

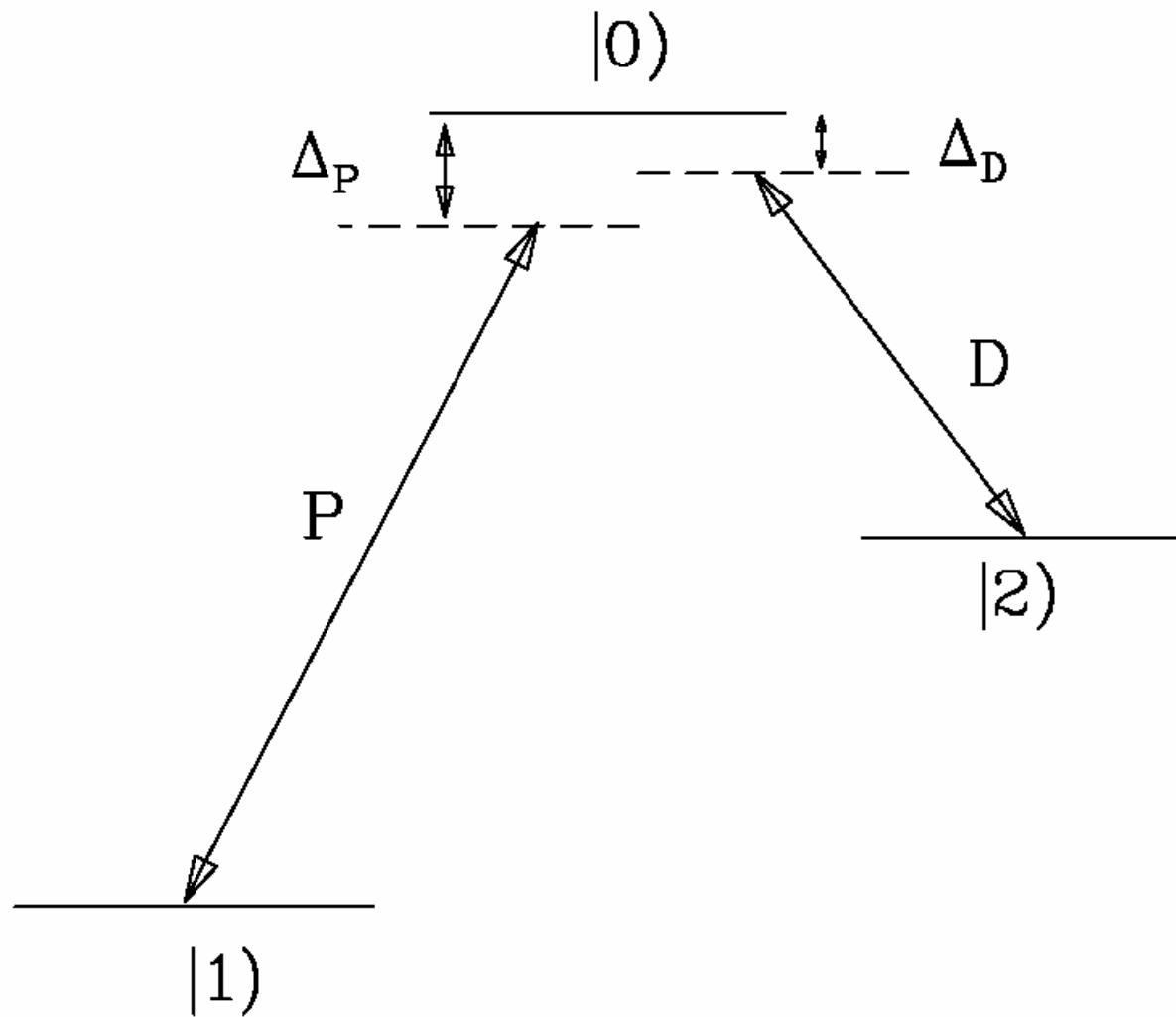
Piecewise adiabatic passage – theory and experiment

Moshe Shapiro, University of British Columbia

KITP Quantum Control, Jun 1, 2009

Piecewise adiabatic passage – theory and experiment

Three level adiabatic passage



$$H=H_M-2\mathbf{d}_1\cdot\hat{\boldsymbol{\epsilon}}_P\varepsilon_P(t)\cos(\omega_Pt)-2\mathbf{d}_2\cdot\hat{\boldsymbol{\epsilon}}_D\varepsilon_D\cos(\omega_Dt)$$

$$\left|\,\Psi(t)\,\right\rangle=\sum_{i=1,0,2}b_i(t)|\psi_i\rangle e^{-iE_it/\hbar}$$

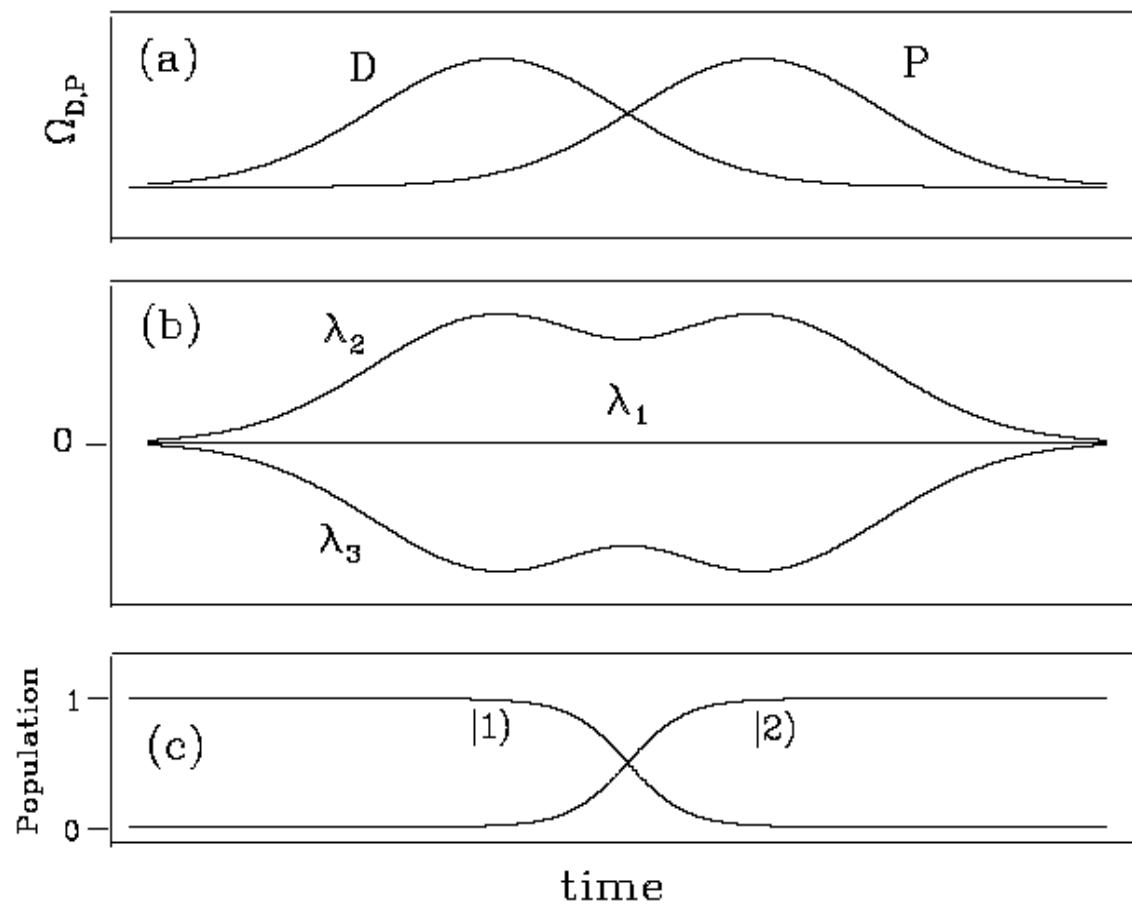
$$|\Psi(t)\rangle = \sum_{i=1,0,2} b_i(t) |\psi_i\rangle e^{-iE_i t/\hbar}$$

$$\frac{d}{dt}\underline{\mathbf{b}}(t)=i\underline{\underline{\mathbf{H}}}\cdot\underline{\mathbf{b}}(t)$$

$$\underline{\underline{\mathbf{H}}}=\left(\begin{array}{ccc}0&\Omega_P^*(t)e^{-i\Delta_P t}&0\\\Omega_P(t)e^{i\Delta_P t}&0&\Omega_D^*(t)e^{i\Delta_D t}\\0&\Omega_D(t)e^{-i\Delta_D t}&0\end{array}\right)$$

$$\underline{\underline{H}} \cdot \underline{\underline{U}} = \underline{\underline{U}} \cdot \hat{\underline{\lambda}}$$

$$\lambda_1 = 0 , \lambda_{2,3}(t) = \pm [\Omega_P(t)^2 + \Omega_D(t)^2]^{1/2}$$



The Λ configuration adiabatic passage.

$$\underline{\mathbf{U}}^{(1)} = \begin{pmatrix} \cos \theta(t) \\ 0 \\ -e^{i\chi(t)} \sin \theta(t) \end{pmatrix}$$

where

$$\theta(t) = \arctan \left(\left| \frac{\Omega_P(t)}{\Omega_D(t)} \right| \right)$$

$$\chi(t) \equiv (\Delta_P - \Delta_D)t - \phi_S(t) + \phi_P(t) = \phi_P(t) - \phi_D(t)$$

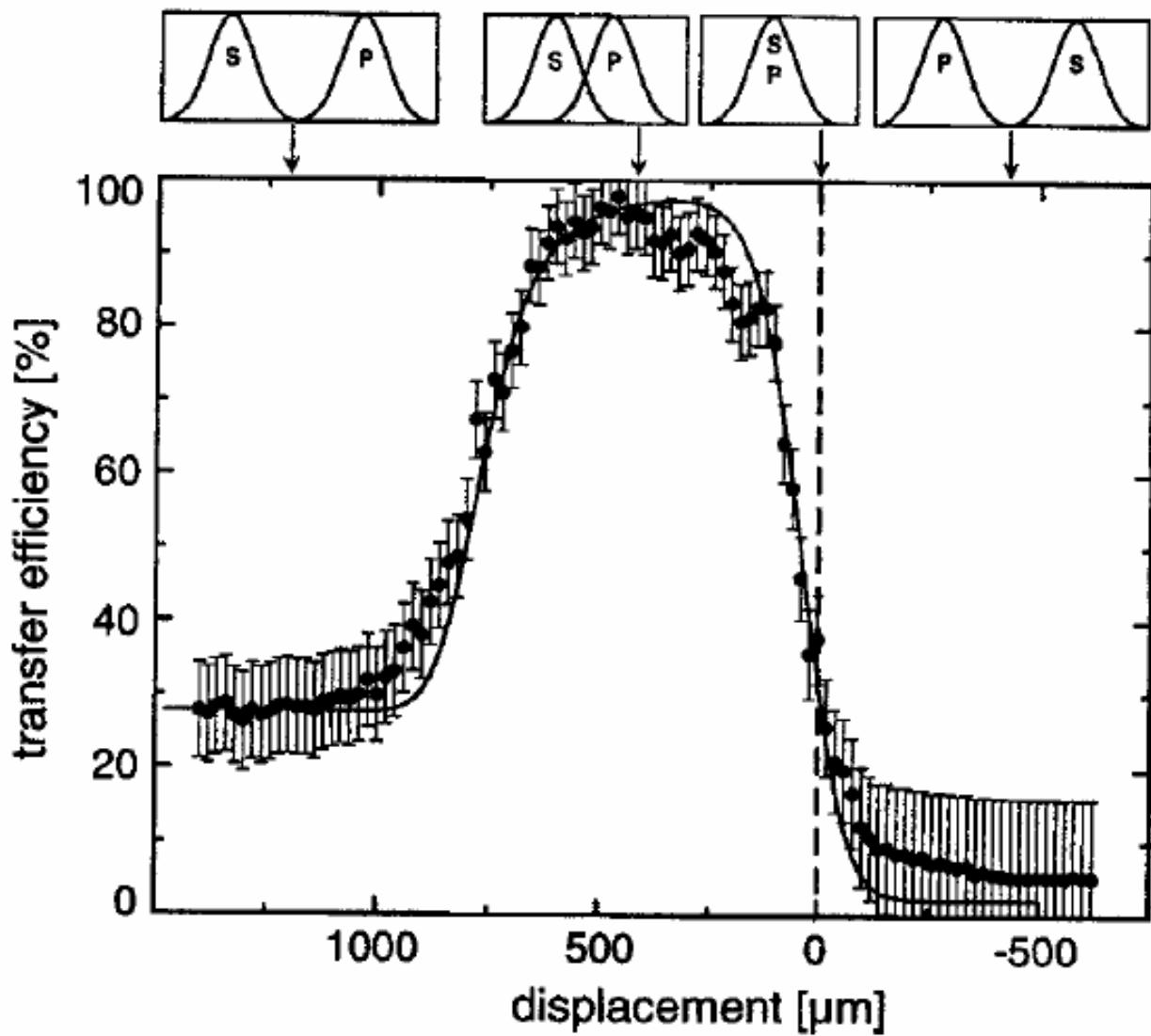
where we assumed two-photon resonance ($\Delta_P - \Delta_D = 0$)
 $\phi_i(t)$ is the phase of the Rabi frequency,

$$\Omega_i(t) \equiv |\Omega_i(t)| e^{i\phi_i(t)}, \quad i = P, D$$

The $\lambda_1 = 0$ eigenvector

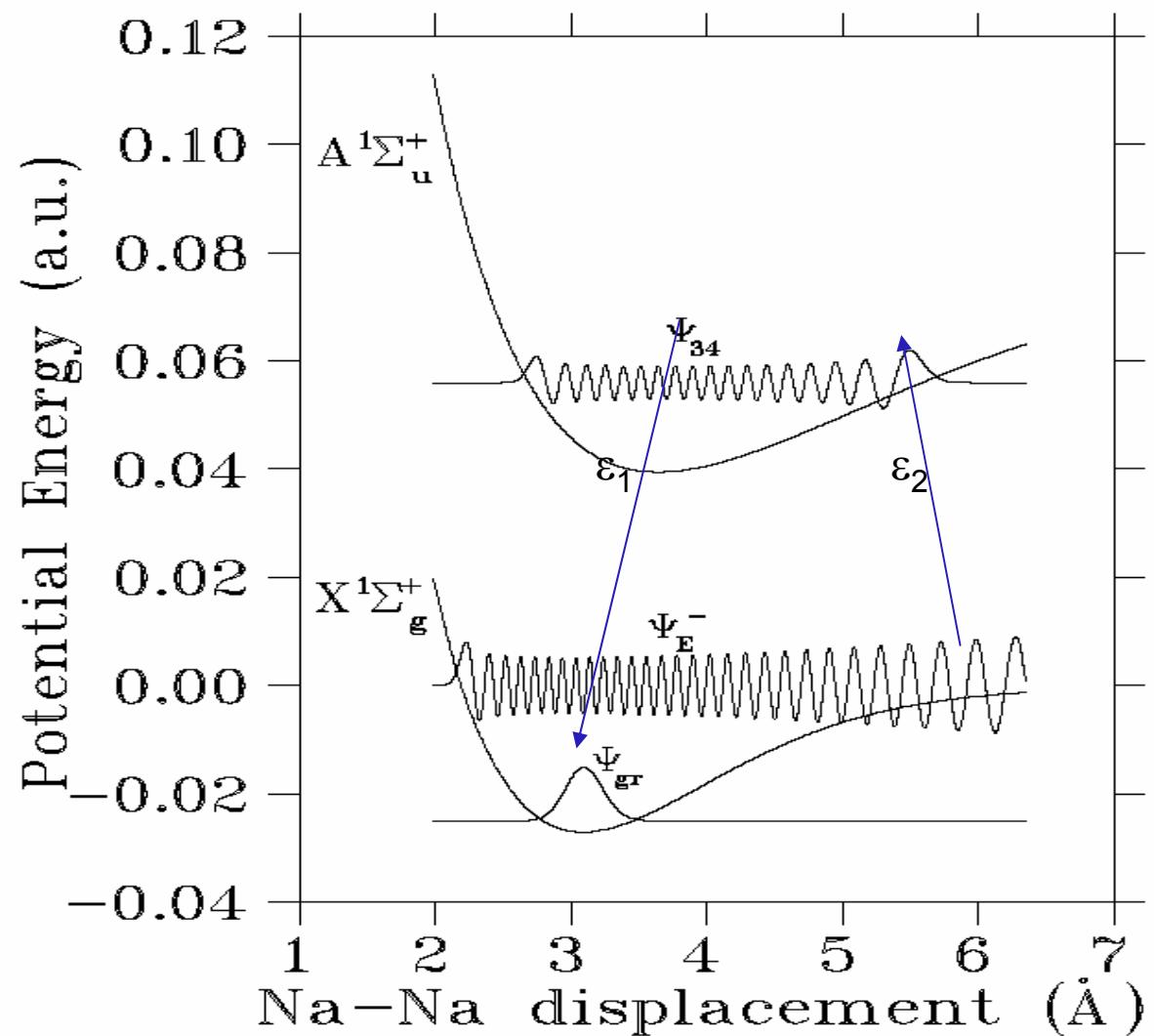
$$|\lambda_1(t)\rangle = \cos\theta(t)e^{-iE_1t/\hbar}|\psi_1\rangle - e^{i\chi(t)}\sin\theta(t)e^{-iE_2t/\hbar}|\psi_2\rangle.$$

The system goes smoothly and completely from state $|\psi_1\rangle$ to state $|\psi_2\rangle$, while maintaining 0 population of level $|\psi_0\rangle$.

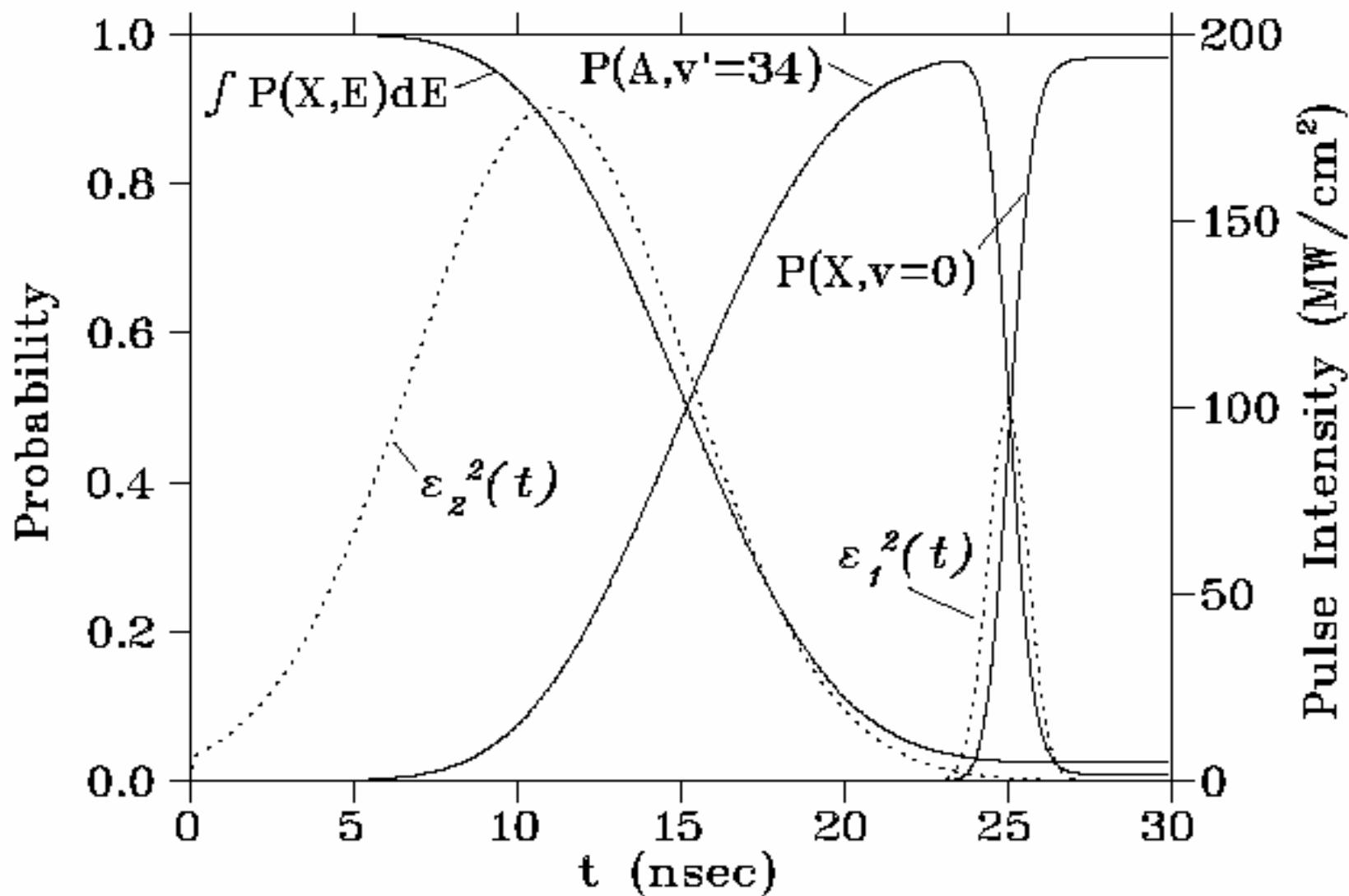


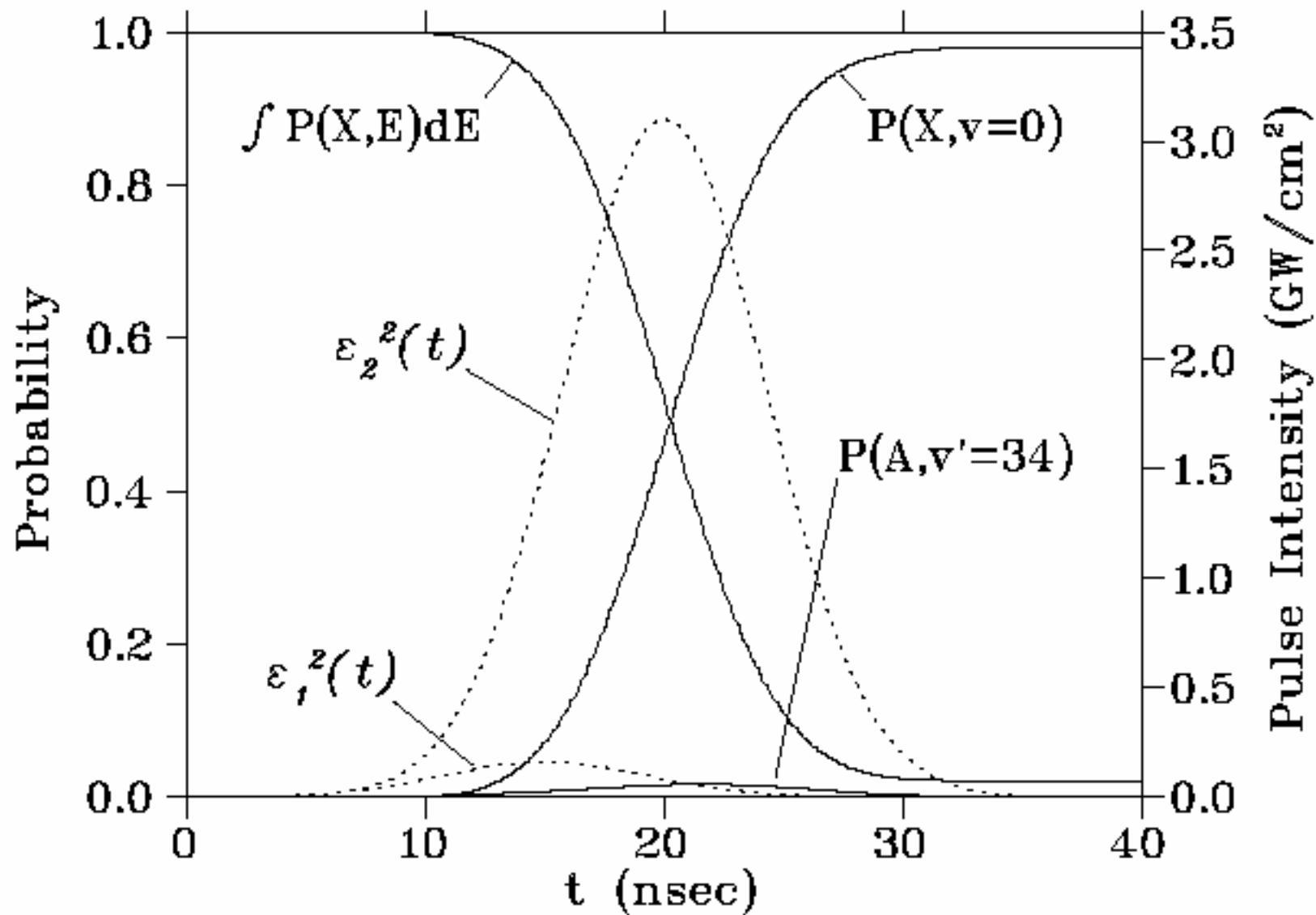
Adiabatic transfer from Ne*

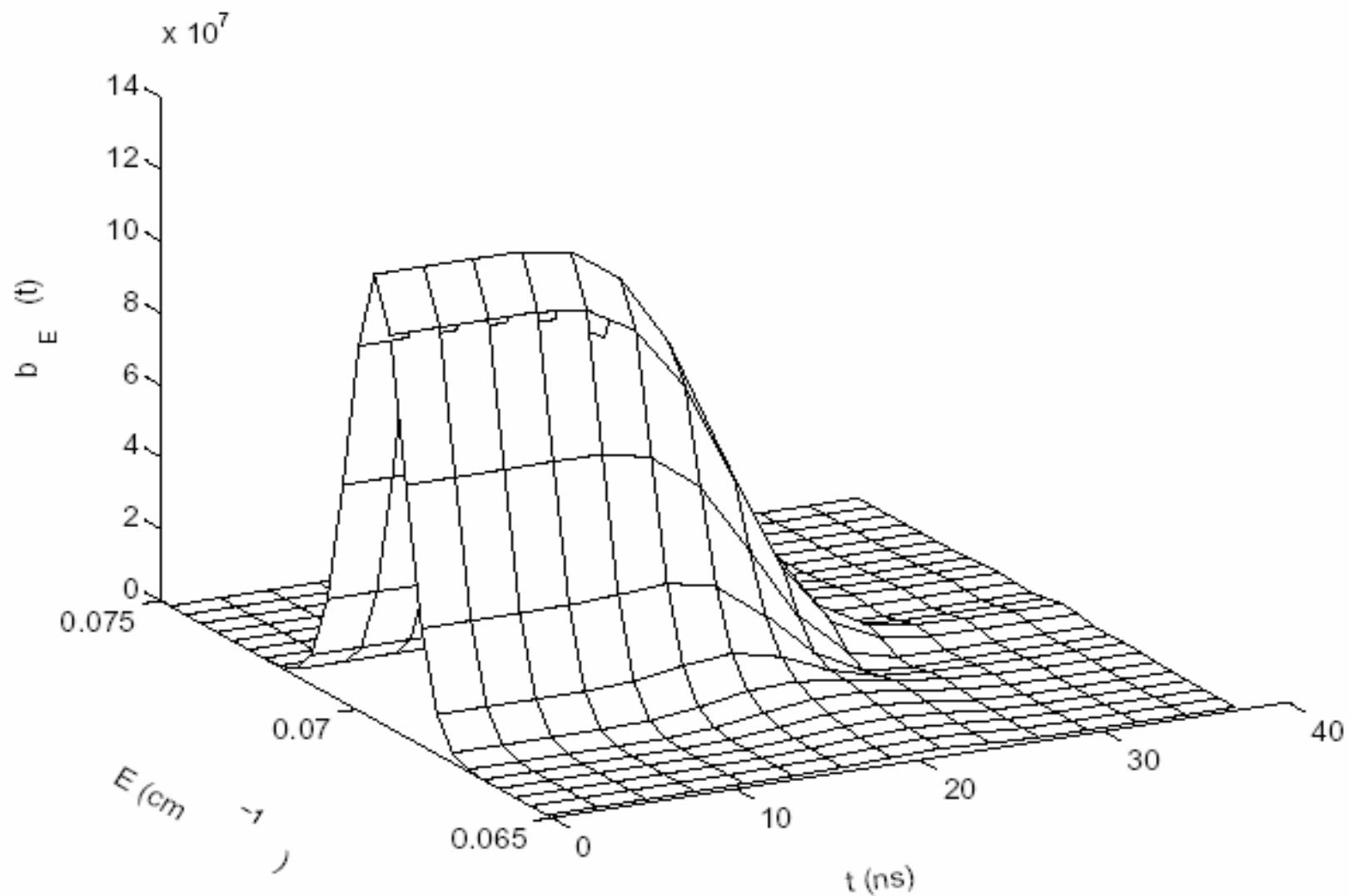
(Bergmann et al. Rev. Mod. Phys. **70**, 1003 (1998)).

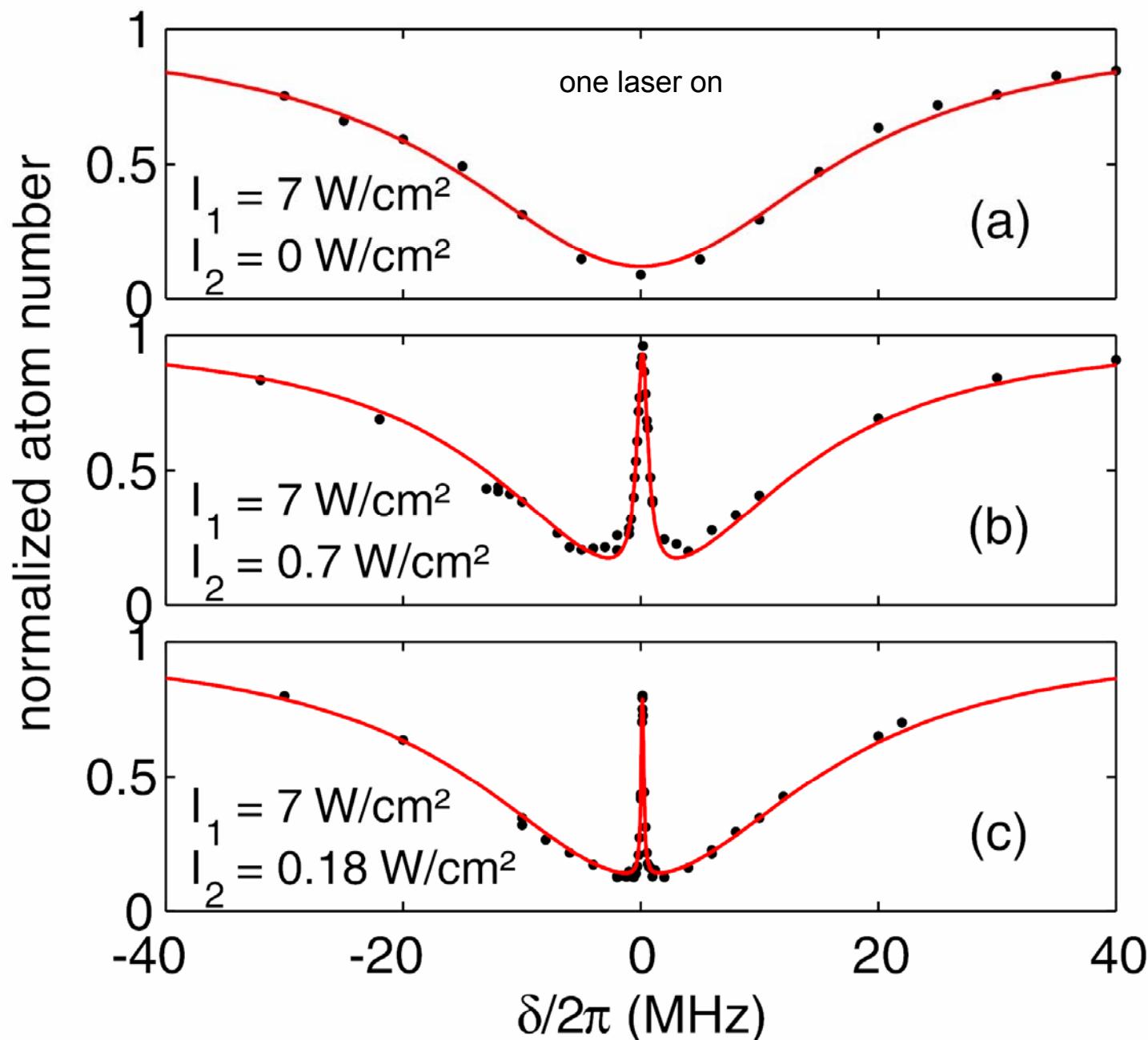


A. Vardi, D. Abrashkevitch, E. Frishman, and M. Shapiro,
J. Chem. Phys. **107**, 6166 (1997)



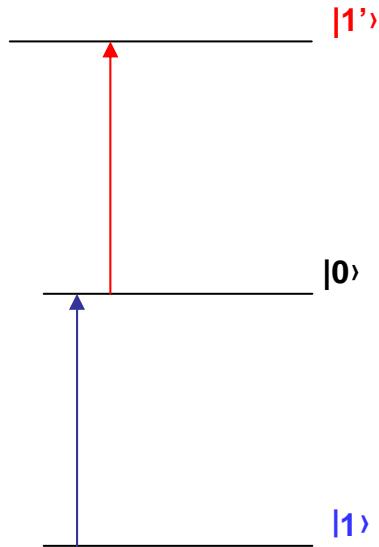






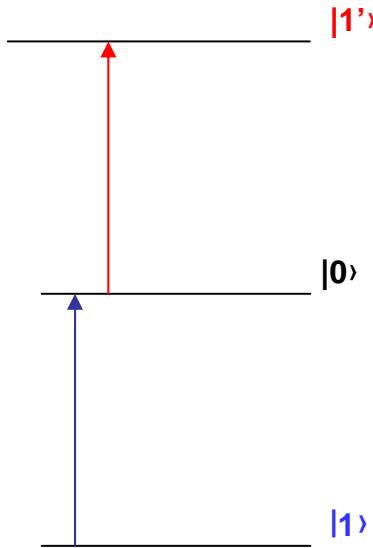
Controllability and adiabatic passage with shaped pulses

Controllability and adiabatic passage with shaped pulses



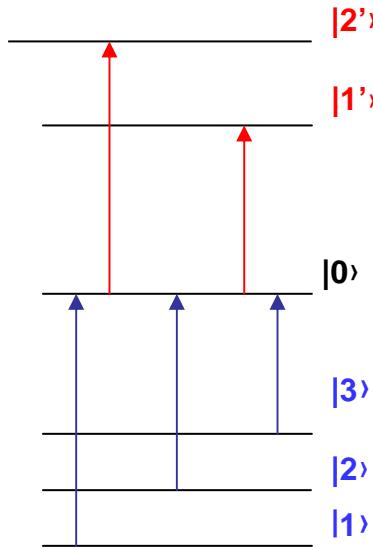
3 states adiabatic passage

K. Bergmann et al.
Rev. Mod. Phys., 70, 1003 (1998).



3 states adiabatic passage

K. Bergmann et al.
Rev. Mod. Phys., 70, 1003 (1998).



**wave packet-to-wave packet
adiabatic passage via a single state**

P. Kral, I. Thanopoulos and M.S.,
Rev. Mod. Phys., 79, 53 (2007).

Analytic solution of the non-degenerate controllability problem

Analytic solution of the non-degenerate controllability problem

$$\hat{C}_k^{\dagger} C_k e^{-\omega_k t} |k\rangle \longrightarrow \hat{C}_{k'}^{\dagger} C_{k'} e^{-\omega_{k'} t} |k'\rangle$$

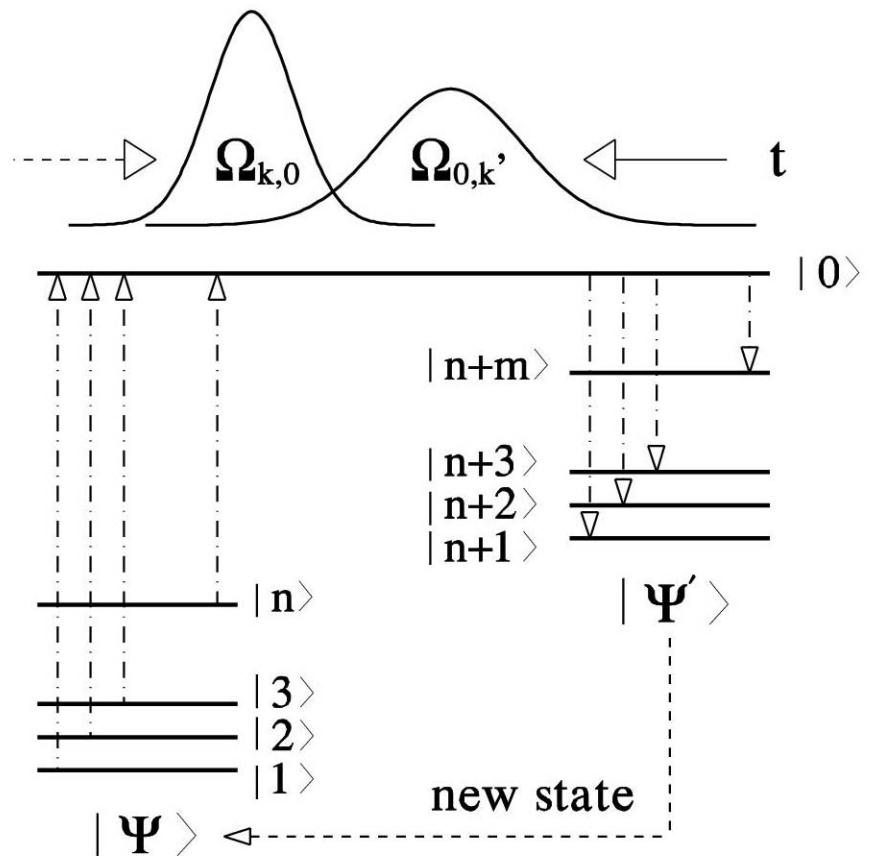
Analytic solution of the non-degenerate controllability problem

$$\sum_k C_k e^{\omega_k t} |k> + |0> + \sum_{k'} C'_{k'} e^{-\omega_{k'} t} |k'>$$

Analytic solution of the non-degenerate controllability problem

$$\sum_k C_k e^{-\omega_k t} |k\rangle \langle 0| + \sum_k C'_k e^{-\omega'_k t} |k'\rangle$$

P. Král, Z. Amitay and M. Shapiro,
PRL 89, 063002 (2002)



The Hamiltonian is ($\omega_{0,k} \equiv \omega_0 - \omega_k$):

$$H = \sum_{k=0}^{n+m} \omega_k |k\rangle\langle k| + \sum_{k=1}^{n+m} [\Omega_{0,k}(t) e^{-i\omega_{0,k} t} |0\rangle\langle k| + h.c.]$$

Here $\Omega_{0,k}(t) \equiv \mu_{0,k} \mathcal{E}_{0,k}(t) \equiv \mathcal{O}_{0,k} f_{D(P)}(t)$, where

$0 < f_{D(P)}(t) < 1$ are the “dump” (“pump”) envelopes

The wave function $|\Psi\rangle = \sum_{i=1}^{n+2} c_i(t) e^{-i\omega_i t} |i\rangle$ follows

the matrix Schrödinger equation $\dot{\mathbf{c}}^T(t) = -i \mathsf{H}(t) \mathbf{c}^T(t)$,

where $\mathbf{c}(t) = (c_0, c_1, \dots, c_n, c_{n+1}, \dots, c_{n+m})$ and

$$\mathsf{H} = \begin{bmatrix} 0 & \Omega_{0,1} & \dots & \Omega_{0,n} & \Omega_{0,n+1} & \dots & \Omega_{0,n+m} \\ \Omega_{1,0} & 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \Omega_{n,0} & 0 & \dots & 0 & 0 & \dots & 0 \\ \Omega_{n+1,0} & 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \Omega_{n+m,0} & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}$$

Of the $n + m + 1$ eigenvalues of $\mathsf{H}(t)$, $n + m - 1$ are zero and two are nonzero,

$$\begin{aligned}\lambda_1 &= \lambda_2 = \dots = \lambda_{n+m-1} = 0 , \\ \lambda_{n+m}(t) &= -\lambda_{n+m+1}(t) = \left(\sum_{k=1}^{n+m} |\Omega_{0,k}(t)|^2 \right)^{1/2} .\end{aligned}$$

Zero eigenvalues correspond to *three types* of “null states”:

“Initial Null States” ($k \neq k' = 1, \dots, n$)

$$|D_{kk'}^I\rangle = \Omega_{0,k'} |k\rangle - \Omega_{0,k} |k'\rangle , \quad (\text{INS})$$

“Mixed Null States” ($k = 1, \dots, n; l = n + 1, \dots, n + m$)

$$|D_{kl}^M\rangle = \Omega_{0,l} |k\rangle - \Omega_{0,k} |l\rangle , \quad (\text{MNS})$$

“Final Null States” ($l \neq l' = n + 1, \dots, n + m$)

$$|D_{ll'}^F\rangle = \Omega_{0,l'} |l\rangle - \Omega_{0,l} |l'\rangle , \quad (\text{FNS})$$

The basis vectors are re-defined as $e^{-i\omega_j t} |j\rangle \rightarrow |j\rangle$

We form an MNS state that correlates with:

$$|\Psi(t=0)\rangle = \sum_k c_k^0 |k\rangle, \quad |\Psi(t=t_{end})\rangle = \sum_l c_l^e |l\rangle$$

The combination that satisfies these conditions is,

$$\begin{aligned} |D^M\rangle &= \sum_{k,l} t_{kl} |D_{kl}^M\rangle \\ &= \sum_{k=1}^n |k\rangle \sum_{l=n+1}^{n+m} t_{kl} \Omega_{0,l} - \sum_{l=n+1}^{n+m} |l\rangle \sum_{k=1}^n t_{kl} \Omega_{0,k}, \end{aligned}$$

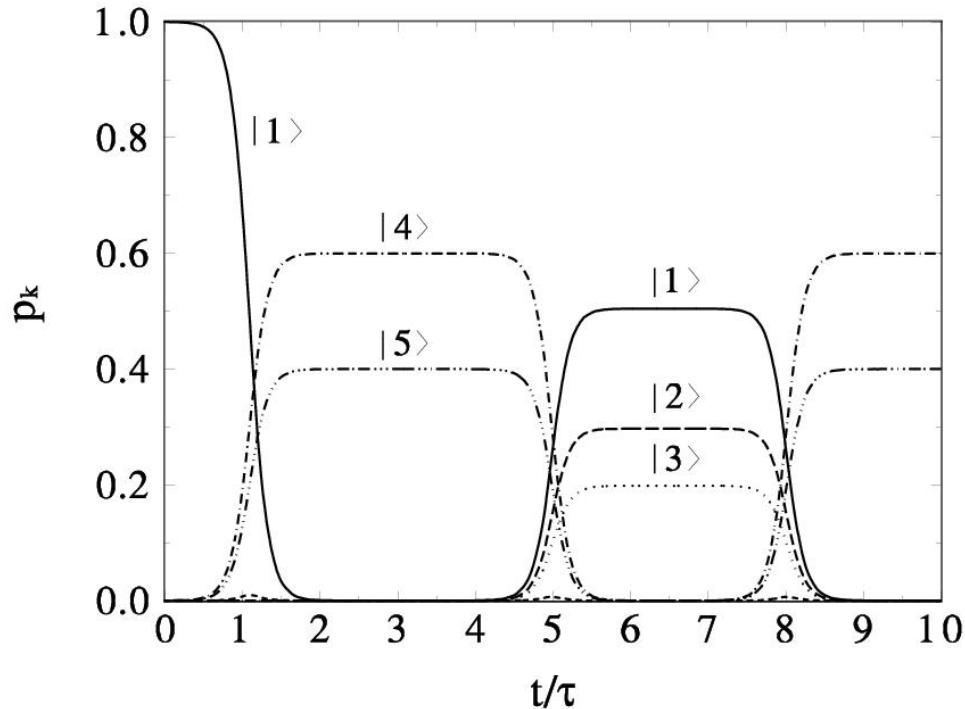
where we need to have

$$\sum_{l=n+1}^{n+m} t_{kl} \Omega_{0,l} \propto c_k^0, \quad \sum_{k=1}^n t_{kl} \Omega_{0,k} \propto c_l^e.$$

This can be satisfied by $(\Omega_{k,0}(t) \equiv \mathcal{O}_{k,0} f_{D(P)}(t))$

$$t_{kl} = \mathcal{O}_{k,0} \mathcal{O}_{l,0}, \quad \text{and} \quad \mathcal{O}_{k,0} = \mathcal{C} c_k^0, \quad \mathcal{O}_{l,0} = \mathcal{C}' c_l^e,$$

where $\mathcal{C}, \mathcal{C}'$ are arbitrary complex numbers (adiabaticity).



In the first transfer of the chain, we use Gaussian pulses:

$$\Omega_{0,1}(t) = \mathcal{O}_{0,1} \exp[-(t - t_0)^2/\tau^2],$$

$$\Omega_{0,4(5)}(t) = \mathcal{O}_{0,4(5)} \exp[-t^2/\tau^2],$$

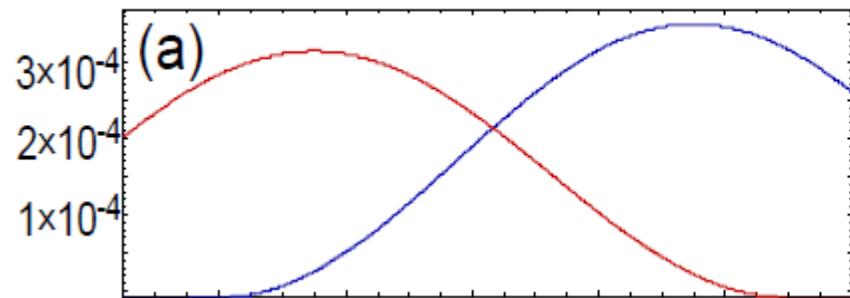
with $t_0 = 2\tau$ and $\mathcal{C} = \mathcal{C}' = 50/\tau$.

Next transfers use **different** times t_0 and amplitudes $\mathcal{O}_{0,k}$
(we also use zero phases).

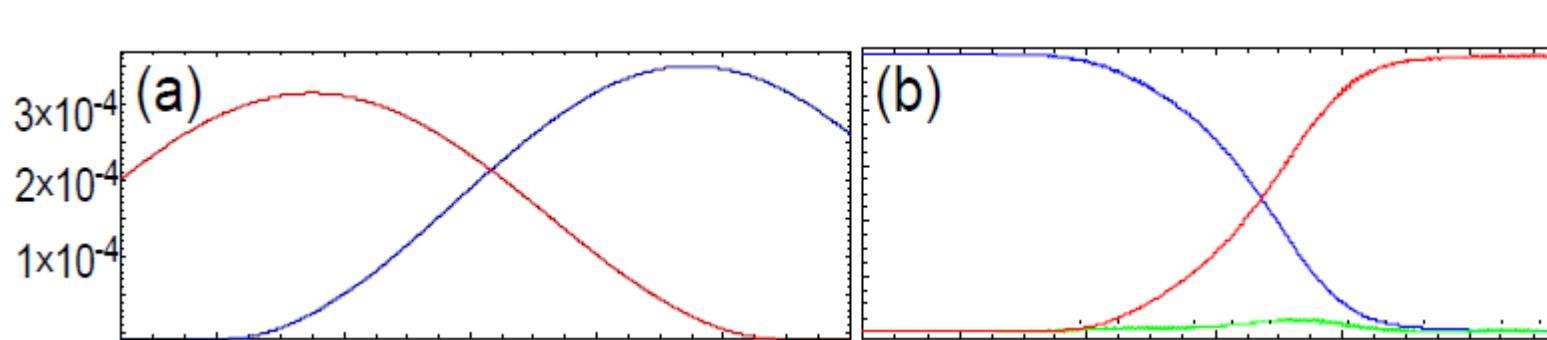
How to use ultrashort pulses? Piecewise adiabatic passage

E.A. Shapiro, C. Menzel-Jones, V. Milner, M. Shapiro, PRL **99**, 033002 (2007)

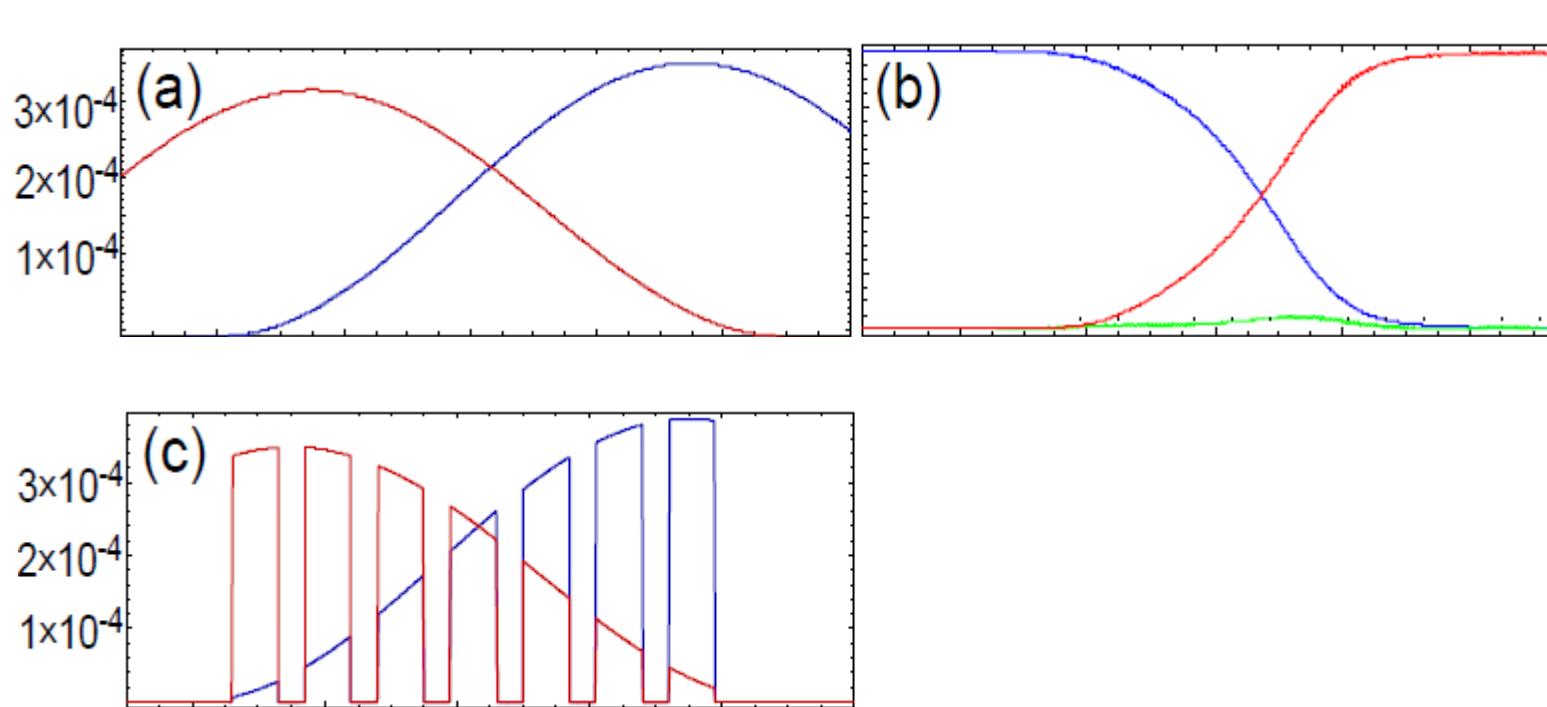
Piecewise adiabatic passage



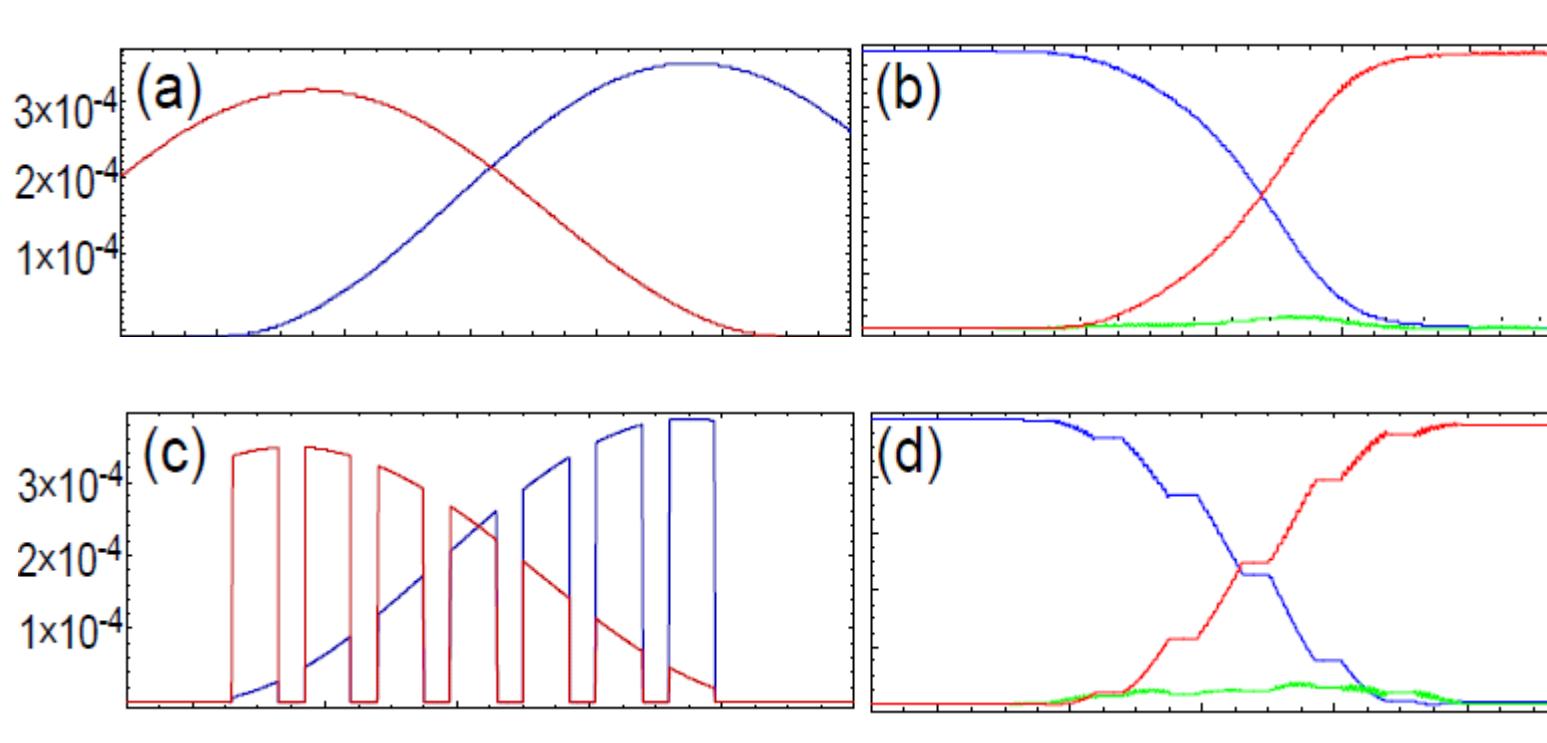
Piecewise adiabatic passage



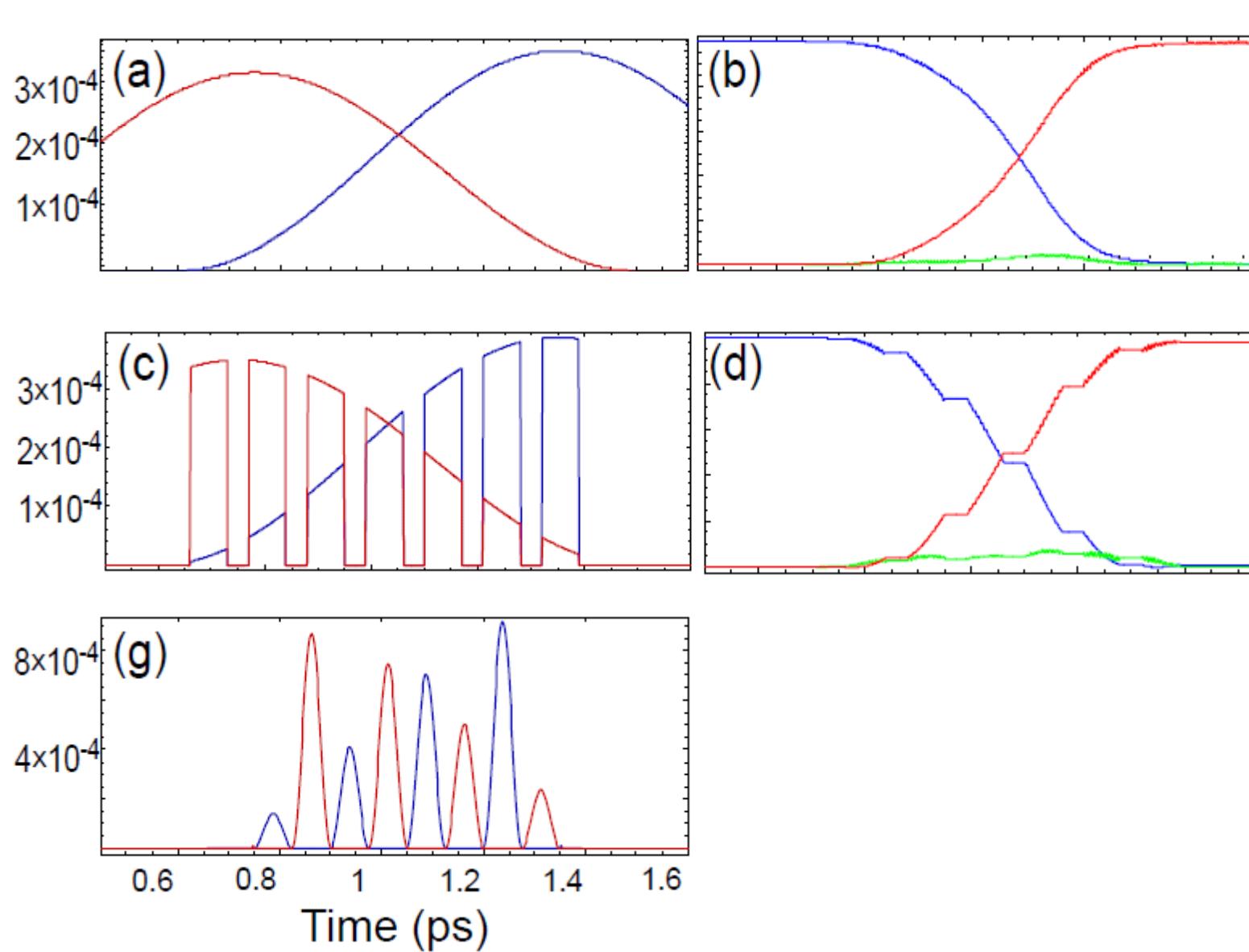
Piecewise adiabatic passage



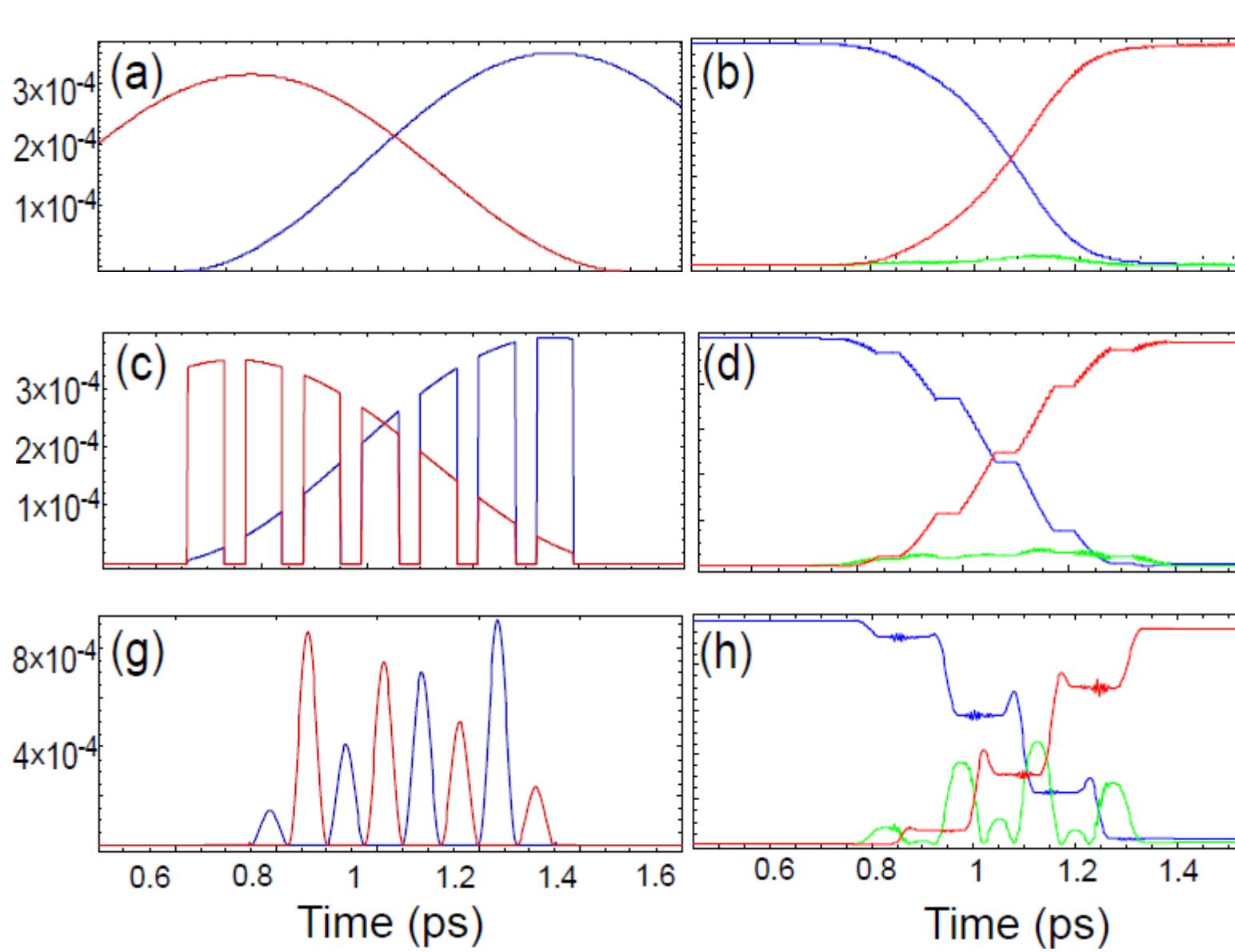
Piecewise adiabatic passage



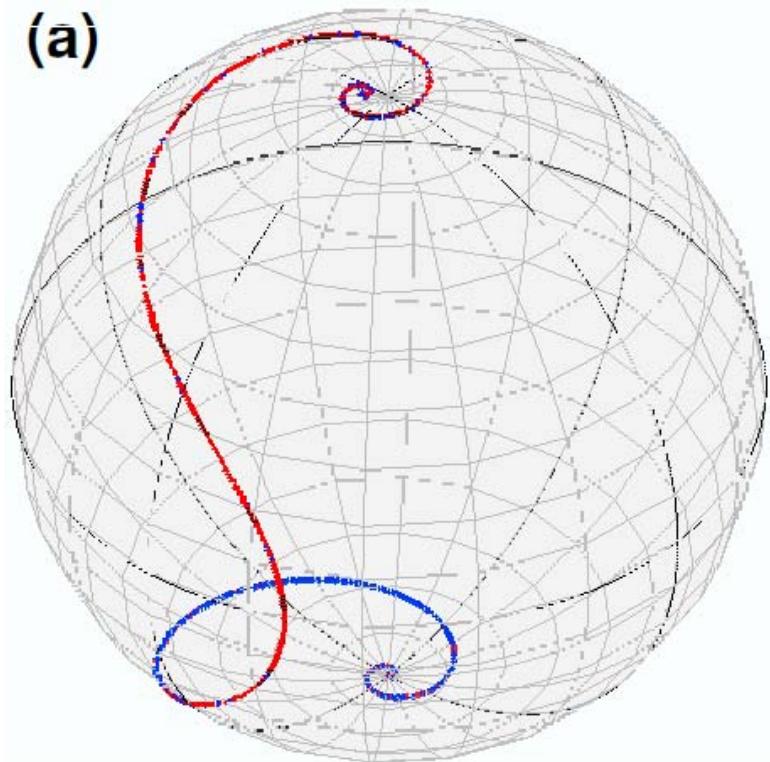
Piecewise adiabatic passage



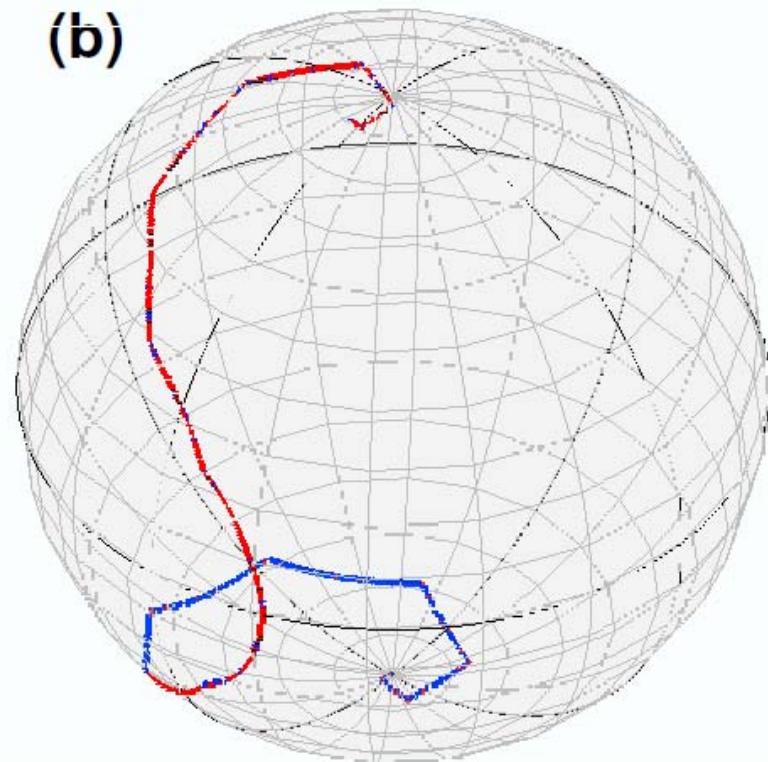
Piecewise adiabatic passage



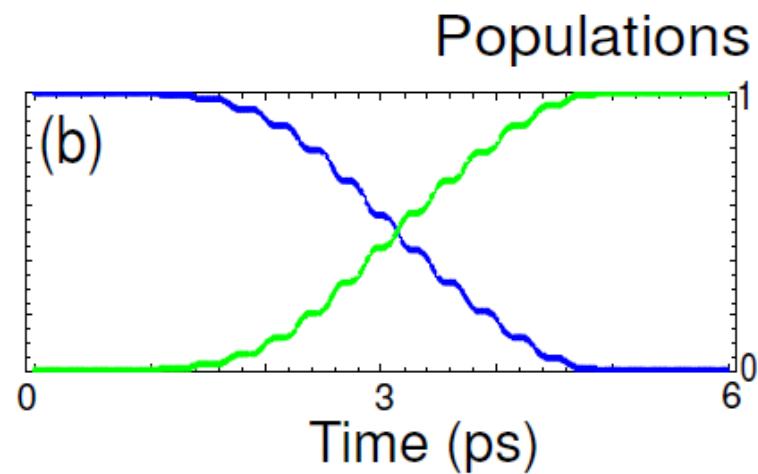
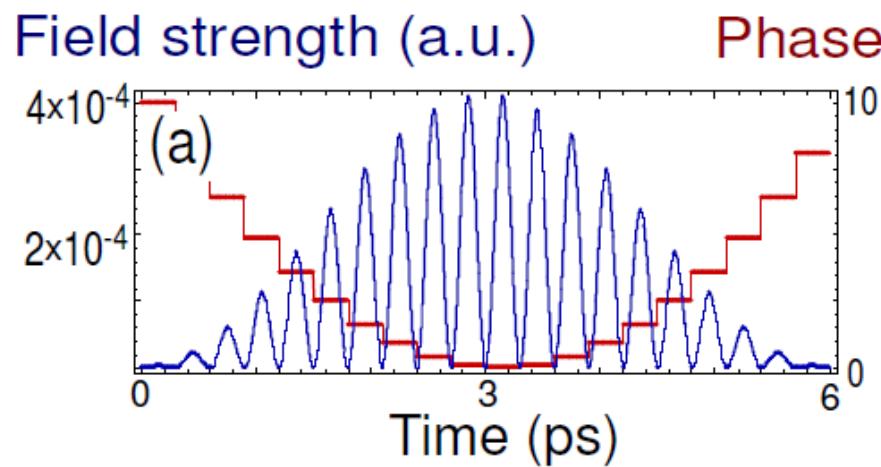
(a)



(b)

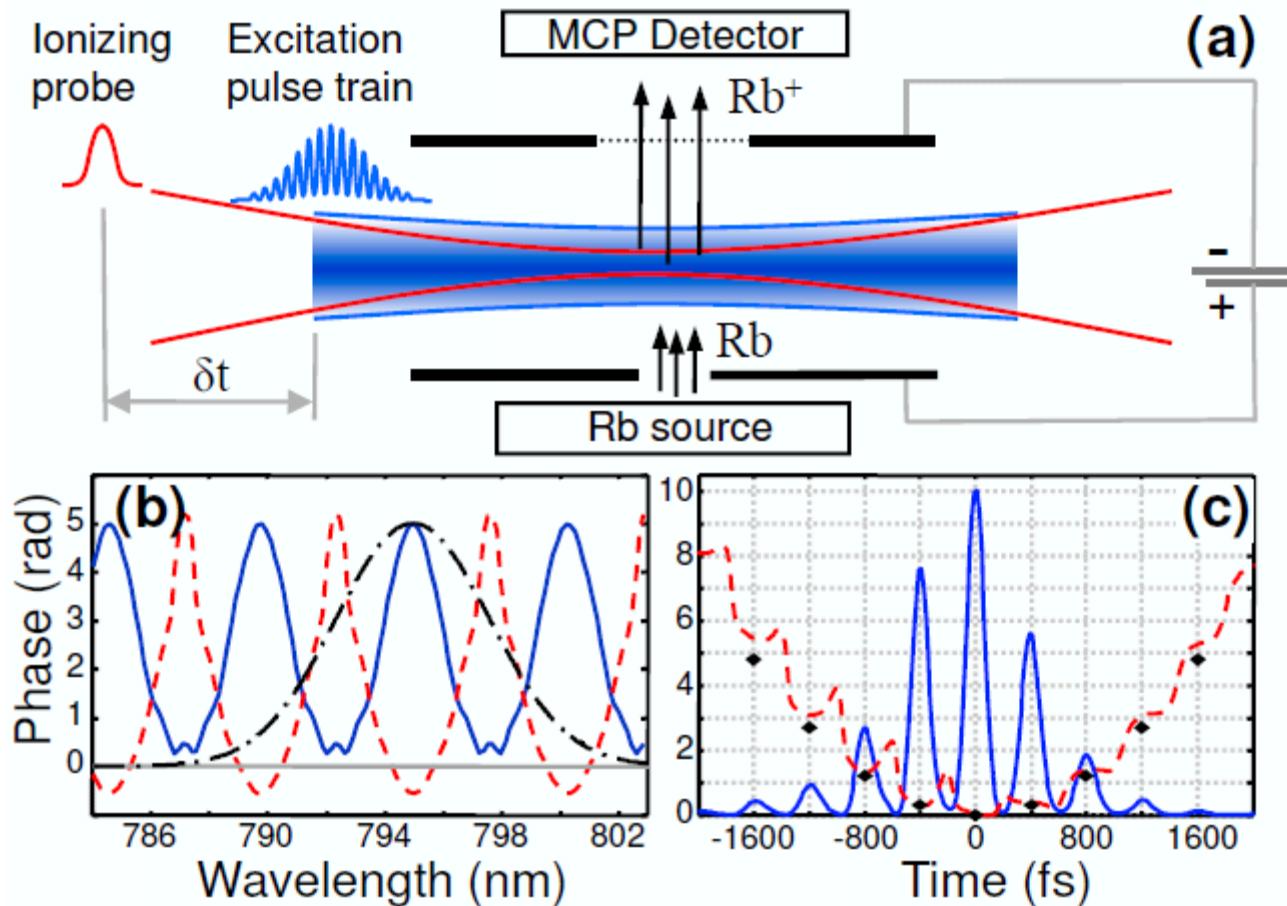


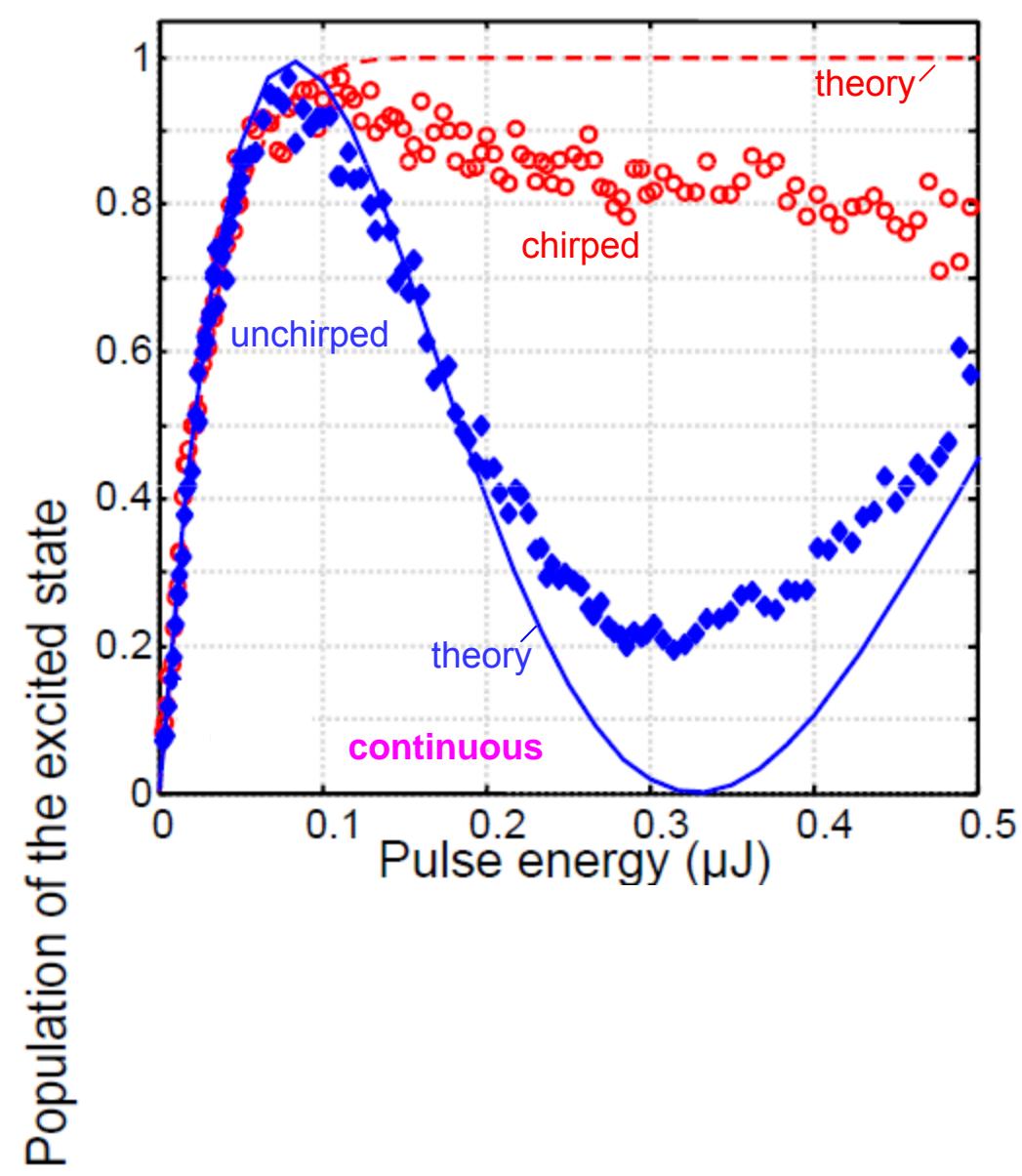
Use of a “chirped” train of pulses - theory

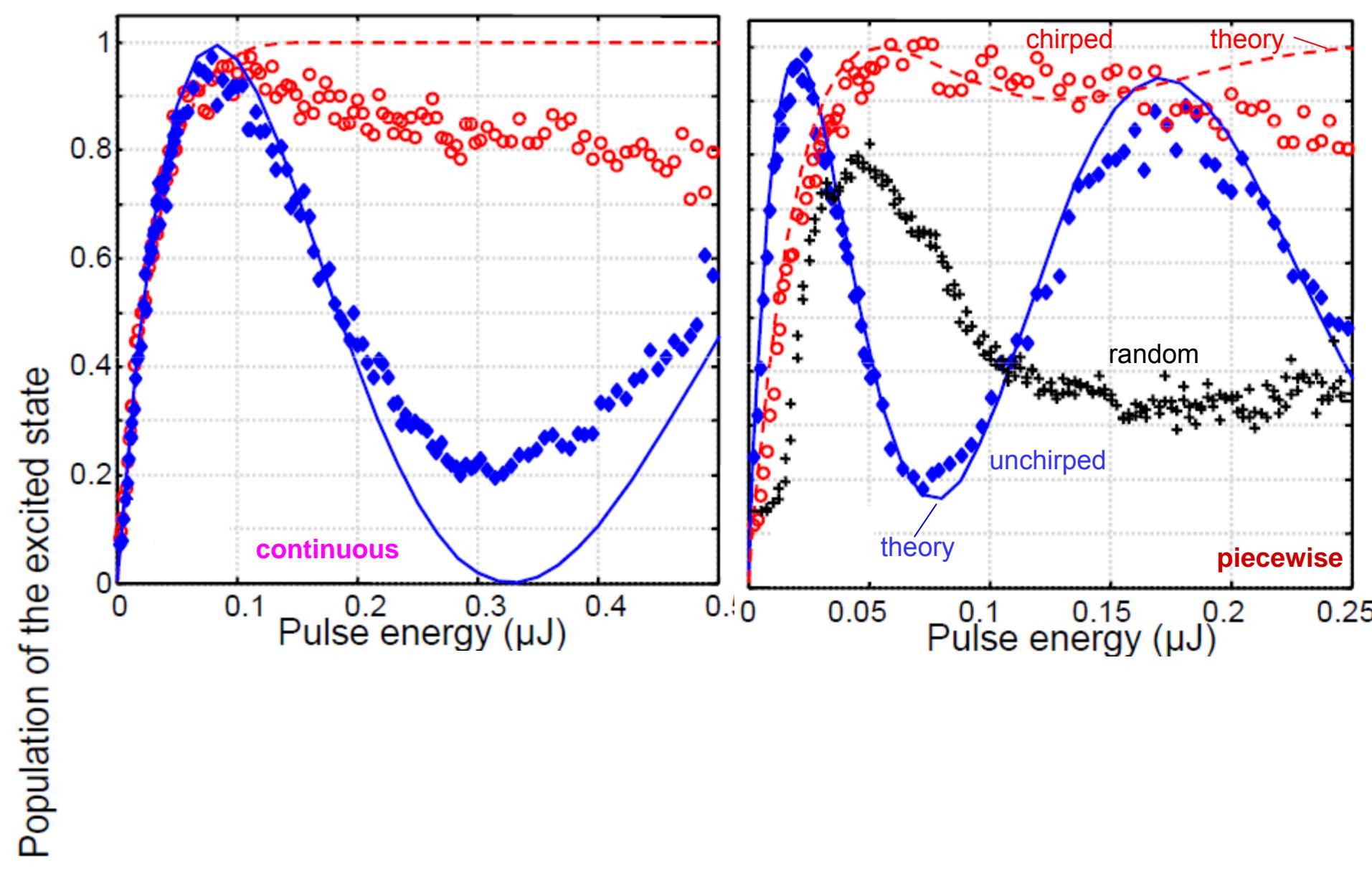


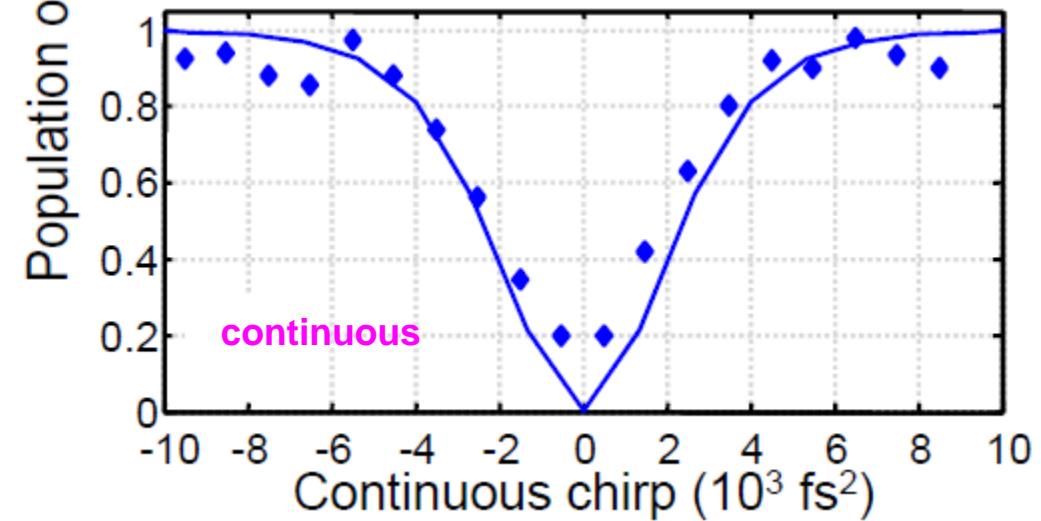
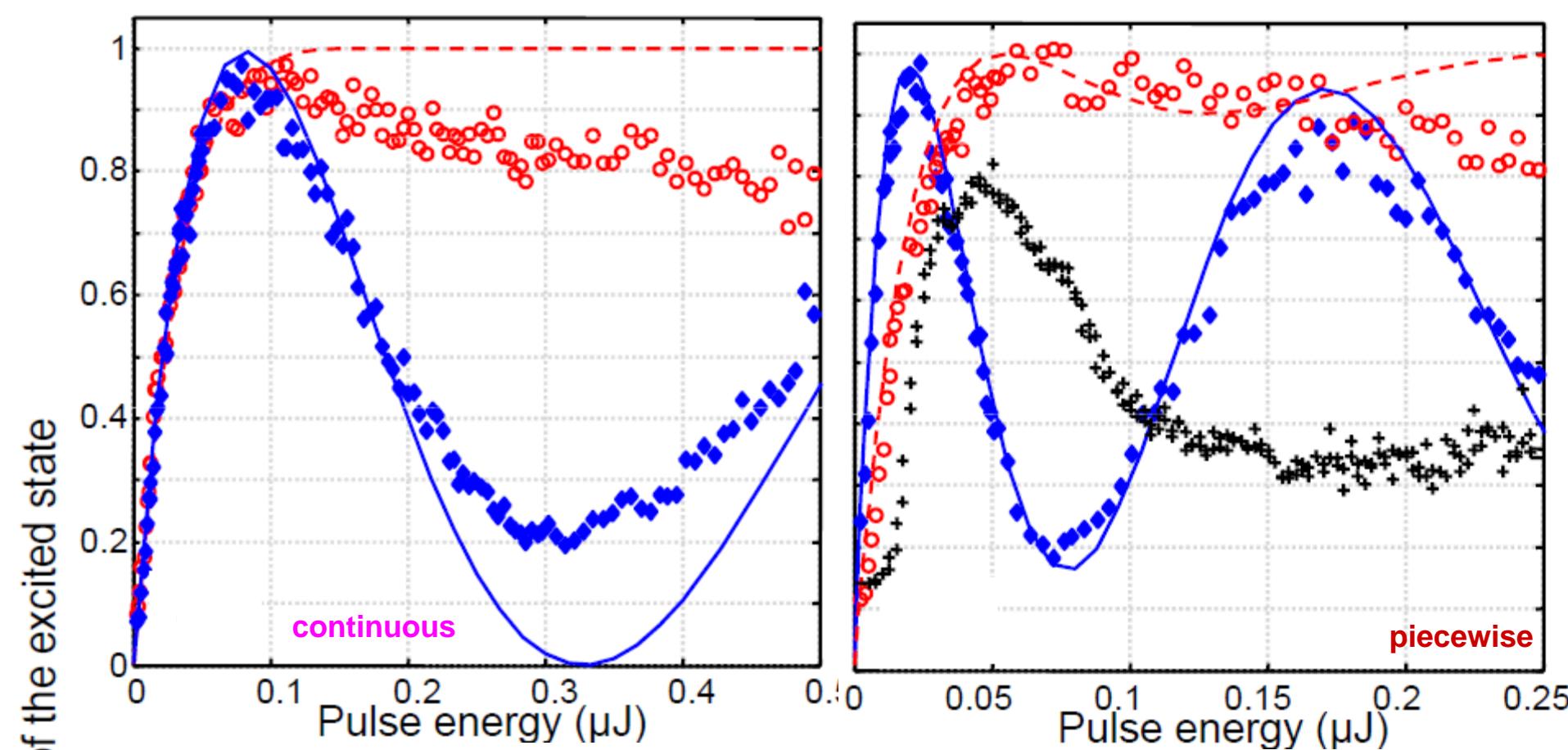
Experiment: S. Zhdanovich, E.A. Shapiro, M. Shapiro, J. Hepburn, V. Milner

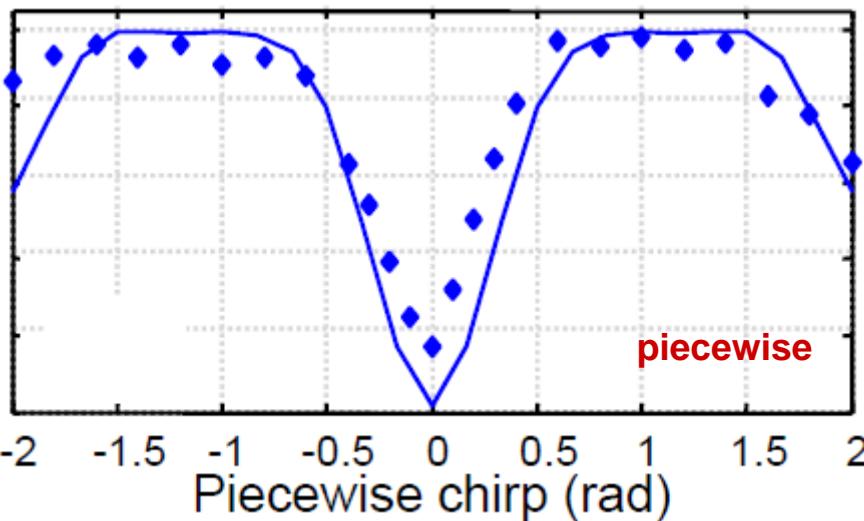
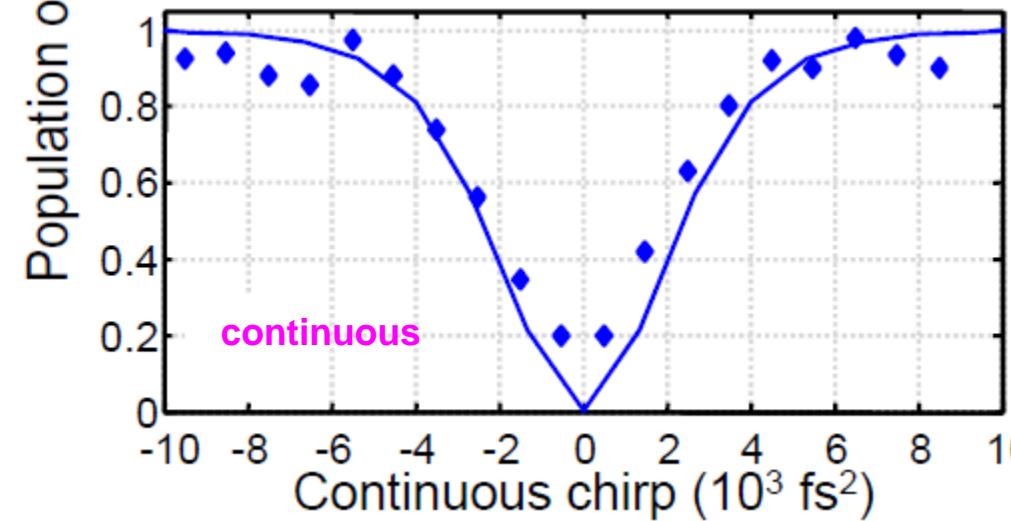
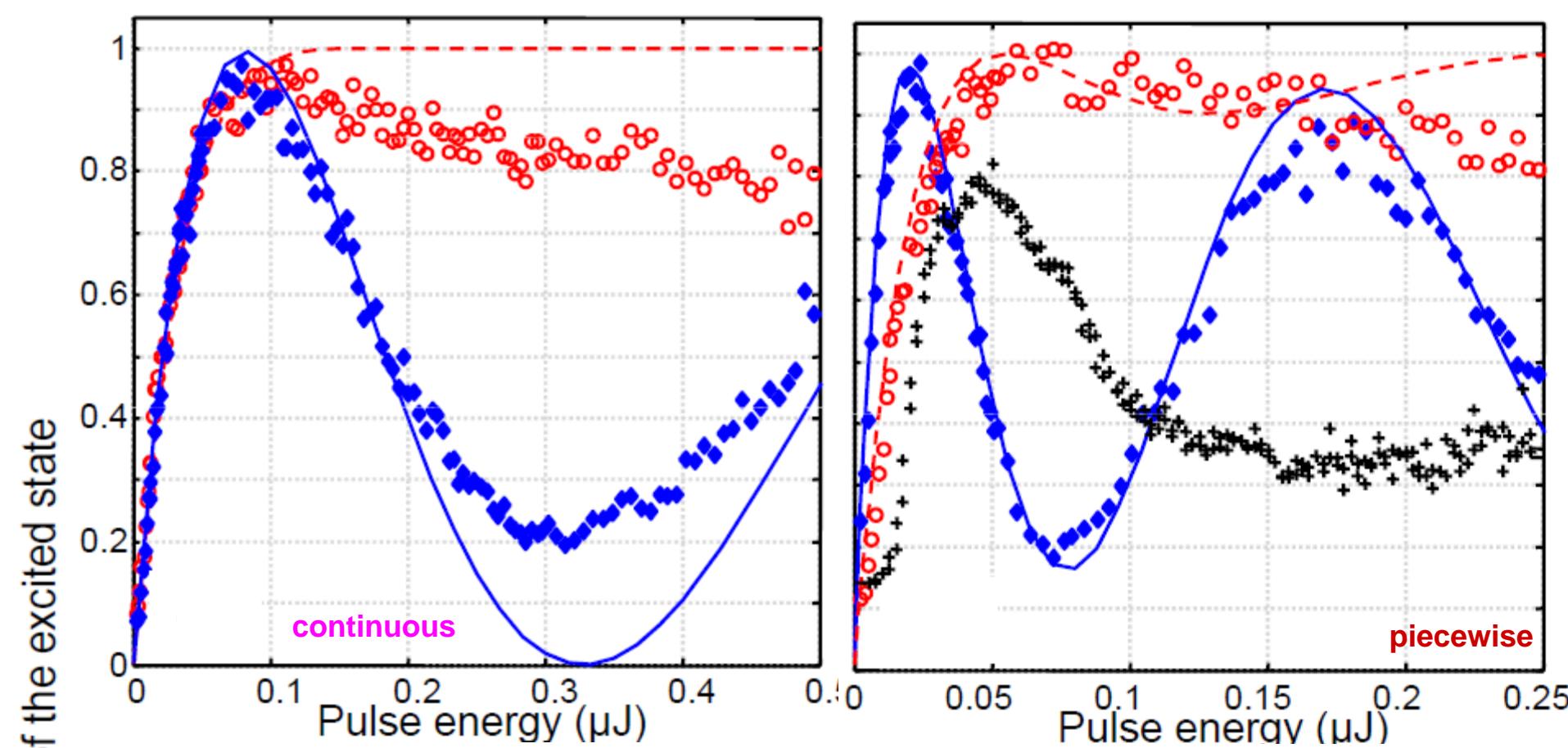
PRL **100**, 103004 (2008)





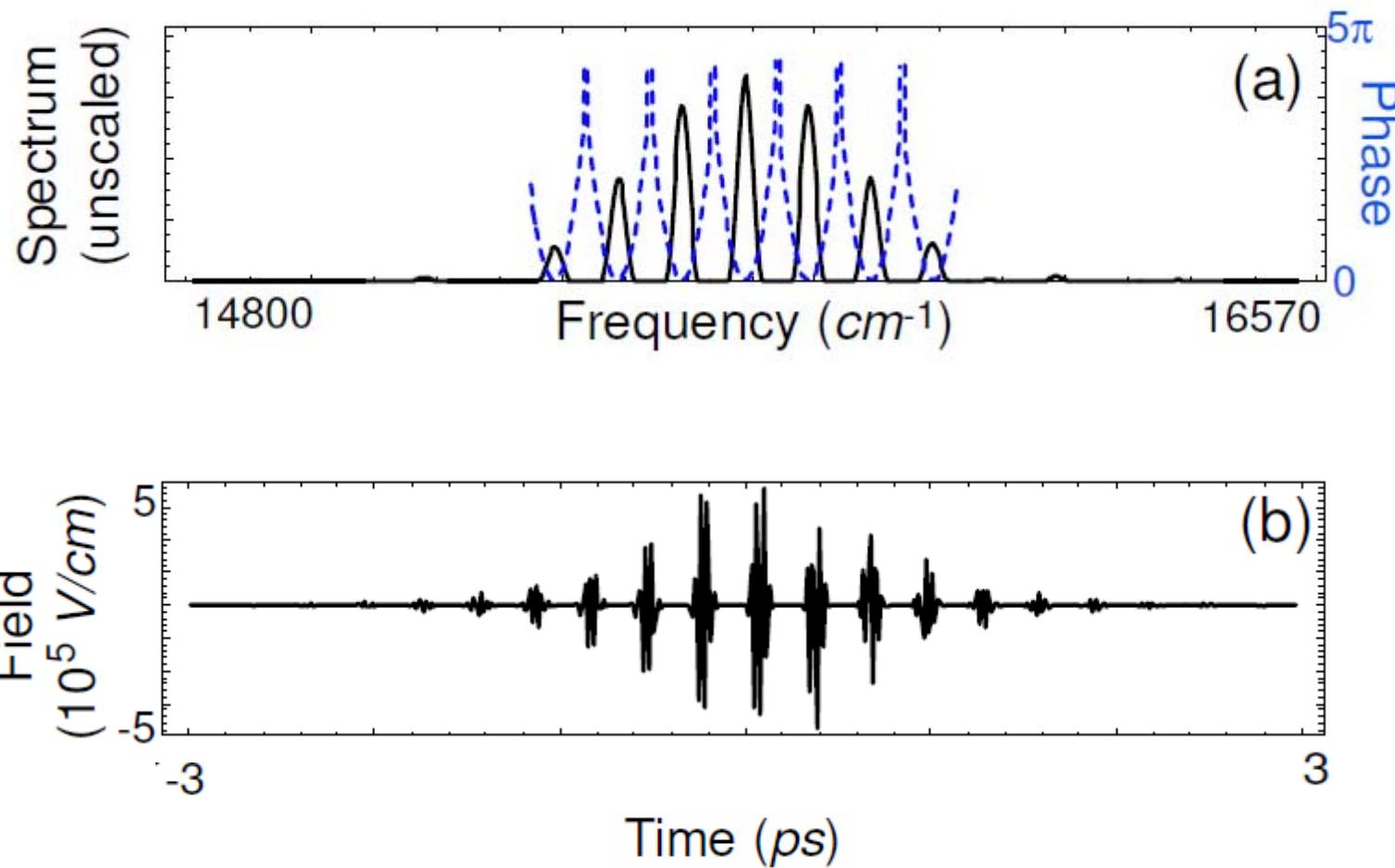


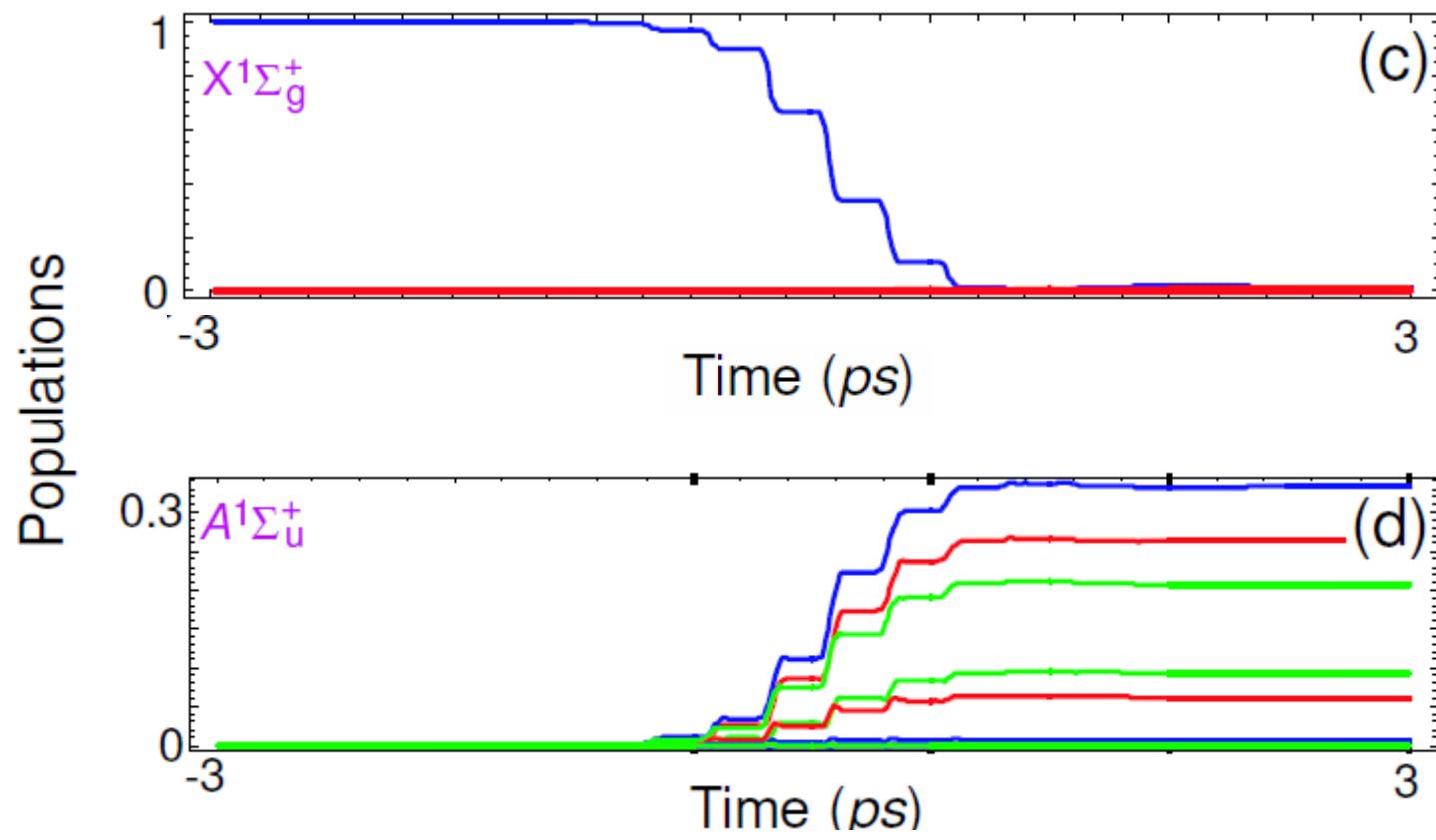


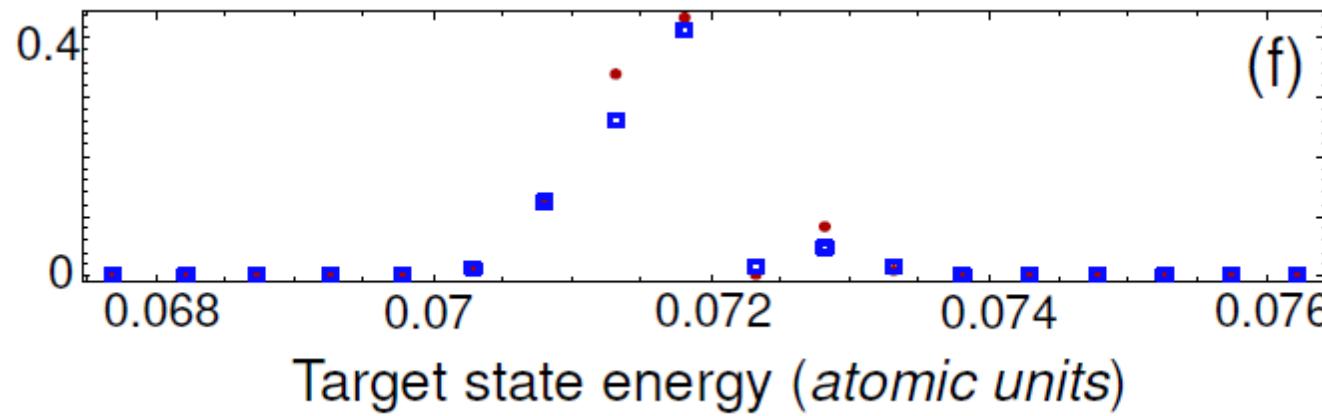
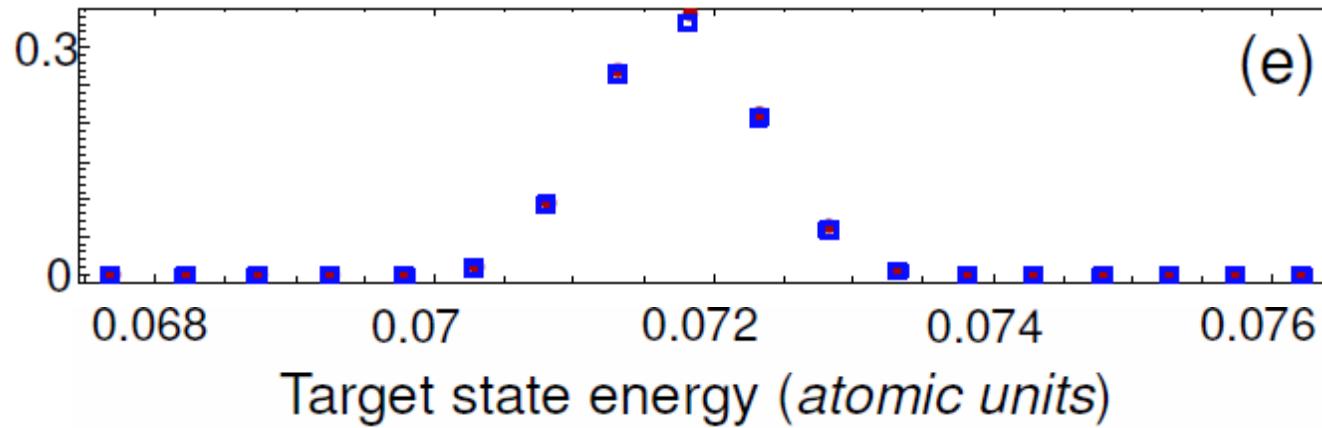


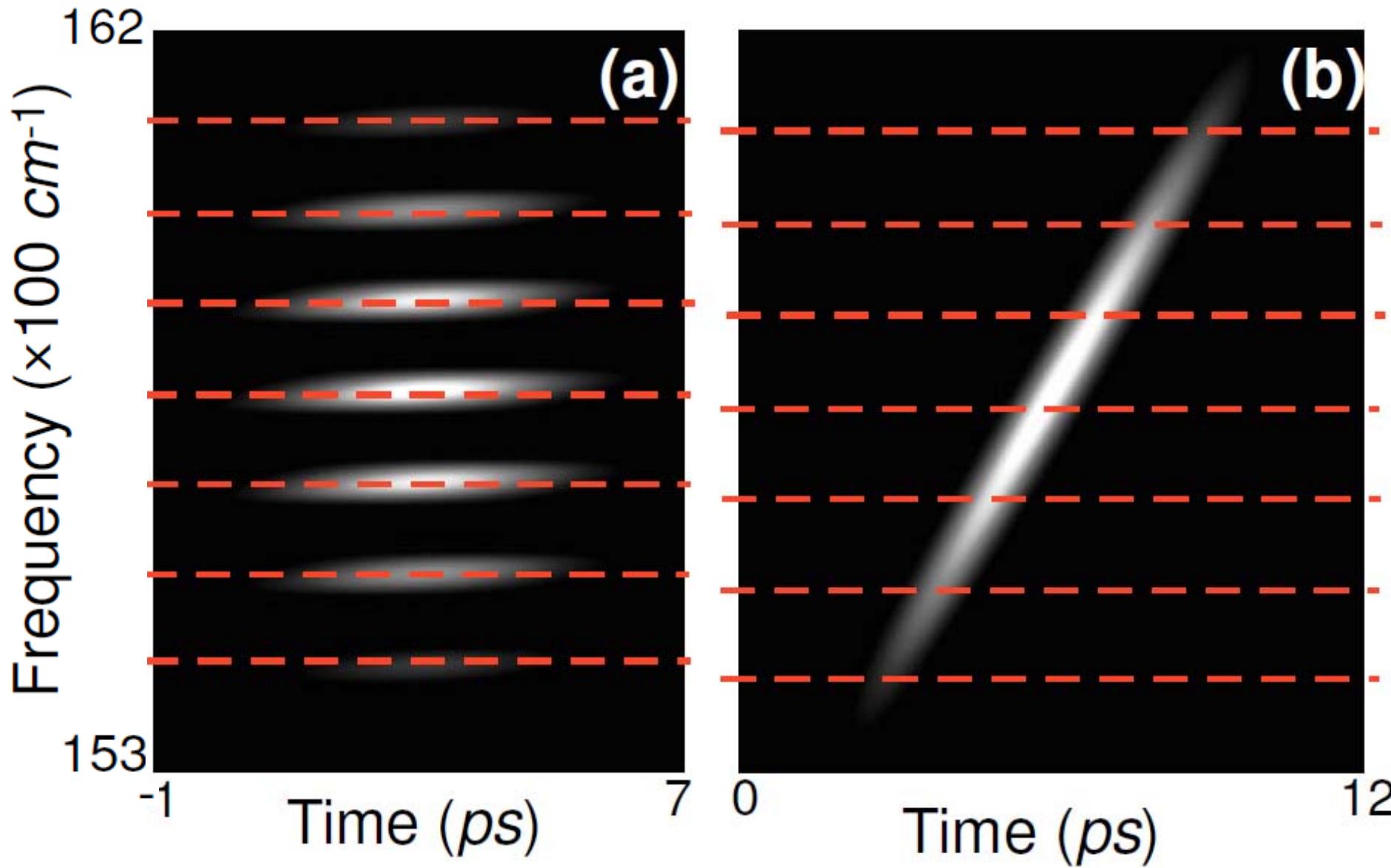
Population transfer to a superposition of many states

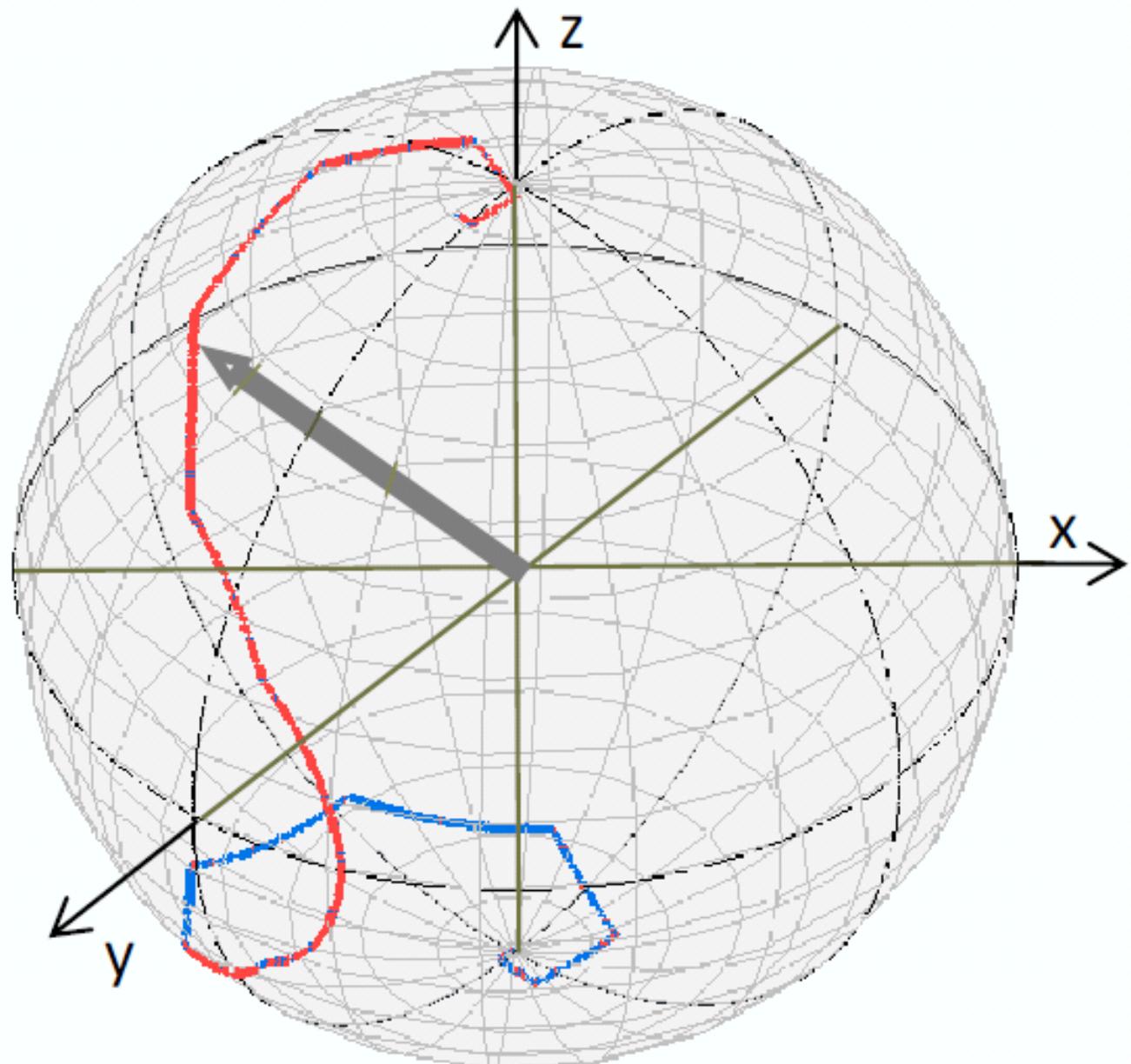
E.A. Shapiro, M. Shapiro, V. Milner, PRA 79, 023422 (2009)

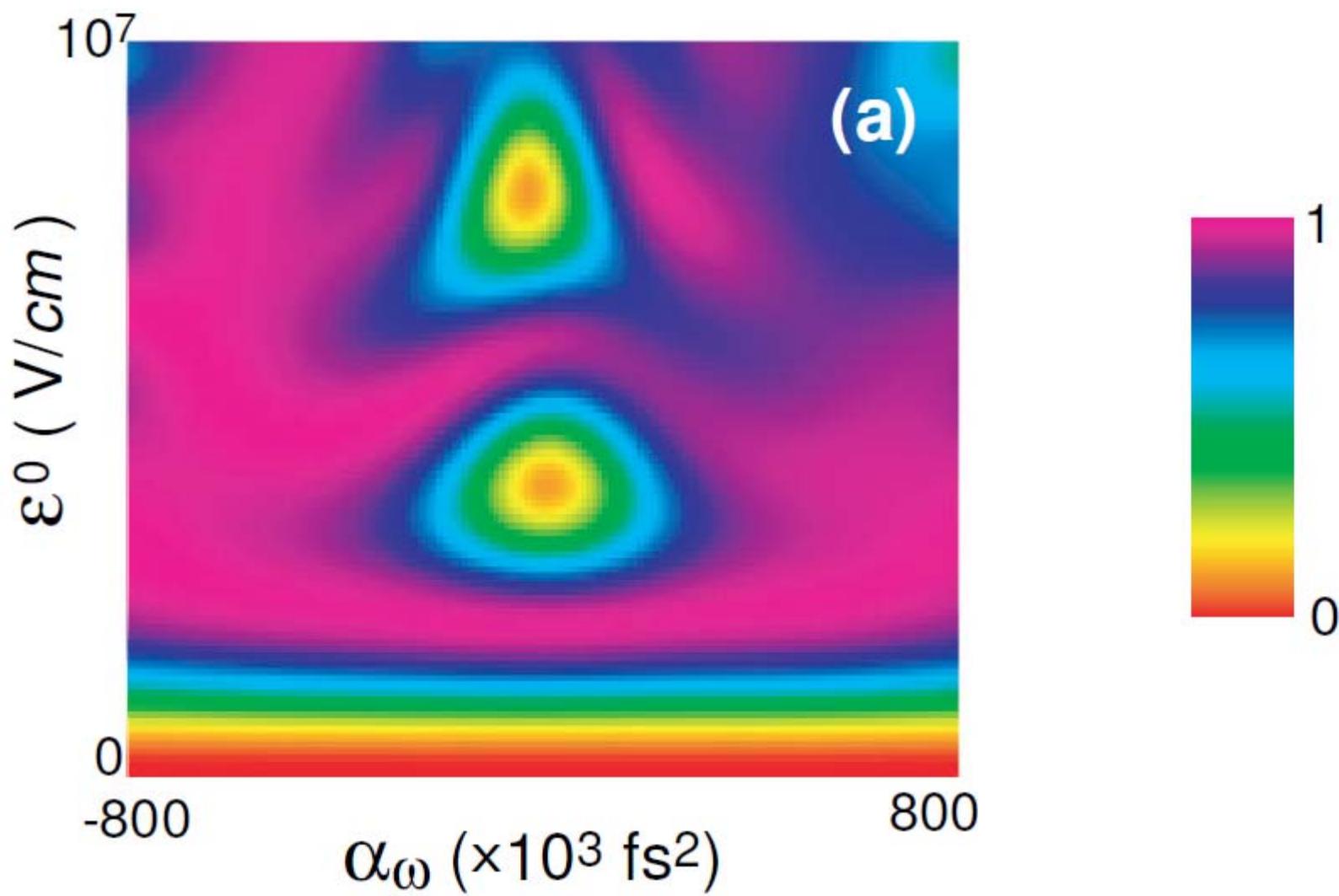


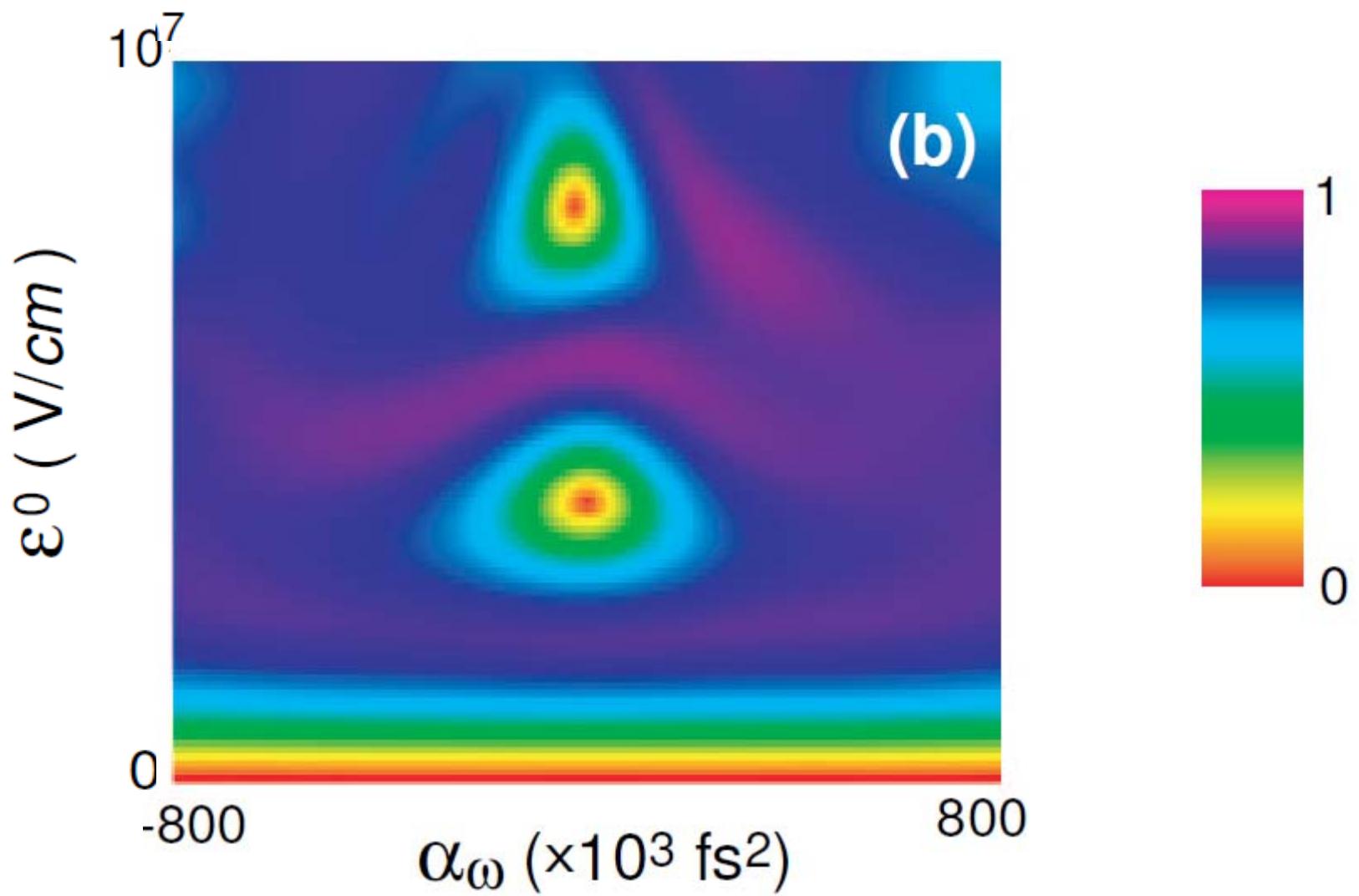






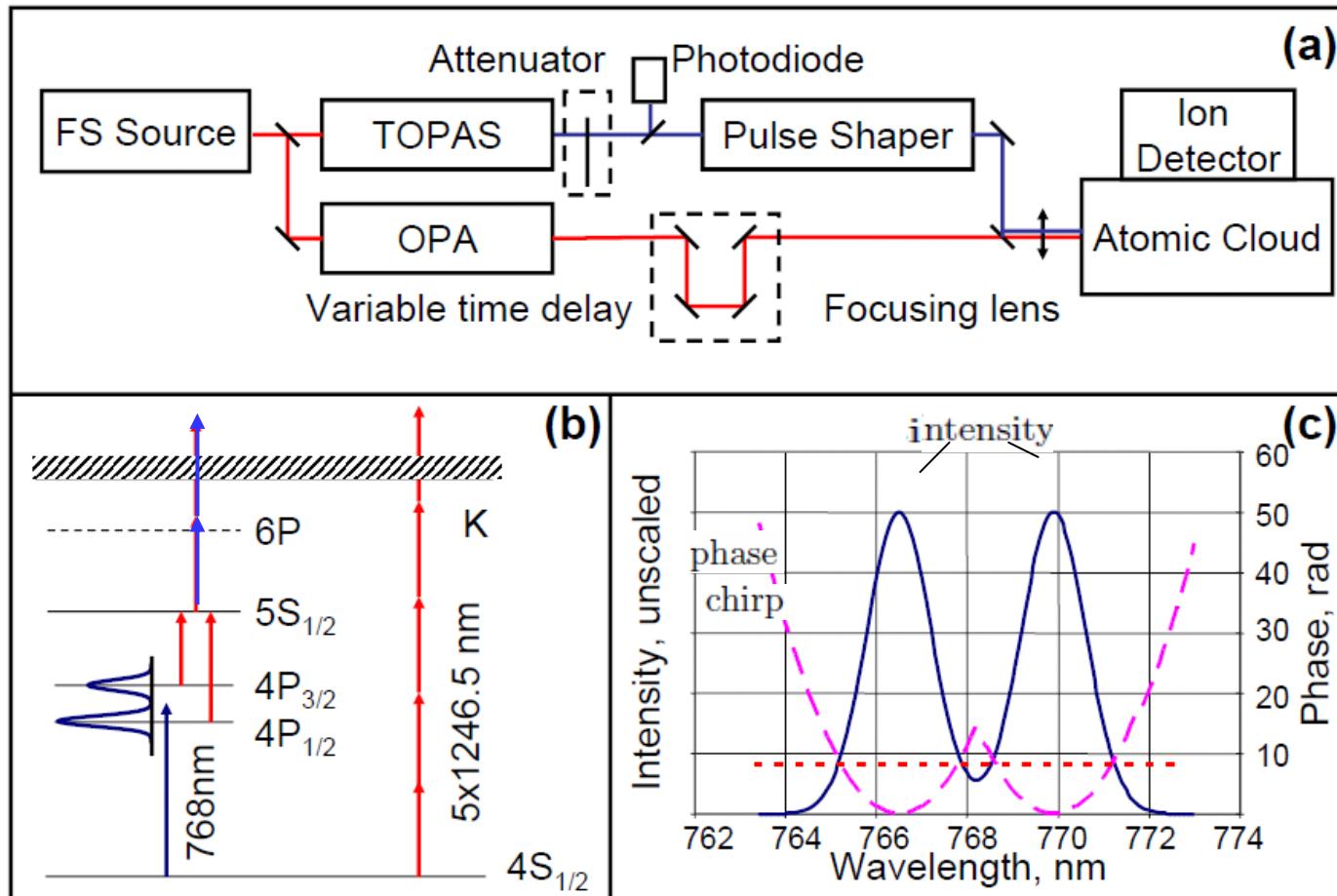




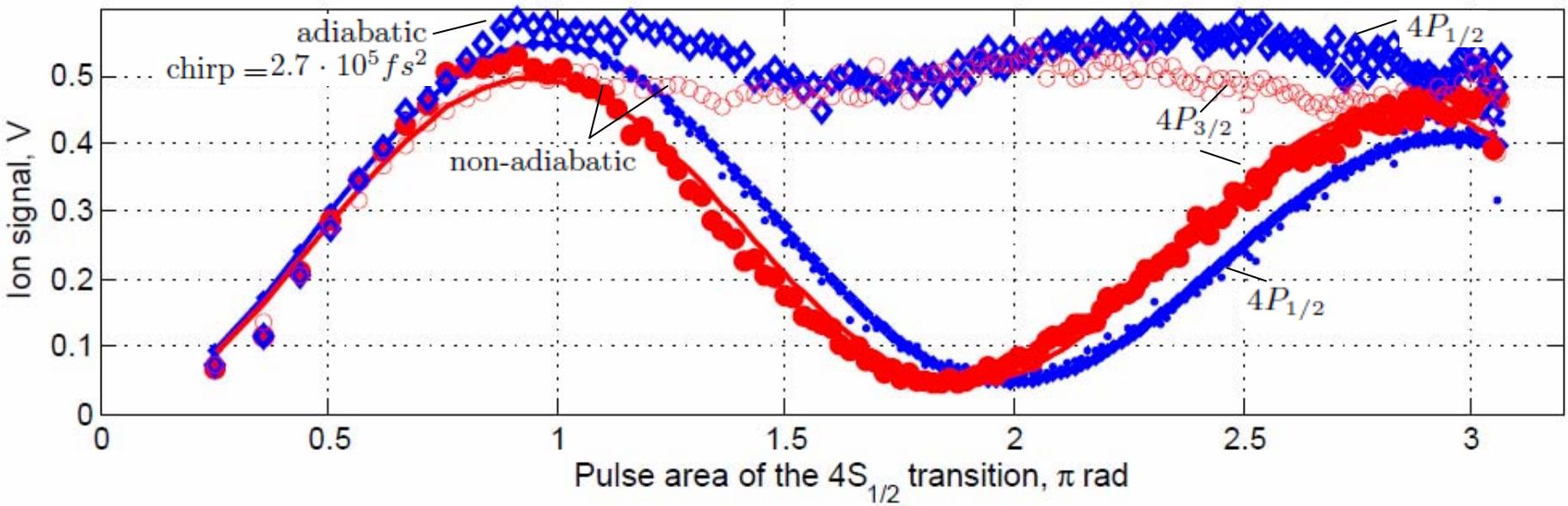
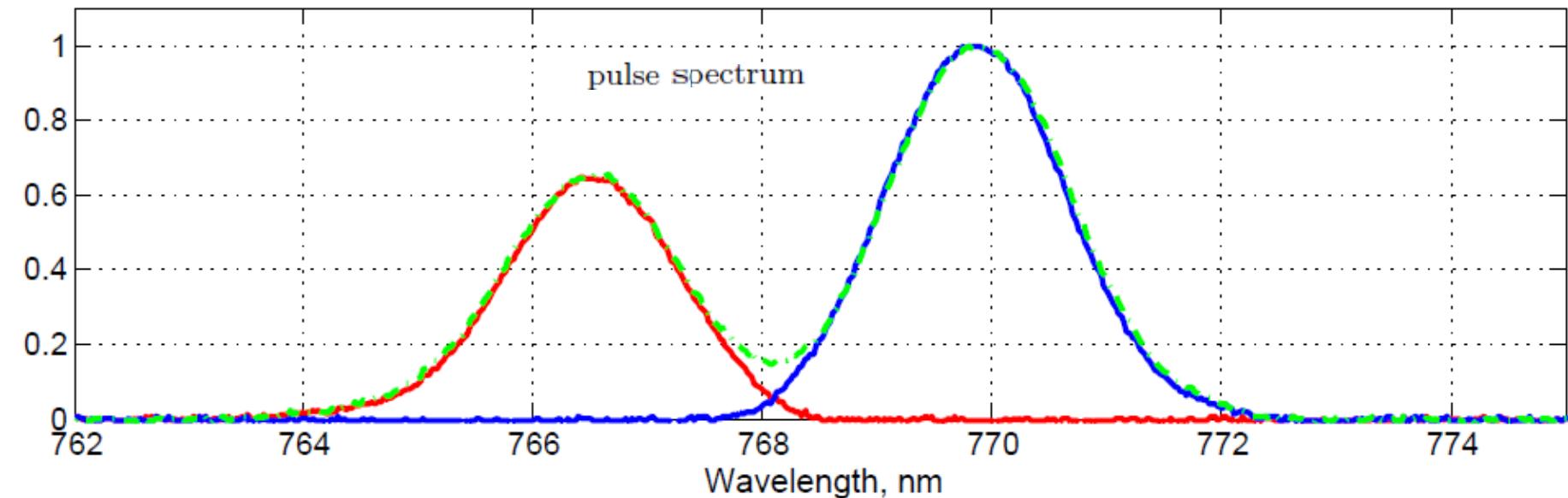


Multi-level piecewise adiabatic passage control: experiment

S. Zhdanovich, J.W. Hepburn, M. Shapiro, and V.Milner, submitted



Calibration experiment: $b_1=1$, $b_2=0$; $b_1=0$, $b_2=1$



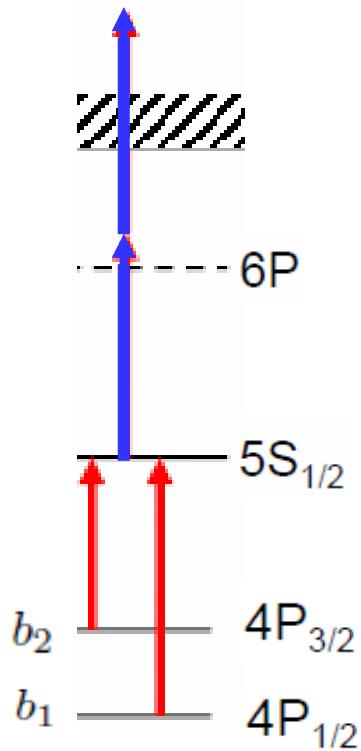
Detection of superposition state by bichromatic control

Detection of superposition state by bichromatic control

$$\Psi = b_1|4P_{1/2}\rangle + b_2|4P_{3/2}\rangle$$

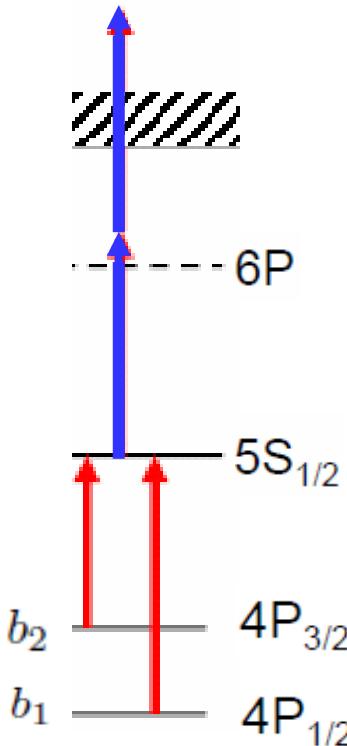
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Detection of superposition state by bichromatic control

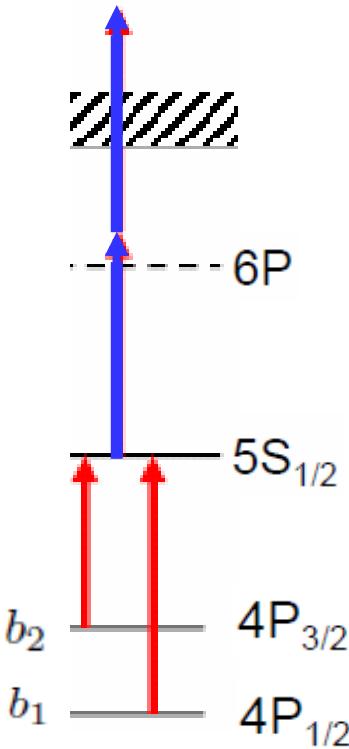
$$\Psi = b_1|4P_{1/2}\rangle + b_2|4P_{3/2}\rangle$$



$$S(t) = |b_1|^2 |\epsilon(\omega_1)|^2 d_{11} + |b_2|^2 |\epsilon(\omega_2)|^2 d_{22} + 2 |b_1 b_2 d_{12} \epsilon(\omega_1) \epsilon(\omega_2)| \sin[(\omega_1 - \omega_2)t + \varphi_{1,2}]$$

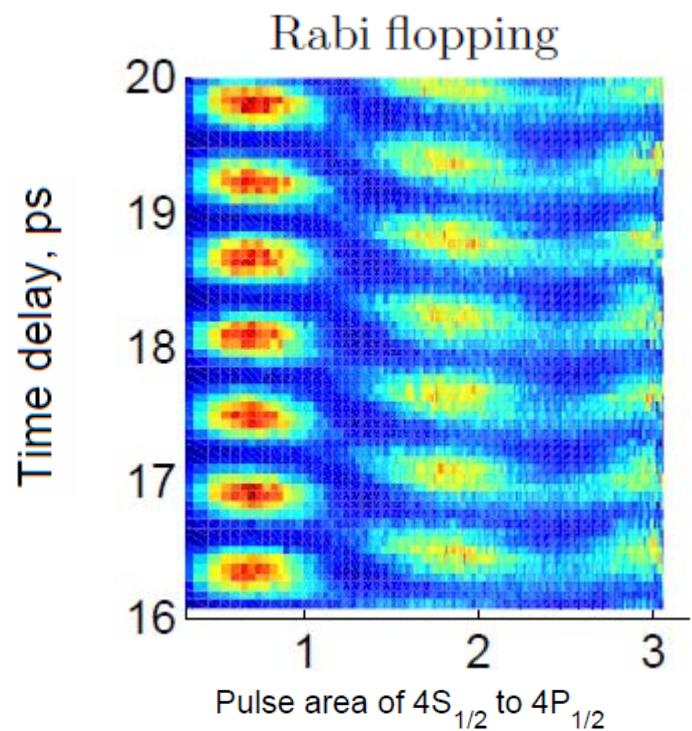
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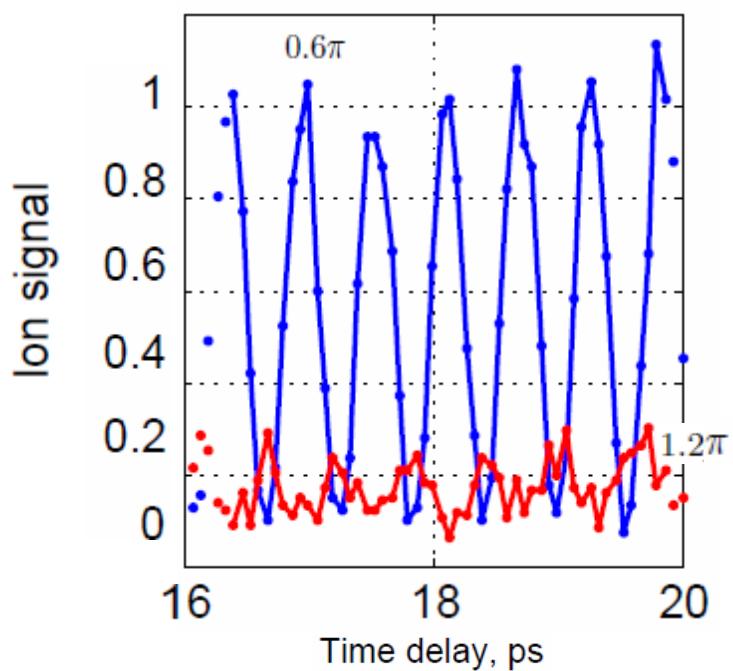
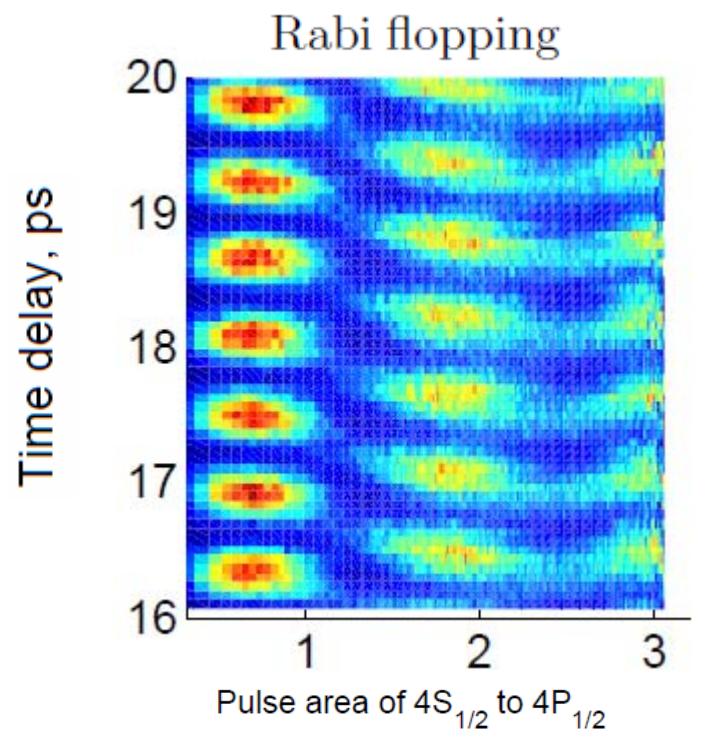
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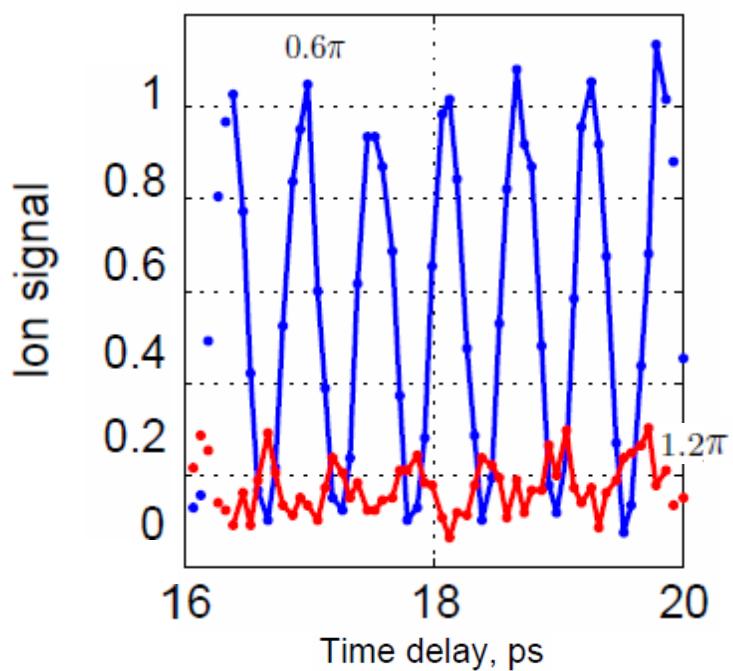
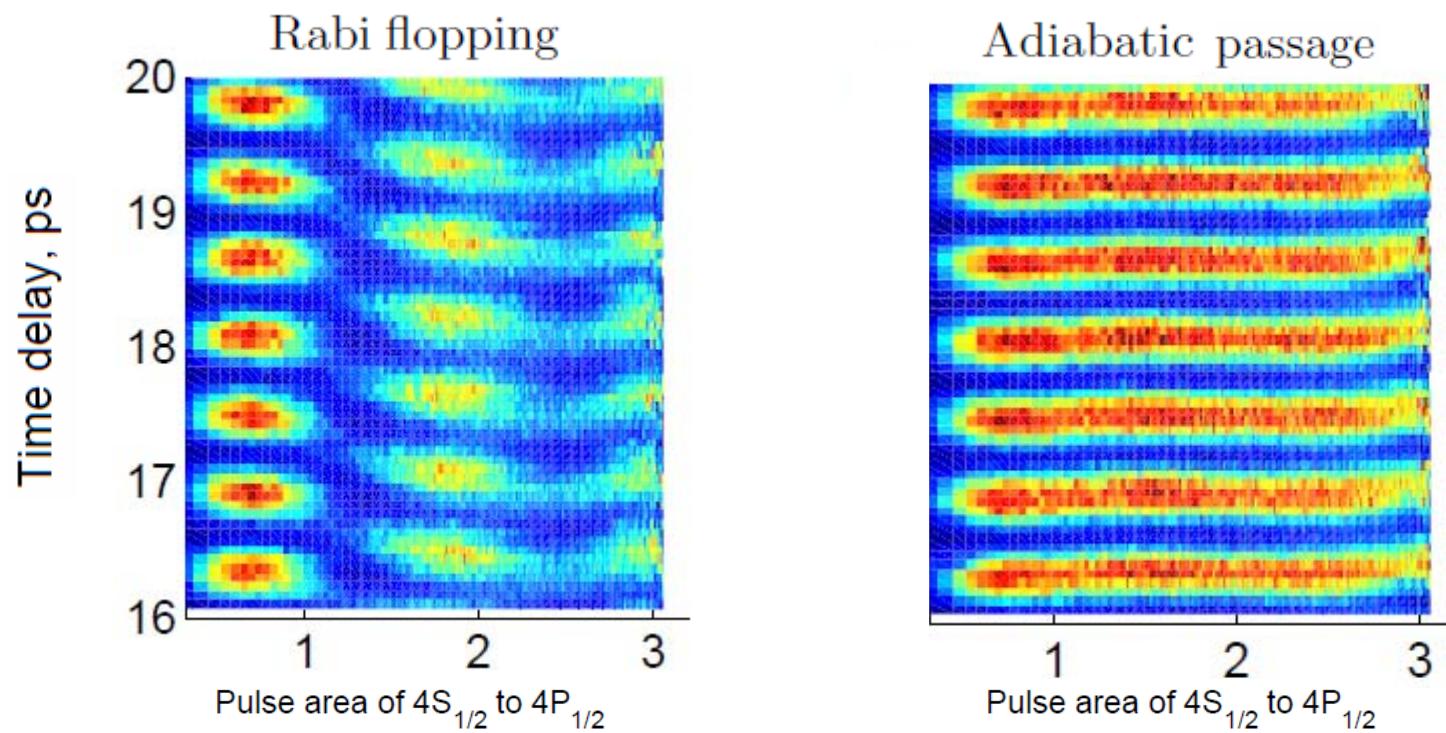


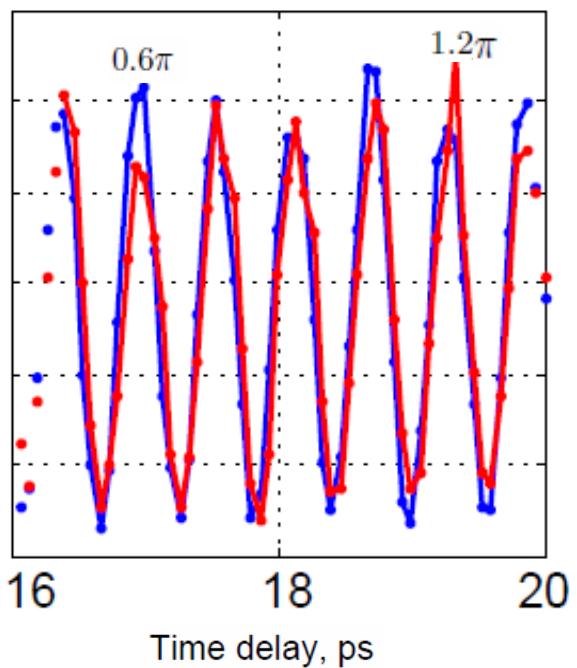
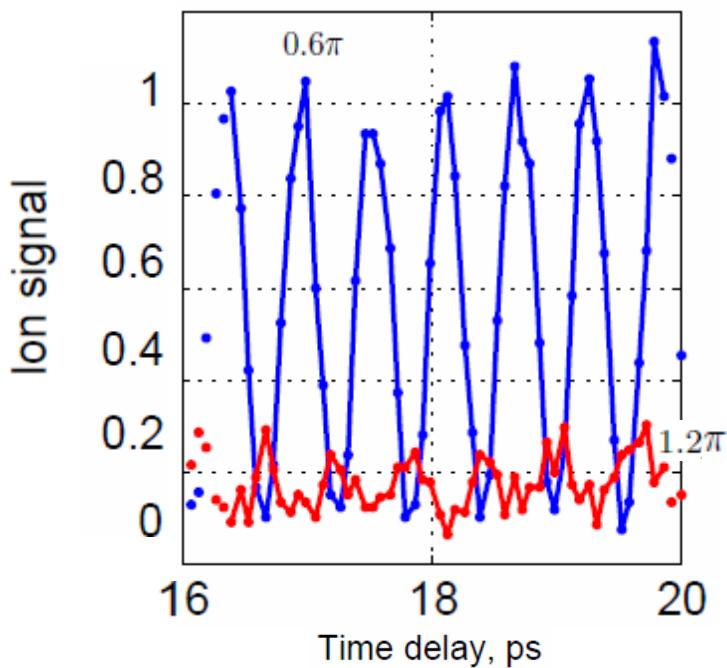
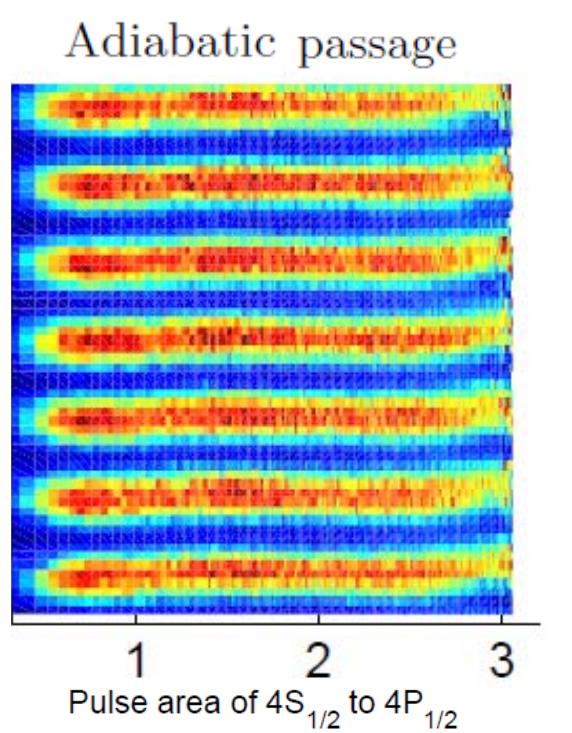
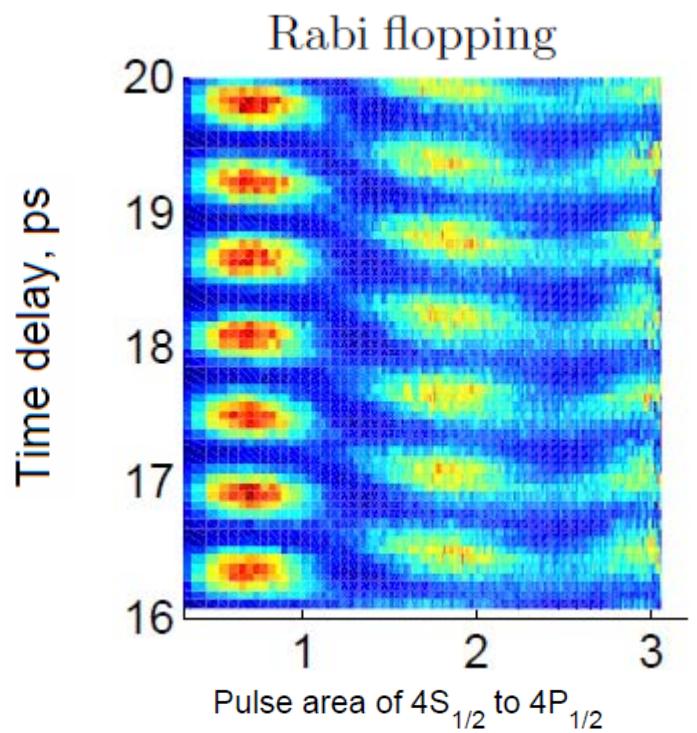
$$S(t) = |b_1|^2 |\epsilon(\omega_1)|^2 d_{11} + |b_2|^2 |\epsilon(\omega_2)|^2 d_{22} + 2 |b_1 b_2 d_{12} \epsilon(\omega_1) \epsilon(\omega_2)| \sin[(\omega_1 - \omega_2)t + \varphi_{1,2}]$$

$$d_{ij} = \langle \Psi_j | d | 5S_{1/2} \rangle \langle 5S_{1/2} | d | \Psi_i \rangle$$

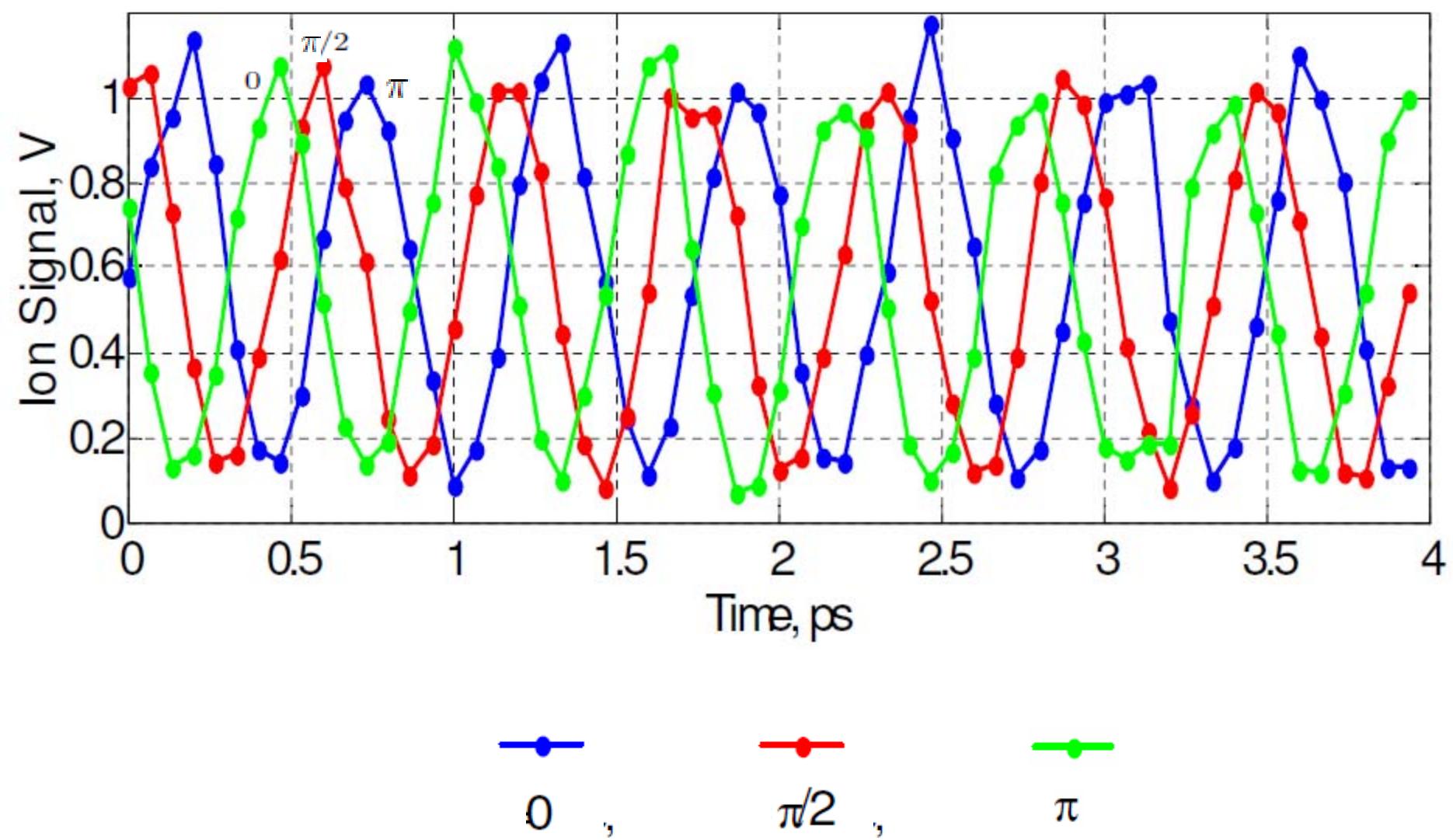




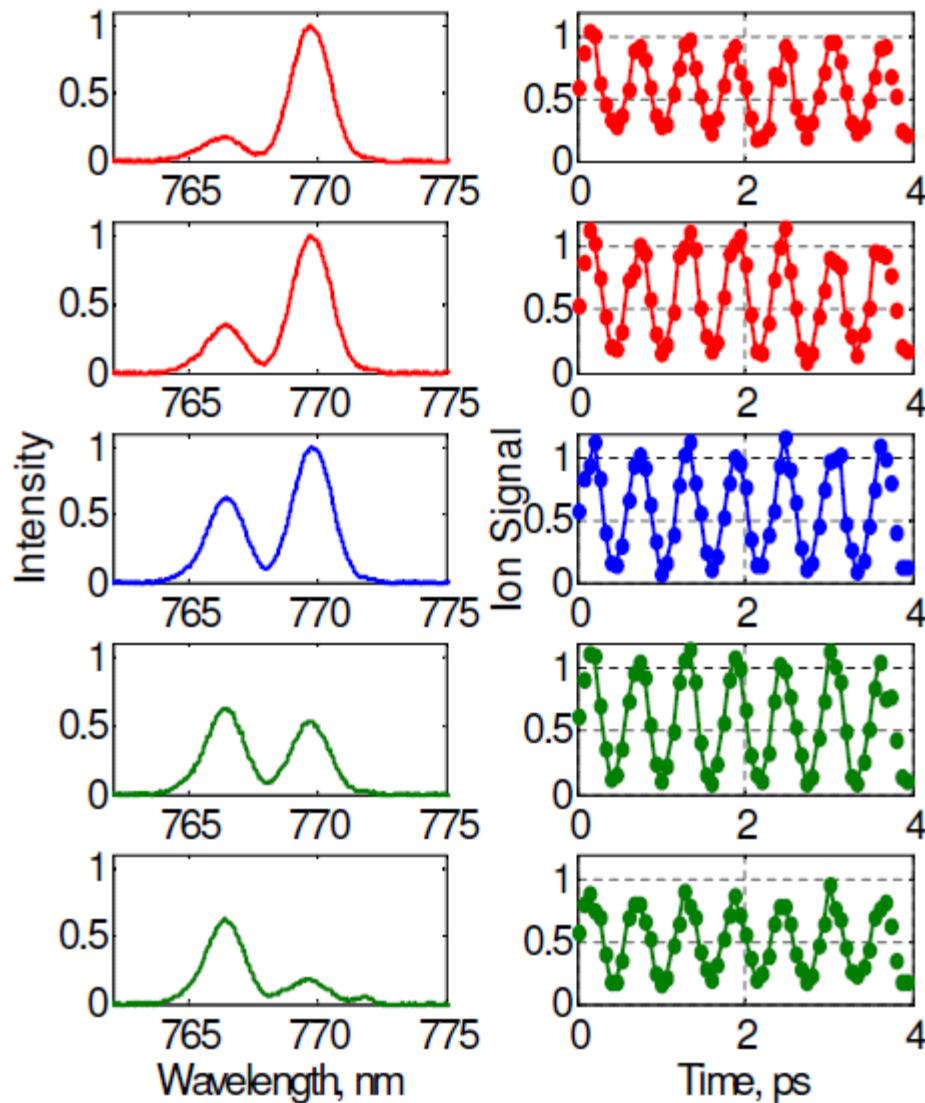




Addition of a constant relative optical phase $\theta_{1,2}$

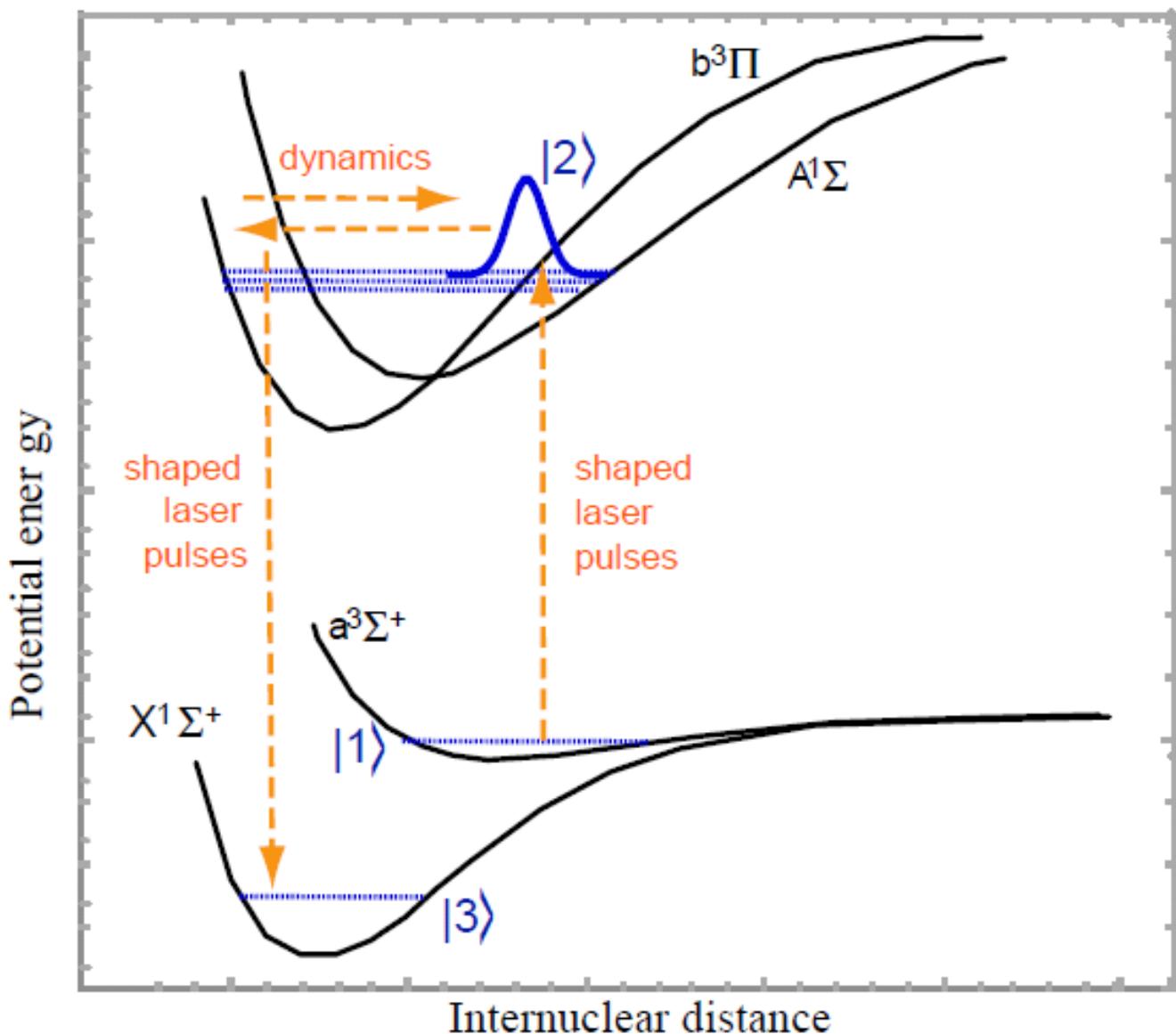


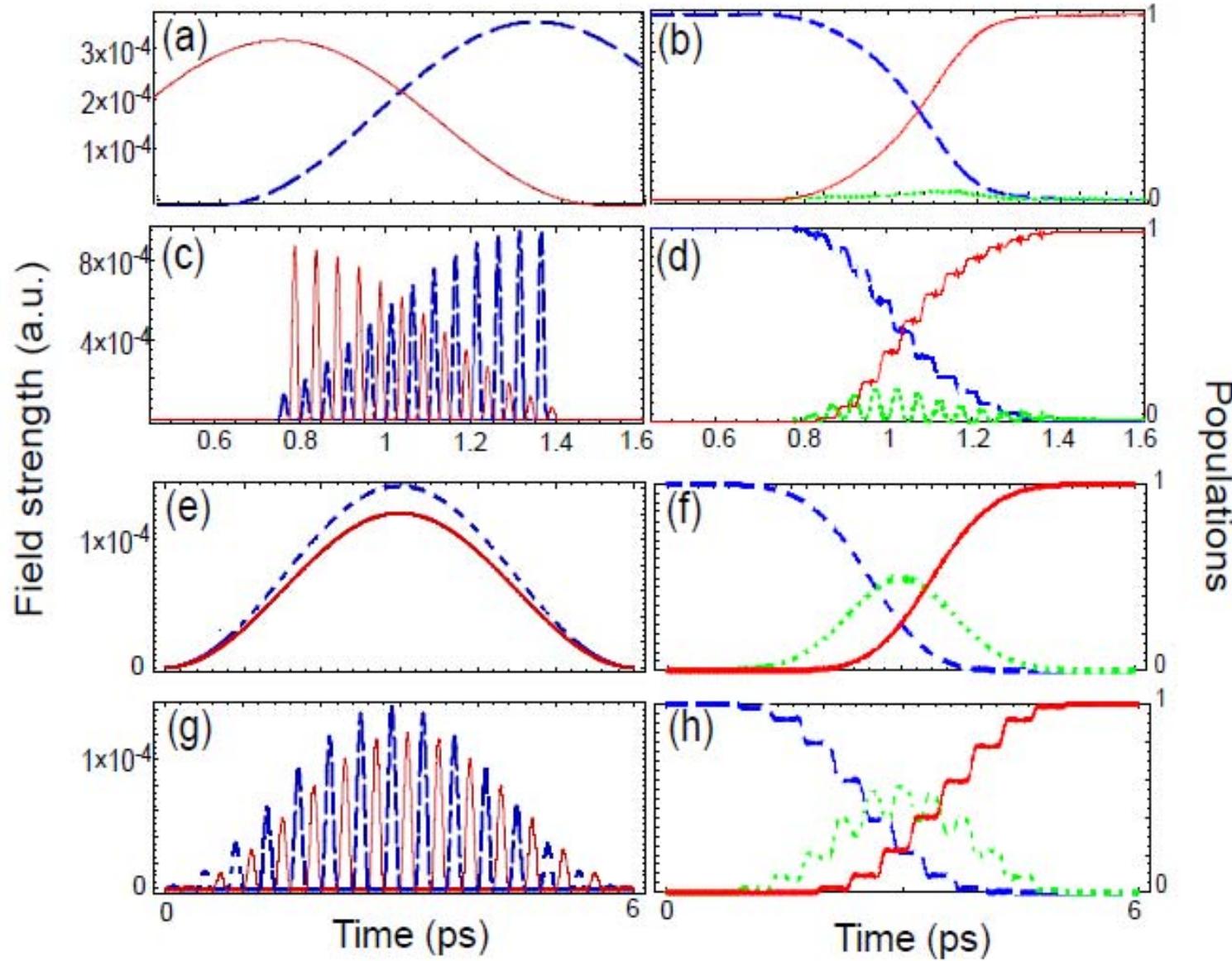
Amplitude ($|b_1|^2$, $|b_2|^2$) control

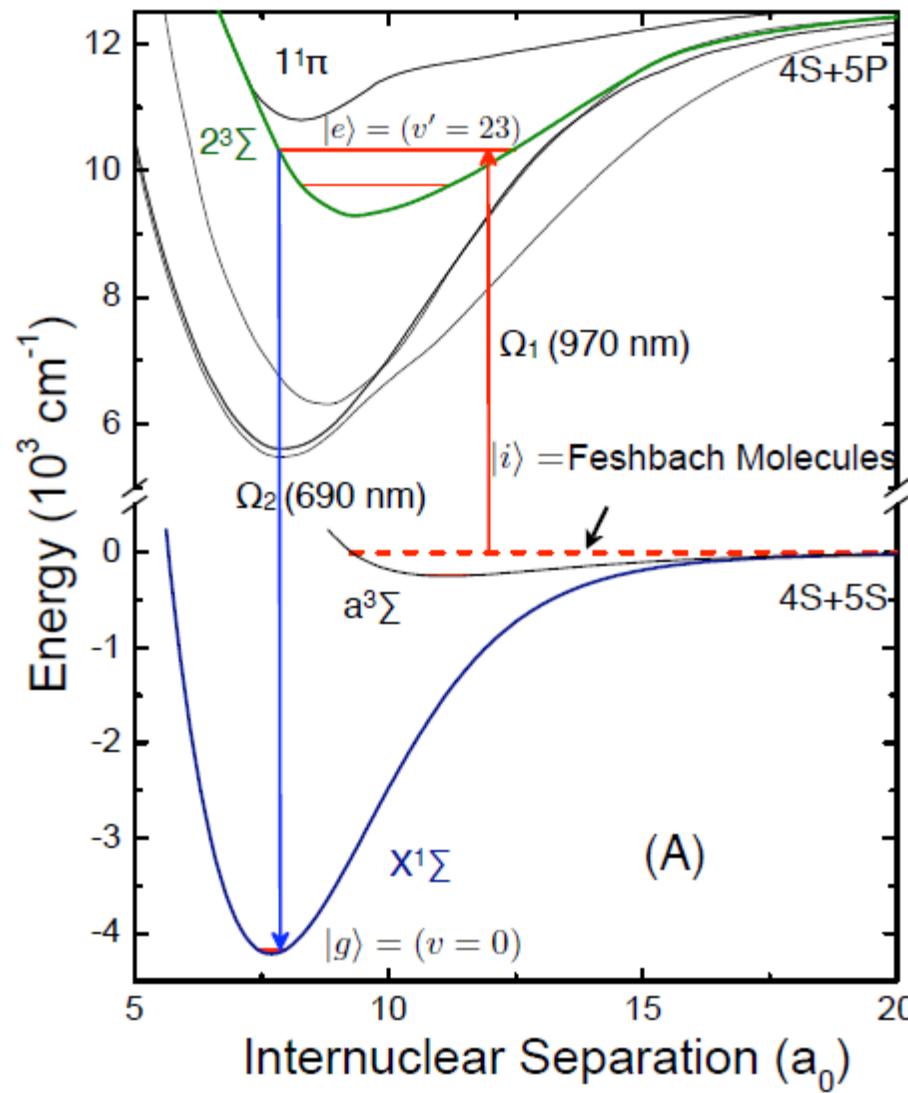


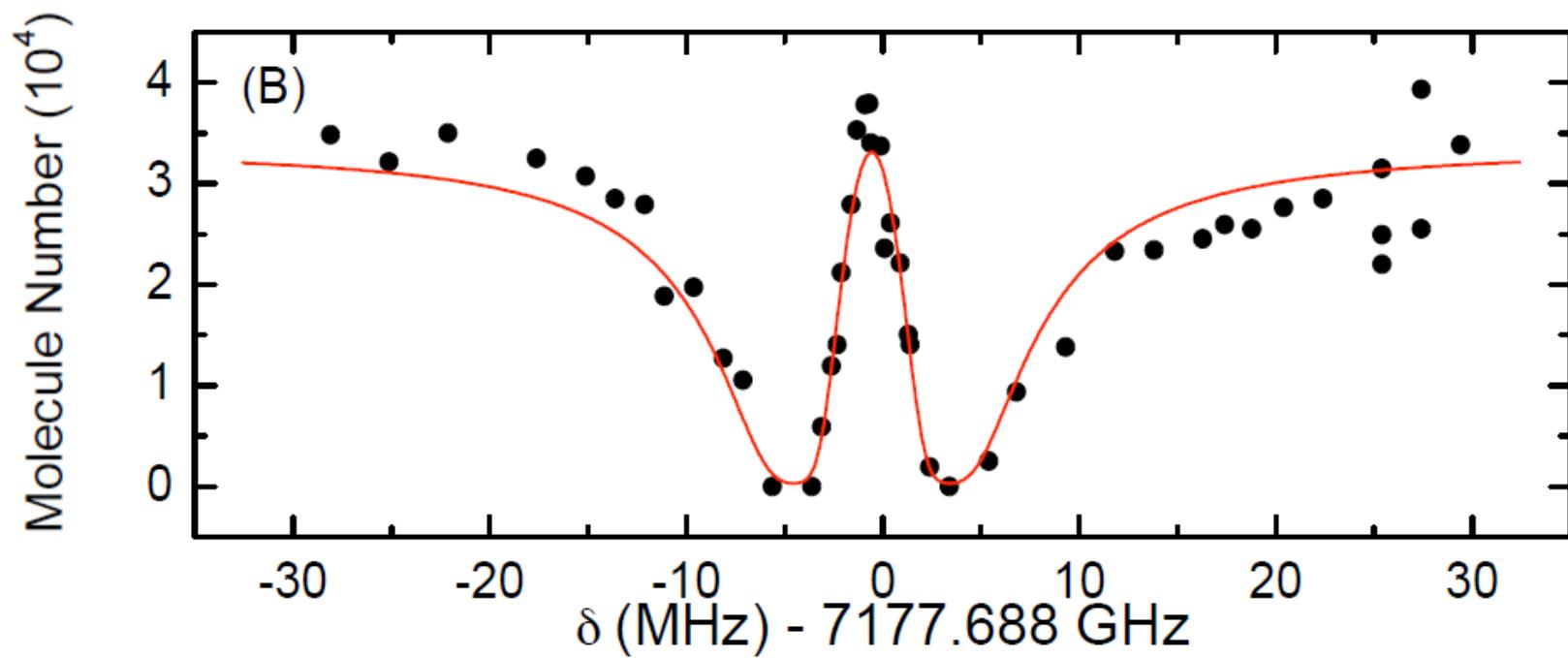
Piecewise photoassociation

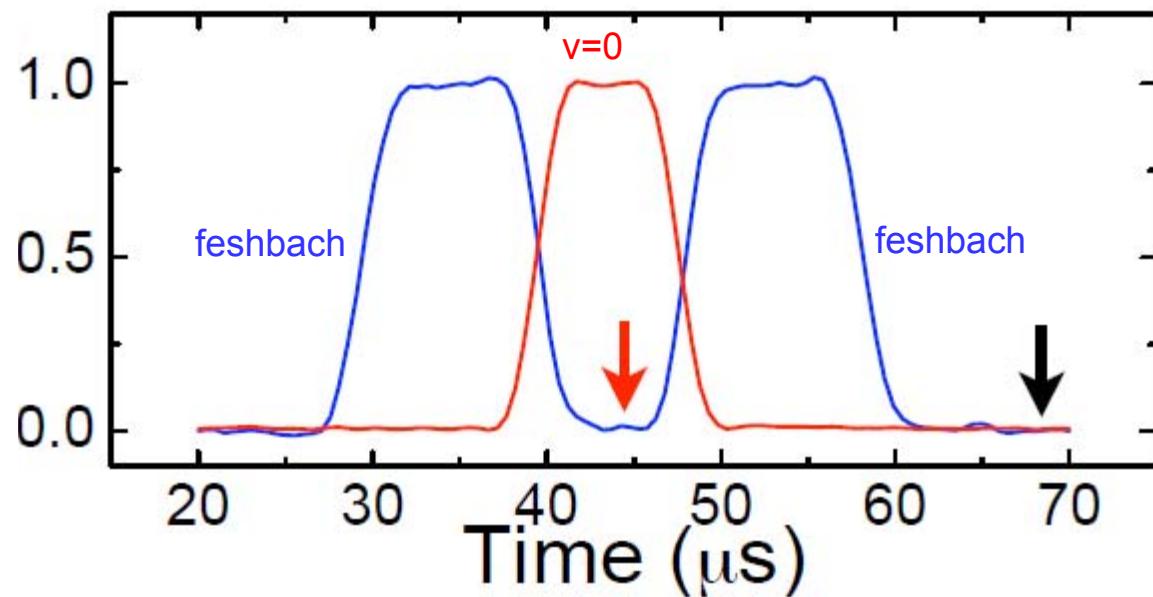
E.A. Shapiro, A. Pe'er, J. Ye, M. Shapiro "Piecewise adiabatic population transfer in a molecule via a wave packet" Phys. Rev. Lett. **101**, 023601 (2008).

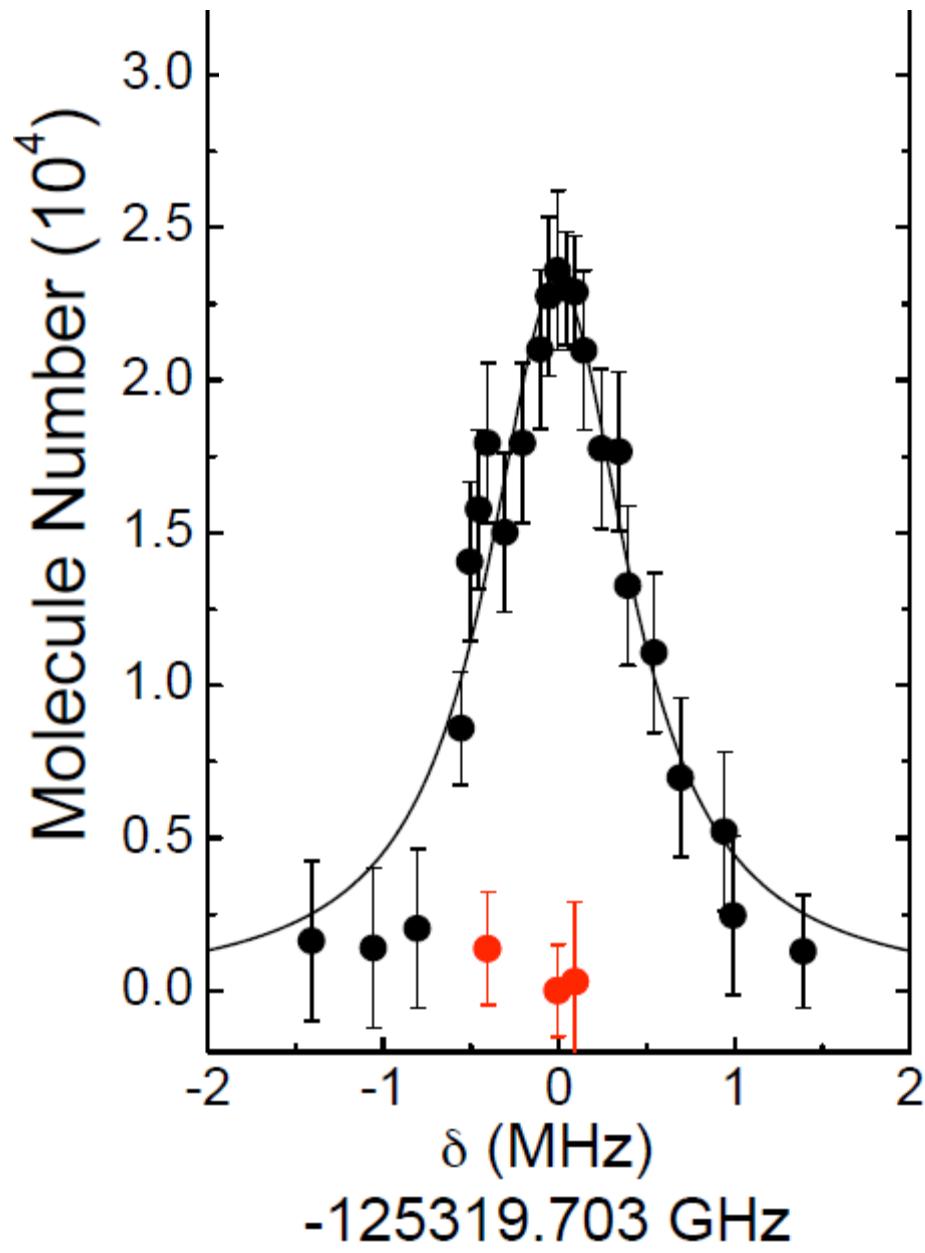












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