QUANTUM SYSTEM IDENTIFICATION

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KITP 15/1/12

OUTLINE

* MOTIVATION
* USING CONTROL
* EQUIVALENCE VS. SIMILARITY
* CONSEQUENCES
* EXAMPLE
* CONCLUSION
* OUTLOOK

MOTIVATION

WHERE DO EXPERIMENTALISTS GET THEIR HAMILTONIANS FROM ?

* UNDERLYING PHYSICS
* INTUITION
* FITTING DATA
* PROCESS TOMOGRAPHY (REALLY ?)

HOW SURE CAN WE BE SYSTEM MODEL IS CORRECT ?

* SYSTEMATIC ERRORS

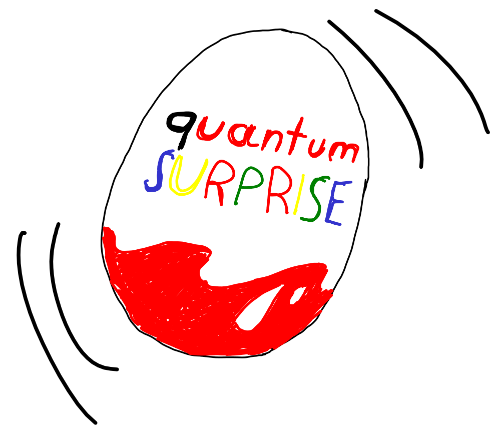
⇒ GOAL: FIND A WAY OF CHARACTERIZING HOW WELL A SYSTEM CAN BE ESTIMATED

⇒ WITHOUT HAVING TO BOTHER ABOUT THE PROTOCOL

⇒ ANALOGOUS OF REACHABILITY

HOW DOES QUANTUM CONTROL HELP FOR SYSTEM IDENTIFICATION ?

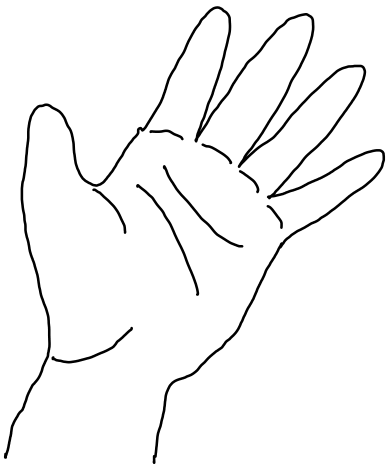
* SHAKE SYSTEM TO FIND CONTENT

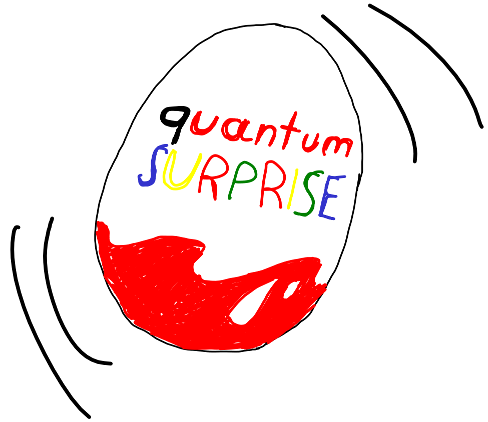


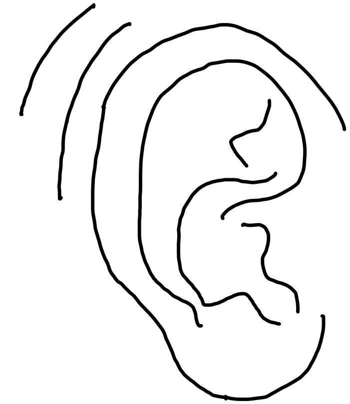
... TRIVIAL IF WE KNOW HOW TO FULLY CONTROL SYSTEM ...

... BUT WHAT IF ACCESS IS LIMITED ?

OUR MODEL (FOLLOWING E. SONTAG)







PULSES

$f_k(t)$

SYSTEM

ρ_0 H_0

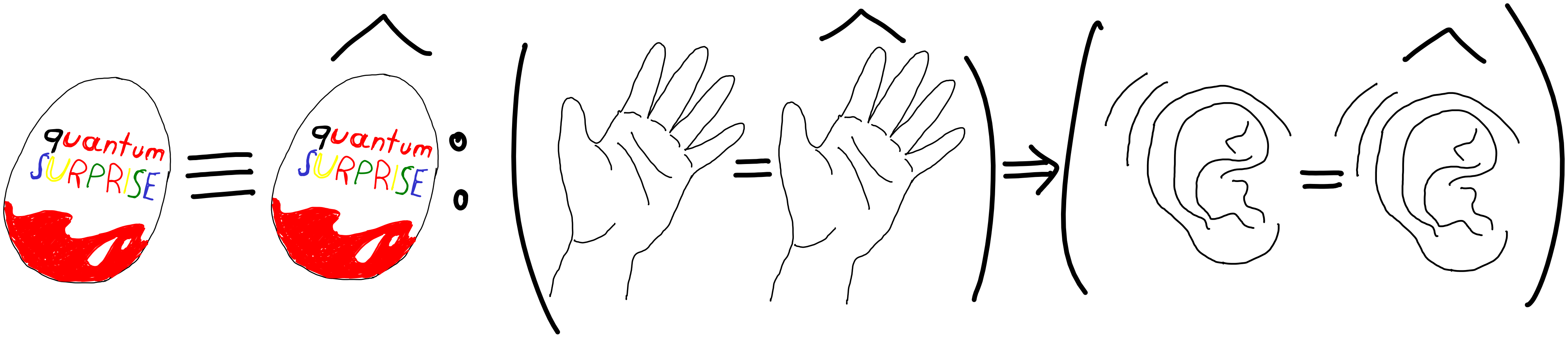
H_k M_ℓ

MEASUREMENTS

$\mathrm{tr}(M_\ell\, \rho(t))$

ASSUMPTIONS: FINITE KNOWN DIMENSION, CLOSED SYSTEM

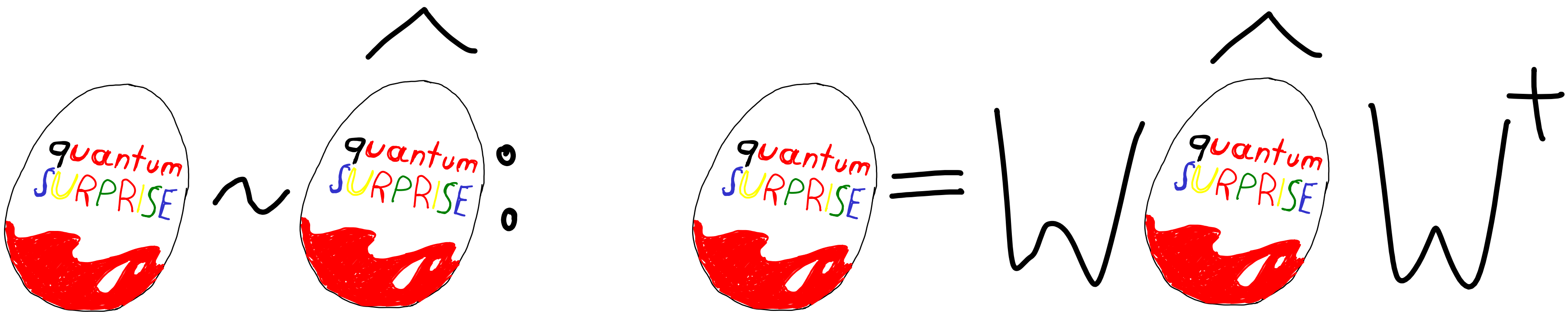
EQUIVALENT SYSTEMS



* BY DEFINITION, ESTIMATING EQUIVALENCE CLASS IS BEST WE CAN HOPE FOR

* BUT CLASSES HARD TO CHARACTERISE

SIMILAR SYSTEMS



(W UNITARY)

MORE PRECISELY:

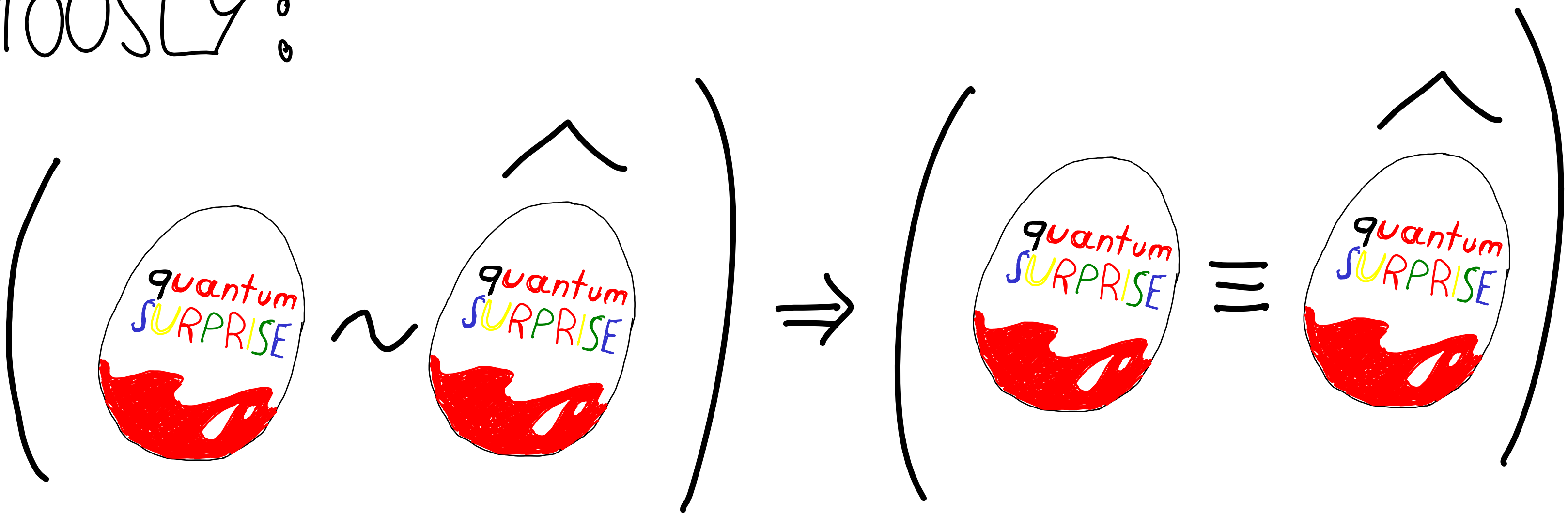
$$H_0 = W \hat{H}_0 W^{+} \quad , \quad H_k = W \hat{H}_k W^{+} \quad , \quad M_\ell = W \hat{M}_k W^{+}$$

$$\rho_0 = W \hat{\rho}_0 W^{+}$$

* EASY DESCRIPTION

A THEOREM

OBVIOUSLY:



"HIDDEN TRANSLATOR"

CONVERSE WRONG:

$$H_0 = A \otimes \mathbb{1} \qquad H_k = A_k \otimes \mathbb{1} \qquad M_\ell = m_\ell \otimes \mathbb{1} \qquad \rho_0 = \rho_A \otimes \rho_B$$

$$\hat{H}_0 = A \otimes \mathbb{1} + \mathbb{1} \otimes B \qquad \hat{H}_k = A_k \otimes \mathbb{1} \qquad \hat{M}_\ell = \hat{m}_\ell \otimes \mathbb{1} \qquad \hat{\rho}_0 = \rho_A \otimes \rho_B$$

EQUIVALENT BUT NOT SIMILAR

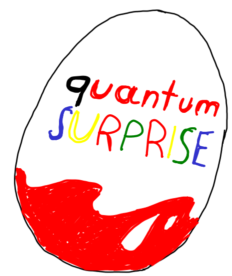
REASON: UNCOUPLED

NEED PULSES TO "REACH" WHOLE SPACE

IF $\langle iH_0, iH_k \rangle_{LIE} = su(d)$ THEN

$\Bigg($ 

$\sim$ 

$\Bigg) \Leftrightarrow \Bigg($ 

$\equiv$ 

$\Bigg)$

$\langle iH_0, iH_k \rangle_{LIE} = su(d)$: TESTABLE,

"LIKELY"

LET'S PLAY!

* UNDER PREMISE OF REACHABILITY, SYSTEM IDENTIFIABLE UP TO SIMILARITY

* ASSUME COMPONENT X IS KNOW
  (FOR INSTANCE, $X = M_\ell$ or $X = H_k$)

  $\Rightarrow$ ESTIMATED $\hat{X} = X = W X W^+$

  $\Rightarrow$ $[W, X] = 0$

* EACH KNOWN X CONSTRAINTS POSSIBLE W'S

* X, Y KNOWN $\Rightarrow$ $[[X,Y], W] = 0$

(JACOBI)

* LIE ALGEBRA GENERATED BY KNOWN THINGS

$$[L_{KNOW}, W] = 0$$

* $\boxed{\{L_{KNOW}\}' = \{c \cdot \mathbb{1}\} \Leftrightarrow \text{IDENTIFIABLE}}$

* WEIRD OBJECT, EG. $\langle i g_0, i M_\ell, i H_k \rangle_{LIE}$

* IN „GENERAL" TWO KNOWN ELEMENTS SUFFICE

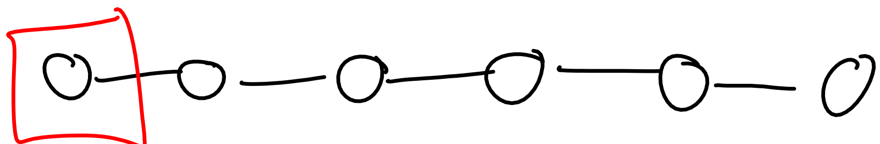
* CONTROLLING & MEASURING 2N PAULIS

IS ENOUGH FOR ALL COUPLED SYSTEMS

(EXPONENTIAL ADVANTAGE)

EXAMPLE:

$$H_0 = \sum_{n=1}^{N-1} \left(XX + YY + ZZ \right)_{n\,n+1}$$



$$\hat{H}_1 = H_1 = X_1 \qquad L = \langle iH_0, iH_1, iH_2 \rangle = su(2^N) \quad \checkmark$$

$$\hat{H}_2 = H_2 = Y_1 \qquad L_{KNOW} = su(2)_1$$

$$M_1 = X_1 \qquad \Rightarrow \quad W = \mathbb{1}_1 \otimes V_{Rest}$$

$\rightsquigarrow$ MANY FREE PARAMETERS

HOWEVER, MANY $\mathbb{1} \otimes V \; H_{Heisenberg} \; \mathbb{1} \otimes V^{+}$ WOULD BE REJECTED:

* DIFFERENT SYMMETRIES
* NOT 2-BODY
* DIFFERENT TOPOLOGY

HOW TO INCLUDE IN L_{KNOWN}?

→ SPECIFIC SOLUTION FOR THIS MODEL (DB, K. MARUYAMA, F. NORI)

CONCLUSION

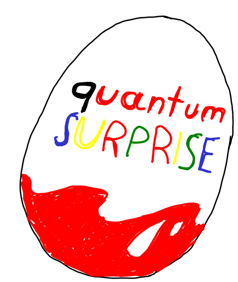
* IN PRESENCE OF CONTROL, 2 KNOWN THINGS ARE GENERALLY ENOUGH FOR I.D.

  ⇒ MUCH LESS THAN PROCESS TOMOGRAPHY

* EASY CRITERION TO CHECK EXPERIMENT, PROTOCOL INDEPENDENT

OPEN SYSTEMS ?

SYMMETRIES, 2-BODY ?



THANKS !