QUANTUM CONTROL OF LIGHT AND MATTER

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# Dynamical Quantum Error Correction: From dynamical decoupling (DD) to dynamically corrected gates (DCGs)


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# The quest for high-fidelity dynamical control

Scalable QIP requires that information is realized fault-tolerantly

- Physical QIP devices are:
  - ➞ Imperfectly isolated: Environmental errors (decoherence, leakage...)
  - ➞ Imperfectly controllable: Operational errors (systematic, random...)

*Methods for quantum error control need to remove more noise than they introduce!*

- Accuracy threshold theorem(s):

Shor 1996; Kitaev 1997; Knill et al 1998; Aharonov&Ben-Or 1998; Preskill 1998; Steane 1999, 2003; Knill 2005...

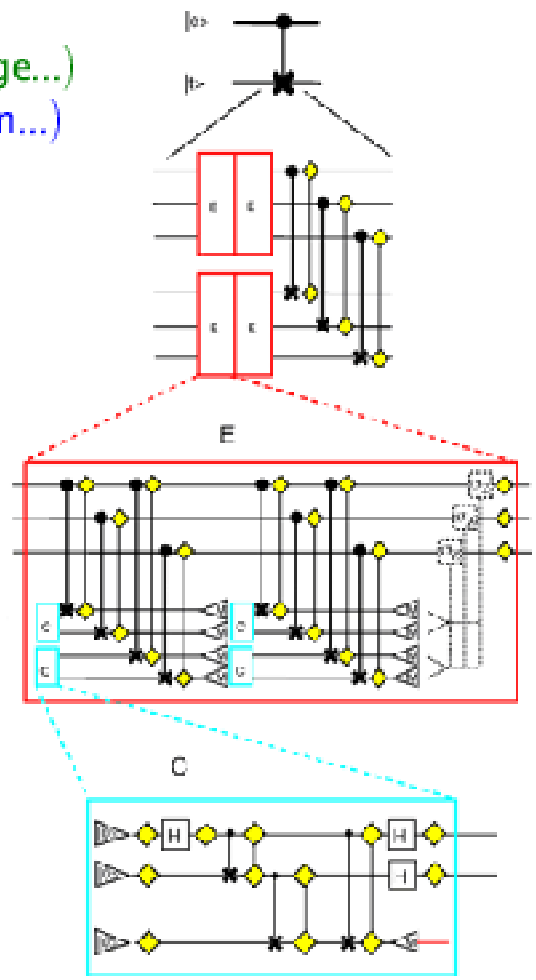
Fault-tolerant architectures require a small error per gate,

$$\text{EPG} < \text{EPG}_{\text{thres}} \approx 10^{-6} \text{ to } 3 \times 10^{-2}$$

➞ Experimentally achieved EPGs $\geq 10^{-2}$...

➞ Estimated number of physical CNOTs needed at EPG=1% for $10^3$ logical gates on 100 qubits $\approx 10^{14}$...

Lower EPGs are imperative



# Open-loop control to the rescue...

Advantages of 'open-loop' error mitigation include
(i) Design simplicity: No measurement and memory overheads; (ii) Established tradition in high-resolution NMR; (iii) Increasing availability in QI technologies.

Key idea: Coherent averaging of interactions

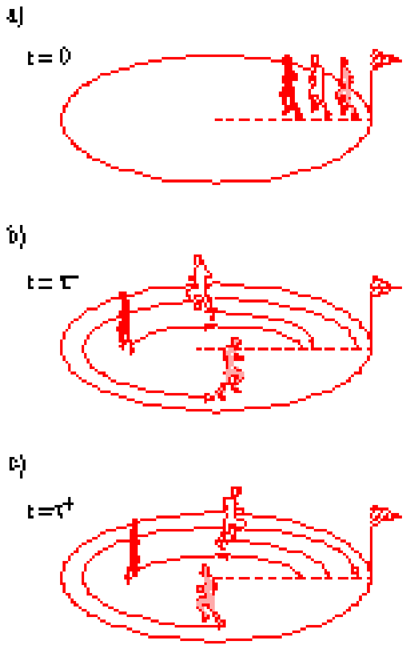
Simplify spectra by removing the splittings due to unwanted interactions.

Paradigmatic example: Spin echo ⟷ Effective time-reversal

Hahn, PR 80 (1950).

Theory: Average Hamiltonian formalism

Haeberlen & Waugh, PR 175 (1968); Waugh, J. Magn. Res. 50 (1982).



QIP tasks: Engineering of closed- and open- system dynamics

- Halting natural evolution: No-op/quantum memory...
- Switching off qubit couplings: Hamiltonian simulation...
- Switching off coupling to environment: Decoherence control...
- Symmetrizing coupling to environment: DFS/NS synthesis...

# A paradigmatic example: Phase noise

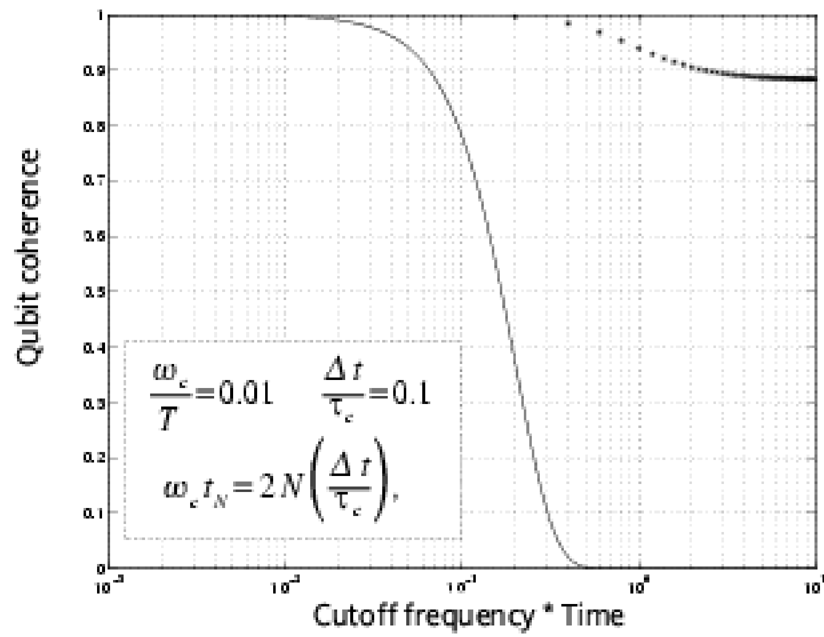
Dephasing spin-boson model:

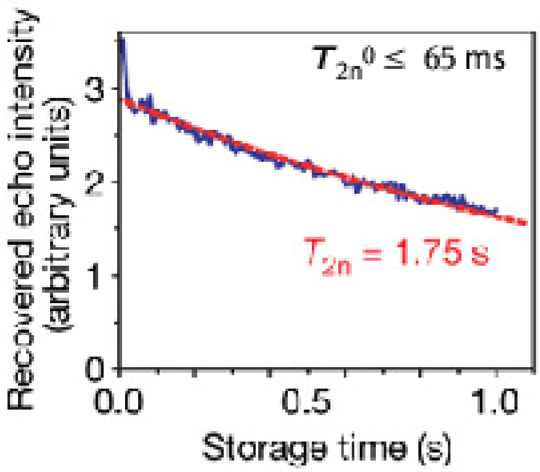
LV & Lloyd, PRA 58 (1998).

$$H = \Omega_0 \sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sigma_z \otimes \sum_k g_k \left(b_k + b_k^\dagger\right)$$

Control action (PDD):
A train of identical, resonant $\pi_x$ pulses, with separation $\Delta t$ – arbitrarily strong and fast (BB).

Decoherence suppression if control period $T_c = 2\Delta t$ shorter than memory correlation time.




Experimental demonstrations:

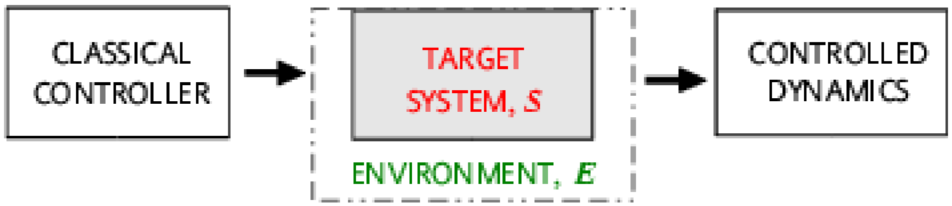
✔ BB control of fullerene qubits.

Morton et al, Nature Phys. 2, Jan 2006.

✔ DD-enhanced quantum storage of electron-spin state in the nuclear-spin state of $^{31}$P in $^{28}$Si single crystal.

Morton et al, Nature 455, Oct 2008.

# Dynamical decoupling (DD) framework




$$H_{\text{tot}}(t) = (H_S + H_{\text{ctrl}}(t)) \otimes I_E + I_S \otimes H_E + \sum_a E_a \otimes B_a \equiv H_{\text{ctrl}}(t) + H_{\text{error}}$$

Reduced system dynamics: $\rho_S(t) = Trace\left\{U(t)\rho_S(0) \otimes \rho_E(0)\, U^\dagger(t)\right\}$

- Environment E is uncontrollable: Adjoin (semiclassical) controller acting on S only,

$$H_c(t) = H_{ctrl}(t) \otimes \boldsymbol{I}_E = \sum_m \left(H_m \otimes \boldsymbol{I}_E\right) u_m(t) \longleftarrow \text{Control inputs}$$

- DD objective: To actively correct a set of error Hamiltonians $\Omega = \{H_S, E_a\}$ by unitary operations drawn from a finite control repertoire so that

$$U(T) \approx \boldsymbol{I}_S \otimes U_E(T), \quad T > 0 \;\Rightarrow\; \rho_S(T) = \rho_S(0) = |\psi\rangle\langle\psi|$$

→ BB setting: $H_c(t)$ realizes a set of instantaneous pulses – Unbounded controls,

$$H_{\text{error}} \approx 0 \text{ during each control operation}$$

→ Physical prerequisite: Time-scale separation – Non-Markovian error regime,

$$T_{DD} \ll \tau_c = \min\{\tau^{\text{corr}}\}$$

# Dynamical error control: (Some) theory challenges

- What about long-time high-fidelity quantum storage?

  DD performance for finite delay/long time depend critically on 'averaging' accuracy...

  → Errors must be removed to high-order while keeping complexity reasonable.

- What about error-corrected quantum computation?

  Different schemes for combining DD with universal control exist in the BB limit: 'intercalate' gates with DD pulses... 'spread' gate operation over DD cycle...

  LV, Lloyd, Knill, PRL 83 (1999); Khodjasteh & Lidar, PRA 78 (2008).

  → Performance bounds only derived for simplest DD schemes...

  → Shortcomings: (i) Stringent synchronization; (ii) Encoding overheads; (iii) BB resources.

- What about (more) realistic control pulses?

  $\delta$-pulse assumption too unrealistic for many control systems of interest...

  → Open-loop engineering with bounded control inputs substantially more challenging.

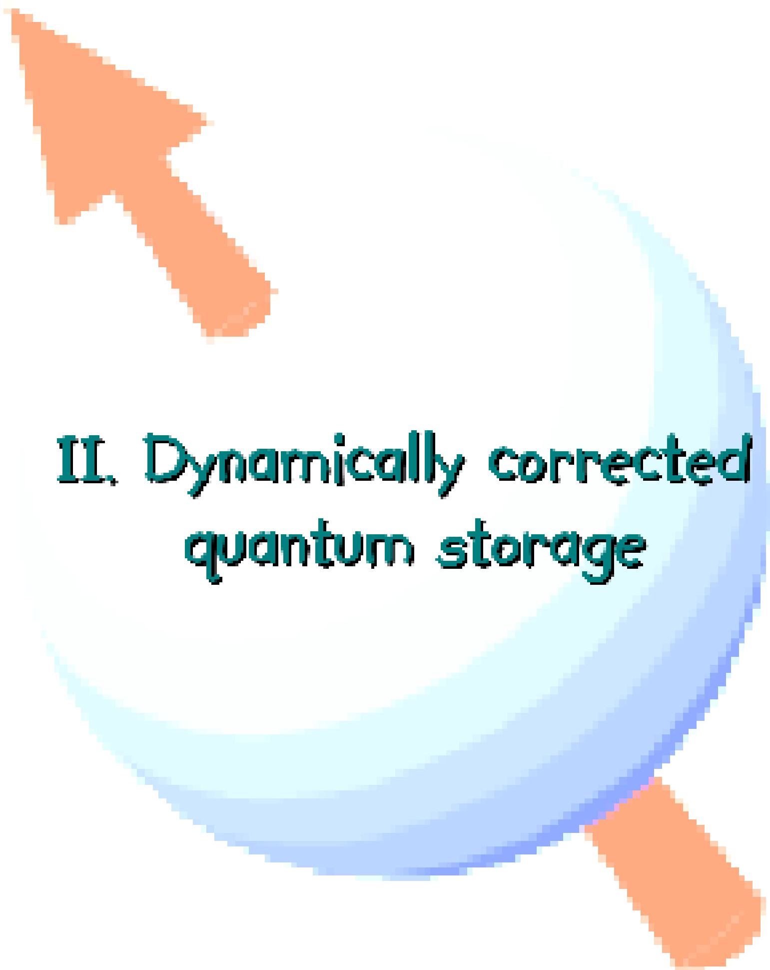
Outline:

II. Case study: Long-time electron spin storage in a QD –

W. Zhang et al., PRB-RC 75, 201302 (2007); PRB 77, 125336 (2008).

III. Dynamically corrected universal quantum gates –

K. Khodjasteh & LV, PRL 102, 080501(2009); arXiv: 0906.0525.



# II. Dynamically corrected quantum storage

# Low-level DD: Periodic DD

LV, Knill, Lloyd, PRL 82, 2417 (1999); Zanardi, PLA 258 (1999).

- Control assumptions: (i) Cyclic controller $U_c(t + T_c) = U_c(t),\ T_c > 0,$

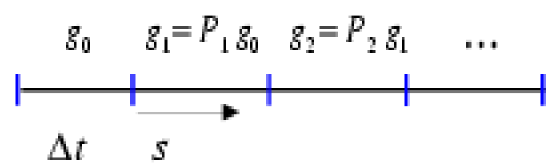
  (ii) Constant, norm-bounded Hamiltonian, $\|H\| \equiv \|H_{error}\| \leq k$

- Group-based DD: Choose $\mathcal{G}_{DD} = \{g_j\}, j = 0,\ldots,|\mathcal{G}_{DD}| - 1,\ g_0 = I_S$, a discrete group.

  Periodic DD (PDD) implemented by letting $T_c = |\mathcal{G}_{DD}|\Delta t$ and by assigning $U_c(t)$ as

$$U_c\big((l-1)\Delta t + s\big) = g_{l-1}$$

Fixed group path --
Sequence of BB control pulses




- Cycle propagator: Compute via Magnus expansion, convergent for $kT_c < 1$ [fast control limit]

$$U(T_c) = e^{-i\overline{H}T_c},\ \overline{H} = \sum_{m=0}^{\infty} \overline{H}^{(m)} \Rightarrow \overline{H} \approx \overline{H}^{(0)} = \frac{1}{|\mathcal{G}_{DD}|}\sum_j g_j^\dagger H g_j \equiv \Pi_{\mathcal{G}_{DD}}(H)$$

$$\Pi_{\mathcal{G}_{DD}}(E_a) = 0,\quad \forall E_a \in \Omega,\quad E_a \text{ traceless}$$

Decoupling condition

Symmetrization of controlled dynamics: 'Filter out' unwanted contributions by symmetry.

# Principles of high-level DD design

- PDD suffers from coherent error accumulation due to higher-order Magnus corrections...

$$F_T = 1 - O\left(T^2 \Delta t^2 \|H_{error}\|^4\right)$$

- Design of high-level DD protocols [BB limit]:

(1) Concatenated DD: Recursively apply a lower-order periodic sequence.
Optimize short-time performance by effective renormalization of $H_{\text{error}}$:

$$F_T = 1 - O\left(T^2 \|H_{error}^{eff}\|^2\right)$$

Khodjasteh & Lidar, PRL 95 (2005); PRA 75 (2007).

→ Number of required pulses grows exponentially with concatenation level...

(2) Optimal DD: Achieve exact cancellation of $H_{\text{error}}$ to desired order:

$$\Delta t_k = T \sin^2 \frac{k\pi}{2(N+1)}, \quad k = 1, 2, \ldots, N$$

Uhrig, PRL 98 (2007).

→ Linear complexity, however only applicable to pure dephasing...

(3) Randomized DD: Pick control operations and/or path at random.
Optimize long-time performance by enforcing probabilistic cancellation of $H_{\text{error}}$:

$$F_T = 1 - O\left(T \Delta t^5 \|H_{error}\|^6\right)$$

LV & E. Knill, PRL 94 (2005);
Santos & LV, PRL 97 (2006); NJP 10 (2008).

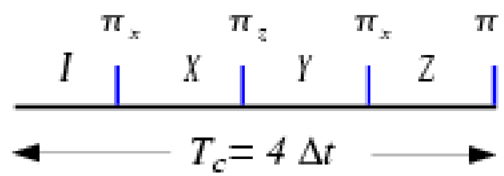
→ Robust against model uncertainty, however requires tracking of control trajectory...

# DD by examples: Single-qubit setting

$$H_{\text{error}} = I_S \otimes H_E + X \otimes B_x + Y \otimes B_y + Z \otimes B_z$$

- The basic PDD sequence: 'Universal DD' based on Pauli group

PDD=fXfZfXfZ=$C_1$




$$\mathcal{G}_{DD} = \mathbb{Z}_2 \times \mathbb{Z}_2 \rightarrow \{I, X, Y, Z\}$$

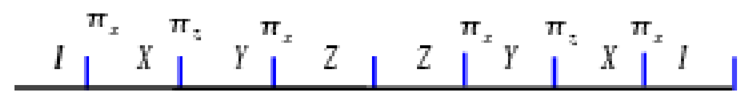
$$\Pi_{\mathcal{G}_{DD}}(\sigma_a) = \frac{1}{4}\left(I\,\sigma_a\,I + X\,\sigma_a\,X + Y\,\sigma_a\,Y + Z\,\sigma_a\,Z\right) = 0,\quad a = x, y, z$$

→ Decoherence error removed to lowest order, $\overline{H}^{(0)} = I_S \otimes H_E$ – but $\overline{H}^{(1)}$ couples S-E...

→ Different 'group paths' give different sequences with same $\overline{H}^{(0)}$ but different $\overline{H}^{(1)}$.

- Improve PDD averaging by invoking...

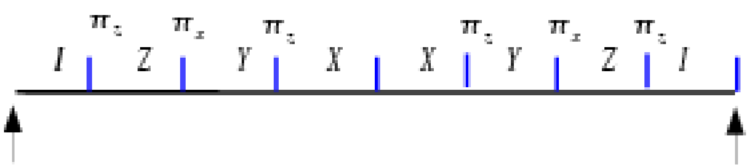
SDD=[fXfZfXfZ][Time-reverse]




→ Symmetrization of control path: System operators removed in all odd order terms...

→ Randomization of control path: At each cycle pick path at random and symmetrize...

SRPD=[fZfXfZfX][Time-reverse]



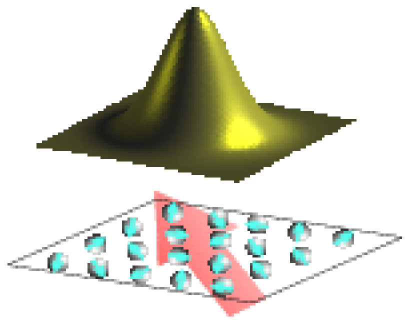

→ Concatenation: Recursively apply Pauli DD...

$$C_{m+1} = C_m P_1 C_m P_2 C_m P_3 C_m P_4$$

e.g.: $\text{PCDD}_2 = C_1 X C_1 Z C_1 X C_1 Z$ = [fXfZfXfZ]X[fXfZfXfZ]Z... = [fXfZfXfYfXfZfXfI][Repeat] $T_c = 16\,\Delta t$

Operators coupling S-E appear at order $\overline{H}^{(4)}$ and higher.

# DD of hyperfine-induced decoherence



$N \sim 10^6$

- Electron spin in a quantum dot: Central spin problem

$$H_0 = \hbar\,\Omega_{El}\,S_z \otimes I_{Nucl} + I_{El} \otimes \sum_{k\neq l}^{N} \Gamma_{kl}\,\vec{I}_k\cdot\vec{I}_l + \vec{S}\otimes\frac{1}{2}\sum_{k=1}^{N} A_k\,\vec{I}_k$$

Zeeman splitting — Intrabath dipolar interaction — Hyperfine contact interaction

→ GaAs QD @ sub-K temperature, sub-T bias:

$A_k = A_0 \frac{V}{N}|\Psi(\vec{r}_k)|^2$, total strength $A_0 \approx 90\ \mu$eV, $T_2^* \sim \frac{1}{B_{Ov}} \sim \frac{1}{\sqrt{N}\,A}$, $A = \sqrt{\frac{1}{N}\sum_k A_k^2} \approx 10^{-4}\ \mu$eV,

→ Consistent with experimentally measured free induction decay times:

$$T_2^* \approx 10\ ns \leqslant T_2$$

Johnson et al., Nature 2005; Koppens et al., Science 2005; Petta et al., ibid. 2005...

- A fairly peculiar DD problem: 'Pure-bath' dipolar timescale $\tau \approx$ 10-100 μs.
  - → (Approximately) non-dynamical and (strongly) non-Markovian nuclear spin reservoir.
  - → Simultaneous dephasing and relaxation dynamics in the limit of weak bias fields.

Questions: (1) What time scale suffices for good DD? $\omega_c \sim N\,A \sim$ 20 GHz... $B_{Ov} \sim \sqrt{N}\,A \sim$ 20 MHz...
(2) What are best DD performers in realistic regimes?

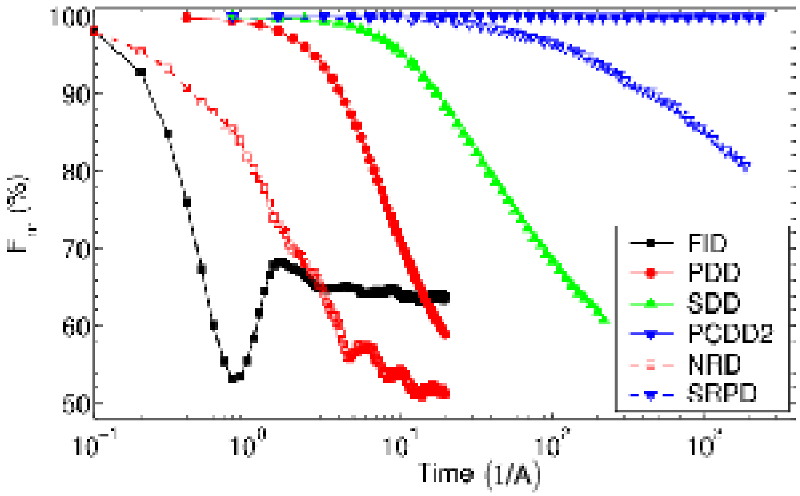
# Control of electron spin coherence: Results

Focus on zero bias field and unpolarized initial bath state.

I. Objective: Arbitrary state preservation.

Realistic pulse delays (> 1 ns) are well outside Magnus convergence domain...

$$\Delta t \sim \frac{1}{\sqrt{N}A} = \frac{1}{\sigma}, \quad kT_c \sim \omega_c T_c \sim \sqrt{N}$$




$N = 15$

$\Delta t = 0.1 \quad (\approx 3\, ns)$

→ DD efficiency determined by spectral width $\sigma$ – not upper spectral cutoff:
Fidelity better than 90% achievable with pulse delays up to √N longer than worst-case estimate.

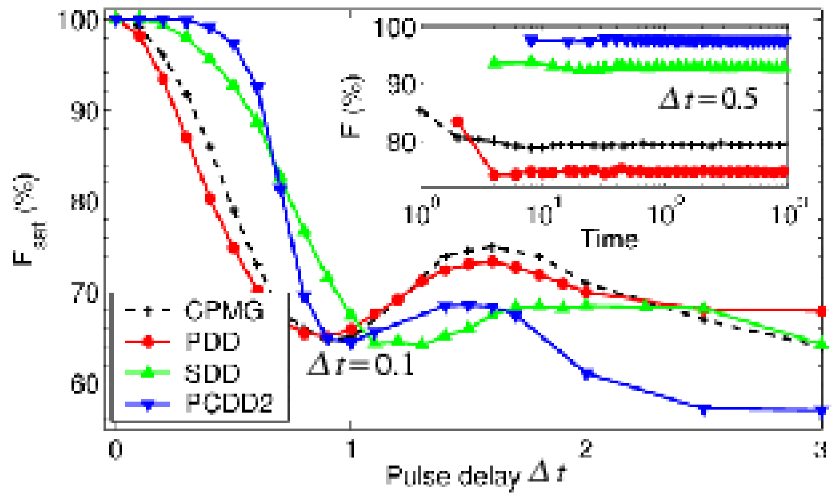
→ Truncated CDD protocol (m=2) shows best performance, as long as pure bath $H_{Nuc} \approx 0$ .
SRPD shows best randomized performance, over as few as 5 control realizations.

# Control of electron spin coherence: Results

Focus on cyclic DD protocols.

II. Objective: Pure state stabilization/DFS synthesis.

Initial electron spin may be aligned with 'effective field' created by control: Eigenstates of dominant Magnus corrections are (approx) preserved. Orthogonal components decay in the long-time limit.




$n^{sat}_{pulse} \sim 50$

[Analogy with spin-locking physics...]

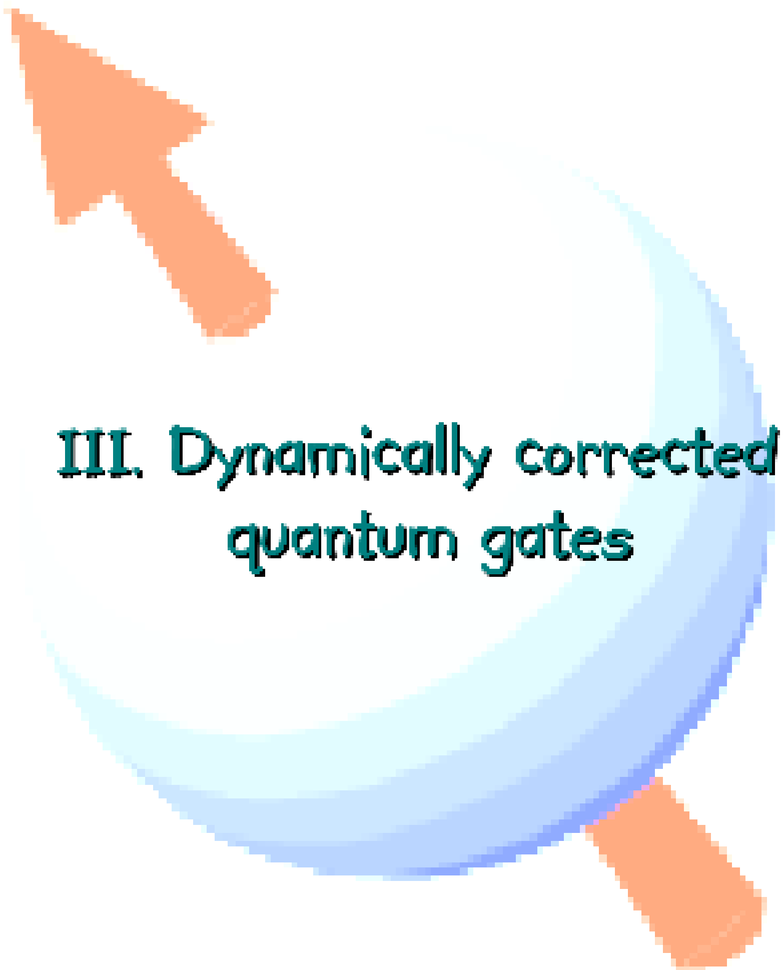
➞ Fidelity saturation indicates open-loop generation of a stable one-dim DFS.

➞ Analytical prediction for CPMG saturation value in the uniform limit $A_k = A$:

$$F_{sat} = 1 - \frac{1}{16} N \left(A \Delta t\right)^2 = 1 - \frac{1}{2} \left(\Delta t / T_2^*\right)^2 ,$$

✔ DD-protected storage of exciton qubits in self-assembled QDs –

Hodgson, LV, D'Amico, PRB 78 (2008).



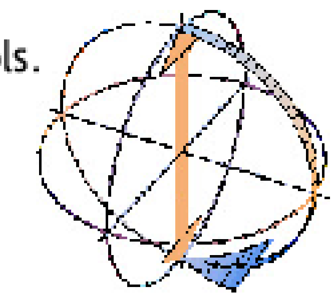
# III. Dynamically corrected quantum gates

# Towards error-correcting quantum gates...


$$H_{\rm tot}(t) = (H_S + H_{\rm ctrl}(t)) \otimes I_E + I_S \otimes H_E + \sum_a E_a \otimes B_a \equiv H_{\rm ctrl}(t) + H_{\rm error}$$

$H_{\rm error} \approx 0$ during each BB pulse, whereas EPG = O($\tau\, ||H_{\rm error}||$) for real-life finite $\tau$...

- Goal: Reduce EPG in a generic gate while avoiding unphysical BB controls.



- [Some] hints from NMR:

  (1) Composite pulses: $H_{\rm error}$ due to systematic faults – purely classical...

  ➞ Exploit non-linear composition properties of rotation errors...

  Levitt (1983); Tycko (1983); Wimperis (1994); Brown, Harrow & Chuang, PRA 70 (2004).

  (2) Strongly-modulating pulses: $H_{\rm error}$ due to internal spin Hamiltonian – fully known...

  ➞ Exploit coherent averaging of Hamiltonian error...

  Fortunato et al, JCP 116 (2002); Boulant et al, PRA 68 (2003).

Unintended error component includes coupling to a dynamical environment, over which no control/minimal knowledge may be available...

# System and control assumptions

- Target system $S$: $n$-qubit drifless register undergoing linear [non-Markovian] decoherence.

$$H_{SE} = \sum_{i=1}^{n} \sum_{\alpha = x,y,z} \sigma_\alpha^{(i)} \otimes B_\alpha^{(i)}$$

$$H_{\text{error}} = I_S \otimes H_E + H_{SE},$$

Bath operators bounded but otherwise unknown

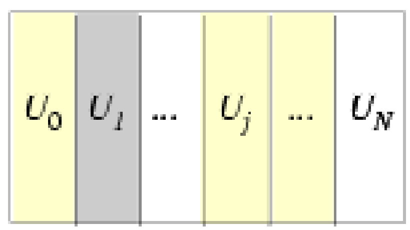
- Controller $C$: Implemented by time-dependent 'primitive' Hamiltonians acting on $S$ only,

$$\left\{ h_x(t)\sigma_x^{(i)},\ h_y(t)\sigma_y^{(i)},\ h_z(t)\sigma_z^{(i)}\otimes\sigma_z^{(j)} \right\}, \quad i, j = 1, \ldots, n \qquad \text{subject to}$$

(i) Finite-power constraint: Bounded control amplitude, $|h_a(t)| \leq h_{\text{max}}$;

(ii) Finite-bandwidth constraint: Minimum switching time for modulation, $\tau_{\text{min}} > 0$.

DCG block structure:




Each $U_j$ generates $U_{\text{ctrl}}(t_j, t_{j-1})$ with error phase $\Phi_j$.

$$U_j(t_j, t_{j-1}) \equiv U_{\text{ctrl}}(t_j, t_{j-1}) \exp[-i\Phi_j(t_j, t_{j-1})]$$

Error action operator

$$\Phi_j^{[1]} = \int_{t_{j-1}}^{t_j} dx\ U_{ctrl}^\dagger(x, t_{j-1}) H_{error} U_{ctrl}(x, t_j) + \Phi_j^{[2+]}$$

Task: Reduce total EPG

$$\text{EPG}_{\text{phys}} \simeq \Phi_j = O(\tau_{\text{min}} \|H_{\text{error}}\|)$$

$$\downarrow$$

$$\text{EPG}_{\text{corrected}} \simeq \Phi_{\text{tot}} = O(\tau^2_{\text{min}} \|H_{\text{error}}\|^2)$$

# Error cancellation via Eulerian DD

Step 1: Seek a combination which removes error while achieving NOOP gate.

LV & Knill, PRL 90 (2003).

- Eulerian DD (EDD): Assume ability to implement group generators, $\mathcal{G} = \{h_l\}$, $l=1,\dots,L$, via bounded-strength primitive control Hamiltonians.
  - → EDD rule for applying generators: Follow an Eulerian cycle on the (Cayley) graph of $\mathcal{G}_{DD}$.

Def. 1 [Cayley graph]: Vertex $g_i$ connected to vertex $g_j$ w edge labeled by $h_l$ iff $g_j = g_i h_l$

Def. 2 [Eulerian cycle]: Closed sequence of $L$x$D$ edges that uses each edge exactly once

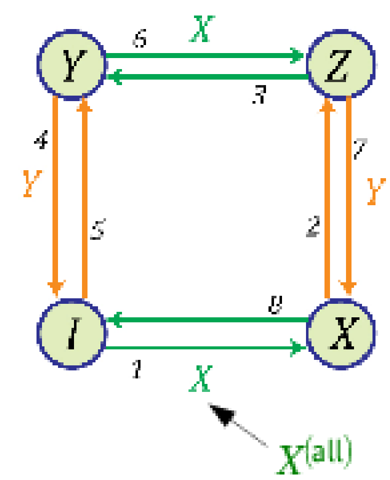
- Example: Arbitrary linear decoherence on n qubits

$$\mathcal{G}_{DD} = \mathbb{Z}_2 \times \mathbb{Z}_2 \rightarrow \{I^{(\text{all})}, X^{(\text{all})}, Y^{(\text{all})}, Z^{(\text{all})}\}, \quad \mathcal{G} = \{X, Y\}$$

→ Collective generators can be implemented by collective primitive Hamiltonians – e.g.

$$X^{(all)} = X_1 \otimes \ldots \otimes X_n = \exp\left[-i\int_0^\tau h_x(s)\,ds\left(\sum_j^n X_j\right)\right]$$

$$\Phi_{EDD} = \sum_{i=1}^{|G|}\sum_{l=1}^{L} U_{g_i}^\dagger \Phi_{h_l} U_{g_i} + \Phi_{EDD}^{[2+]}$$




Euler cycle: X Y X Y Y X Y X

Significantly smaller error compared to free evolution.

# Error cancellation beyond NOOP

<u>Step 2:</u> Seek a combination which removes error while achieving generic gate.

- Additional knowledge of errors to be cancelled is needed: Exploit different gate combinations sharing same error phase →

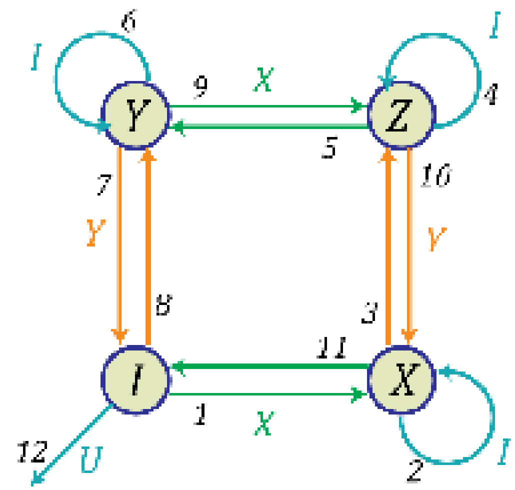
$$M_U = U\exp(-i\Phi),\quad M_I = \exp(-i\Phi)$$

- Modified Eulerian construction: Implement control path which begins at $I$ and ends at $U$ on modified graph →
  - (i) To non-identity vertex, attach edge labeled by $M_I$
  - (ii) To identity vertex, attach edge labeled by $M_U$

$$\Phi_{DCG} = \Phi_{EDD} + \sum_{i=1}^{|G|} U_{g_i}^\dagger \Phi U_{g_i} + \Phi_{DCG}^{[2+]}$$

Total 1st-order error vanishes as long as primitive errors $\Phi_{h_j}$ and $\Phi$ obey DD condition

$\Rightarrow$ EPG $= \|\Phi_{DCG}{}^{[2+]}\| = O[\max(\|\Phi_{h_j}\|^2, \|\Phi\|^2)]$




Euler path: X I Y I X I Y Y X Y X U

Significantly smaller error compared to direct switching.

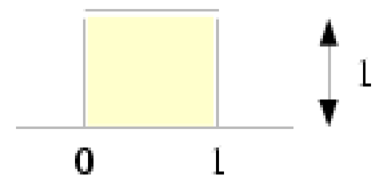
# DCG resourse requirements

Explicit constructions depend on specific 'pulse shape' assumptions:
Focus on piecewise constant controls – rectangular pulses:

→ Assume that control profile over $[t_1, t_2]$ is obtained by stretching & scaling of a fixed reversible pulse shape over $[0, 1]$:

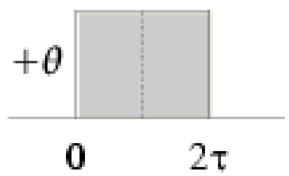
$$h(\theta, t) = \theta\, h_0\left(\frac{t - t_1}{t_2 - t_1}\right), \quad t \in [t_1, t_2]$$

Rotation angle



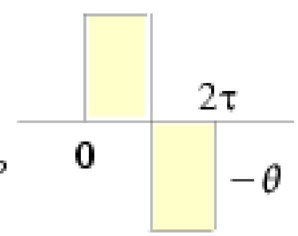

→ Example of gate combinations sharing the same [leading] error phase:

$h_1(\theta, t)$:




$M_U = C_{2\theta}$

$h_2(\theta, t)$:

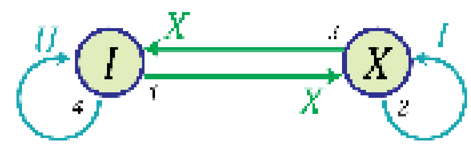



$M_I = NOOP$

$$\Phi_{M_U}^{[1]} \equiv \Phi_{M_I}^{[1]}$$

DCG time overheads:

2 x 4 = 8 ⇒ 16 time slots per DCG for linear decoherence
2 x 2 = 4 ⇒ 6 time slots per DCG for pure dephasing

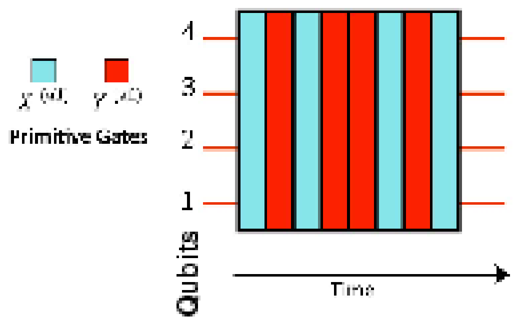



Euler path: *X I X U*

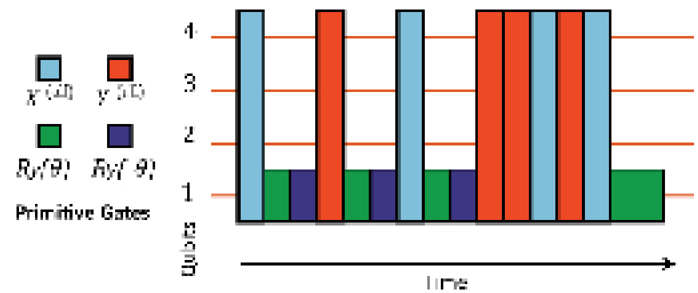
# DCG circuits: Examples

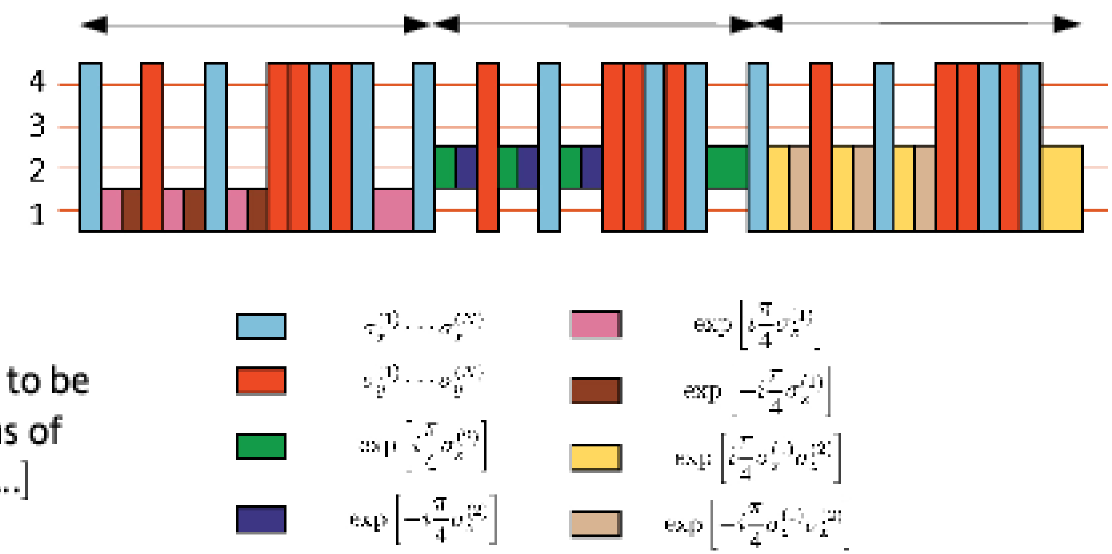
Arbitrary linear decoherence on $n = 4$ qubits.

$U_{gate} = \text{NOOP}$




$U_{gate} = R_y^{(1)}(90)$



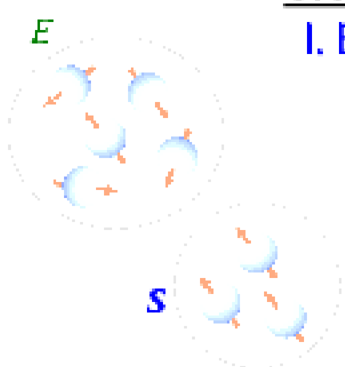

$U_{gate} = C^{(1)}Z^{(2)}$

[Each single-qubit z rotation to be further decomposed in terms of primitive x- and y- rotations...]

## DCG performance: Results

Case study: Cat-state benchmark under spin-bath decoherence.

I. Bath-induced error with ideal [bounded-strength] controls.




$$H_{error} = I_S \otimes \sum_{k=1}^{N} \Gamma_{kl} \vec{I}_k \cdot \vec{I}_l + \sum_{i=1}^{n} \vec{\sigma}_i \otimes \sum_{k=1}^{N} A_k \vec{I}_k$$

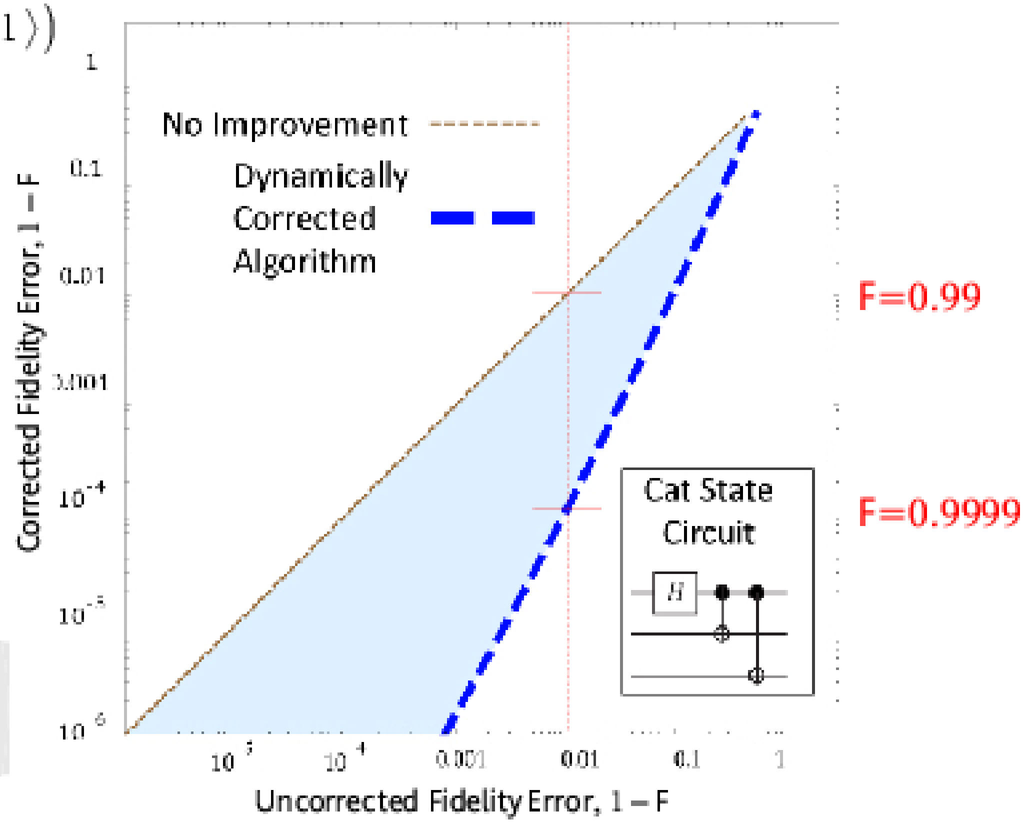
$$|\psi_S\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

→ DCG implementation consists of $[2 + 2 \times 6] \times 16 = 256$ primitive gates

→ Performance indicator:
Change in error-corrected 'slope'

$$\mathrm{EPG}_{corrected} = (k\tau_{min} \|H_{error}\|)\ \mathrm{EPG}_{phys}$$

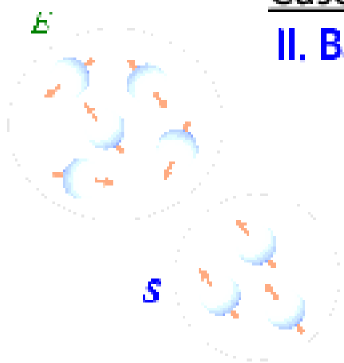
Large region of improvement exists



# DCG performance: Results

Case study: Cat-state benchmark under spin-bath decoherence.

II. Bath-induced error with faulty [bounded-strength] controls.




$$H_{error} = I_S \otimes \sum_{k=1}^{N} \Gamma_{kl} \vec{I}_k \cdot \vec{I}_l + \sum_{i=1}^{n} \vec{\sigma}_i \otimes \sum_{k=1}^{N} A_k \vec{I}_k$$

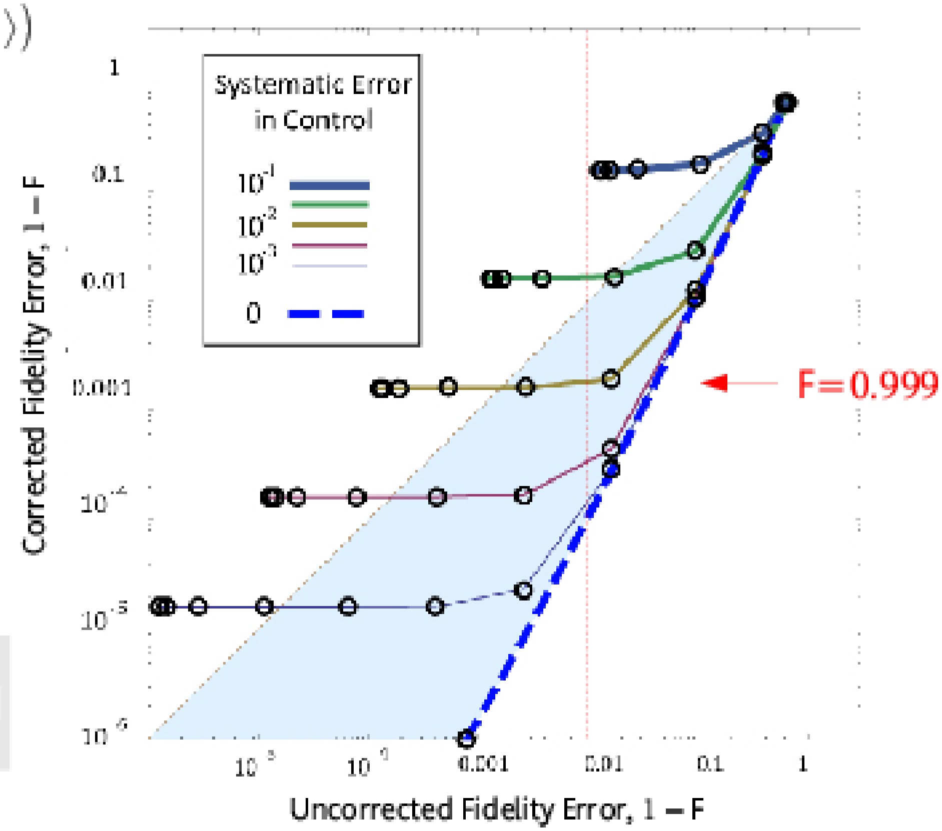
$$|\psi_S\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

➞ Pulse length error included:

$$h_0(t) \rightarrow h_0(t)(1+\epsilon)$$

➞ DCG performance plateau once uncompensated systematic error dominates over bath-induced error

Large region of improvement exists



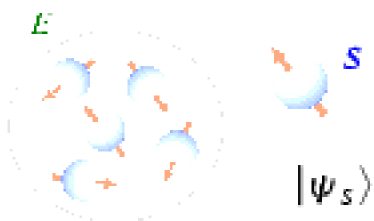
# What about arbitrarily accurate DCGs?

K. Khodjasteh, D.A. Lidar & LV, forthcoming.

- Can decoherence suppression be pushed to arbitrarily high order in principle?
  - ➞ Combine DCG constructions with recursive design: Concatenated DCGs (CDCGs).
  - ➞ Hint: Embed lower-order DCGs as components for EDD sequences and 'balance pairs'...

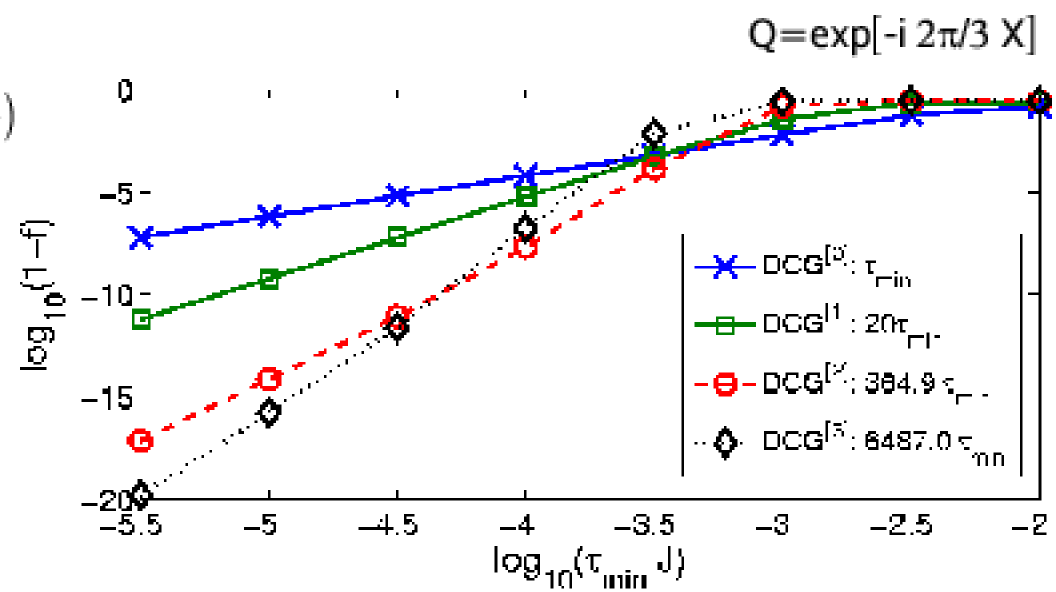
$$\text{EPG}^{[m]} = (k\tau_{\text{min}} \|H_{\text{error}}\|)^{m+1}$$

- Solution is constructive and fully analytic, however plenty of room for optimization...




$$|\psi_S\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

➞ Increasing slopes are achieved as concatenation level grows, if sufficiently small primitive switching times are available.



# Conclusions and outlook

- High-level DD protocols (both deterministic and randomized) can offer viable decoherence control venues in realistic settings:
  - → Solid-state systems: Quantum dots, rare-earth doped ions in crystals...
  - → Bosonic systems: Nanomechanical resonators...
  - → Optical systems: Flying polarization qubits...

    Damodarakurup et al, arXiv:0811.2654, Nov 2008.
  - → Atomic /molecular systems: Rydberg atoms, trapped ions...

  ⋮

    Biercuk et al, Nature 458, 996 (2009).

- DCGs approximate ideal gates in a universal set with error that scales quadratically in the physical EPG without encoding or measurement overheads:
  - → Use for 'low-level' error correction within fault-tolerant architectures...
  - → Concatenate with composite pulses for additional robustness...
  - → Extend construction to different open-system models/control resources...
  - → Explore 'control landscape'/make contact with optimal-control theory approaches... ☞

  ⋮

- Additional experimental implementations of open-loop error control benchmarks much needed and welcome!...

*Thanks for your attention...*