AN INTRODUCTION CONTROL +

QUANTUM CIRCUITS AND “COHERENT” QUANTUM FEEDBACK CONTROL

Josh Combes

The Center for Quantum Information and Control,
University of New Mexico
Quantum feedback control
- model imperfections (inefficient detections, time delays, control field etc.)
- purification / stabilization / rapid measurement
- optimal control (HJB eqn)

Parameter estimation & tomography
- frequentist and Bayesian
- adaptive
- sample or time efficient

Open loop control
+ conditional measurements

Quantum limited amplifiers
- linear
- "noiseless"

Wavepacket Fock states
- master equations / stochastic master equations
- Scattering (S-matrix)
Assumptions:
1. All quantum control can be represented and understood through quantum circuits
2. If you draw these quantum circuits, patterns emerge (flow of information).

Therefore: we can categorize and understand quantum control protocols based on the flow of information.

(I) Introduction to control theory

(II) Quantum measurements & trajectories

(II) Categorizing quantum control with circuits
   Open loop control
   Measurement & coherent feedback control
   Non commutative quantum control
Collaborators

Carlton M. Caves
University of New Mexico, US

Gerard J. Milburn
University of Queensland, Australia

Everything

Non commutative control
What is control theory?

The study of dynamical systems, with inputs, in order to manipulate them in a desired way.
What is control theory?

The study of dynamical systems with inputs, in order to manipulate them in a desired way.

Some history:

Controllers:

Tesibius of Alexandria (285–222 BC)

Theory:


The following communications were read:—

   Received Feb. 20, 1868.

A Governor is a part of a machine by means of which the velocity of the machine is kept nearly uniform, notwithstanding variations in the driving-power or the resistance.
Types of controllers

Cooling a room to certain temperature at a single time is pointless.
Types of controllers

- Setpoint
- Monitoring
- Actuator

Average Daytime Temperature

Setpoint

Morning Lunch Afternoon

Airconditioner Output Temperature

Setpoint

Morning Lunch Afternoon

Deterministic
Types of controllers

Self regulating

• Control relative to setpoint or “system state”
• if T>23 cool for 1 min
• if T<23 heat for 1 min

Triggered / Event driven

• if event X occurs do Y
• e.g.
  IF door is open for > 10 seconds AND outside temp>30
  THEN output 16 degree air for 1 min.
What is an actuator?

Trick: choose input to represent desired output

Input

Actuator

System

Output

Plant /System

Limited control

Electricity

Air

Air

Air inside room

Disturbance

voltage \propto a set of possible actions

particular action

voltage \propto temperature

Thursday, January 10, 13
Types of controllers

**Deterministic**  
**Open Loop Control**
- system evolution is nearly deterministic
- unstable when disturbed
- e.g. washing machine, irrigation sprinklers

**Triggered / Event driven**  
**Feedforward Control**
- system evolution is nearly deterministic
- disturbances can be detected
- the effect of disturbance on the system is well characterized

**Self regulating**  
**Feedback Control**
- system evolution can be anything
- system state can be monitored
- feedback delay is small

**Learning/Adaptive/Intelligent Control**
- system evolution can be anything
- multiple trials
- algorithm must be trained
- if the noise has not been seen before any single run can be bad
Representing any Q-systems with Q-circuits

The Liberal interpretation of quantum circuits:

★ Finite / infinite dimensional
★ A collection of systems / Mode
★ etc.

\[ U \]

\[ \rho \]

\[ \sigma \]

\[ \sigma_N \]

\[ \sigma_3 \]

\[ \sigma_2 \]

\[ \sigma_1 \]
Translating schematics into Q-circuits

Dictionary:

(a) (c) (e) (g)
(b) (d) (f) (h)

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Translating schematics into Q-circuits

Dictionary:

(a) 
(b) 
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(i) 
(j)
Translating schematics into Q-circuits

Dictionary:

Diagram:
(II) Quantum Measurements & Trajectories

Assumptions:
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Therefore: we can categorize and understand quantum control protocols based on the flow of information.
Conditional measurements in the circuit model

\[ \Pi_{k_1} = |k_1\rangle\langle k_1|, \quad k_1 \in [0, 1] \]
\[ M_{k_1} = \langle k_1|U_I|\psi_1\rangle \]

SWAP \(|\phi\rangle \otimes |\psi\rangle = |\psi\rangle \otimes |\phi\rangle\)
Relation to the quantum trajectory description (1)

\[ H_{\text{int}}^{(k)} = i\Theta(\sigma_- \otimes \sigma_+^{(k)} - \sigma_+ \otimes \sigma_-^{(k)}) \]

\[ \sigma_\pm^{(k)} \equiv I \otimes (k-1) \sigma_\pm I \otimes (N-k) \]

Relation to the quantum trajectory description

\[ H^{(k)}_{\text{int}} = i \Theta (\sigma_- \otimes \sigma_+^{(k)} - \sigma_+ \otimes \sigma_-^{(k)}) \]

\[ \sigma_\pm^{(k)} \equiv I \otimes (k-1) \sigma_\pm \otimes (N-k) \]

\[ U = \exp[\theta (\sigma_- \otimes \sigma_+ - \sigma_+ \otimes \sigma_-)] \quad \theta = \Theta t, \quad \theta \ll 1 \]

\[ U = I \otimes I + \theta (\sigma_- \otimes \sigma_+ - \sigma_+ \otimes \sigma_-) - \frac{1}{2} \theta^2 (\sigma_+ \sigma_- \otimes \sigma_- \sigma_+ + \sigma_- \sigma_+ \otimes \sigma_+ \sigma_-) \]

Relation to the quantum trajectory description (2)

\[ M_0 = \langle g | U | g \rangle = I - \frac{1}{2} \theta^2 \sigma_+ \sigma_- \]
\[ M_1 = \langle e | U | g \rangle = \theta \sigma_- \]
\[ M^\dagger_0 M_0 + M^\dagger_1 M_1 = I \]
\[ \delta t = \theta^2 / N \]

Binomial approximation on the denominator
\[ (1 - \frac{1}{2} \delta t \text{Tr}[\sigma_+ \sigma_- + \rho \sigma_+ \sigma_-])^{-1} \approx 1 + \frac{1}{2} \delta t \text{Tr}[\sigma_+ \sigma_- + \rho \sigma_+ \sigma_-] \]

\[
\rho_0(t + \delta t) = \frac{M_0 \rho M_0^\dagger}{\text{Tr}[M_0^\dagger M_0 \rho]} = \frac{\rho - \frac{1}{2} \delta t \sigma_+ \sigma_- \rho - \frac{1}{2} \delta t \rho \sigma_+ \sigma_-}{1 - \frac{1}{2} \delta t \text{Tr}[\sigma_+ \sigma_- + \rho \sigma_+ \sigma_-]}.
\]

\[
\delta \rho_0 = \rho_0(t + \delta t) - \rho_0(t) = -\frac{1}{2} \delta t \sigma_+ \sigma_- \rho - \frac{1}{2} \delta t \rho \sigma_+ \sigma_- + \frac{1}{2} \delta t \text{Tr}[\sigma_+ \sigma_- + \rho \sigma_+ \sigma_-] \rho
\]

\[
\delta \rho_1 = \rho_1(t + \delta t) - \rho_1(t) = \frac{\sigma_- \rho \sigma_+}{\text{Tr}[\sigma_+ \sigma_-]} - \rho
\]

\[
\delta \rho = dN \left( \frac{\sigma_- \rho \sigma_+}{\text{Tr}[\sigma_+ \sigma_-]} - \rho \right) + (1 - dN) \frac{1}{2} \delta t (-\sigma_+ \sigma_- - \rho \sigma_+ \sigma_- + \text{Tr}[\sigma_+ \sigma_- + \rho \sigma_+ \sigma_-] \rho)
\]

\[ dN \in \{0, 1\} \]

\[ \rho(t + \delta t) = \rho(t) + \delta \rho \]
\[ \delta \rho = dN \left( \frac{\sigma_- \rho \sigma_+}{\text{Tr}[\sigma_+ \sigma_- \rho]} - \rho \right) + (1 - dN) \frac{1}{2} \delta t \left(-\sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_- + \text{Tr}[\sigma_+ \sigma_- \rho + \rho \sigma_+ \sigma_-] \rho \right) \]
Relation to the quantum trajectory description

\{\ket{g}, \ket{e}\}

Other field states are equally easy to deal with e.g.

\[ |\tilde{\alpha}\rangle = (1 + \delta t \alpha \sigma_+) |g\rangle \]

\[ |1_\xi\rangle = \xi_1 |ggg\ldots g\rangle + \xi_2 |gge\ldots g\rangle + \xi_3 |gge\ldots e\rangle + \cdots + \xi_N |ggg\ldots e\rangle \]
(II) Categorizing quantum control with circuits

Open loop control
Measurement & coherent feedback control
Non commutative quantum control

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Open loop control

Classically controlled unitaries:
if \( k_i \) apply \( U_{k_i} \)
Measurement based feedback control

- **Estimator**: $r$ (the desired output aka setpoint)
- **Controller**: $y$ (output)
- **Plant**: $x$ (input)
- **Disturbances**: $u$ (control)
- **Measurements**: $y_m$ (measured variable)
- **Error/Innovations**: $e = r - y_m$ (error/innovations)
Measurement based feedback control

$r$ (the desired output aka setpoint)

$y_m$ (measured variable)

$e = r - y_m$ (error / innovations)

$y$ (output)

$x$ (input)

Plant

Measurements

Estimator

Controller

Probes $|\psi\rangle_1$, $|\psi\rangle_2$, $|\psi\rangle_3$

$M_{K_1}$, $M_{K_2}$

$U_1$, $U_{k_1}$, $U_{k_2}$

$\rho$, $\rho_{k_1,k_2}$

Classically controlled unitaries: if $k_i$ apply $U_{k_i}$

FIG. 14: proportional, direct, or Wiseman–Milburn type “Hamiltonian” feedback.
Measurement based feedback control

FIG. 14: proportional, direct, or Wiseman–Milburn type “Hamiltonian” feedback.

FIG. 15: Information flow in proportional Hamiltonian feedback.
Bayesian / State based control
“Coherent” feedback control

FIG. 14: proportional, direct, or Wiseman–Milburn type “Hamiltonian” feedback.

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Principles of deferred measurement [Wiseman & Milburn 94, Griffiths & Niu 96]:
(1) Measurement can always be moved from an intermediate state of an evolution (circuit) to the end of an evolution (circuit).
(2) If the measurement results are used at any stage of the evolution (circuit) then the classically controlled operations can be replaced by conditional coherent quantum operations.

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Coherentized feedback control

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback, Phys. Rev. A 49 4110 (1994)

FIG. 14: proportional, direct, or Wiseman–Milburn type “Hamiltonian” feedback.

Coherentized proportional, direct, or Wiseman–Milburn type “Hamiltonian” feedback.
Example: coherent feedback control
Example: coherent feedback control

J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi,
Designing Quantum Memories with Embedded Control: Photonic Circuits for Autonomous Quantum Error Correction,
Example: coherent feedback control
Example: coherent feedback control

J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi,
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Non commutative quantum control

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,

“Complex amplitude feedback”

Principles of deferred measurement
[Wiseman & Milburn 94, Griffiths & Niu 96]

Non commutative / Complex amplitude

Represents control of both quadratures.

NB: must assume you have limited access to some part of the system or plant otherwise we can use measurement based quantum computation techniques and the distinction breaks down.
Principle of non commutative control [Wiseman & Milburn 94]:
Non commutative quantum control can always be approximated by approximate measurement of non commuting observables (e.g. Heterodyne measurements, which necessarily introduce additional vacuum fluctuations) then controlling unitaries off those measurements.
From the principle of non commutative control the measurement based approximation is:

\[ \exists l, f : [U_l, V_f] \neq 0 \quad \text{and} \quad |\langle f|l \rangle|^2 = \frac{1}{d} \quad \forall f, l \]
Non commutative quantum control

S. Lloyd
Coherent quantum feedback

We don’t care if the interaction unitary is or is not mediated by fields. We can still determine if non commutative quantum control is taking place.

examples of SWAP... not really non commutative control
Non commutative quantum control + sensing

Feedforward control

Coherent Quantum-Noise Cancellation for Optomechanical Sensors
M. Tsang and C. M. Caves

Feedback control

Advantages of Coherent Feedback for Cooling Quantum Oscillators
R. Hamerly and H. Mabuchi

Sensing

Achieving minimum-error discrimination of an arbitrary set of laser-light pulses
M. P. da Silva, S. Guha, Z. Dutton
arXiv:1208.5758

uses optimization & design principles from

Coherent quantum LQG control
H. I. Nurdin, M. R. James, and I. R. Petersen
Automatica 45, 1837 (2009).

optical application of

Ideal state discrimination with an O(1)-qubit quantum computer
R. Blume-Kohout, S. Croke, M. Zwolak
arXiv:1201.6625
END
Quantum control system design and performance

Information is Physical
-Landauer
Quantum control system design and performance

determined by physics and balanced by budget
Control system design and performance

- Mathematical model of plant
  - $n^{th}$ order linear Differential Equation (DE)
  - nonlinear DE e.g ODE or PDE
  - linear / nonlinear stochastic DE

- Controllability
- Stability

- Performance / Objectives
  - Steady state response
  - transient response

\[\text{determined by physics and balanced by budget}\]
Open loop control

\[ H = \gamma \Delta B \frac{\sigma_x}{2} \]
\[ \omega \in [0, \omega_0] \]

\[ \Delta B_{\text{nuc}} = B_{\text{nuc,L}} - B_{\text{nuc,R}} \]

\[ Z |S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2} \]

\[ |T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} \]

\[ U^{(t_0 + \tau)} \quad Z(\pi) \quad U^{(t_0 + 2\tau)} \]

Slowly varying