Quantum Computational Phases of Matter

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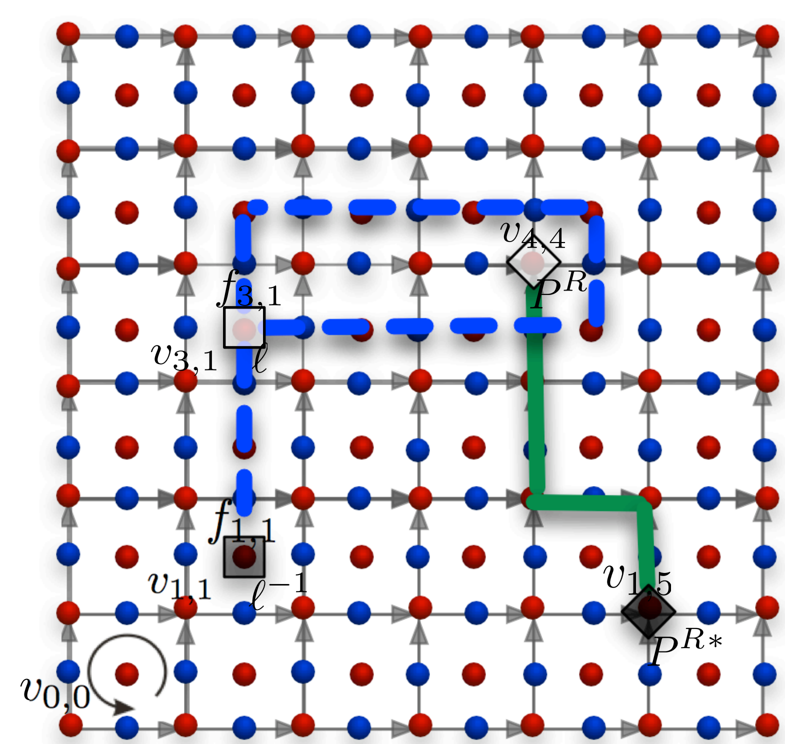
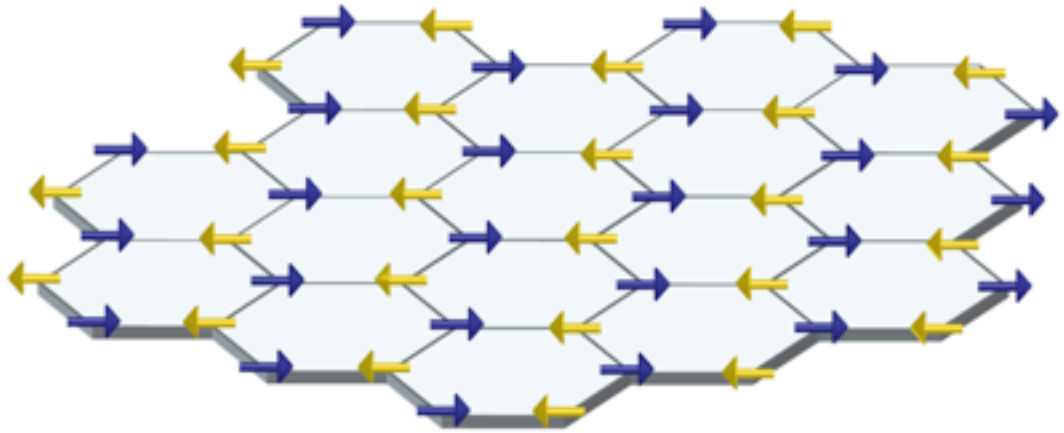
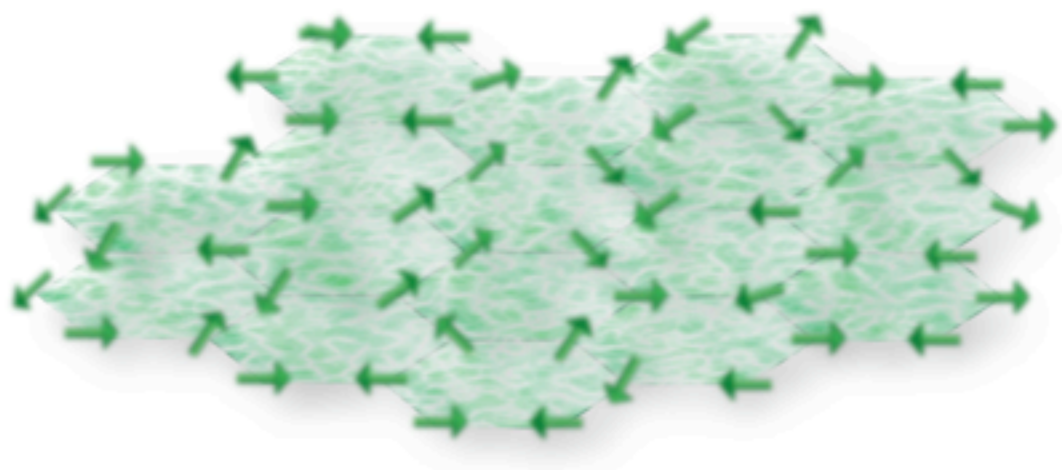
- All models of quantum computing must fight decoherence
- But nature does allow for some stable phases of strongly correlated matter
  - e.g. Mott insulators, Haldane gapped phases, superconducting phases
- Can we use such phases for quantum memories/gates?
- Not obvious:
  - Nature abhors a degeneracy that would protect q. info
  - Is dynamical processing antithetical to equilibrium phases?
- One option Topological Order
  - Very difficult to engineer



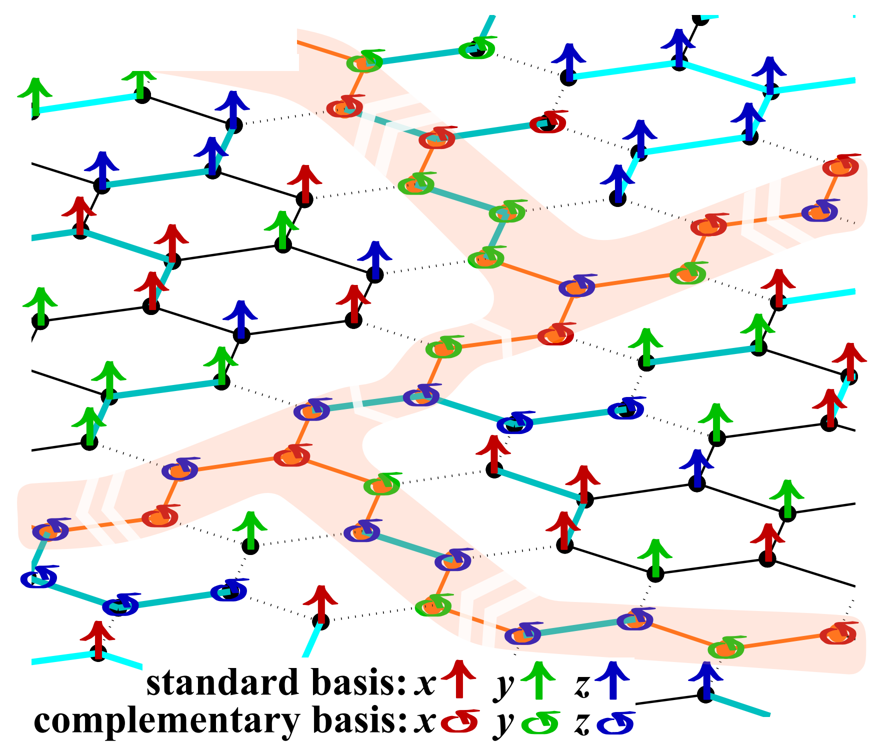


Antiferromagnet



Spin liquid

Quantum Processor



Images:
A. Miyake, Ann. Phys. 326, 1656 (2011)
E. Edwards: http://www.newswise.com/articles/searching-for-spin-liquids

Outline

- Ground code measurement based computing
  - 1D Haldane phase
    - Quantum computational renormalization
  - 2D AKLT phase
- Symmetry Protected Topological Order
  - Holonomic computing in the Haldane Phase
- Summary

GKB, A. Miyake, PRL **101**, 010502 (2008)

S.D. Bartlett, GKB, A. Miyake, and J. Renes, PRL **105**, 110502 (2010)

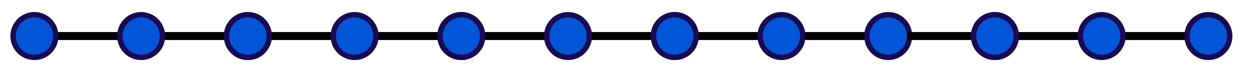
A. Darmawan, GKB, and S. Bartlett, New J. Phys. **14**, 013023 (2012)

J. Renes, A. Miyake, GKB, and S.D. Bartlett, New J. Phys. (in press); arXiv: 1108.4741

Ground code computing

- A start: 1D AKLT Hamiltonian*

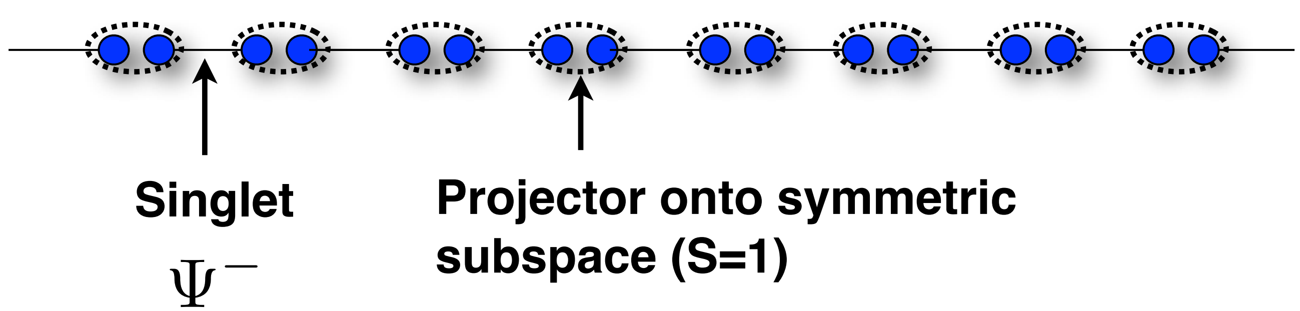
*I. Affleck, T. Kennedy, E.H. Lieb, H. Tasaki, CMP **115**, 477 (1988)



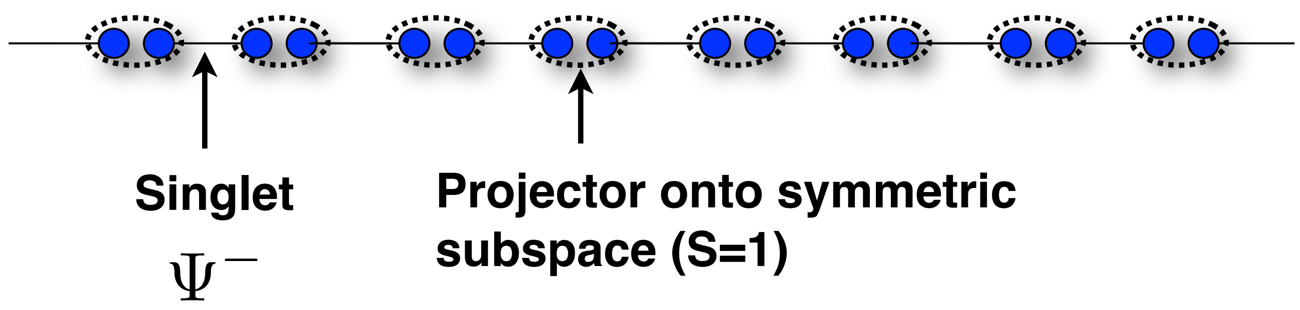
$$H = \frac{J}{2}\sum_j \left(\vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3}(\vec{S}_j \cdot \vec{S}_{j+1})^2 + \frac{2}{3}\mathbf{1}\right) = J\sum_j P^2_{j,j+1}$$

  - gapped in thermodynamic limit
  - frustration free: global ground state is also locally a ground state

- Ground state is a valence bond solid



  - Degeneracy:
    - Open boundaries (4 fold=2 qubit edge modes)
    - Closed or infinite (1 fold)



- Representation of ground state as a matrix product state (MPS)

$$|GS_B\rangle = \sum_{\{\alpha_j\}\in\{1,2,3\}} Tr[BA^{[1]}[\alpha_1]A^{[2]}[\alpha_2]\cdots A^{[N]}[\alpha_N]]|\alpha_1\rangle|\alpha_2\rangle\cdots|\alpha_N\rangle$$

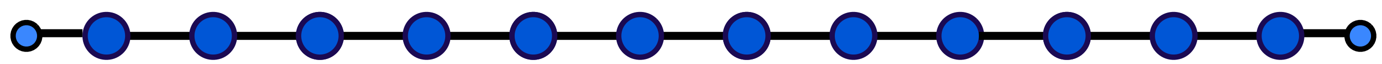
boundary op

$$A[1] = \frac{1}{\sqrt{3}}X \quad A[2] = \frac{1}{\sqrt{3}}XZ \quad A[3] = \frac{1}{\sqrt{3}}Z$$

$$|1_j\rangle = -\frac{1}{\sqrt{2}}(|S_j^z = 1\rangle - |S_j^z = -1\rangle)$$
$$|2_j\rangle = \frac{1}{\sqrt{2}}(|S_j^z = 1\rangle + |S_j^z = -1\rangle)$$
$$|3_j\rangle = |S_j^z = 0\rangle$$

Protocol

- Add boundary spin-1/2 particles (Not actually needed)



$$H = J[\sum_{j=1}^{N-1} P^{2}_{j,j+1} + P^{3/2}_{0,1} + P^{3/2}_{N,N+1}]$$

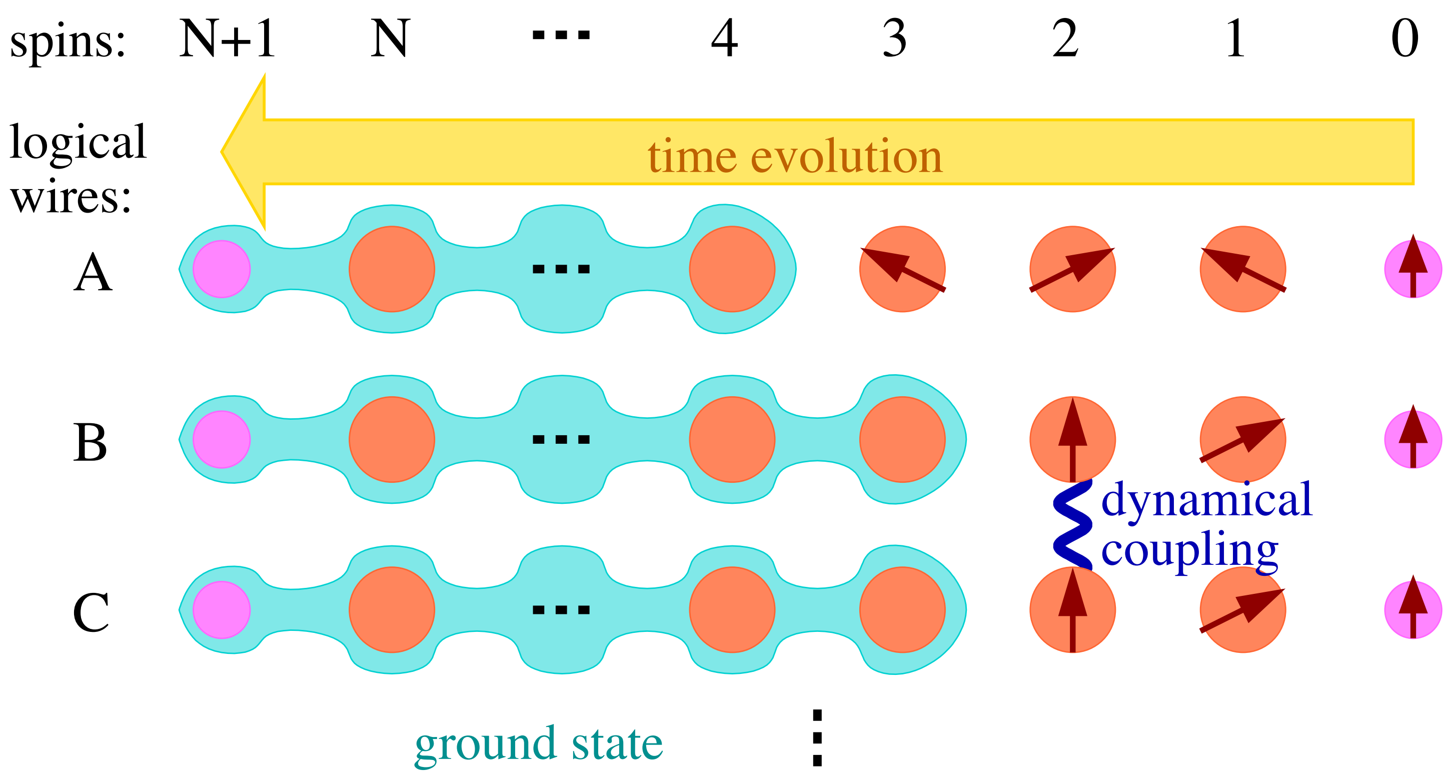
$$P^{3/2}_{j,j'} = \frac{2}{3}(\mathbf{1}_6 + \mathbf{s}_j \cdot \mathbf{S}_{j'})$$

  - Ground state is unique

- Qubit initialization

  - Turn off boundary term $\vec{s}_0 \cdot \vec{S}_1$ and measure right spin-1/2
  - For outcome $S^z_0 = -\frac{1}{2}$ initialize logical 0

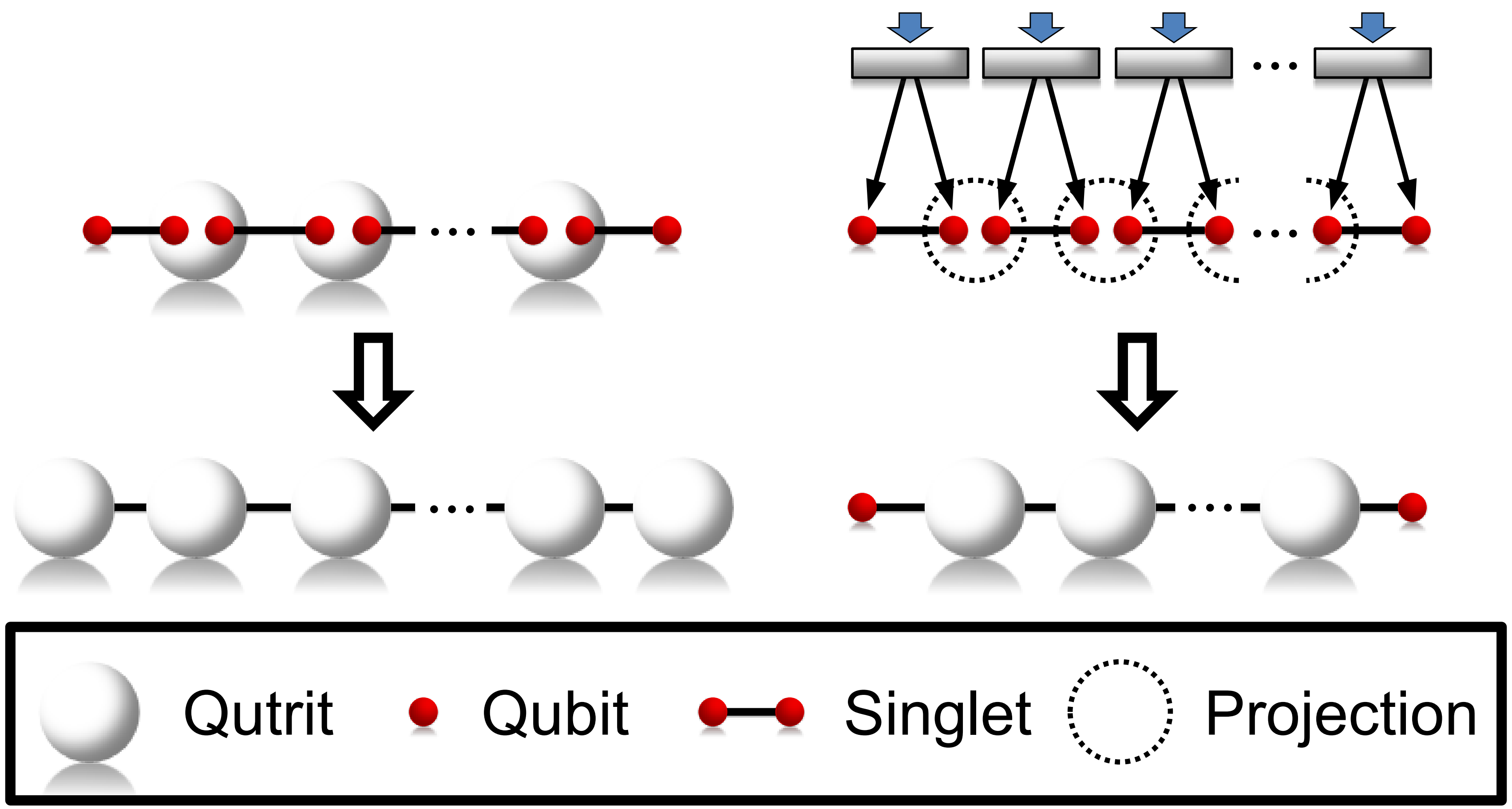
$$|0^L\rangle = \sum_{\{\alpha_j\}\in\{1,2,3\}} |\alpha_1\rangle|\alpha_2\rangle\cdots|\alpha_N\rangle A^{[N]}[\alpha_N]A^{[N-1]}[\alpha_{N-1}]\cdots A^{[1]}[\alpha_1]]|0_{N+1}\rangle$$



- Single qubit rotations
  - For $R^Z(\theta) = |0^L\rangle\langle 0^L| + e^{i\theta}|1^L\rangle\langle 1^L|$ measure in basis $\{|\gamma_j^Z(\theta)\rangle\} = \{\frac{1}{2}((1 \pm e^{-i\theta})|1_j\rangle + (1 \mp e^{-i\theta})|2_j\rangle), |3_j\rangle\}$
    - For (+) outcome performs $XR^Z(\theta)$, for (-) outcome $XZR^Z(\theta)$
    - Otherwise no rotation with Z byproduct. If this happens (prob=1/3) try again
- Two qubit CPHASE
  - Dynamical gate $\exp(iH^{\text{int}}\pi/\chi)$ with $H^{\text{int}} = \chi|S^z_{A_j} = 1\rangle\langle S^z_{A_j} = 1| \otimes |S^z_{B_j} = 1\rangle\langle S^z_{B_j} = 1|$ + measure
    - If outcome $|1_{A_j}1_{B_j}\rangle,\ |1_{A_j}2_{B_j}\rangle,\ |2_{A_j}1_{B_j}\rangle$ or $|2_{A_j}2_{B_j}\rangle$ then performs CPHASE
    - Otherwise fail with Pauli byproduct. If this happens (prob=5/9) try again
- Readout by measuring left spin-1/2 particles

- Experimental realization in entangled photonic networks

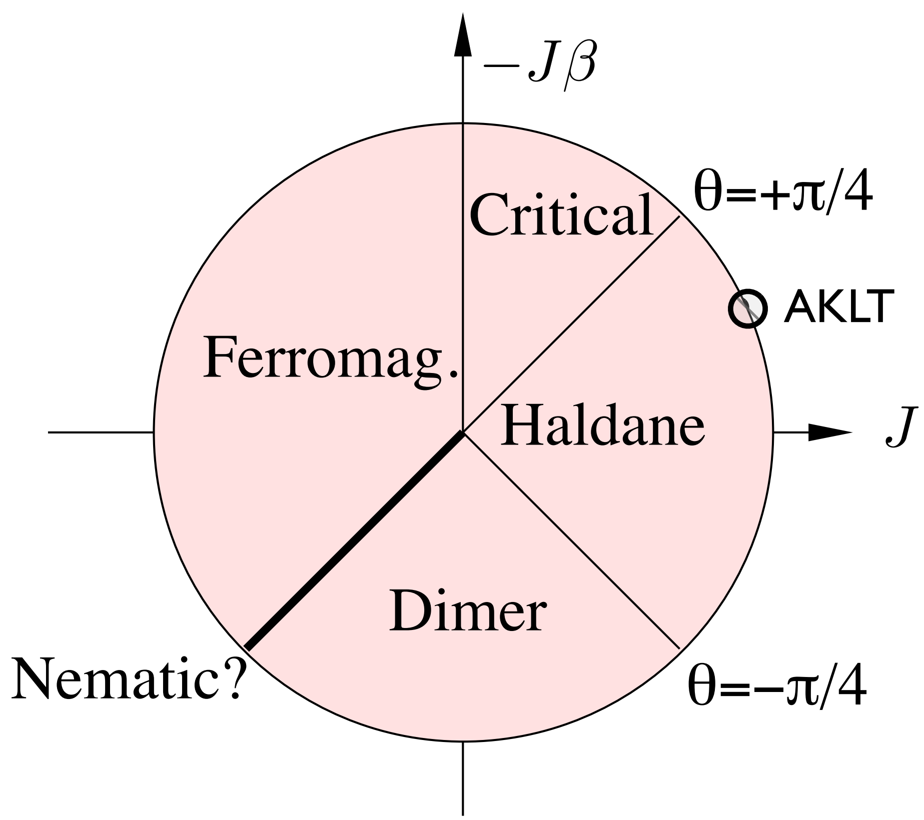
[**Optical one-way quantum computing with a simulated valence-bond solid**]
Rainer Kaltenbaek, Jonathan Lavoie, Bei Zeng, Stephen D. Bartlett, Kevin J. Resch
Nature Physics (17 October 2010)



Quantum computational renormalization in the Haldane phase

- AKLT Hamiltonian is one point in a family of SO(3) symmetric spin-1 chains

$$H(\beta) = J \sum_j \left[\mathbf{S}_j \cdot \mathbf{S}_{j+1} - \beta(\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2\right]$$



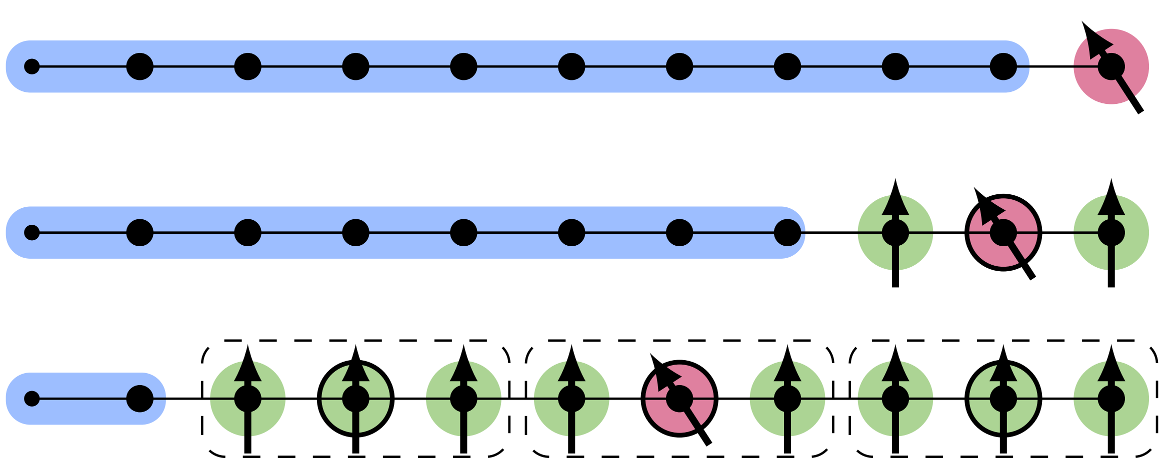
- The entire Haldane phase is gapped and has exponentially decaying correlation functions. But only at AKLT does the ground state have the simple MPS description we need for measurement based computing.
- Can we use other ground states in the Haldane phase?

- Haldane phase $H(\beta) = J \sum_j \left[\mathbf{S}_j \cdot \mathbf{S}_{j+1} - \beta (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2 \right]$ $J > 0$ $-1 < \beta < 1$
- Note: the SWAP operator between two spin-1 particles

$$\mathcal{S}_{j,j+1} := \mathbf{S}_j \cdot \mathbf{S}_{j+1} + (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2 - \mathbb{1}$$

  - Perturbations away from AKLT point $H(\beta = -1/3)$ are perturbations by SWAP, so the ground states are roughly coherent superpositions (up to kth order) of SWAP on spins separated by k.

- Buffering protocol
  - Measure spins flanking target spin in basis commuting with target rotation

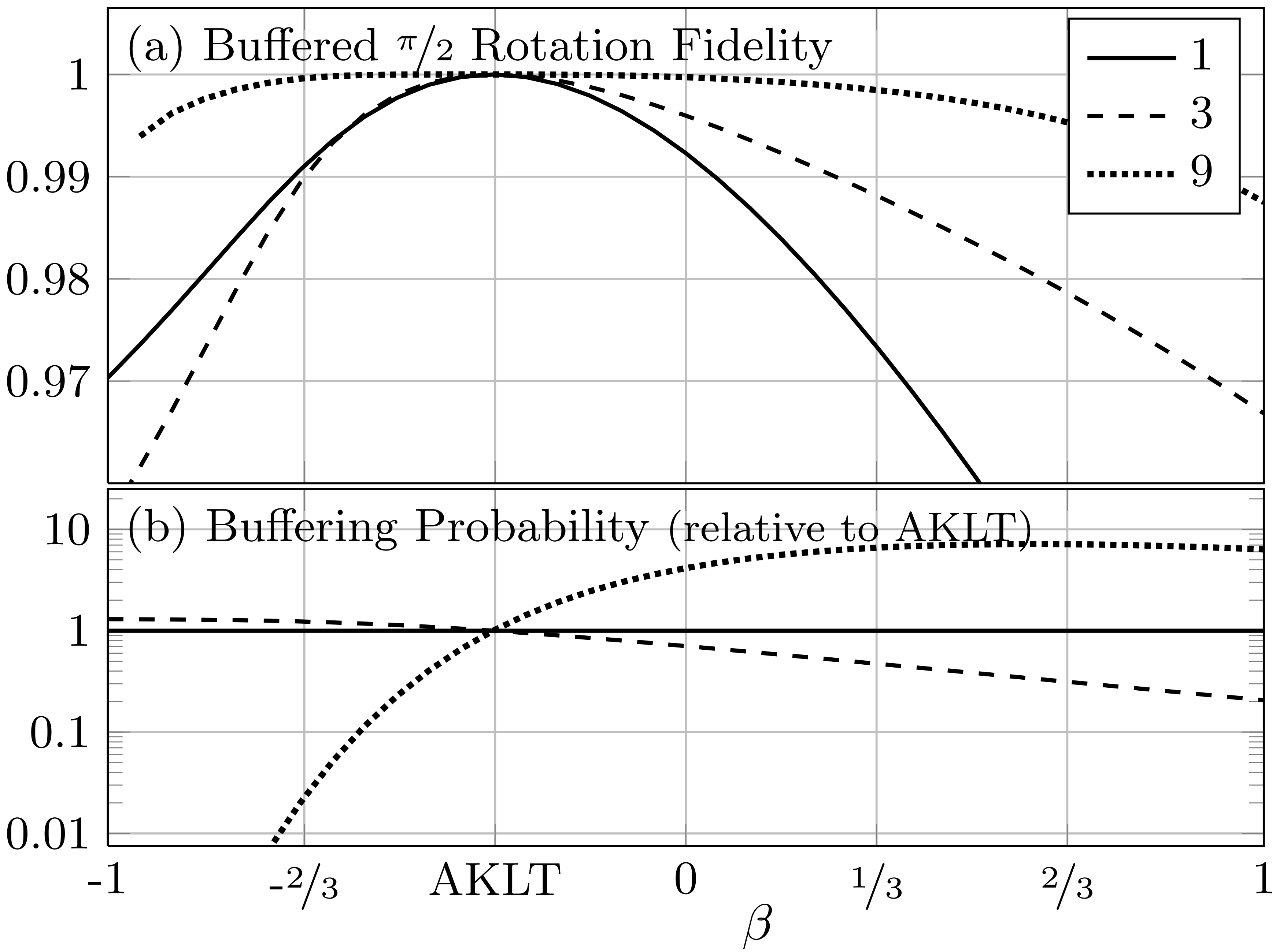


No buffering

L=3 buffering

L=9 buffering=concatenated L=3 buffering

- Results on N=12 Haldane chain

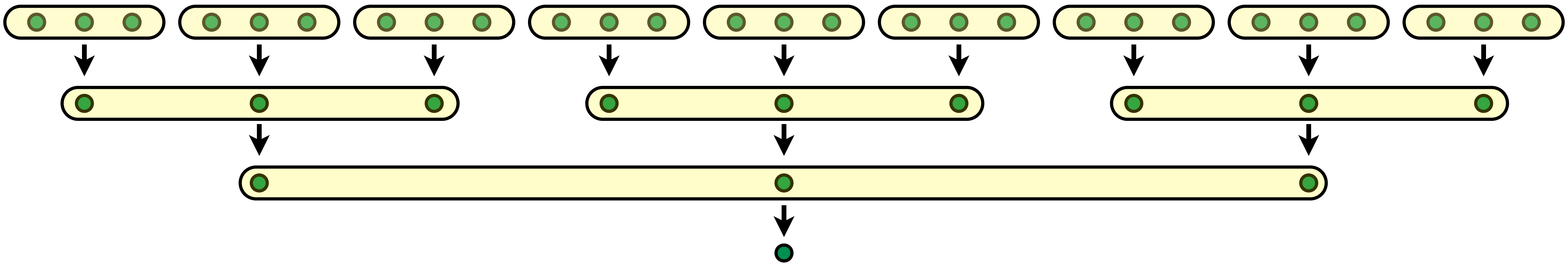


Fidelity: Overlap of target qubit state with obtained measured state

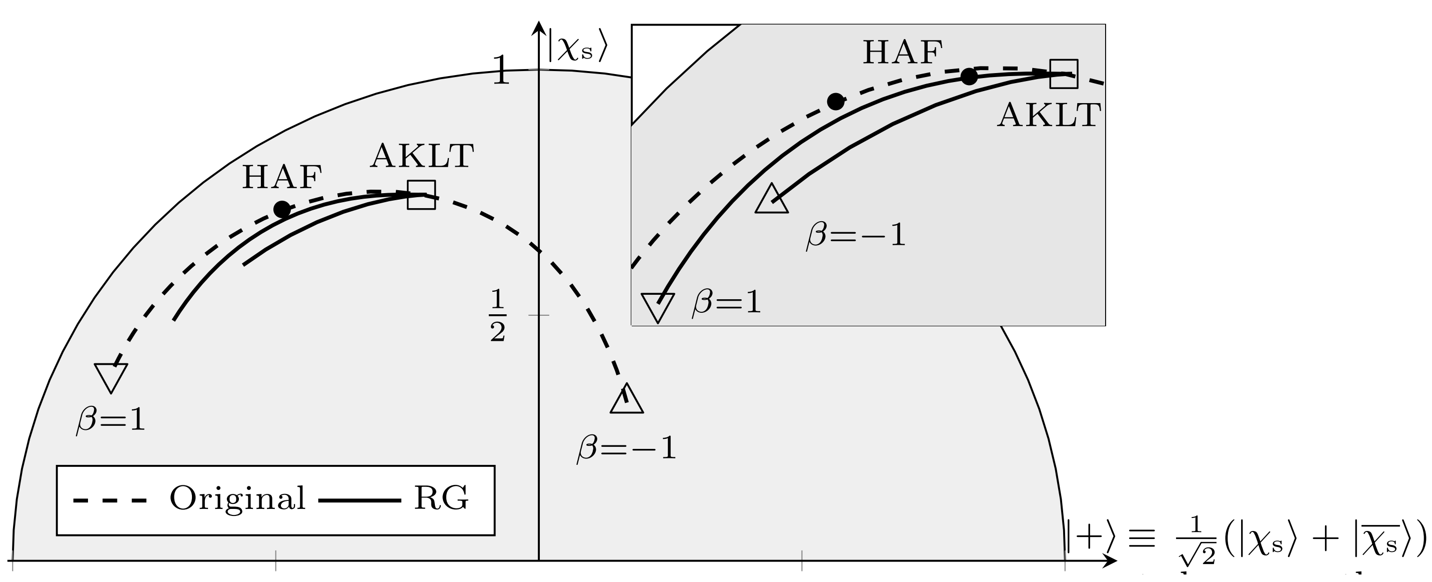
AKLT probability=3^{-L} for L buffering

- Buffering is insensitive to short range variations near AKLT point and keeps the long range degrees of freedom characterized by the Haldane phase

- Consider L=3 buffering as an RG flow



- Buffered rotation measurements act as the desired measurements on the renormalized spin via postselection
- RG map
  - Block spins into L=3 blocks, project on J=1 subspace of 3 spin-1s, trace over irrep label
  - Yields new spin-1s and a chain 1/3 the length



Radial length of Bloch vector is weight of J=1 subspace on g.s.

Vertical height is buffering success probability given projection onto J=1

2D AKLT state

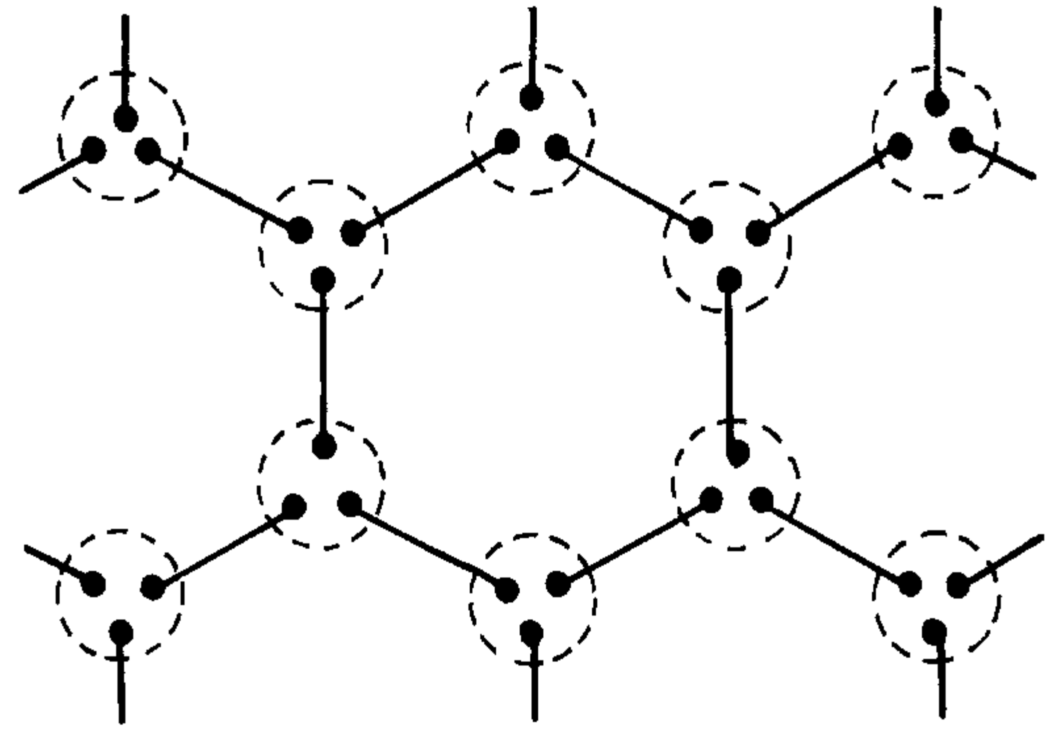
- Spin-3/2 on honeycomb lattice

$$H = \sum_{\langle j,k \rangle} P^3_{j,k}$$

  - Exponentially decaying correlation functions
  - Ground state has tensor network description

$$|\mathcal{G}\rangle = \sum_{\alpha_k,\alpha_{k'}} \mathrm{tr}\left[B \prod_{k\in\top} A_\top[\alpha_k]\,|\alpha_k\rangle \prod_{k'\in\perp} A_\perp[\alpha_{k'}]\,|\alpha_{k'}\rangle\right],$$

boundary condition



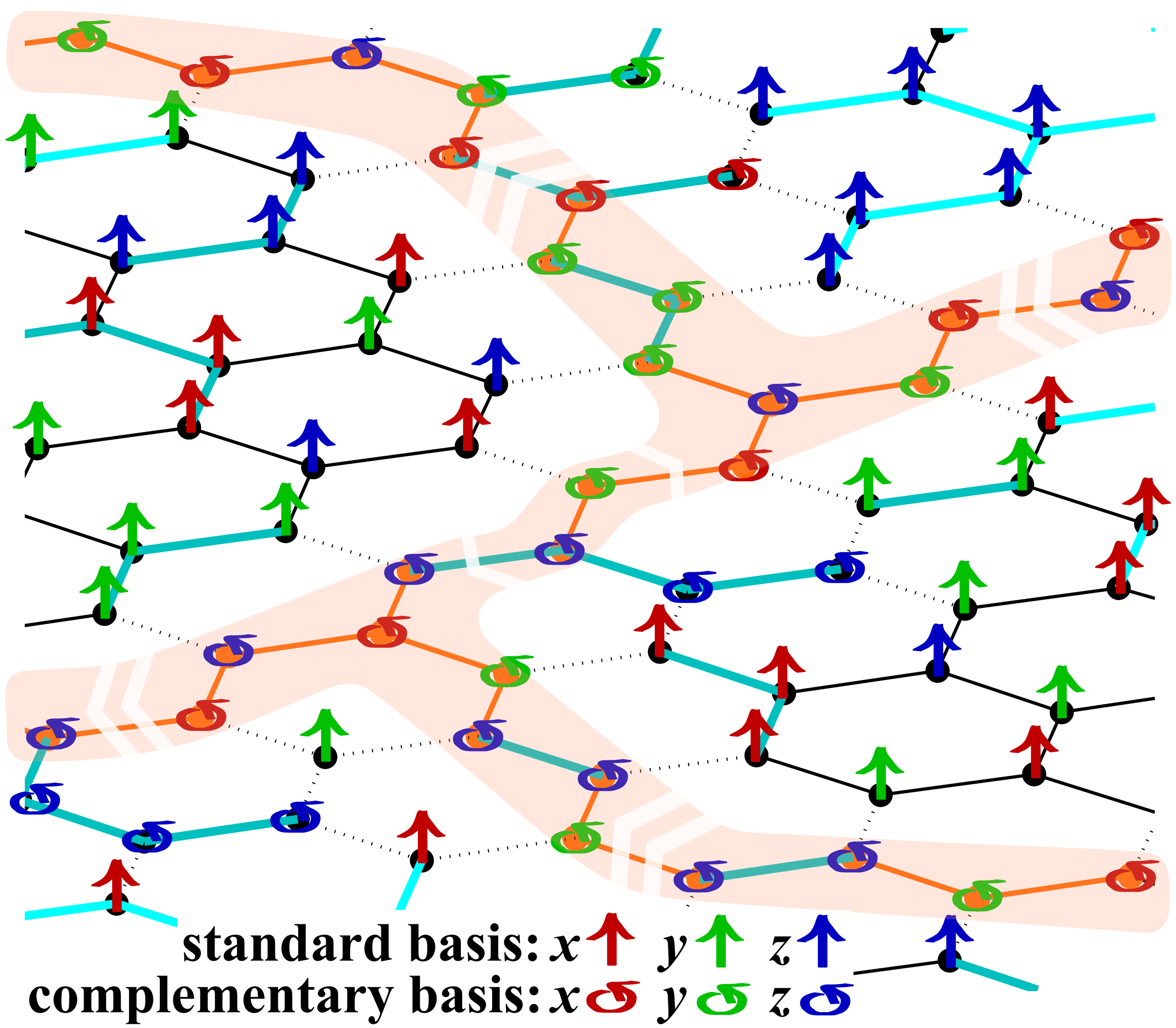
$$A_\perp[3/2] = |1\rangle_u|1\rangle_l\langle 1|_r,$$
$$A_\perp[1/2] = \frac{1}{\sqrt{3}}\left(|1\rangle_u|1\rangle_l\langle 0|_r + |1\rangle_u|0\rangle_l\langle 1|_r + |0\rangle_u|1\rangle_l\langle 1|_r\right)$$
$$A_\perp[-1/2] = \frac{1}{\sqrt{3}}\left(|1\rangle_u|0\rangle_l\langle 0|_r + |0\rangle_u|1\rangle_l\langle 0|_r + |1\rangle_u|0\rangle_l\langle 0|_r\right)$$
$$A_\perp[-3/2] = |0\rangle_u|0\rangle_l\langle 0|_r,$$

$A_\top[m]$ the same but with all bits flipped

  - Gapped?

MBQC in 2D AKLT state

- Recently 2 groups showed how to use a 2D AKLT state for measurement based computing



T.-C. Wei, I. Affleck, R. Raussendorf, PRL 106, 070501 (2011)

A. Miyake, Ann. Phys. 326, 1656 (2011) (also fig. source)

Spins reduced to qubits by filtering POVM

$$\{F_x, F_y, F_z\} \qquad \sum_\mu F_\mu^\dagger F_\mu = \mathbf{1}$$

$$F_\mu = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}_\mu\right\rangle\left\langle\frac{3}{2}_\mu\right| + \left|-\frac{3}{2}_\mu\right\rangle\left\langle-\frac{3}{2}_\mu\right|\right)$$

Identify logical wires based on correlations of filtering outcomes

Deformations of 2D AKLT Hamiltonian

- A one parameter family of Hamiltonians which are frustration free
  - Homogeneous and Isotropic: All summands $h_{j,k}$ are the same for all nearest neighbor pairs.
  - Parity invariant: $[h_{j,k}, SWAP(j,k)] = 0$.
  - $U(1)$ symmetry: $[h_{j,k}, e^{i\phi(S_j^z + S_k^z)}] = 0$.
  - $\mathbb{Z}_2$ symmetry: $h_{j,k}$ invariant under spin flip: $S_j^z + S_k^z \to -S_j^z - S_k^z$.
- Changes tensor network for ground state

$$
\begin{aligned}
A[3/2] &= |1\rangle_u |1\rangle_l \langle 1|_r, \\
A[1/2] &= \frac{1}{\sqrt{3}} (|1\rangle_u |1\rangle_l \langle 0|_r + |1\rangle_u |0\rangle_l \langle 1|_r + |0\rangle_u |1\rangle_l \langle 1|_r) \\
A[-1/2] &= \frac{1}{\sqrt{3}} (|1\rangle_u |0\rangle_l \langle 0|_r + |0\rangle_u |1\rangle_l \langle 0|_r + |1\rangle_u |0\rangle_l \langle 0|_r) \\
A[-3/2] &= |0\rangle_u |0\rangle_l \langle 0|_r,
\end{aligned}
$$

$$\frac{1}{\sqrt{3}} \longrightarrow -\frac{1}{a}$$

- Two phases*
  - $a^2 < 6.52$ Disordered phase
  - $a^2 > 6.52$ Neel ordered

*H. Niggemann, A. Klümper, J. Zittartz, Z. Phys. B 104, 103 (1997).

Strategy for MBQC in disordered phase

- Filtering to project spins onto qubit subspaces
  - At AKLT point ($a = \sqrt{3}$)

$$\{F_x, F_y, F_z\} \qquad F_\mu = \sqrt{\frac{2}{3}}\Big(\Big|\frac{3}{2}_\mu\Big\rangle\Big\langle\frac{3}{2}_\mu\Big| + \Big| -\frac{3}{2}_\mu\Big\rangle\Big\langle -\frac{3}{2}_\mu\Big|\Big)$$

  - At other points in the phase need to balance weights of tensors toward AKLT state

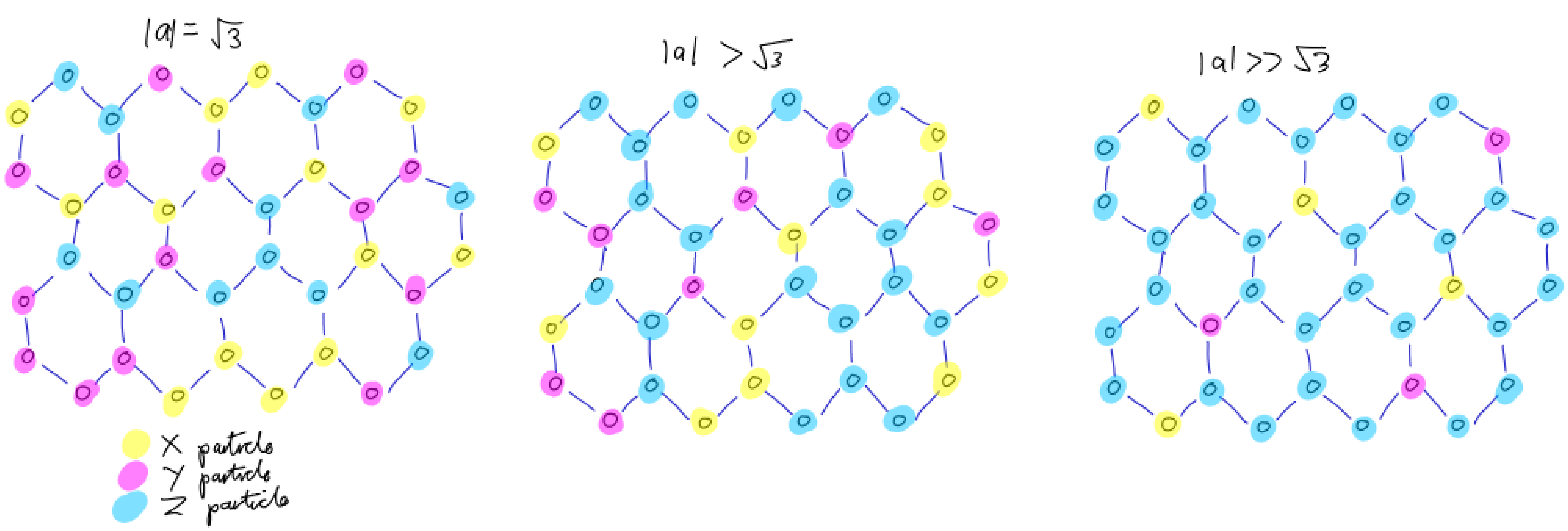
$$F_x(a) = \sqrt{\frac{4}{3}\left(\frac{a^2}{1+a^2}\right)} D(a) F_x D(a)$$

$$F_y(a) = \sqrt{\frac{4}{3}\left(\frac{a^2}{1+a^2}\right)} D(a) F_x D(a)$$

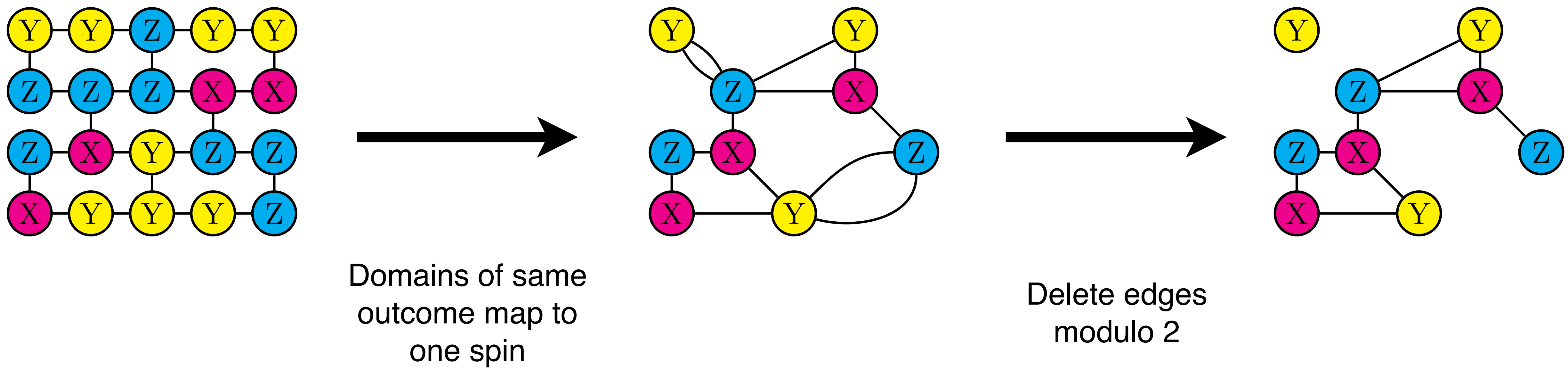
$$F_z(a) = a\sqrt{\frac{(a^2-1)}{6}} D(a) F_z D(a)\,.$$

$$D(a) = \mathrm{diag}(\sqrt{3}/a, 1, 1, \sqrt{3}/a) \text{ in the } S_z \text{ basis}$$

$$F_x(a)^\dagger F_x(a) + F_y(a)^\dagger F_y + F_z(a)^\dagger F_z(a) = I$$



- Converting filter outcomes to a graph state



- A good graph will have no percolating clusters and many percolating superclusters

- Statistical model

  - Probability of obtaining filter outcome set σ with deformation *a* is

$$p(\sigma) = \frac{1}{\mathcal{Z}} \left(\frac{a^2 - 1}{2} \right)^{N_z(\sigma)} 2^{|V(\sigma)| - |E(\sigma)|}$$

$|E(\sigma)|$ is the number of inter-cluster bonds

$|V(\sigma)|$ is the number of clusters for a given outcome

$N_z(\sigma)$ is the total number of Z filter outcomes

  - Equivalent to FKSW classical statistical mechanics model (3 state Potts + Random cluster)

$$p(\sigma) = \frac{1}{\mathcal{Z}} e^{-\beta(V(\sigma) + E(\sigma) + BN_z(\sigma))}$$

$E(\sigma)$ term is the Potts Hamiltonian

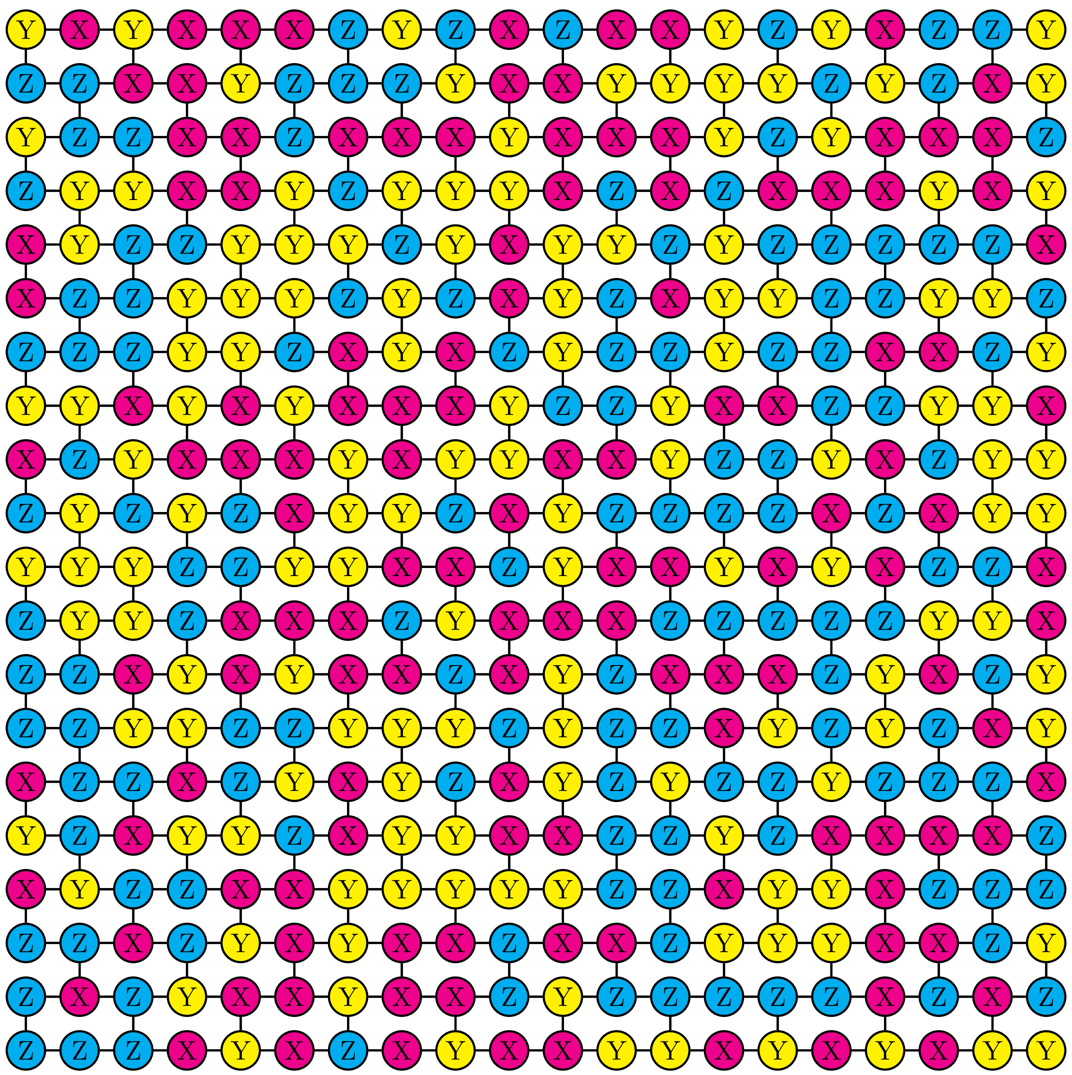
$V(\sigma)$ is a non-local cluster counting term

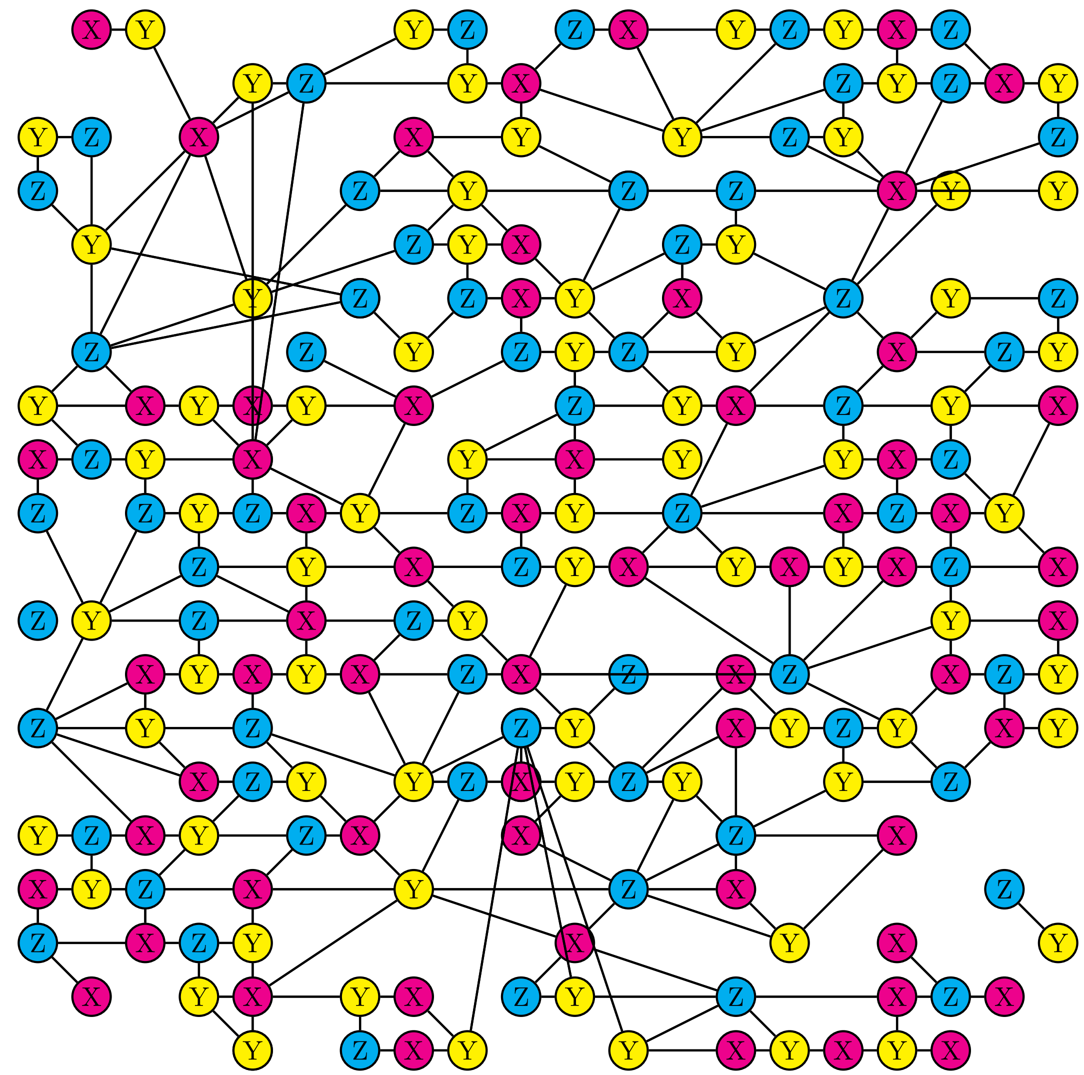
$BN_z(\sigma)$ is an external field $\quad B = \log_2(a^2 - 1) - 1$

$\beta = \log_e 2$

Numerical Results

- Reduction at AKLT point (20x20 spin lattice)

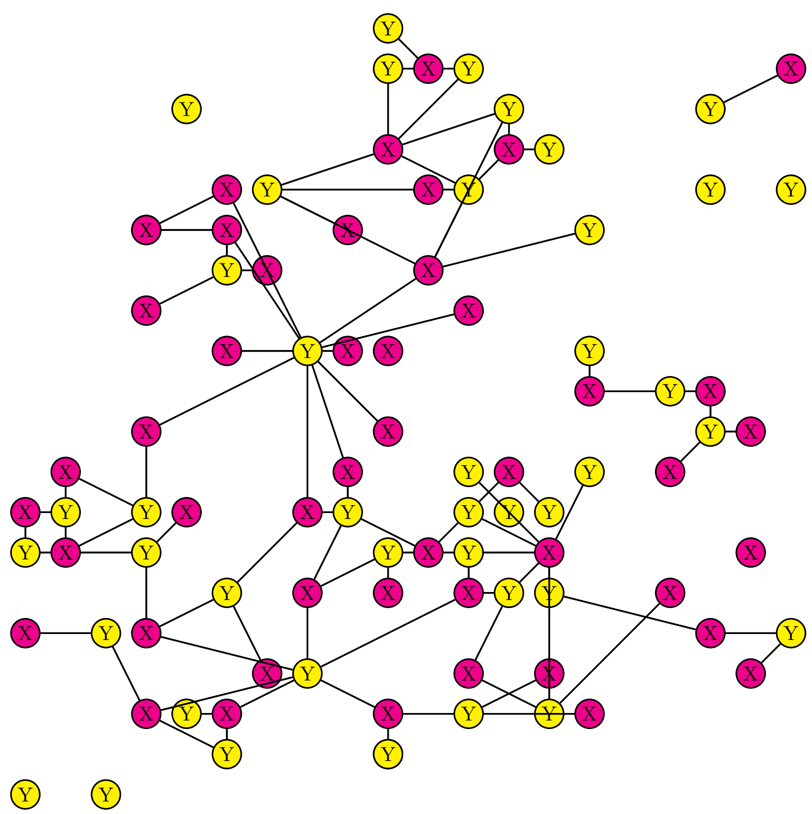




$$a^2 = 3$$

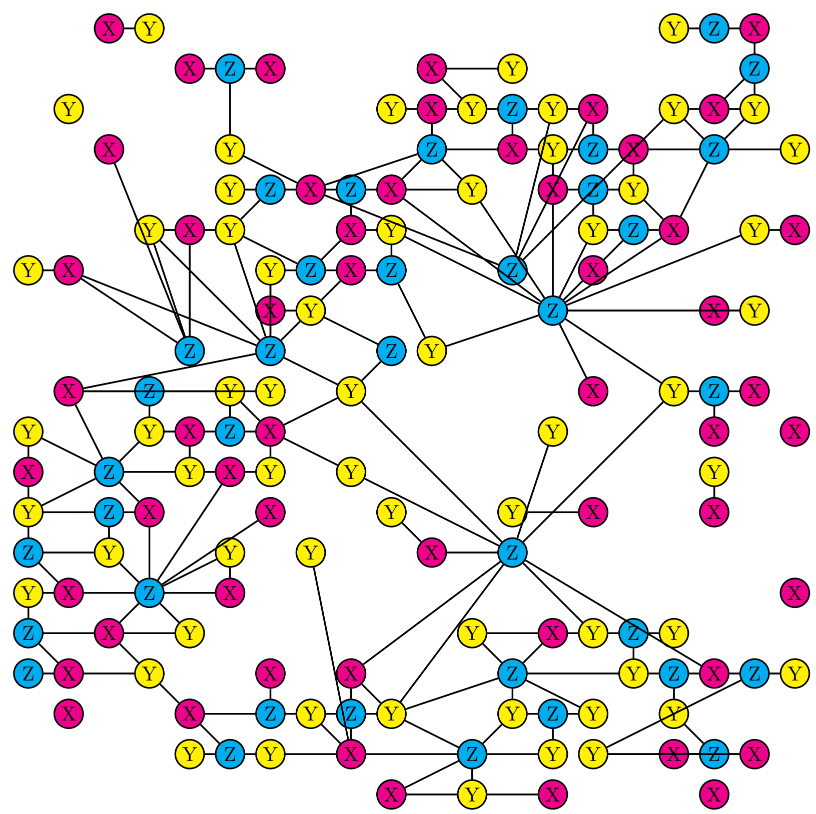
  - Universal resource for MBQC

- Reduction on 20x20 spin lattice



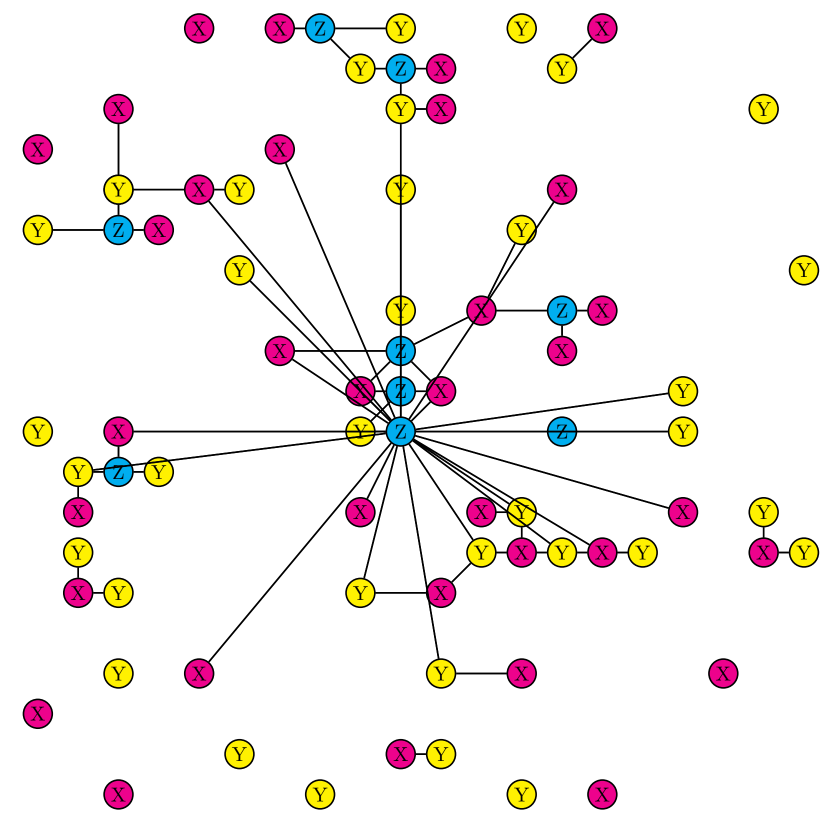
$$a^2 = 1$$

Universal



$$a^2 = 5.70$$

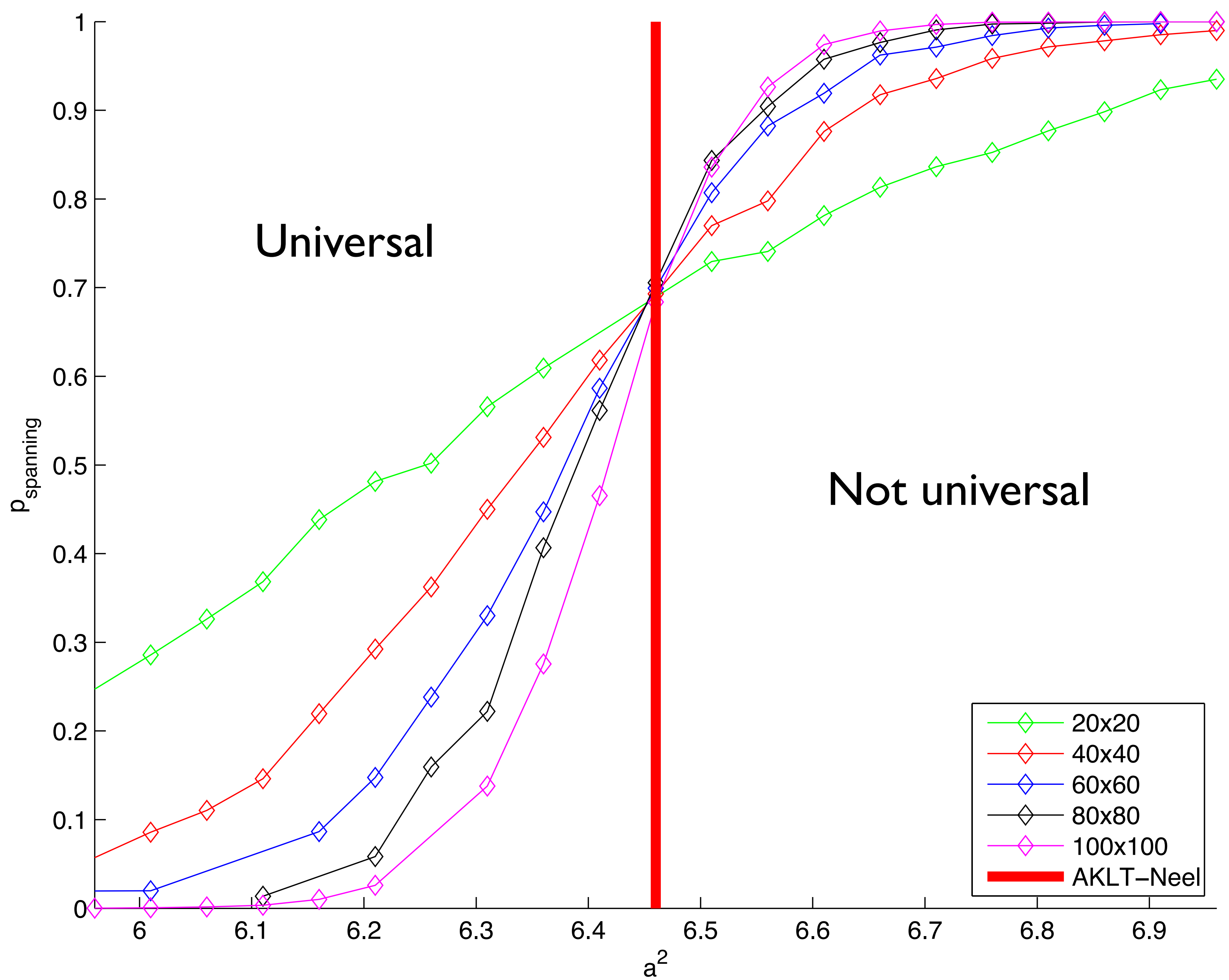
Universal



$$a^2 = 6.96$$

Not universal

- Probability of spanning cluster as function of deformation a



Monte-carlo samplings

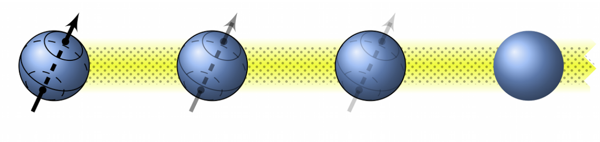
- Logarithmic sized clusters inside disordered (universal) phase

Symmetry Protected Topological Order

- Quantum Order
  - Gapless: Critical systems
    - e.g.: gs of transverse Ising model or Heisenberg model at criticality
  - Gapped
    - Short range entangled: Locally unitarily connected to product states
      - e.g.: cluster states, ferromagnetic gs
    - Long range entangled
      - Topological ordered (2D,3D...): No local order parameter
        - e.g. quantum Hall states, p+ip superconductors, string-net models
      - Symmetry Protected Topological Order (1D,2D,3D,...): gs degeneracy protected by a symmetry
        - e.g. topological insulators, Haldane phase
- For up to date classifications follow Xiao-Gang Wen on cond-mat

Holonomic QC in Haldane chains

- Spin chain qubit
  - Spin-1 chain with boundary spin-1/2: degenerate logical qubit in ground states

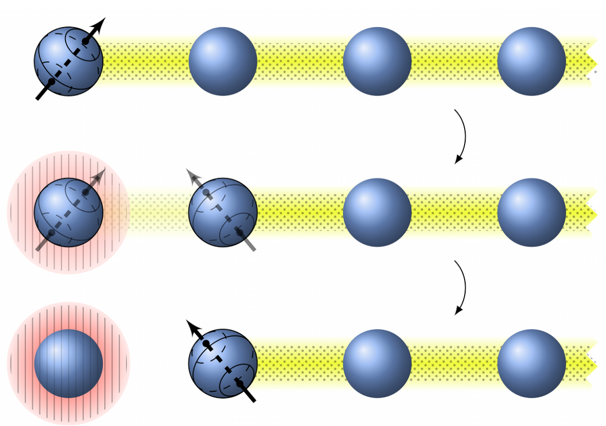


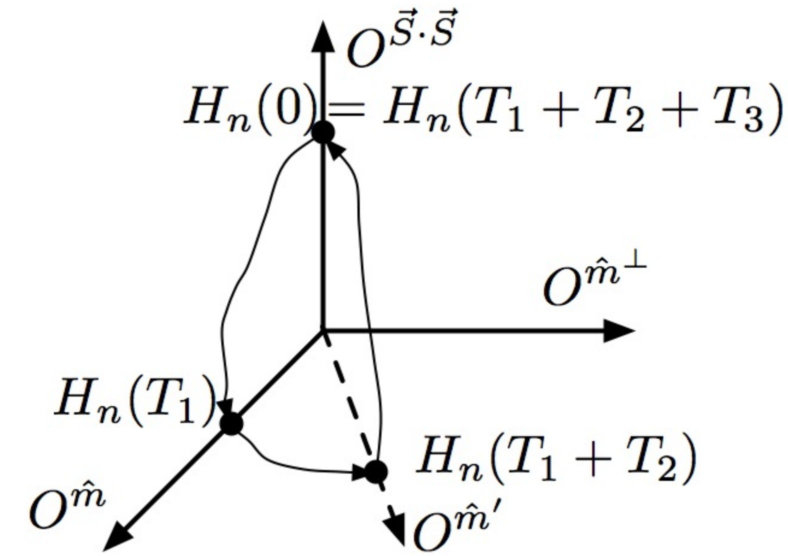
$$H_n = J\sum_{j=1}^{n-1} \vec{S}_j \cdot \vec{S}_{j+1} + J\vec{S}_n \cdot \vec{s}_{n+1}$$

  - Hamiltonian has D_2 symmetry (π rotations about any orthogonal axis triad)

$$\Sigma_n^{\widehat{m}} = \left(\bigotimes_{j=1}^{n} e^{i\pi S_j^{\widehat{m}}}\right) \otimes \sigma^{\widehat{m}}$$

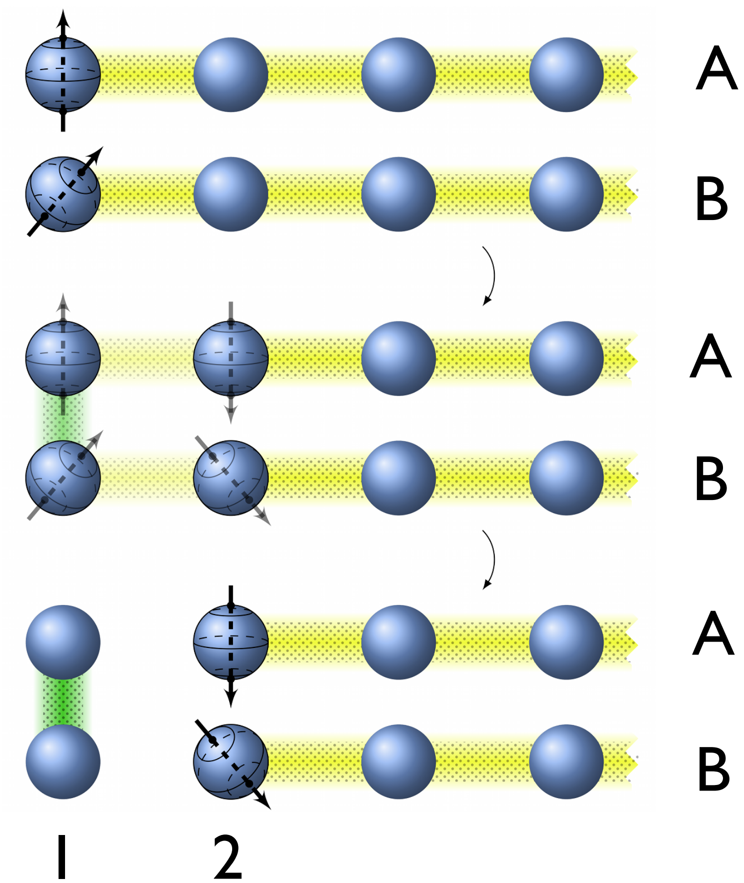
- Single qubit rotations





  - Apply $(S_1^{\hat{m}})^2$ during adiabatic drag out then dragreverse process with $(S_1^{\hat{m}'})^2$
  - Geometric gate: $R_{\hat{m}\times\hat{m}'}(2\cos^{-1}(\hat{m}\cdot\hat{m}'))$

- Two qubit CPHASE gate

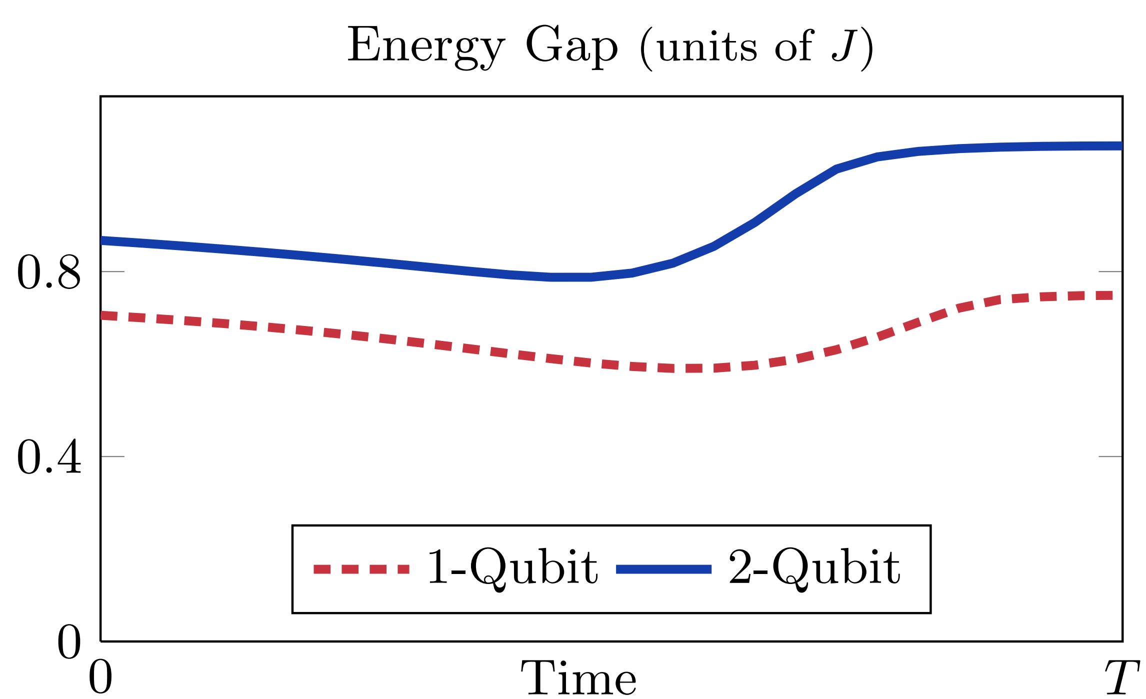


$$H^A_{n-1} + H^B_{n-1} + H^{AB}(t)$$

$$H^{AB}(t) = f(t)W^{AB} + g(t)(\vec{S}^A_1 \cdot \vec{S}^A_2 + \vec{S}^B_1 \cdot \vec{S}^B_2)$$

$$W^{AB} = [(S^{\hat{x}}_1)^2 - (S^{\hat{y}}_1)^2]^A \otimes [S^{\hat{z}}_1]^B + [S^{\hat{z}}_1]^A \otimes [(S^{\hat{x}}_1)^2 - (S^{\hat{y}}_1)^2]^B$$

- Energy gap



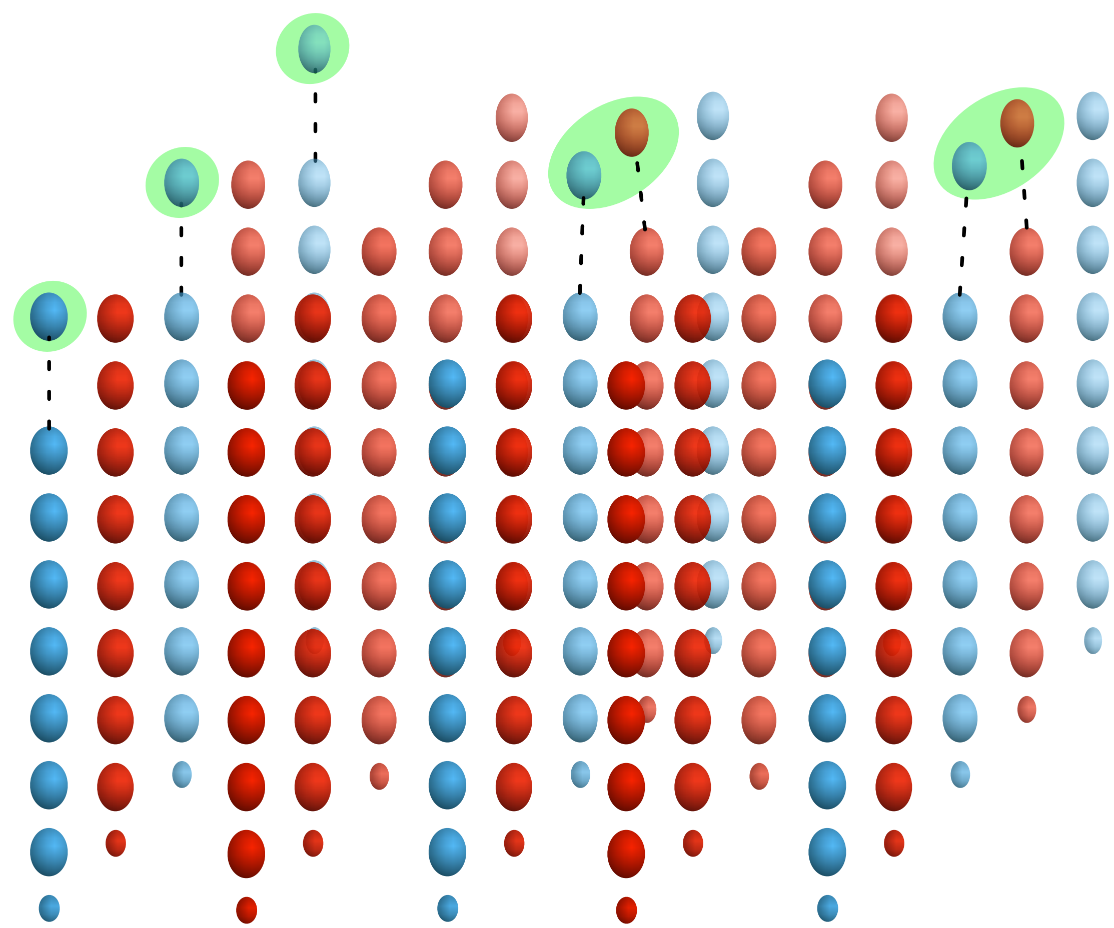
- Error budget for Symmetry Protected Topological Order

Error type		Effect
Memory	D_2-invariant	Logically protected
	Bulk	$p_L = 0,\ p_\ell \sim \left(\frac{\|\|h\|\|}{\Delta}\right)^2$
	Boundary	$p_L \sim \frac{\|\|h\|\|}{\Delta}$
Gate	D_2-invariant	Logically protected
	Quenched	Systematic $p_L \sim \frac{\|\|h\|\|}{\Delta}$
	Stochastic	$p_L \sim \frac{\|\|h\|\|}{\Delta}$

Δ gap

h perturbation

- Sketch of a fault tolerant architecture
  - 3 x 3 Bacon Shor Code



Summary & Outlook

- We should take ``hidden order” and phase stability seriously as a resource for quantum computation
- Renormalization as a physical process
    - Works in 1D without knowing exact value of perturbation
    - Works in 2D if you know the perturbation (e.g. by tomography on a coupled pair)
- Quantum gates in a1D symmetry protected topological phase
    - Easier to engineer than full topological order
- Is there a *nice* way to make ground code computing fault tolerant?
    - Could just import standard concatenated codes but is there a better way?
- Quantum gates in 2D SPTO?
    - Gapless edge modes better protected