COHERENT CONTROL VIA QUANTUM FEEDBACK NETWORKS

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PRIFYSGOL ABERYSTWYTH UNIVERSITY

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Pioneering research and skills

Papers

- J.G. M.R. James, *Commun. Math. Phys., 287, 1109-1132* (2009)
- J. G., M.R. James, *IEEE Transactions on Automatic Control*, (2009)
- O.G. Smolyanov, A. Truman, Doklady Math, Vol 8, No. 3, 974-977(2010)h, Vol 8, No. 3, 974-977(2010)

Quantum Technology: The 2nd Quantum Revolution*

Organizing and controlling the components of complex systems governed by the laws of quantum physics.

New principles:

- uncertainty principle
- superposition of states
- tunneling
- entanglement
- decoherence

∗ J.P. Dowling and G.J. Milburn, Phil Trans Roy. Soc. London (2003)

Why networks?

Figure : single transistor

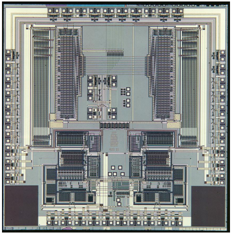


Figure : integrated network

NETWORKS AND FEEDBACK CONTROL

Types of closed loop control:

- Coherent feedback control
- Measurement-based feedback control

The distinction is fundamental in quantum control!

Coherent Feedback

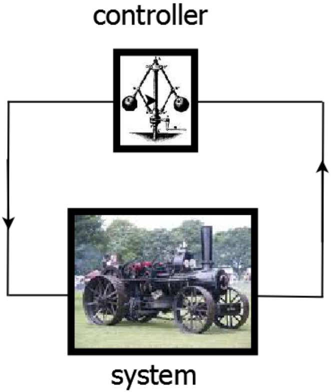


Figure : system and controller

Measurement Based Feedback

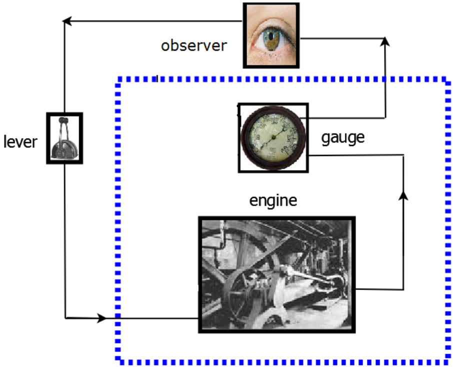


Figure : System controlled by measurement

Control through Interconnection!

Denmark's great conribution to technology ...

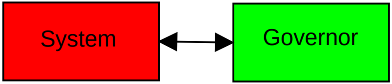
Control through Interconnection!

Denmark's great conribution to technology ...

Figure : Lego!!!!!

Connections through direct coupling

Given an system with Hamiltonian H_S on Hilbert space $\mathfrak{h}_S$, couple the system directly to a second system (the governor) with Hilbert space $\mathfrak{h}_G$.



The total evolution on $\mathfrak{h}_S \otimes \mathfrak{h}_G$ is of the form

$$H = H_S \otimes 1_G + 1_S \otimes H_G + V.$$

Design problems of this type first promoted by Seth Lloyd.

Connections mediated by quantum fields

We consider formal "white noise" processes

$$\left[b(t), b^{\dagger}(s)\right] = \delta(t-s)$$

with

$$B(t) = \int_0^t b(s)\, ds, \quad B^{\dagger}(t) = \int_0^t b^{\dagger}(s)\, ds.$$

It is possible to build a non-commutative version of the Itō calculus (**Hudson-Parthasarathy**) on the Fock space over $L^2[0, \infty)$ with respect to differentials $dB(t)$ and $dB^{\dagger}(t)$, and we have

$$dB(t)\ dB^{\dagger}(t) = dt.$$

A **unitary** system + noise dynamics:

$$dU = \left\{ L \otimes dB^\dagger - L^\dagger \otimes dB - iH \otimes dt \right\} \circ U$$

Weyl-Stratonovich form,

$$\equiv \left\{ L \otimes dB^\dagger - L^\dagger \otimes dB - \left(\frac{1}{2} L^\dagger L + iH \right) \otimes dt \right\} U$$

Wick-Itō form.

Mathematically, the Itō version is well-defined and one has $X \circ dY = XdY + \frac{1}{2} dXdY$. Itō differentials are future pointing: $dX(t) := X(t + dt) - X(t)$.

The flow of system observables $j_t(X) = U^\dagger(t)[X \otimes 1]U(t)$:

$$dj_t(X) = j_t(\mathcal{L}X)\,dt + j_t([X,L])dB^\dagger + j_t(\left[L^\dagger, X\right])dB,$$

where the (Gorini-Kossakowski-Sudarshan-Lindblad) generator is

$$\mathcal{L}X = \frac{1}{2}[L^\dagger, X]L + \frac{1}{2}L^\dagger[X,L] - i[X,H].$$

Taking averages in the vacuum state:

$$\frac{d}{dt}\langle j_t(X)\rangle = \langle j_t(\mathcal{L}X)\rangle,$$

as the forward pointing differentials average to zero.

For example, we may have an optical cavity with coupling

$$L = \sqrt{\gamma} a.$$

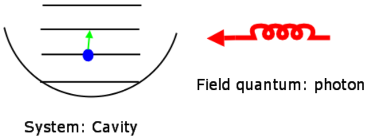


Figure : Absorption of field quanta

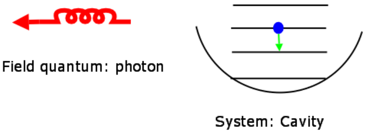


Figure : emission of field quanta

It is possible to introduce a **scattering process**

$$\Lambda(t) = \int_0^t b^{\dagger}(s)\, b(s)\, ds$$

and we have the **quantum Itō table**

$\times$	$d\Lambda$	$dB^{\dagger}$
dB	dB	dt
$d\Lambda$	$d\Lambda$	$dB^{\dagger}$

We have the quantum Itō product rule

$$d(XY) = X \circ dY + (dX) \circ Y = X\, dY + dX\, Y + dX\, dY\,.$$

All done in 1984 by Hudson and Parthasarathy!

The general unitary is $\left(E^{\dagger}_{\alpha\beta} = E_{\beta\alpha} \right)$

$$dU = -i \left\{ E_{11} \otimes d\Lambda + E_{10} \otimes dB^{\dagger} + E_{01} \otimes dB + E_{00} \otimes dt \right\} \circ U$$

with formal Hamiltonian

$$\Upsilon(t) = E_{11} \otimes b^{\dagger}(t) b(t) + E_{10} \otimes b^{\dagger}(t) + E_{01} \otimes b(t) + E_{00} \otimes 1.$$

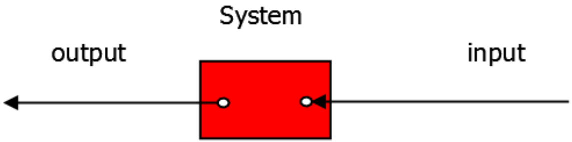
The Itō form is

$$dU = \left\{ (S - I) \otimes d\Lambda + L \otimes dB^\dagger - L^\dagger S \otimes dB - (\frac{1}{2} L^\dagger L + iH) \otimes dt \right\} U$$

where

$$\begin{aligned} S &= \frac{1 - \frac{i}{2} E_{11}}{1 + \frac{i}{2} E_{11}} \quad \textbf{(unitary!)}, \; L = i \frac{1}{1 + \frac{i}{2} E_{11}} E_{10}, \\ H &= E_{00} + \frac{1}{2} E_{01} \mathrm{Im}\{ \frac{1}{1 + \frac{i}{2} E_{11}} \} E_{10} \quad \textbf{(self-adjoint!)}. \end{aligned}$$

We represent a system (S, L, H) as a single component with input and output field:



- **System Hamiltonian** H.
- **Coupling operator** L between the system and the field.
- **Scattering operator** S, unitary.

The Markov Property: *the past is statistically independent of the future given the present*.

We note that the Fock space $\mathfrak{F}$ for the Bose field decomposes for each times $s < t$ as

$$\mathfrak{F} = \mathfrak{F}_{\leq s} \otimes \mathfrak{F}_{[s,t]} \otimes \mathfrak{F}_{\geq t},$$

where $\mathfrak{F}_{[s,t]}$ is the Fock space for the degrees of freedom of the field passing through the system from time s to time t.

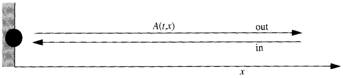


Figure : Gardiner's input formalism

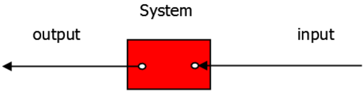


Figure : (S,L,H)

Quantum Itō Evolution

Closed evolution (Schrödinger equation) $dU_t = -iHU_t dt$.

Itō QSDE

Unitary adapted quantum stochastic evolution

$$\begin{aligned} dU_t &= (S-I)U_t d\Lambda(t) + LU_t dB(t)^\dagger \\ &\quad - L^\dagger S U_t dB(t) - (\frac{1}{2}L^\dagger L + iH)U_t dt. \end{aligned}$$

Heisenberg equations $j_t(X) = U_t^\dagger (X \otimes 1) U_t$

$$\begin{aligned} dj_t(X) &= j_t(S^\dagger X S - X) d\Lambda(t) \\ &\quad + j_t(S^\dagger [X, L]) dB(t)^\dagger + j_t([L^\dagger, X] S) dB(t) + j_t(\mathcal{L}X) dt. \end{aligned}$$

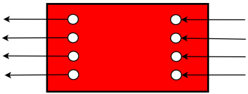
Output fields $B_{\text{out}}(t) = U_t^\dagger (1 \otimes B(t)) U_t$

$$dB_{\text{out}}(t) = j_t(S) dB(t) + j_t(L) dt.$$

Multiple inputs/outputs

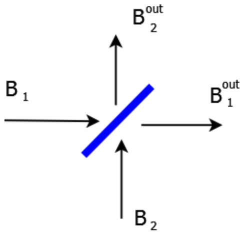
We may also represent the multi-channel case

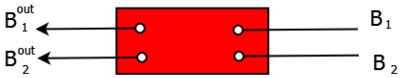
$$B = \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix}, S = \begin{bmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{nn} \end{bmatrix}, L = \begin{bmatrix} L_1 \\ \vdots \\ L_n \end{bmatrix}$$

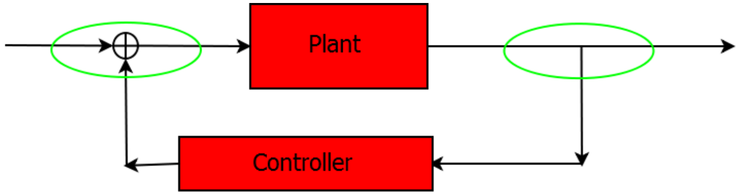


These are a special case where $L = 0$ amd $H = 0$.

$$\begin{bmatrix} B_1^{\text{out}} \\ B_2^{\text{out}} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$







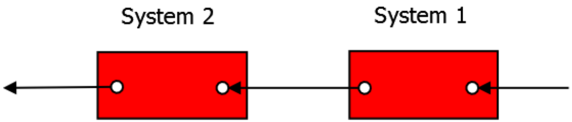
cannot happen in the quantum setting!!!

must use unitary junctions (e.g., beamsplitters)

Figure : classical feedback diagram

Cascades: Systems in Series

We generalize the notion of cascade introduced by H.J. Carmichael†.



$$
\begin{aligned}
dB_{\text{out}}^{(2)} &= S_2 dB_{\text{in}}^{(2)} + L_2 dt \\
&= S_2 (S_1 dB_{\text{in}}^{(1)} + L_1 dt) + L_2 dt \\
&= S_2 S_1 B_{\text{in}}^{(1)} + (S_2 L_1 + L_2) dt
\end{aligned}
$$

† H.J. Carmichael, Phys. Rev. Lett., 70(15):2273 2276, 1993.

The Series Product*

The cascaded system in the **instantaneous feedforward** limit is equivalent to the single component

$$(S_2, L_2, H_2) \lhd (S_1, L_1, H_1) = \left(S_2 S_1, L_2 + S_2 L_1, H_1 + H_2 + \mathrm{Im}\left\{ L_2^\dagger S_2 L_1 \right\} \right).$$

∗ J. G., M.R. James, *The Series Product and Its Application to Quantum Feedforward and Feedback Networks* IEEE Transactions on Automatic Control, 2009.

Modeling double-pass atom-field coupling

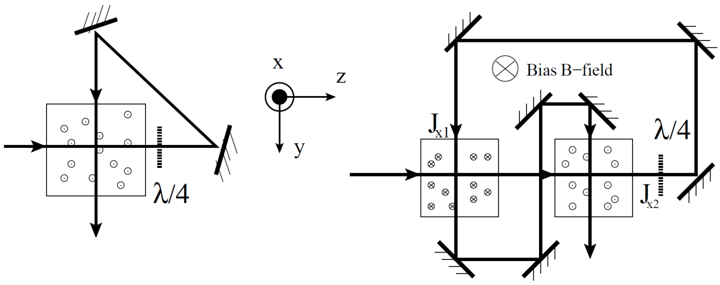
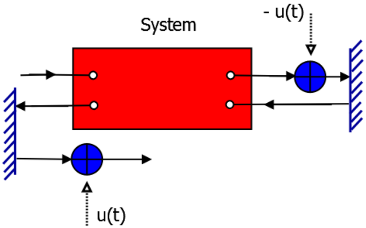


Figure : Production of Squeezed Light

J. F. Sherson and K. Moelmer, Phys. Rev. Lett. 97, 143602 (2006).
Gopal Sarma, Andrew Silberfarb, and Hideo Mabuchi Phys. Rev. A 78, 025801 (2008)

Bilinear Control Hamiltonian*



Based on H. M. Wiseman and G. J. Milburn. *All-optical versus electro-optical quantum-limited feedback.* Phys. Rev. A, 49(5):41104125, 1994.

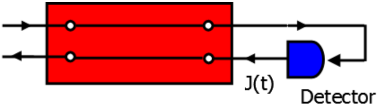
$$(I, u(t), 0) \triangleleft (-I, 0, 0) \triangleleft (I, L, 0) \triangleleft (-I, 0, 0) \triangleleft (I, -u(t), 0) \triangleleft (I, L, 0) = (I, 0, H(t))$$

where

$$H(t) = \mathrm{Im}\{L^{\dagger} u(t)\} = \frac{1}{2i} L^{\dagger} u(t) - \frac{1}{2i} L u(t)^{*} .$$

$*$ J. G., *Construction of bilinear control Hamiltonians using the series product and quantum feedback* Phys. Rev. A 78, 052311 (2008)

Direct Measurement Feedback



- 1st pass - (I, L, H_0)
- 2nd pass - corresponding to

$$U(t+dt,t) = \exp\{-iFdJ(t)\}.$$

Homodyne detection, $J_t = B(t) + B(t)^\dagger$, $(dJ)^2 = dt$, 2nd pass is $(I, -iF, 0)$

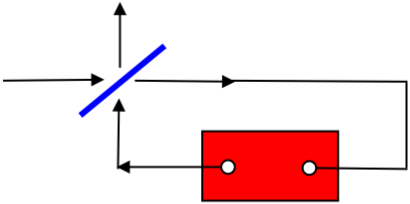
$$\text{closed loop } (I, -iF, 0) \lhd (I, L, H_0) = \left(I, L - iF, H_0 + \frac{1}{2}\left(FL + L^\dagger F \right) \right);$$

Photon counting, $J_t = \Lambda_t$, $(dJ)^2 = dJ$, 2nd pass is $(S = e^{-iF}, 0, 0)$

$$\text{closed loop } (S, 0, 0) \lhd (I, L, H_0) = (S, SL, H_0).$$

Components in-loop

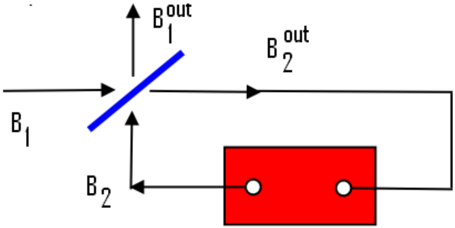
Model considered by M. Yanagisawa utilizing a beamsplitter



Feedback loops introduce topologically nontrivial paths!
Which way did the signal go?

Example

beamsplitter $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$,
and *in-loop component* $(S_0, L_0, 0)$:

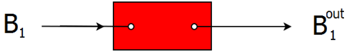


$$dB_2 = S_0 dB_2^{\rm out} + L_0 dt = S_0(S_{21}dB_1 + S_{22}dB_2) + L_0 dt$$
$$\Rightarrow dB_1^{\rm out} = S_{11}dB_1 + S_{12}dB_2 \equiv \hat{S}_0 dB_1 + \hat{L}_0 dt$$

where

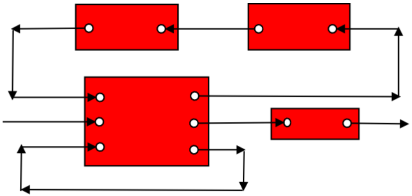
$$\hat{S}_0 = S_{11} + S_{12}(I - S_0 S_{22})^{-1} S_0 S_{21}, \quad \hat{L}_0 = S_{12}(I - S_{22})^{-1} S_0 L_0.$$

Equivalent component $(\hat{S}_0, \hat{L}_0, \hat{H}_0)$:



General Networks

More generally how do we build arbitrary networks from multiple components.

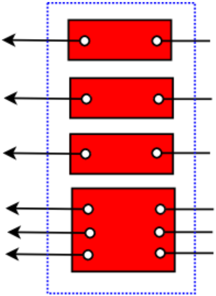


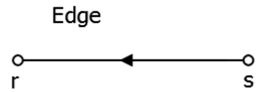
How do we obtain the limit of instantaneous feedback/forward, i.e., eliminate the internal connections?

Open loop system in parallel

Concatenation

$$\boxplus_{j=1}^{n}(S_j, L_j, H_j) = \left(\begin{bmatrix} S_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & S_n \end{bmatrix}, \begin{bmatrix} L_1 \\ \vdots \\ L_n \end{bmatrix}, H_1 + \cdots + H_n \right).$$





Edge elimination*

The reduced model obtained by eliminating the edge (r_0, s_0) is

$$\begin{aligned} S^{\mathrm{red}}_{sr} &= S_{sr} + S_{sr_0}\left(1 - S_{s_0 r_0}\right)^{-1} S_{s_0 r}, \\ L^{\mathrm{red}}_{s} &= L_s + S_{sr_0}\left(1 - S_{s_0 r_0}\right)^{-1} L_{s_0}, \\ H^{\mathrm{red}} &= H + \sum_{\text{inputs } s} \mathrm{Im} L_s^\dagger S_{sr_0}\left(1 - S_{s_0 r_0}\right)^{-1} L_{s_0}. \end{aligned}$$

∗ J. G., M.R. James, *Quantum Feedback Networks: Hamiltonian Formulation* Commun. Math. Phys., 1109-1132, Volume 287, Number 3 / May, 2009.

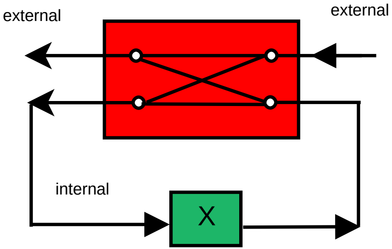
Properties of the Feedback Reduction Formula

- Mathematically a Schur complement of the matrix of coefficient operators

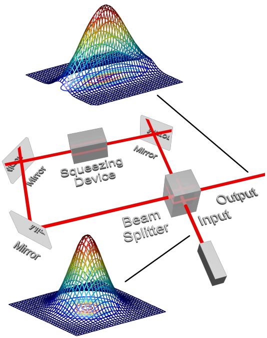
$$\mathbf{G} = \begin{bmatrix} -\frac{1}{2}L^*L - iH & -L^*S \\ L & S - I \end{bmatrix}.$$

  Equivalently formulated as a fractional linear transformation.
- Independent of the order of edge-elimination.
- Commutes with adiabatic elimination of fast degrees of freedom of components (see talk by Hendra Nurdin).

The contruction of system with controller forming a coherent feedback system mediated by Bose fields is now routine.



In-loop degenerate parametric amplifier



$H_{\mathrm{DPA}} = \frac{i\varepsilon}{4}\left(a^{\dagger 2} - a^2\right)$, $L = \sqrt{\kappa}a$.
Beamsplitter with matrix

$$T = \begin{bmatrix} \alpha & \sqrt{1-\alpha^2} \\ \sqrt{1-\alpha^2} & -\alpha \end{bmatrix}.$$

In-loop renormalized coupling strength

$$\kappa(\alpha) = \frac{1-\alpha}{1+\alpha}\kappa.$$

Squeezing parameter $r_{\mathrm{DPA}}(\alpha) = \ln \dfrac{\kappa(\alpha)+\varepsilon}{\kappa(\alpha)-\varepsilon}$.

J.G, S. Wildfeuer *Enhancement of Field Squeezing Using Coherent Feedback*, Phys. Rev. A 80, 042107 (2009)

S. Iida, M. Yukawa, H. Yonezawa, N. Yamamoto, and A. Furusawa, *Experimental demonstration of coherent feedback control on optical field squeezing*, IEEE TAC 2011

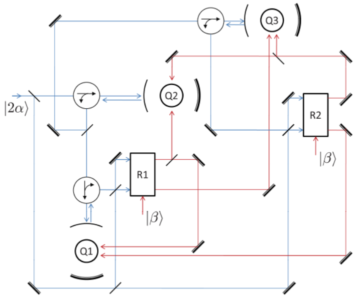


Figure : Schematic diagram of a coherent-feedback

J. Kerckhoff, H.I. Nurdin, D. Pavlichin, H. Mabuchi *Designing Quantum Memories with Embedded Control: Photonic Circuits for Autonomous Quantum Error Correction*, PRL 105, 040502 (2010)