

Coherent control as a springboard for quantum gate design

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Plan

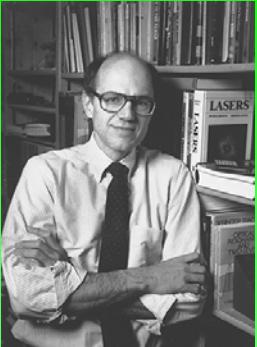
- Overview of Coherent Control
- Entanglement of Two Qubits
- Fast Single Qubit Gates
- Conclusions

History/Methods of Coherent control

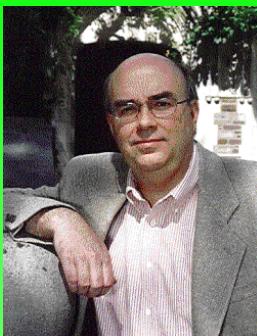
- Control of chemical reactions
- Weak field control
- Control by initial state preparation
- Pump-dump control
- Coherent control using interference of several pathways (relative phase control)
- Strong field control
 - a) Rabi oscillation
 - b) adiabatic passage
 - c) STIRAP
- Local coherent control
- Global Optimal control
- ?



David Tannor



Joe Eberly



Warren Warren



Philip Bucksbaum



Paul Brumer



André D. Bandrauk



Klaas Bergmann



Moshe Shapiro



Ahmed Zewail



Hersch Rabitz



Ronnie Kosloff



Robert Gordon



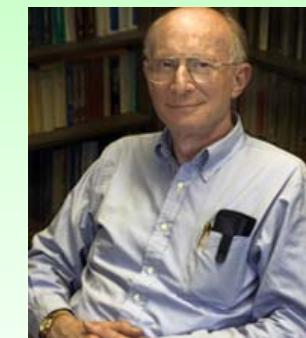
Kent R. Wilson



Jeff Krause



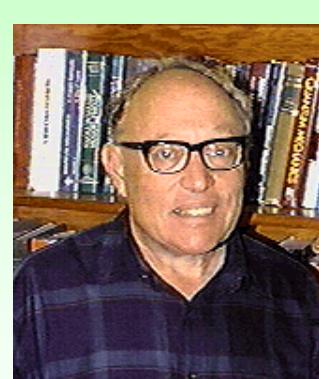
Marcos Dantus



Stuart Rice



Gustav Gerber



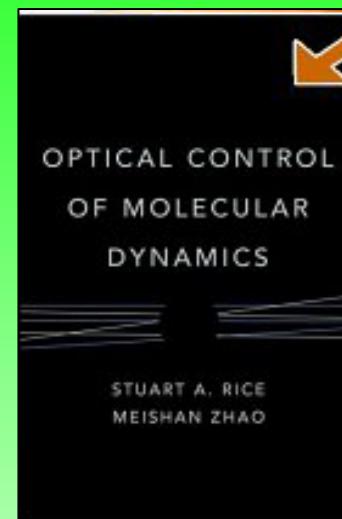
Stephen E. Harris



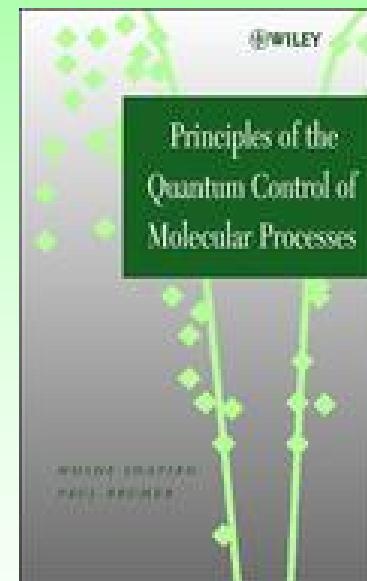
Yaron Silberberg

Textbooks

Rice / Zhao
Optical Control of Molecular Dynamics,
Wiley 2000

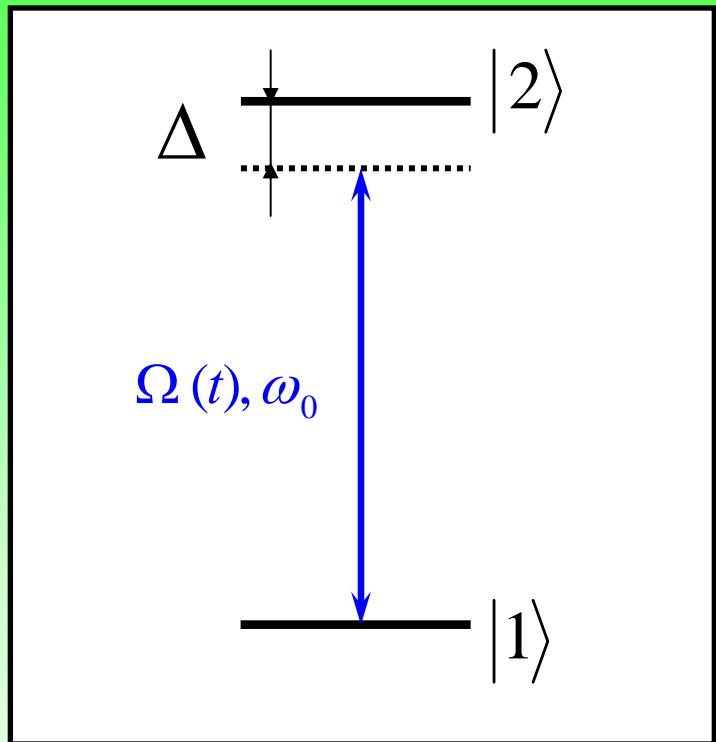


Brumer / Shapiro
Principles of the Quantum Control
of Molecular Processes
Wiley 2003

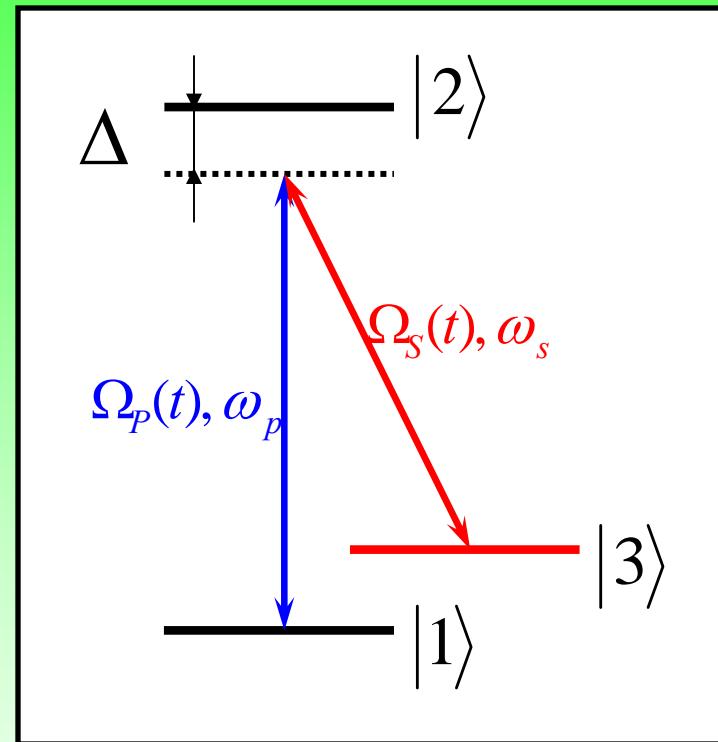


◊ Overview of coherent control

two-level system



three-level system



$$\Omega_{ij}(t) = \mu_{ij} E_k(t) / \hbar \quad \text{Rabi frequency}$$

$$\Delta = (E_2 - E_1) / \hbar - \omega_0 \quad \text{detuning}$$

$$E_k(t) = E_{k0}(t) \cos(\omega_0 t + \phi_0) \quad \text{e.-m. field}$$

•Time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = \hat{H}(t) \Psi(t)$$

$$\hat{H}(t) = -\frac{\hbar}{2} \begin{pmatrix} 0 & \Omega(t) \\ \Omega(t) & 2\Delta \end{pmatrix}$$

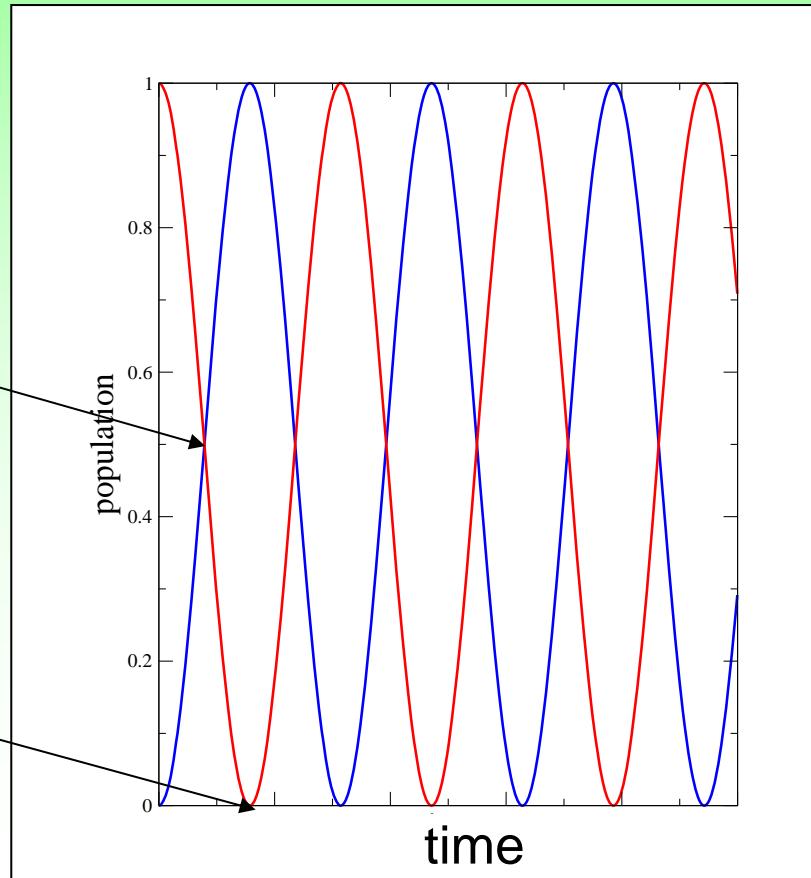
$$\Psi(t) = a_1(t)|1\rangle + a_2(t)|2\rangle$$

total wave function

$\pi/2$ -pulse

•Rabi oscillations

π -pulse



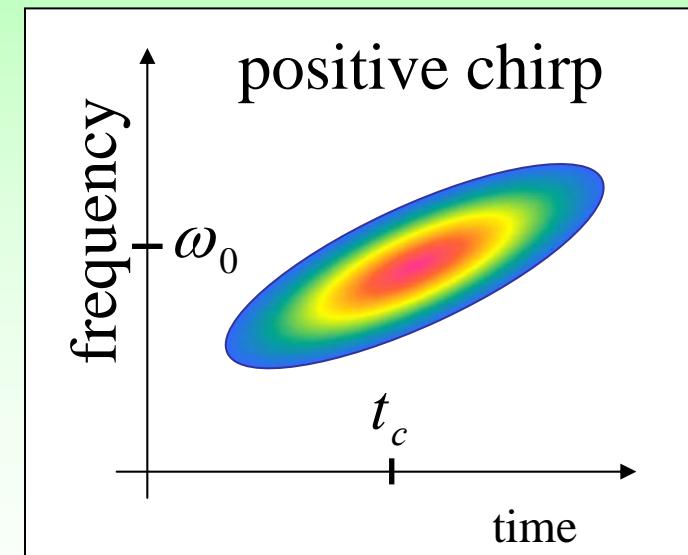
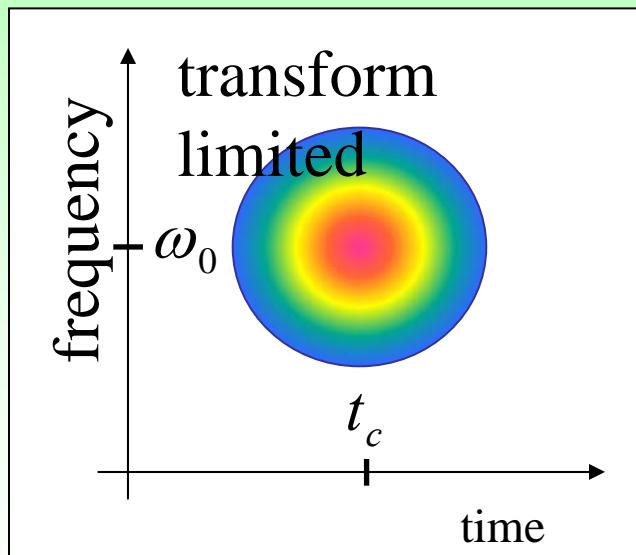
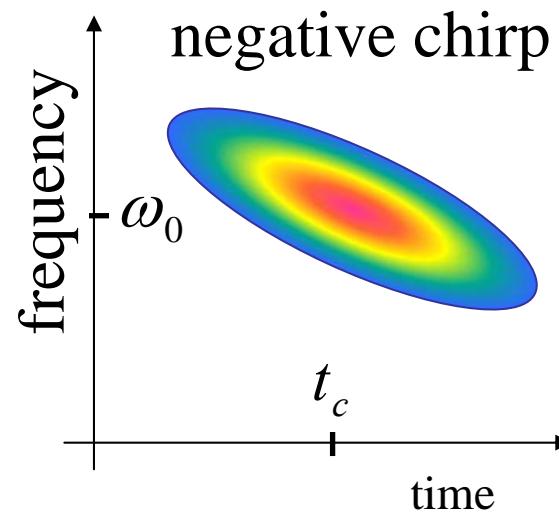
I.I. Rabi
Nobel Prize (1944)

Adiabatic passage; Chirped pulses

$$\varphi(t) = \varphi_0 + \omega_0(t - t_c) + \frac{\alpha(t - t_c)^2}{2} + \frac{\beta(t - t_c)^3}{6} + \dots$$

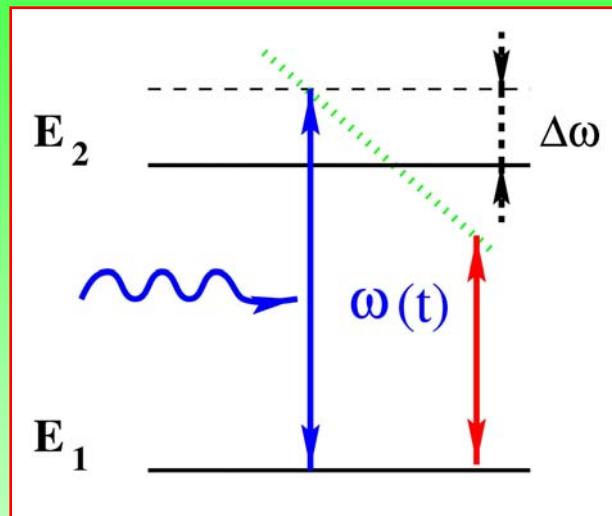
$$\omega(t) = \frac{\partial}{\partial t} \varphi(t) = \omega_0 + \alpha(t - t_c) + \frac{\beta(t - t_c)^2}{2} + \dots$$

- Husimi plots

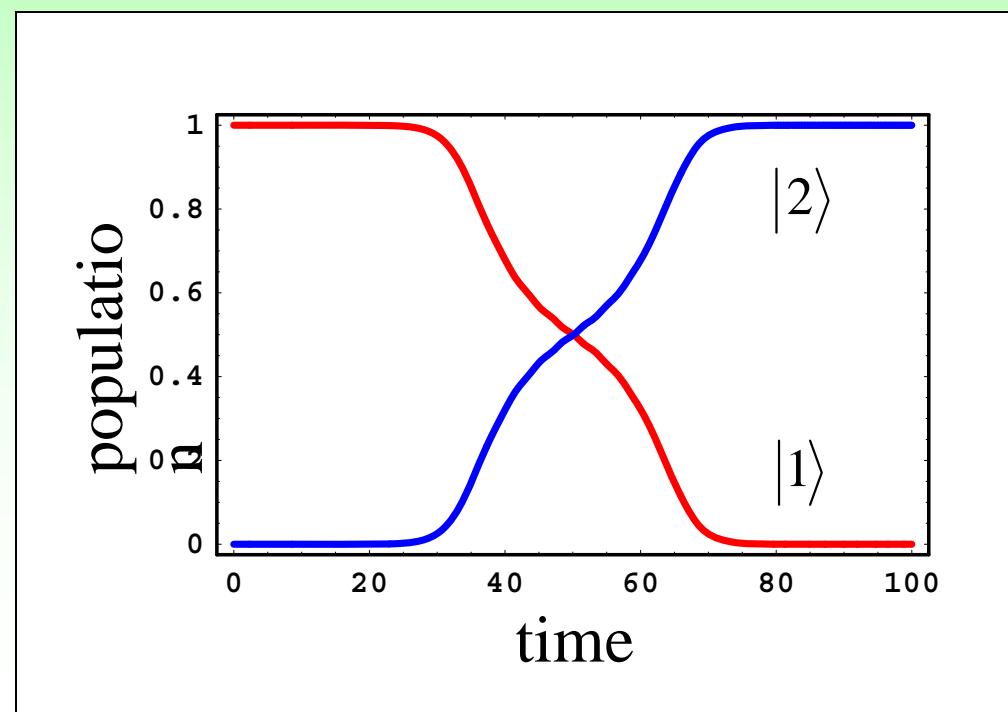
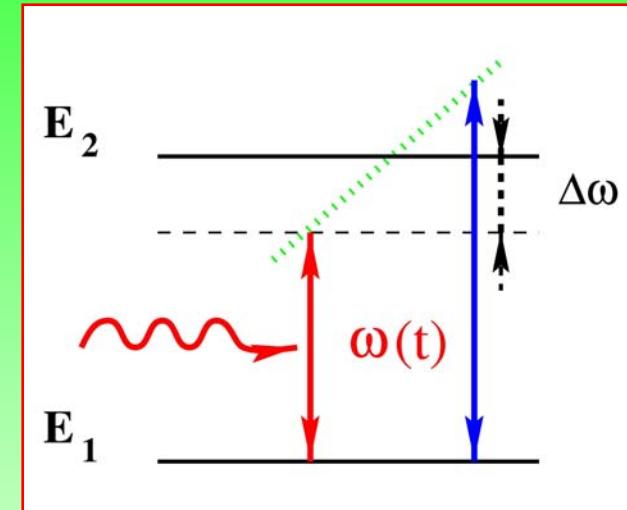


•Adiabatic passage

Landau-Zener-Stueckelberg formula



$$P_2 \approx 1 - \exp \left\{ -\pi \frac{\Omega_0^2}{2\alpha} \right\}$$



$$\frac{\Omega_0^2}{2|\alpha|} \gg 1$$

Adiabatic Basis, dressed states

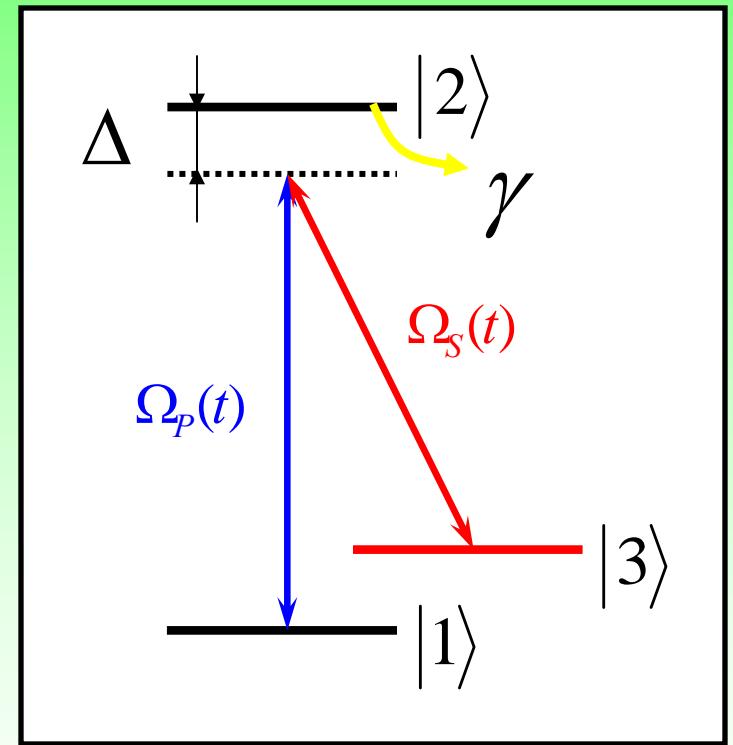
- Time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = \hat{H}(t) \Psi(t)$$

$$\Psi(t) = a_1(t)|1\rangle + a_2(t)|2\rangle + a_3(t)|3\rangle$$

In the rotating wave approximation(RWA)

$$\hat{H}(t) = -\frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_P(t) & 0 \\ \Omega_P(t) & 2\Delta & \Omega_S(t) \\ 0 & \Omega_S(t) & 0 \end{pmatrix}$$



•Counterintuitive pulse sequence

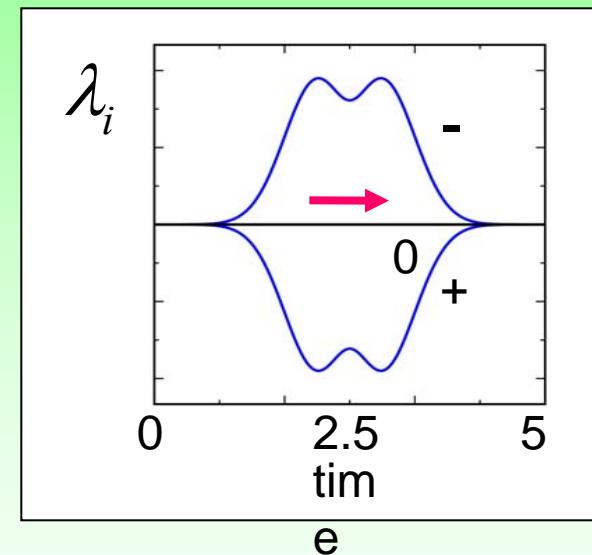
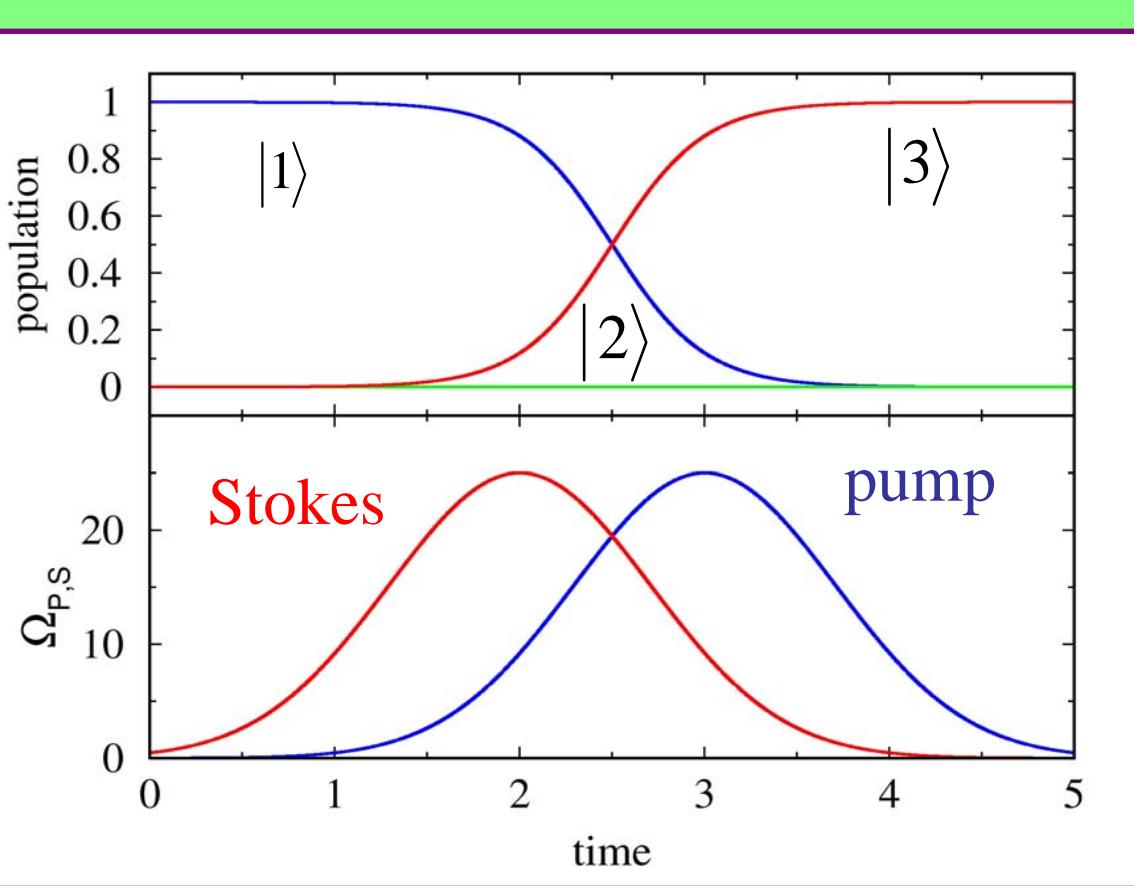
$$|c^0\rangle = \cos \theta |1\rangle - \sin \theta |3\rangle$$

$$\cos \theta = \frac{\Omega_S(t)}{\Omega_e(t)}$$

$$\sin \theta = \frac{\Omega_P(t)}{\Omega_e(t)}$$

$$\Omega_e(t) = \sqrt{\Omega_P^2(t) + \Omega_S^2(t)}$$

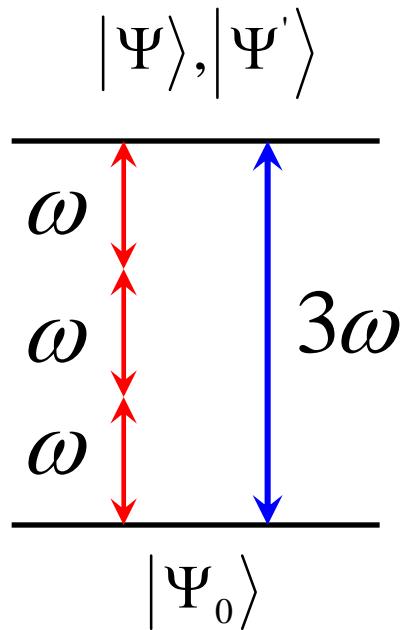
!STIRAP!



K. Bergmann and H. Theuer and B. W. Shore, Coherent population transfer among quantum states of atoms and molecules, Rev. Mod. Phys. 70, 1003(1998).

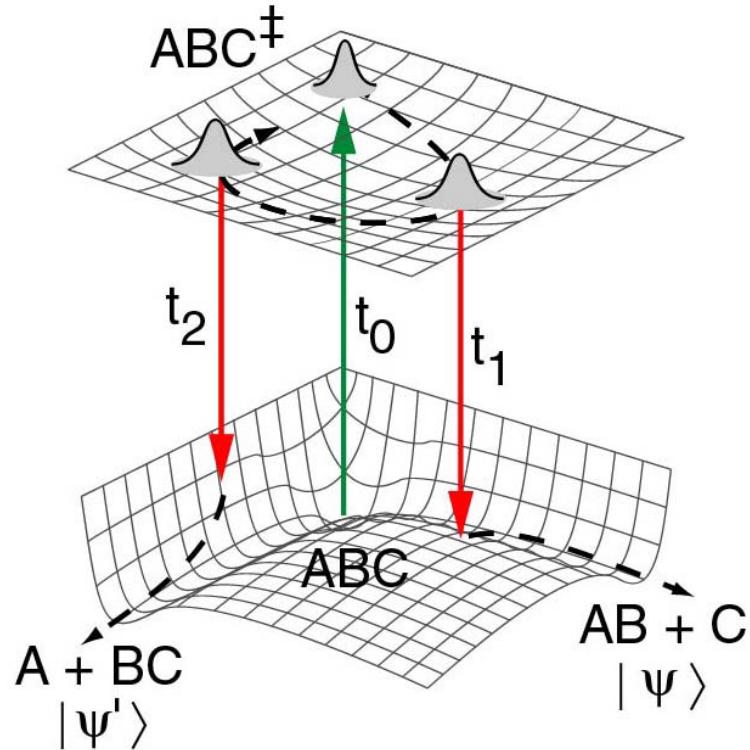
•Control schemes

Brumer-Shapiro
“phase control”
CPL 126, 541(1986)



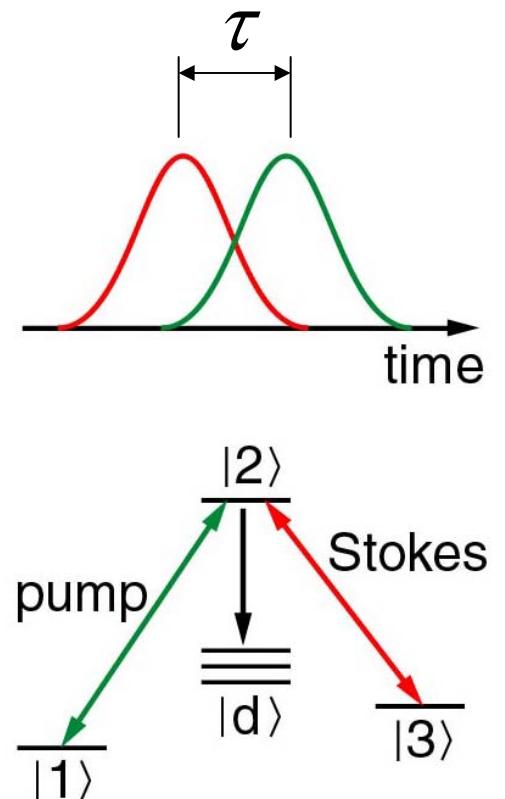
$$\Delta\phi = \phi_\omega - \phi_{3\omega}$$

Tannor-Kosloff-Rice
“pump-dump control”
JCP 85, 5805(1986)



$$\Delta t = t_{1,2} - t_0$$

Bergmann et.al.
“STIRAP control”
CPL 149, 463(1988)



•Local tracking control

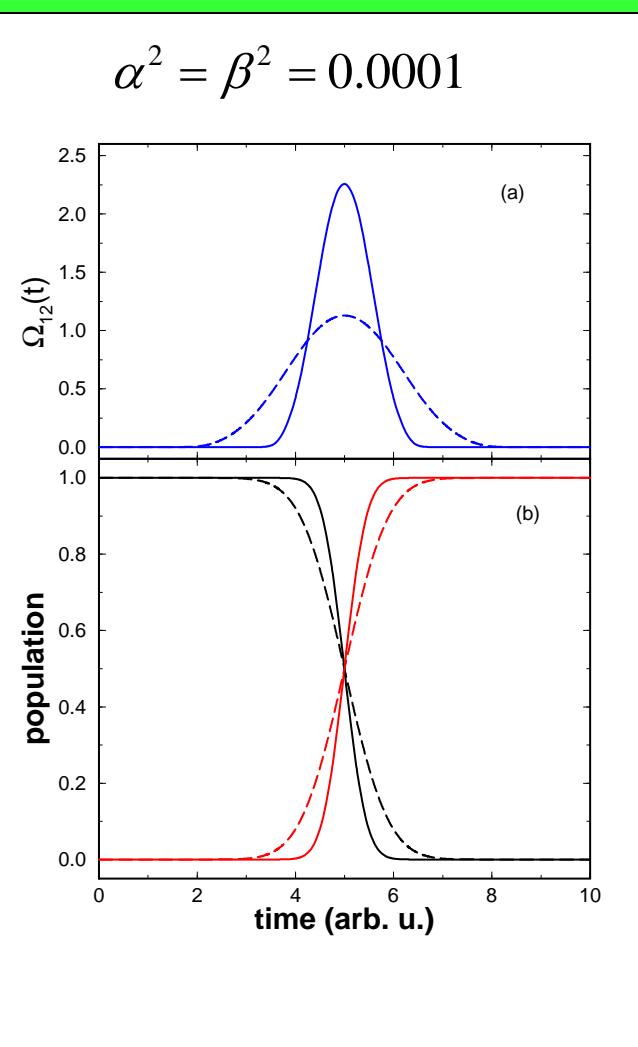
$$i \begin{pmatrix} \dot{a}(t) \\ \dot{b}(t) \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 0 & \Omega_{12}(t) \\ \Omega_{21}(t) & 2\Delta \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\begin{cases} \frac{d|a(t)|^2}{dt} = -\Omega_{12}(t) \operatorname{Im}\{a^*(t)b(t)\}, \\ \frac{d|b(t)|^2}{dt} = \Omega_{12}(t) \operatorname{Im}\{a^*(t)b(t)\}. \end{cases}$$

Lets choose a track for the population

$$|a(t)|^2 = f(t), \text{ then } \Omega_{12}(t) = -\frac{\dot{f}(t)}{\operatorname{Im}\{a^*(t)b(t)\}}$$

$$f(t) = \alpha^2 + \frac{1 - \beta^2 - \alpha^2}{2} \left[1 + \operatorname{erf}\left(\frac{t - t_c}{\tau}\right) \right]$$

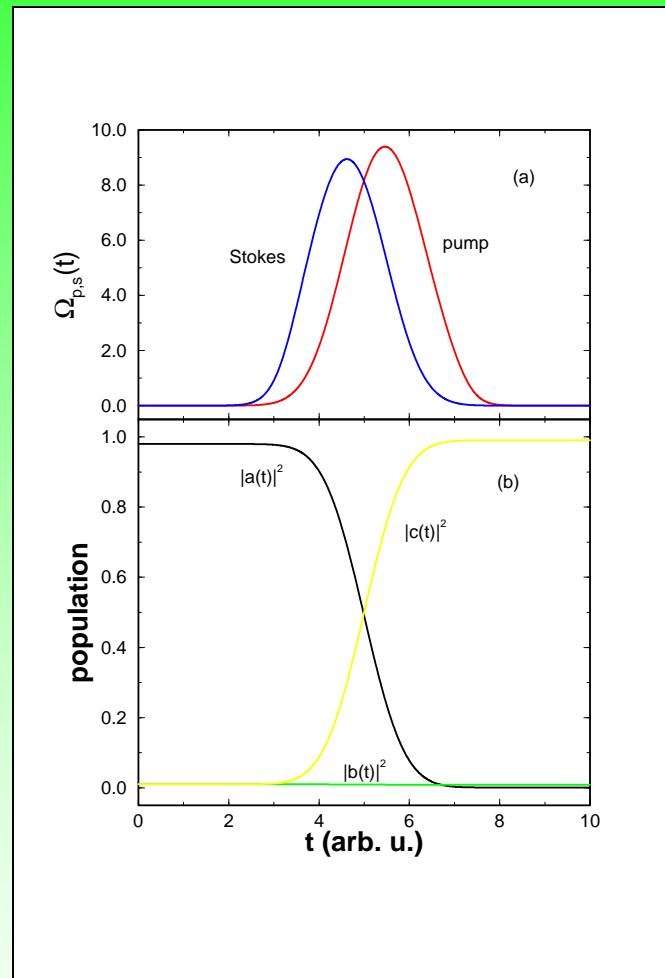


π pulse solution!

• STIRAP <> local tracking control

$$\hat{H}(t) = -\frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 2\Delta & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{pmatrix}$$

$$\begin{cases} \frac{d|a(t)|^2}{dt} = -\Omega_p(t) \operatorname{Im}\{a^*(t)b(t)\}, \\ \frac{d|b(t)|^2}{dt} = \Omega_p(t) \operatorname{Im}\{a^*(t)b(t)\} + \Omega_s(t) \operatorname{Im}\{c^*(t)b(t)\}, \\ \frac{d|c(t)|^2}{dt} = -\Omega_s(t) \operatorname{Im}\{c^*(t)b(t)\}. \end{cases}$$



$$|b(t)|^2 = \text{const, then } \begin{cases} \Omega_p(t) = -\Omega_0(t) \operatorname{Im}\{c^*(t)b(t)\}, \\ \Omega_s(t) = \Omega_0(t) \operatorname{Im}\{a^*(t)b(t)\}. \end{cases}$$

Local control



Global control



- Optimal control method in wave function formalism

Schrödinger equation

$$i \hbar \frac{\partial}{\partial t} |\psi(t)\rangle = [H_0 - \mu E(t)] |\psi(t)\rangle$$

μ : electric dipole moment operator

$E(t)$: electric field (semiclassical approximation)

optimal control method

- (1) Introducing a target operator W to specify a physical objective.
- (2) Adding a penalty term due to pulse fluence in order to reduce pulse energy.
- (3) Introducing a Lagrange multiplier density $\xi(t)$ that constrains the system to obey the equation of motion.

objective functional has the form

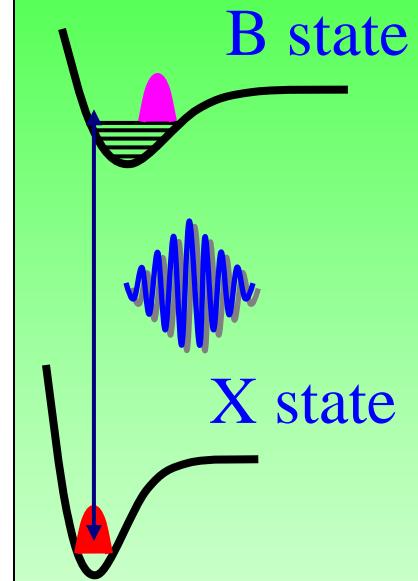
$$\bar{J} = \langle \psi(t_f) | W | \psi(t_f) \rangle \quad \xleftarrow{\text{(1) expectation value}}$$

$$- \int_0^{t_f} dt \frac{1}{\hbar A} [E(t)]^2 \quad \xleftarrow{\text{(2) penalty term}}$$

$$+ Re \left\{ \frac{i}{\hbar} \int_0^{t_f} dt \langle \xi(t) | \left(i\hbar \frac{\partial}{\partial t} - H^t \right) | \psi(t) \rangle \right\}$$

(3) constraint due to the Schrödinger equation

$|\xi(t)\rangle$ Lagrange multiplier



$$\delta \bar{J} = 0 \text{ with respect to variation of all parameters}$$

- 1) Initial guess
- 2) Forward integration
- 3) Project
- 4) Backward integration
- 5) New field generation
- 6) Checking convergence

$$E_0(t)$$

$$i \hbar \frac{\partial}{\partial t} |\psi(t)\rangle = [H_0 - \mu E(t)] |\psi(t)\rangle$$

$$|\xi(t_f)\rangle = \hat{P} |\psi(t_f)\rangle$$

$$i \hbar \frac{\partial}{\partial t} |\xi(t)\rangle = [H_0 - \mu E(t)] |\xi(t)\rangle$$

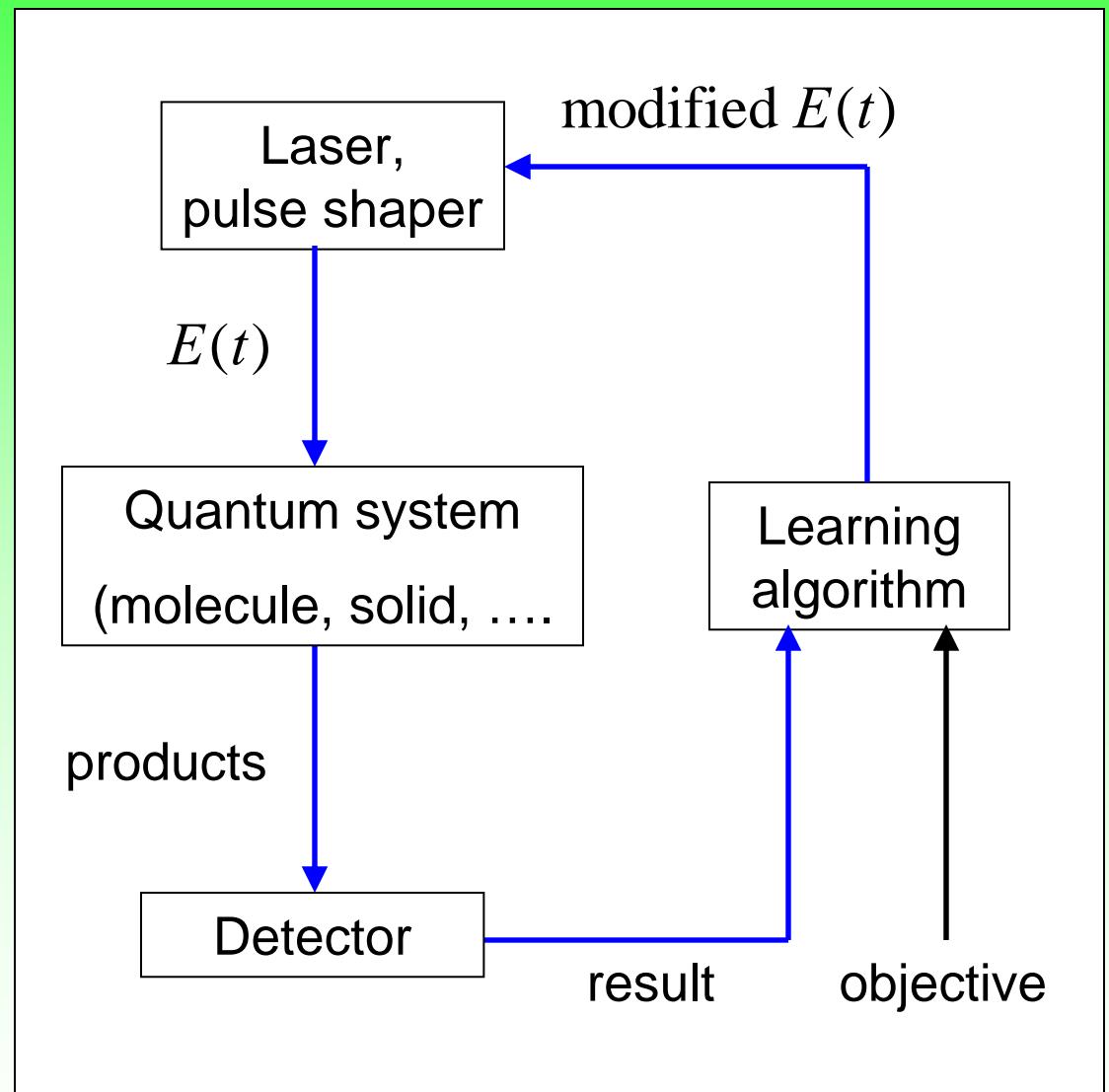
- Adaptive learning algorithm



Hersch Rabitz

“Teaching lasers to control molecules”

Experimental output is included in optimization loop

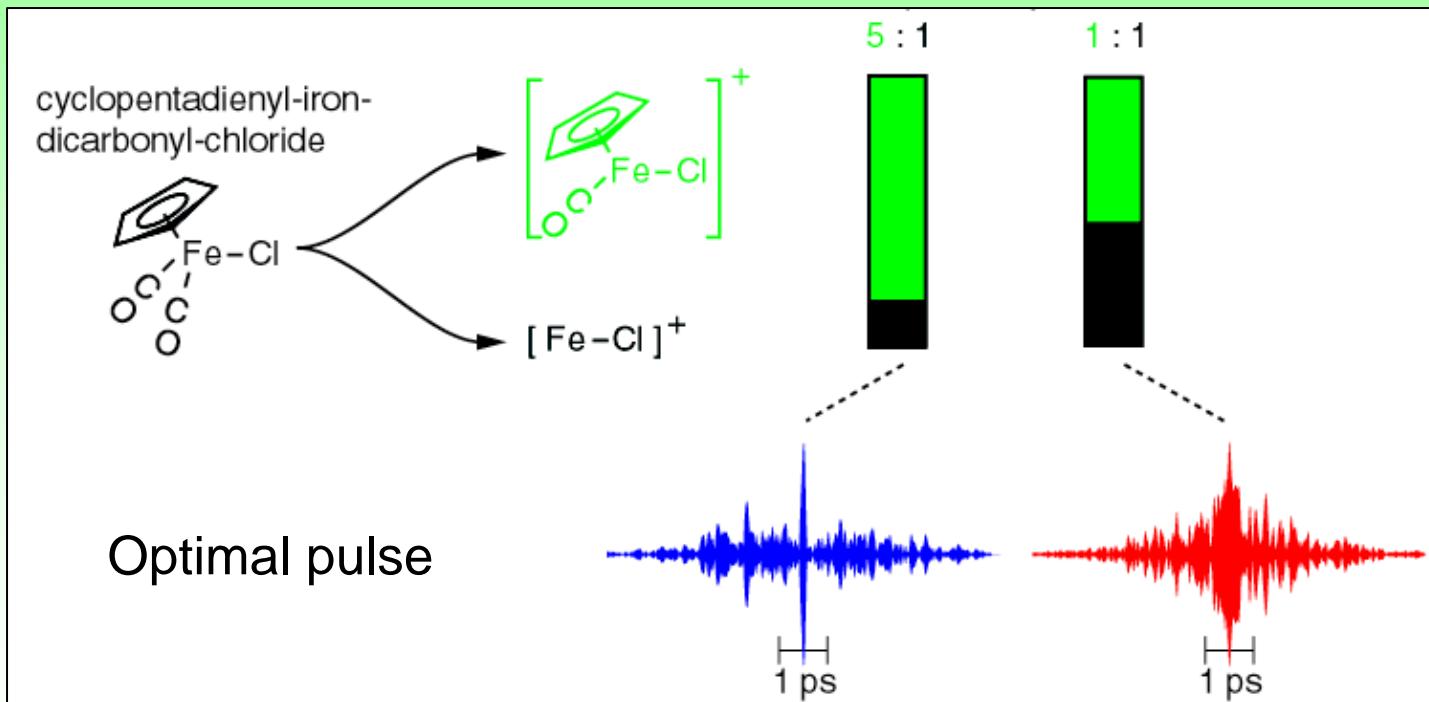


•Quantum control of photodissociation

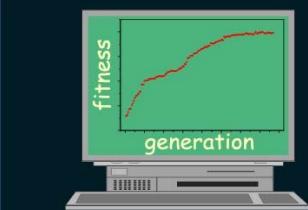
Science 282, 919 (1998)
Chem. Phys. 267, 241 (2001)



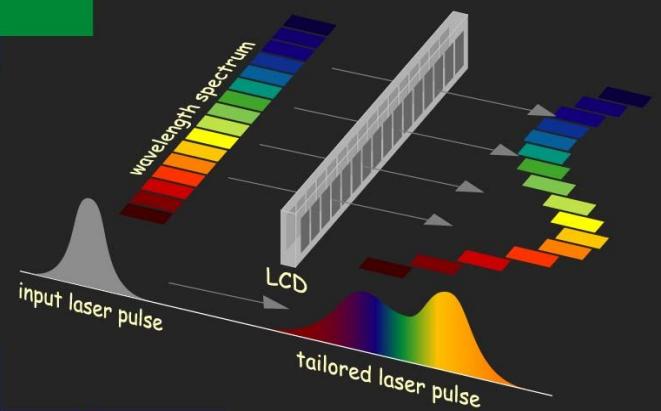
Gustav Gerber



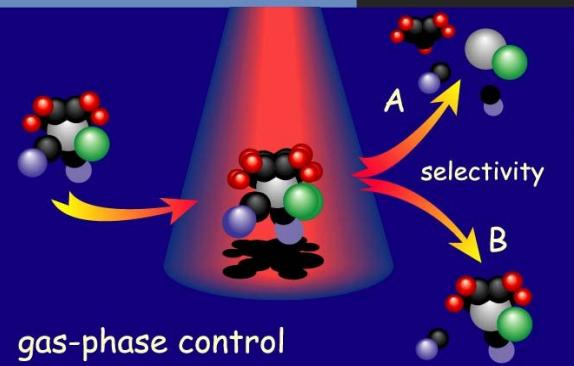
Adaptive Quantum Control



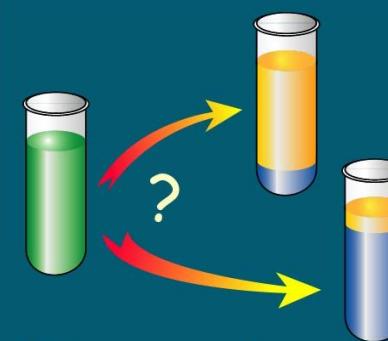
evolutionary optimization



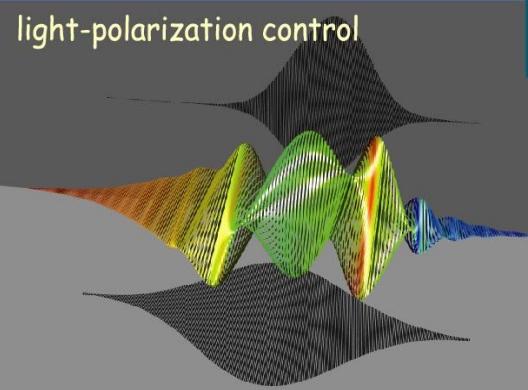
femtosecond pulse shaping



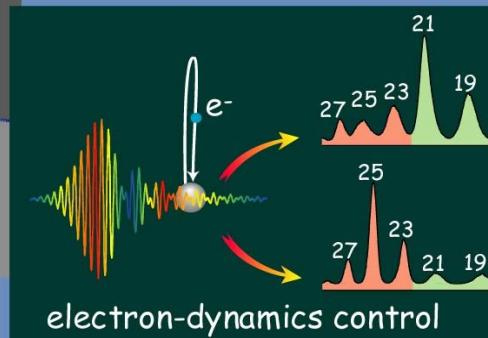
gas-phase control



liquid-phase control

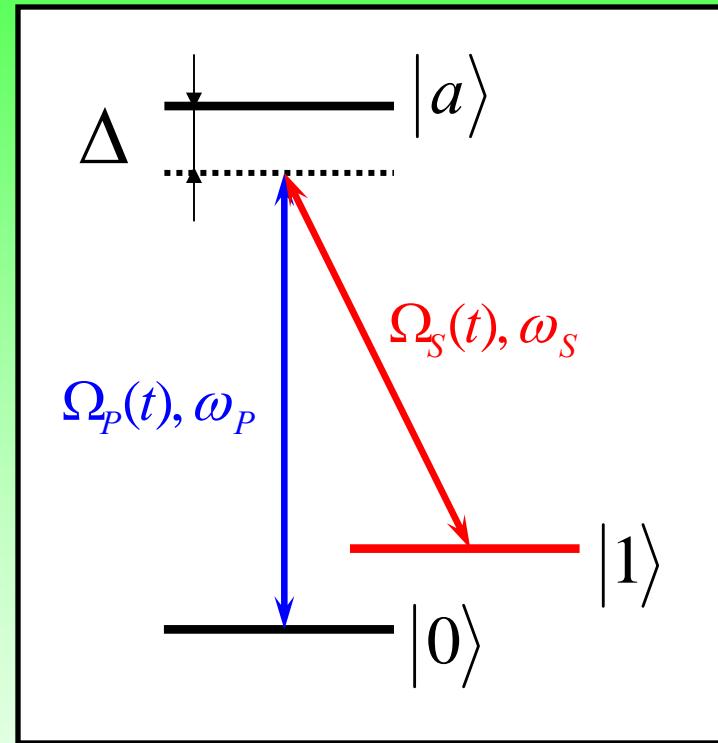
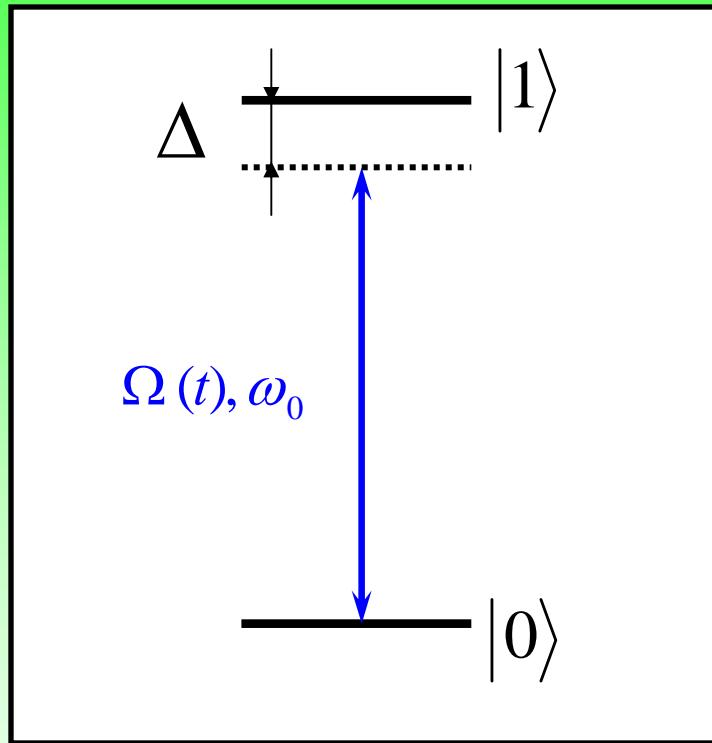


light-polarization control



electron-dynamics control

Qubits!

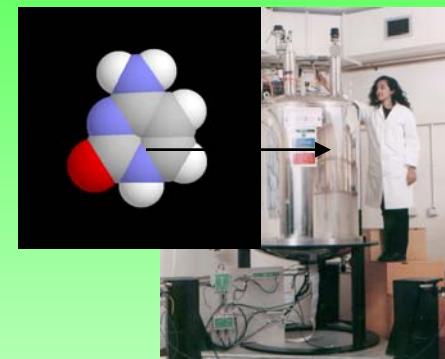
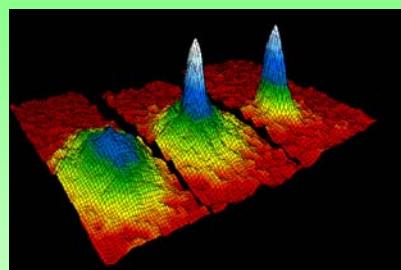


$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Some actual or proposed quantum computers

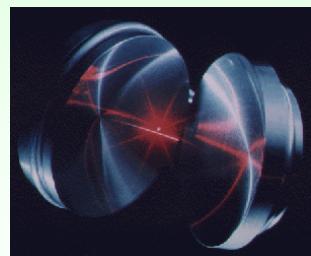
Liquid-state NMR (“quantum computing in a coffee cup”
- has factored 15)

Bose-Einstein condensates

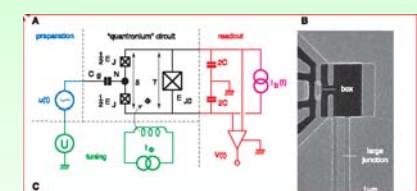
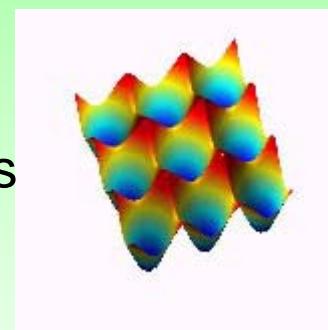


Lattices of cold atoms

Atom/photon interactions in
cavities (“cavity QED”)



Ion traps



Superconducting
circuits

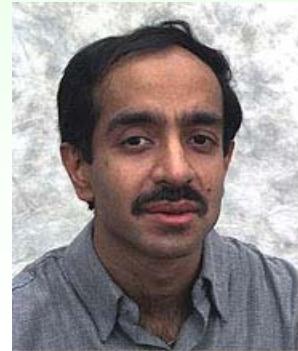


Deutsch-Jozsa algorithm

(constant or balanced)



Peter Shor's factorization algorithm



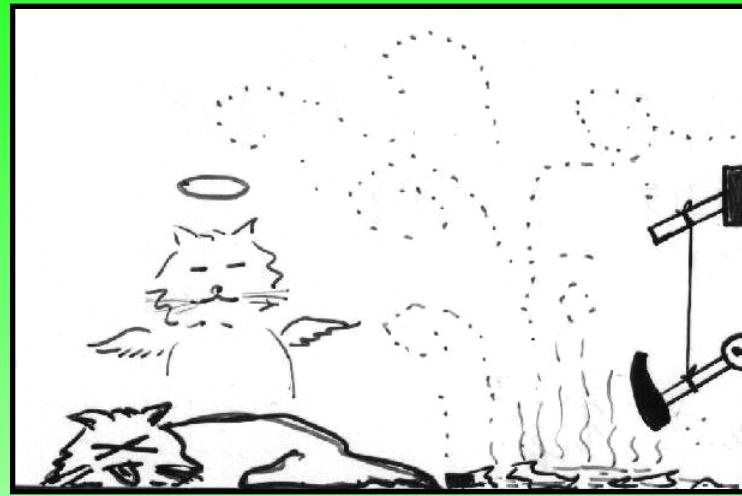
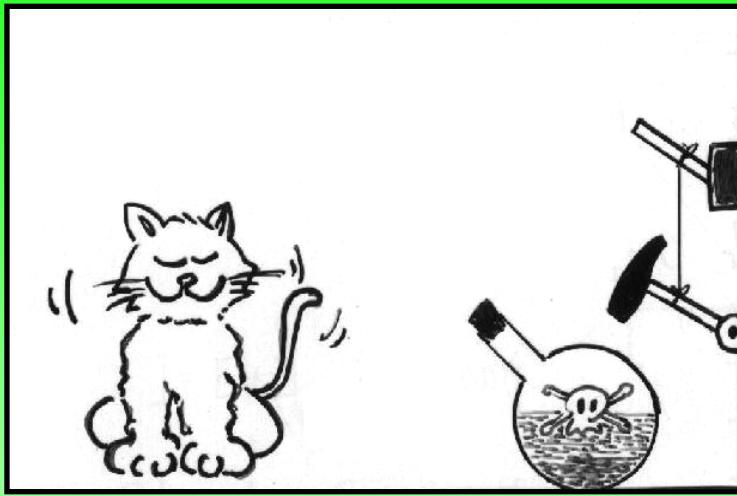
Grover's search algorithm

Many physical systems have been proposed as potential candidates to implement a quantum computer.

Following DiVincenzo the chosen quantum system should satisfy five conditions:

- (1) favorable scalability with the number of well-characterized qubits;**
- (2) ability to initialize the system in a particular qubit state;**
- (3) decoherence times much longer than the quantum gate operation time;**
- (4) control over a universal set of quantum gates;**
- (5) the ability to measure specific qubits.**

.....



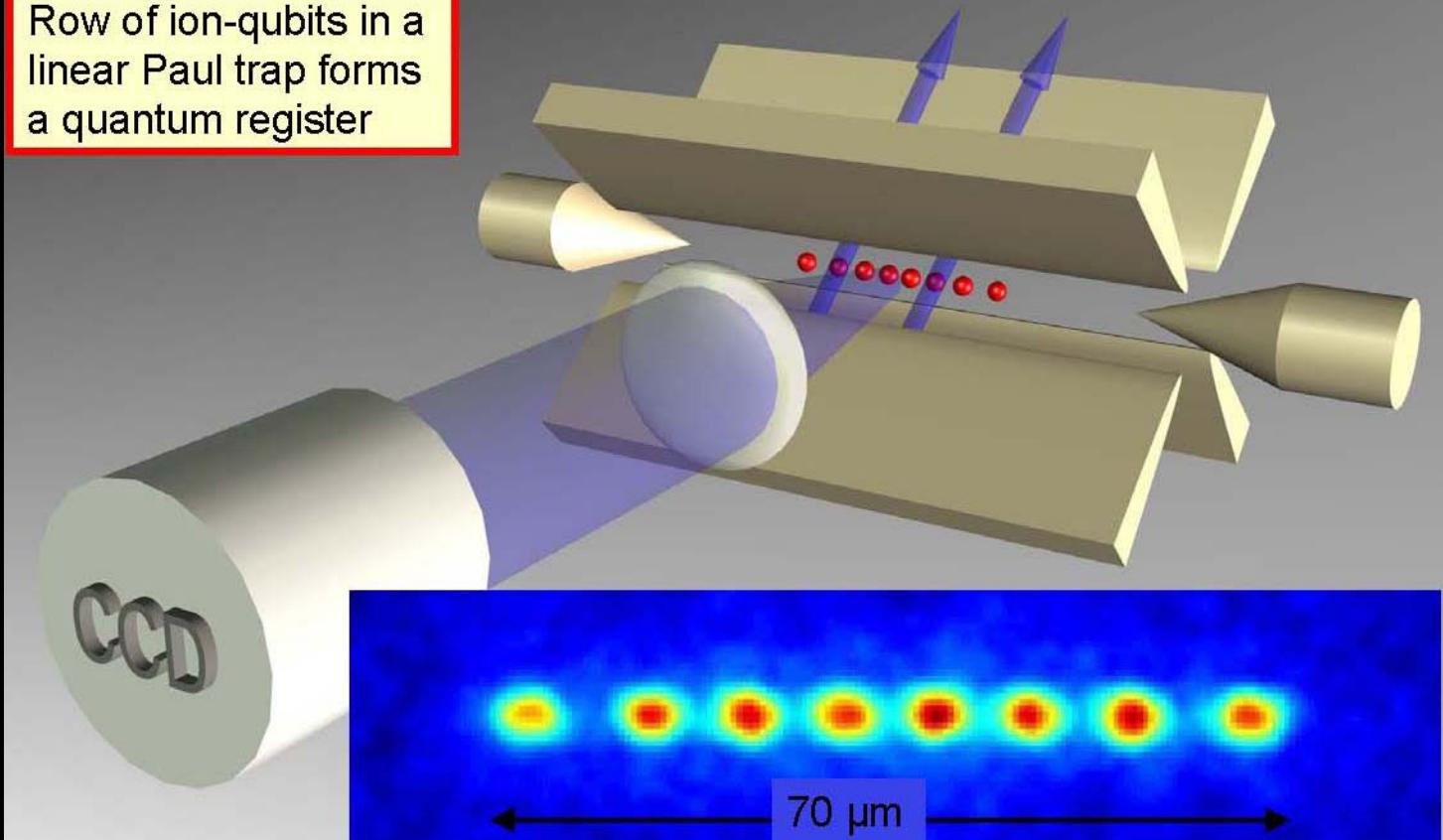
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|alive\rangle + |\downarrow\rangle|dead\rangle)$$

$$\langle \uparrow | \Psi \rangle \Rightarrow |alive\rangle$$

$$\langle \downarrow | \Psi \rangle \Rightarrow |dead\rangle$$

The Ca⁺ ion quantum gate

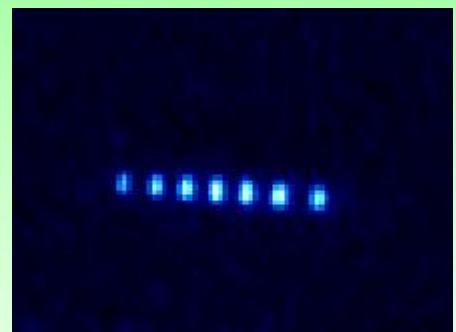
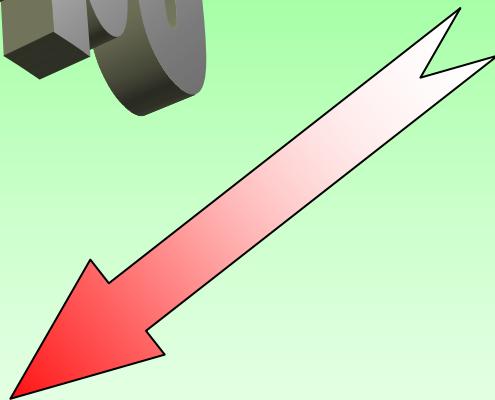
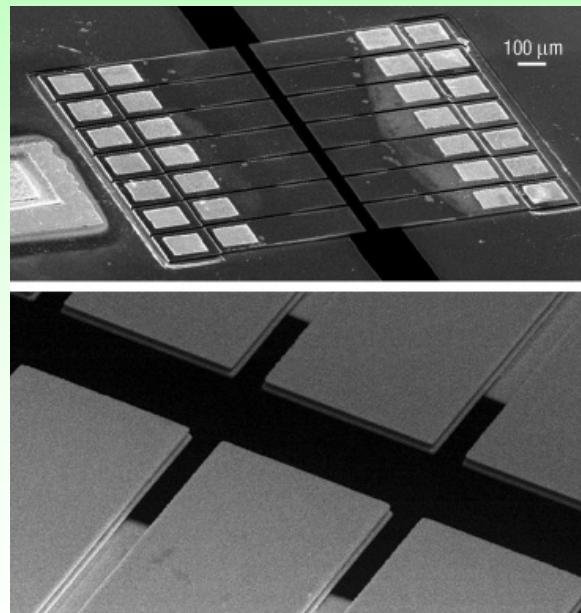
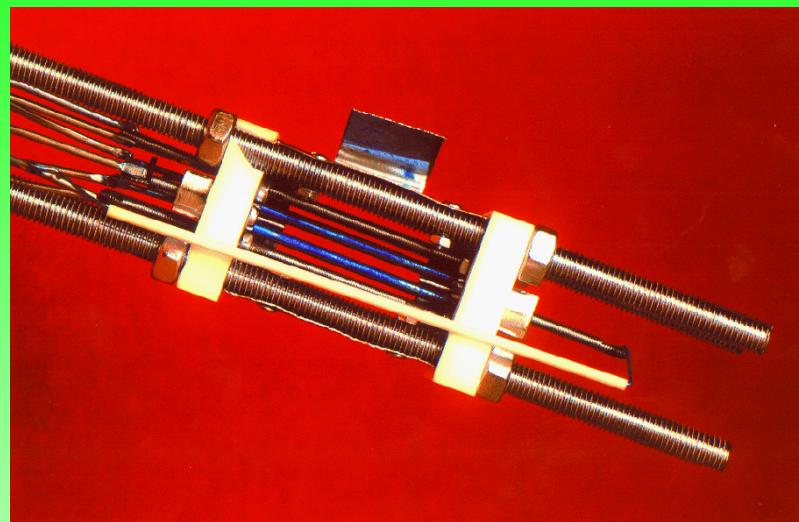
Row of ion-qubits in a linear Paul trap forms a quantum register



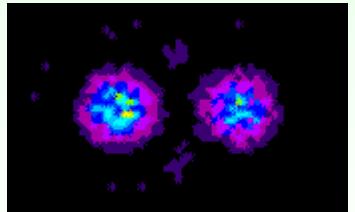
H.C. Nägerl et al., Appl. Phys. B 66, 603 (1998)

<http://nodeps.physics.ox.ac.uk/~mcdonnell/wardPres/wardPres.html>

IonTraps



<http://www.physics.gatech.edu/ultracool/Ions/7ions.jpg>



<http://www.nature.com/nphys/journal/v2/n1/images/nphys171-f2.jpg>

Quantum Computation with Cold Trapped Ions

PRL, 74, 4091(1995).



Ignacio Cirac



Peter Zoller

VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

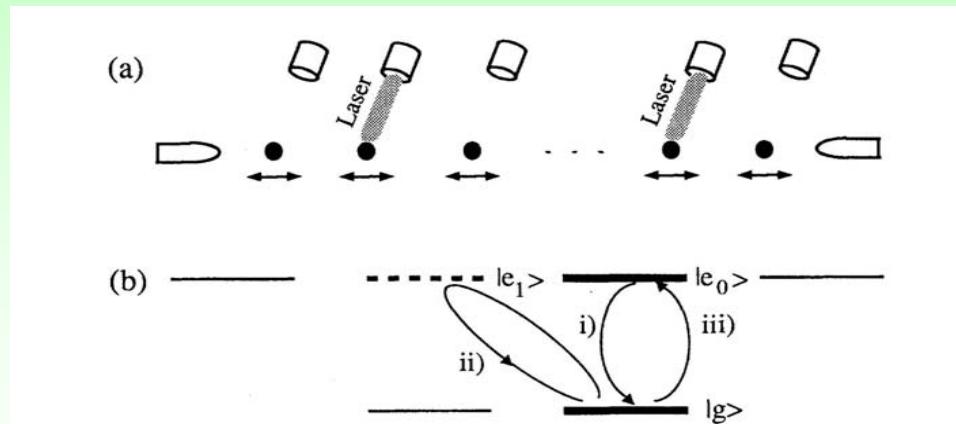


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

Quantum Computation with Ions in Thermal Motion

Anders Sørensen and Klaus Mølmer

Institute of Physics and Astronomy, University of Aarhus, DK-8000 Århus C, Denmark

(Received 26 June 1998; revised manuscript received 25 November 1998)

We propose an implementation of quantum logic gates via virtual vibrational excitations in an ion-trap quantum computer. Transition paths involving unpopulated vibrational states interfere destructively to eliminate the dependence of rates and revolution frequencies on vibrational quantum numbers. As a consequence, quantum computation becomes feasible with ions whose vibrations are strongly coupled to a thermal reservoir. [S0031-9007(99)08589-0]



Anders Sørensen



Klaus Mølmer

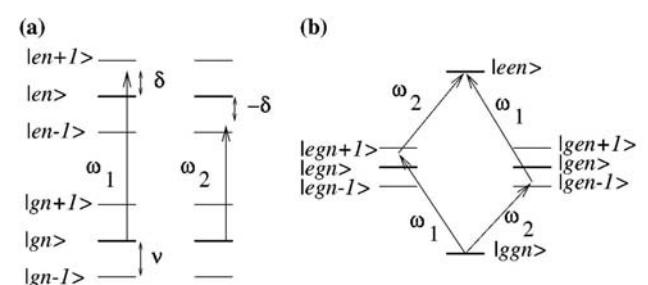


FIG. 1. Energy levels and laser detunings. (a) Two ions with quantized vibrational motion are illuminated with lasers detuned close to the upper and lower sidebands. (b) The ions oscillate in collective vibrational modes, and two interfering transition paths are identified.

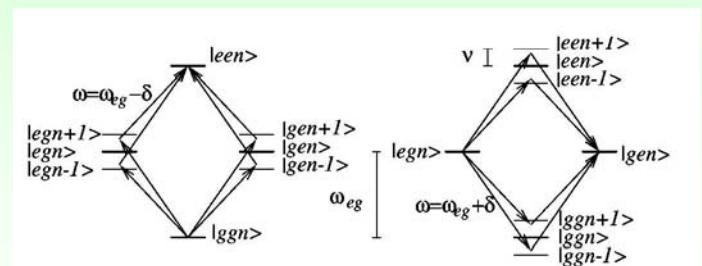


FIG. 1. Energy-level diagram for two ions with quantized vibrational motion illuminated with bichromatic light. The only resonant transitions are from $|ggn\rangle$ to $|een\rangle$ (left) and from $|egen\rangle$ to $|gen\rangle$ (right). Various transition paths involving intermediate states with a vibrational number n differing by unity are identified.

PHYSICAL REVIEW A, VOLUME 62, 022311

Entanglement and quantum computation with ions in thermal motion

Anders Sørensen and Klaus Mølmer

Institute of Physics and Astronomy, University of Aarhus, DK-8000 Århus C, Denmark

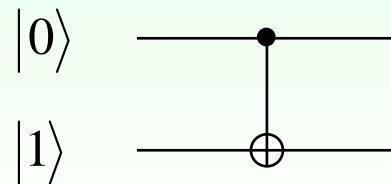
(Received 10 February 2000; revised manuscript received 1 May 2000; published 18 July 2000)

With bichromatic fields, it is possible to deterministically produce entangled states of trapped ions. In this paper we present a unified analysis of this process for both weak and strong fields, for slow and fast gates. Simple expressions for the fidelity of creating maximally entangled states of two or an arbitrary number of ions under nonideal conditions are derived and discussed.

Two qubit CNOT gate

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}$$

$$U_{CNOT} |\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle \quad \text{with} \quad \varepsilon_{1,2} = 0,1$$



- Two spin system

I and S both of spin-1/2 in a static magnetic field \mathbf{B}_0

$$H = \omega_S S_z + \omega_I I_z + J I_z S_z$$

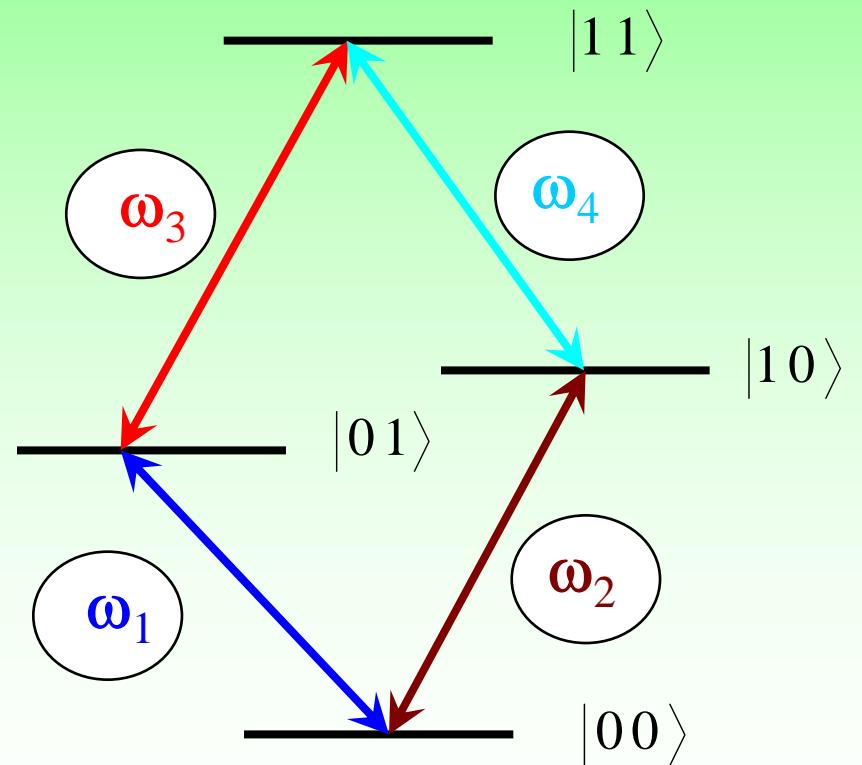
$$\omega_{S,I} = \gamma_{S,I} B_0$$

$$|\downarrow\rangle_S |\downarrow\rangle_I = |00\rangle \quad E_1 = -\frac{\omega_S + \omega_I}{2} + \frac{J}{4}$$

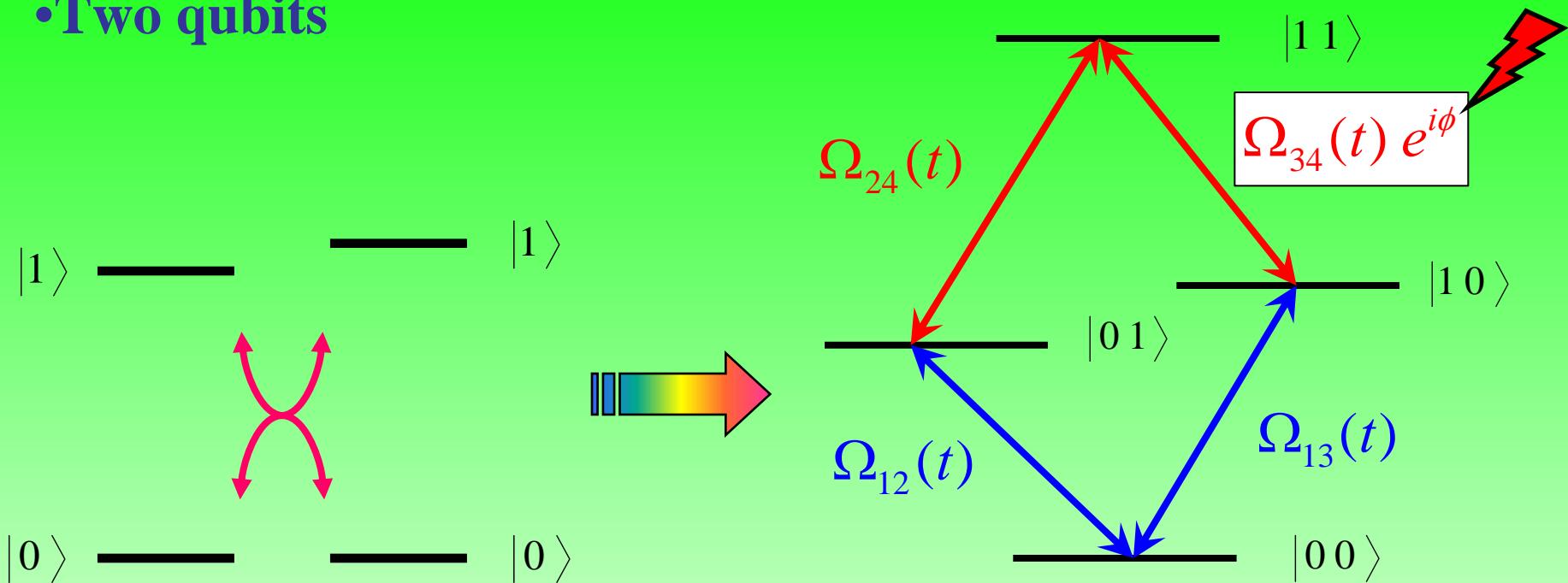
$$|\downarrow\rangle_S |\uparrow\rangle_I = |01\rangle \quad E_2 = -\frac{\omega_S - \omega_I}{2} - \frac{J}{4}$$

$$|\uparrow\rangle_S |\downarrow\rangle_I = |10\rangle \quad E_3 = -\frac{\omega_I - \omega_S}{2} - \frac{J}{4}$$

$$|\uparrow\rangle_S |\uparrow\rangle_I = |11\rangle \quad E_4 = \frac{\omega_I + \omega_S}{2} + \frac{J}{4}$$



•Two qubits



$$|\Xi(t)\rangle = a_1(t)|00\rangle + a_2(t)|11\rangle + b_1(t)|01\rangle + b_2(t)|10\rangle$$

$$\left\{ \begin{array}{l} |\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ |\Phi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \end{array} \right.$$

$$\left\{ \begin{array}{l} |\Psi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ |\Psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{array} \right.$$

•Time-dependent Schrödinger equation

$$i \begin{pmatrix} \dot{a}_1(t) \\ \dot{a}_2(t) \\ \dot{b}_1(t) \\ \dot{b}_2(t) \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 0 & 0 & \Omega_{p1}(t) & \Omega_{p2}(t) \\ 0 & 0 & \Omega_{s1}(t) & \Omega_{s2}(t)e^{i\phi} \\ \Omega_{p1}(t) & \Omega_{s1}(t) & 2\Delta_1 & 0 \\ \Omega_{p2}(t) & \Omega_{s2}(t)e^{-i\phi} & 0 & 2\Delta_2 \end{pmatrix} \begin{pmatrix} a_1(t) \\ a_2(t) \\ b_1(t) \\ b_2(t) \end{pmatrix}$$

$$\Omega_{p,s}(t) = \Omega_0 \exp \left\{ -\frac{(t \mp \tau_d)^2}{\tau^2} \right\}$$

•Dressed states

Resonant case , $\Delta_{1,2} = 0$

$$\begin{aligned} |c_1(t)\rangle &\doteq \frac{ie^{-i\phi/2}}{2} \left(\frac{\lambda_-(\Omega_p^2(t) - \Omega_s^2(t) - \bar{\Omega}^2(t))}{\Omega_p(t)(\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} - \bar{\Omega}^2(t))}, \frac{\Omega_s(t)(1+e^{i\phi})\lambda_-}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} - \bar{\Omega}^2(t)}, \frac{\Omega_p^2(t) + \Omega_s^2(t)e^{i\phi} - \bar{\Omega}^2(t)}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} - \bar{\Omega}^2(t)}, 1 \right), \\ |c_2(t)\rangle &\doteq \frac{ie^{-i\phi/2}}{2} \left(\frac{\lambda_-(\Omega_p^2(t) - \Omega_s^2(t) - \bar{\Omega}^2(t))}{\Omega_p(t)(\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} - \bar{\Omega}^2(t))}, \frac{\Omega_s(t)(1+e^{i\phi})\lambda_-}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} - \bar{\Omega}^2(t)}, \frac{\Omega_p^2(t) + \Omega_s^2(t)e^{i\phi} - \bar{\Omega}^2(t)}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} - \bar{\Omega}^2(t)}, -1 \right), \\ |c_3(t)\rangle &\doteq \frac{\xi}{2} \left(\frac{\lambda_+(\Omega_p^2(t) - \Omega_s^2(t) + \bar{\Omega}^2(t))}{\Omega_p(t)(\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} + \bar{\Omega}^2(t))}, \frac{\Omega_s(t)(1+e^{i\phi})\lambda_+}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} + \bar{\Omega}^2(t)}, \frac{\Omega_p^2(t) + \Omega_s^2(t)e^{i\phi} + \bar{\Omega}^2(t)}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} + \bar{\Omega}^2(t)}, 1 \right), \\ |c_4(t)\rangle &\doteq \frac{\xi}{2} \left(\frac{\lambda_+(\Omega_p^2(t) - \Omega_s^2(t) + \bar{\Omega}^2(t))}{\Omega_p(t)(\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} + \bar{\Omega}^2(t))}, \frac{\Omega_s(t)(1+e^{i\phi})\lambda_+}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} + \bar{\Omega}^2(t)}, \frac{\Omega_p^2(t) + \Omega_s^2(t)e^{i\phi} + \bar{\Omega}^2(t)}{\Omega_p^2(t) + \Omega_s^2(t)e^{-i\phi} + \bar{\Omega}^2(t)}, -1 \right). \end{aligned}$$

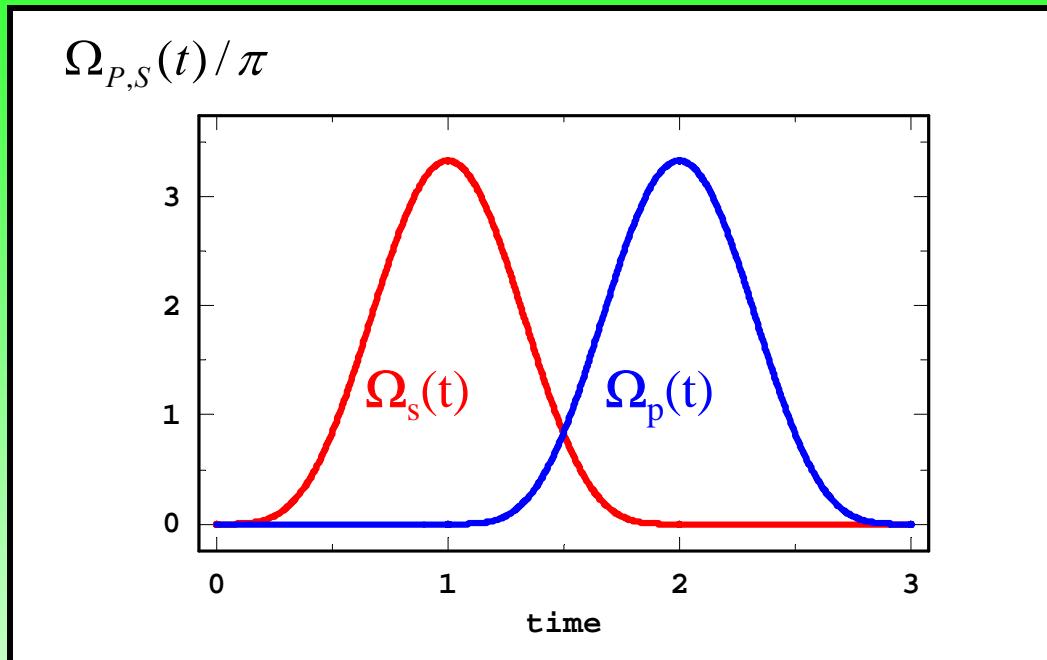
$$\lambda_{1,2}(t) = \mp \lambda_- / 2$$

$$\lambda_{3,4}(t) = \mp \lambda_+ / 2$$

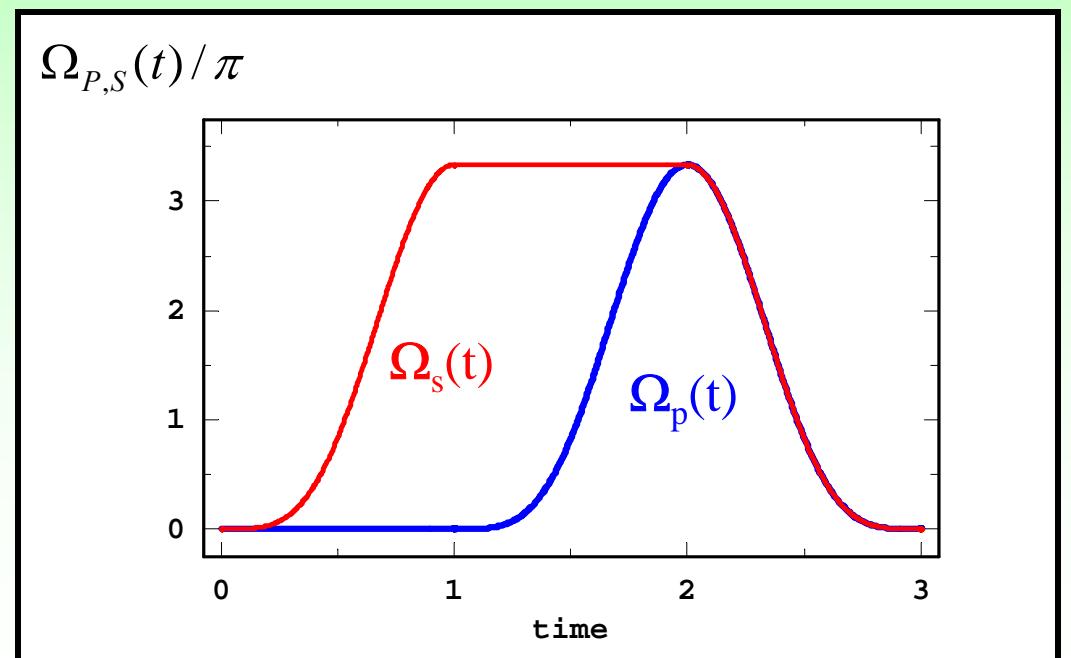
$$\lambda_{\pm}(t) = \sqrt{\Omega_p^2(t) + \Omega_s^2(t) \pm \bar{\Omega}^2(t)}$$

$$\bar{\Omega}^2(t) = \sqrt{\Omega_p^4(t) + \Omega_s^4(t) + 2 \cos \phi \Omega_p^2(t) \Omega_s^2(t)}$$

STIRAP



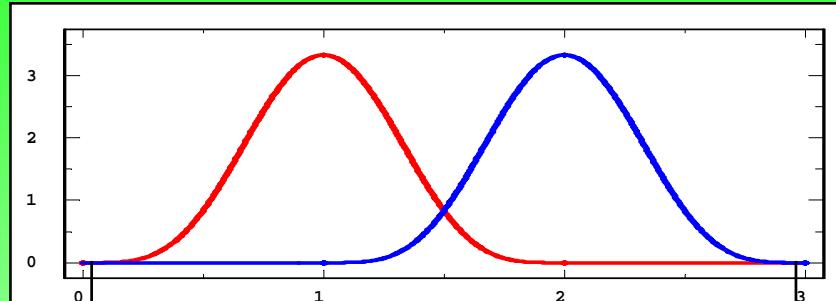
half-STIRAP



STIRAP

★★
counterintuitive

$$\frac{\Omega_p(t)}{\Omega_s(t)} \rightarrow 0$$



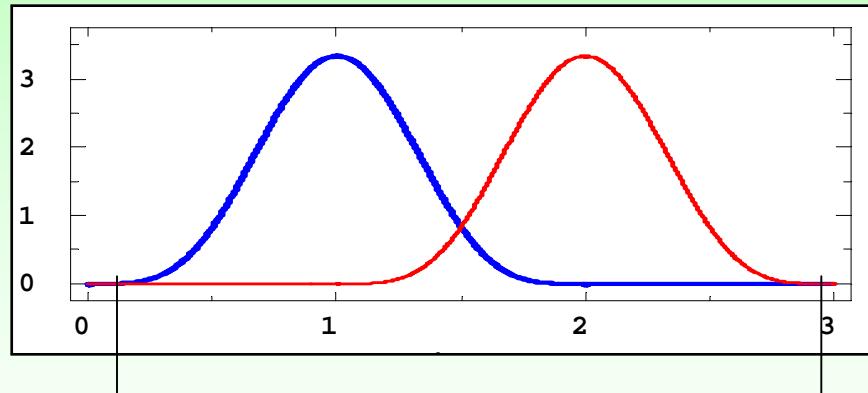
$$\frac{\Omega_s(t)}{\Omega_p(t)} \rightarrow 0$$

$$|00\rangle = \frac{|c_1(0)\rangle + |c_2(0)\rangle}{\sqrt{2}}$$

$$|\Xi(\infty)\rangle = \cos\left[\frac{S_-}{2}\right] |11\rangle + \frac{e^{-i\phi/2}}{\sqrt{2}} \sin\left[\frac{S_-}{2}\right] (|01\rangle - |10\rangle)$$

★★★
intuitive

$$\frac{\Omega_s(t)}{\Omega_p(t)} \rightarrow 0$$

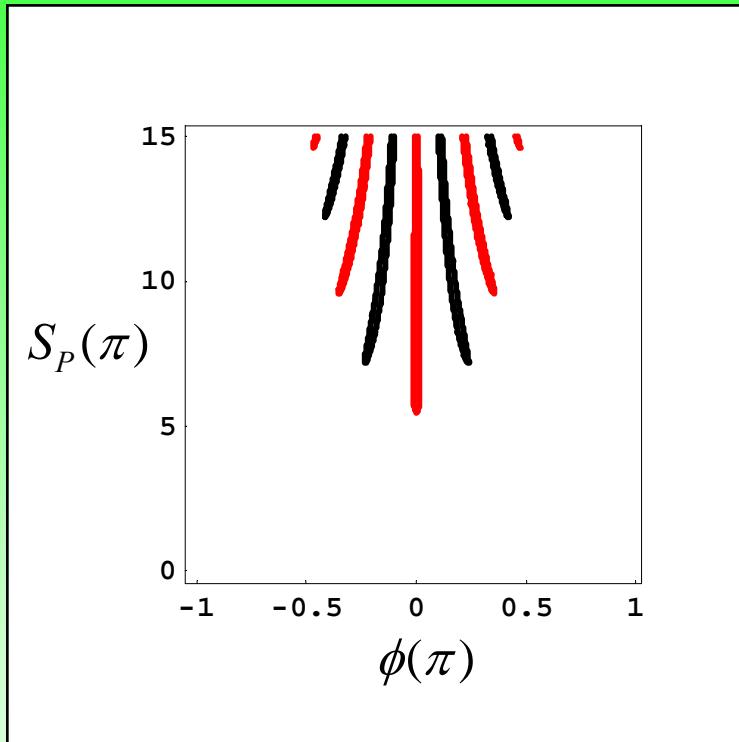


$$\frac{\Omega_p(t)}{\Omega_s(t)} \rightarrow 0$$

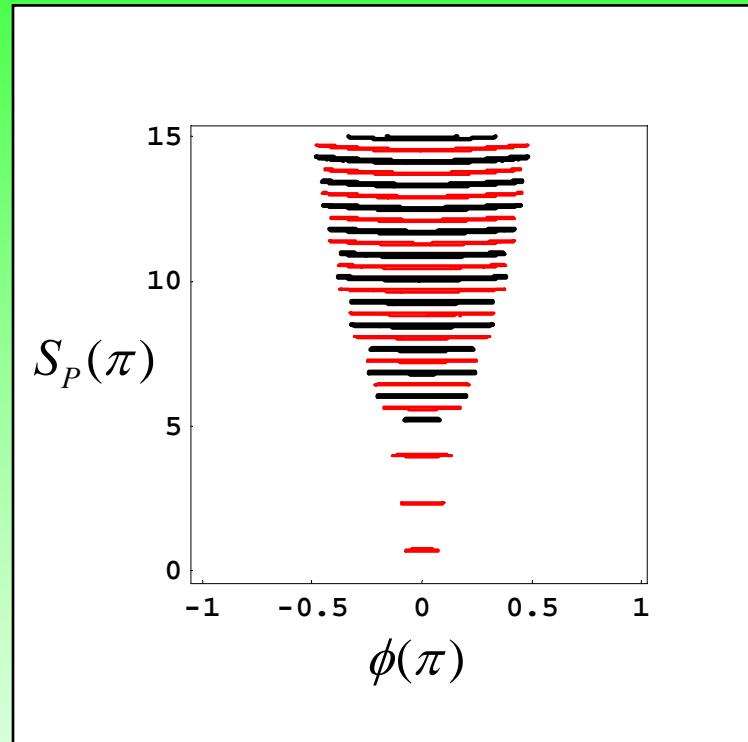
$$|00\rangle = \frac{|c_3(0)\rangle + |c_4(0)\rangle}{\sqrt{2}}$$

$$|\Xi(\infty)\rangle = \cos\left[\frac{S_+}{2}\right] |11\rangle + \frac{i}{\sqrt{2}} \sin\left[\frac{S_+}{2}\right] (|01\rangle + e^{-i\phi} |10\rangle)$$

counterintuitive



intuitive



— $|\Psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$

— $|\Psi\rangle = \frac{|01\rangle + e^{-i\phi} |10\rangle}{\sqrt{2}}$

— $|11\rangle$

•Final states

$$\frac{\Omega_p(t)}{\Omega_s(t)} \rightarrow 1$$

half-STIRAP

$$|c_1(\infty)\rangle \doteq \left(\frac{e^{-i\phi/4}}{2}, -\frac{e^{i\phi/4}}{2}, -\frac{i}{2}, \frac{ie^{-i\phi/2}}{2} \right),$$

$$|c_2(\infty)\rangle \doteq \left(\frac{e^{-i\phi/4}}{2}, -\frac{e^{i\phi/4}}{2}, \frac{i}{2}, -\frac{ie^{-i\phi/2}}{2} \right),$$

$$|c_3(\infty)\rangle \doteq \left(\frac{e^{-i\phi/4}}{2}, \frac{e^{i\phi/4}}{2}, \frac{1}{2}, \frac{e^{-i\phi/2}}{2} \right),$$

$$|c_4(\infty)\rangle \doteq \left(\frac{e^{-i\phi/4}}{2}, \frac{e^{i\phi/4}}{2}, -\frac{1}{2}, -\frac{e^{-i\phi/2}}{2} \right).$$

HCl

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle - e^{i\phi/2}|11\rangle)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - e^{-i\phi/2}|10\rangle)$$

IH

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + e^{i\phi/2}|11\rangle)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + e^{-i\phi/2}|10\rangle)$$

$$\left\{ \begin{array}{l} |\Xi(\infty)\rangle = \frac{e^{-i\phi/4}}{\sqrt{2}} \cos\left[\frac{S_-}{2}\right] (|00\rangle - e^{i\phi/2}|11\rangle) + \frac{1}{\sqrt{2}} \sin\left[\frac{S_-}{2}\right] (|01\rangle - e^{-i\phi/2}|10\rangle) \\ \\ |\Xi(\infty)\rangle = \frac{e^{-i\phi/4}}{\sqrt{2}} \cos\left[\frac{S_+}{2}\right] (|00\rangle + e^{i\phi/2}|11\rangle) + \frac{i}{\sqrt{2}} \sin\left[\frac{S_+}{2}\right] (|01\rangle + e^{-i\phi/2}|10\rangle) \end{array} \right.$$

◊ Harmonic potential: **Collapse and revival of entanglement**

$$\hat{H}_0 = \hbar\nu \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \sum_i \frac{E_2^i}{2} \left(\hat{I} - \hat{\sigma}_{zi} \right)$$

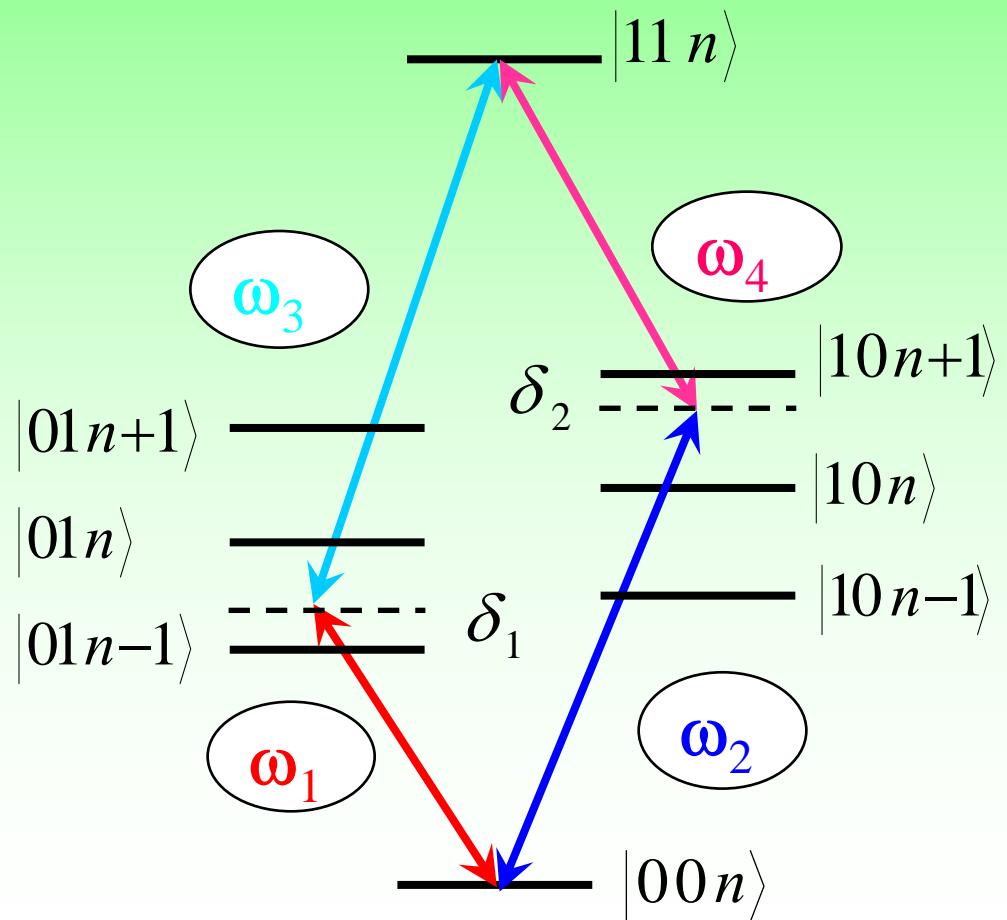
$$\hat{V}_{\text{int}} = -\hbar \sum_{j,i} \Omega_j(t) \cos \left[\omega_j t + \varphi_j - \eta_j (\hat{a}^\dagger + \hat{a}) \right] \hat{\sigma}_{xi} + h.c.$$

\hat{a}^\dagger, \hat{a} -- are the ladder operators of the quantized oscillator

η -- is the Lamb-Dicke parameter

$$|\Xi(t)\rangle = a_1(t)|00n\rangle + a_2(t)|11n\rangle + b_1(t)|01n-1\rangle + b_2(t)|10n+1\rangle$$

$$\hat{H} = -\frac{1}{2} \begin{pmatrix} 0 & \Omega_{1,n}(t)e^{i\varphi_1} & \Omega_{2,n+1}(t)e^{i\varphi_2} & 0 \\ \Omega_{1,n}(t)e^{-i\varphi_1} & -2\delta_1 & 0 & \Omega_{3,n}(t)e^{i\varphi_3} \\ \Omega_{2,n+1}(t)e^{-i\varphi_2} & 0 & -2\delta_2 & \Omega_{4,n+1}(t)e^{i\varphi_4} \\ 0 & \Omega_{3,n}(t)e^{-i\varphi_3} & \Omega_{4,n+1}(t)e^{-i\varphi_4} & 0 \end{pmatrix}$$



Ch.Wunderlich & Ch.Balzer ph/0305129

Population inversion

$$W(t) = \sum_{n=0}^{\infty} P(n) \left(|a_1(t)|^2 - |a_2(t)|^2 \right)$$

Concurrence

$$C(\rho) = \max \left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right)$$

where λ_i are the eigenvalues of the matrix

$$\mathfrak{R} = \rho \left(\sigma_y^A \otimes \sigma_y^B \right) \rho^* \left(\sigma_y^A \otimes \sigma_y^B \right)$$

Renyi entropy

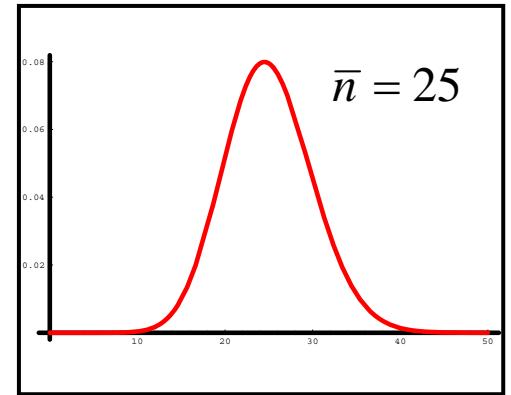
$$\mathbb{R}(\rho) = \text{Tr} \left[\rho^2 \right]$$

- W.K. Wootters, PRL, 80, 2245(1998)
- S.Hill and W.K.Wootters, PRL, 78, 5022(1997)
- T.Yu and J.H.Eberly, PRB, 66,193306(2002); 68,165322(2003)
- J.K.Stockton et.al. PRA, 67, 022112(2003)

Coherent State

$$P(n) = \frac{\exp\{-\bar{n}\} \bar{n}^n}{n!}$$

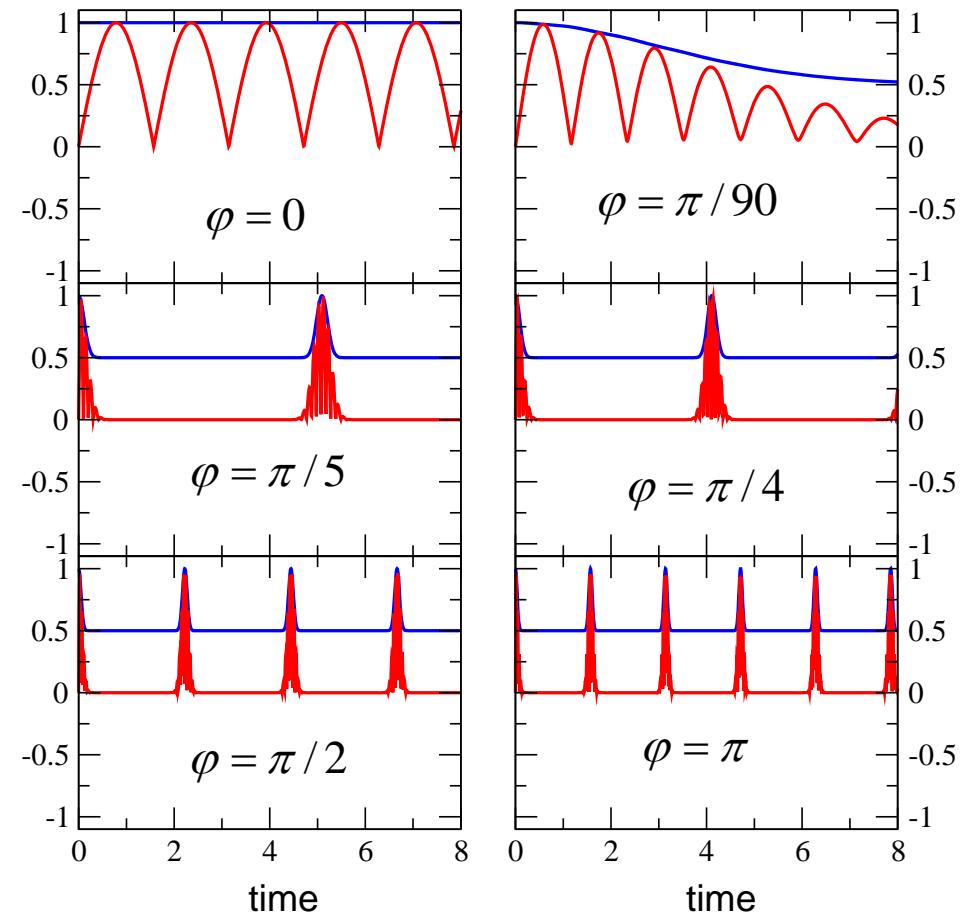
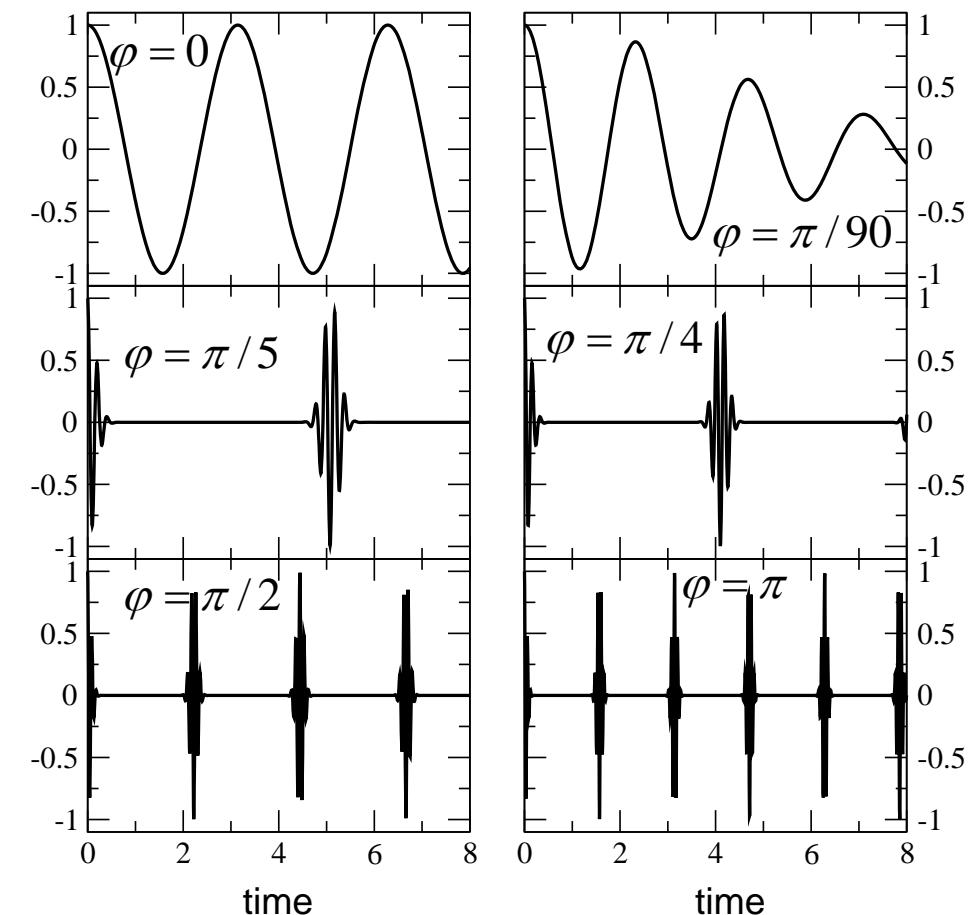
Jaynes-Cummings solution



Inversion

Concurrence

Renyi entropy



Important result, arbitrary φ

$$a_1(t) = e^{iS(t)} \cos \left[S(t) \sqrt{1 + 4n(n+1)\sin^2(\varphi/2)} \right]$$

$$a_2(t) = e^{iS(t)} \sin \left[S(t) \sqrt{1 + 4n(n+1)\sin^2(\varphi/2)} \right] / \alpha(\varphi, n)$$

where

$$S(t) = \frac{\eta^2}{4\delta_0} \int_0^t \Omega_0^2(t') dt'$$

$$\alpha(\varphi, n) = \frac{(2n+1)\sin(\varphi/2) + i\cos(\varphi/2)}{\sqrt{1 + 4n(n+1)\sin^2(\varphi/2)}}$$

$$|00n\rangle \leftrightarrow |11n\rangle$$

Evolution operators

$$i\dot{a}_1 = -a_1\Omega_e - a_2\Omega_e e^{i\phi_+},$$

$$i\dot{a}_2 = -a_2\Omega_e - a_1\Omega_e e^{-i\phi_+},$$

$$U_{ii} = e^{i\xi(t)} \begin{pmatrix} \cos \xi(t) & ie^{-i\phi_+} \sin \xi(t) \\ i \sin \xi(t) & e^{-i\phi_+} \cos \xi(t) \end{pmatrix}$$

$$|01n\rangle \leftrightarrow |10n\rangle$$

$$i\dot{a}_1 = -a_1\Omega_e - a_2\Omega_e,$$

$$i\dot{a}_2 = -a_2\Omega_e - a_1\Omega_e,$$

$$U_{ij} = e^{i\xi(t)} \begin{pmatrix} \cos \xi(t) & i \sin \xi(t) \\ i \sin \xi(t) & \cos \xi(t) \end{pmatrix}$$

where $\phi_+ = \phi_1 + \phi_2$, $\xi(t) = \int_0^t \Omega_e(t') dt'$

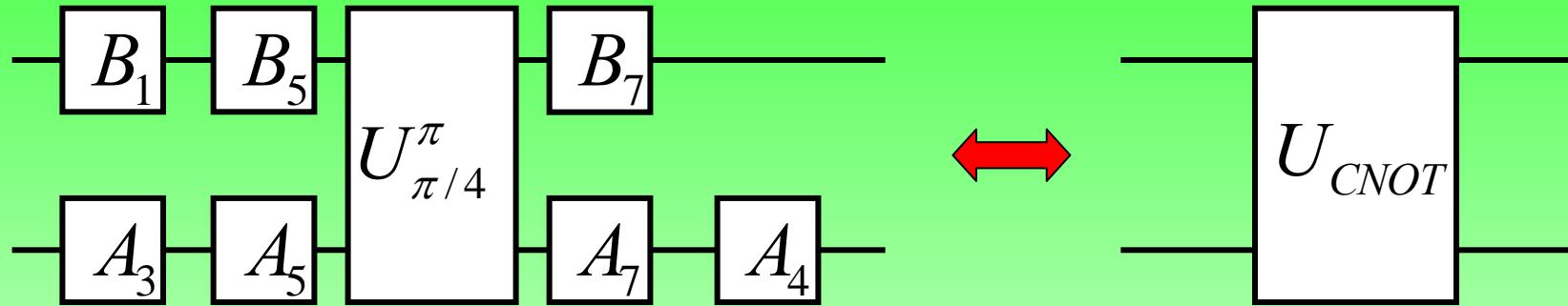
Decomposition of U into canonical form

$$U_{\pi/4}^{\pi} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -i \\ 0 & 1 & i & 0 \\ 0 & i & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix} = C_1 \otimes D_1 e^{i \frac{\pi}{4} \sigma_x \otimes \sigma_x} C_2 \otimes D_2$$

$\left(\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \right)$

where $C_1 = D_1 = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$, $C_2 = D_2 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

Scheme of constructing a CNOT gate



$$U_{CNOT} = A_4 A_7 B_7 U_{\pi/4}^\pi A_5 B_5 B_1 A_3$$

A_i and B_i act locally on the first and second qubit correspondingly

where $B_1 = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$, $A_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$,

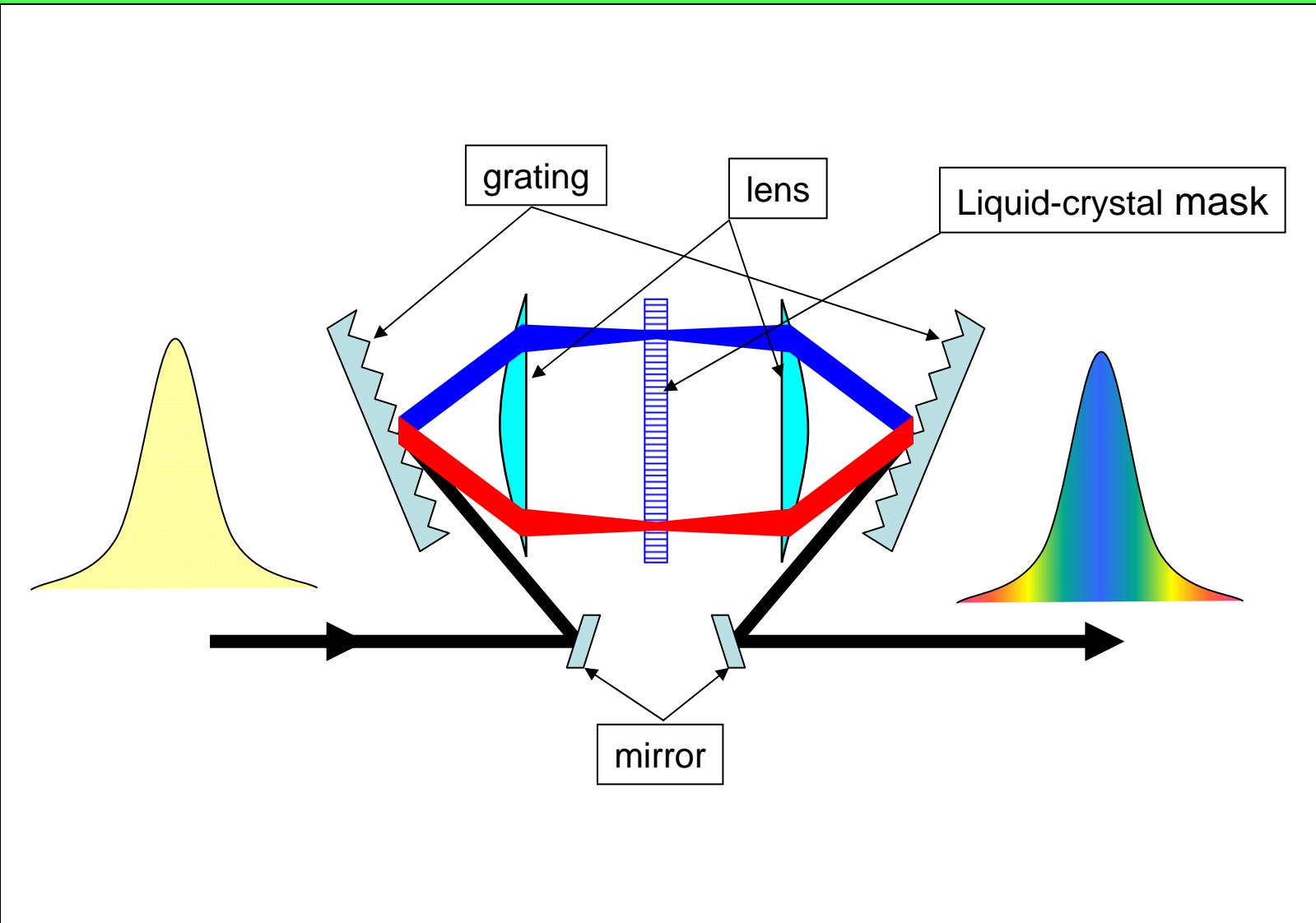
$$A_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad A_5 = \frac{1}{2} \begin{pmatrix} 1+i & 1-i \\ -1+i & -1-i \end{pmatrix},$$

$$A_7 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}, \quad B_5 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix},$$

$$B_7 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

How to make fast quantum gates?

Pulse shaper for QUNATUM INFORMATION



◊ Basic properties of chirped pulses

Time domain

$$E(t) = E_0 \exp \left[-\frac{t^2}{2\tau^2} - i\omega_0 t - i\alpha \frac{t^2}{2} \right]$$

Frequency domain

$$E(\omega) = E'_0 \exp \left[-\frac{(\omega - \omega_0)^2}{2\Gamma^2} + i\alpha' \frac{(\omega - \omega_0)^2}{2} \right]$$

E_0 is the peak amplitude;

$\tau\sqrt{\ln 16}$ is the pulse duration;

ω_0 is the center frequency;

α is the temporal chirp;

E'_0 is the peak amplitude;

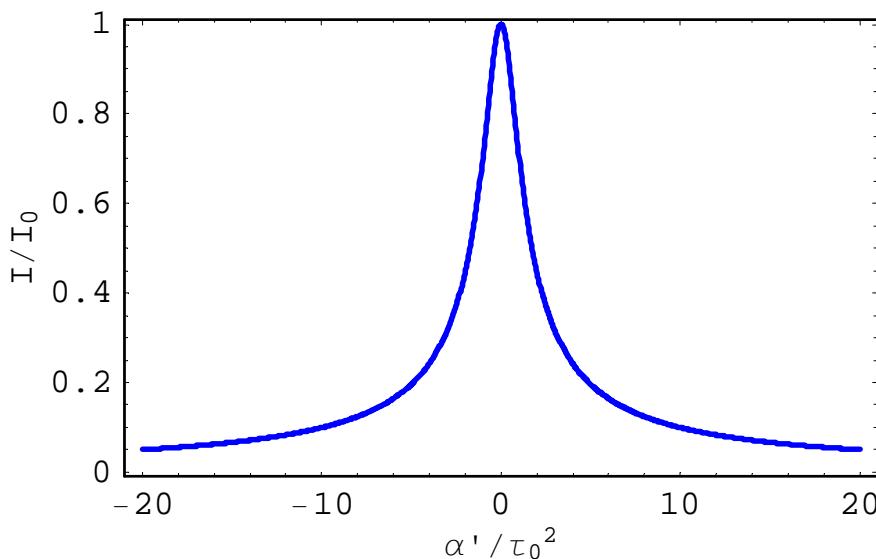
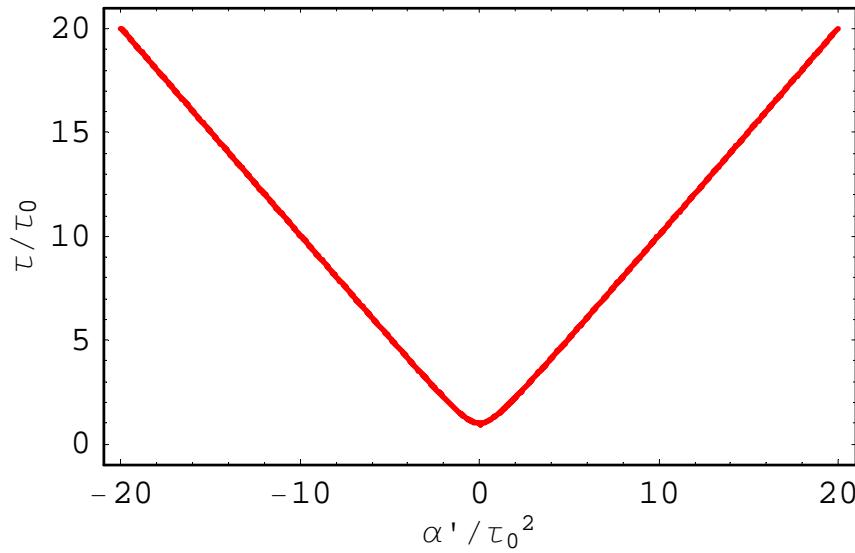
$\Gamma\sqrt{\ln 16}$ is the frequency bandwidth;

ω_0 is the center frequency;

α' is the spectral chirp;

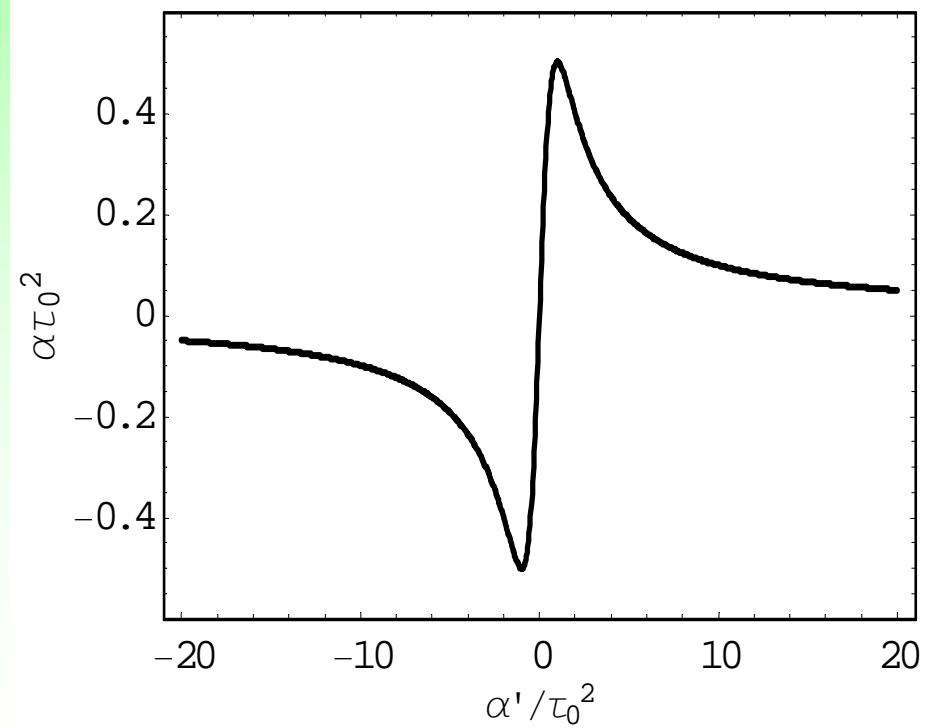
Transform-limited pulse $\rightarrow \alpha = \alpha' = 0, \Gamma = \frac{1}{\tau_0}$

◊ Things to remember about chirped pulses

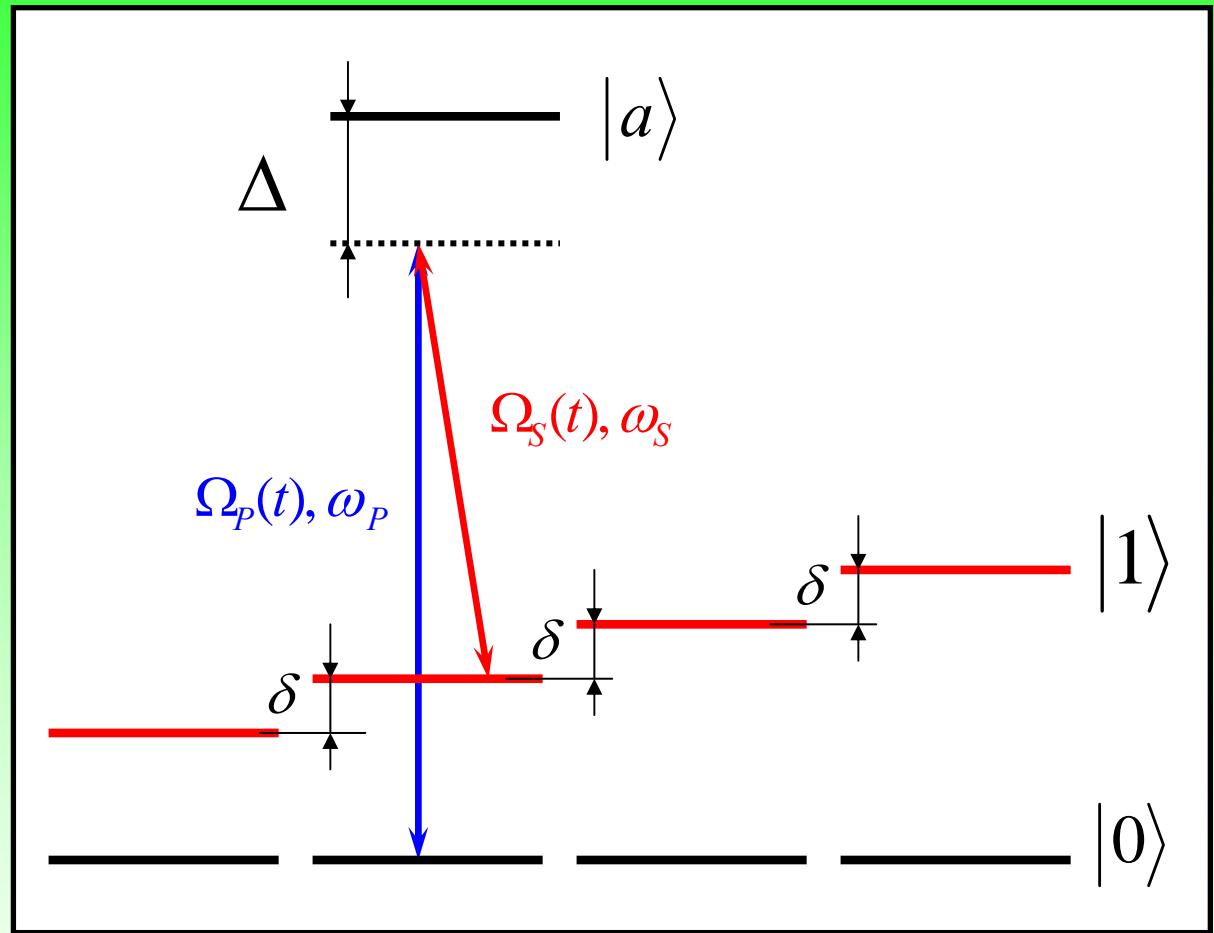
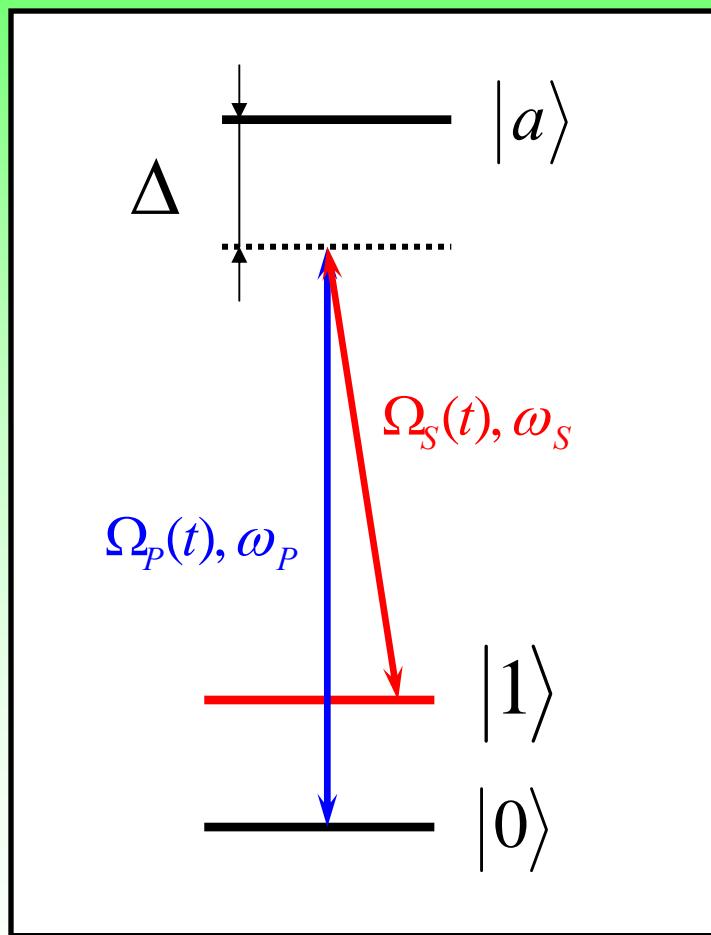


We consider that chirp is applied to the pulse using conventional linear optics, e.g. via a grating or prism pair. The pulse energy is conserved, the bandwidth is fixed.

$$P_0 = E_0^2 \tau = \text{const}, \Gamma = \frac{1}{\tau_0} = \text{const}$$



Quantum register



D. DeMille. *Quantum computation with trapped polar molecules*. Phys. Rev. Lett. 88, 067901 (2002).

$$i \frac{d}{dt} \begin{pmatrix} a_0 \\ a_1 \\ b \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\Omega_p(t) \\ 0 & E_1 & -\Omega_s(t) \\ -\Omega_p(t) & -\Omega_s(t) & E_2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ b \end{pmatrix}$$

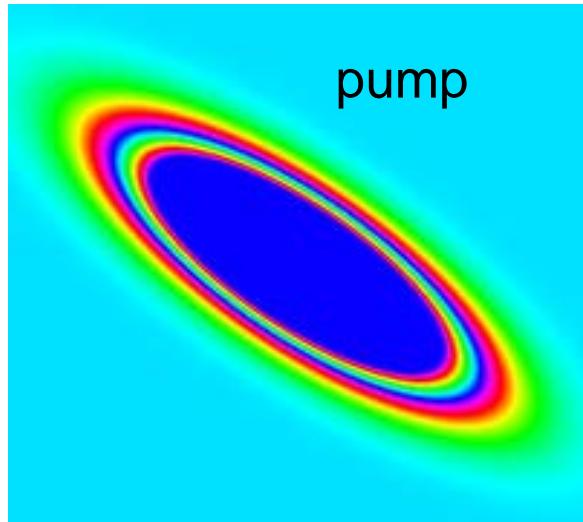
$$\Omega_p(t) = \Omega_{p0}(t) \cos\left(\omega_p t + \frac{\alpha t^2}{2}\right), \quad \Omega_s(t) = \Omega_{s0}(t) \cos\left(\omega_s t + \frac{\beta t^2}{2}\right)$$

In the RWA, after adiabatic elimination of the excited state we have

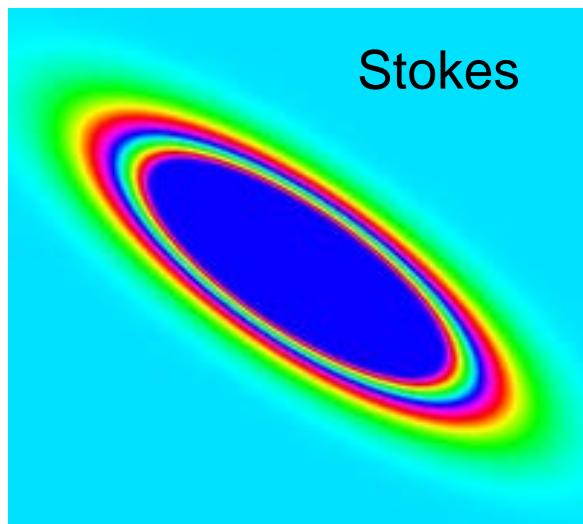
$$i \frac{d}{dt} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \left\{ \frac{1}{2} \left[\delta + (\alpha - \beta)t - \frac{\Omega_{p0}^2(t) - \Omega_{s0}^2(t)}{4\Delta} \right] \hat{\sigma}_z - \frac{\Omega_{p0}(t) \Omega_{s0}(t)}{4\Delta} \hat{\sigma}_x \right\} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

◊ Wigner plots of the chirped pulses

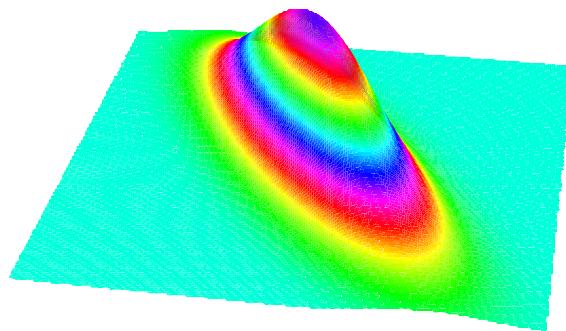
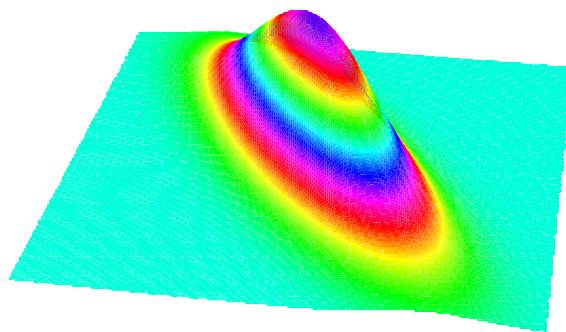
frequency



frequency



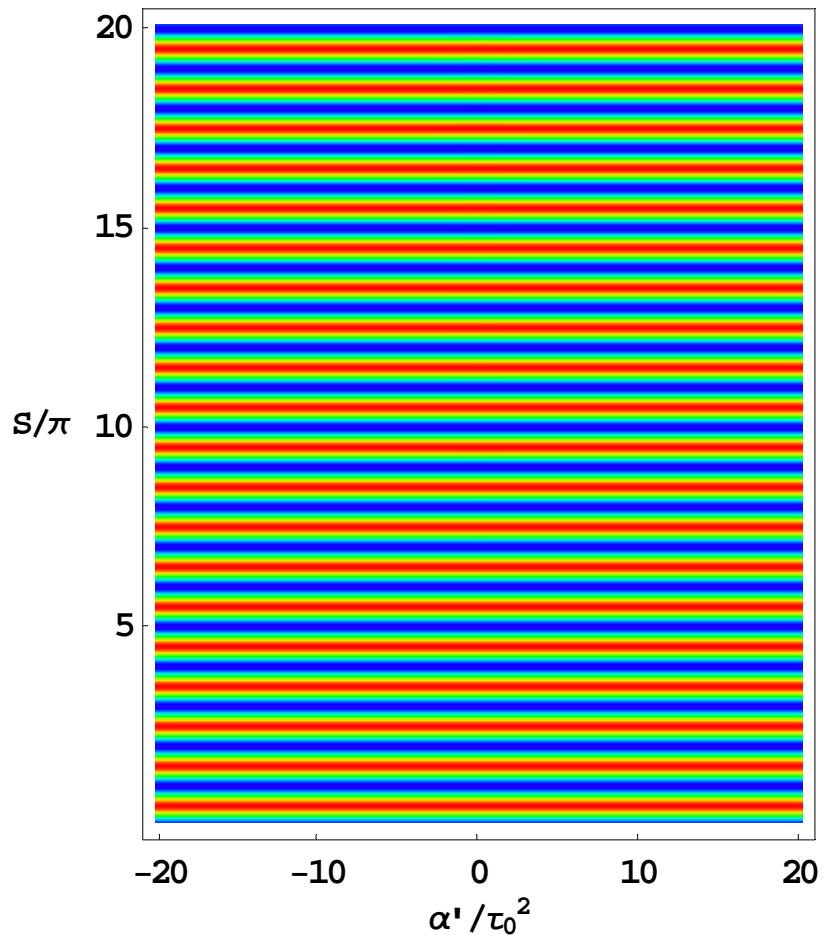
$$\alpha = \beta$$



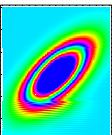
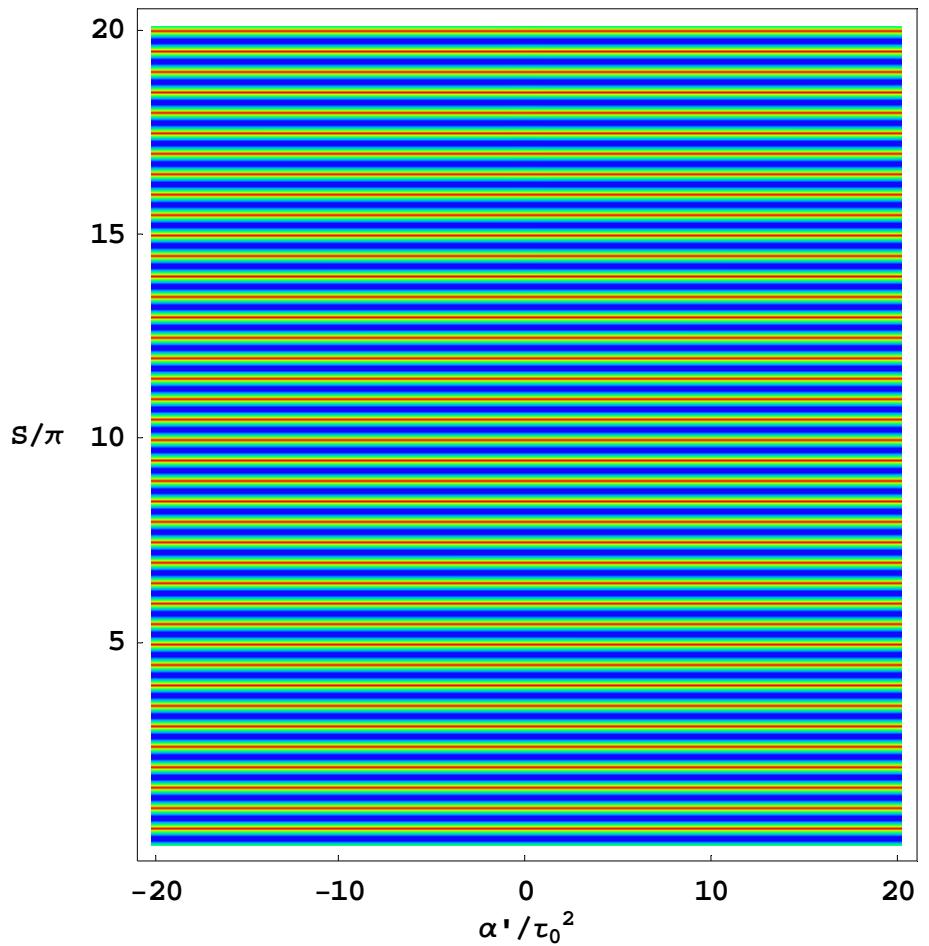
◊ Resonant system, $\delta = 0$



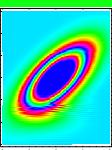
Excited state population



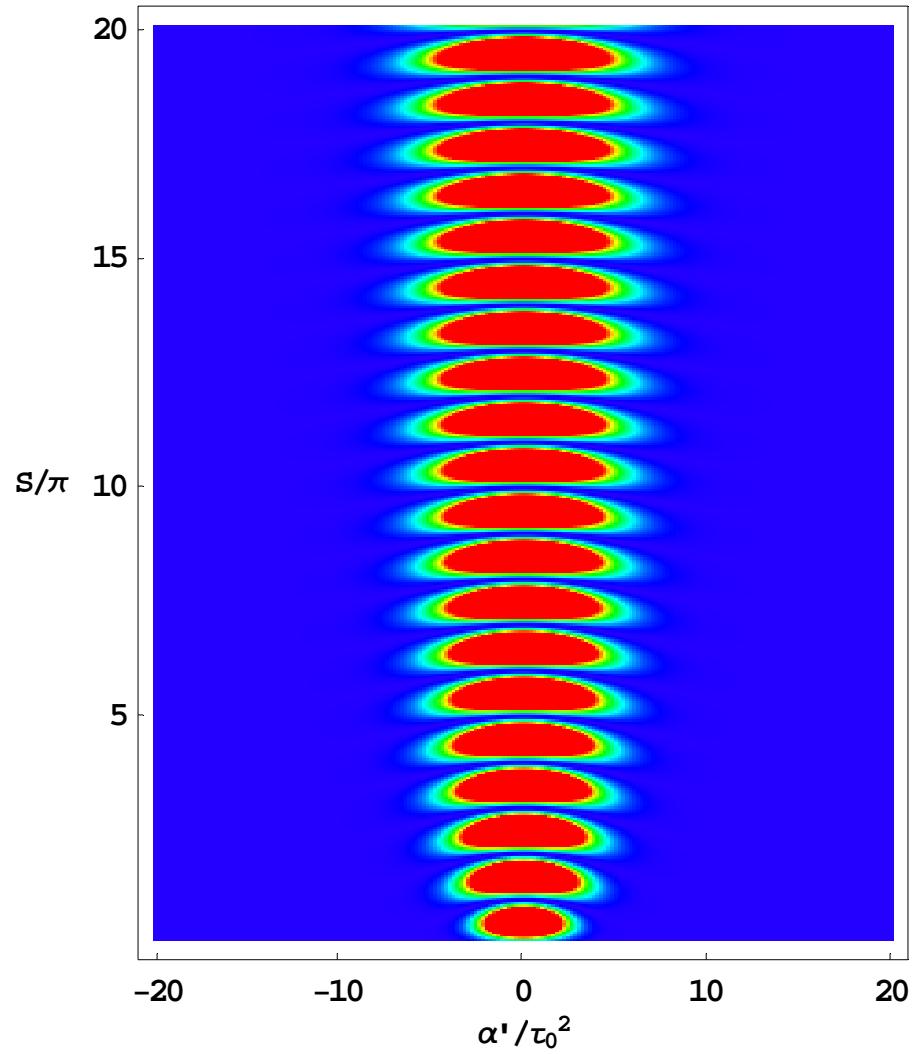
coherence



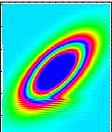
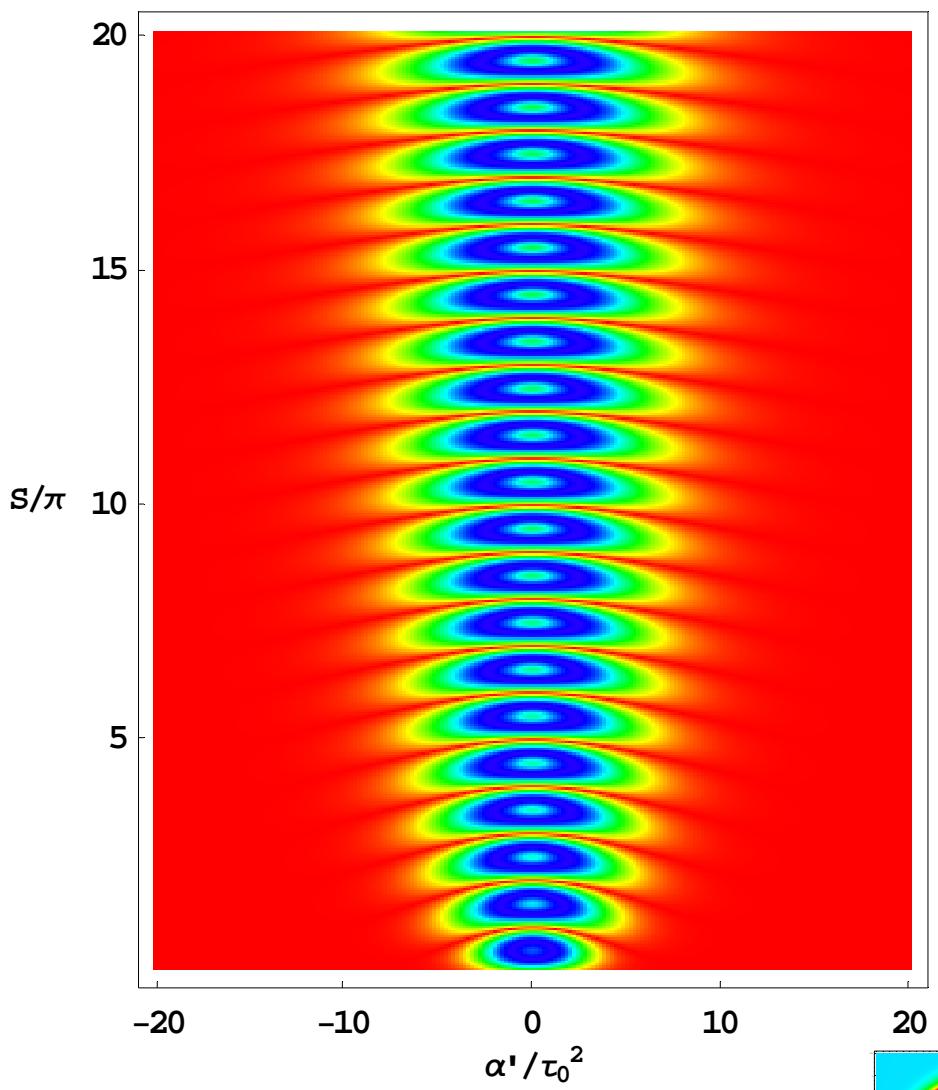
◊ Off-resonant system, $\delta\tau_0 = 0.75$



Excited state population



coherence



◊ Single qubit gates

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{Hadamard}$$
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{Pauli} - X$$
$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{Pauli} - Y$$
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{Pauli} - Z$$

$$U(t) = \begin{pmatrix} \cos \xi(t) & i \sin \xi(t) \\ -i \sin \xi(t) & \cos \xi(t) \end{pmatrix}$$

where $\xi(t) = \int_{-\infty}^t \Omega_e(t') dt'$

Evolution operator

$I : \delta = 0 \leftrightarrow$ resonant case;

$$\begin{aligned} i\dot{a}_0 &= -\Omega_e(t)a_1, \\ i\dot{a}_1 &= -\Omega_e(t)a_0, \end{aligned}$$

where $\Omega_e(t) = \frac{\Omega_{p0}(t)\Omega_{s0}(t)}{4\Delta}$

$$U(t) = \begin{pmatrix} \cos S(t) & -i \sin S(t) \\ -i \sin S(t) & \cos S(t) \end{pmatrix}$$

where $S(t) = \int_{-\infty}^t \Omega_e(t') dt'$

Gates

$$U^{\pi/4}(\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$U^{\pi/2}(\infty) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

Evolution operator

$H : \delta \neq 0 \leftrightarrow$ off-resonant case;

$$U(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\xi(t)} \sqrt{1 + \frac{\delta}{\sqrt{\delta^2 + 4\Omega_e^2(t)}}} & e^{-i\xi(t)} \sqrt{1 - \frac{\delta}{\sqrt{\delta^2 + 4\Omega_e^2(t)}}} \\ e^{i\xi(t)} \sqrt{1 - \frac{\delta}{\sqrt{\delta^2 + 4\Omega_e^2(t)}}} & e^{-i\xi(t)} \sqrt{1 + \frac{\delta}{\sqrt{\delta^2 + 4\Omega_e^2(t)}}} \end{pmatrix}$$

where $\xi(t) = \frac{1}{2} \int_{-\infty}^t \sqrt{\delta^2 + 4\Omega_e^2(t')} dt'$

Phase gate

$$U(\infty) = \begin{pmatrix} e^{i\xi(\infty)} & 0 \\ 0 & e^{-i\xi(\infty)} \end{pmatrix}$$

◊Conclusions

- ◊Adiabatic method to prepare entangled states based on control of relative phase
 - ◊Relative phase is the important parameter which controls a phase of entangled states
 - ◊We demonstrated a pulse-area control method to construct CNOT gate
 - ◊We demonstrated a simple pulse-area control method to construct fast single qubit gates using chirped pulses
 - ◊Looking for experimental realizations
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