AN INTRODUCTION CONTROL +

QUANTUM CIRCUITS AND "COHERENT" QUANTUM FEEDBACK CONTROL

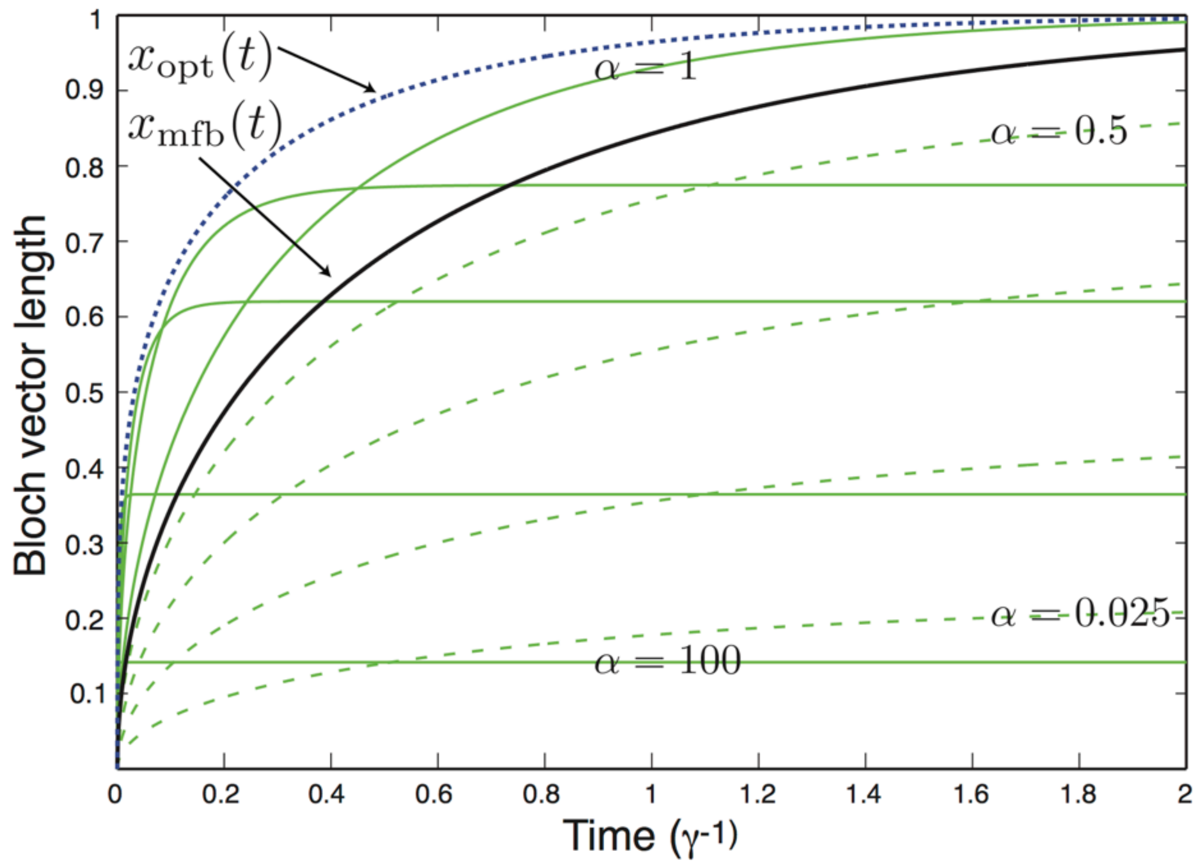
Josh Combes

The Center for Quantum Information and Control,
University of New Mexico

UNM

Quantum feedback control

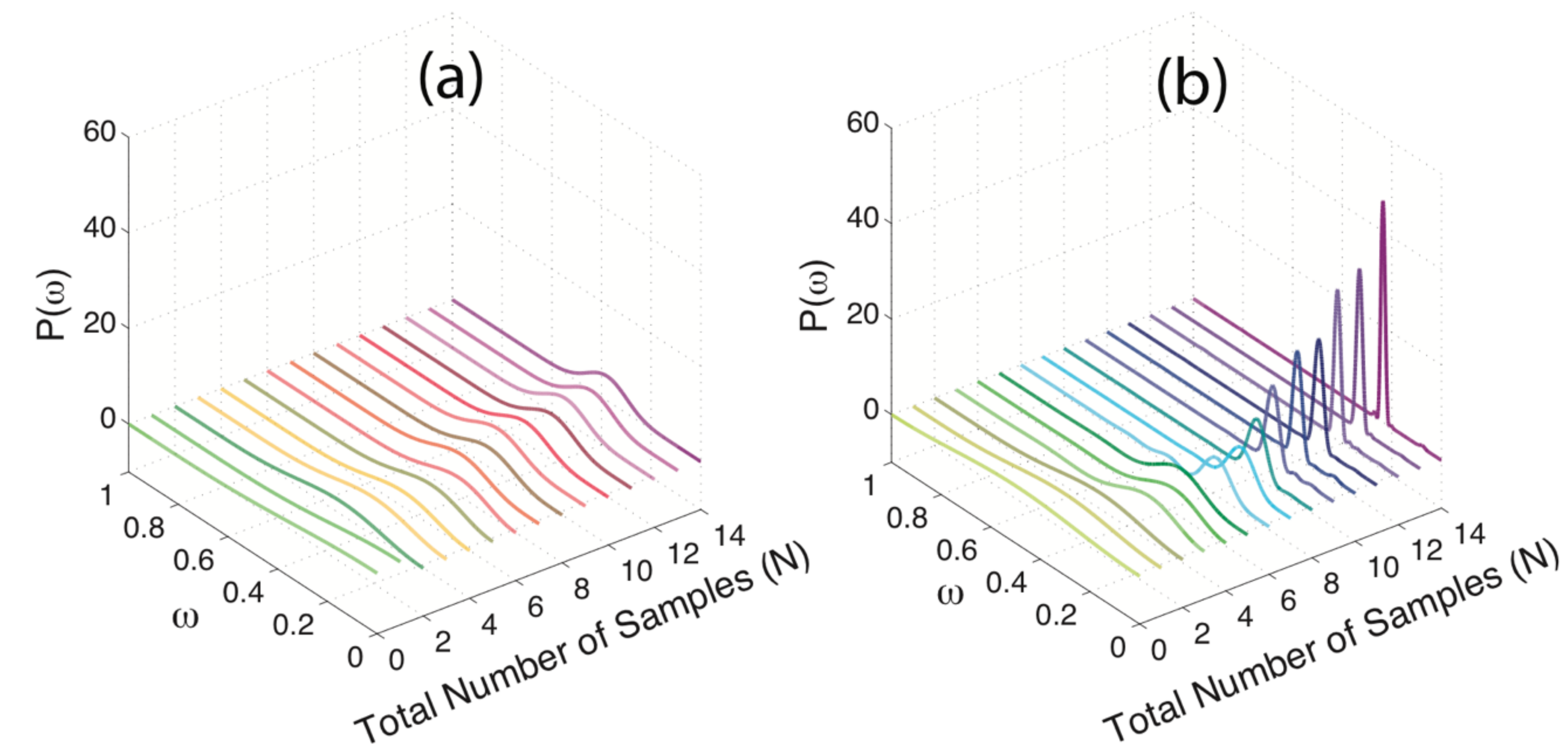
- model imperfections (inefficient detections, time delays, control field etc.)
- purification / stabilization / rapid measurement
- optimal control (HJB eqn)



Open loop control
+ conditional measurements

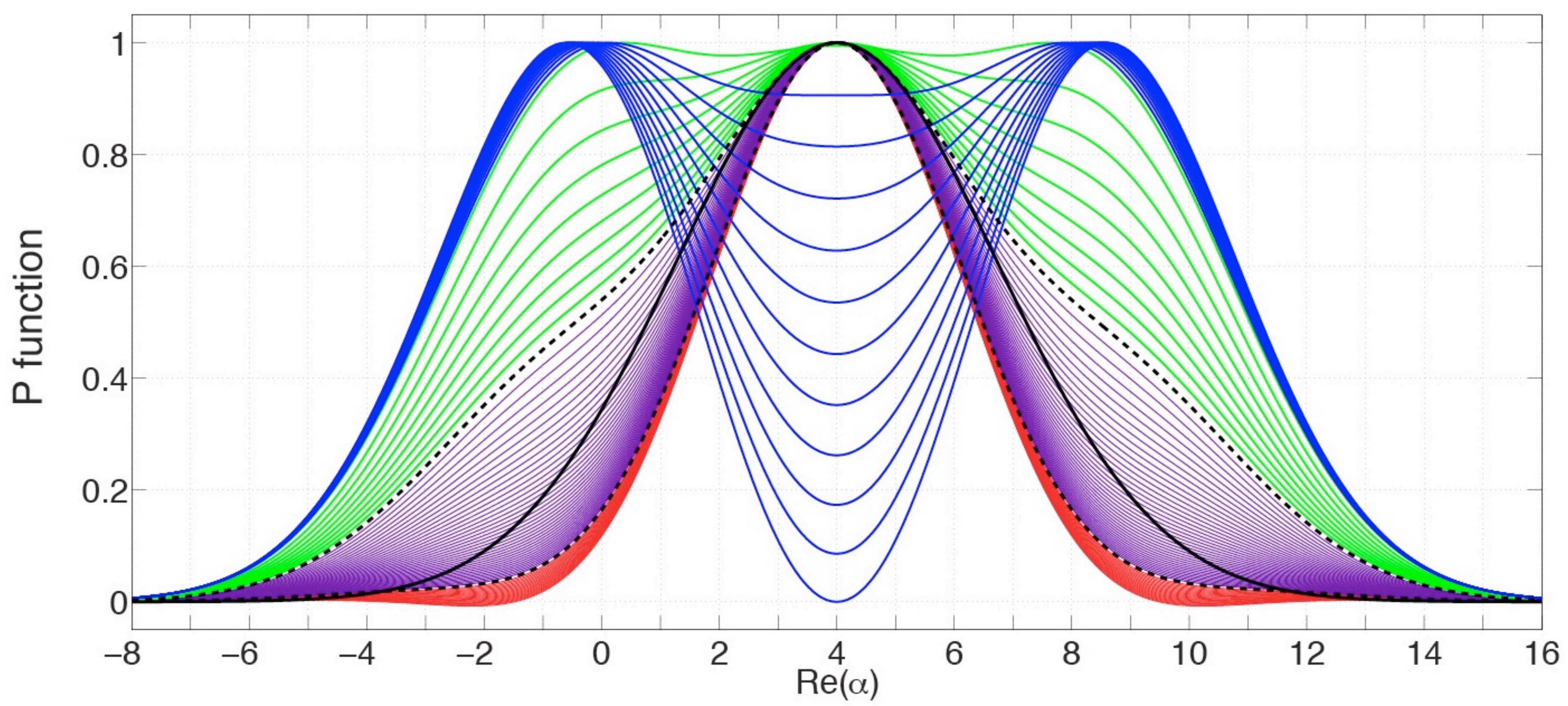
Parameter estimation & tomography

- frequentist and Bayesian
- adaptive
- sample or time efficient



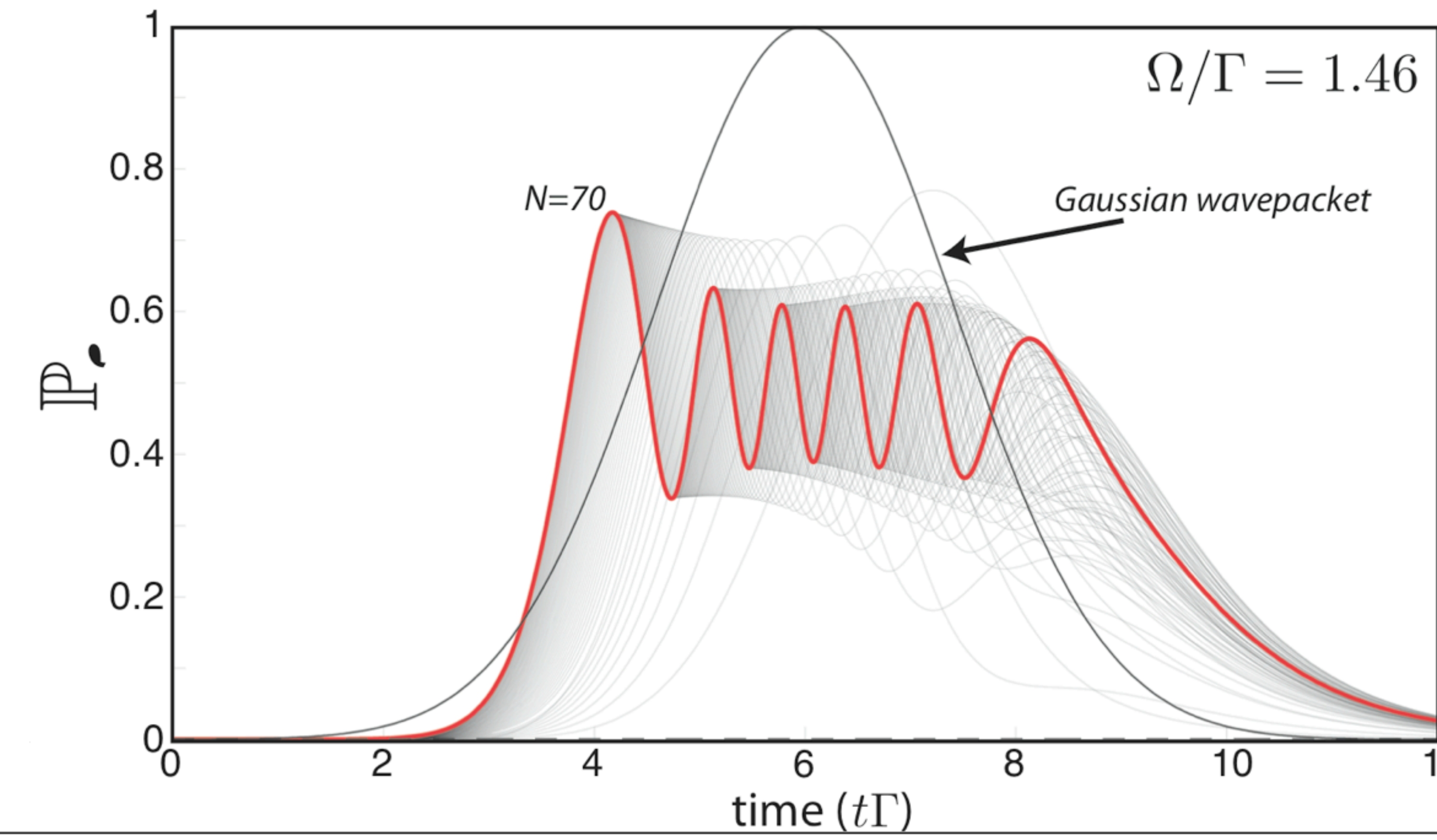
Quantum limited amplifiers

- linear
- ”noiseless”

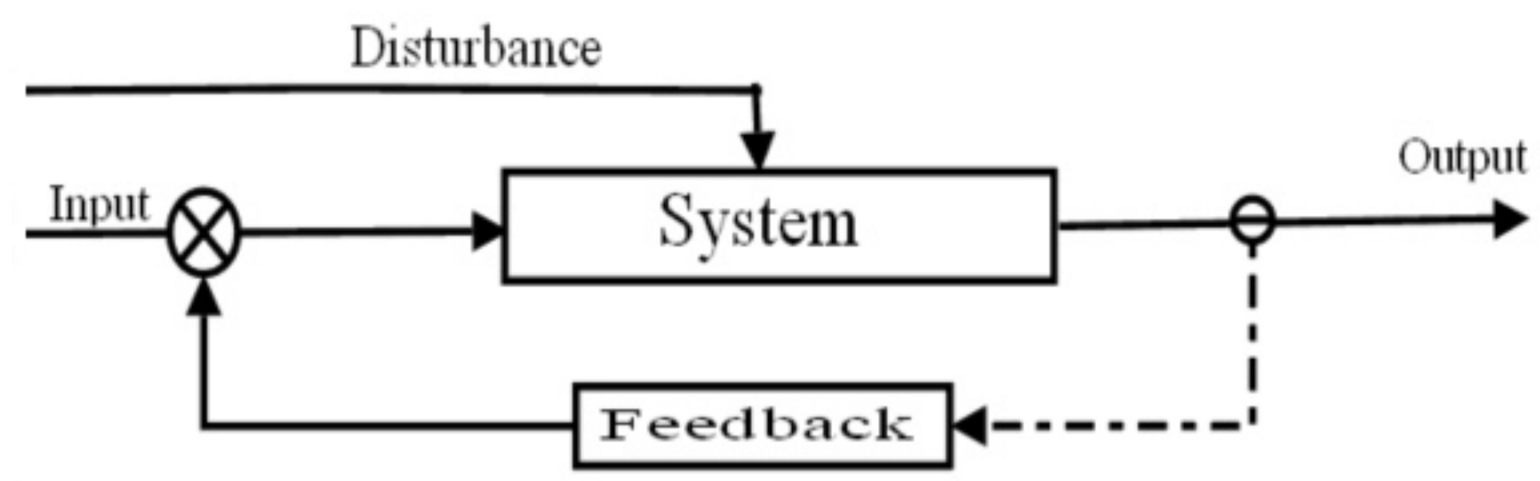


Wavepacket Fock states

-master equations / stochastic master equations
- Scattering (S-matrix)



(I) Introduction to control theory



Assumptions:

1. All quantum control can be represented and understood through quantum circuits
2. If you draw these quantum circuits, patterns emerge (flow of information).

Therefore: we can categorize and understand quantum control protocols based on the flow of information.

~~(II) Quantum measurements & trajectories~~

(II) Categorizing quantum control with circuits

Open loop control

Measurement & coherent feedback control

Non commutative quantum control

Collaborators

Carlton M. Caves
University of New Mexico, US

Everything

Gerard J. Milburn
University of Queensland, Australia

Non commutative control

What is control theory ?

The study of dynamical systems, with inputs,
in order to manipulate them in a desired way.

What is control theory ?

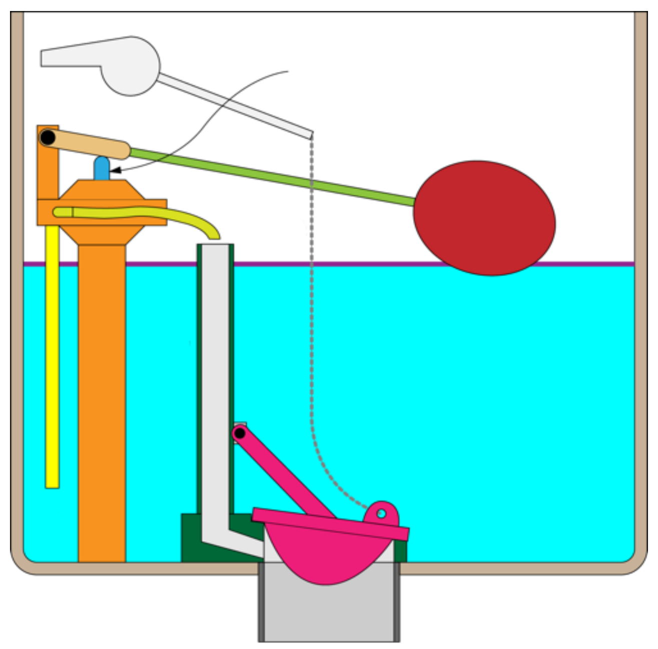
Additional slide
(not related to audio)

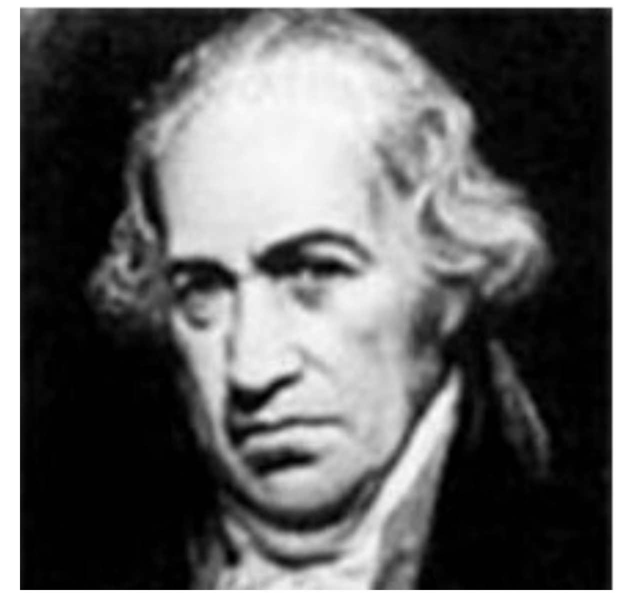
The study of dynamical systems with inputs, in order to manipulate them in a desired way.

Some history:

Controllers:

Tesibius of Alexandria
(285–222 BC)





James Watt
(1736 - 1819)

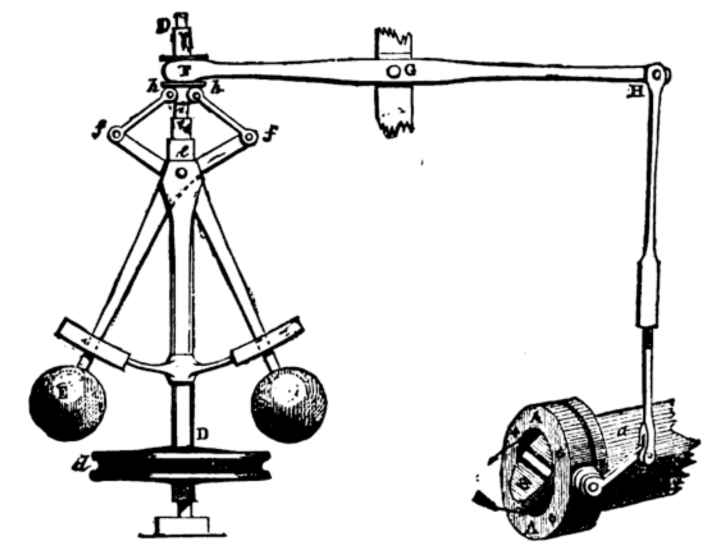
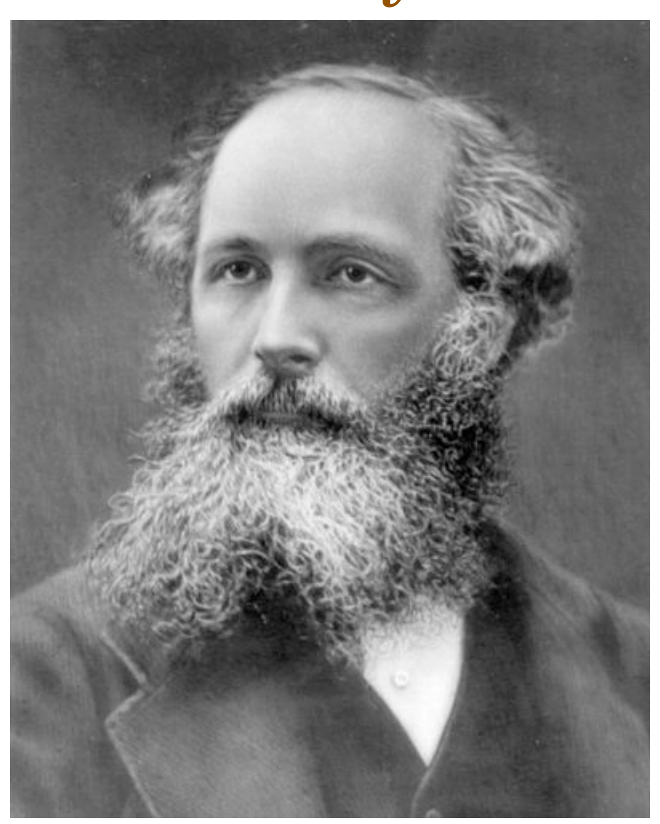


FIG. 4.—*Governor and Throttle-Valve.*

Theory:



James Clerk Maxwell
(1831 - 1879)

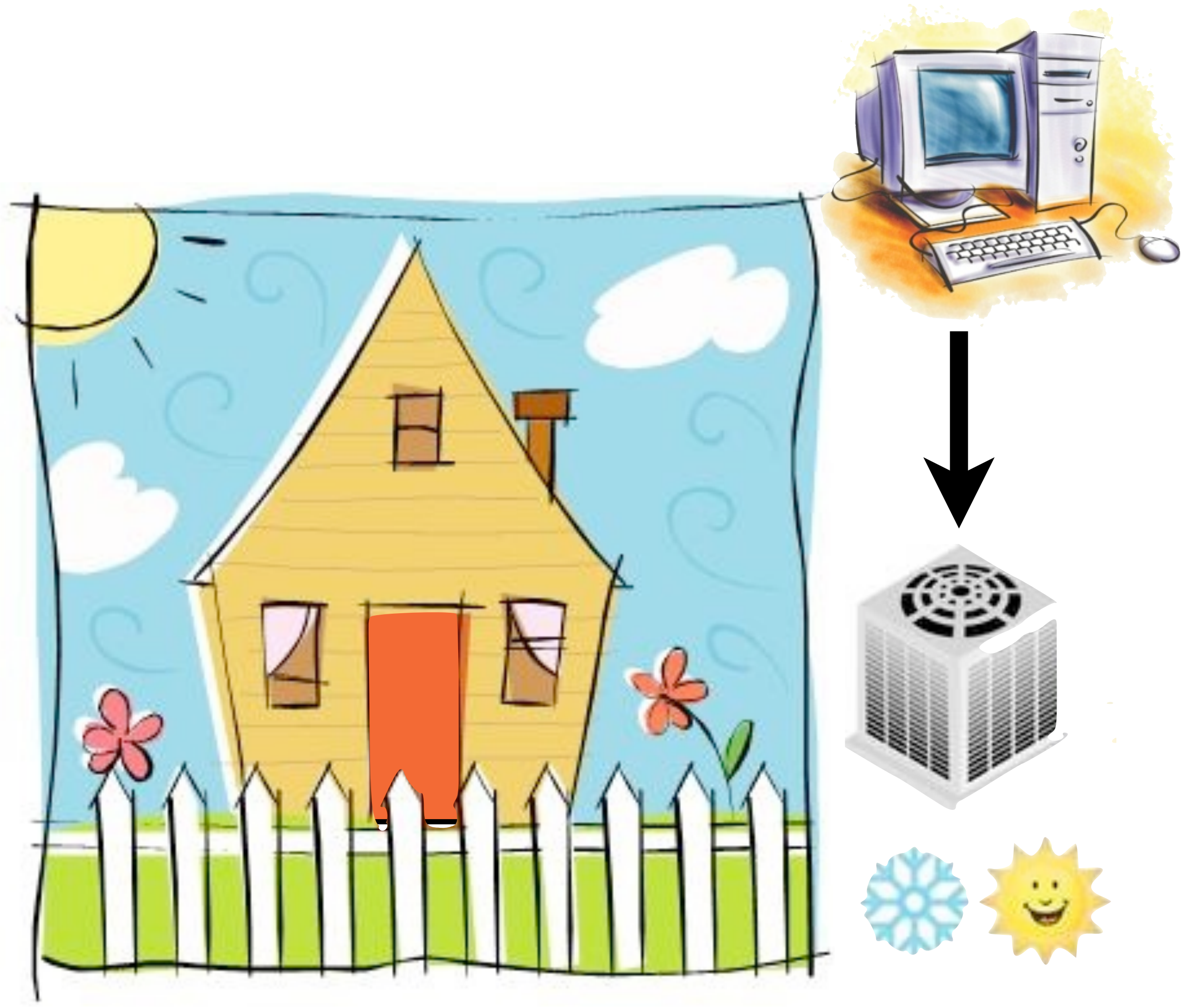
James Clerk Maxwell, On Governors, Proc. of Royal Soc. of London, **16**, 270 (1868).

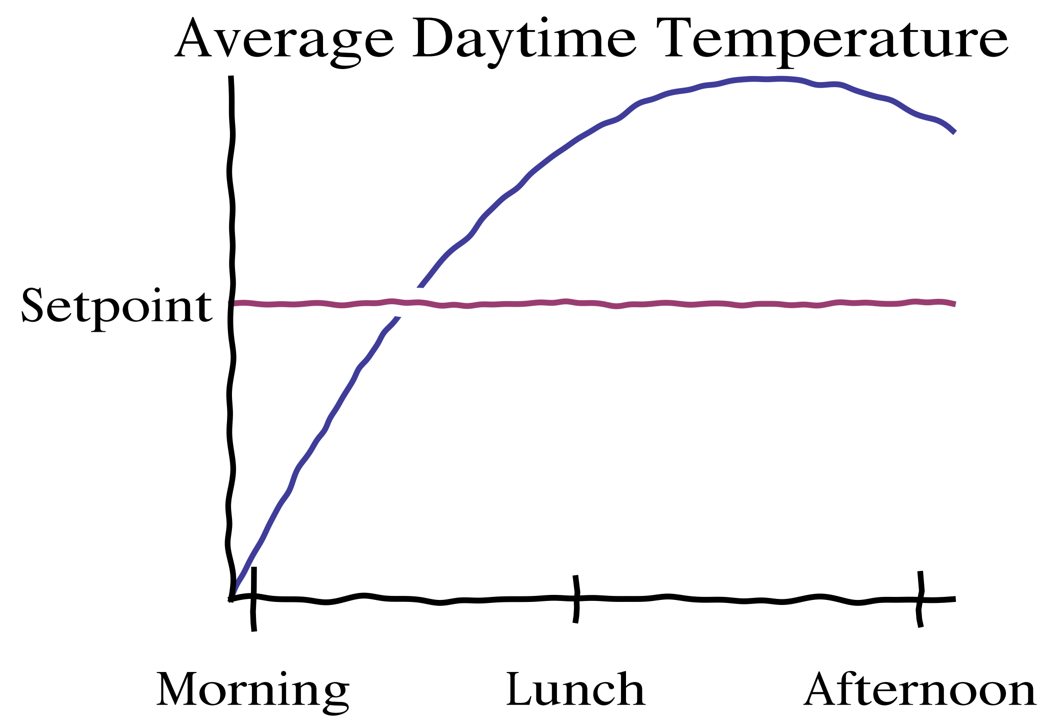
The following communications were read :—

I. " On Governors." By J. CLERK MAXWELL, M.A., F.R.SS.L. & E.
Received Feb. 20, 1868.

A Governor is a part of a machine by means of which the velocity of the machine is kept nearly uniform, notwithstanding variations in the driving-power or the resistance.

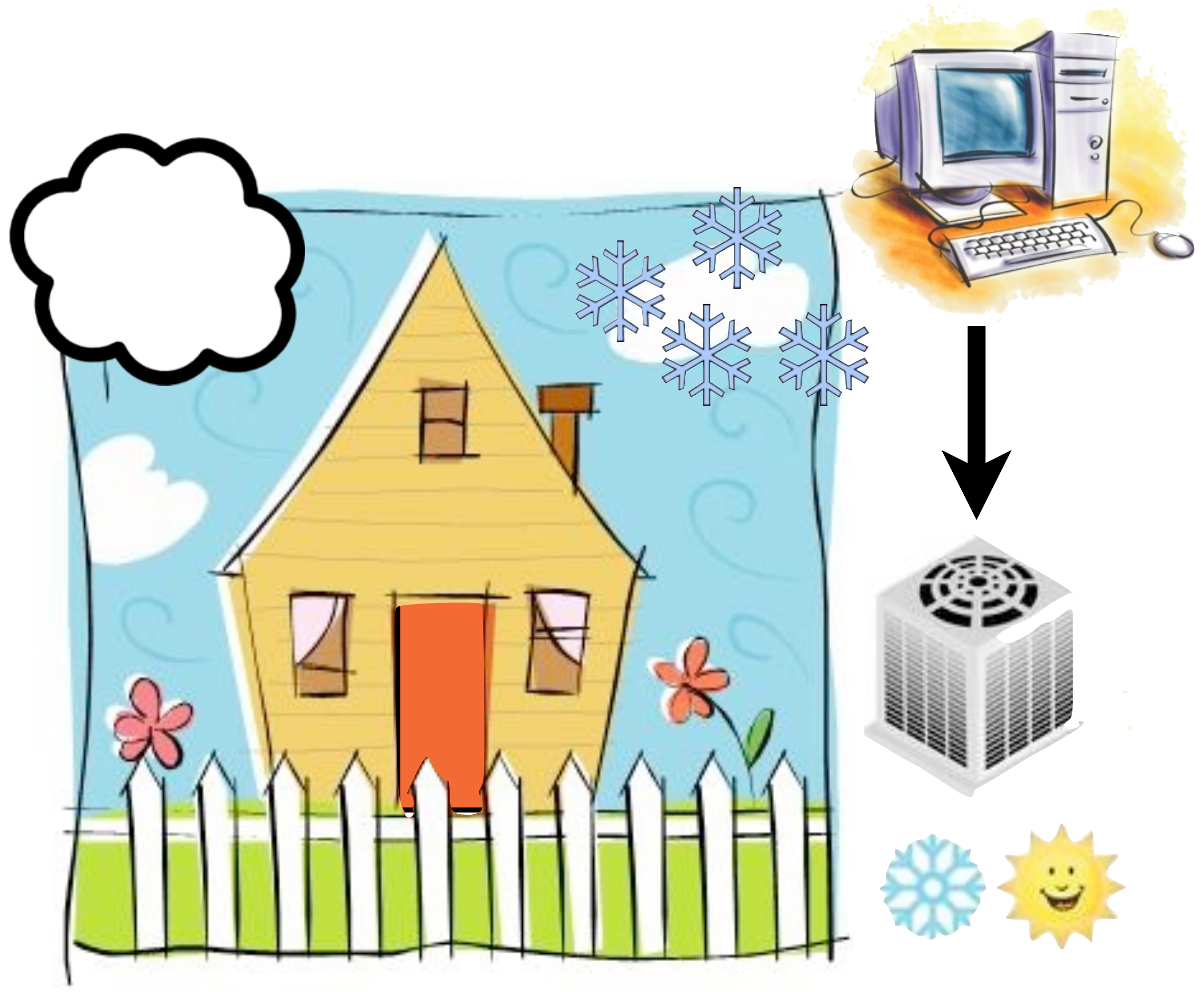
Types of controllers

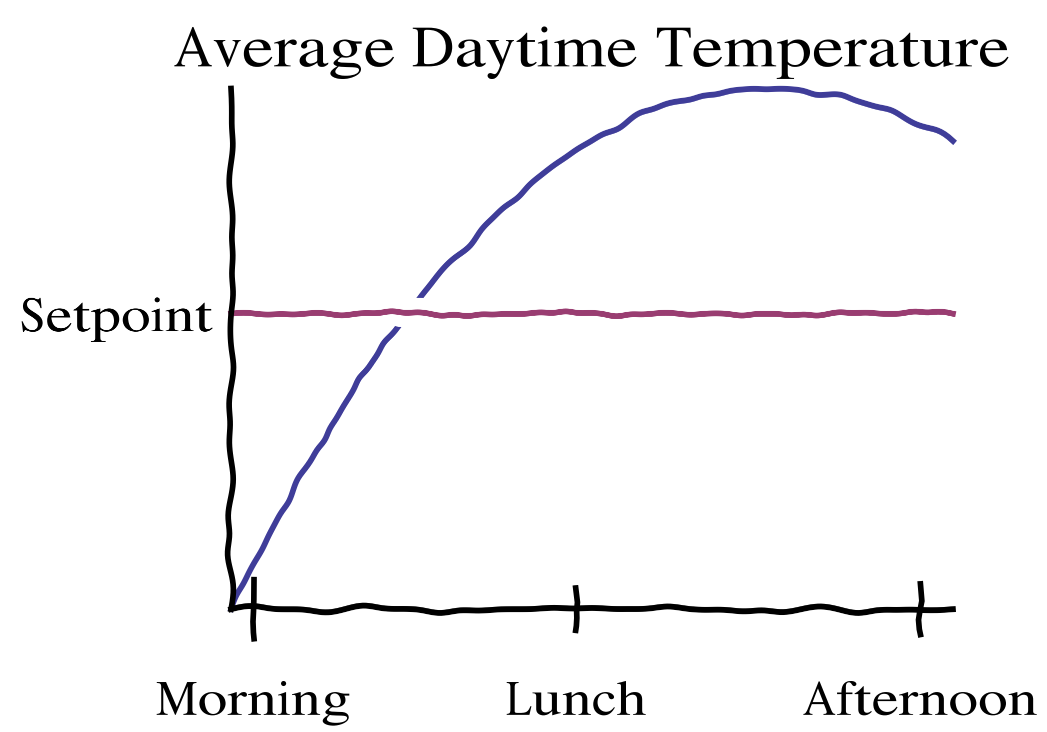




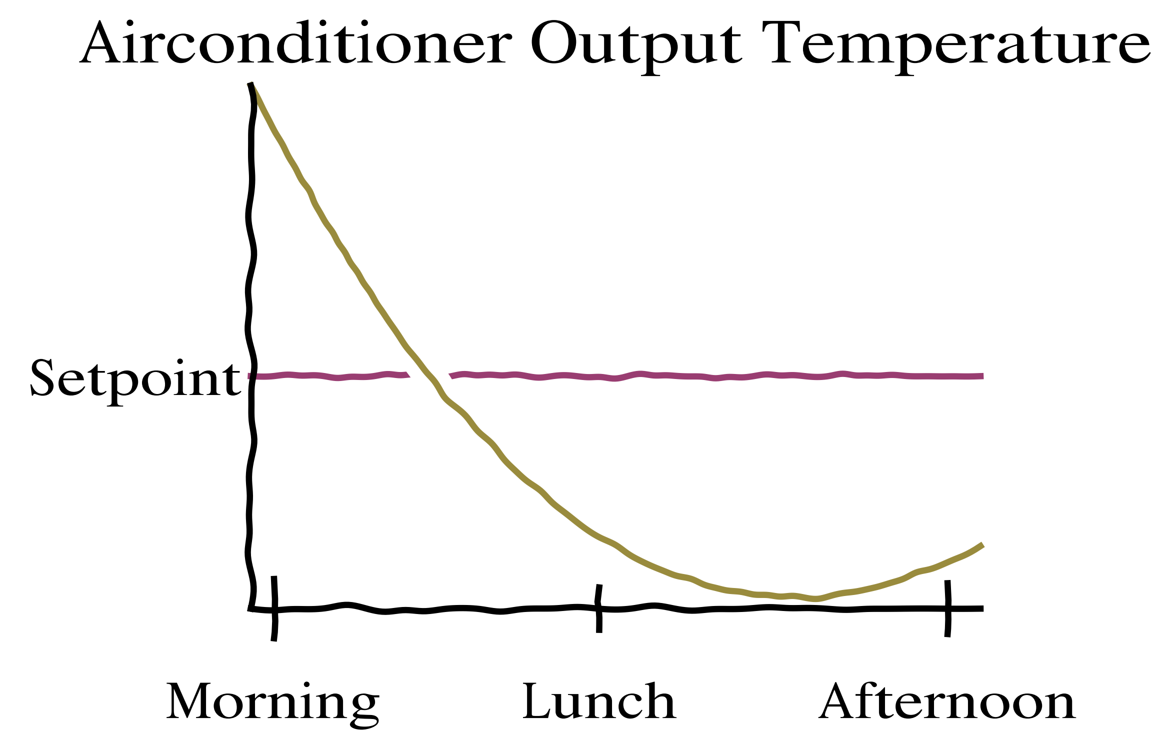
Cooling a room to certain temperature at a single time is pointless.

Types of controllers

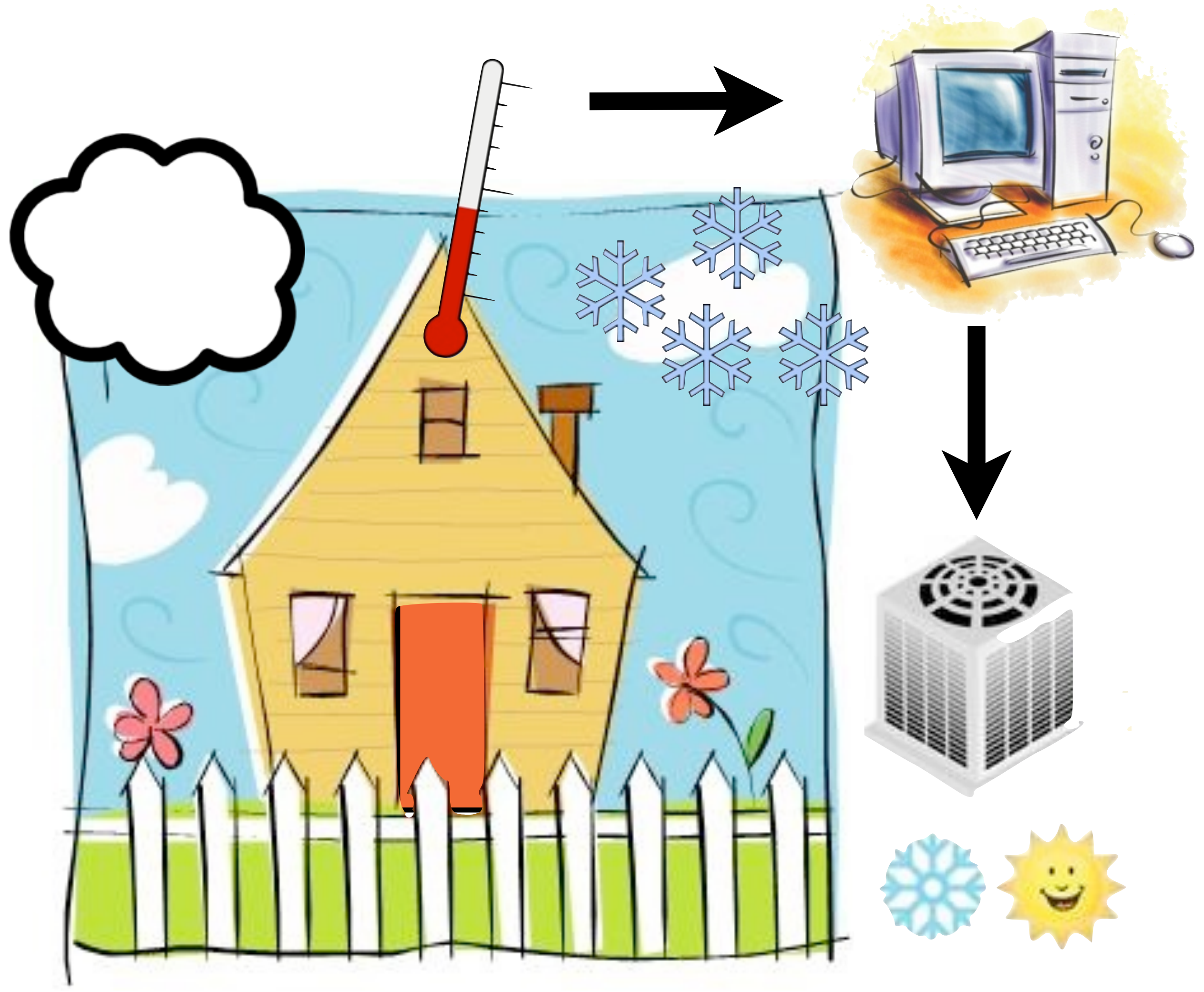


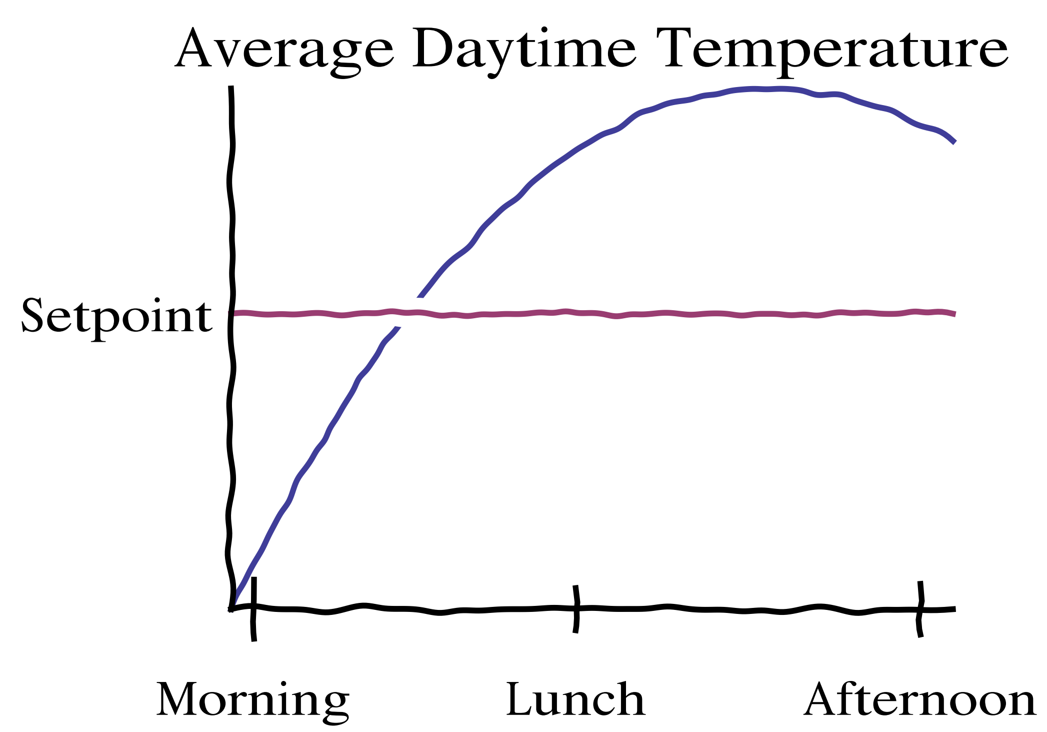


Deterministic

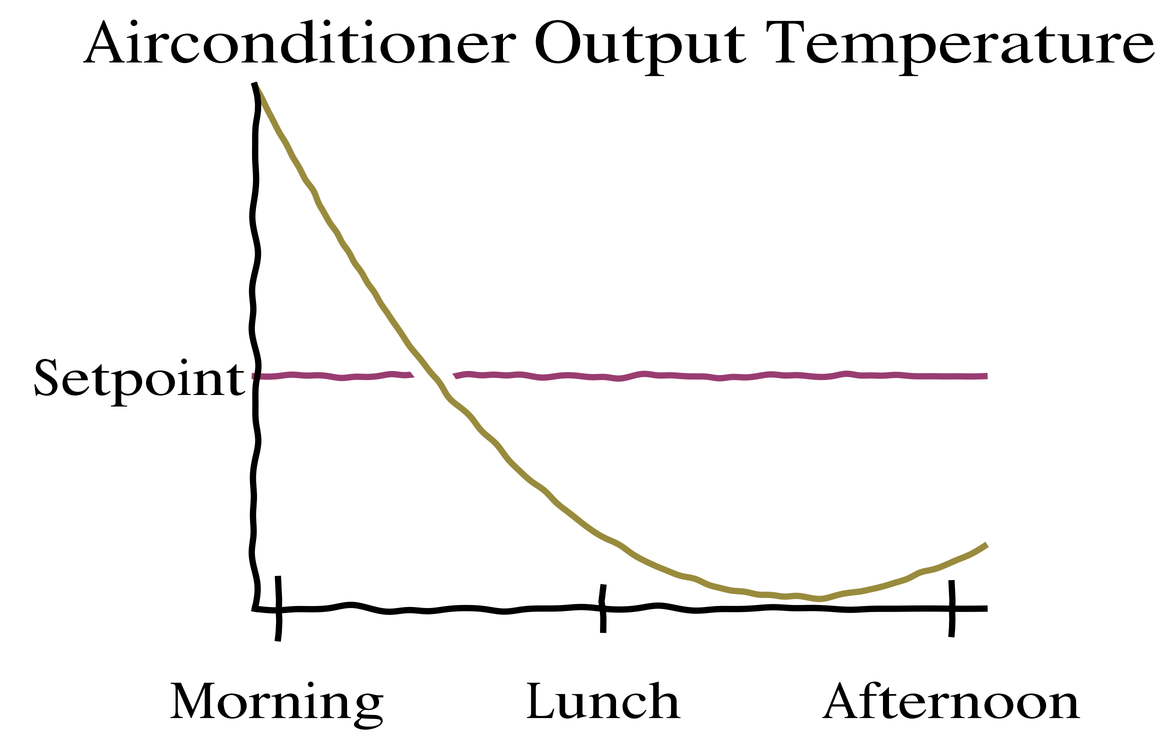


Types of controllers





Deterministic



Self regulating

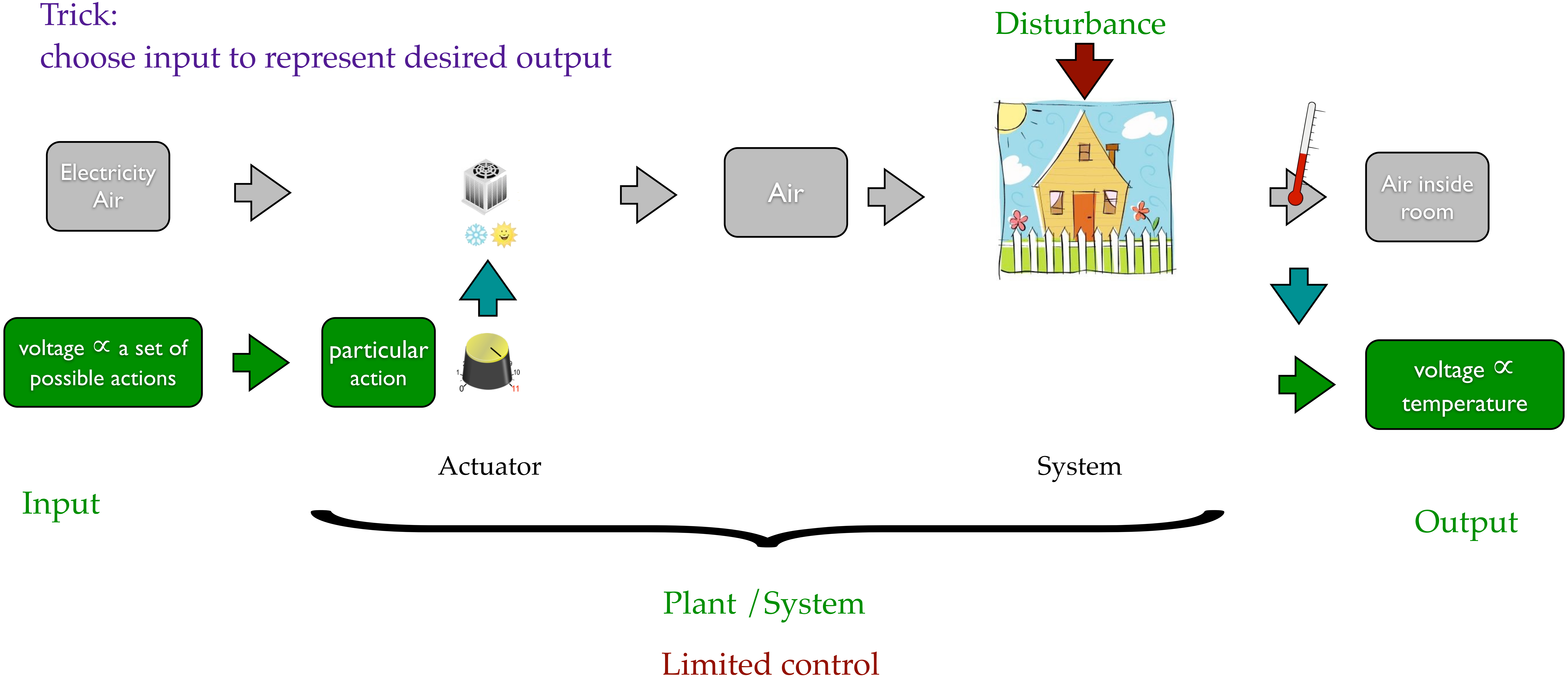
- Control relative to setpoint or “system state”
- if T>23 cool for 1 min
- if T<23 heat for 1 min

Triggered / Event driven

- if event X occurs do Y
- e.g.
  IF door is open for > 10 seconds AND outside temp>30
  THEN output 16 degree air for 1 min.

What is an actuator?

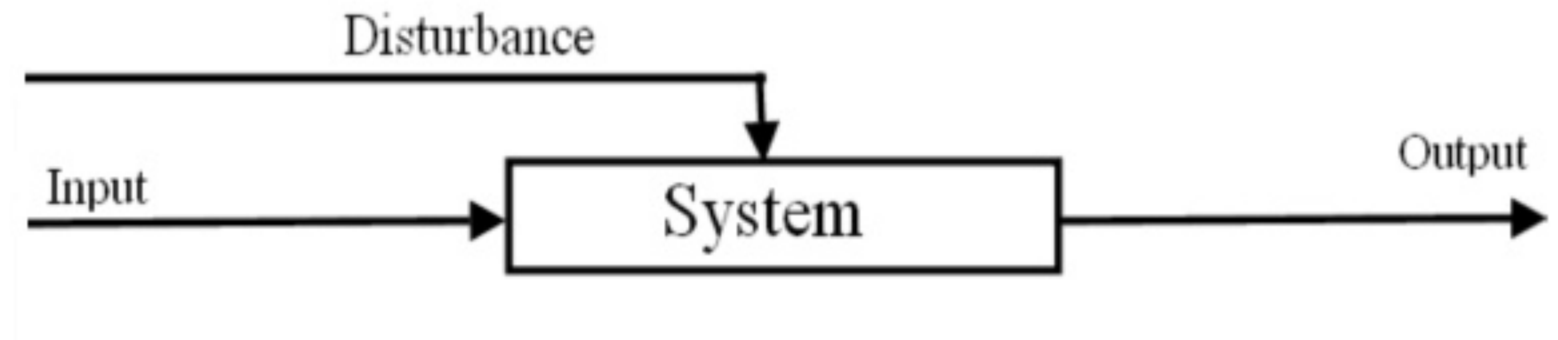
Trick:
choose input to represent desired output



Types of controllers

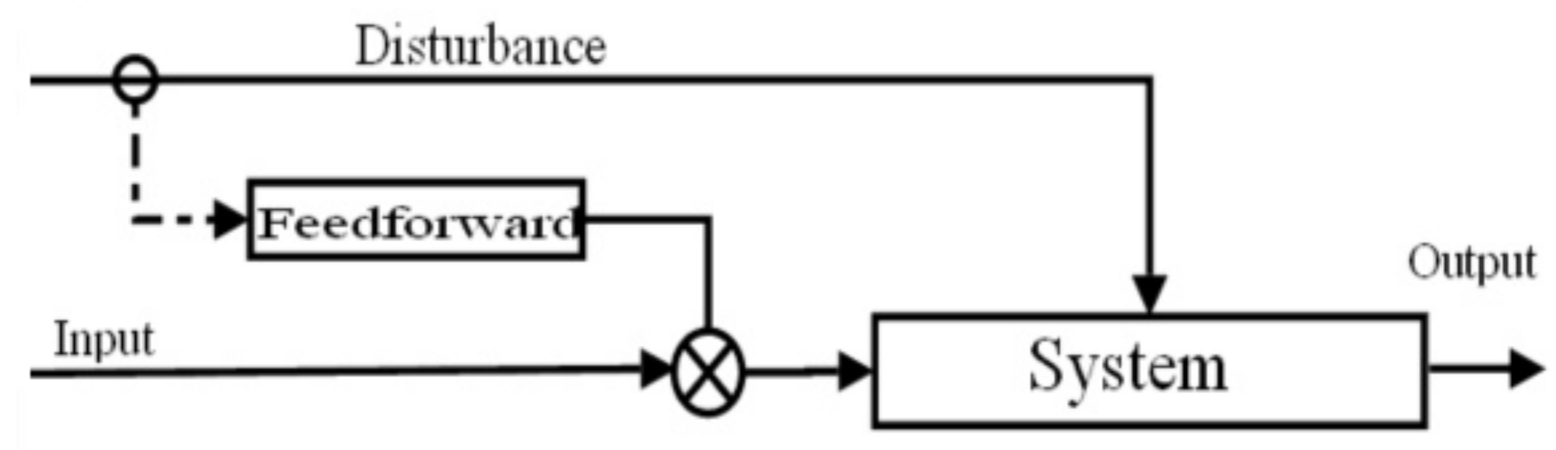
Deterministic Open Loop Control

- system evolution is nearly deterministic
- unstable when disturbed
- e.g. washing machine, irrigation sprinklers



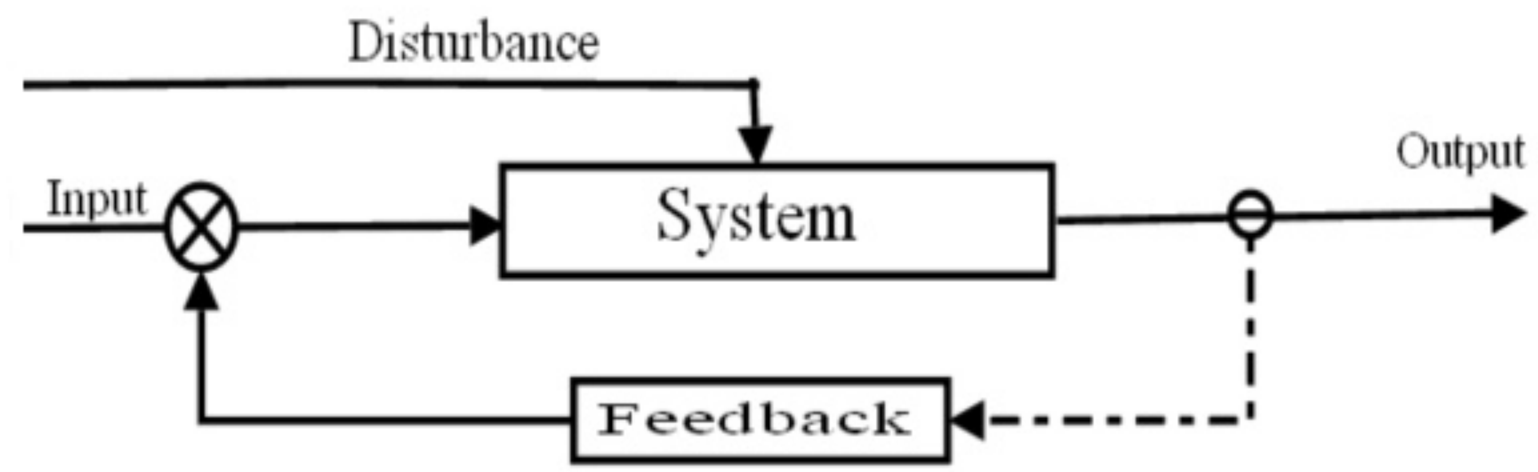
Triggered / Event driven Feedforward Control

- system evolution is nearly deterministic
- disturbances can be detected
- the effect of disturbance on the system is well characterized



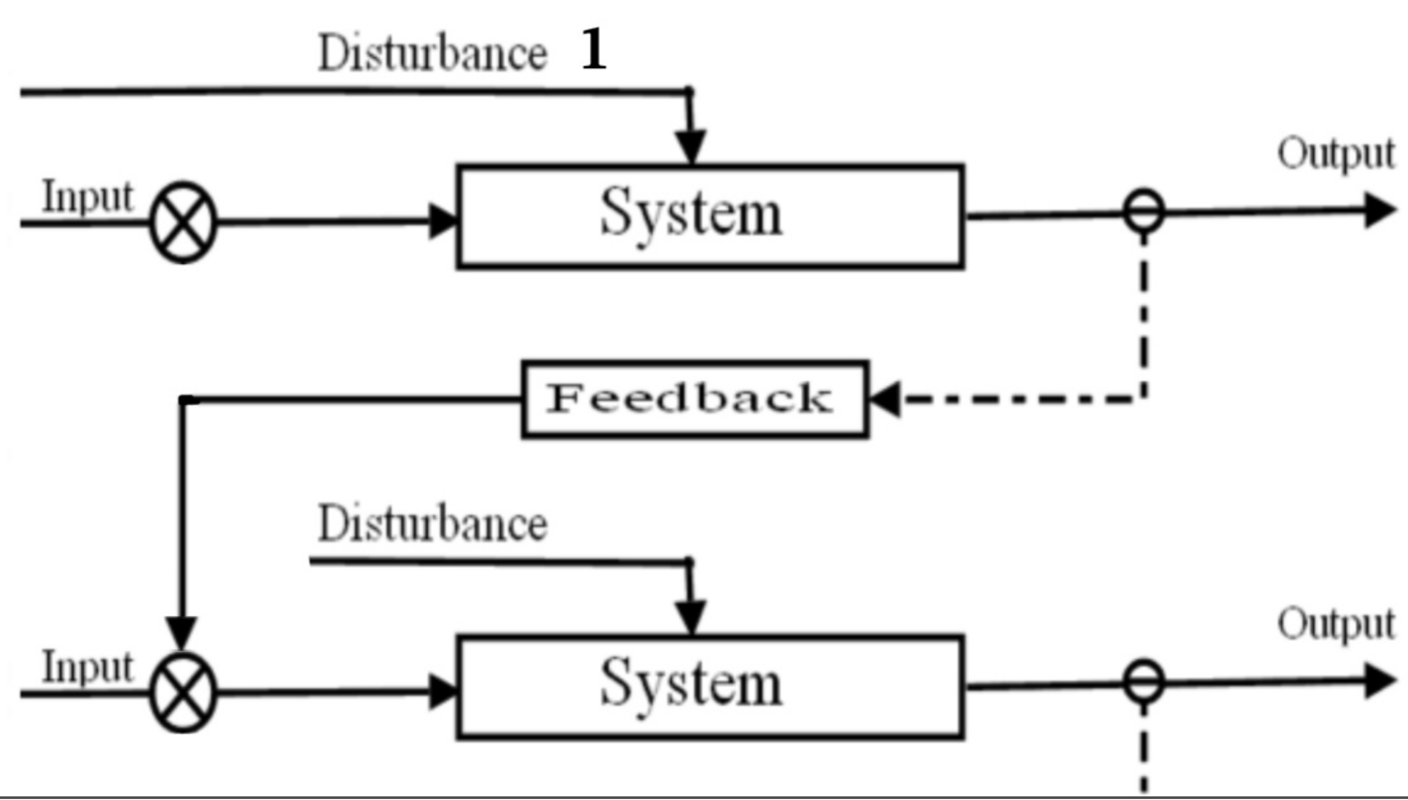
Self regulating Feedback Control

- system evolution can be anything
- system state can be monitored
- feedback delay is small



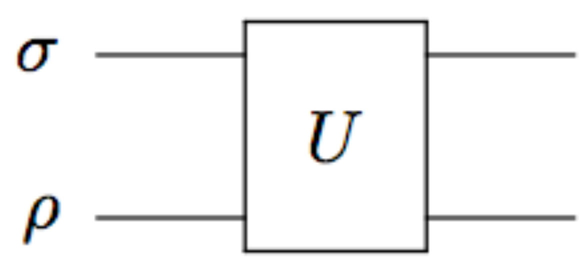
Learning / Adaptive / Intelligent Control

- system evolution can be anything
- multiple trials
- algorithm must be trained
- if the noise has not been seen before any single run can be bad

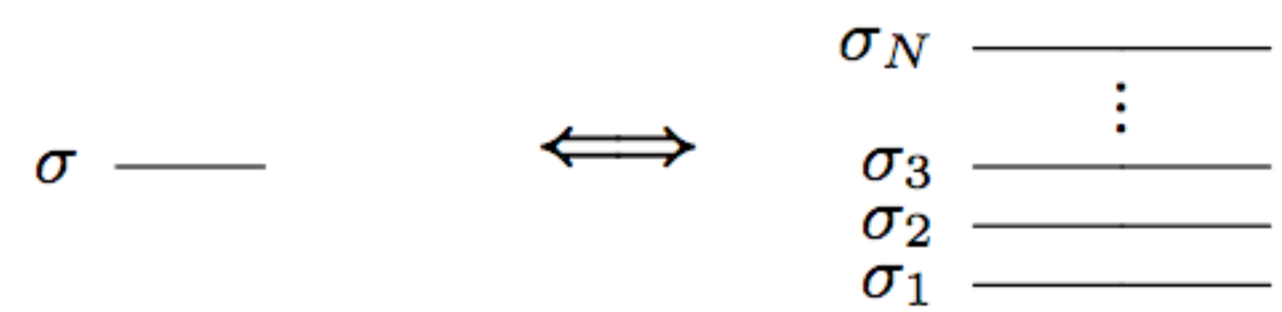


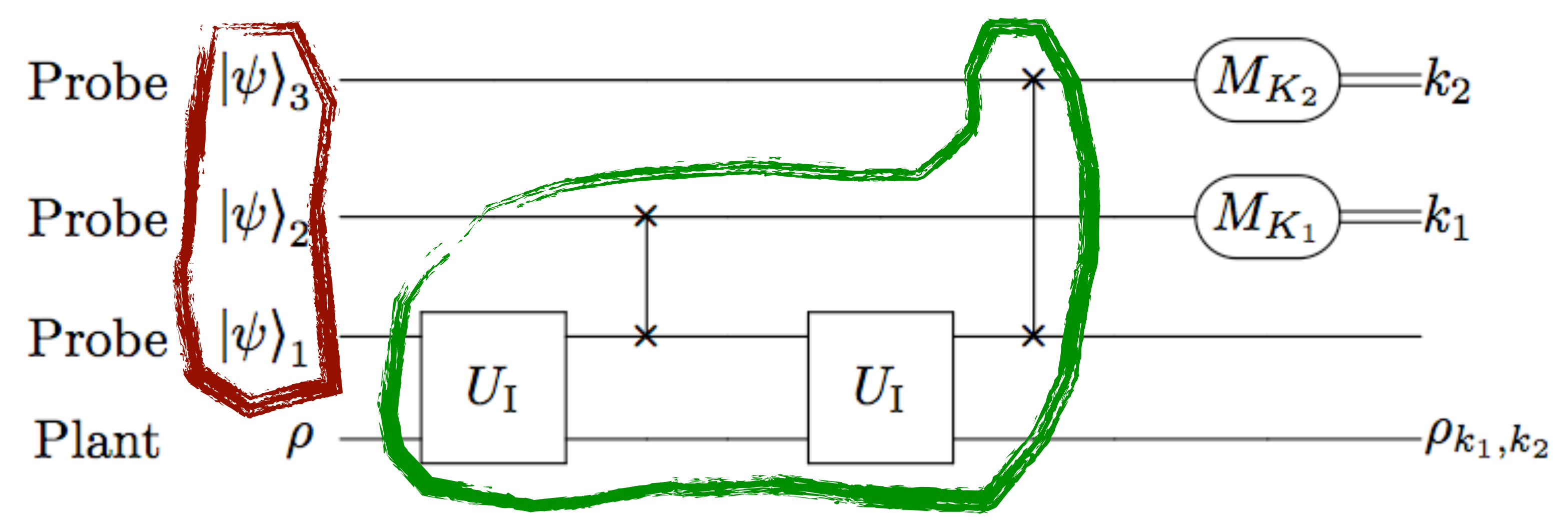
Representing any Q-systems with Q-circuits

The Liberal interpretation of quantum circuits:



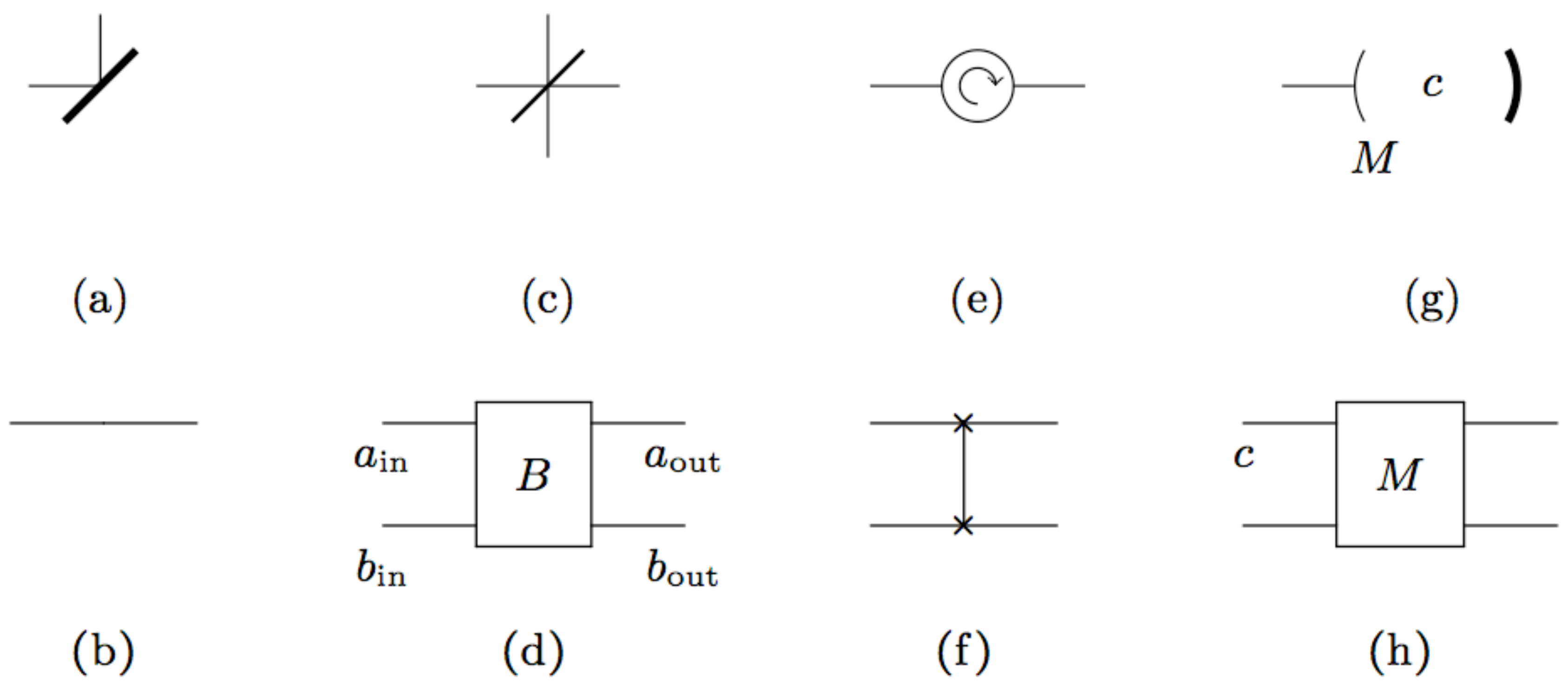
★ Finite / infinite dimensional
★ A collection of systems / Mode
★ etc.

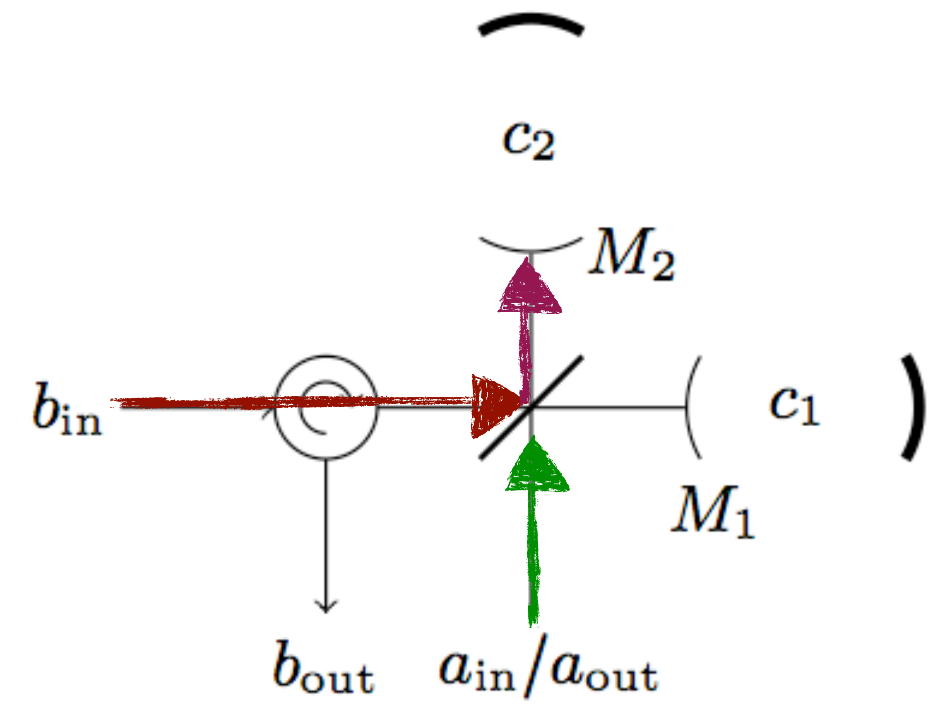


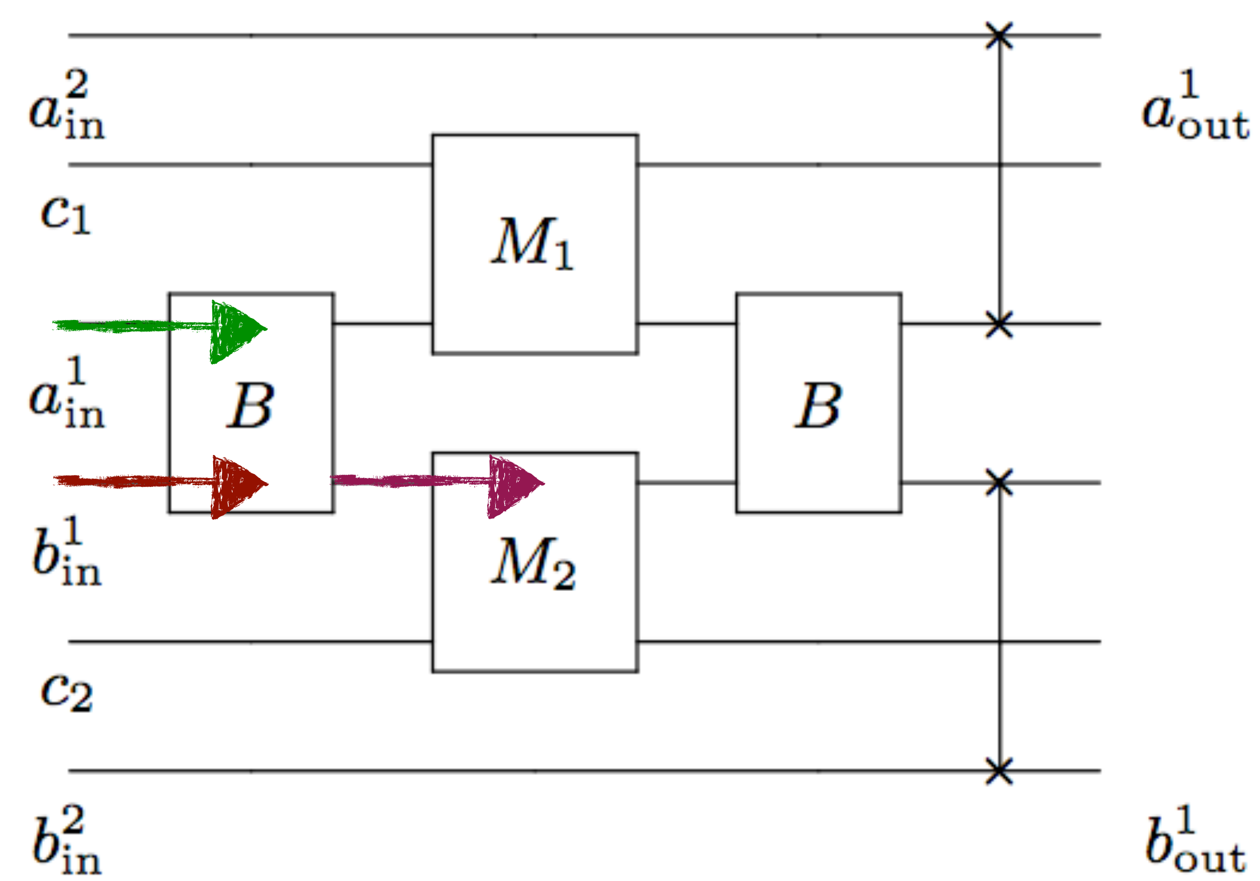


Translating schematics into Q-circuits

Dictionary:

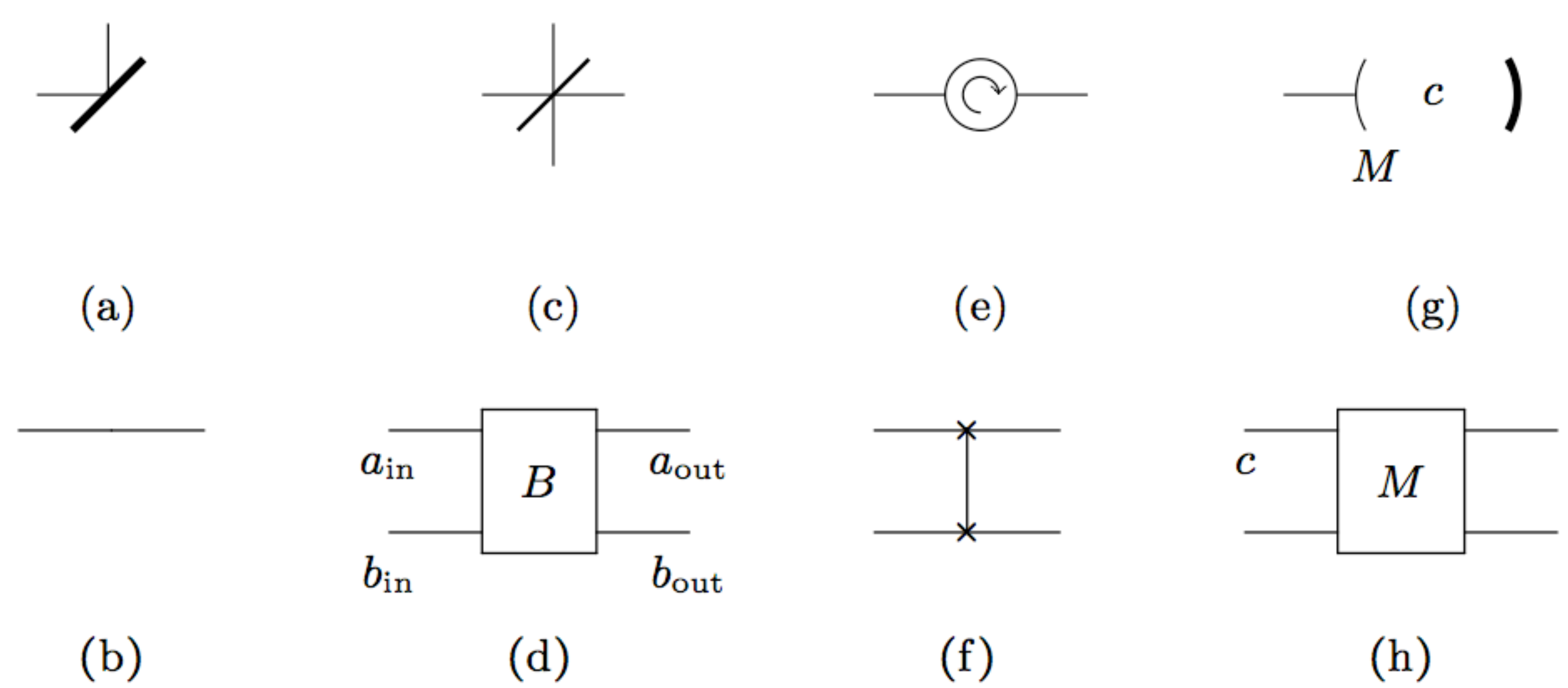


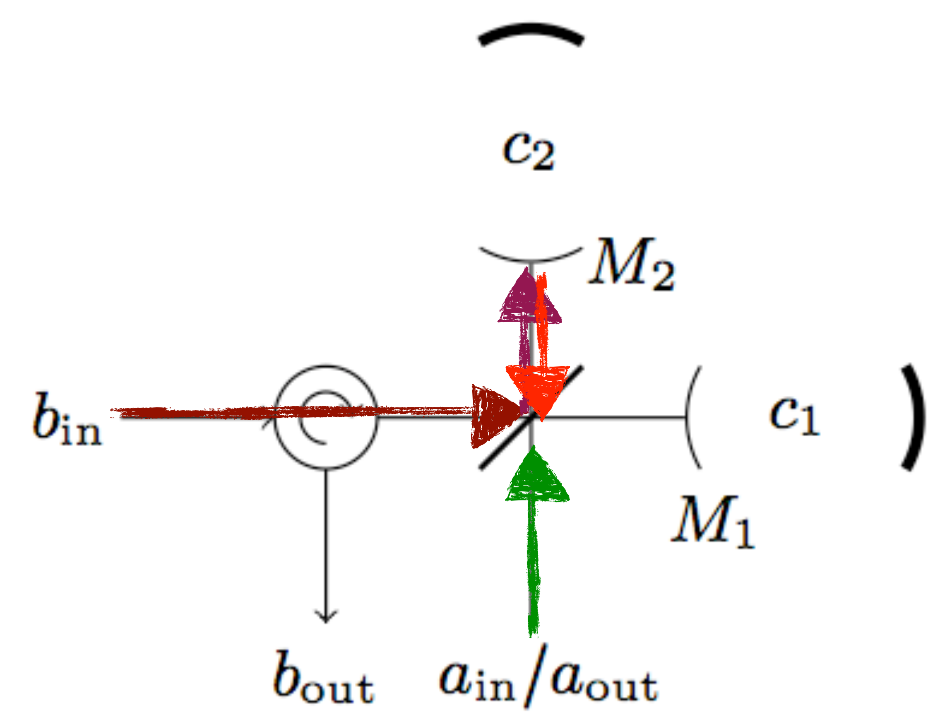


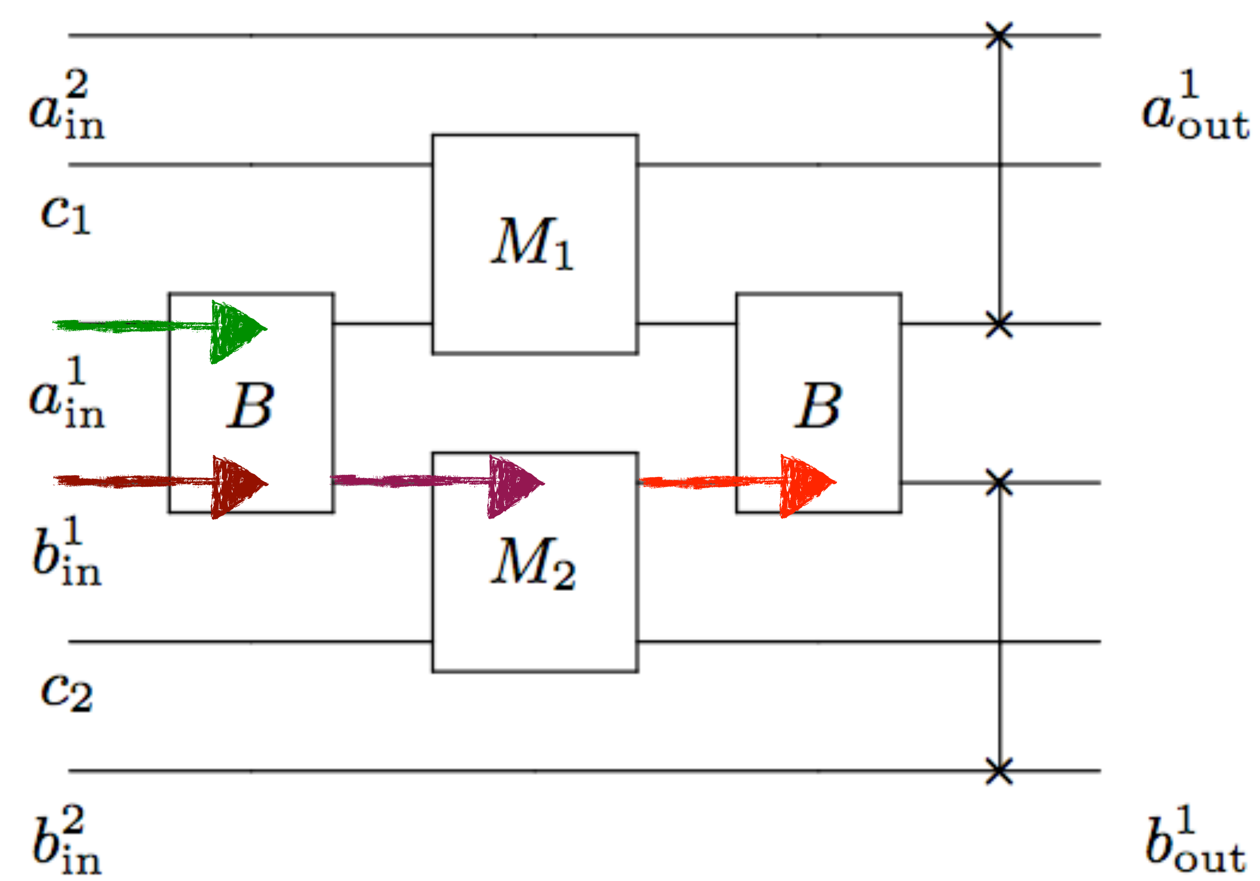


Translating schematics into Q-circuits

Dictionary:

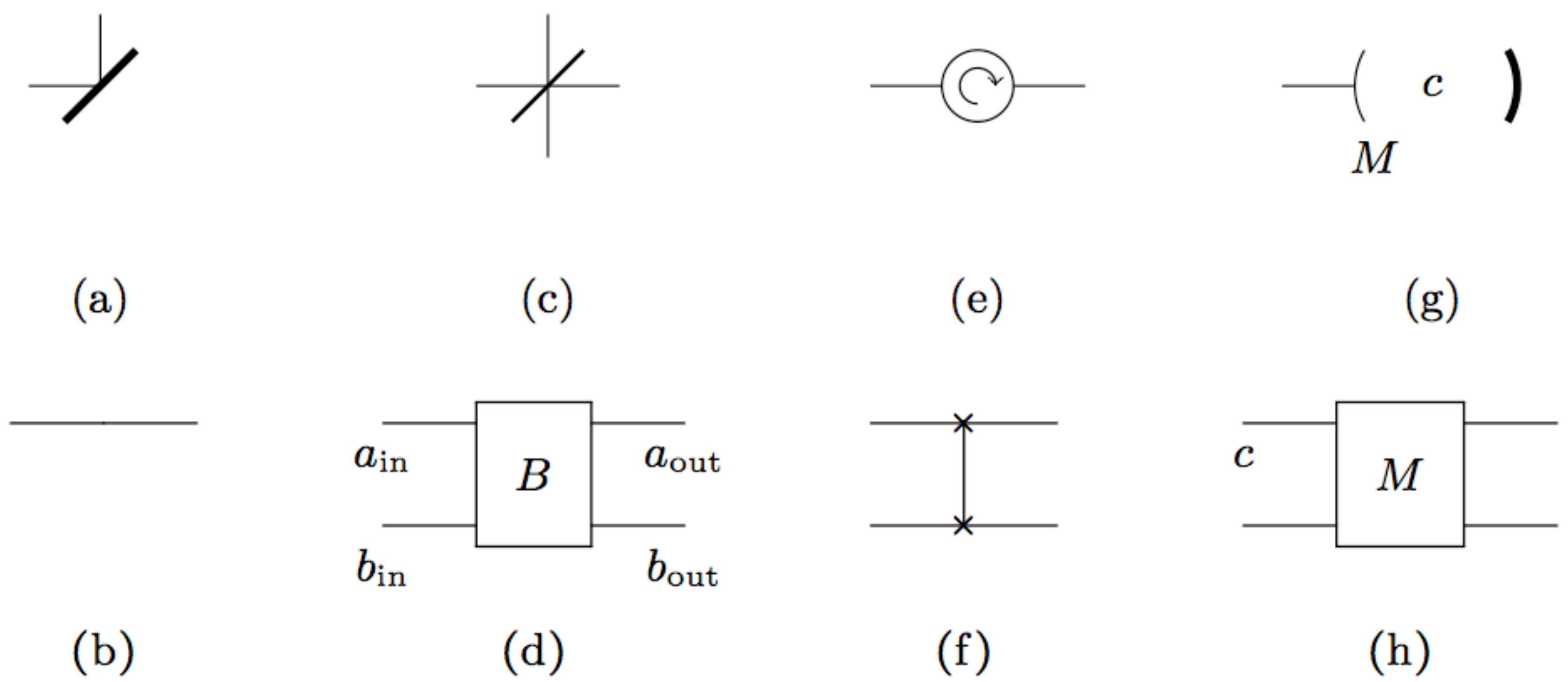


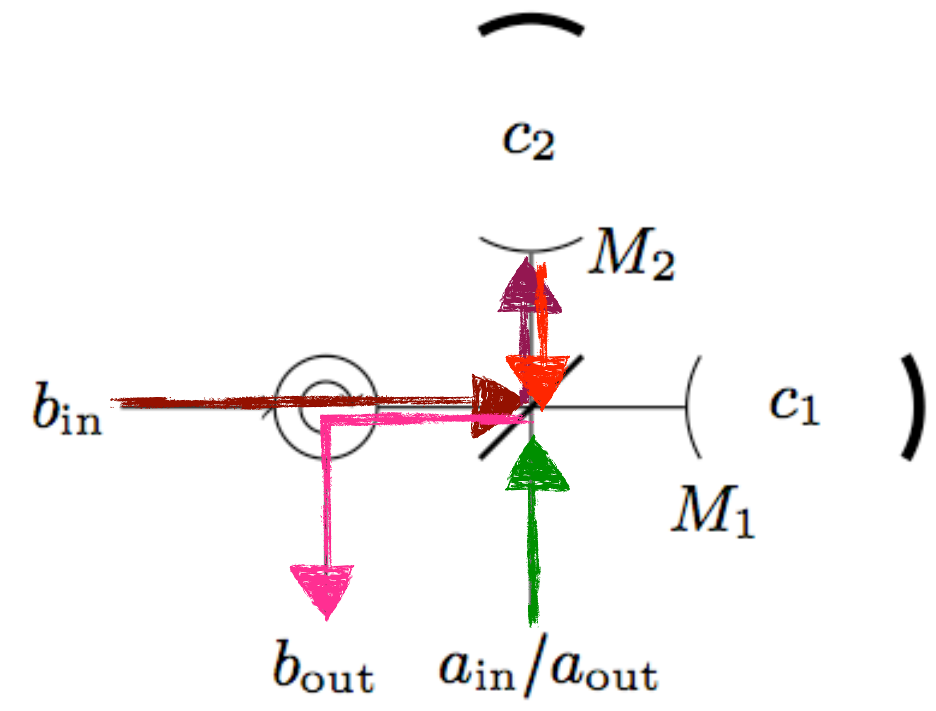


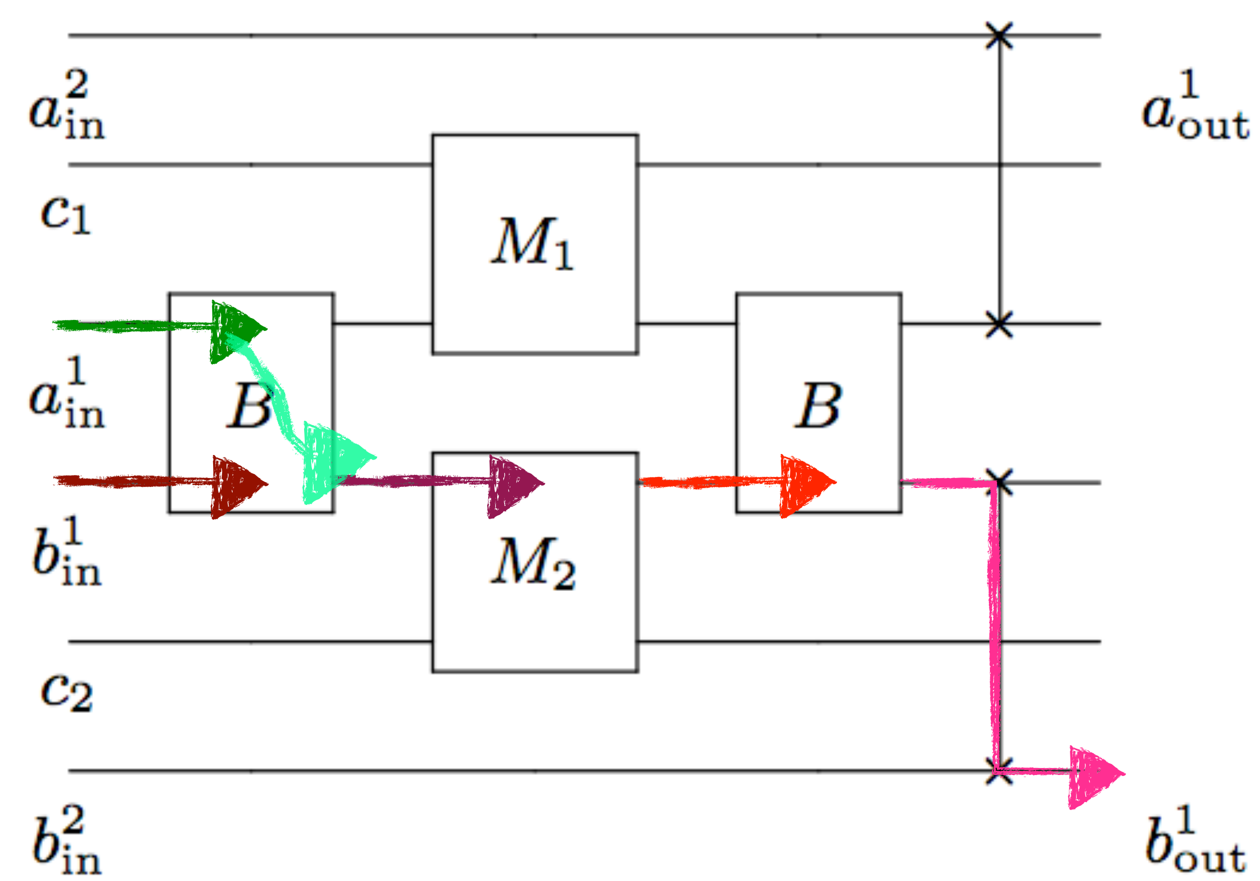


Translating schematics into Q-circuits

Dictionary:







Additional slide
(not related to audio)

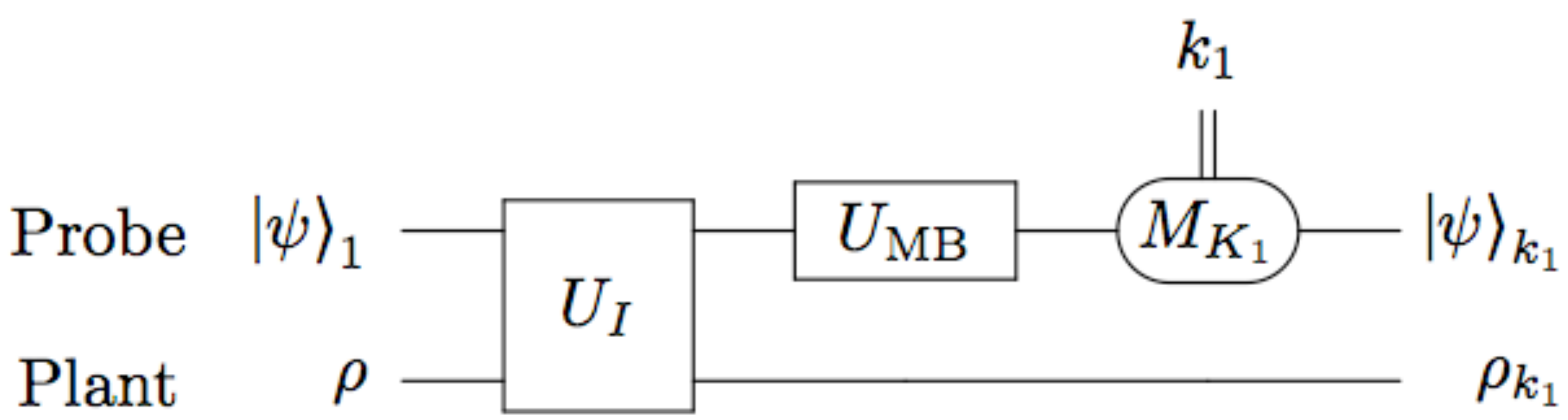
(II) Quantum Measurements & Trajectories

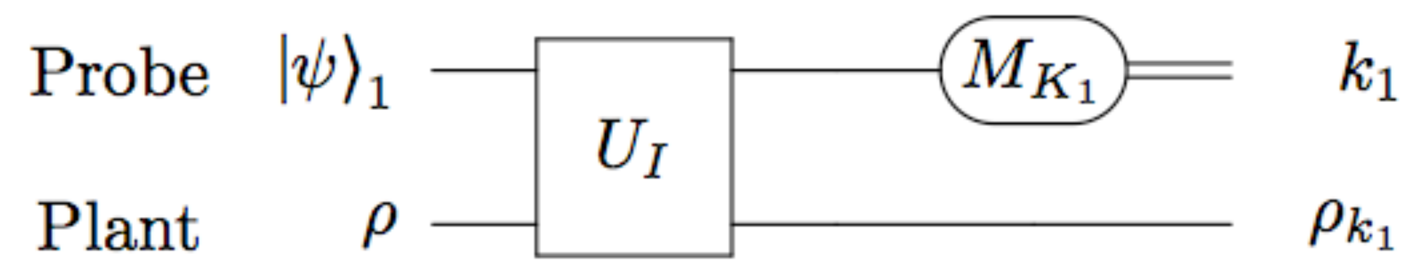
Assumptions:
1. All quantum control can be represented and understood through quantum circuits
3. If you draw these quantum circuits, patterns emerge (flow of information).

Therefore: we can categorize and understand quantum control protocols based on the flow of information.

Conditional measurements in the circuit model

Additional slide (not related to audio)

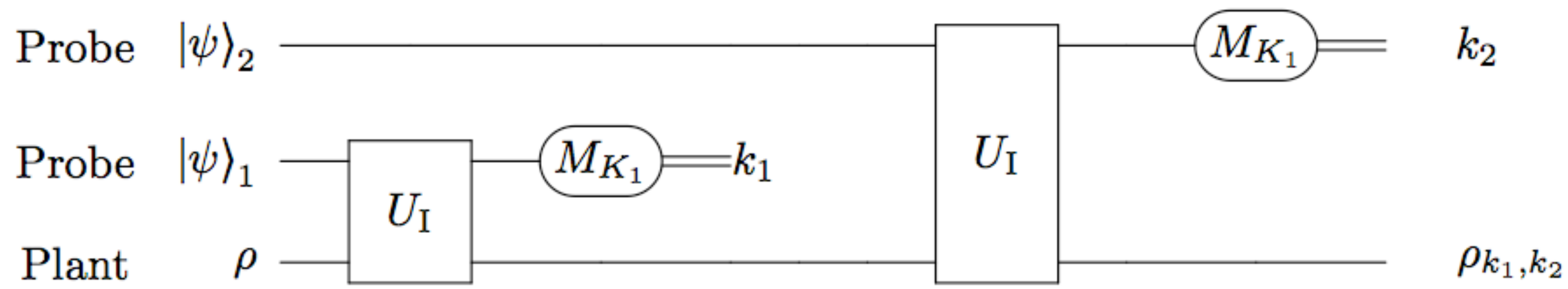




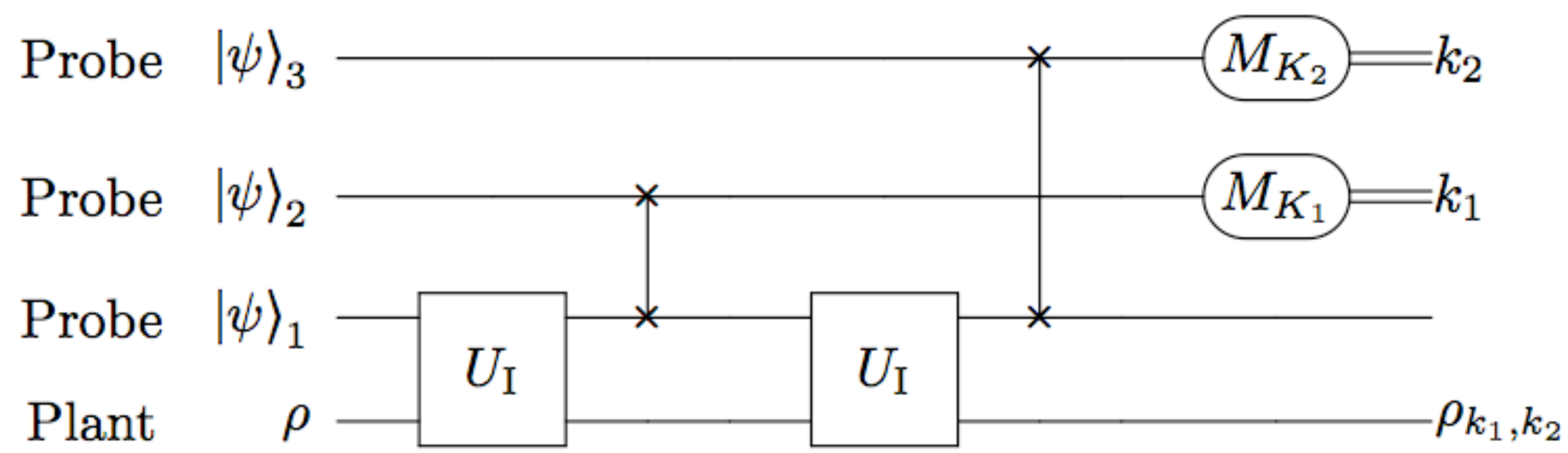
$$\rho_{k_1}(t+\tau) = \frac{M_{k_1}\rho_t M_{k_1}^\dagger}{\mathrm{Tr}[M_{k_1}^\dagger M_{k_1}\rho_t]}$$

$$\Pi_{k_1} = |k_1\rangle\langle k_1|, \quad k_1 \in [0,1]$$

$$M_{k_1} = \langle k_1|U_I|\psi_1\rangle$$

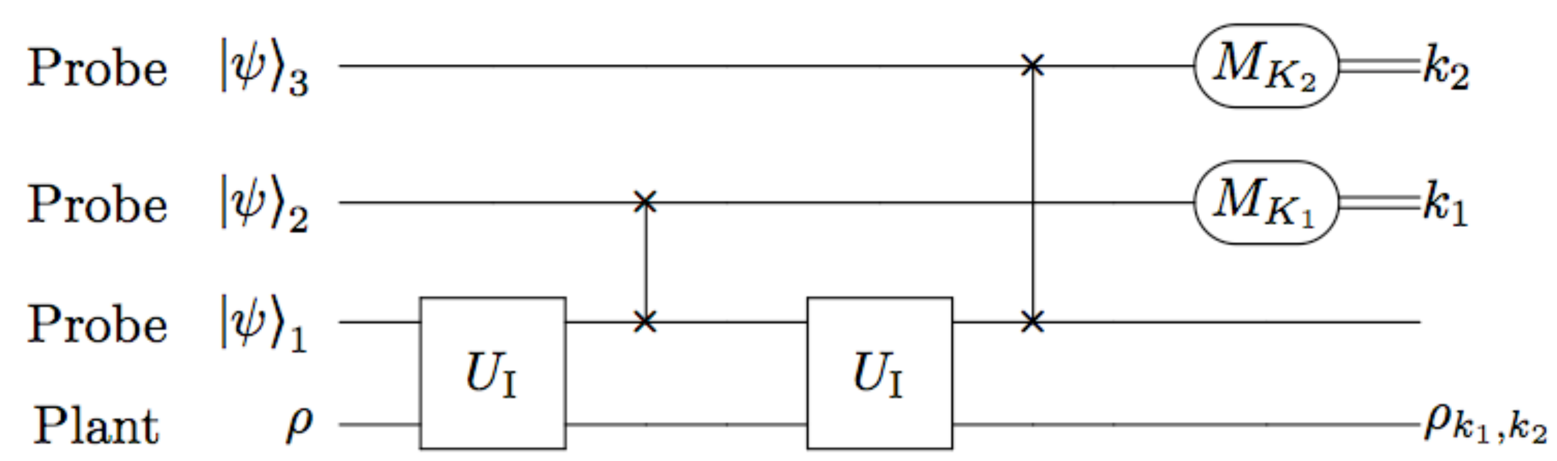


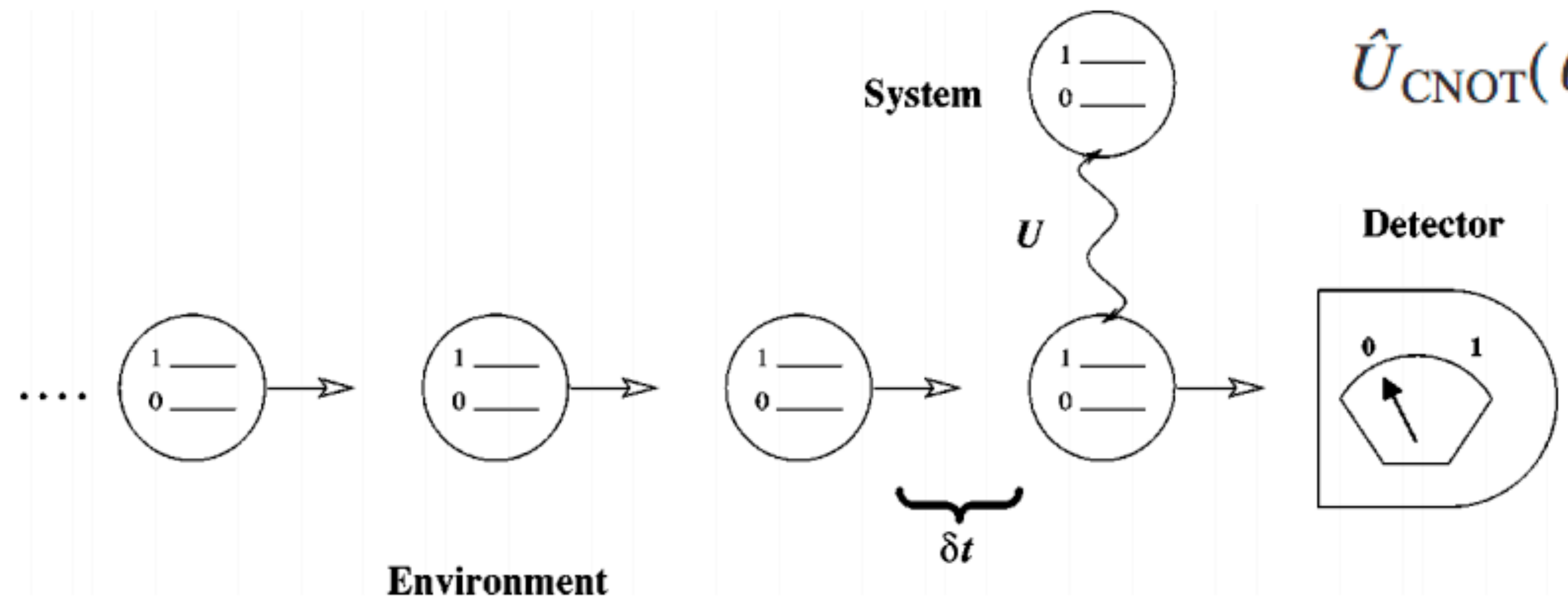
$$\mathrm{SWAP}|\phi\rangle \otimes |\psi\rangle = |\psi\rangle \otimes |\phi\rangle$$



Relation to the quantum trajectory description (1)

Additional slide
(not related to audio)





$$\hat{U}_{\mathrm{CNOT}}(\theta) = \exp\{-i\theta\hat{U}_{\mathrm{CNOT}}\} = \hat{1}\cos\theta - i\hat{U}_{\mathrm{CNOT}}\sin\theta, \quad (15)$$

T. Brun, Am. J. Phys. **70**, 719 (2002)

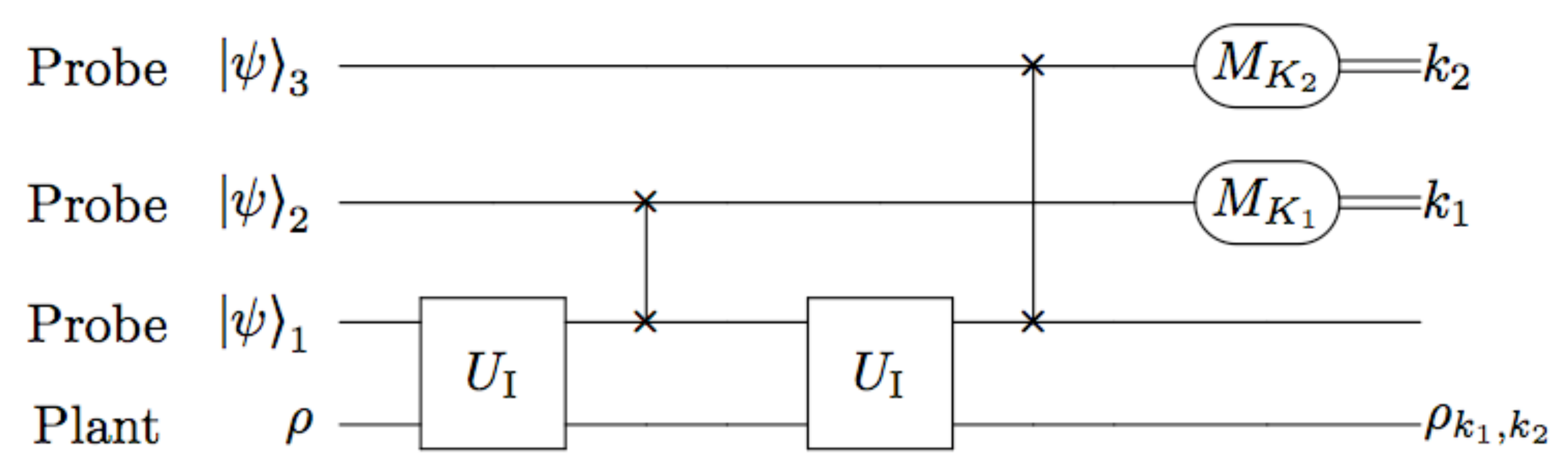
$$H_{\mathrm{int}}^{(k)} = i\Theta(\sigma_- \otimes \sigma_+^{(k)} - \sigma_+ \otimes \sigma_-^{(k)})$$

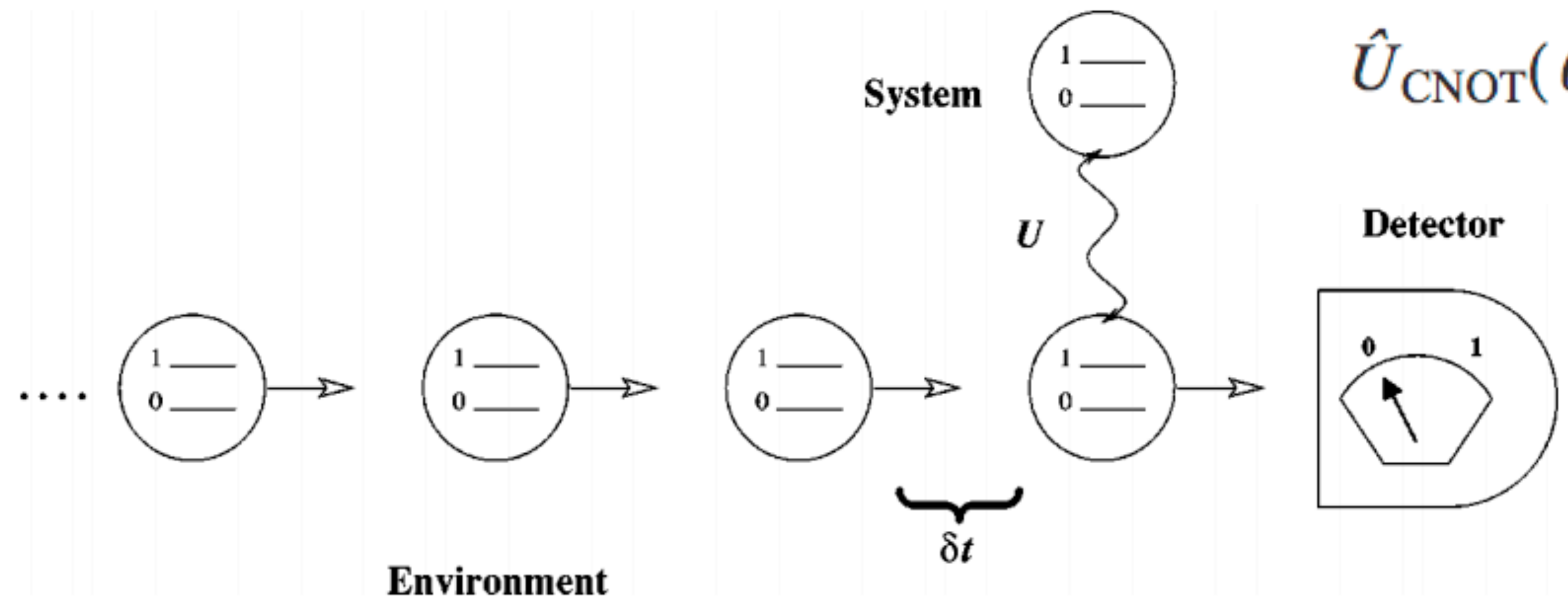
$$\sigma_\pm^{(k)} \equiv I^{\otimes(k-1)}\sigma_\pm^{(k)} I^{\otimes(N-k)}$$

System / Plant

Relation to the quantum trajectory description (1)

(not related to audio) Additional slide





$$\hat{U}_{\rm CNOT}(\theta) = \exp\{-i\theta\hat{U}_{\rm CNOT}\} = \hat{1}\cos\theta - i\hat{U}_{\rm CNOT}\sin\theta, \quad (15)$$

T. Brun, Am. J. Phys. **70**, 719 (2002)

$$H_{\rm int}^{(k)} = i\Theta(\sigma_- \otimes \sigma_+^{(k)} - \sigma_+ \otimes \sigma_-^{(k)})$$

$$U = \exp[\theta(\sigma_- \otimes \sigma_+ - \sigma_+ \otimes \sigma_-)] \qquad \theta = \Theta t, \quad \theta \ll 1$$

$$\sigma_\pm^{(k)} \equiv I^{\otimes(k-1)}\sigma_\pm^{(k)} I^{\otimes(N-k)}$$

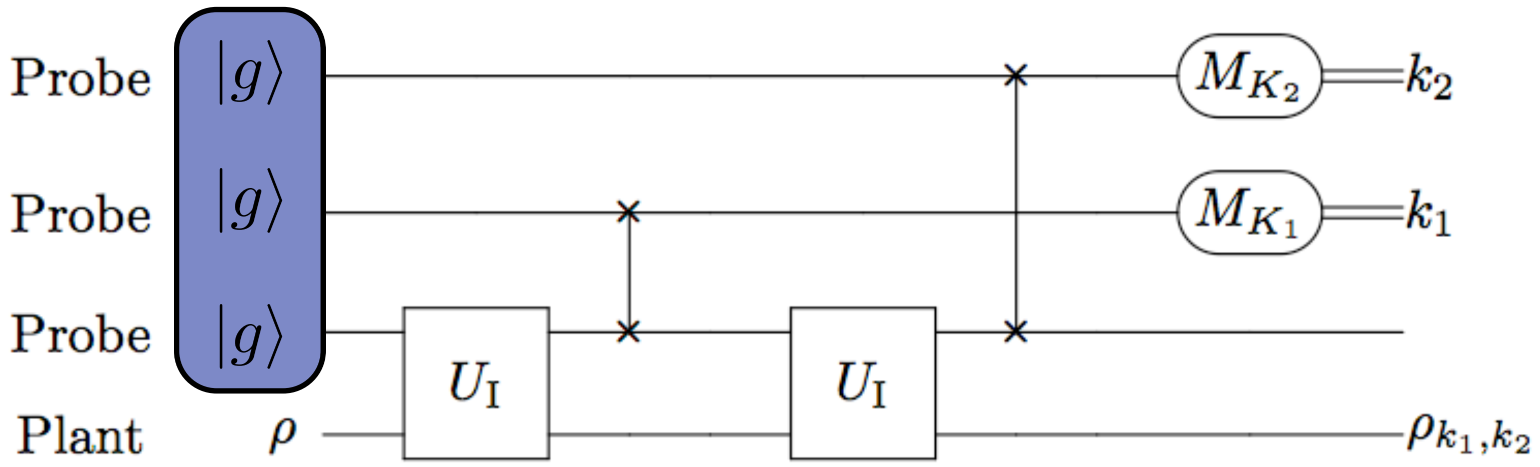
System / Plant

Environment / Bath / Ancilla / Probe

$$U = I \otimes I + \theta(\sigma_- \otimes \sigma_+ - \sigma_+ \otimes \sigma_-) - \frac{1}{2}\theta^2(\sigma_+\sigma_- \otimes \sigma_-\sigma_+ + \sigma_-\sigma_+ \otimes \sigma_+\sigma_-)$$

Relation to the quantum trajectory description (2)

(not related to audio) Additional slide



$$\{|g\rangle, |e\rangle\}$$

$$M_0 = \langle g|U|g\rangle = I - \frac{1}{2}\theta^2\sigma_+\sigma_-$$
$$M_1 = \langle e|U|g\rangle = \theta\sigma_-,$$

$$M_0^\dagger M_0 + M_1^\dagger M_1 = I$$

$$\delta t = \theta^2/N$$

$$\rho_0(t+\delta t) = \frac{M_0\rho M_0^\dagger}{\mathrm{Tr}[M_0^\dagger M_0\rho]} = \frac{\rho - \frac{1}{2}\delta t\sigma_+\sigma_-\rho - \frac{1}{2}\delta t\rho\sigma_+\sigma_-}{1 - \frac{1}{2}\delta t\mathrm{Tr}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-]}.$$

Binomial approx. on the denominator

$$\left(1 - \frac{1}{2}\delta t\mathrm{Tr}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-]\right)^{-1} \approx 1 + \frac{1}{2}\delta t\mathrm{Tr}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-]$$

$$\delta\rho_0 = \rho_0(t+\delta t) - \rho_0(t) = -\frac{1}{2}\delta t\sigma_+\sigma_-\rho - \frac{1}{2}\delta t\rho\sigma_+\sigma_- + \frac{1}{2}\delta t\mathrm{Tr}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-]\rho$$

$$\delta\rho_1 = \rho_1(t+\delta t) - \rho_1(t) = \frac{\sigma_-\rho\sigma_+}{\mathrm{Tr}[\sigma_+\sigma_-\rho]} - \rho$$

$$dN \in \{0,1\}$$

$$\rho(t+\delta t) = \rho(t) + \delta\rho$$

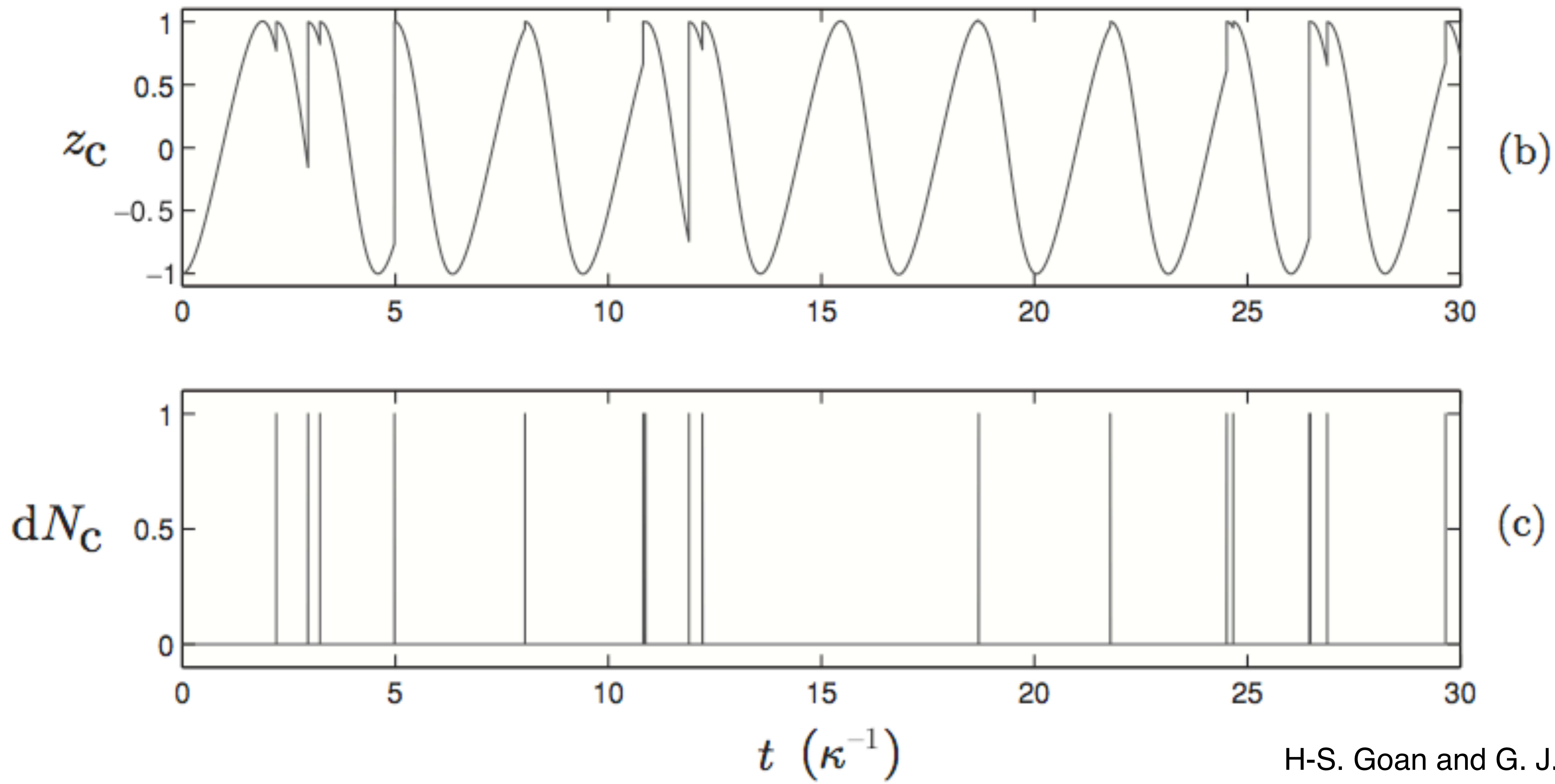
$$\delta\rho = dN\left(\frac{\sigma_-\rho\sigma_+}{\mathrm{Tr}[\sigma_+\sigma_-\rho]} - \rho\right) + (1-dN)\frac{1}{2}\delta t(-\sigma_+\sigma_-\rho - \rho\sigma_+\sigma_- + \mathrm{Tr}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-]\rho)$$

Relation to the quantum trajectory description (2)

(Additional slide (not related to audio))

Trajectory

Measurement Record

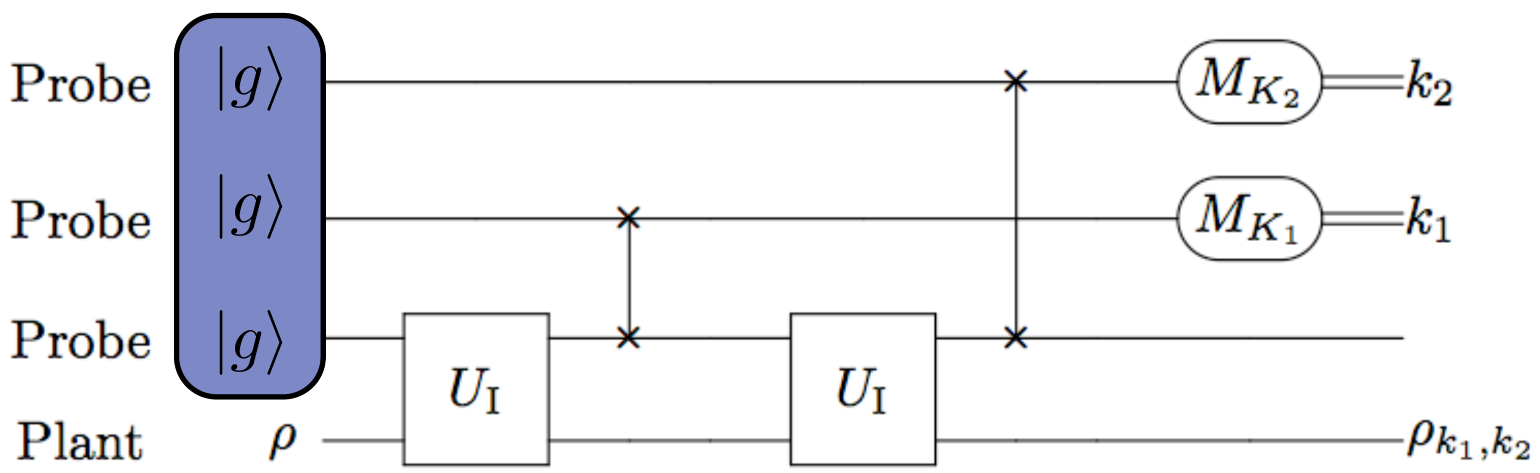


H-S. Goan and G. J. Milburn,
Phys. Rev. B **64**, 235307, (2001)

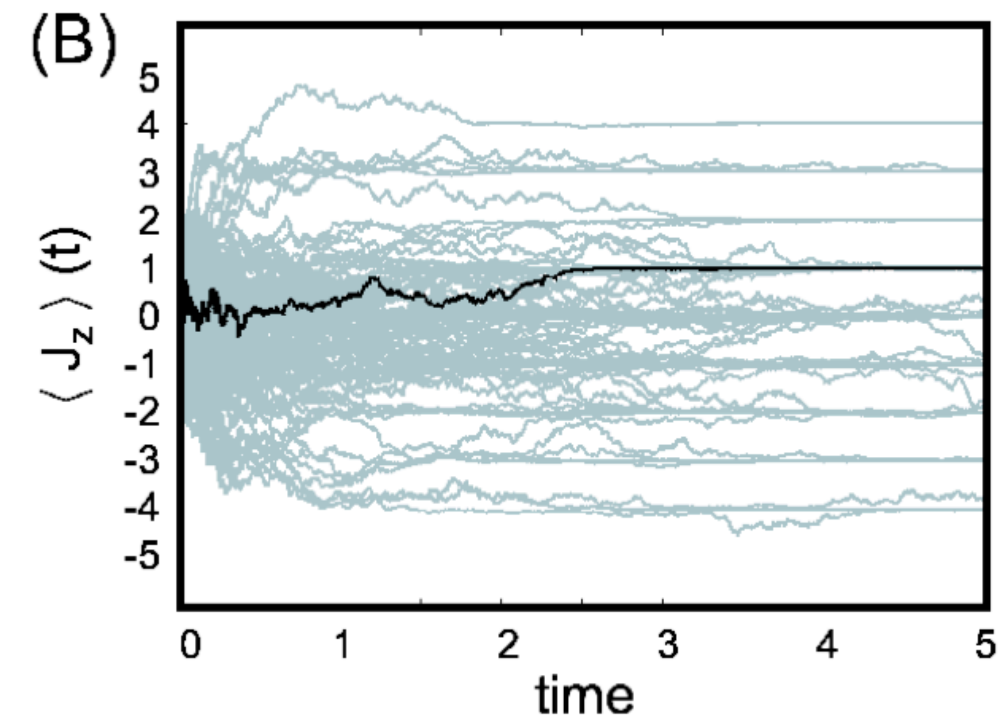
$$\delta\rho = dN\left(\frac{\sigma_-\rho\sigma_+}{\mathrm{Tr}[\sigma_+\sigma_-\rho]} - \rho\right) + (1-dN)\frac{1}{2}\delta t(-\sigma_+\sigma_-\rho - \rho\sigma_+\sigma_- + \mathrm{Tr}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-]\rho)$$

Relation to the quantum trajectory description (3)

Additional slide (not related to audio)



$$\{|g\rangle, |e\rangle\}$$



Stockton, van Handel, & Mabuchi, PRA **70** 022106 (2004)

Initial state	Final state	Kraus operators	SME
$\|g\rangle$	$\{\|e\rangle, \|g\rangle\}$	$M_e = \theta\sigma_-,\ M_g = I - \frac{1}{2}\theta^2\sigma_+\sigma_-$	Jump
$\|g\rangle$	$\|\psi\rangle = (\|g\rangle \pm \|e\rangle)/\sqrt{2}$	$M_\pm = (I \pm \theta\sigma_- - \frac{1}{2}\theta^2\sigma_+\sigma_-)/\sqrt{2}$	Homodyne X
$\|g\rangle$	$\|\phi\rangle = (\|g\rangle \pm i\|e\rangle)/\sqrt{2}$	$M_\pm = (I \pm i\theta\sigma_- - \frac{1}{2}\theta^2\sigma_+\sigma_-)/\sqrt{2}$	Homodyne Y
$\|g\rangle$	$\|\Psi\rangle_{\pm,\tilde{\pm}} = (\|\psi\rangle + \|\phi\rangle)/\sqrt{2}$	$M_\pm = \frac{1}{\sqrt{2}}\left[I + \frac{1}{\sqrt{2}}\theta(\pm 1 \tilde{\mp} i)\sigma_- - \frac{1}{2}\theta^2\sigma_+\sigma_-\right]$	Hetrodyne

Other field states are equally easy to deal with e.g.

$$|\tilde{\alpha}\rangle = (1 + \delta t\, \alpha\, \sigma_+)|g\rangle$$

$$|1_\xi\rangle = \xi_1|egg\ldots g\rangle + \xi_2|geg\ldots g\rangle + \xi_3|gge\ldots g\rangle + \cdots + \xi_N|ggg\ldots e\rangle$$

(II) Categorizing quantum control with circuits

Open loop control

Measurement & coherent feedback control

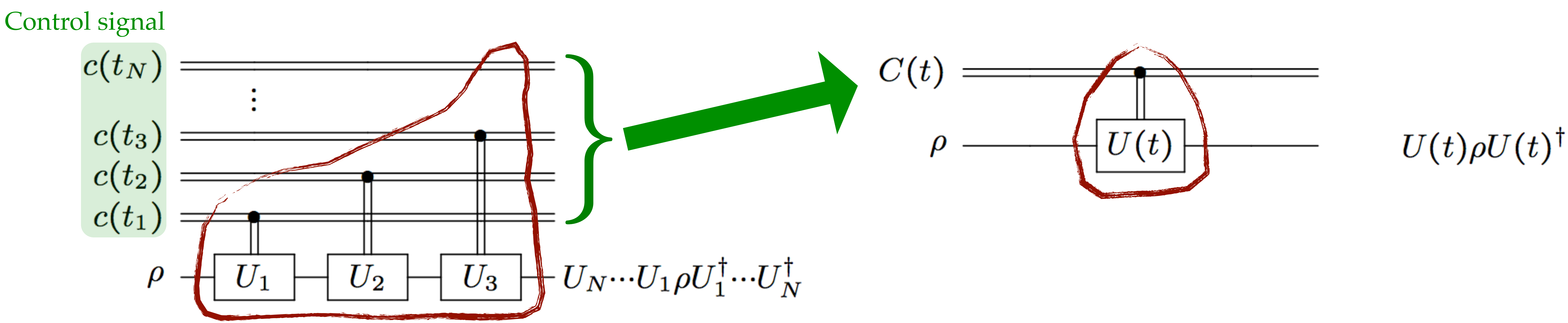
Non commutative quantum control

Assumptions:

1. All quantum control can be represented and understood through quantum circuits
2. If you draw these quantum circuits, patterns emerge (flow of information).

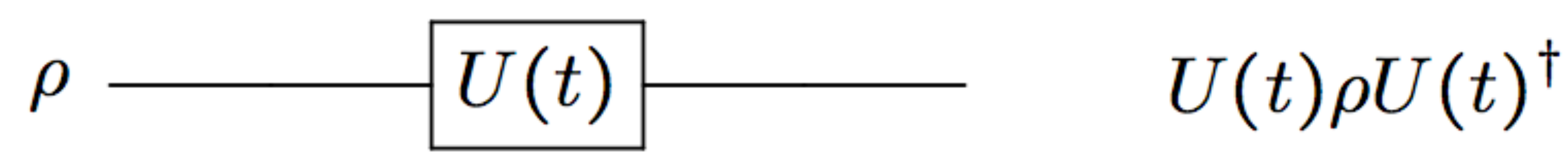
Therefore: we can categorize and understand quantum control protocols based on the flow of information.

Open loop control

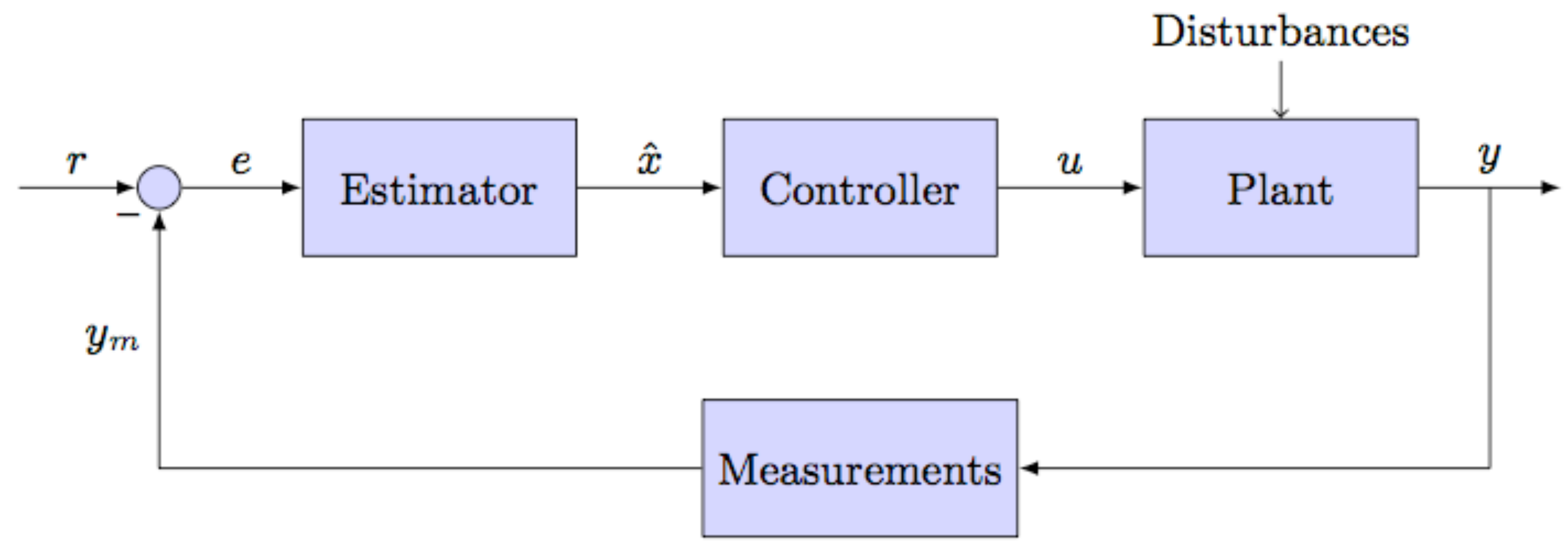


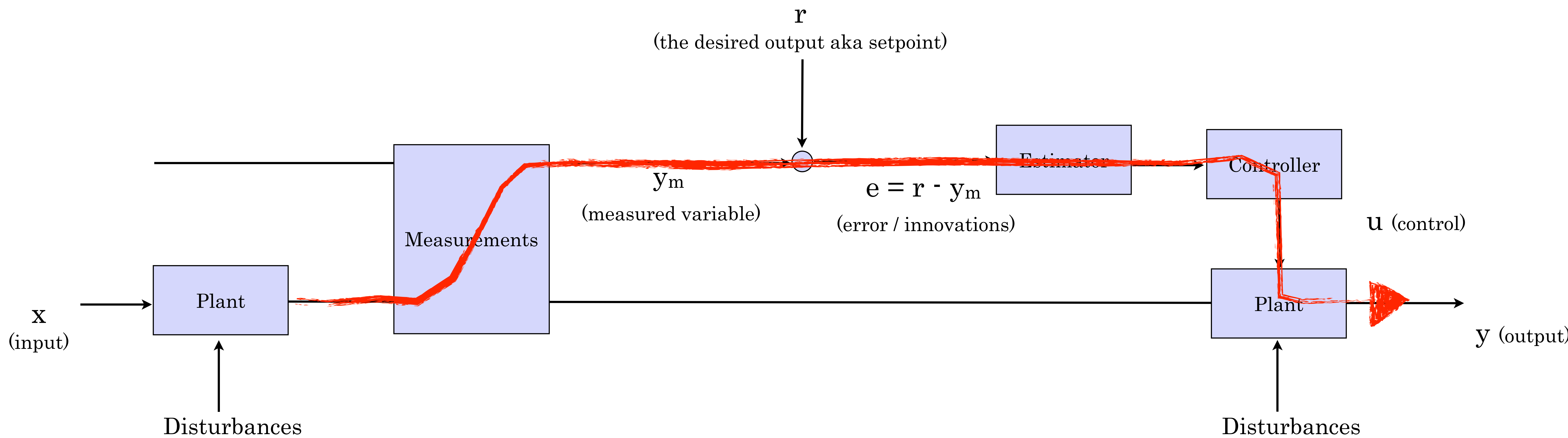
Classically controlled unitaries :

if k_i apply U_{k_i}

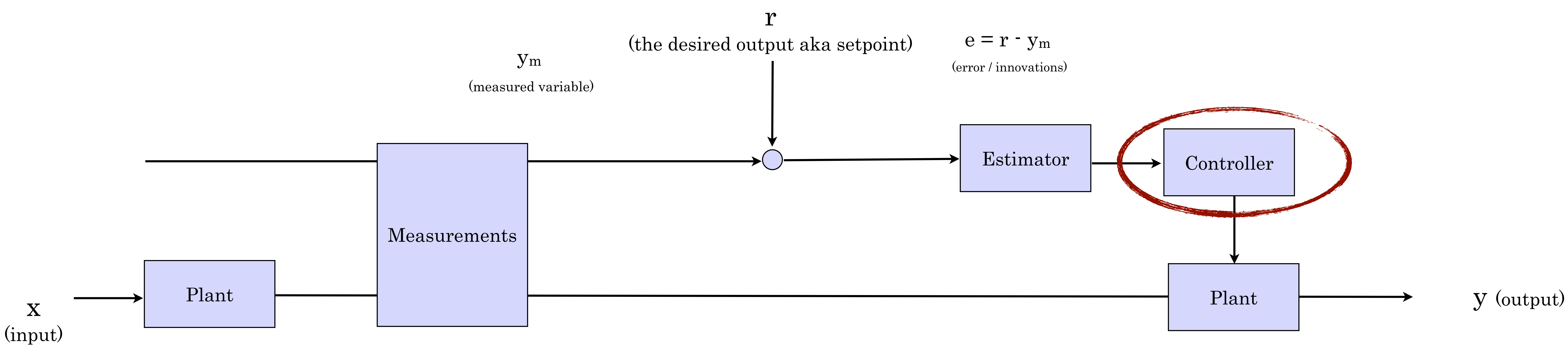


Measurement based feedback control





Measurement based feedback control



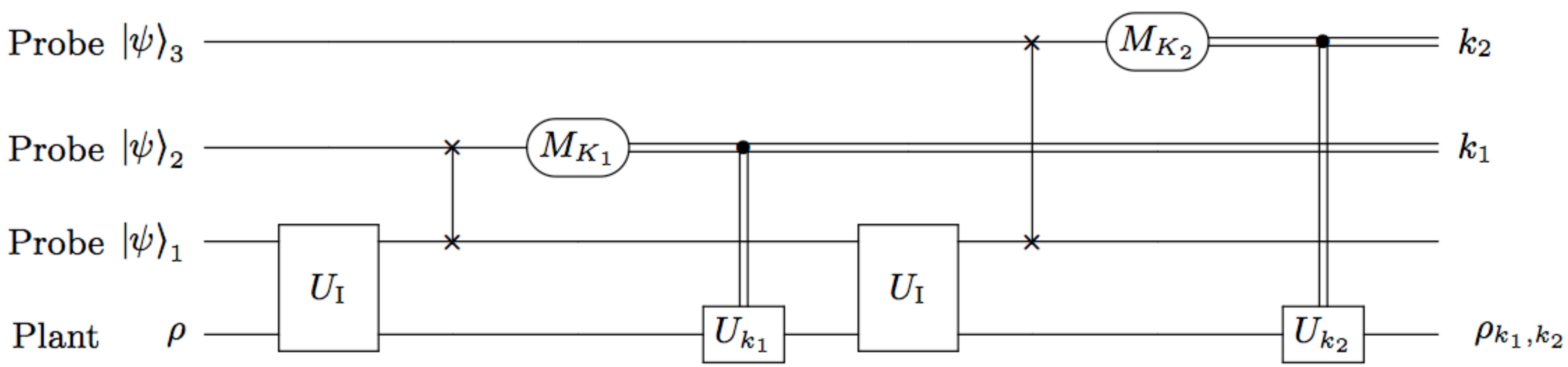


FIG. 14: proportional, direct, or Wiseman–Milburn type "Hamiltonian" feedback.

Classically controlled unitaries :

if k_i apply U_{k_i}

Measurement based feedback control

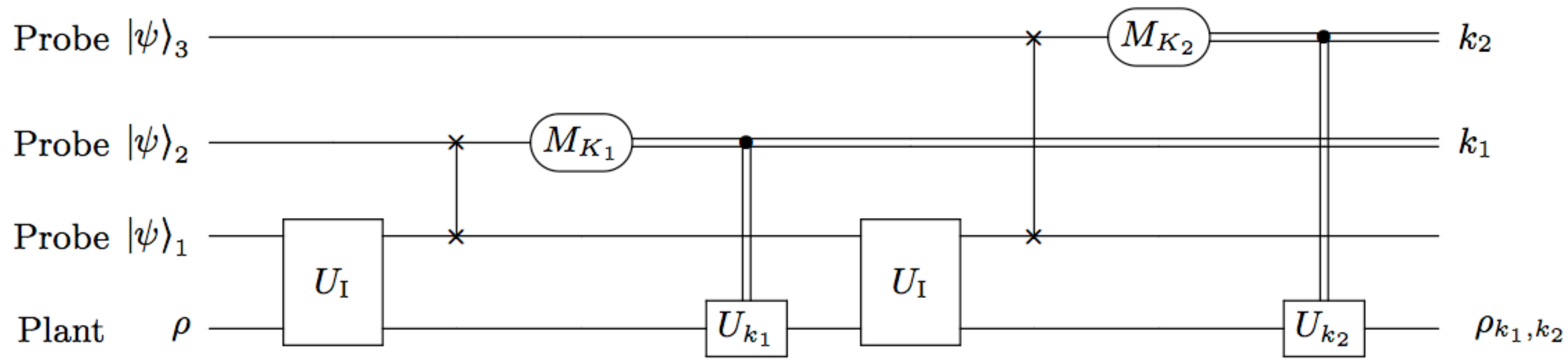


FIG. 14: proportional, direct, or Wiseman–Milburn type "Hamiltonian" feedback.

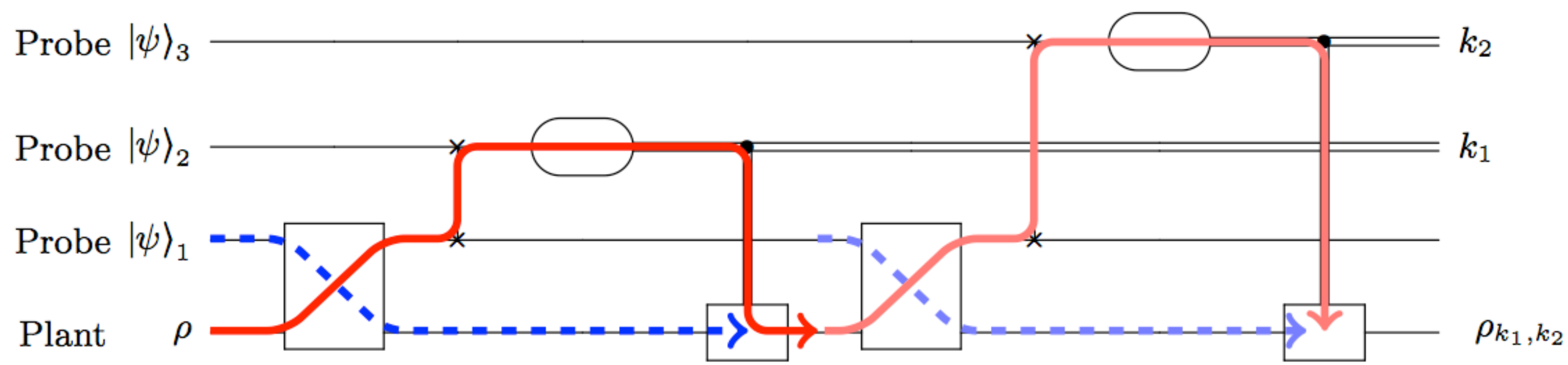
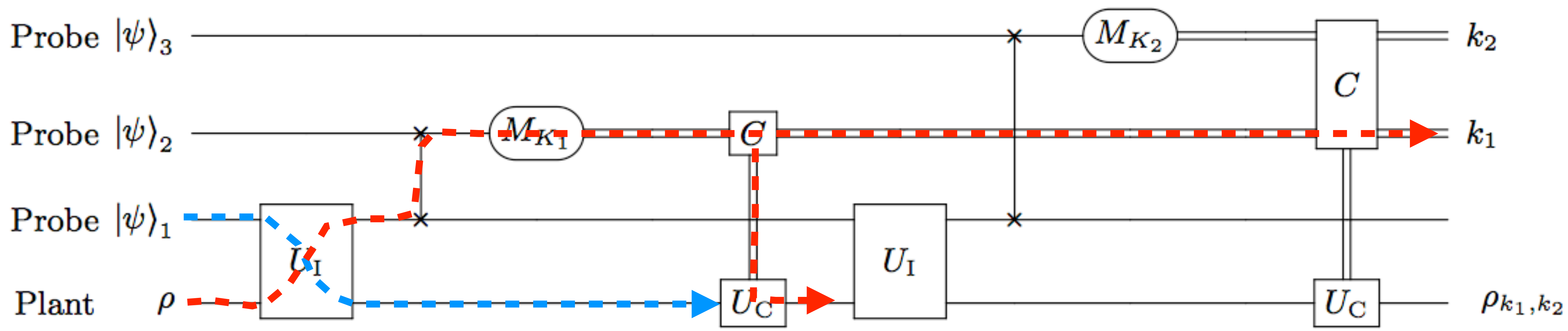


FIG. 15: Information flow in proportional Hamiltonian feedback.

Thursday, January 10, 13

Bayesian / State based control



"Coherent" feedback control

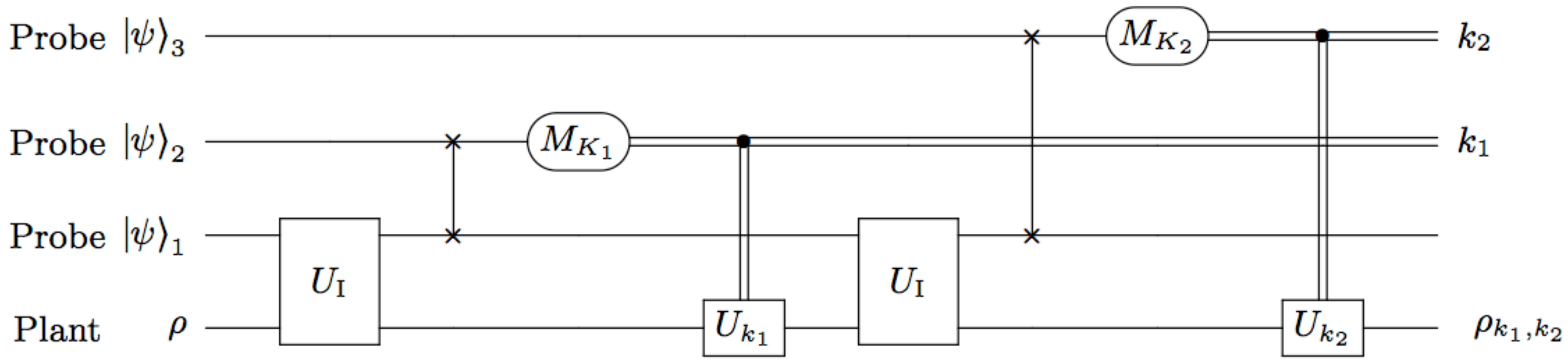


FIG. 14: proportional, direct, or Wiseman–Milburn type "Hamiltonian" feedback.

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Phys. Rev. A **49** 4110 (1994)

Coherentized feedback control

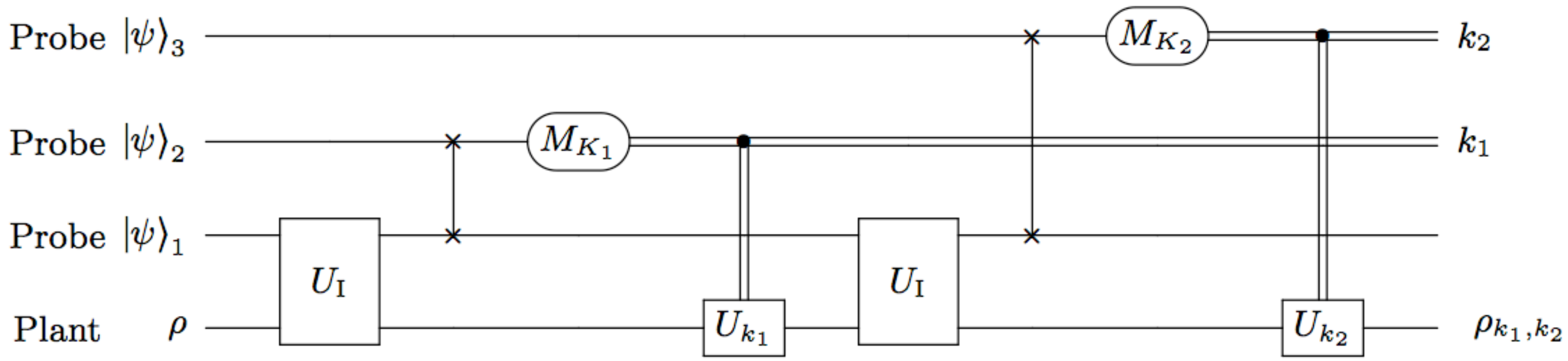


FIG. 14: proportional, direct, or Wiseman–Milburn type "Hamiltonian" feedback.

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Phys. Rev. A **49** 4110 (1994)

Principles of deferred measurement [Wiseman & Milburn 94, Griffiths & Niu 96]:

(1) Measurement can always be moved from an intermediate state of an evolution (circuit) to the end of an evolution (circuit).

(2) If the measurement results are used at any stage of the evolution (circuit) then the classically controlled operations can be replaced by conditional coherent quantum operations.

Coherentized feedback control

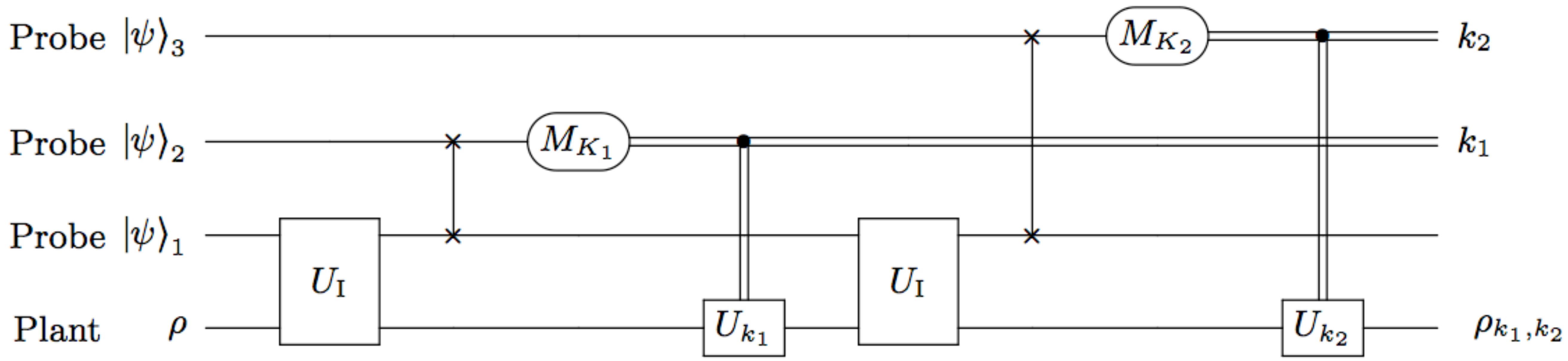
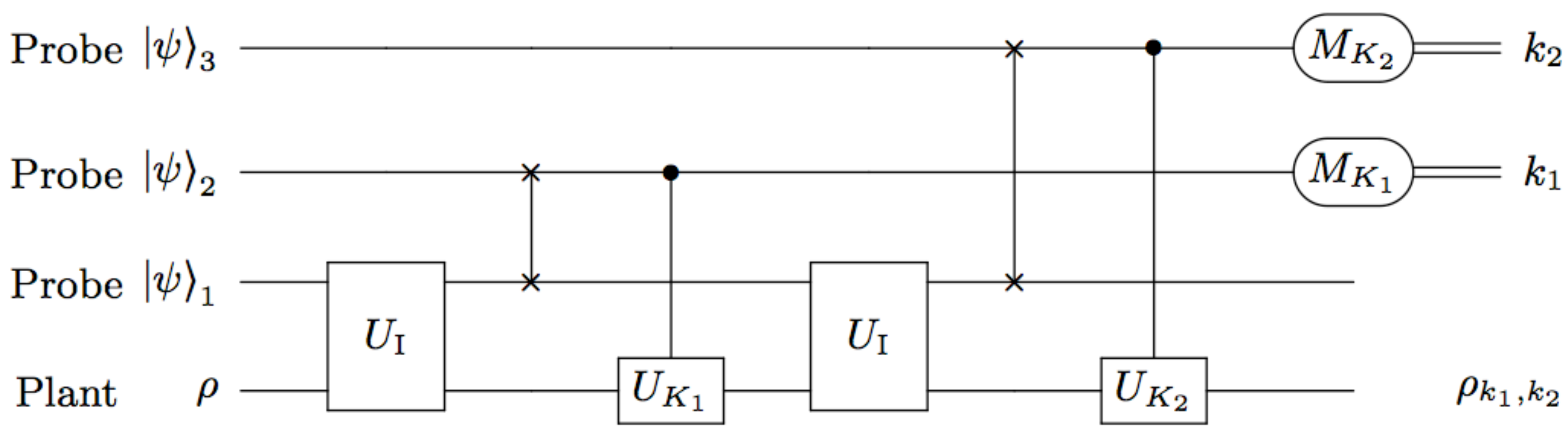


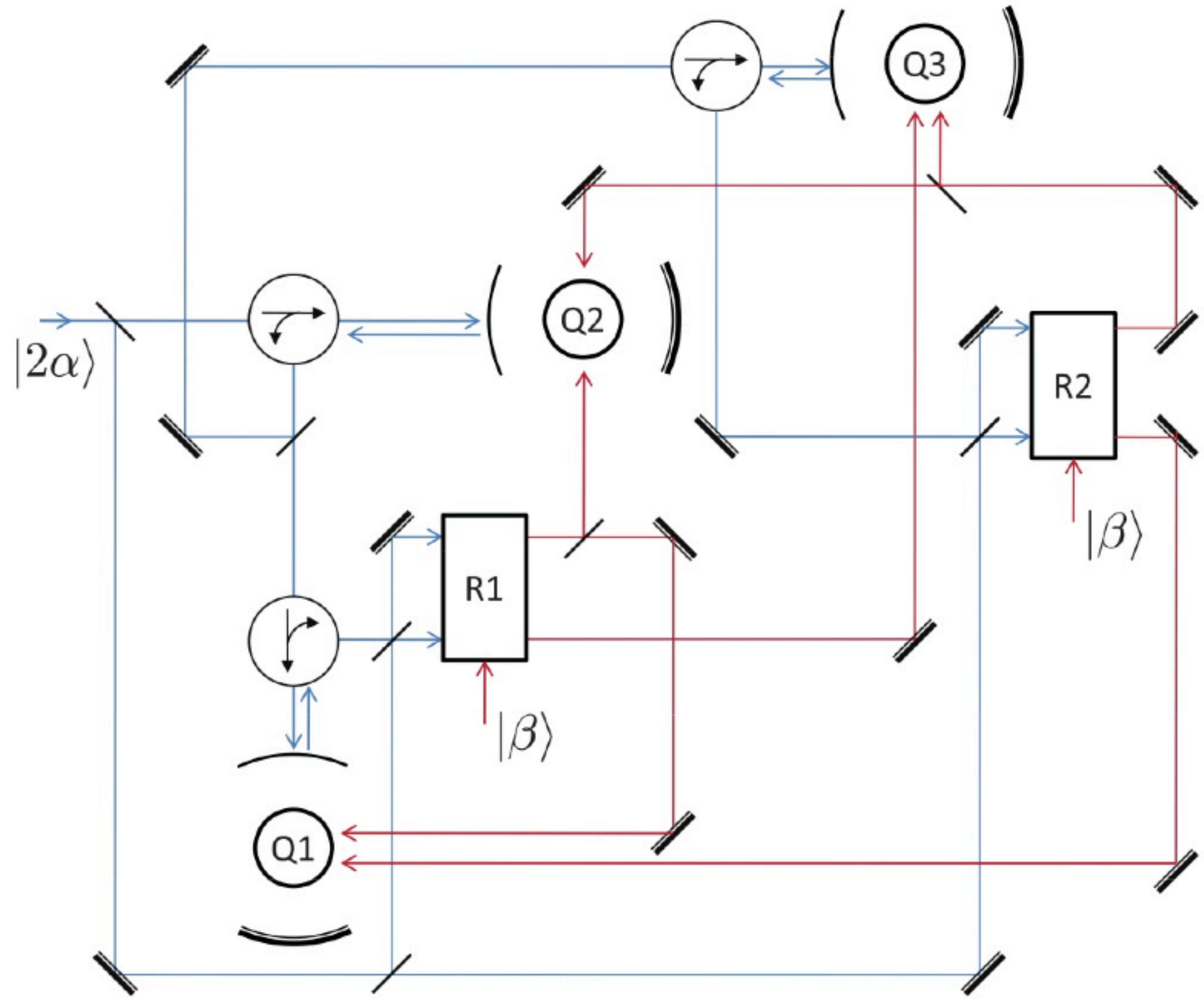
FIG. 14: proportional, direct, or Wiseman–Milburn type "Hamiltonian" feedback.

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Phys. Rev. A **49** 4110 (1994)

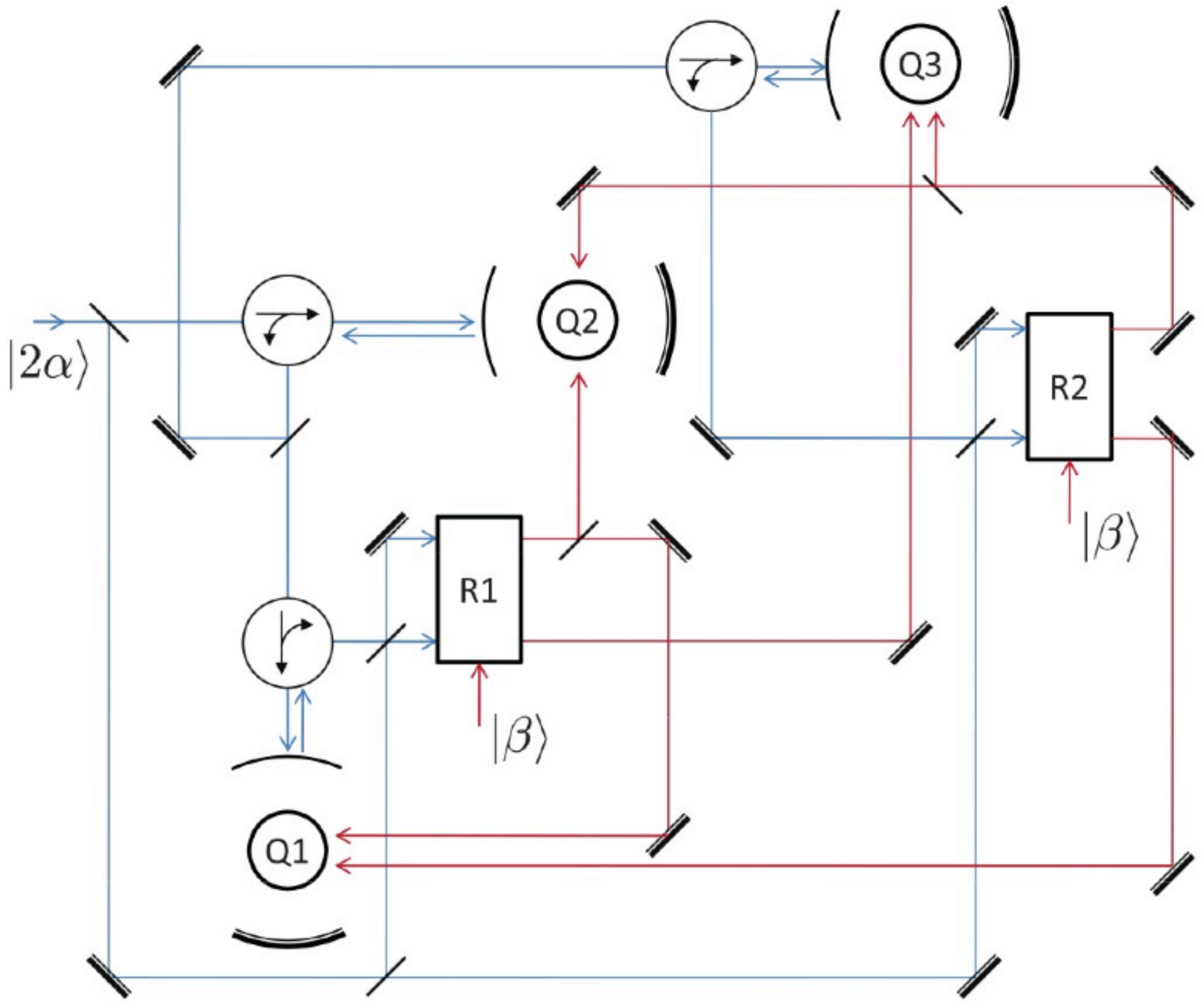


Coherentized proportional, direct, or Wiseman–Milburn type "Hamiltonian" feedback.

Example: coherent feedback control

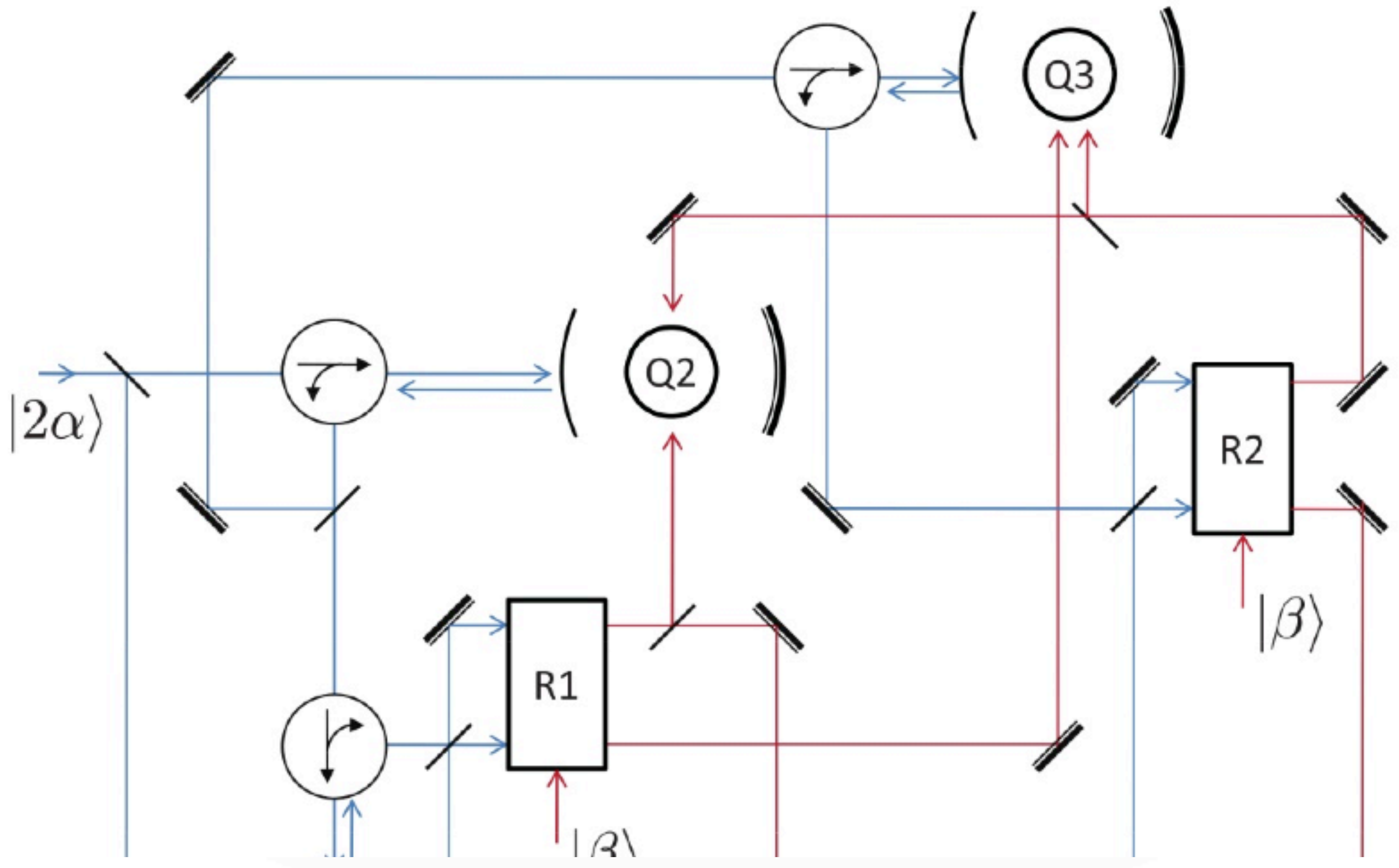


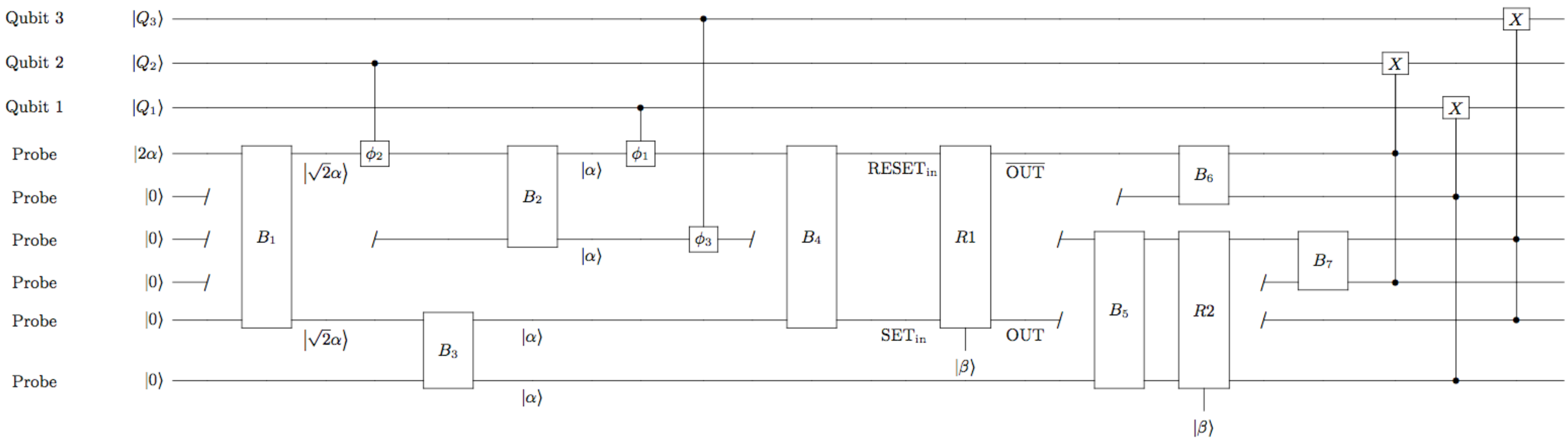
Example: coherent feedback control



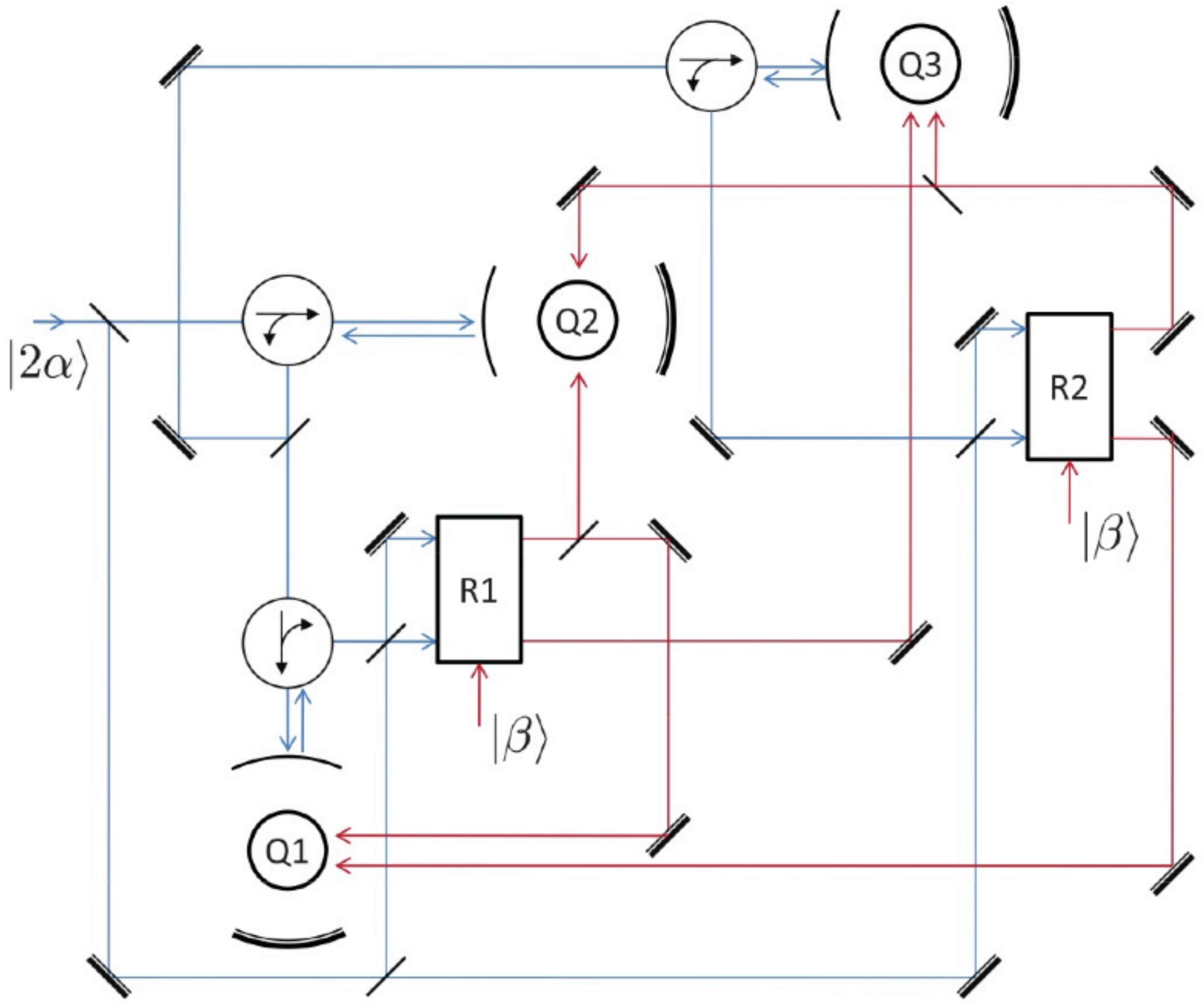
J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi,
Designing Quantum Memories with Embedded Control: Photonic Circuits for Autonomous Quantum Error Correction,
Phys. Rev. Lett. **105**, 040502 (2010)

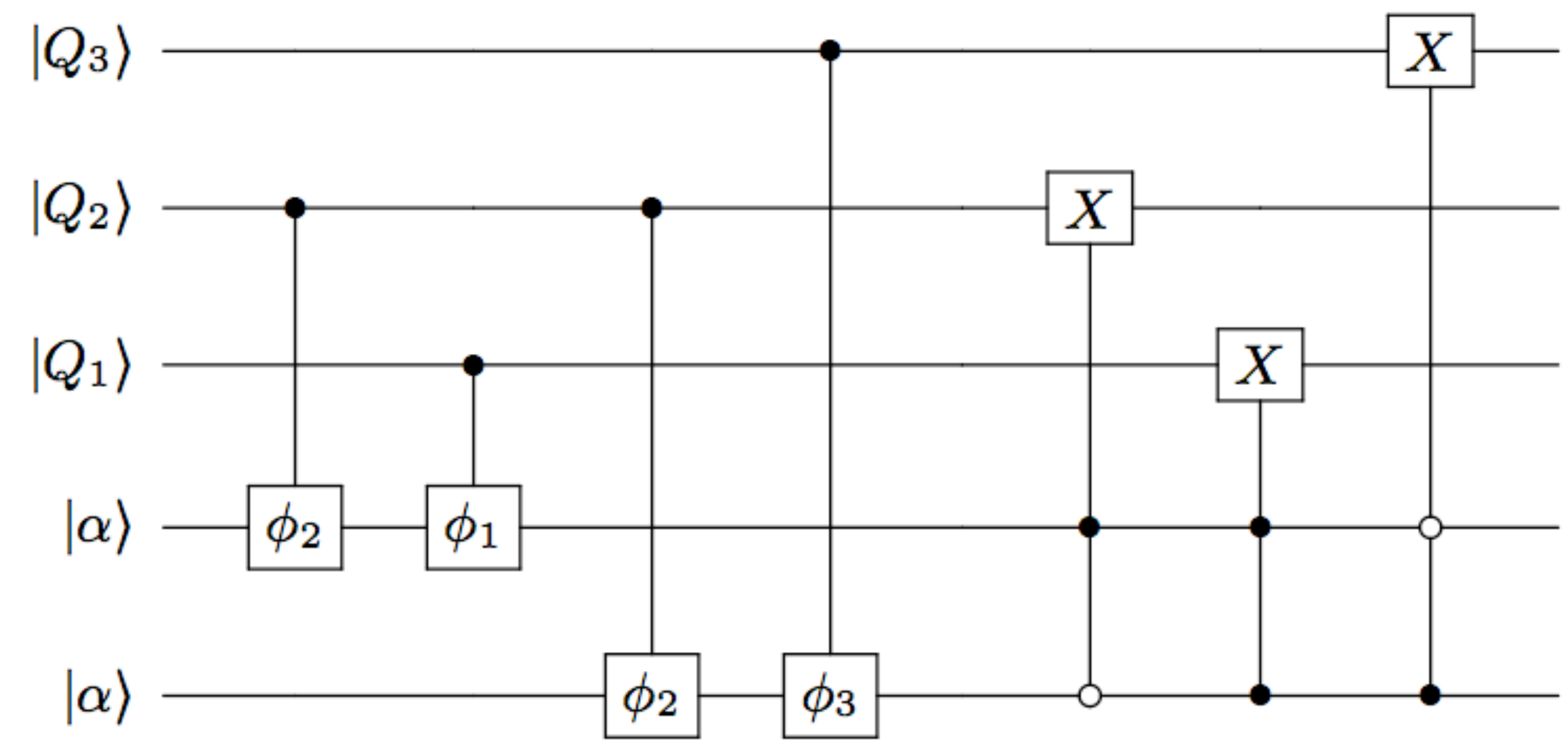
Example: coherent feedback control



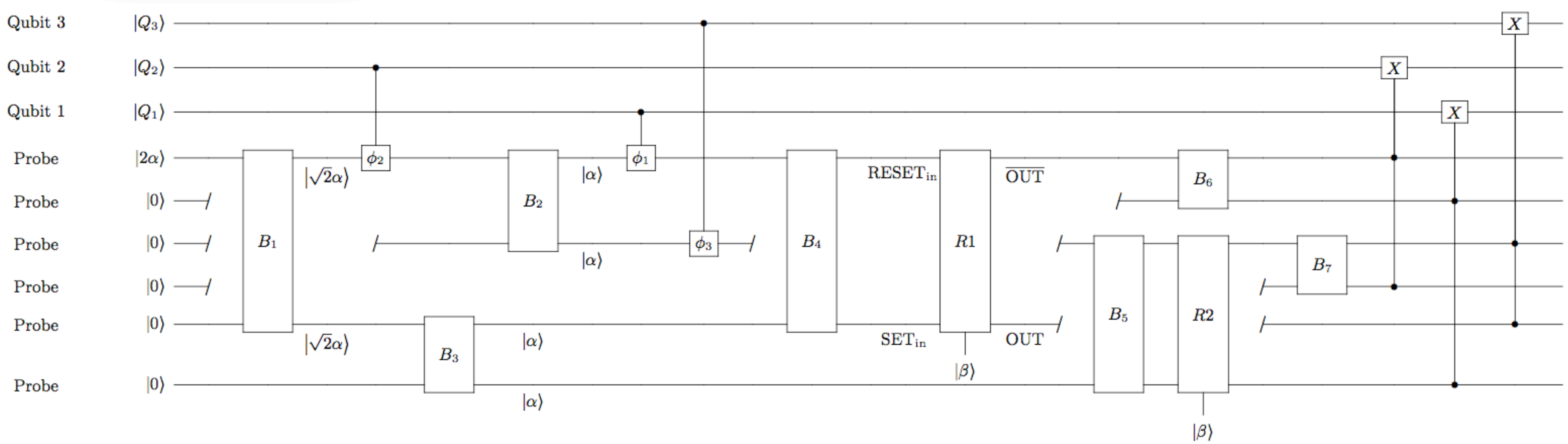


Example: coherent feedback control





J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi,
Designing Quantum Memories with Embedded Control: Photonic Circuits for Autonomous Quantum Error Correction,
Phys. Rev. Lett. **105**, 040502 (2010)

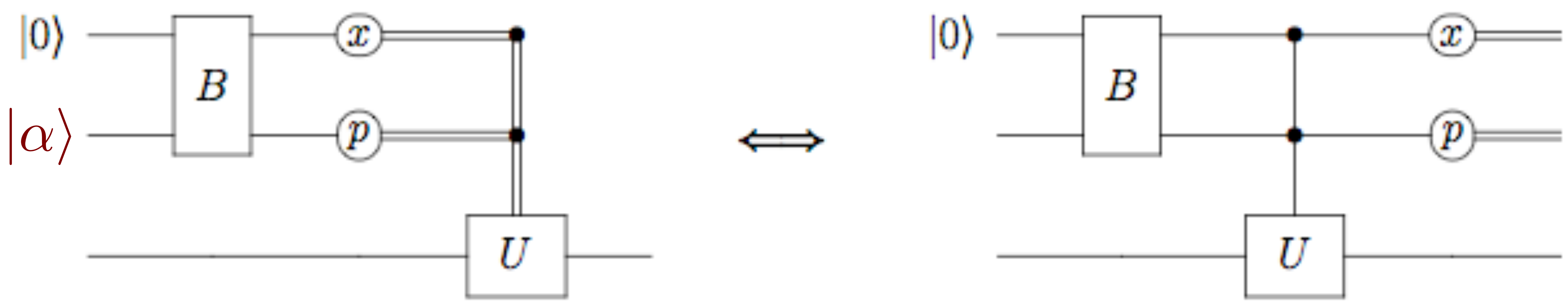


Non commutative quantum control

H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Phys. Rev. A **49** 4110 (1994)

“Complex amplitude feedback”

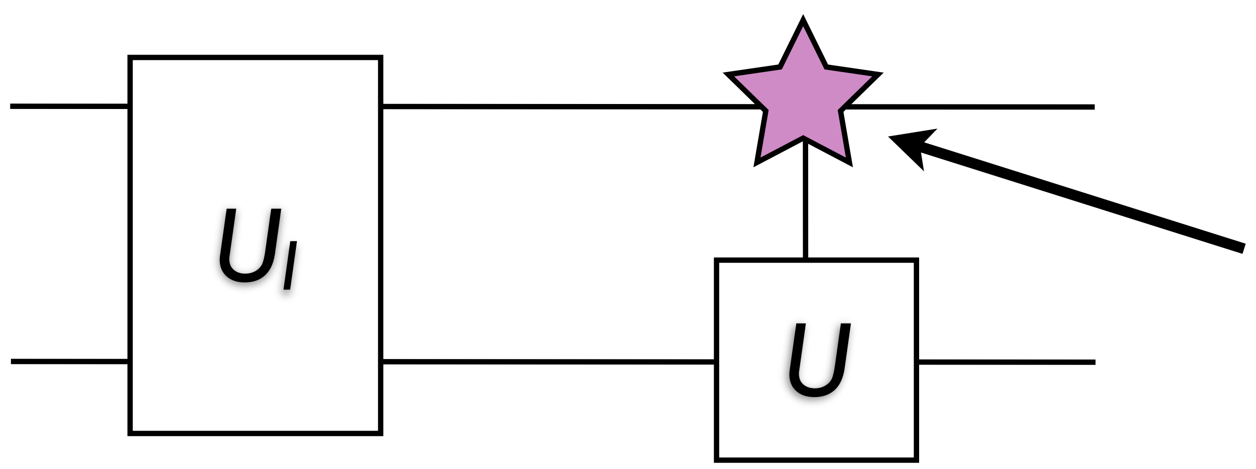
Measurement   Coherentized



1 quanta of noise
(vacuum fluctuations)

Principles of deferred measurement
[Wiseman & Milburn 94, Griffiths & Niu 96]

Non commutative / Complex amplitude



Represents control of both quadratures.

NB: must assume you have limited access to some part of the system or plant otherwise we can use measurement based quantum computation techniques and the distinction breaks down.

Non commutative quantum control

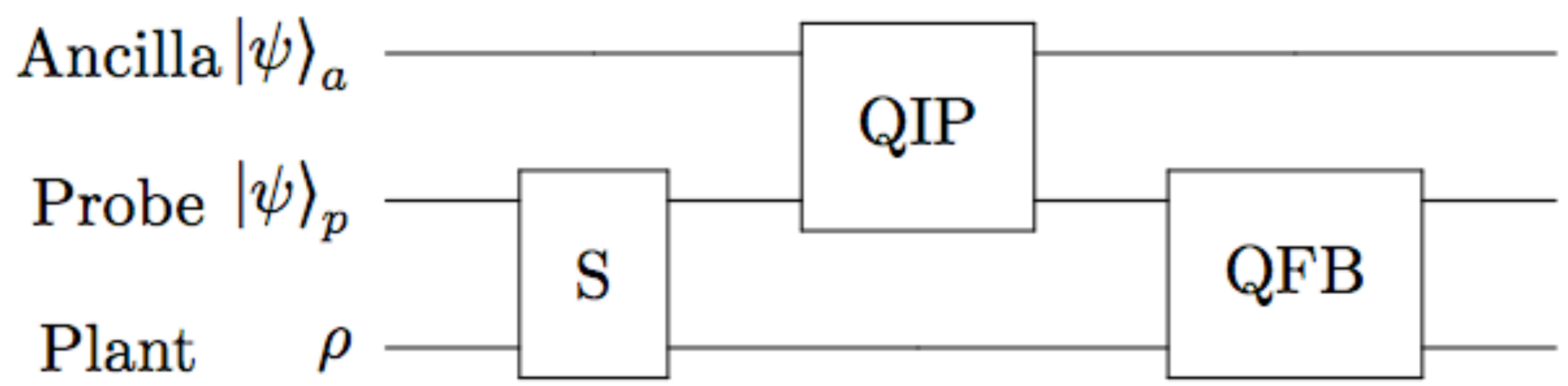
H. M. Wiseman & G. J. Milburn
All-optical versus electro-optical quantum-limited feedback,
Phys. Rev. A **49** 4110 (1994)

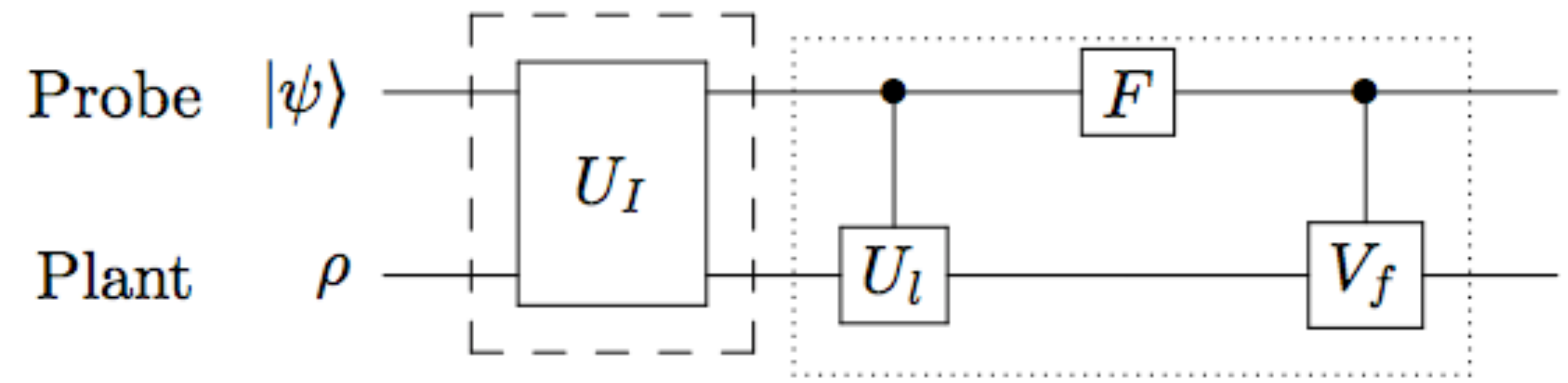
"Complex amplitude feedback"

Principle of non commutative control [Wiseman & Milburn 94]:
Non commutative quantum control can always be approximated by approximate measurement of non commuting observables (e.g. Heterodyne measurements, which necessarily introduce additional vacuum fluctuations) then controlling unitaries off those measurements.

Non commutative quantum control

Additional slide (not related to audio)



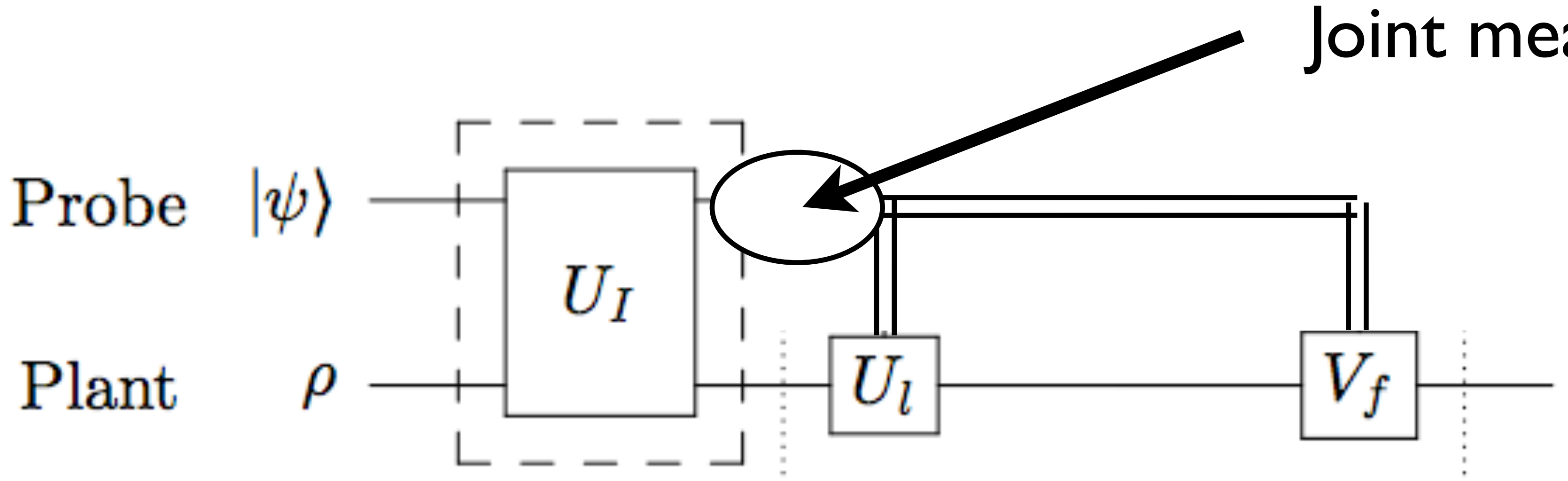


$$\sum_l |l\rangle\langle l| \otimes U_l \qquad \sum_f |f\rangle\langle f| \otimes V_f \qquad |f\rangle = F|l\rangle$$

$$\exists \; l, f : \quad [U_l, V_f] \neq 0 \qquad |\langle f|l\rangle|^2 = \frac{1}{d} \;\; \forall f, l$$

From the principle of non commutative control the measurement based approximation is:

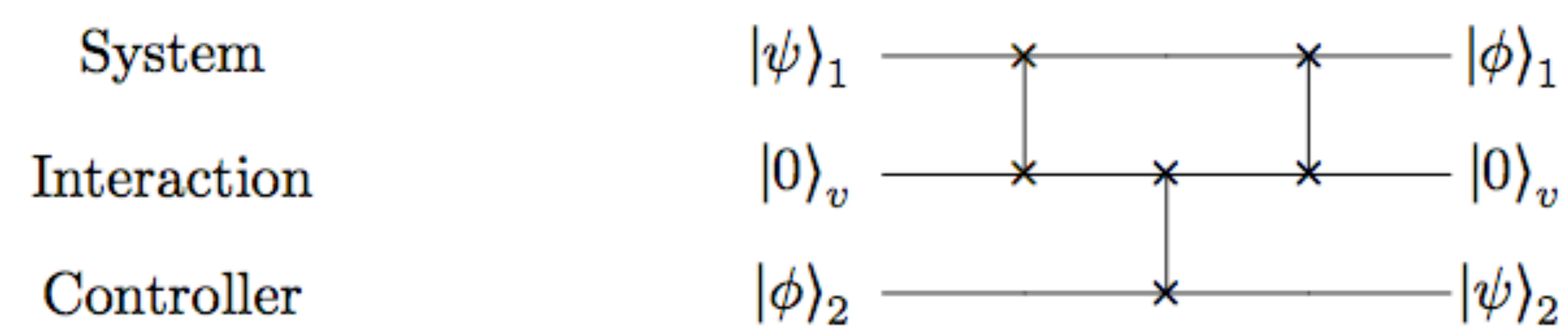
Joint measurement of "*l*" and "*f*"

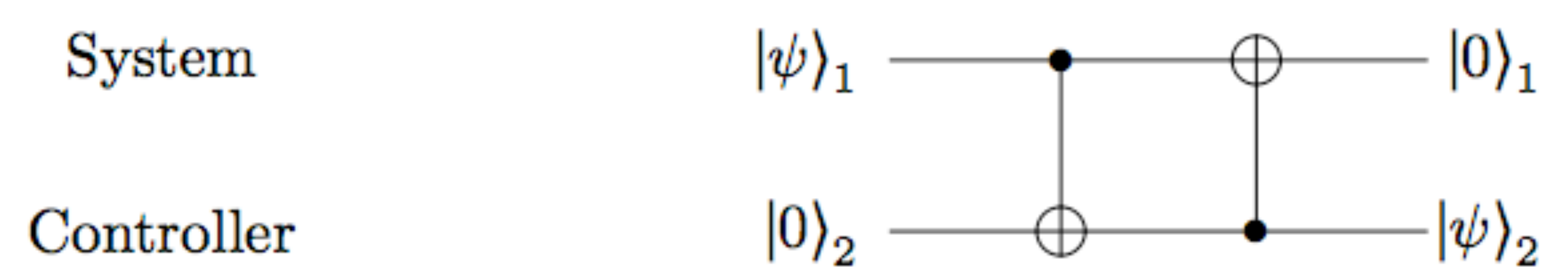


Non commutative quantum control

Additional slide
(not related to audio)

S. Lloyd
Coherent quantum feedback
Phys. Rev. A **62** 022108 (2000)





examples of SWAP... not really non commutative control

We don't care if the interaction unitary is or is not mediated by fields. We can still determine if non commutative quantum control is taking place.

Non commutative quantum control + sensing

Feedforward control

Coherent Quantum-Noise Cancellation for Optomechanical Sensors
M. Tsang and C. M. Caves
Phys. Rev. Lett. **105**, 123601 (2010)

Feedback control

Advantages of Coherent Feedback for Cooling Quantum Oscillators
R. Hamerly and H. Mabuchi
Phys. Rev. Lett. **109**, 173602 (2012)

uses optimization & design principles from →

Coherent quantum LQG control
H. I. Nurdin, M. R. James, and I. R. Petersen
Automatica **45**, 1837 (2009).

Sensing

Achieving minimum-error discrimination of an arbitrary set of laser-light pulses
M. P. da Silva, S. Guha, Z. Dutton
arXiv:1208.5758

optical application of →

Ideal state discrimination with an O(1)-qubit quantum computer
R. Blume-Kohout, S. Croke, M. Zwolak
arXiv:1201.6625

END

Quantum control system design and performance

Information is Physical
-Landauer

Quantum control system design and performance

determined by physics
and balanced by budget

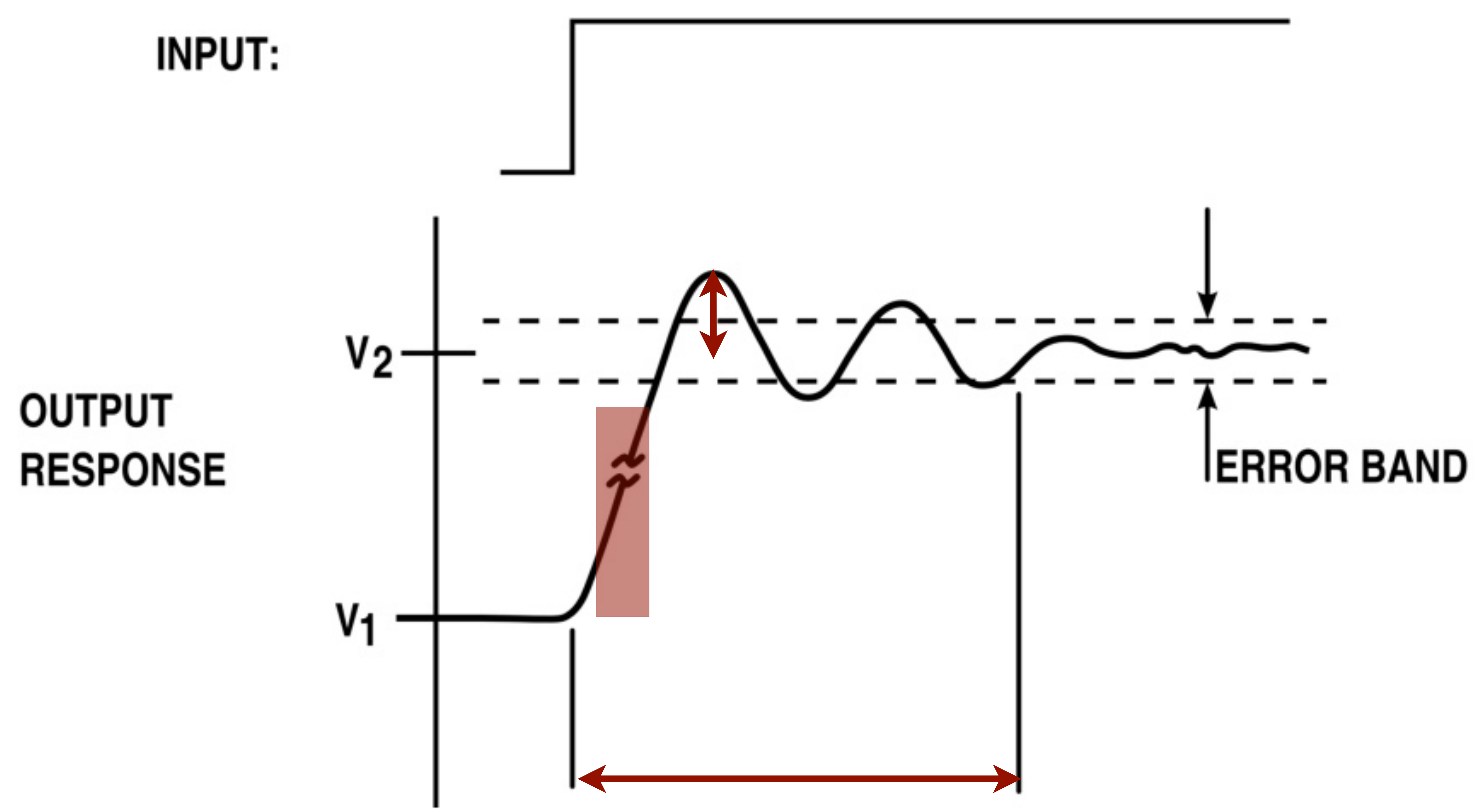
Control system design and performance

- Mathematical model of plant
  - n^{th} order linear Differential Equation (DE)
  - nonlinear DE e.g ODE or PDE
  - linear / nonlinear stochastic DE

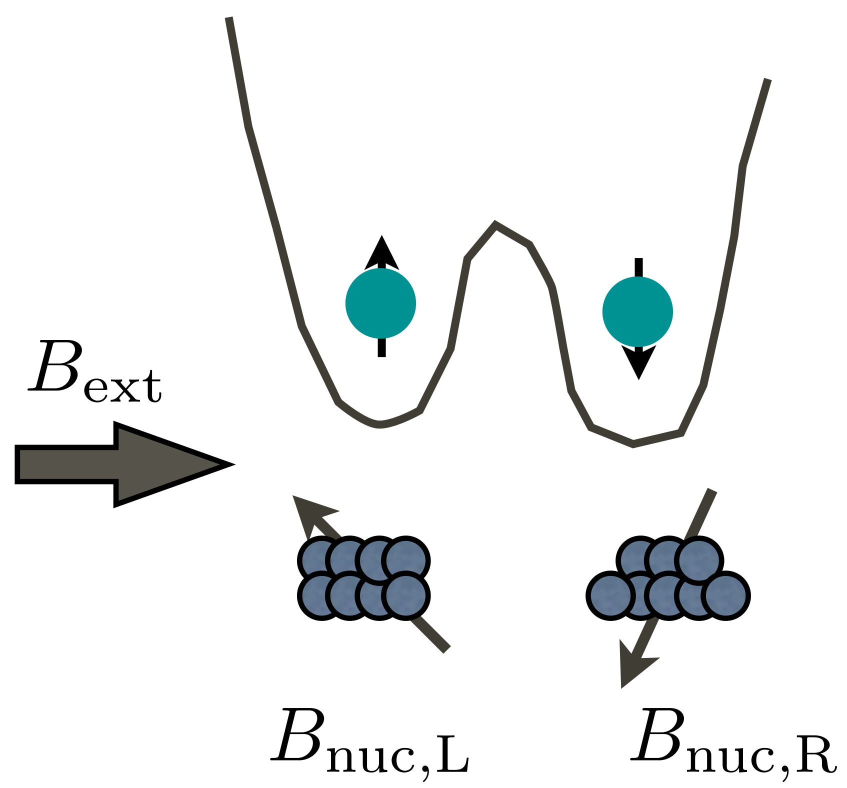
- Controllability
- Stability

- Performance / Objectives
  - Steady state response
  - transient response

determined by physics
and balanced by budget



Open loop control



$$\Delta B_{\text{nuc}} = B_{\text{nuc,L}} - B_{\text{nuc,R}}$$

$$H = \gamma \Delta B \frac{\sigma_x}{2}$$

$$\omega \in [0, \omega_0]$$

Slowly varying