Do we really want to adiabatically eliminate?

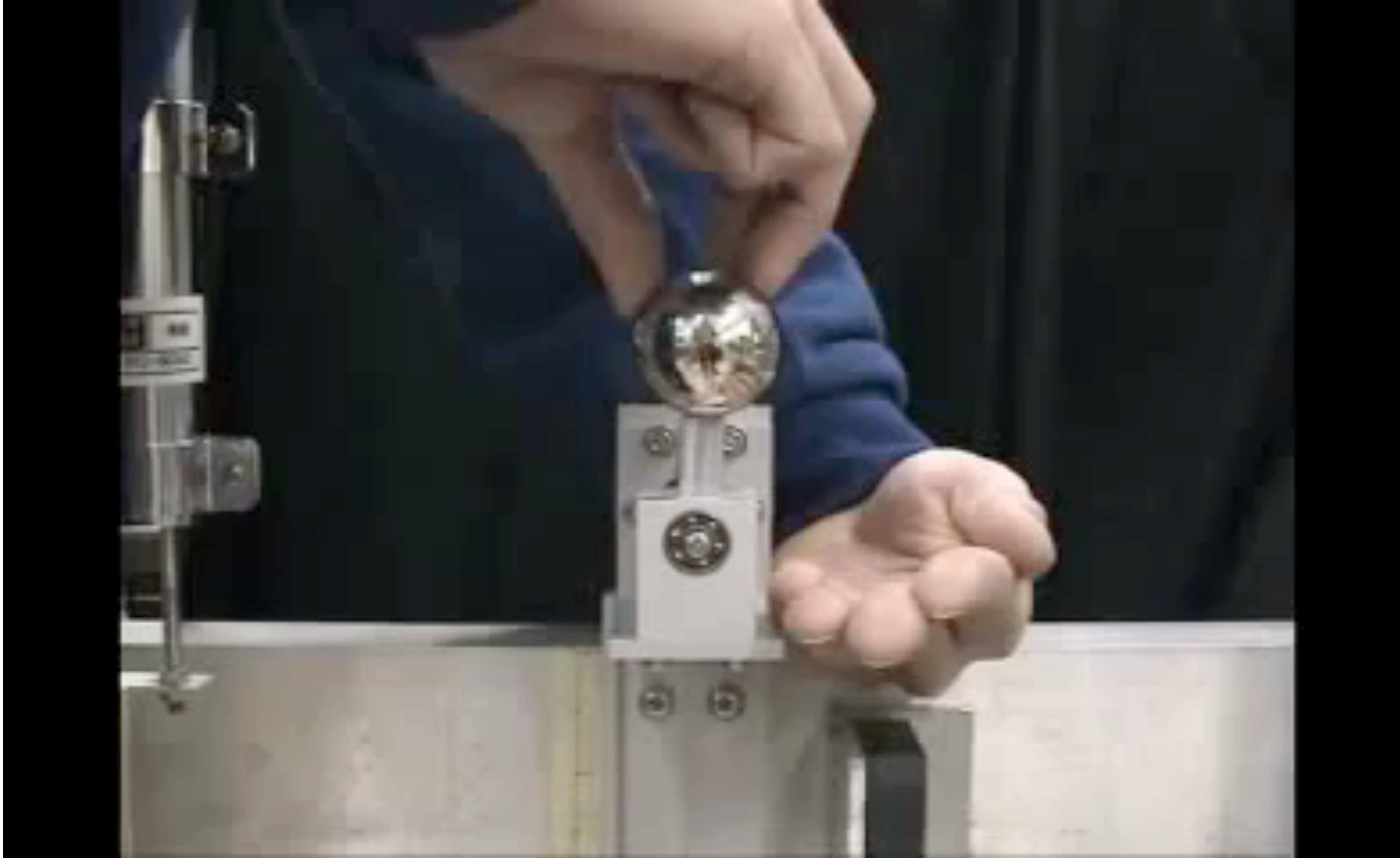
Controlling wiggles,
wiggling control

Outline

- Wiggles
  - in classical control
  - in quantum control
- Noise
  - classical
  - quantum
- Cutting off wiggles
- Wiggles as primitives

Wiggles

in classical control



The Kapitza pendulum

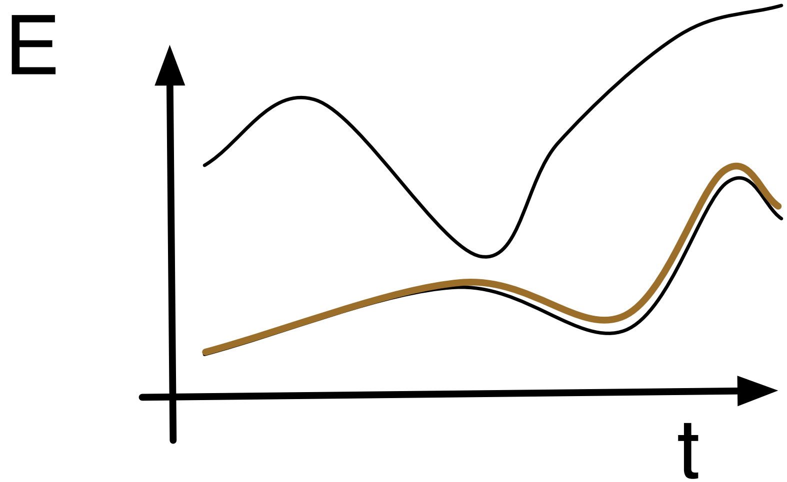
Stabilization without feedback

More wiggles

in quantum control

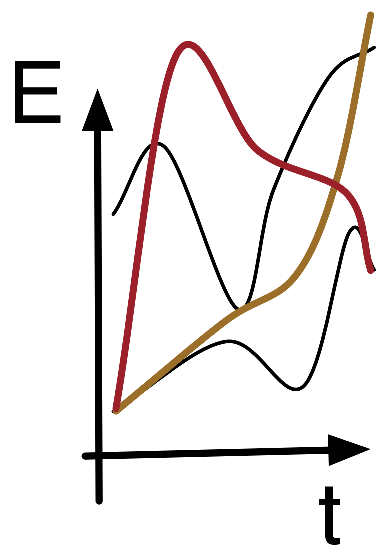
Optimal dynamics: a cartoon

Slow



Adiabatic strategy

Fast

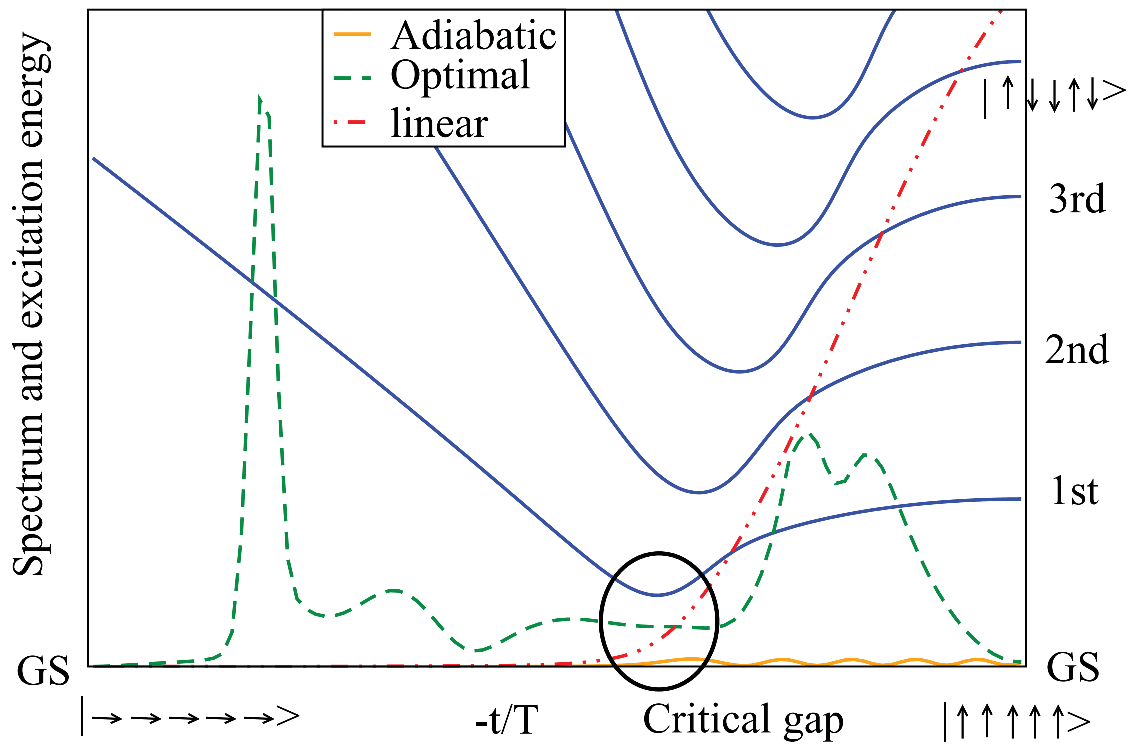


Optimal control

Optimal dynamics: a real (many-body) example

Lipkin-Meshkov-Glick model

$$H^{\mathrm{LMG}} = -\sum_{i<j}^{N} J_{ij}\sigma_i^x\sigma_j^x - \Gamma(t)\sum_i^N \sigma_i^z$$

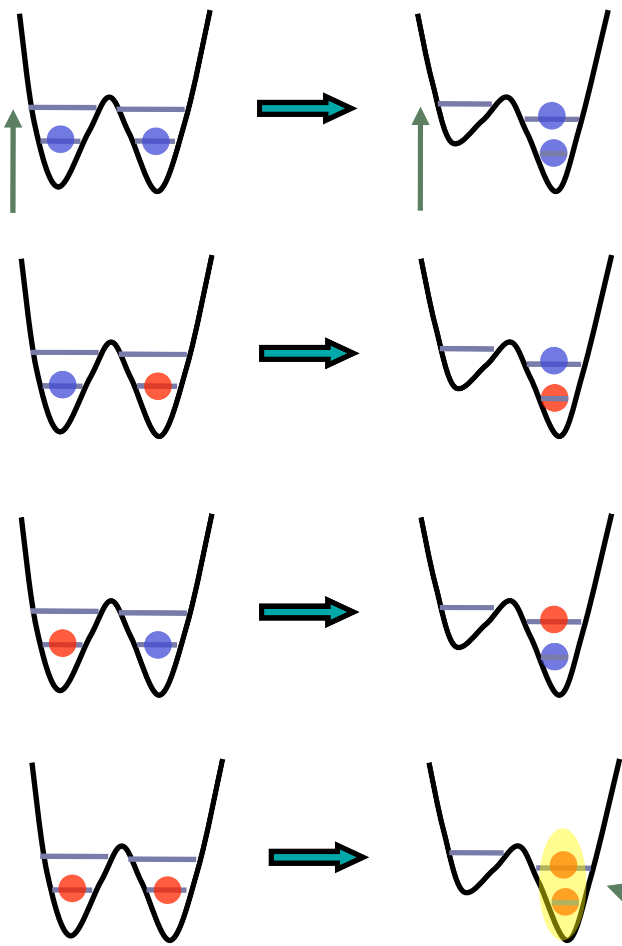


Caneva, TC, Fazio, Santoro, Montangero, Phys. Rev. A 84, 012312 (2011)

Optimal control in superlattices

Calarco, Dorner, Julienne, Williams, Zoller PRA 70, 12306 (2004)

elevator



$|0\rangle|0\rangle \longrightarrow |0\rangle|0\rangle$

$|0\rangle \quad |1\rangle$

state independent superlattice

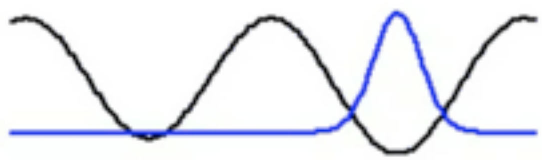
$|0\rangle|1\rangle \longrightarrow |0\rangle|1\rangle$

$|1\rangle|0\rangle \longrightarrow |1\rangle|0\rangle$

$|1\rangle|1\rangle \longrightarrow e^{i\phi} \; |1\rangle|1\rangle$

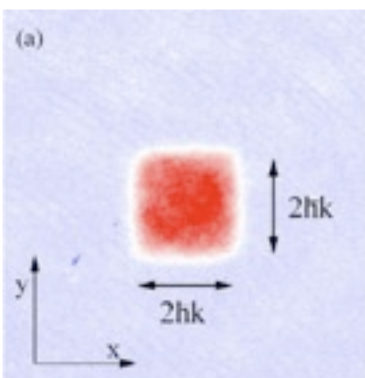
state dependent Feshbach resonances

Transport in dipole traps

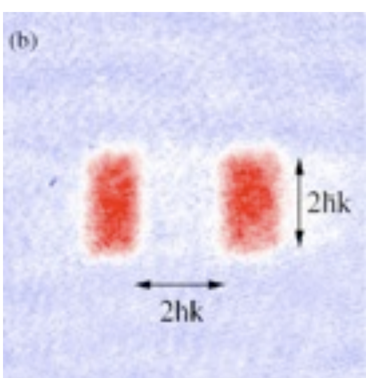


© T. Porto, W. Phillips 2005

Realization of (not time-optimized) transport in an optical lattice

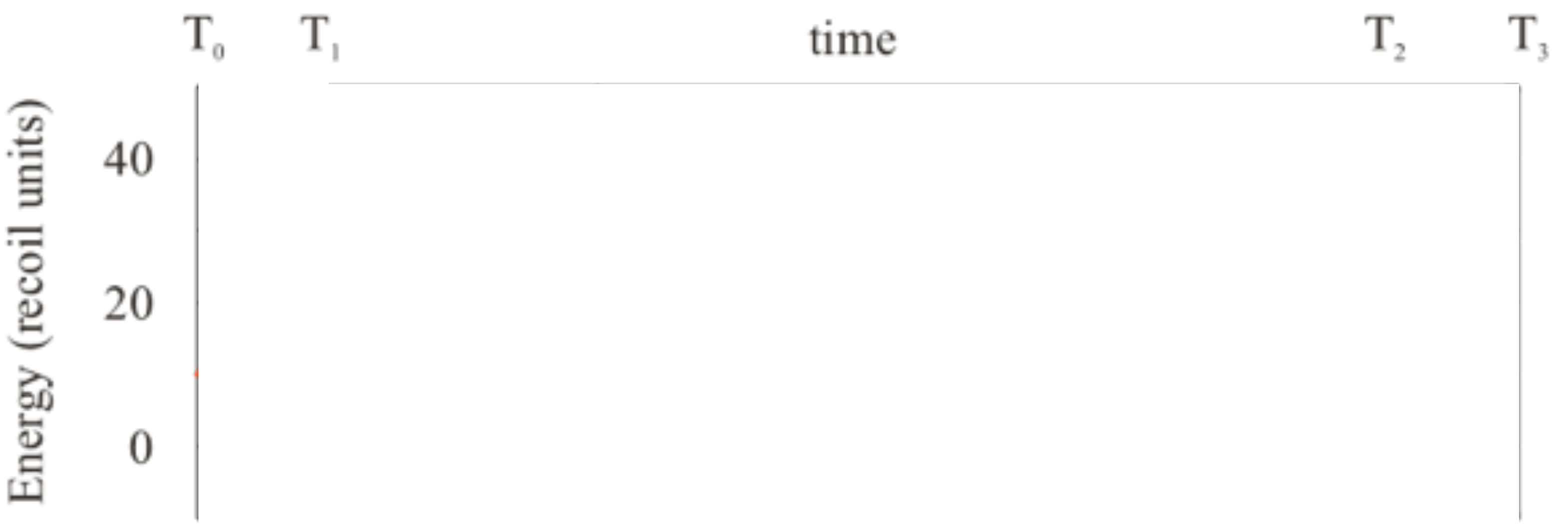


few ms transfer time



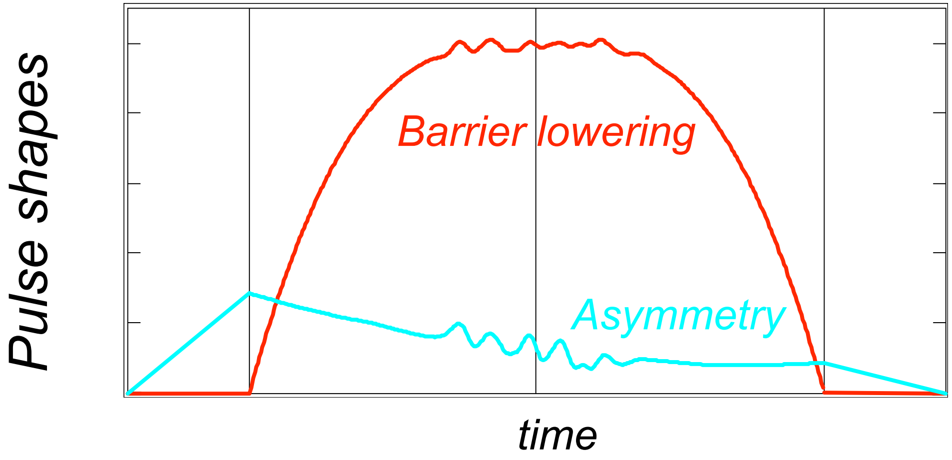
...two-qubit gate: W. Phillips, Nature 2007

Dipole traps - connection diagram



Dipole traps - optimized pulses in detail

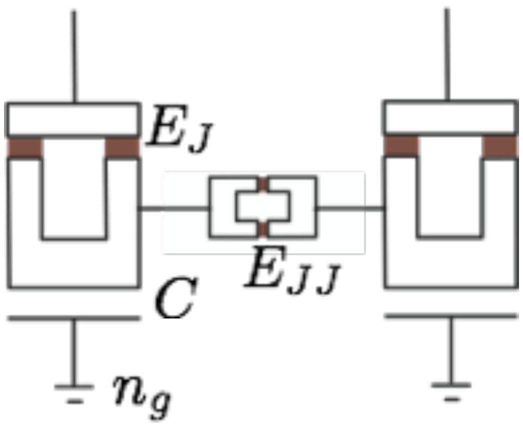
- Optimization algorithm introduces wiggles in pulse shapes
- “Shaking” helps exciting-deexciting
- Frequency higher than gate operation rate



Classical control noise

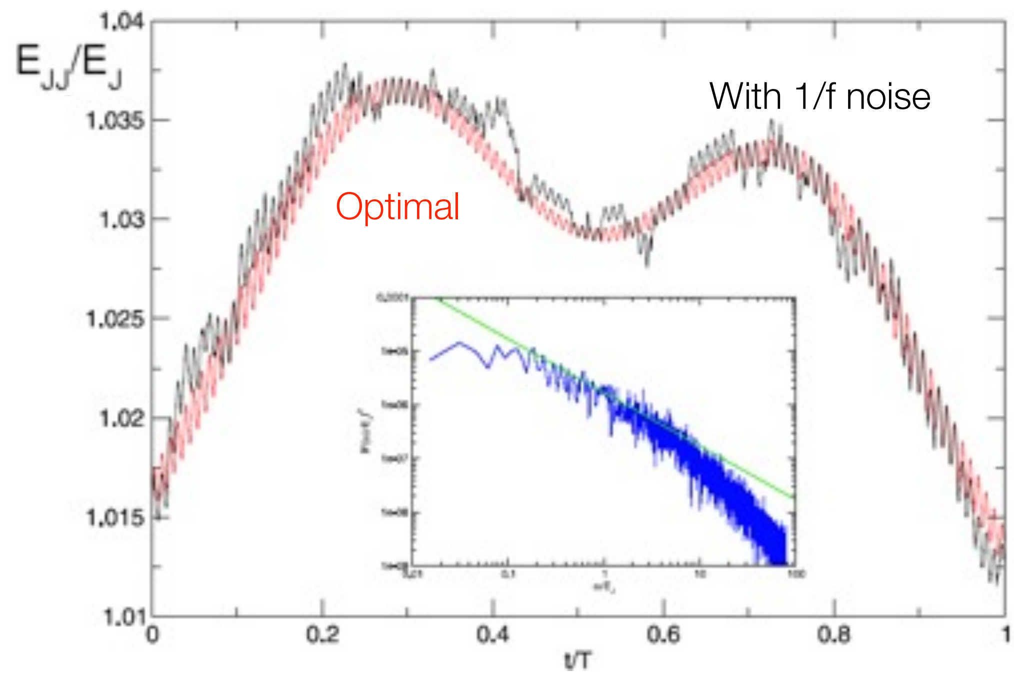
What if there is no such timescale separation?

with R. Fazio, PRL '07



$$G_{JJ} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & \pm i & 0 & 0 \\ 0 & 0 & \pm i & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- Qubit: 0 or 1 excess Cooper pair
- Control parameter: Josephson energy E_{JJ}



Error with/without control

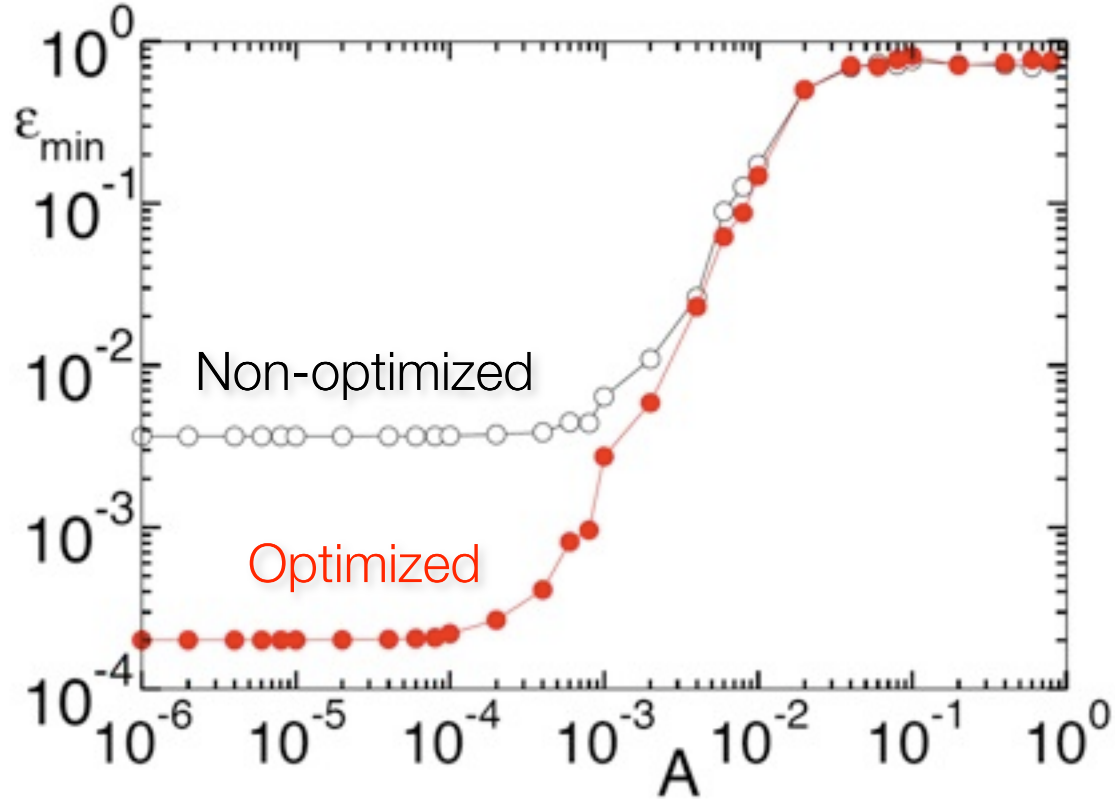
1/f noise

$$S(\omega) \propto A/\omega$$

Typical exp. values

$$A \sim 10^{-5}$$

Fault tolerance with realistic noise?



Why does it work? ...noise - frequency separation

Legendary *Titanic* band

[edit]

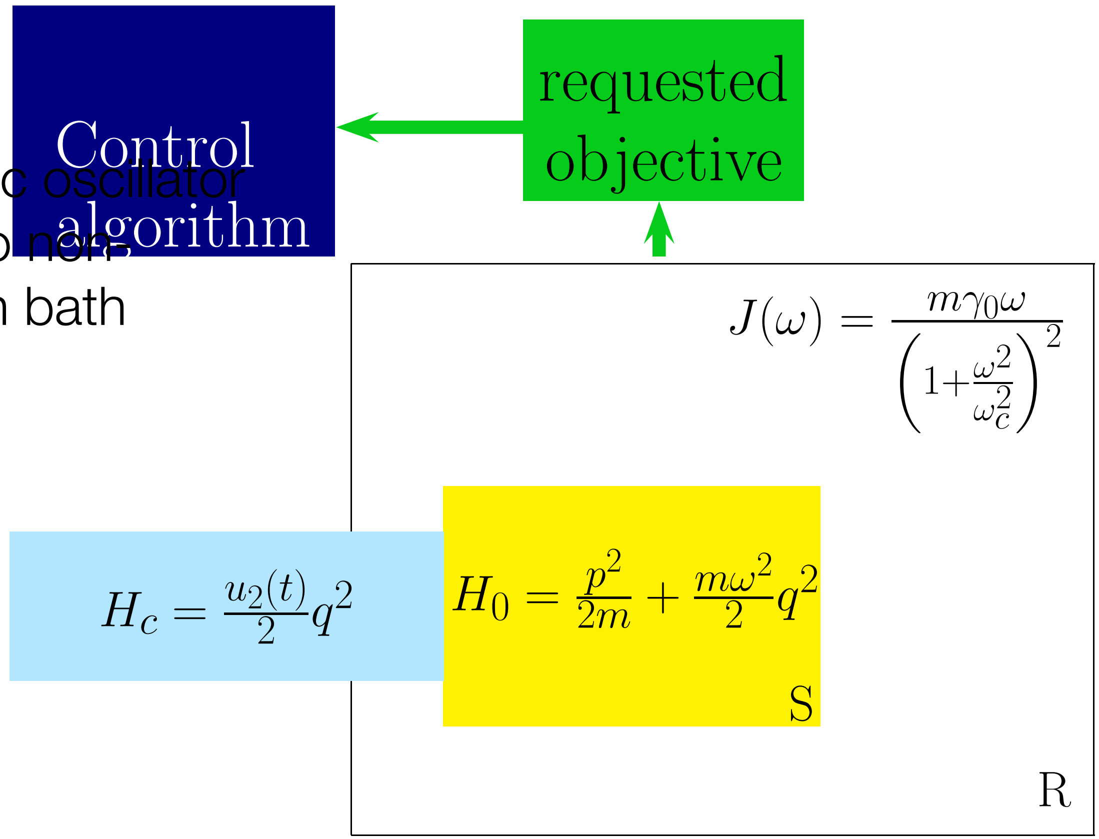
From Wikipedia, the free encyclopedia

Some events during the *Titanic* disaster have had a legendary impact. One of the most famous stories of the *Titanic* is of the band. On 15 April, the *Titanic*'s eight-member band, led by Wallace Hartley, had assembled in the first class lounge in an effort to keep passengers calm and upbeat. Later they would move on to the forward half of the boat deck. Band members had played during Sunday worship services the previous morning, and the band continued playing music even when it became apparent the ship was going to sink.

Quantum non-Markovian noise

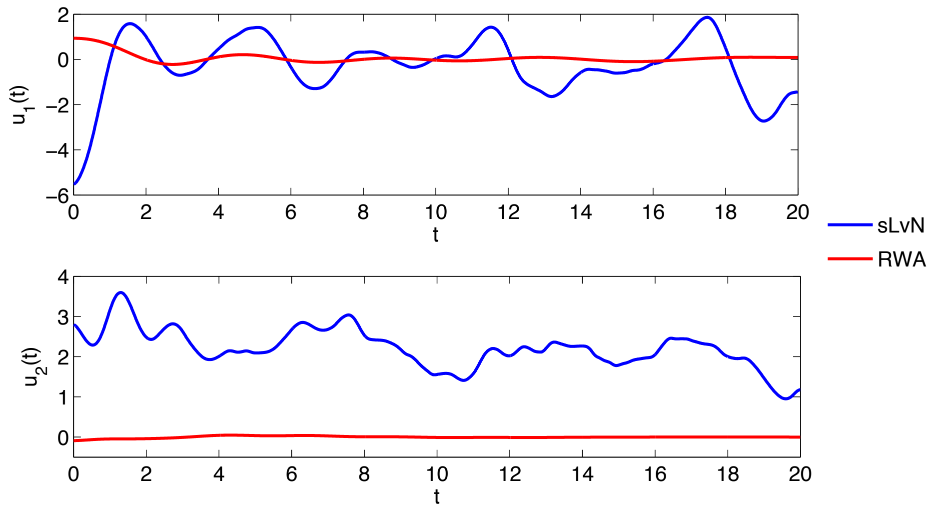
Optimal dynamics: a simple open system

Driven harmonic oscillator coupled to non-Markovian bath

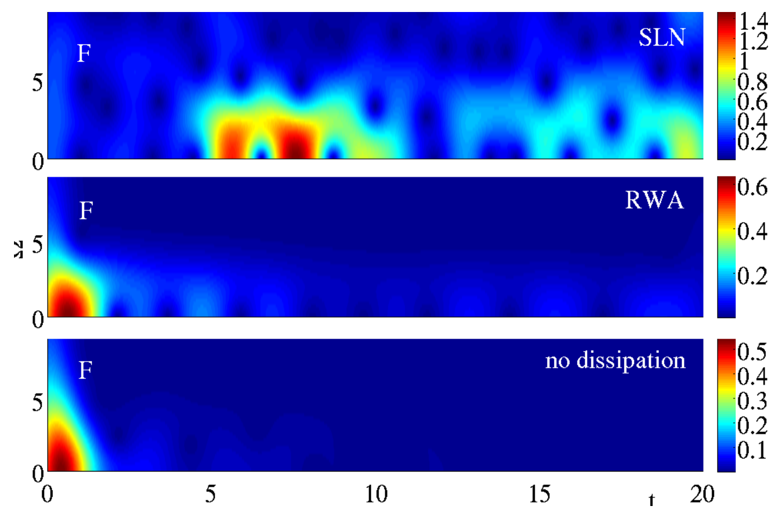


$$H = H_S + H_R + H_I \qquad \text{with: } H_S = H_0 + H_C$$

Open-system control results

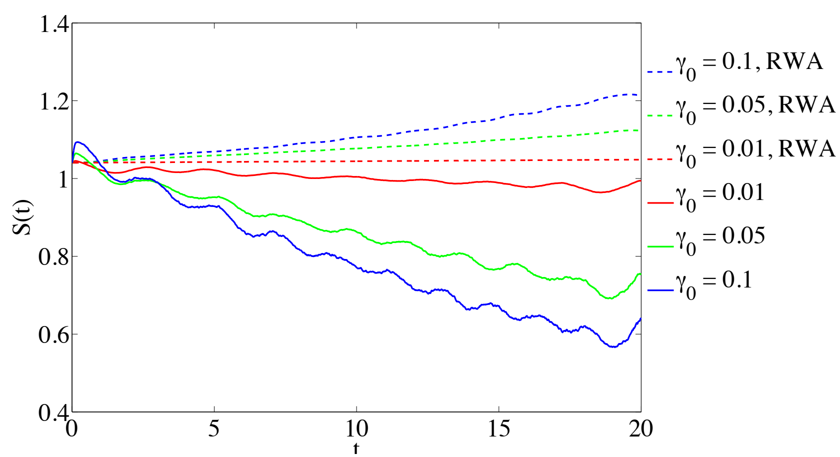


Control pulse



Short-time FT

Entropy loss

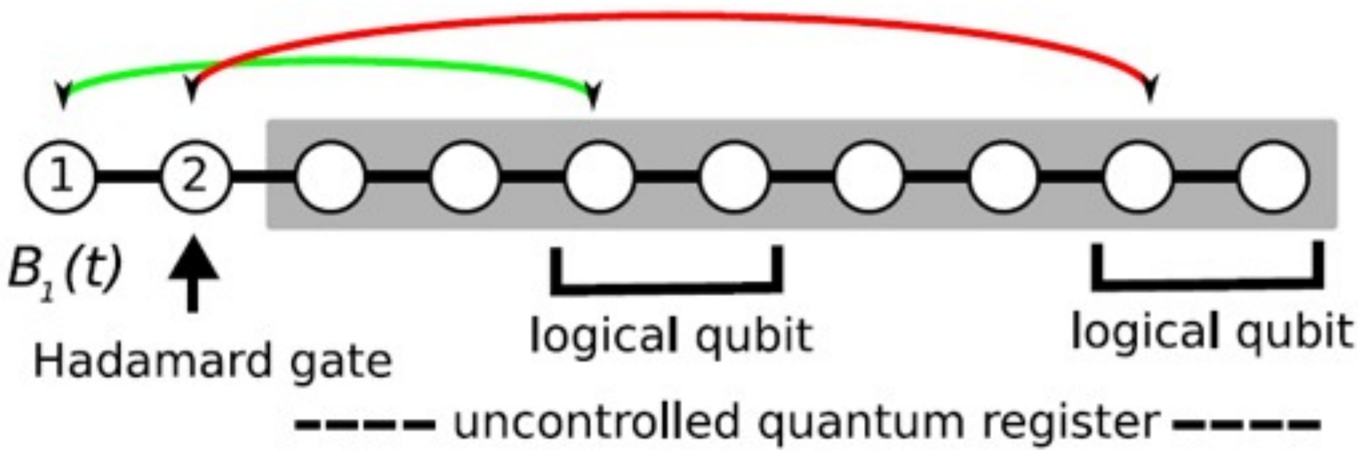


Cutting off wiggles

up to a certain extent

Scalable quantum computation via local control of only two qubits

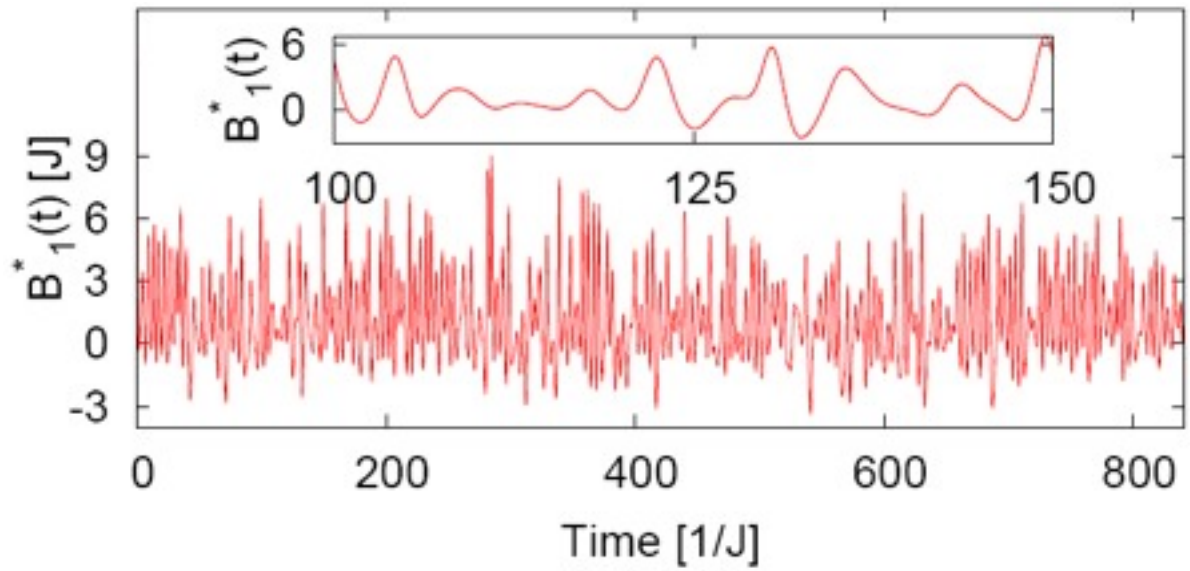
$$H = \frac{1}{2}\sum_{n=1}^{N-1} c_n[(1-\gamma)XX + (1-\gamma)YY]_{n,n+1} + \sum_{n=1}^{N} B_n Z_n$$

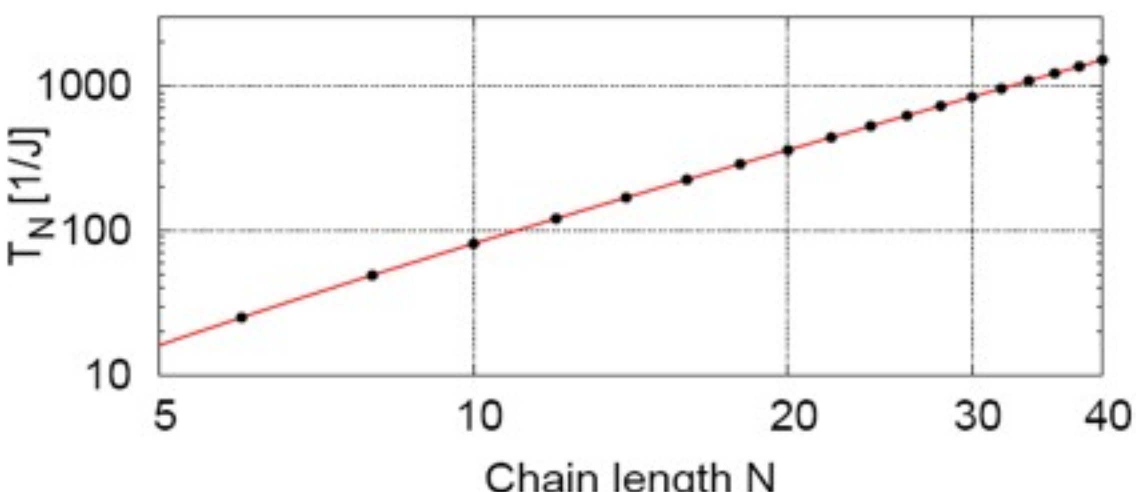


D. Burgarth, K. Maruyama, M. Murphy, S. Montangero, T. Calarco, F. Nori, M. Plenio, Phys. Rev. A 81, 040303 (2010)

Scaling of the operation time

Sample
control pulse

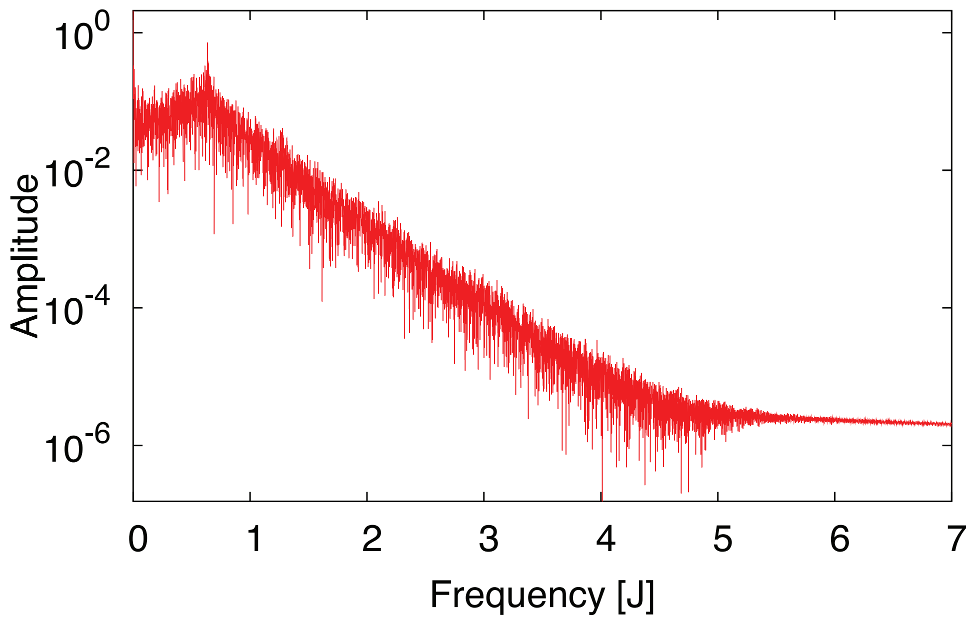


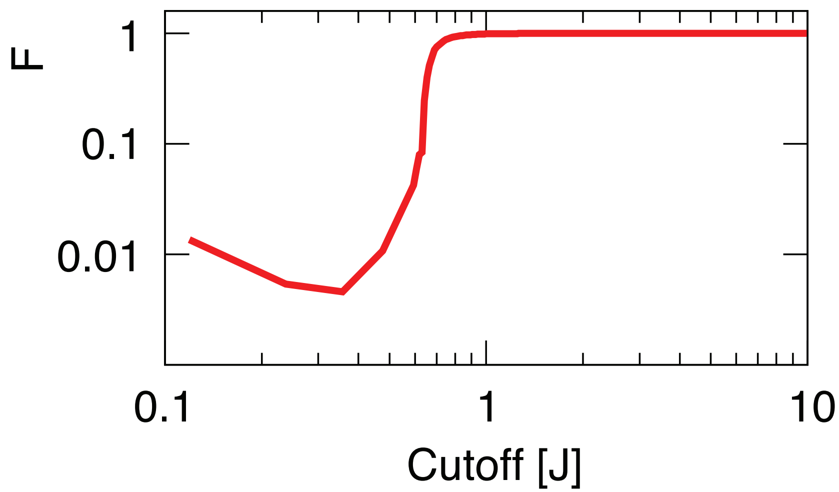


$$T_N = (N-1)^2$$

How many wiggles are needed?

Control pulse spectrum





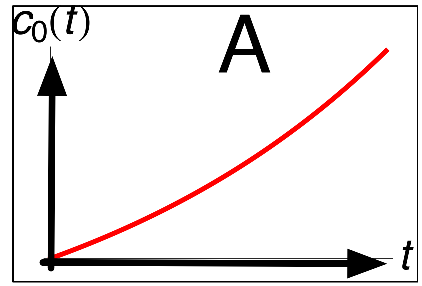
Only frequencies up to the natural scale J are needed

Wiggles as primitives

A load of CRAB

Chopped RAndom Basis (CRAB) algorithm

Initial guess: $c_0(t)$



Correction:

$$g(t) = \sum_{k=1}^{n} a_k \tilde{f}_k(t)$$

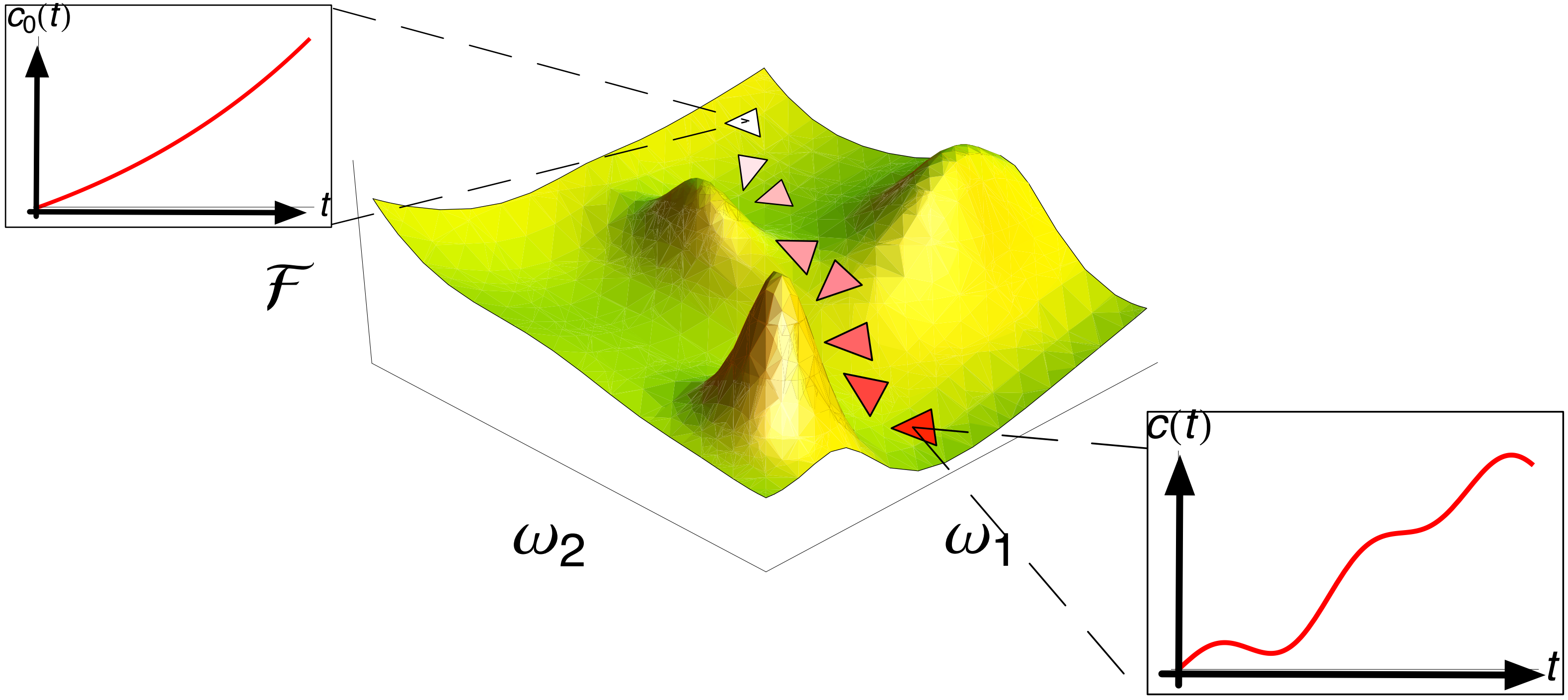
$\tilde{f}_k(t)$ “randomized” basis functions

Examples: $f_k(t) = sin(\omega_k t),\ x_k^{\alpha},\ H_k(x), ...$

Trial pulse: $c(t) = c_0(t) g(t)$

Optimize n=O(10) parameters!

Direct search optimization

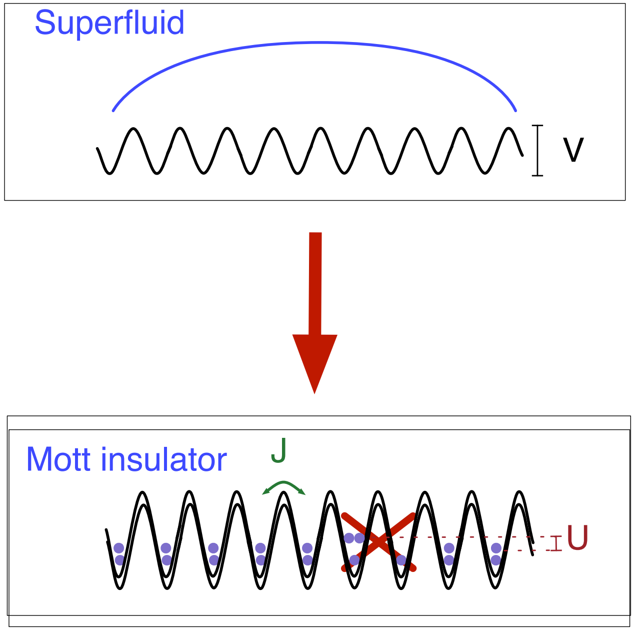


- No need of gradient (Nelder-Mead, simplex, etc.)
- No need of (semi-)analytical solutions
- Figures of merit: energy, fidelity, purity, entanglement.

Application: Mott-Superfluid transition with cold atoms in optical lattices

Bose Hubbard model

$$\mathrm{H} = \sum_j [-J(b_j^\dagger b_{j+1} + \mathrm{h.c.}) + \Omega(j - \frac{N}{2})^2 n_j + \frac{U}{2}(n_j^2 - n_j)]$$



J/U >> 0.1

J/U << 0.1

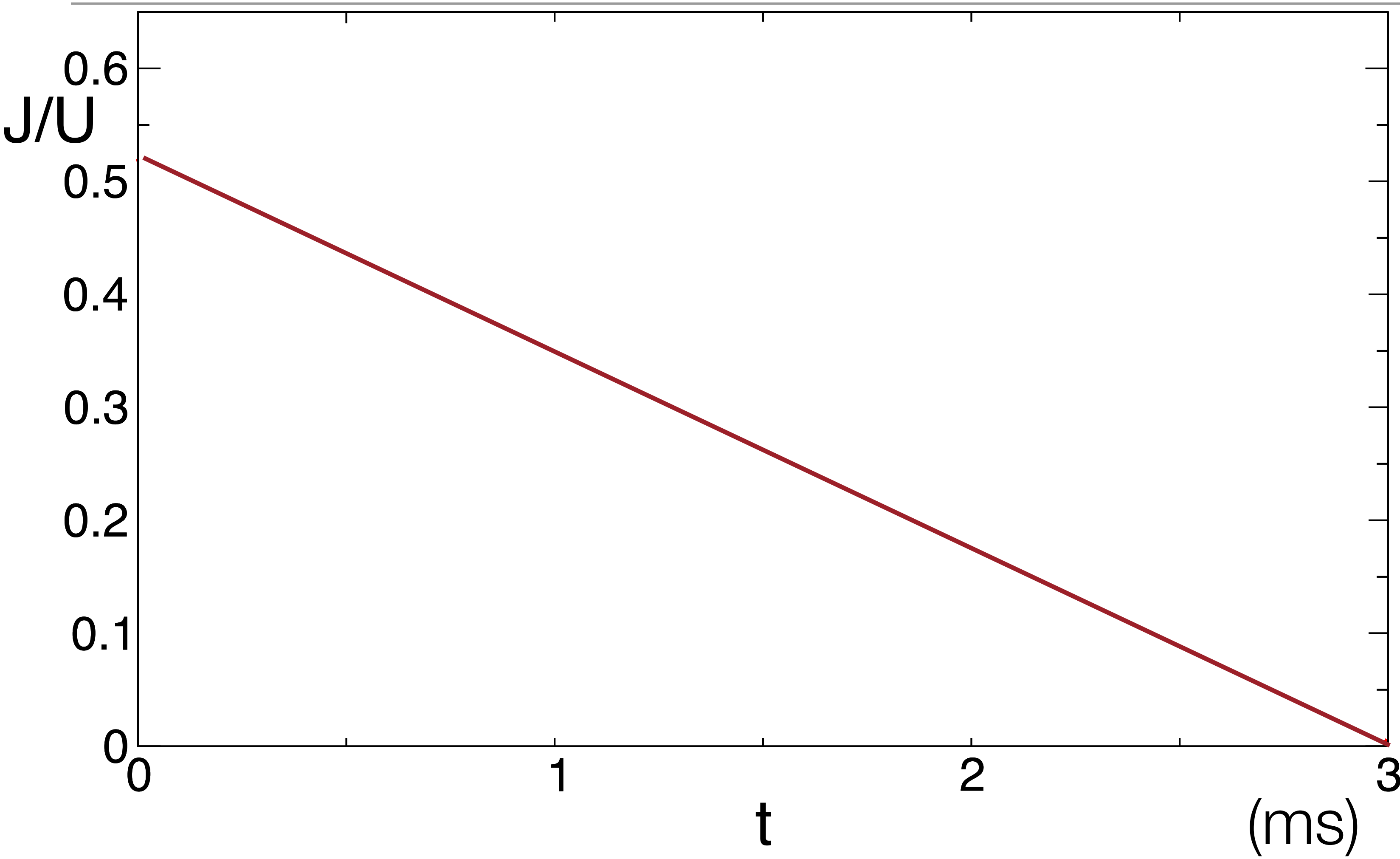
J Hopping

U Onsite energy

Ω Trapping

M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch and I. Bloch, Nature 415, 39 (2002).

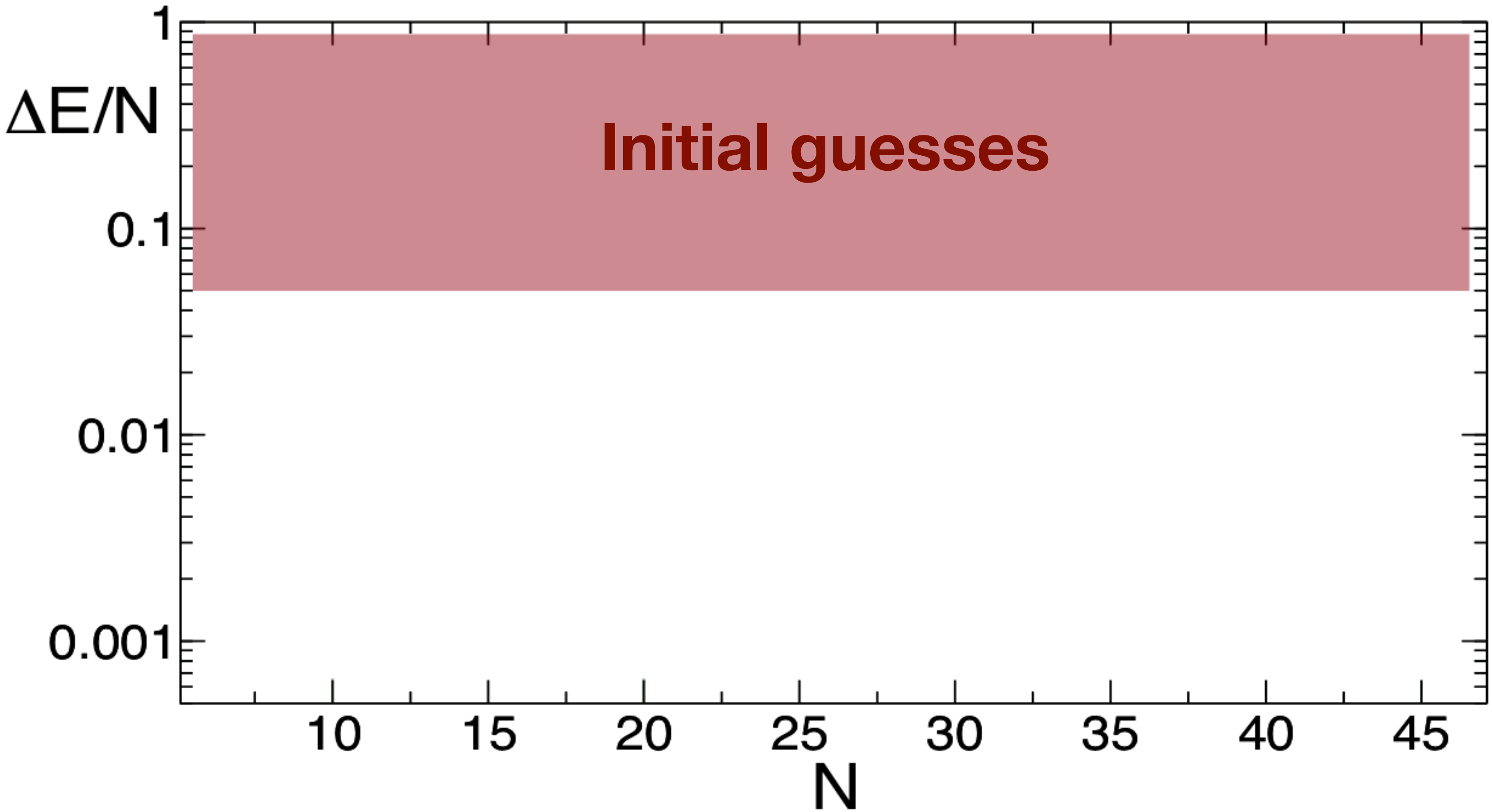
Optimal pulse



Filling one

L=30

Density of defects



T= 3ms

Homogeneous system

Trapping potential
T. Esslinger group
PRL (2004)

Do we really want to adiabatically eliminate?

Why do wiggles work so well?