Do we really want to adiabatically eliminate?

Controlling wiggles,
wigglng control
Outline

• Wiggles
  • in classical control
  • in quantum control
• Noise
  • classical
  • quantum
• Cutting off wiggles
• Wiggles as primitives
Wiggles

in classical control
The Kapitza pendulum

Stabilization without feedback
More wiggles in quantum control
Optimal dynamics: a cartoon

Slow

Fast

Adiabatic strategy

Optimal control
Optimal dynamics: a real (many-body) example

Lipkin-Meshkov-Glick model

\[ H^{\text{LMG}} = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x - \Gamma(t) \sum_i \sigma_i^z \]

Caneva, TC, Fazio, Santoro, Montangero, Phys. Rev. A 84, 012312 (2011)
Optimal control in superlattices

Calarco, Dorner, Julienne, Williams, Zoller PRA 70, 12306 (2004)

$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$

$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$

$|1\rangle|0\rangle \rightarrow |1\rangle|0\rangle$

$|1\rangle|1\rangle \rightarrow e^{i\phi}|1\rangle|1\rangle$
Transport in dipole traps

Realization of (not time-optimized) transport in an optical lattice

...two-qubit gate: W. Phillips, Nature 2007
Dipole traps - connection diagram
Dipole traps - optimized pulses in detail

- Optimization algorithm introduces wiggles in pulse shapes
- “Shaking” helps exciting-deexciting
- Frequency higher than gate operation rate

Pulse shapes

- Barrier lowering
- Asymmetry

(time)
Classical control noise
What if there is no such timescale separation?

- Qubit: 0 or 1 excess Cooper pair
- Control parameter: Josephson energy

\[ E_{JJ} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & \pm i & 0 & 0 \\ 0 & 0 & \pm i & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \]

Optimal

With 1/f noise

with R. Fazio, PRL ‘07
Error with/without control

1/f noise

\[ S(\omega) \propto \frac{A}{\omega} \]

Typical exp. values

\[ A \sim 10^{-5} \]

Fault tolerance with realistic noise?
Why does it work? ...noise - frequency separation

**Legendary Titanic band**

From Wikipedia, the free encyclopedia

Some events during the *Titanic* disaster have had a legendary impact. One of the most famous stories of the *Titanic* is of the band. On 15 April, the *Titanic*'s eight-member band, led by Wallace Hartley, had assembled in the first class lounge in an effort to keep passengers calm and upbeat. Later they would move on to the forward half of the boat deck. Band members had played during Sunday worship services the previous morning, and the band continued playing music even when it became apparent the ship was going to sink.
Quantum non-Markovian noise
Optimal dynamics: a simple open system

$H = H_S + H_R + H_I$  \[\text{with: } H_S = H_0 + H_C\]
Open-system control results

Control pulse  Short-time FT

Entropy loss
Cutting off wiggles

up to a certain extent
Scalable quantum computation via local control of only two qubits

\[ H = \frac{1}{2} \sum_{n=1}^{N-1} c_n[(1 - \gamma)XX + (1 - \gamma)YY]_{n,n+1} + \sum_{n=1}^{N} B_n Z_n \]

Scaling of the operation time

Sample control pulse

\[ T_N = (N - 1)^2 \]
How many wiggles are needed?

Control pulse spectrum

Only frequencies up to the natural scale $J$ are needed.
Wiggles as primitives

A load of CRAB
Chopped RAndom Basis (CRAB) algorithm

Initial guess: \( c_0(t) \)

Correction:

\[
g(t) = \sum_{k=1}^{n} a_k \tilde{f}_k(t)
\]

\( \tilde{f}_k(t) \) “randomized” basis functions

Examples: \( f_k(t) = \sin(\omega_k t), x_k, H_k(x), ... \)

Trial pulse: \( c(t) = c_0(t) g(t) \)

Optimize \( n=O(10) \) parameters!
Direct search optimization

- No need of gradient (Nelder-Mead, simplex, etc.)
- No need of (semi-)analytical solutions
- Figures of merit: energy, fidelity, purity, entanglement.
Application: Mott-Superfluid transition with cold atoms in optical lattices

Bose Hubbard model

\[ H = \sum_j \left[ -J (b_j^+ b_{j+1} + \text{h.c.}) + \Omega (j - \frac{N}{2})^2 n_j + \frac{U}{2} (n_j^2 - n_j) \right] \]

\[ \frac{J}{U} >> 0.1 \]

\[ \frac{J}{U} << 0.1 \]

- \( J \): Hopping
- \( U \): Onsite energy
- \( \Omega \): Trapping

The system is due to sites with occupation number being... for the system. This quantity is directly related to the density \( N \) per site fluctuations, clearly demonstrating the convergence to a... produced by the CRAB optimization. In the inset we... with respect to the initial exponential guess and no... of the confining trap is shown for the parameter values... set the total time... the exact final ground state energy as a function of the system size... fulfilling the adiabatic condition, given that this parameter... the experimental setup of [23]. We optimized numerical simulations and experimental results [23, 24], we... studied the ideal homogeneous system and reproduced...\( J/U \) [17]. Finally, in Fig. 4 we show the final residual energy... for different initial ramp shapes. Correspondingly, in Fig. 4... as...001. The red... energy in the two cases (without and with trap) are... it can be seen from the inset of Fig. 3, fluctuations are... size...

**FIG. 3**: Optimal ramp for the system in the presence of defects: for any additional energy present in... of one-dimensional quantum systems composed by... tensor structure, i.e. a Matrix Product State (MPS). DMRG is based on the assumption that it is possible to... represent a major design tool for future quantum... e.g. by means of recently introduced numerical tech... optimization strategy introduced here can in principle... an improvement with respect to the initial guess by... reported: they are well below one per cent, demonstrat... size...t-DMRG -

**Filling one**

\[
\frac{J}{U} \propto \langle n^2 \rangle - \langle n \rangle^2 \approx \frac{1}{N}.
\]
The system is due to sites with occupation number be-

\[
N \text{ per site}
\]

Mott insulator state after the ramp optimization. Introduced by the CRAB optimization. In the inset we dis-

\[
N \text{ corresponding to the experiment [8] and for a system size}
\]

\[
N \text{ of the confining trap is shown for the parameter values}
\]

\[
\text{lattice depth} \quad \text{reaches a given threshold}
\]

\[
\text{the Mott insulator phase. When the density of defects}
\]

\[
\text{set the total time}
\]

\[
\text{with}
\]

\[
\text{the presence of the trap (experimental parameters from [8])}
\]

\[
\text{We optimize the ramp to obtain the minimal residual en-
}\]

\[
\text{ergy per site}
\]

\[
\text{We consider a starting value of the lattice depth}
\]

\[
\text{an optimal ramp shape for the optical lattice depth}
\]

\[
\text{the Superfluid-Mott insulator transition and we obtained}
\]

\[
\text{cally the time dependence of the ratio}
\]

\[
(0) = 2, \ldots, 30 \text{ sites, average occupation}
\]

\[
0.2, 0.5, 0.5, 0.6, 0.6, 0.3, 0.1, 0.1
\]

\[
\text{Finally, in Fig. 4 we show the final residual energy}
\]

\[
\text{FIG. 3: Optimal ramp}
\]

\[
\text{for the homogeneous and for the trapped}
\]

\[
\text{system composed by}
\]

\[
\text{DMRG is based on the assumption that it is possible to}
\]

\[
\text{DMRG} \text{ classical simulator.}
\]

\[
\text{techniques [25]. Perhaps an even more stimulating perspec-
}\]

\[
\text{be applied also to open quantum many-body systems,}
\]

\[
\text{optimization strategy introduced here can in principle}
\]

\[
\text{gave residual density of defects at least one order of mag-
}\]

\[
\text{exponential guess – like other guesses: linear, random –}
\]

\[
\text{drastically reduced. Correspondingly, in Fig. 4 the resid-
}\]

\[
\text{it can be seen from the inset of Fig. 3, fluctuations are}
\]

\[
\text{fluctuations. Indeed, one can relate the residual energy}
\]

\[
\Delta E/N \text{ as}
\]

\[
\text{Initial guesses}
\]

\[
\text{Density of defects}
\]

\[
\text{Initial guesses}
\]

\[
\text{Homogeneous system}
\]

\[
\text{Trapping potential}
\]

\[
T = 3 \text{ms}
\]

\[
\text{Pauli limit}
\]

\[
\text{residual density of defects}
\]

\[
\text{order of magnitude bigger (red region in Fig. 4).}
\]

\[
\text{with experimental parameters from [8]} \text{and for a system size}
\]

\[
\text{of 40, for the homogeneous and for the trapped}
\]

\[
\text{time dependence of the ratio}
\]

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\[
\text{initial lattice}
\]

\[
\text{However, the initial lattice}
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\text{time dependence of the ratio}
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Do we really want to adiabatically eliminate?

Why do wiggles work so well?