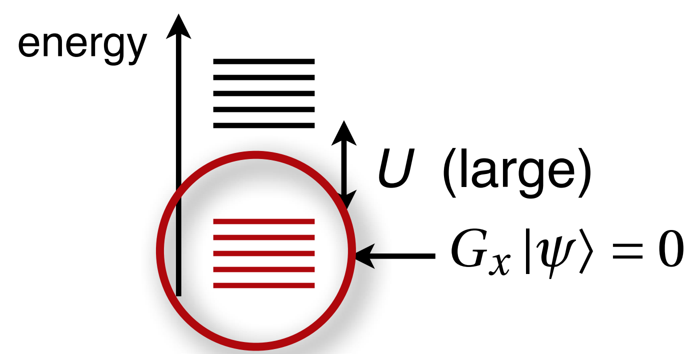


Strategies for Microscopic Implementation

1. Energy Constraints (as in cond mat)

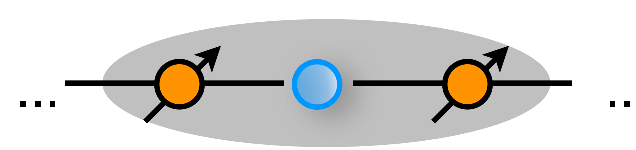
$$H_{\text{micro}} = U \sum_x G_x^2 + \ldots$$



✓interaction

✓emergent lattice gauge theory

see example below



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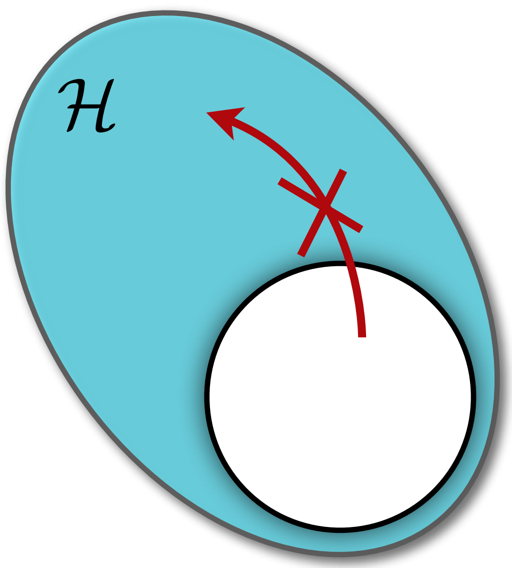


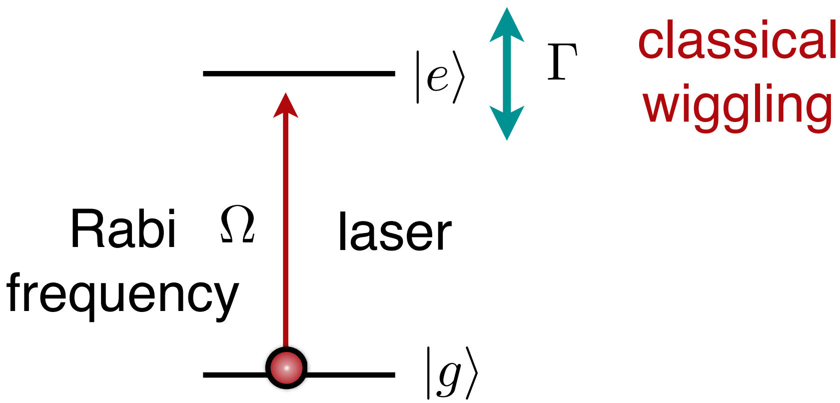
<u>Strategies for Microscopic Implementation</u>

2. “Classical Zeno effect” K. Stannigel

A toy model: “motional narrowing”

classical noise

Two-level atom +
dephasing



Fermi’s Golden Rule

$$P_g(t) = \exp\left(-\frac{\Omega^2}{\Gamma}\,t\right)$$

$\rightarrow 1$ for $\Gamma \rightarrow \infty$

population frozen in ground state

Compare “Zeno effect” by losses
in optical lattices

Rempe, Cirac et al.
Daley et al.

A first remark on implementation: enforcing "Gauss law"

- **Lattice Gauge Theory:** gauge symmetry fundamental
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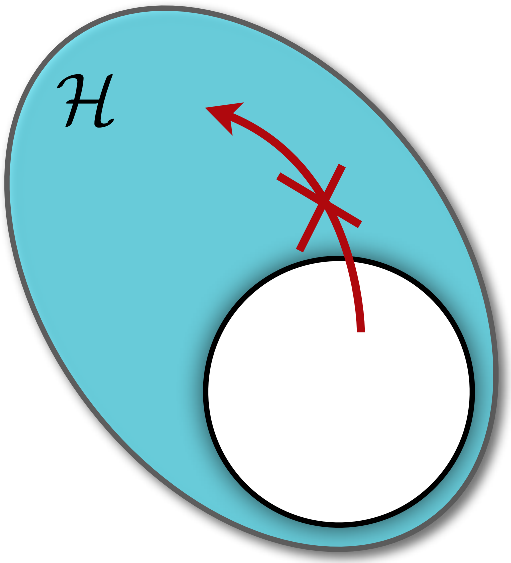


Strategies for Microscopic Implementation

2. "Classical Zeno effect" K. Stannigel

$$H_{\text{micro}} = \sum_x \xi_x(t) G_x + \dots$$

see example below

✓ white noise

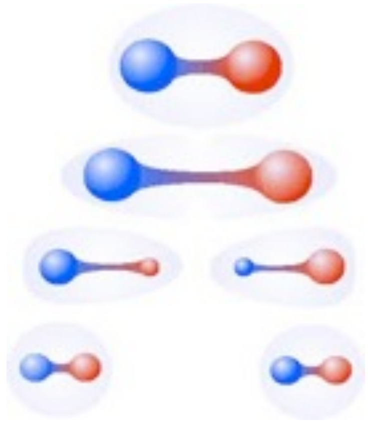
✓ *linear* in G ~ *single* particle terms ☺

Rem.: decoherence free subspace, bang-bang

Example 1:

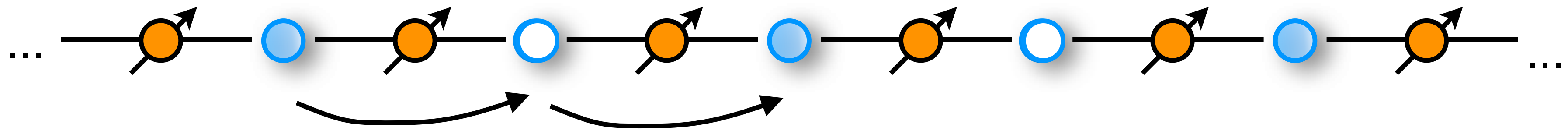
The simplest (meaningful) quantum link model:
1D Schwinger model

AMO Implementation:
Bose-Fermi Mixtures in Optical Lattices



string breaking

Boson (gauge field)



Fermion (quark)

“quantum spin ice coupled to matter”

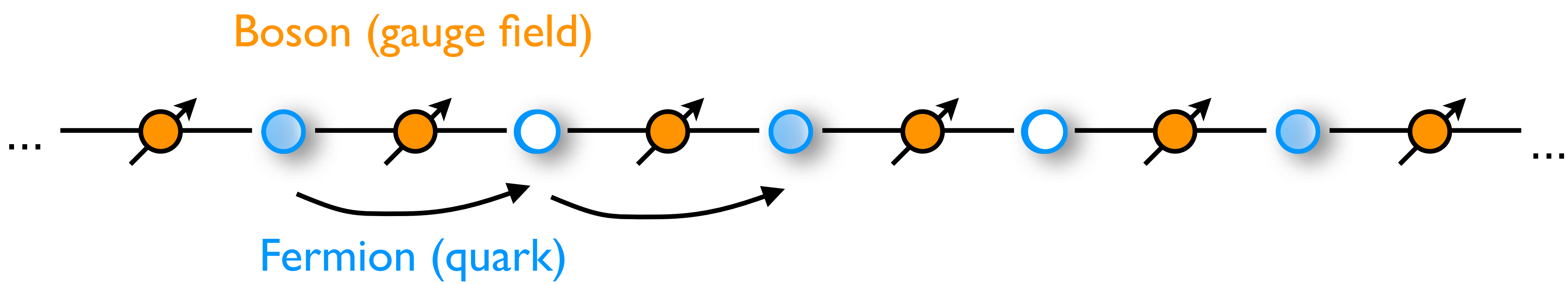
D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U. J. Wiese, and P.Z., PRL 2012

Schwinger Model: U(1) fermions + gauge bosons in 1D

- **Hamiltonian:** staggered fermions in 1D coupled to quantum link spin S

$$H = \frac{g^2}{2}\sum_x E^2_{x,x+1} - t\sum_x \left[\psi^\dagger_x U_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi^\dagger_x \psi_x$$

electric flux | hopping | staggered fermions



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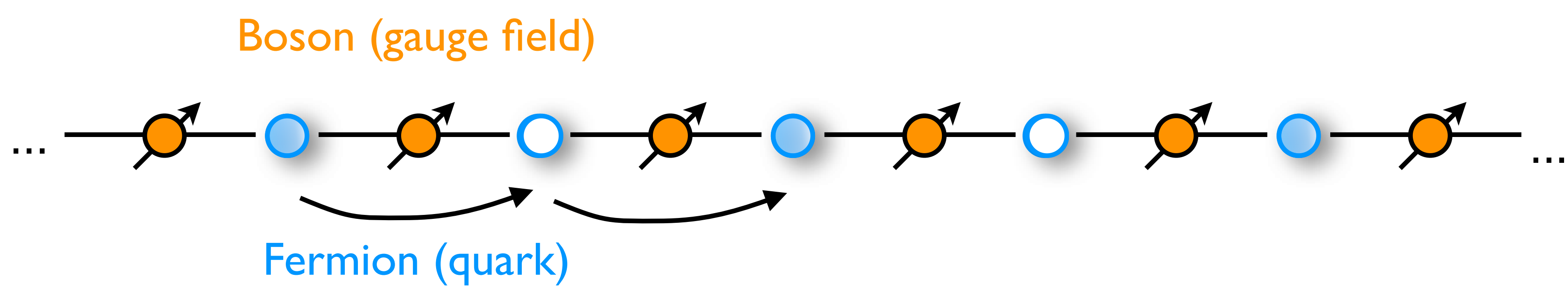
electric flux — hopping — staggered fermions



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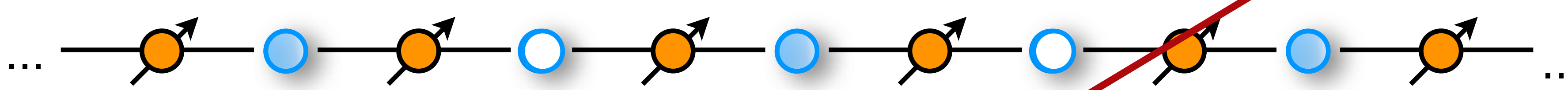
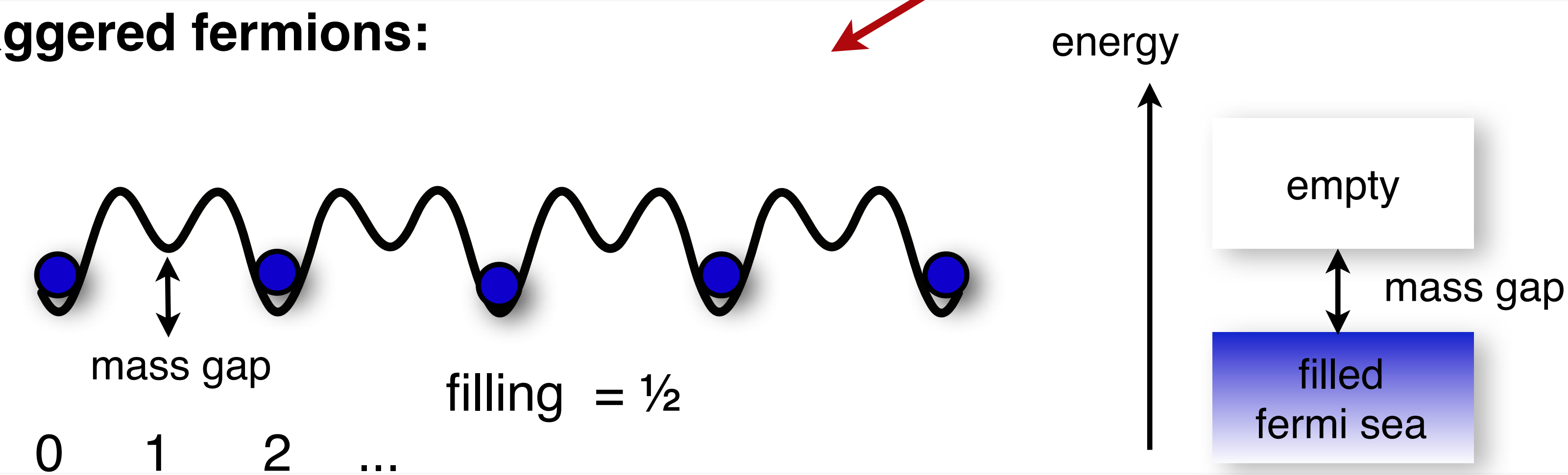


Staggered fermions:



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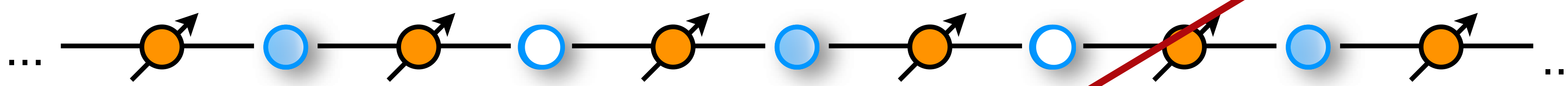
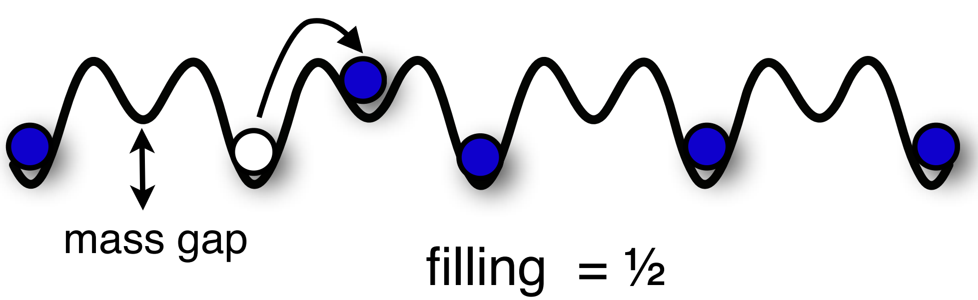
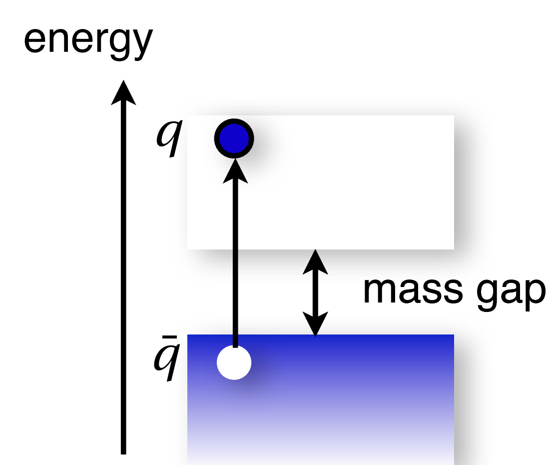
electric flux   hopping   staggered fermions



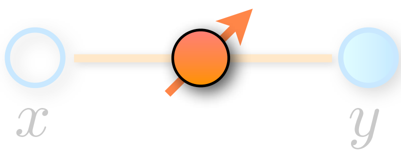
Staggered fermions:





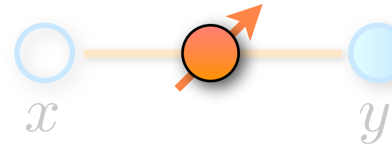
Implementation with Atoms: Hamiltonian

- **building blocks**



Spin S=½,1,...

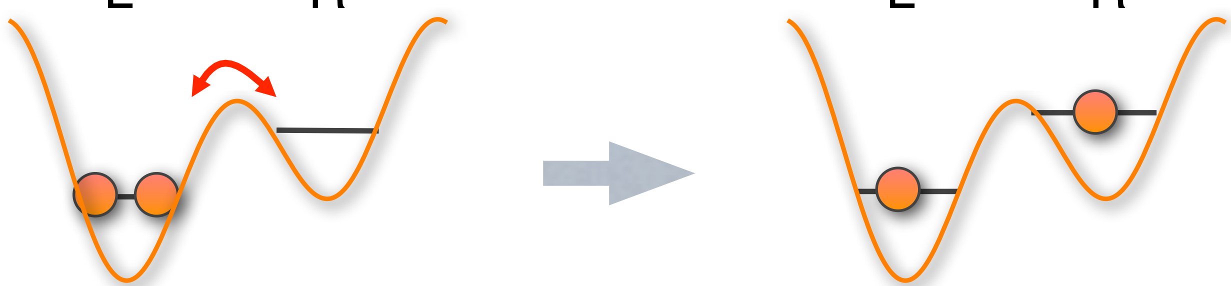
Implementation with Atoms: Hamiltonian



Spin S=½,1,...

- **Spin as Schwinger bosons**

bosons



N bosons in double well (S=N/2)

$$S^{+}_{x,y} = b^{\dagger}_{L} b_{R}$$

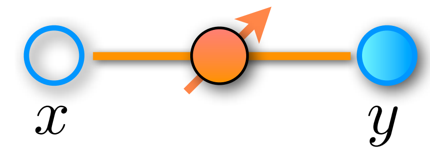
$$S^{z}_{x,y} = \frac{1}{2}\left(b^{\dagger}_{R} b_{R} - b^{\dagger}_{L} b_{L}\right)$$

Hamiltonian

$$h_B = -t_B(S^{+}_{x,y} + \mathrm{h.c.}) + U_B(S^{z}_{x,y})^2$$

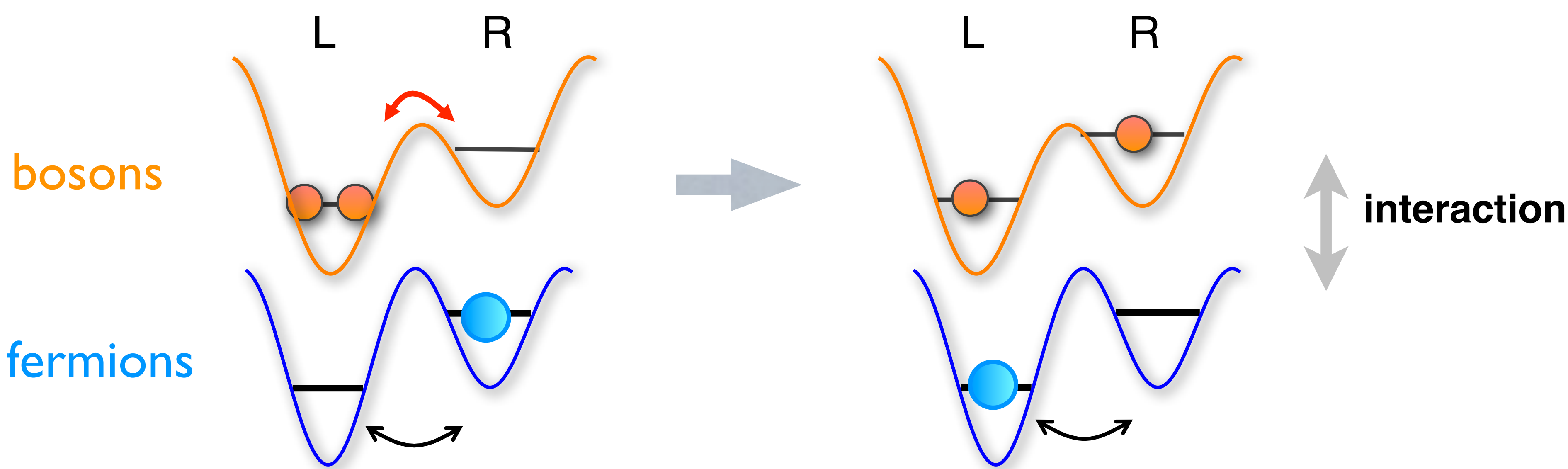
hopping electric energy

Implementation with Atoms: Hamiltonian

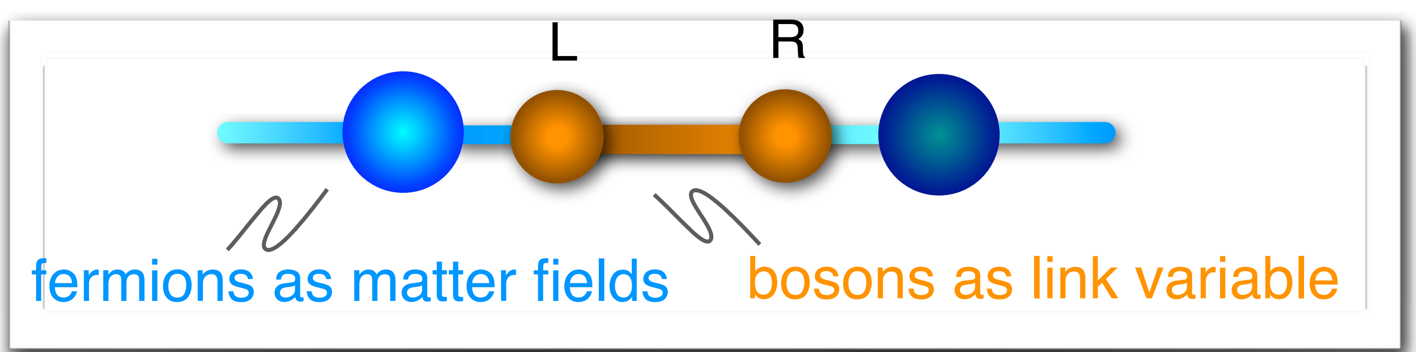


$$H = -t\psi_x^\dagger S_{xy}^+ \psi_y + \text{h.c.}$$

- **correlated hopping**



$$H = -\frac{t_B t_F}{U}\, \psi_x^\dagger\, b_R^\dagger b_L\, \psi_y + \text{h.c.}$$



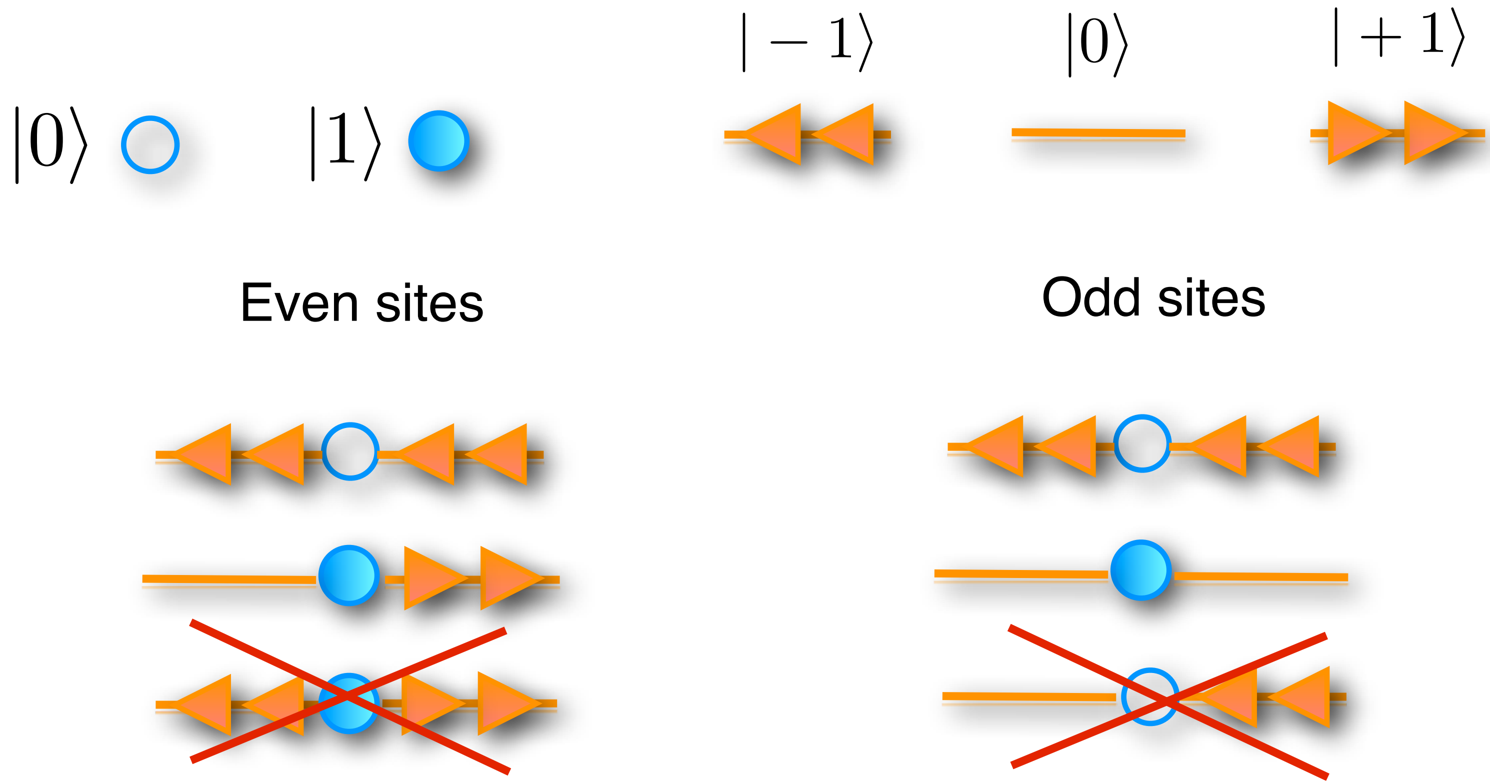
Gauss Law (as a constraint)

- **Gauss' law**

$$G_x = \psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2} - (E_{x,x+1} - E_{x-1,x})$$

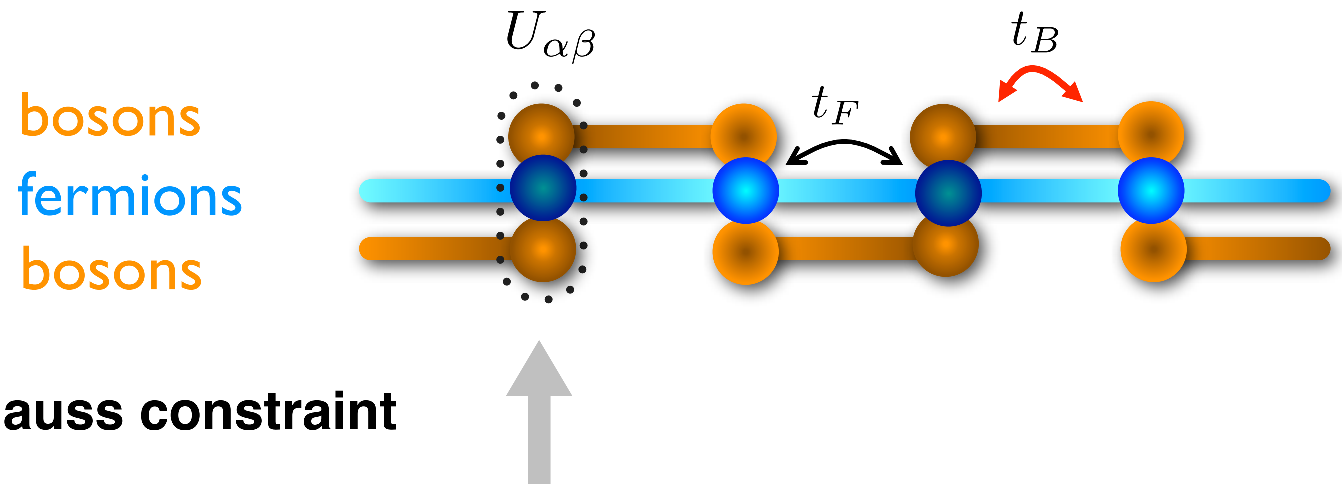
$$G_x \,|\, \text{physical states}\rangle = 0 \;\longleftrightarrow\; \rho - \nabla \cdot E = 0$$

- **Example: Spin 1 representation**



Implementation with Atoms: Gauss Constraints

- **Bose-Fermi mixtures in superlattices**



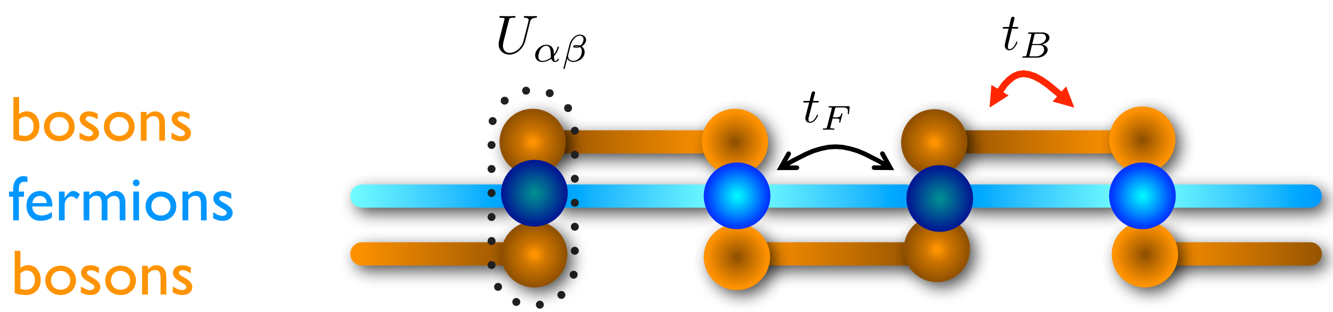
- **Gauss constraint**

$$\tilde{G}_x = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}\left[(-1)^x - 1\right]$$

~ total number of atoms on site *x* fixed: "super-Mott insulator"

Implementation with Atoms: Gauss Constraints

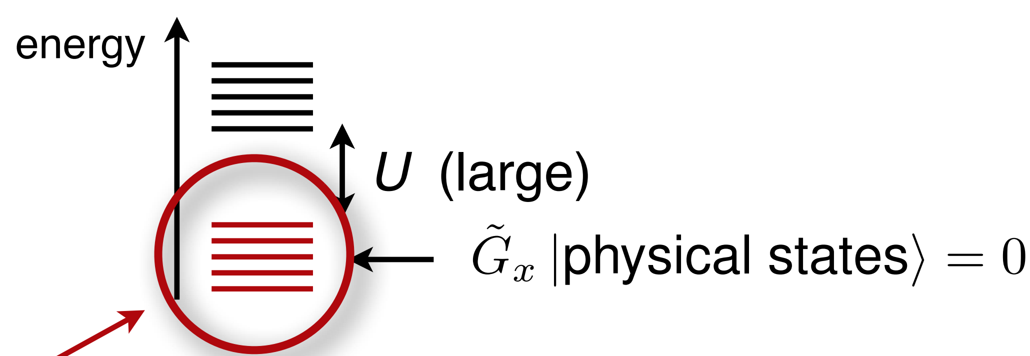
- **Bose-Fermi mixtures in superlattices**



- **enforcing the Gauss Law as an *energy constraint***

$$H_{\text{microscopic}} = U \sum_x \tilde{G}_x^2 + \ldots$$

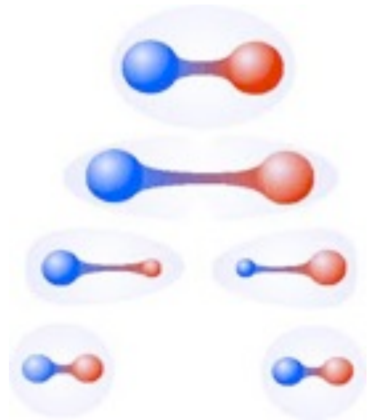
Bose + Fermi Hubbard model



- emergent lattice gauge theory

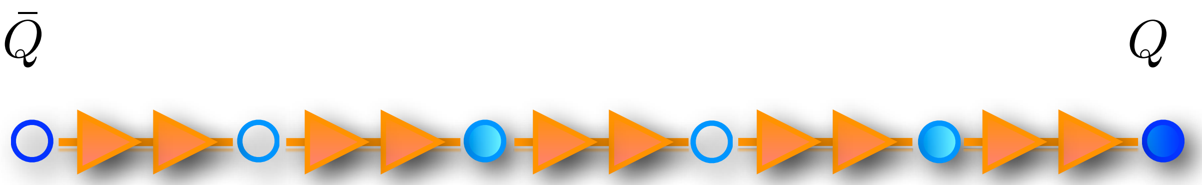
- dynamics in physical subspace: analogous to t-J model
- we have verified the reduction: microscopic to the quantum link model at the few- and many-body level

String breaking and confinement



string breaking

Unbroken String



$$E_{\text{string}} - E_0 = \frac{g^2}{2}(L-1)$$

Mesons + vacuum

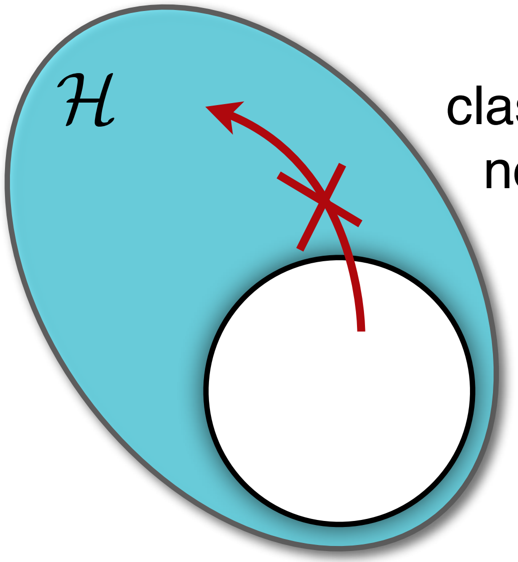
$$E_{\text{mesons}} - E_0 = 2\left(\frac{g^2}{2} + m\right)$$

Critical string length $L^{(c)} = 2 + 2m/g^2$

We have verified this dynamics in the microscopic model

Rem.: string breaking 1D vs. 3D / non-Abelian

Example 2:



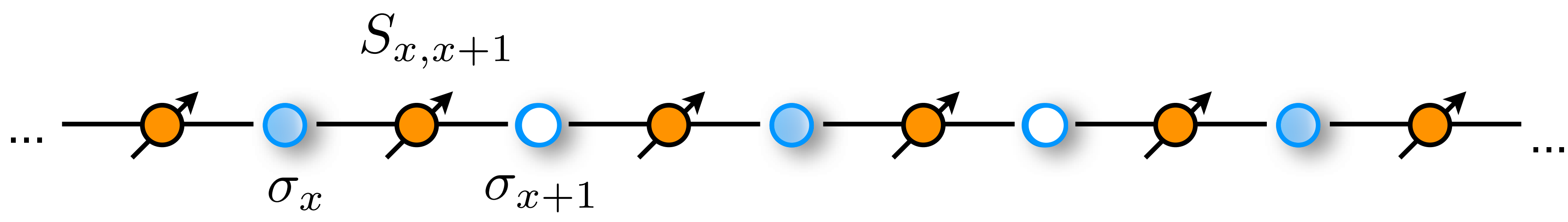
Enforcing Gauss' Law by Classical Noise

K. Stannigel

K. Stannigel

1D Schwinger Model: Fermions → Spins

- **Hamiltonian**



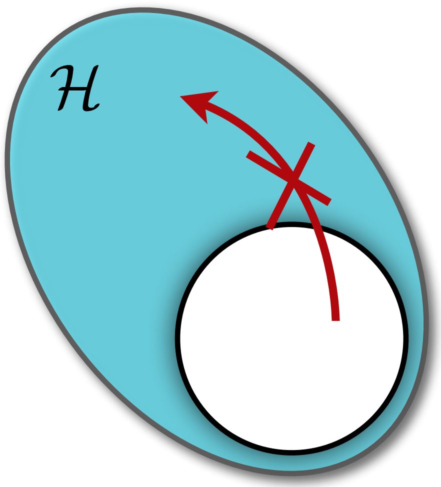
$$H_0 = -J \sum_{x=1}^{3} \left(\sigma_x^- S_{x,x+1}^- \sigma_{x+1}^+ + \text{h.c.}\right) + m(t) \sum_{x=1}^{4} (-1)^x \sigma_x^z$$

- **Gauss Law**

$$G_x = S_{x-1,x}^z + \sigma_x^z - S_{x,x+1}^z + \frac{1}{2}(-1)^x$$

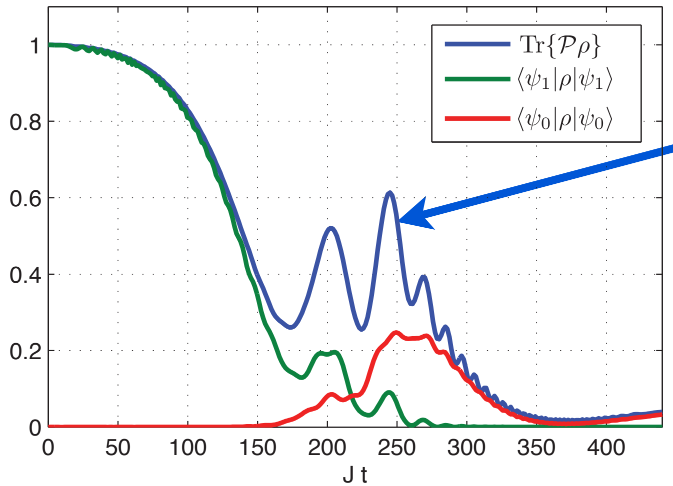
- **We add a *gauge-variant* term to the Hamiltonian ...**

$$H_1 = \lambda \sum_{x=1}^{3} (\sigma_x^- \sigma_{x+1}^+ + \text{h.c.})$$



imperfection

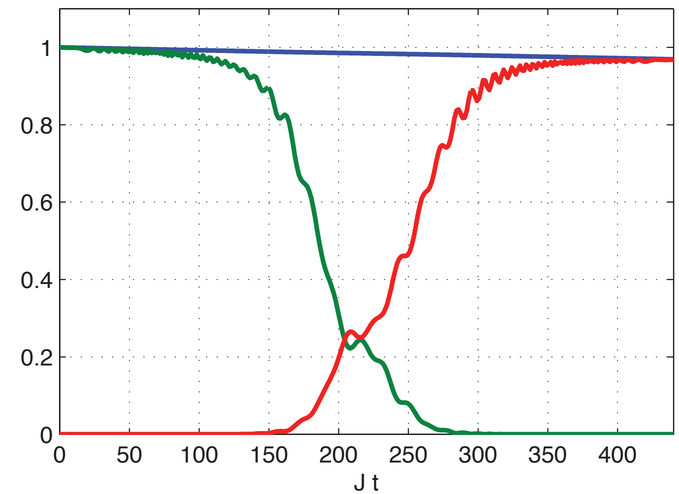
$\lambda/J = 0.2$



system leaves Gauge invariant subspace

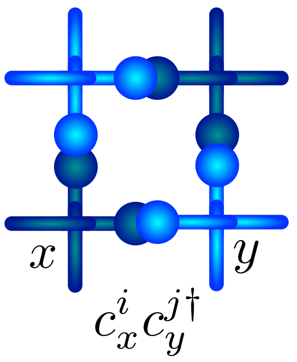
noise imperfection

$\kappa/\lambda = 100$



- **… and *suppress it by noise***

K. Stannigel



Non-Abelian U(N) and SU(N) Quantum Link Models with Fermionic Atoms

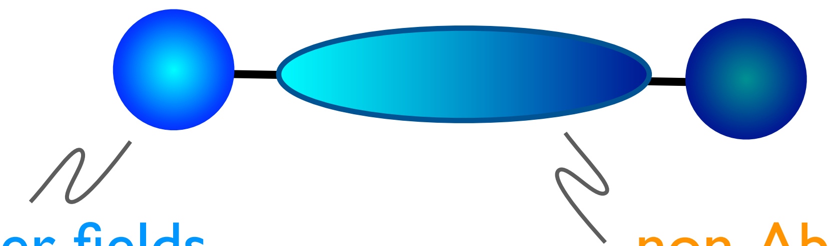
… a few basic aspects

D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U. J. Wiese, & PZ,
PRL in print.

[Innsbruck - Bern]

Non-Abelian Lattice Gauge Theory

- **Example: U(N), SU(N), ...**



fermions as matter fields

$$\psi_x^i \quad (i = 1, \ldots, N)$$

(color)

non-Abelian gauge fields

$$U_{x,y}^{ij}$$

- NxN unitary matrices
- generators

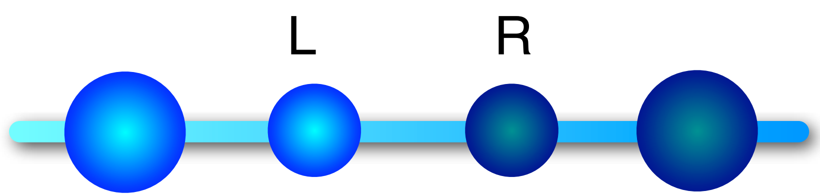
$$\lambda_{ij}^a \quad (a = 1, \ldots N^2 - 1)$$

$$[\lambda^a, \lambda^b] = i2 f_{abc} \lambda^c$$

$$H = -t \sum_{i,j=1}^{N} \psi_x^{i\dagger} U_{xy}^{ij} \psi_y^j + \text{h.c.} + \ldots$$

Non-Abelian Lattice Gauge Theory

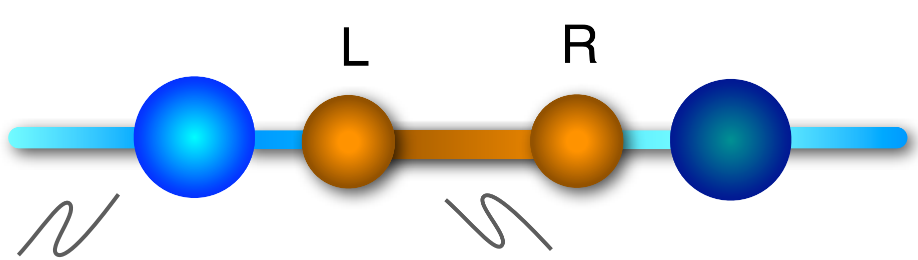
- **Quantum Link Models**



$$U^{ij} = c_R^i c_L^{j\dagger} \qquad (i = 1, \ldots, N)$$

representation as *fermionic* rishons

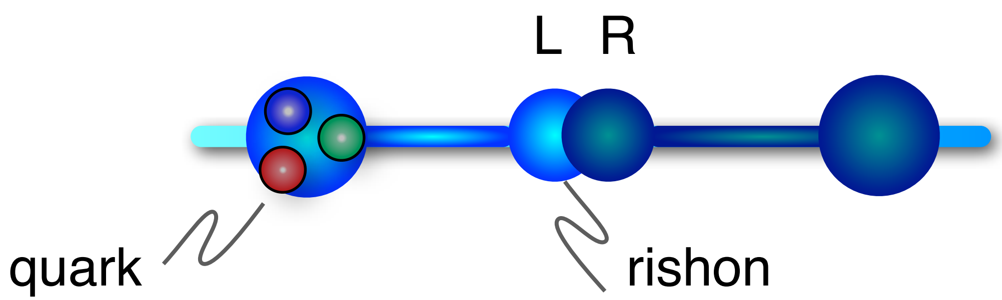
- **Abelian U(1)**



fermions as matter fields bosons as link variable

Non-Abelian Lattice Gauge Theory

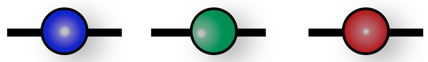
- **Quantum Link Models**



$$H = -t \left(\sum_{i=1}^{N} \psi_x^{i\dagger} c_R^i \right) \left(\sum_{j=1}^{N} c_L^{j\dagger} \psi_y^j \right) + \text{h.c.}$$

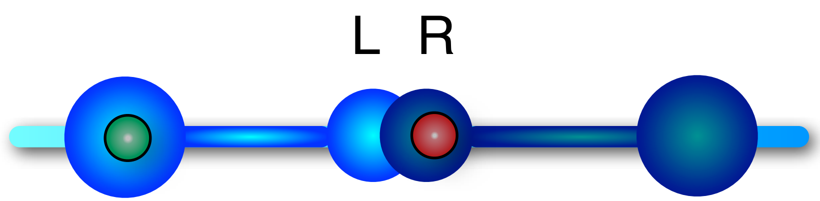
(separable interaction ☺)

Implementation

✓ fermionic atoms with N internal states (color) 

Non-Abelian Lattice Gauge Theory

- **Quantum Link Models**



$$H = -t \left(\sum_{i=1}^{N} \psi_x^{i\dagger} c_R^i \right) \left(\sum_{j=1}^{N} c_L^{j\dagger} \psi_y^j \right) + \text{h.c.}$$

(separable interaction ☺)

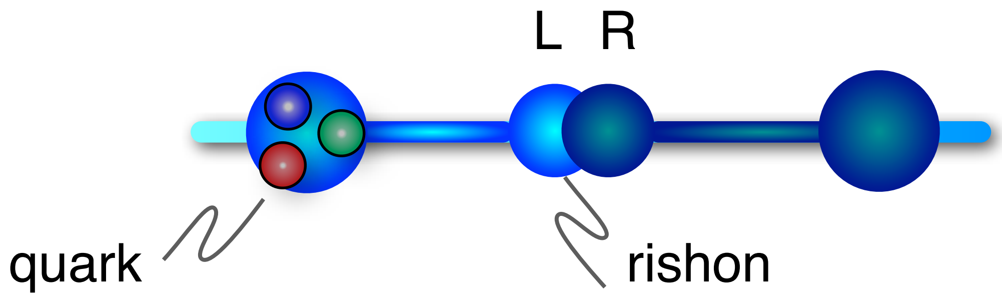
Dynamics

✓ fermionic atoms converts itself from quark to rishon & vice versa
✓correlated hop

Bern
Innsbruck

Non-Abelian Lattice Gauge Theory

- **Quantum Link Models**



$$H = -t \left(\sum_{i=1}^{N} \psi_x^{i\dagger} c_R^i \right) \left(\sum_{j=1}^{N} c_L^{j\dagger} \psi_y^j \right) + \text{h.c.}$$

(separable interaction ☺)

“nuclear and particle physics”

$\psi^a g^a$ nucleon

$g^a g^a$ glueball

$\psi^{a\dagger} \psi^a$ meson

$\psi g \psi$ deuteron

$\psi\psi\psi$ triton

We know how to implement gauge groups SU(2), SU(3), SO(3), ...
[at the moment in 1D]

PRL in print

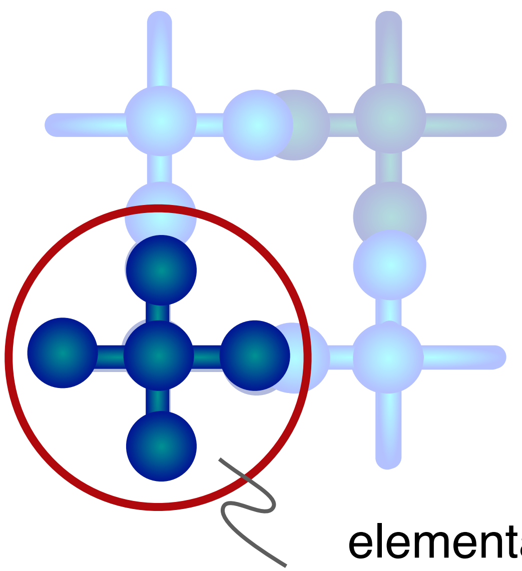
Implementation: enforcing Gauss Law / *local* symmetry

- **strategy 3:**

U(N),SU(N) Non-Abelian LGT

implementation

fundamental local symmetry / conservation law



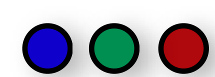
M. Dalmonte

elementary building block

local particle # conservation

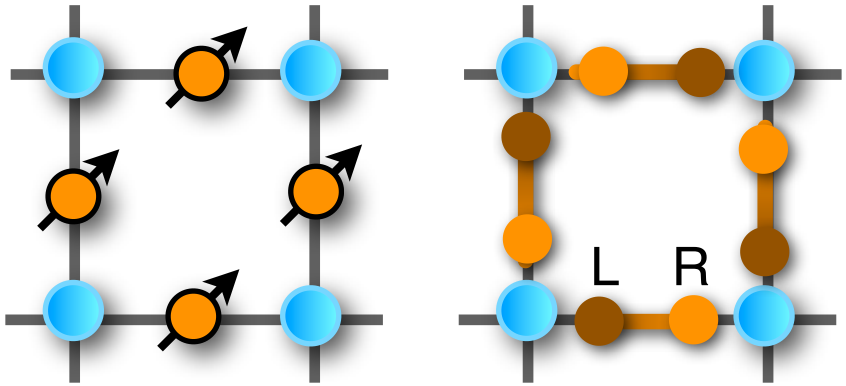
(implemented with high precision)

fermions



Summary

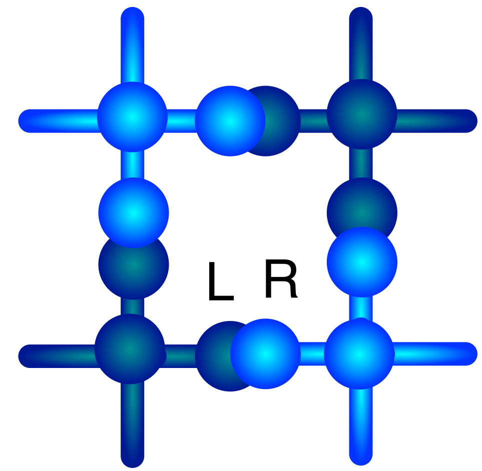
U(1) Abelian LGT



atomic boson-fermi mixtures

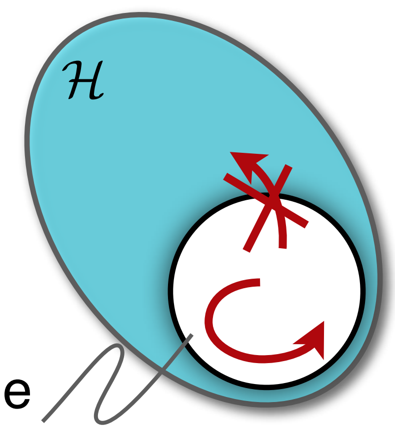
D. Banerjee et al., PRL in print

U(N),SU(N) Non-Abelian LGT



multi-species fermi gases

D. Banerjee et al., PRL 2012



physical gauge invariant subspace

Hamiltonian + Gauss constraint:

1.Energy constraints

2.Classical Noise

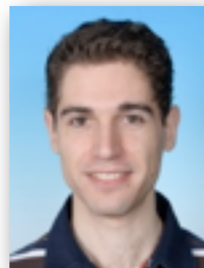
3.Microscopic Hamiltonian *is* gauge invariant

We are on our way to find simpler, and thus more realistic implementations

The Collaboration

- **IQOQI - Innsbruck University**






M. Dalmonte E. Rico D. Marcos K. Stannigel

- **Ibk → Madrid**

M. Müller

- **Albert Einstein Center - Bern University**

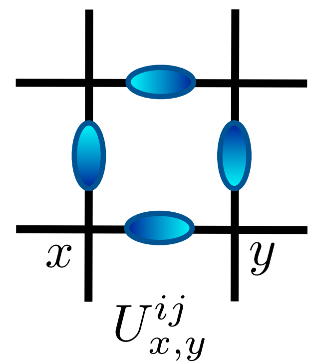


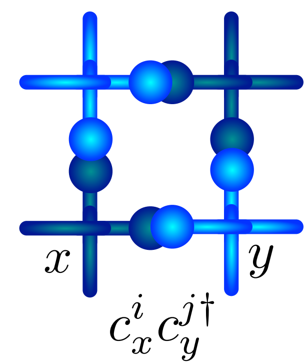


D. Banerjee M. Bögli P. Stebler P. Widmer U.-J. Wiese

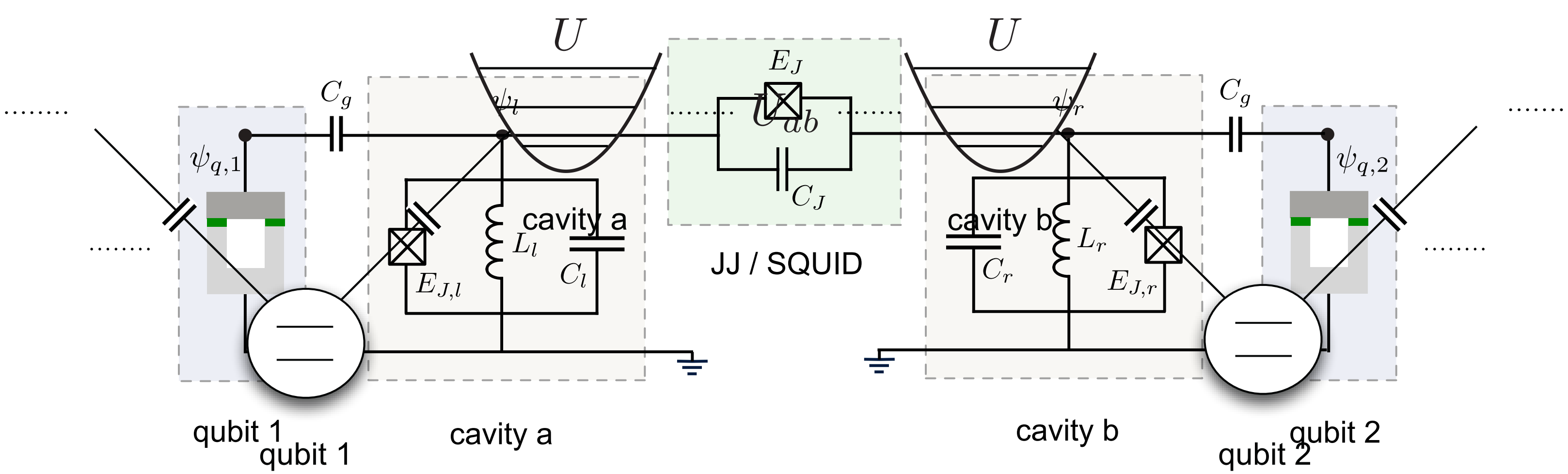
Outlook

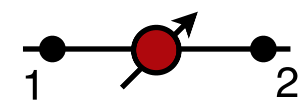
- **gauge fields in 2D, 3D**





- **superconducting qubits**





D. Marcos et al., unpublished

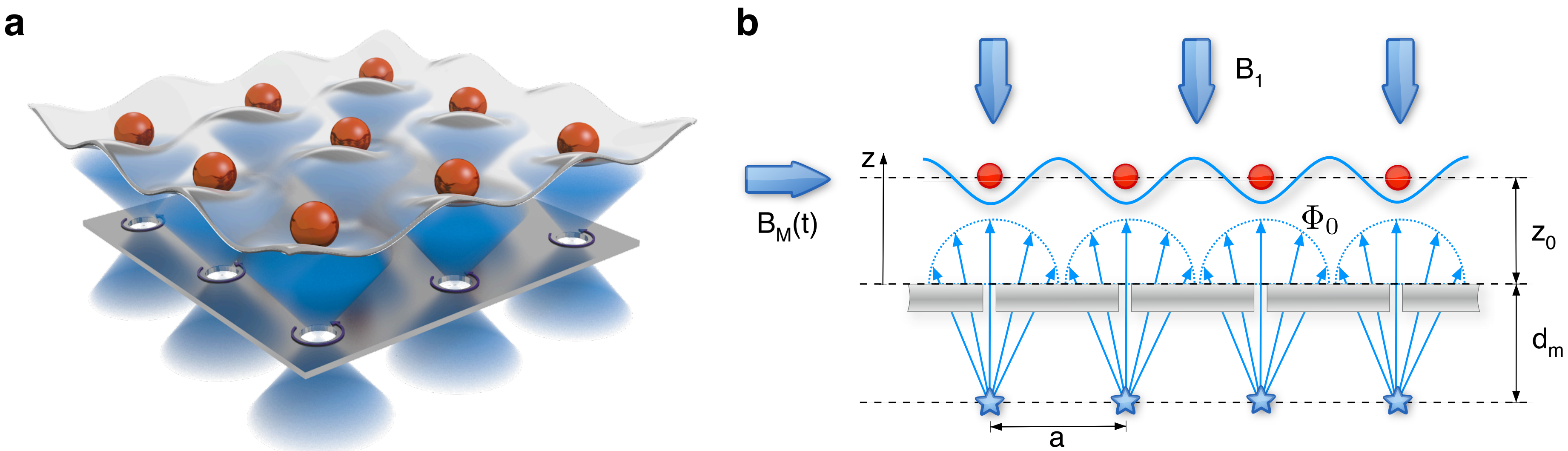
- **Is there life after optical lattices ...**

Nano-Scale Trap Arrays from Superconducting Vortices

O. Romero-Isart, C. Navau, A. Sanchez, P. Zoller, and J. I. Cirac, arXiv 2013

Oriol Romero-Isart

- **Pinned vortices in type-II superconductors as magnetic trap arrays**
  - lattice spacings down to 50 nm
  - nano-structuring of surfaces & holes



Superconducting Vortex Lattice for Atoms

Mesocopic size magnetic / superconducting chip traps (Zimmermann, Dumke, Haroche, …)