SHAKING WAVEPACKE

Suppression of decoherence in a wavepacket

(*with a bucket*)

and

Control of quantum chaos (*with a sieve*)

Evgeny Shapiro

KITP, May 2009

UBC

ADVERTISEMENT

Adiabatic Passage

driven by a few <u>kicks</u>

by femtosecond laser pulses

aka

Coherently Controlled Adiabatic Passage

Talk by Moshe Shapiro @ the conference

Done with

M. Ivanov, M. Spanner, I Walmsley

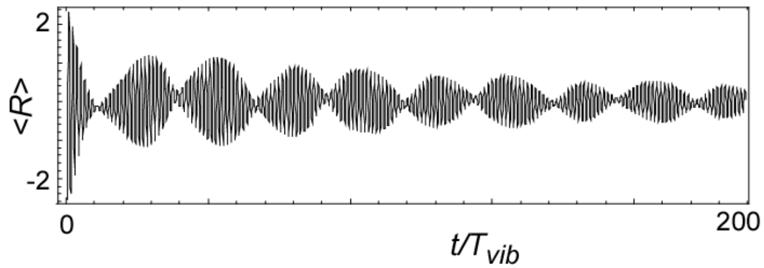
Yu. Billig – theory of wave packet controllability

K. Lee, D. Villeneuve, P. Corkum – experiment on wave packet quantum gates

The plan

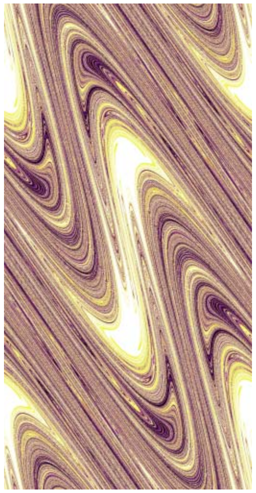
- Intro: Wavepacket QI-QC program

- Suppression of decoherence in a wavepacket *with a bucket*



- Few basics on chaos

- Control of quantum chaos: *Wavepackets in a sieve*



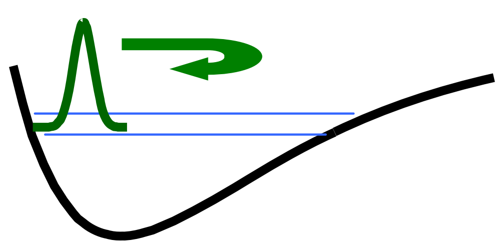
Background: Wavepacket QI-QC program

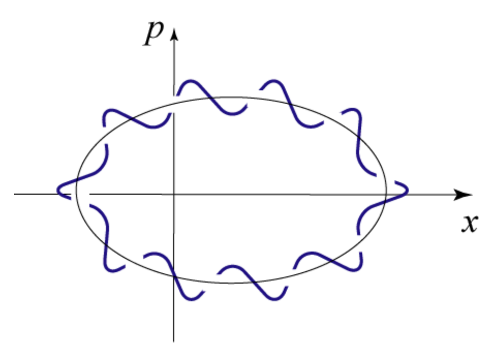
- Number of levels involved is not known, not fixed.
  Amplitudes of the levels are not of interest.
  Track the flow of probability and phase

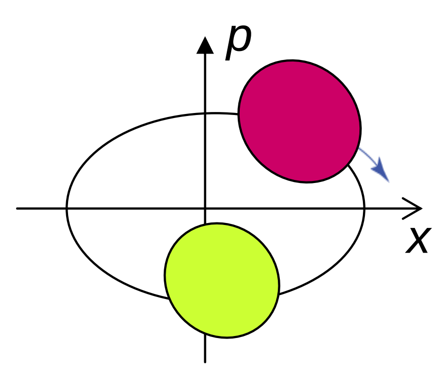
- Look for coarse-grained quantum controls: chunks of phase space.
  Scale with the amount of interesting information, not with the number of levels involved

- Control by applying coordinate-dependent, time-dependent potentials

- Encoding and control robust to initial conditions

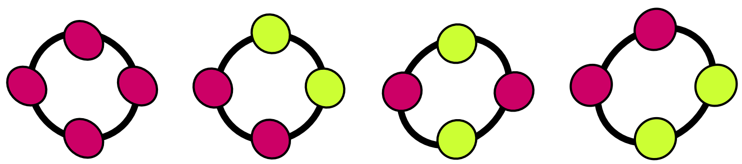






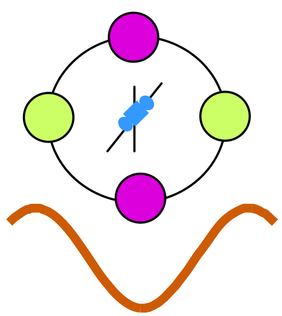
Background: Wavepacket QI-QC program

- Encode bitwise information in symmetries of the wave function *envelope*



PRL **91** 237901 (2003), JMO **52** 897 (2005)

- Control by phase kicks and free evolution



Molecular alignment:

$$V \sim -\cos^2 \theta$$

PRL **92** 093991 (2004); **93** 233601 (2004)

- Controllability with free evolution and *smooth* coordinate dependance of the phase kicks? YES!

JCP **120** 9925 (2004)

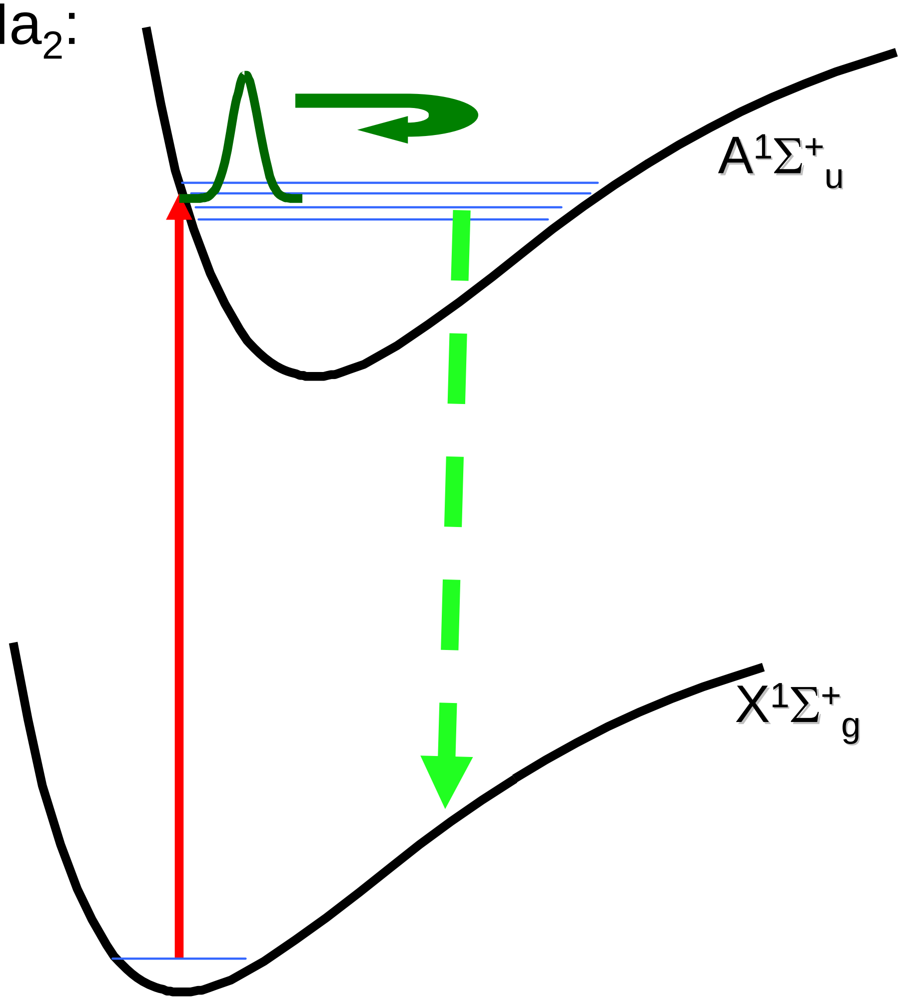
Suppression of decoherence in a wavepacket with the help of a bucket

PRL **98**, 050501 (2007)

Experiment

Na_2, gas at 450^{o}C from heat pipe

Na_2:



excitation of the wavepacket
by short pulse

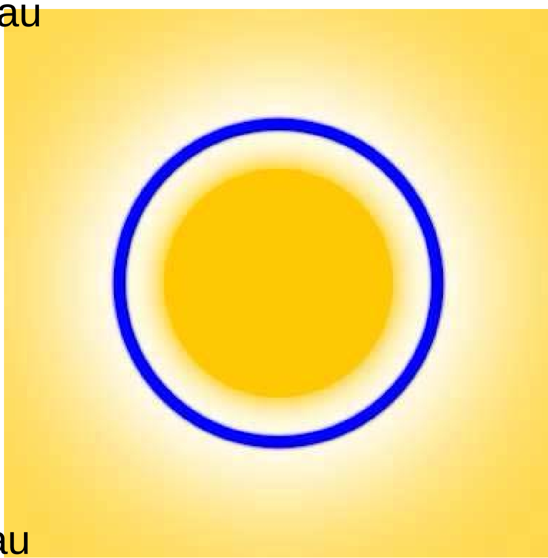
monitoring the state
by emission tomography
$T_{vib} \sim 330\,\mathrm{fs}$

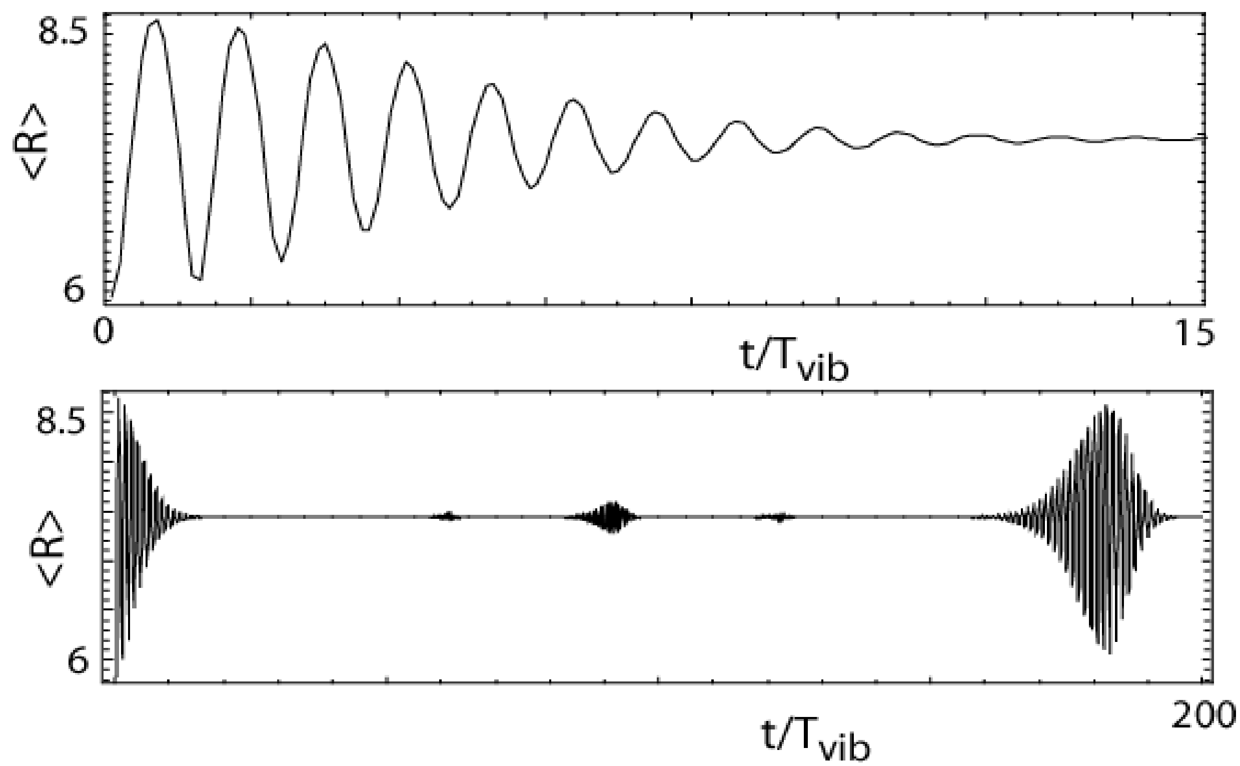
Walmsley, Waxer, JPB **31** 1825 (1998)

Dynamics of the vibrational wavepacket

2D: $$E_{vJ} = \omega_e (v+1/2) - \omega_e x_e (v+1/2)^2 + (B - \alpha_e (v+1/2)) J^2 - D J^4$$

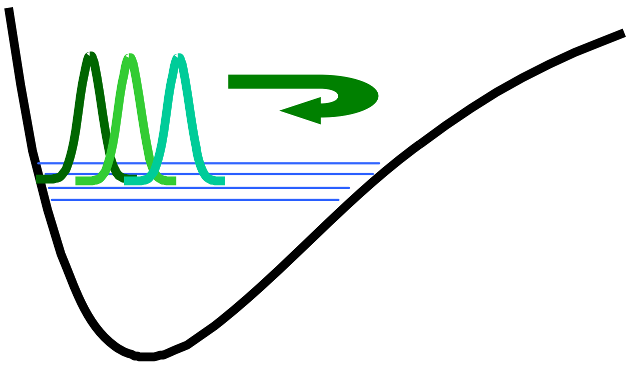
$$\omega_{vib}(v,J) = \omega_e - 2\,\omega_e x_e (v+1/2) - 2\alpha_e J^2$$





Revival: $E_{v+1} - E_v = 2\pi k$ for all v

Temperature brings decoherence

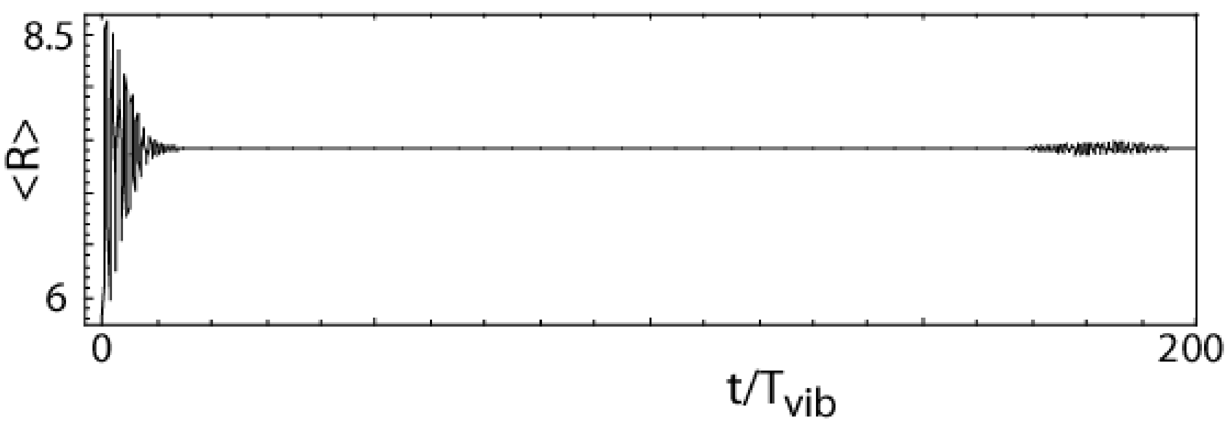


Rotational temperature:

$$\omega_{vib}(v_0,J) = \omega_e - 2\,\omega_e x_e\,(v_0 + 1/2) - \alpha_e J^2$$

oscillators in the hot rotational ensemble mutually dephase

formally = decoherence, $\rho_{vv'}(t) = C_v C_{v'}^* \left(\sum_J P_J e^{i(E_{v'J} - E_{vJ})t} \right)$



T=450°C

$t_{dec} \sim 30\ T_{vib}$

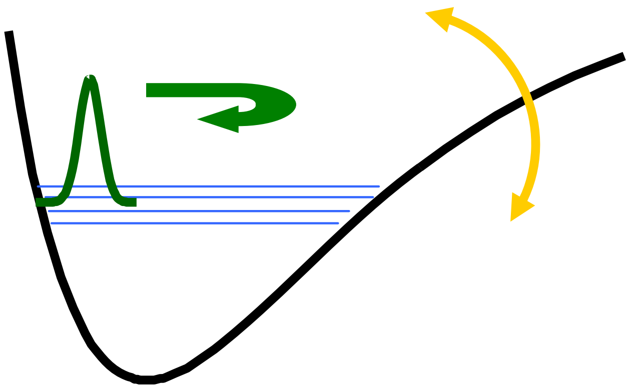
Vibrational temperature in combination with anharmonicity works the same way

Well-known methods

would not work to fight decoherence in a wavepacket

decoherence free subspaces	do not exist here
"bang-bang"	can work well only with few-level systems
methods to stabilize wavepackets against decay	require knowledge of the state to be stabilized and/or carefully arranged level-by-level interferences

Place it in the bucket



Drive it periodically

Nonlinear resonance (‘*bucket*’):

effective potential moving along the resonance phase space orbit.

“Lucky” vs. “unlucky” initial conditions

classical motion:

- with the bucket, along the resonance orbit
- in the bucket, relative to the resonance orbit

Encode information in quantum motion *relative* to the resonance orbit.

This motion will be stabilized

Ideal case: we act only on vibrations

$$H = H_0(R;\theta) + V(R)\cos\Omega t$$

Quantum nonlinear resonance:

- quasienergy states in the rotating frame:

$$\chi(R,t) \sim e^{-i\gamma t}\sum_v C_v(t)e^{-i\Omega(v-v_i)t}$$

- Taylor expand $E(v)$ near v_i,
- envelope: $\Psi(\lambda) = \sum_v C_v e^{i(v-v_i)\lambda} e^{i\kappa\lambda}, \qquad \kappa = \dfrac{\Delta\omega}{2\omega_e x_e}$

$$-\omega_e x_e \Psi'' - V_{v,v\pm1}\cos\lambda\Psi = \left(\frac{\kappa^2}{2} - \gamma\right)\Psi$$

Berman, Zaslavsky, Phys. Lett. **61**A, 295 (1977)
Fedorov, Zh. Exp. Theor. Phys. **73**(1) 134 (1977)

Decoupling

$$-\omega_e x_e \Psi'' - V\cos\lambda\Psi = \left(\frac{\kappa^2}{2} - \gamma\right)\Psi \qquad \kappa = \Delta\omega \big/ 2\omega_e x_e$$

different states in the initial thermal ensemble => different detunings => different excitations in the “bucket” lattice

Buckets are nearly harmonic at the bottom

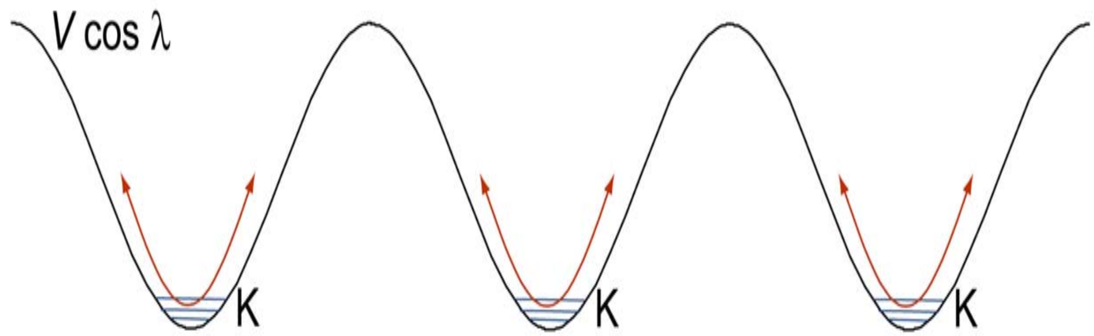
different initial (J, v_0) states have the same frequency in the bucket and so do not decohere

excitations are near the bottom:

$$\Delta\omega_T << 2\sqrt{V\omega_e x_e}$$

excitations are in a single QE zone:

$$\Delta\omega_T < 32^{1/4} V^{1/4} \omega_e x_e^{3/4}$$

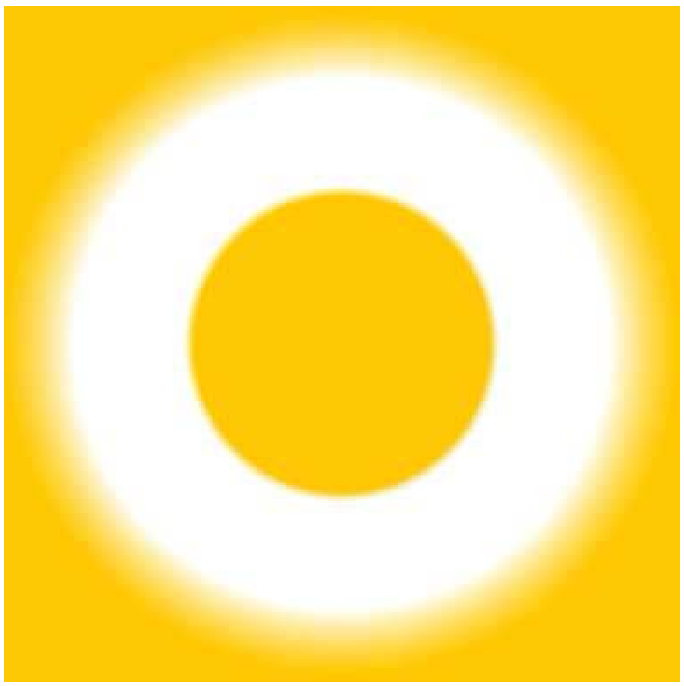


Driving by polarizability

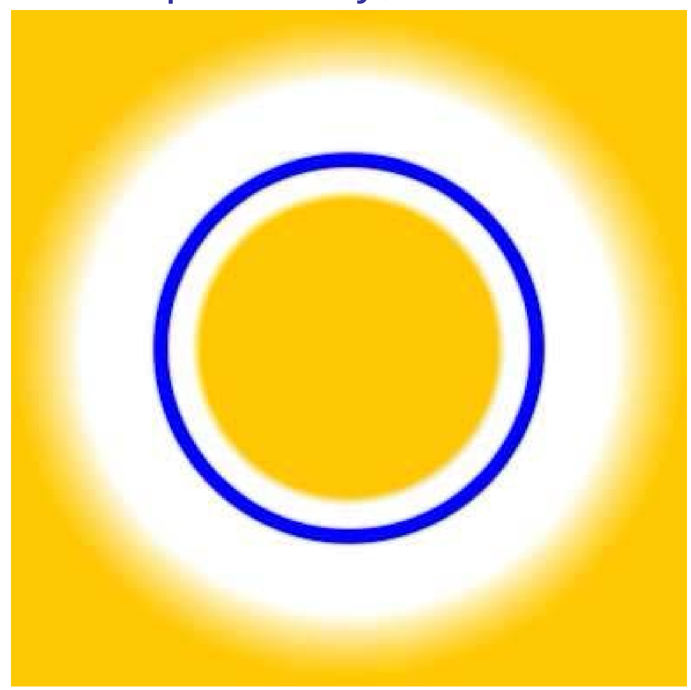
two beams, frequencies $\omega_{\mathrm{L}} \pm \Omega$, $I_1 = I_2 = I/2$

$$H = H_0(R,\theta) - \frac{E^2}{2}\cos^2\frac{\Omega t}{2}\left(\alpha_\perp(R)\sin^2\theta + \alpha_\parallel(R)\cos^2\theta\right)$$

potential



probability, $J = 48$

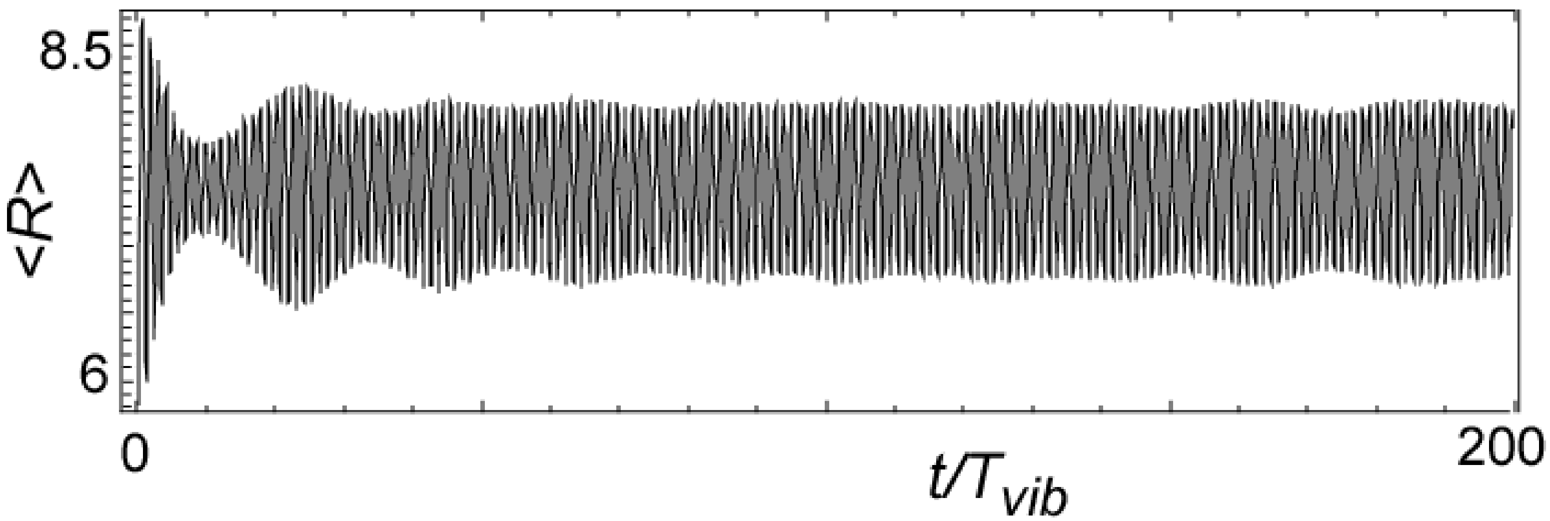


Na_2: $I = 2 + 10^{11}$ W/cm^2

linear approximation for $\alpha(R)$ near R_0

polarizability from Dr. S. Patchkovskii, NRC

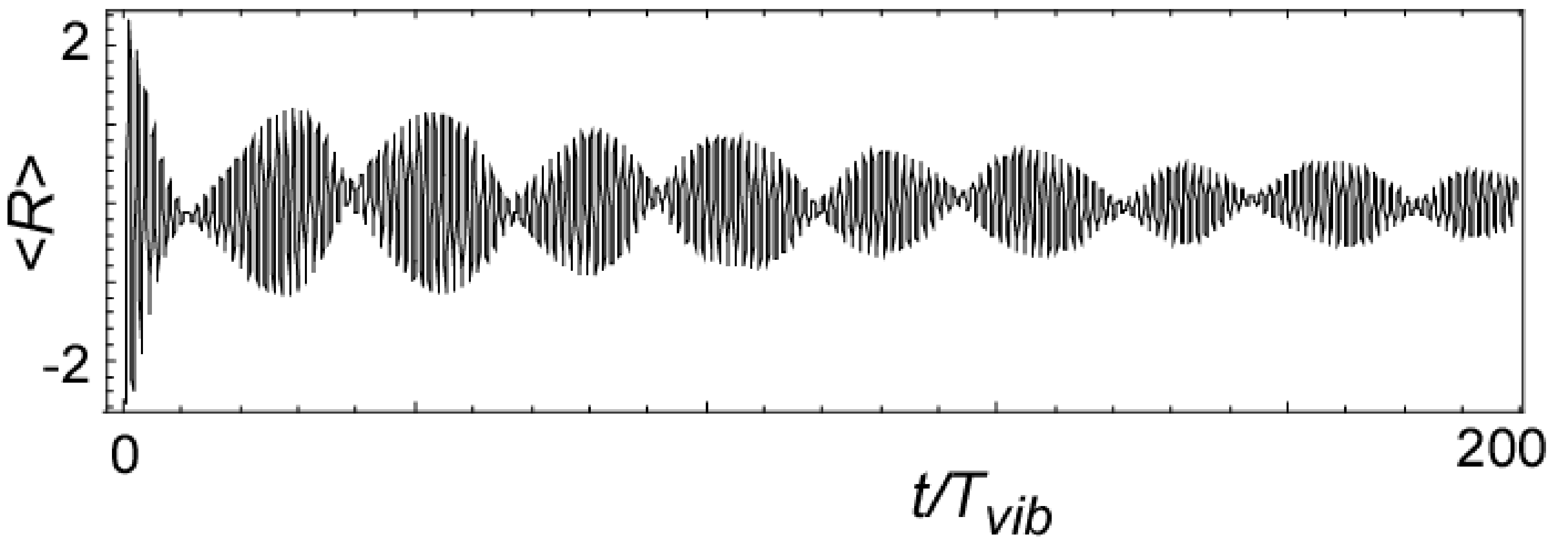
In the bucket



weighted with rotational temperature

signal for the WP on $A^1\Sigma^+_u$ excited at $\lambda = 0$ $(\Omega t_0 = 0)$

In the bucket



difference of signals for $\lambda = \pi/3$ and $\lambda = -\pi/3$, T = 450°C

Time scales:

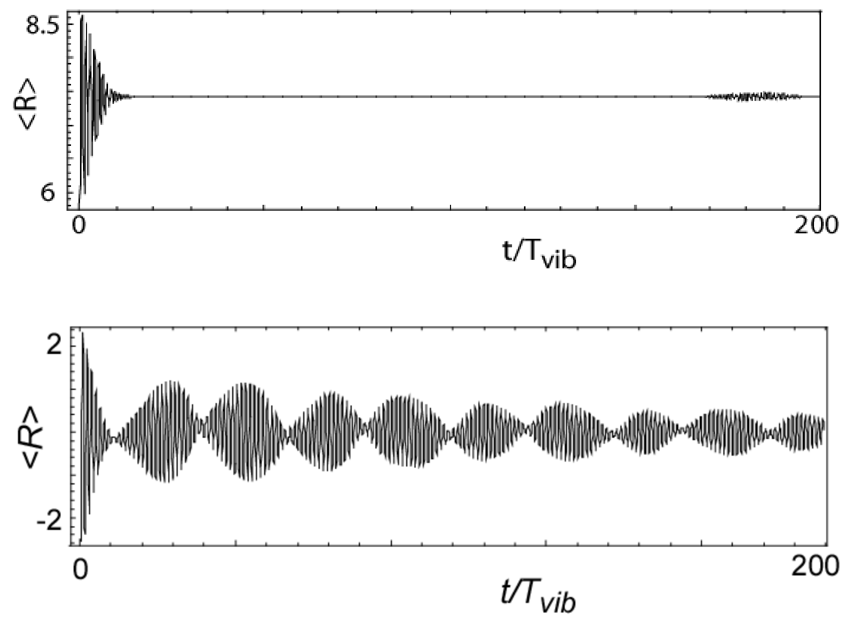
- oscillations with the bucket
- oscillations inside the bucket
- spreading inside the bucket

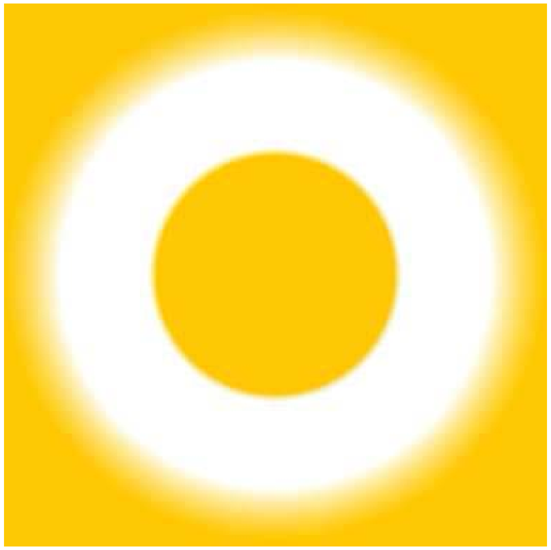
Non-ideal case

$$H = H_0(R,\theta) - \frac{E^2}{2}\cos^2\frac{\Omega t}{2}\left(\alpha_\perp(R) + \Delta\alpha(R)\cos^2\theta\right)$$

- angular dynamics
- different *J* behave feel different buckets
- additional ro-vibrational coupling

The scheme still works





Conclusions

- WP case: strong off-resonance field to modify the potential.
- Not only the non-linear resonance stabilizes the motion, but it also can suppress the thermal dephasing: Different initial conditions are transferred into different excitations in the nearly harmonic bucket.
- In the non-ideal case the effect can still work.

The plan

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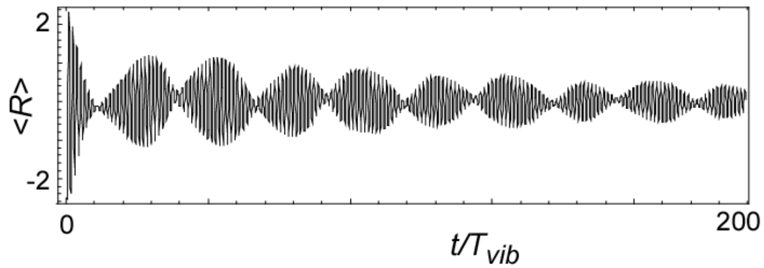
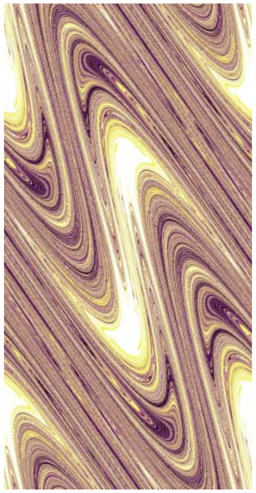


you are here

- Few basics on chaos

- Control of quantum chaos: *Wavepackets in a sieve*



Kicked rotor

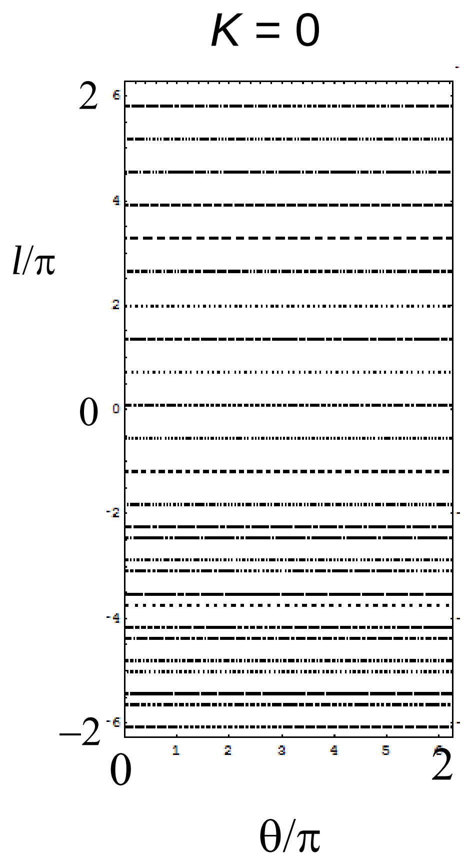
$$H = \frac{L^2}{2I} + A\cos\theta \sum_n \delta\left(\frac{t}{T} - n\right)$$

$$K = \frac{AT^2}{I}; \qquad l_n = \frac{L_n T}{I}$$

$$l_{n+1} = l_n + KT\sin\left(\theta_n + l_n/2\right)$$

$$\theta_{n+1} = \theta_n + \left(l_{n+1} + l_n\right)/2$$

the standard map

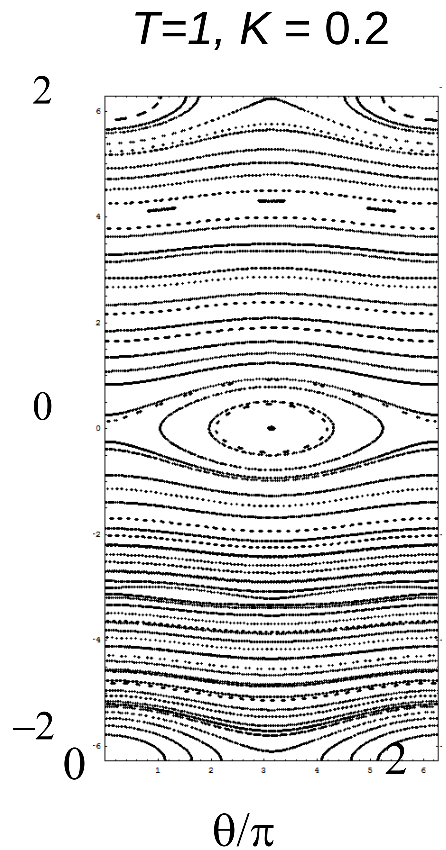


The perturbation scales with KT

stroboscopic map
aka Poincare section

Resonances

$$H = \frac{L^2}{2I} + A\sum_n \cos\left(\theta - j\omega t\right)$$

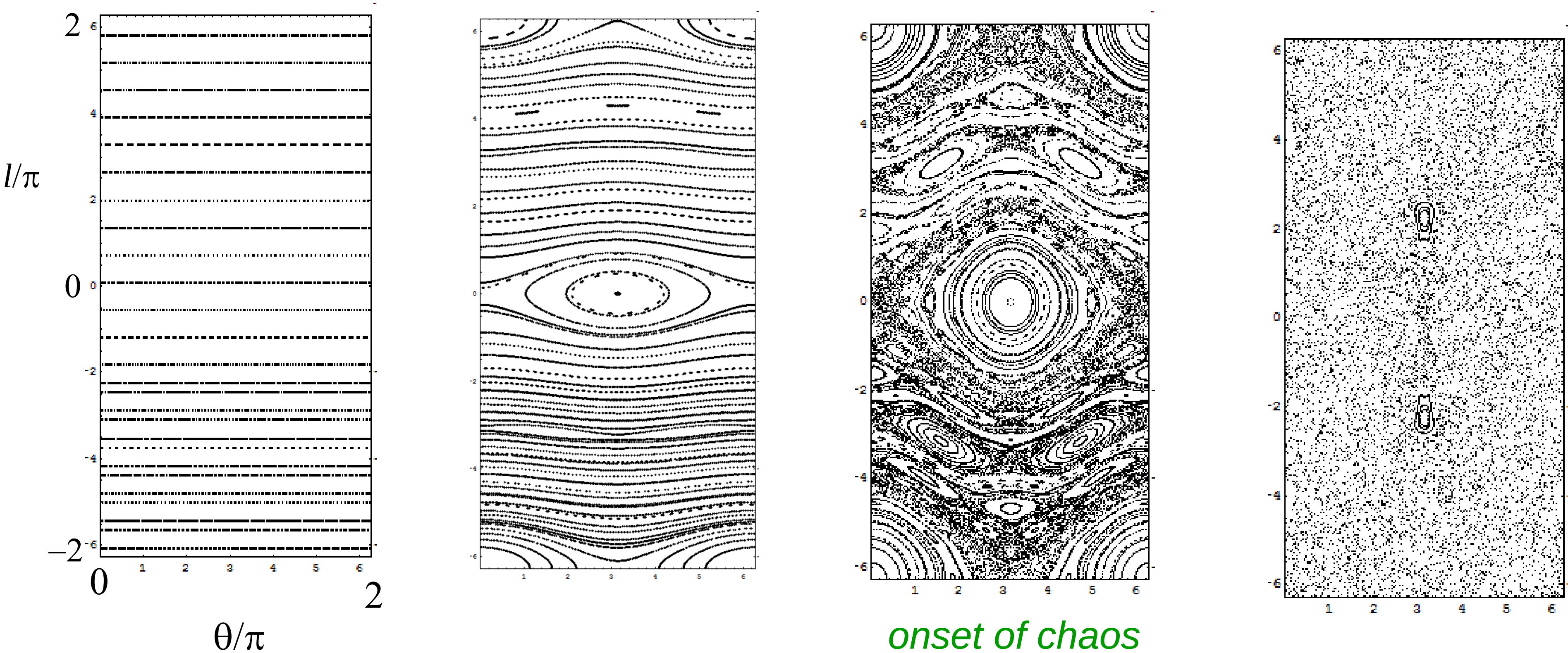


From regular motion to chaos

As *K* grows, the resonances grow and overlap

The sea of chaos and the resonance islands

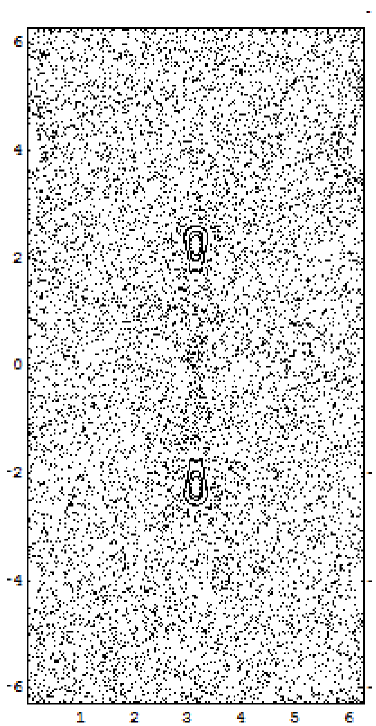
K = 0, T=1 | *K* = 0.2 | *K* = 1 | *K* = 5



$$\langle E_n \rangle = \frac{1}{M} \sum_M \frac{L_n^2}{2I} \sim Dn$$

Quantum vs. classical diffusion

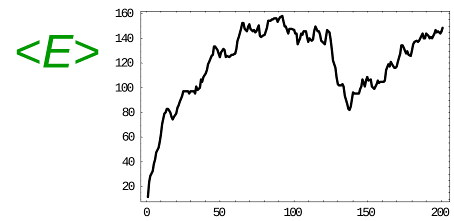
Classical



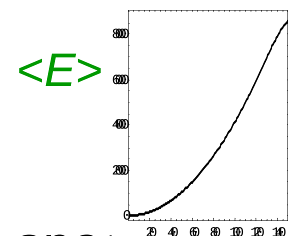
$$\langle E_n \rangle \sim Dn$$

Quantum

- saturation after $t \sim 1/\Delta E_{Floquet}$



- *quantum resonances at $T = 4\,\pi\, m/n$*



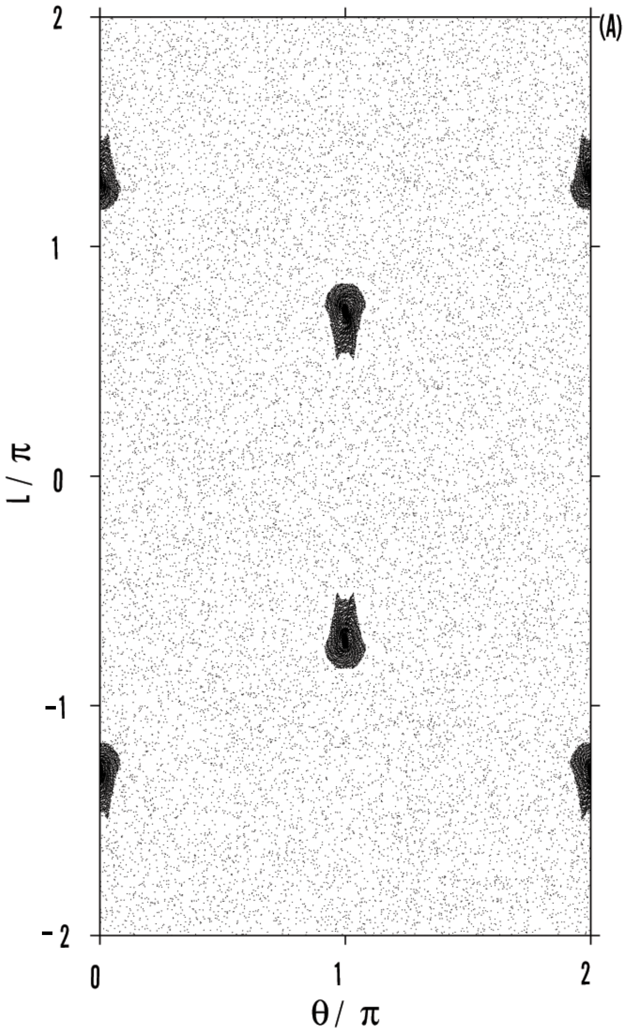
- *structures in phase space:*
  - *cantori impede the diffusion*
  - *"scarred" eigenstates*

Can one control quantum states under strong chaoticity?

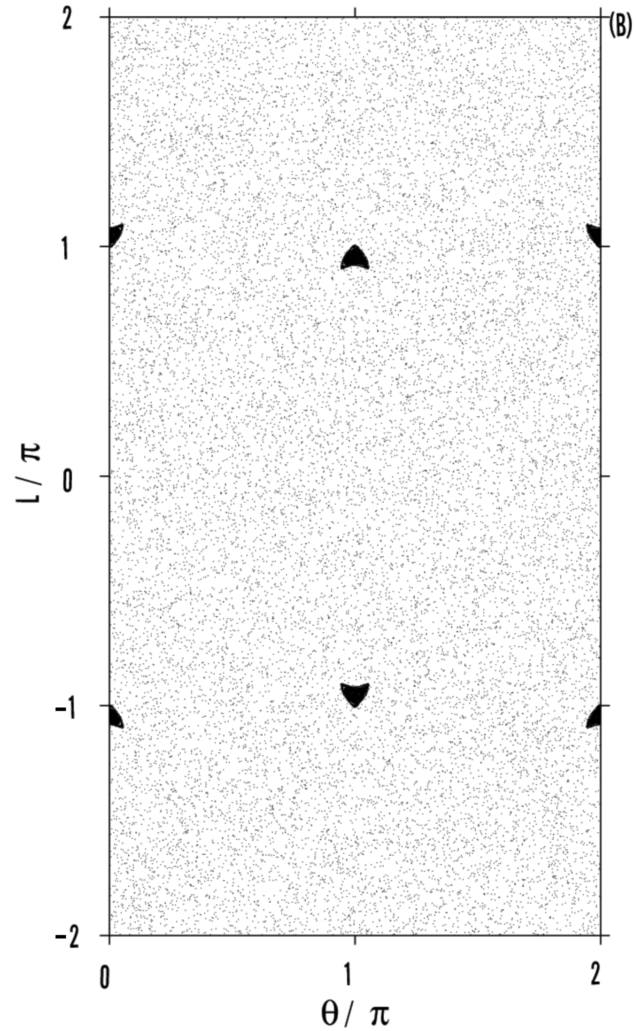
Wave packets in a sieve: quantum control at the edge of strong chaos

J. Mod. Optics **54** 2161 (2007)

Poincare sections

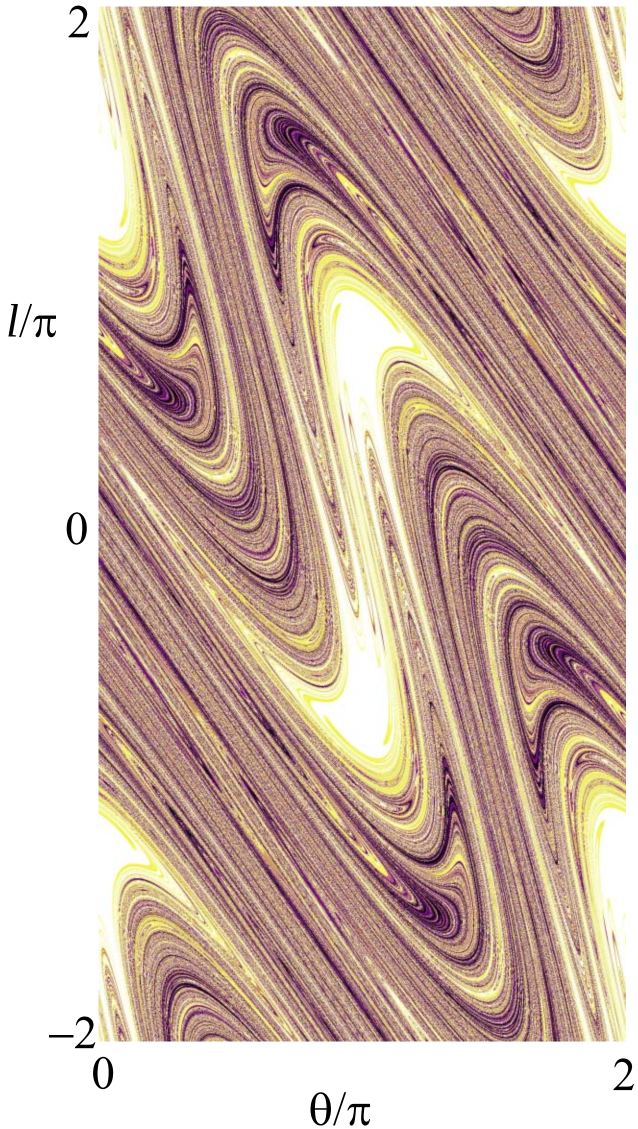


$T = 1; K = 5$

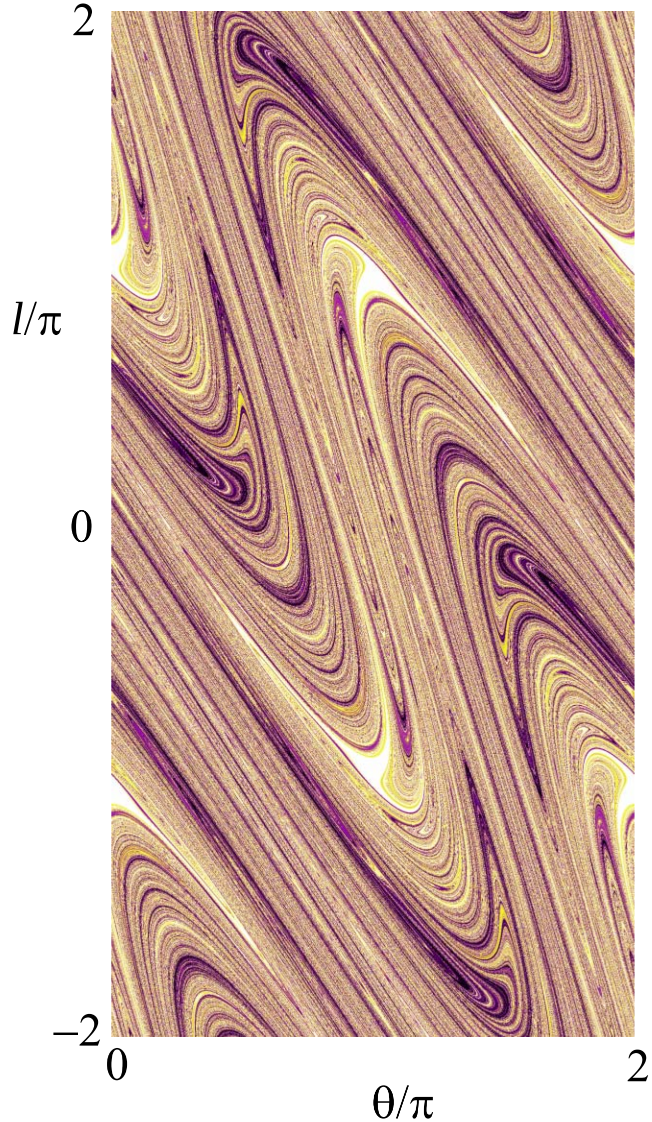


$T = 1; K = 6$

Diffusion rate is not uniform!

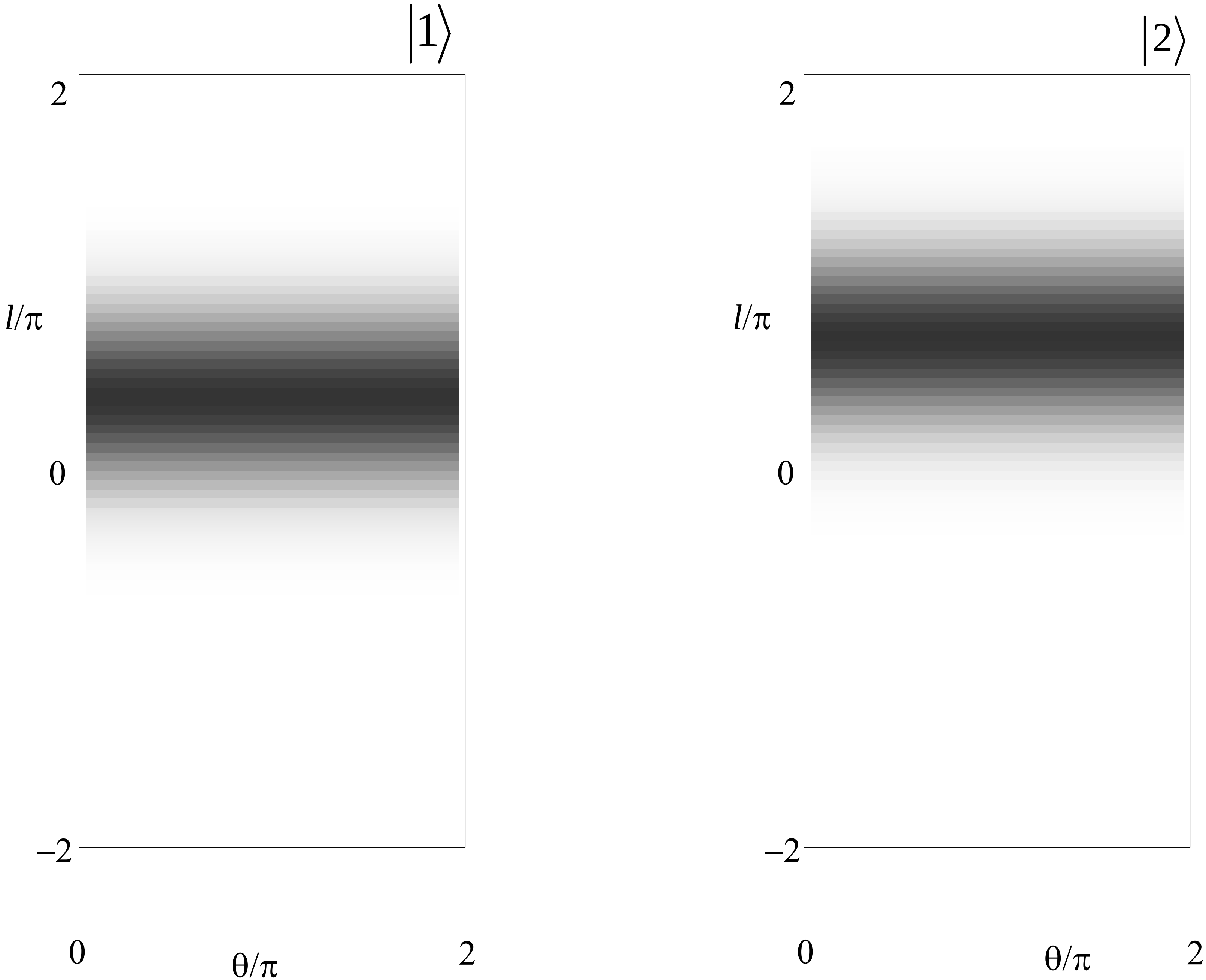


$T = 1$; $K = 5$



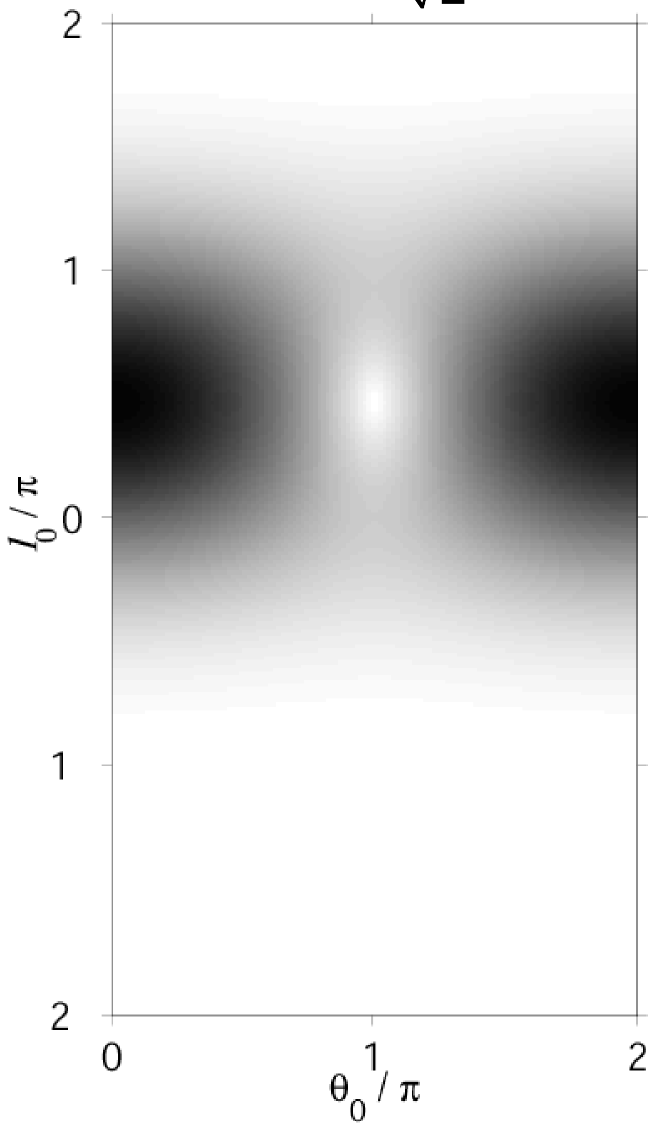
$T = 1$; $K = 6$

Husimi distributions.

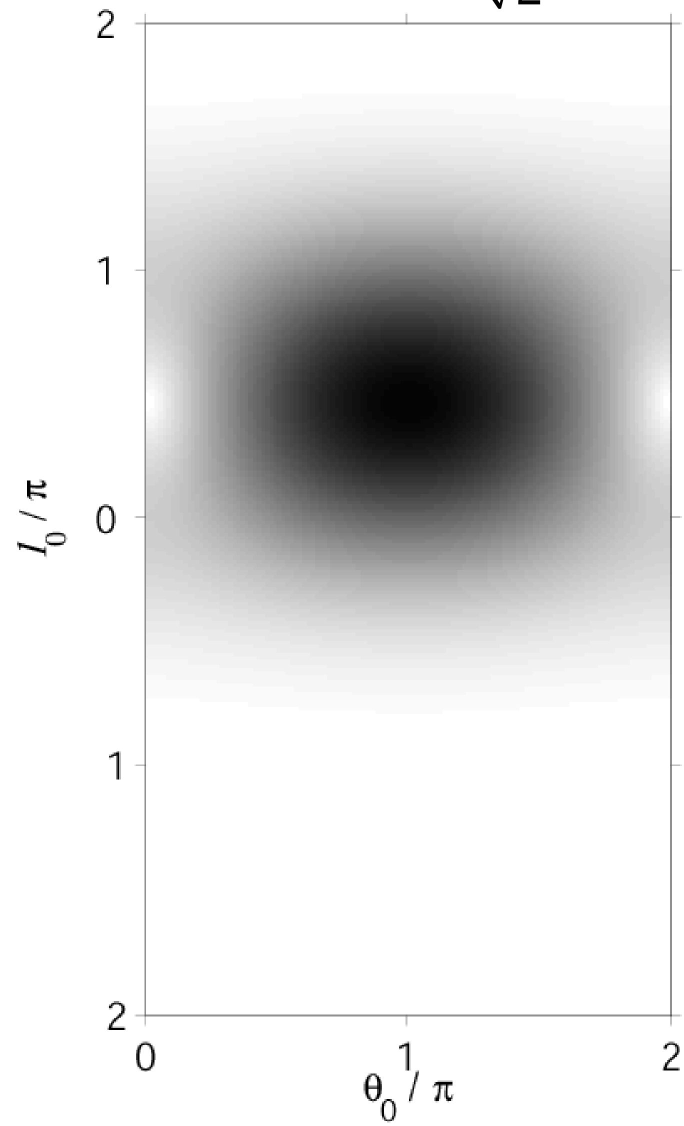


Husimi distributions.

$$\Psi_+ = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$$

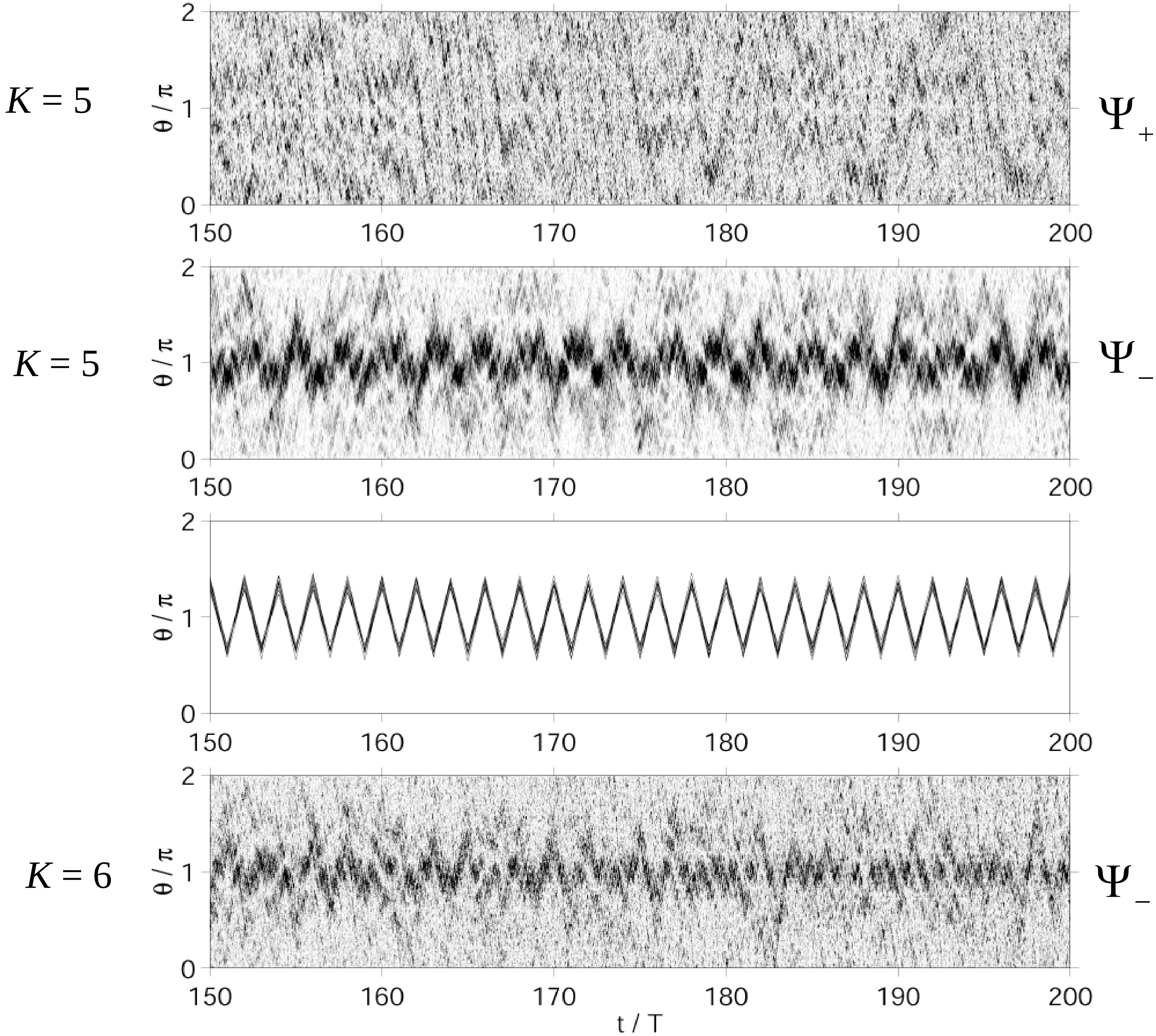


$$\Psi_- = \frac{1}{\sqrt{2}}\left(|1\rangle - |2\rangle\right)$$

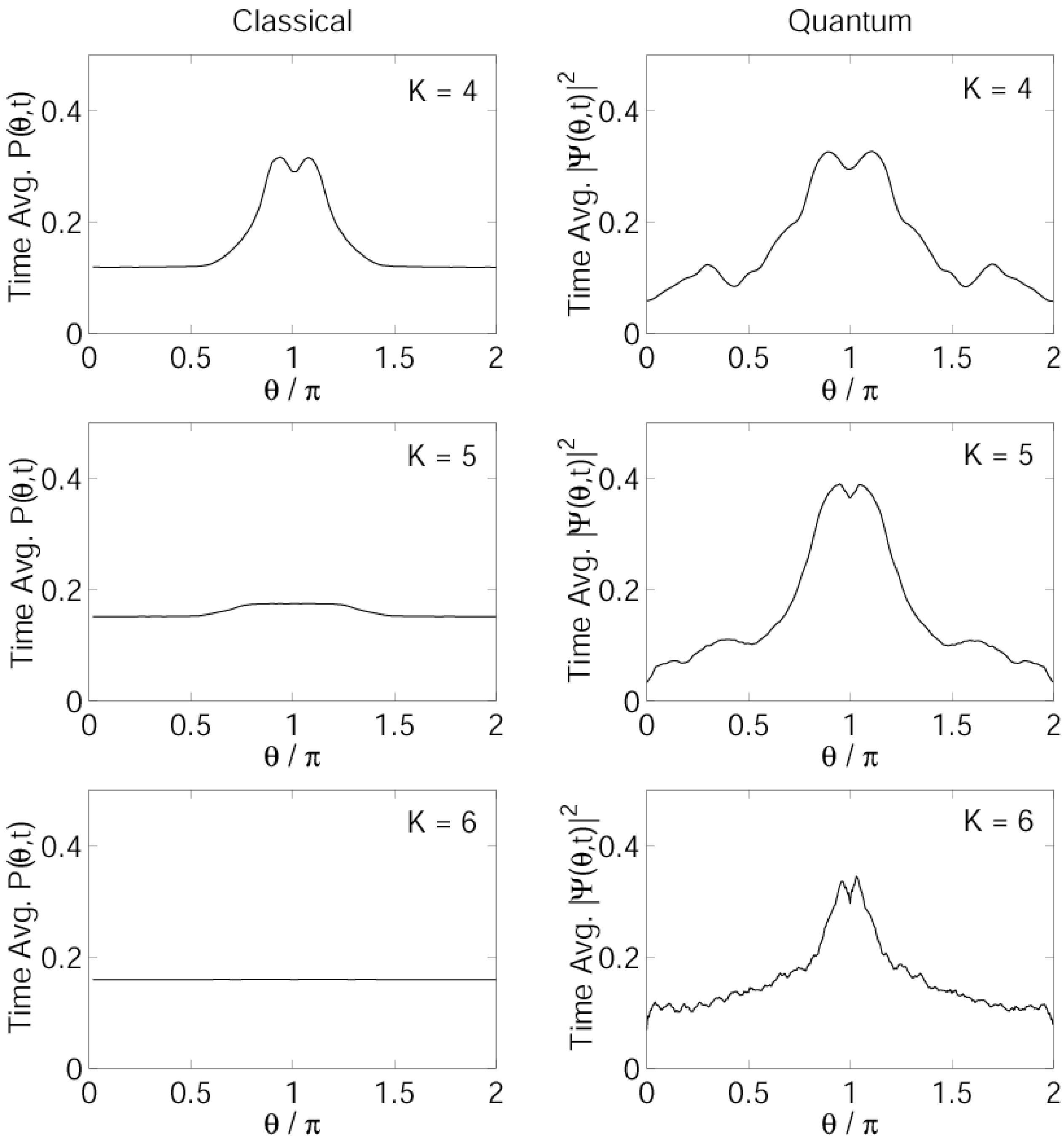


Wave function is much wider than the stable islands

Dynamics

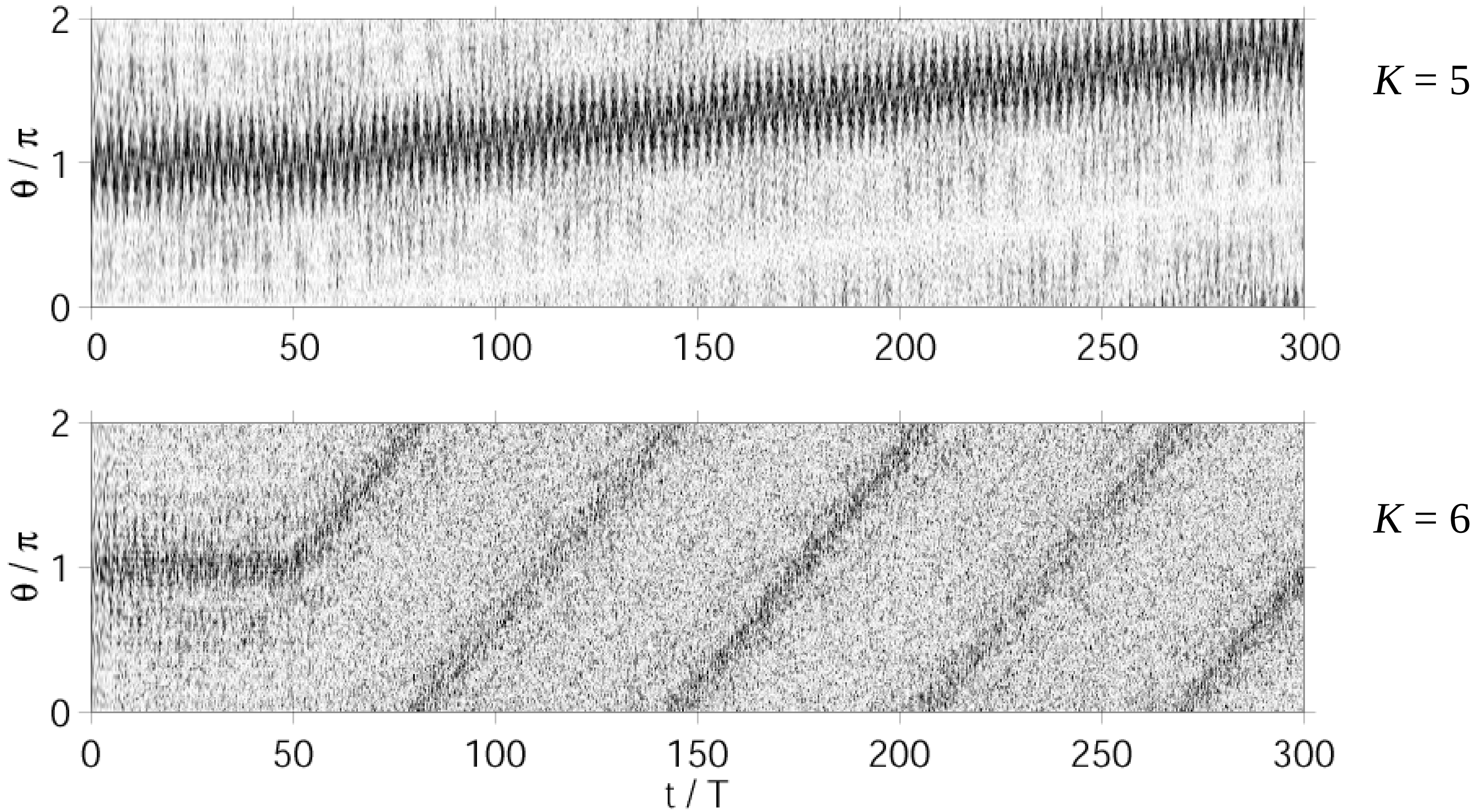


Quantum vs. classical localization

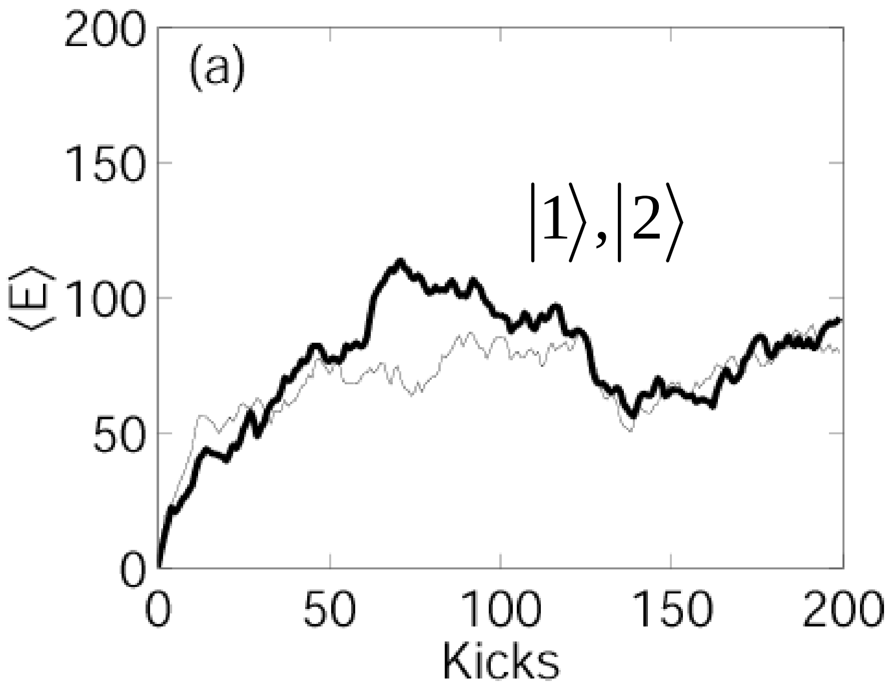


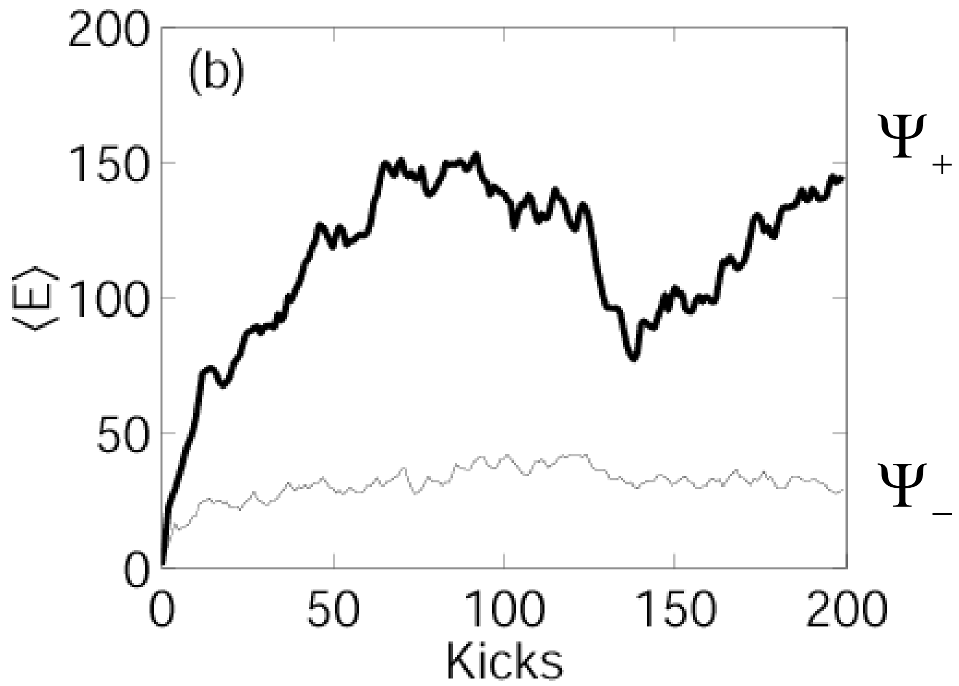
$$\left\langle \left| \Psi_{-} \right|^{2} \right\rangle_{t=100..200}$$

Drag the low-diffusion areas across the phase space



Diffusion in energy

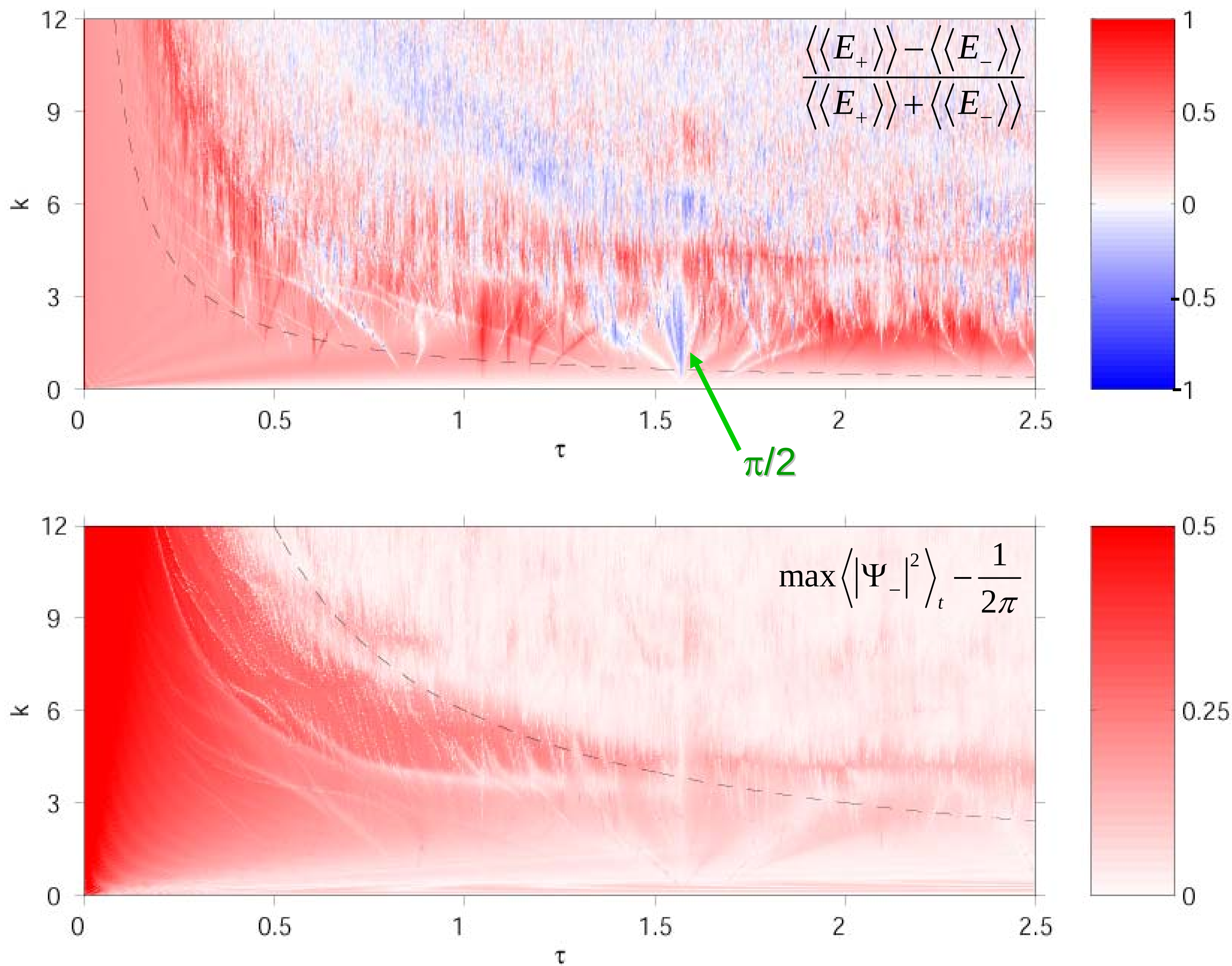




Coherent control of quantum chaos?

J. Gong, P. Brumer PRL 86 1741 (2001), JCP 115 3590 (2001).

Diffusion in energy



Conclusions

- The low-diffusion areas of phase space can keep and drag quantum population even after the resonance islands are gone.
- Control over the localization energy is questionable. Most probably, due to quantum resonances.