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Verification and Validation of Controlled Quantum Information Systems

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Robust Control Design

For quantum information systems, a *robust optimization problem* that we often want to solve can be expressed as

$$\max_{\theta} \min_{\delta} \mathcal{F}[\theta, \delta]$$

subject to $\theta \in \Theta$ and $\delta \in \Delta$

$\mathcal{F}[\theta, \delta] = |\mathrm{Tr}(U[\theta, \delta] V^\dagger)|,$ quantum gate fidelity

$U[\theta, \delta]$: actual unitary operation; V : target unitary operation

δ : uncertain and stochastic parameters; Δ : corresponding set

θ : control and design parameters; Θ : corresponding set

Caveat emptor: This is often not a convex optimization problem!

Robust Control Design

- Robust control and optimization of uncertain systems are essential in science and engineering.

- QIP requires an unprecedented degree of control!
  - Active area of research
  - Many control protocols involve some form of numerical optimization
  - High-fidelity results are possible, e.g., $1 - \mathcal{F} \in [10^{-6}, 10^{-4}]$, which is “1” for most engineering problems, but not QIP!

Sequential Convex Programming

Initialize

Initialize control: $\theta \in \Theta \subseteq \mathbf{R}^N$; Sample uncertainties: $\delta_i \in \Delta$;
Set trust region: $\tilde{\Theta} \subseteq \mathbf{R}^N$

Repeat

1. Calculate fidelities, gradients, and Hessians with respect to θ:

$$\mathcal{F}[\theta, \delta_i], \quad \nabla_\theta \mathcal{F}[\theta, \delta_i], \quad \nabla_\theta^2 \mathcal{F}[\theta, \delta_i]$$

2. Solve convex optimization for $\tilde{\theta}$ using linearized fidelity:

$$\max_{\tilde{\theta}} \ \min_{\delta_i} \ \mathcal{F}[\theta, \delta_i] \ + \ \left(\nabla_\theta \mathcal{F}[\theta, \delta_i]\right)^{\mathrm{T}} \tilde{\theta} \ - \ \tilde{\theta}^{\mathrm{T}} \nabla_\theta^2 \mathcal{F}[\theta, \delta_i] \tilde{\theta}/2$$

subject to $\theta + \tilde{\theta} \in \Theta$ and $\|\tilde{\theta}\|_\infty \leq \tilde{\Theta}$

3. Update

IF	$\min_i \ \mathcal{F}(\theta + \tilde{\theta}, \delta_i) > \min_i \mathcal{F}(\theta, \delta_i)$
THEN	$\theta \leftarrow \theta + \tilde{\theta}$ and increase trust region $\tilde{\Theta}$
ELSE	decrease trust region $\tilde{\Theta}$

Until Convergence criteria satisfied

Sequential Convex Programming

- Incorporates convex constraints exactly, e.g.,

magnitude: $\theta_{\min} \le \theta(t) \le \theta_{\max}, \;\; 0 \le t \le T$

fluence: $\int_0^T \theta^2(t)dt \le \alpha$

area: $\int_0^T |\theta(t)|dt \le \beta$

slew rate: $\left|\dfrac{d\theta(t)}{dt}\right| \le \gamma, \;\; 0 \le t \le T$

- Incorporates uncertainties and stochasticity by sampling, e.g.,
  - An uncertain coefficient ω: $\delta = [\omega_1, \; \omega_2, \cdots]^{\mathrm{T}}$
  - A stochastic process Ω: $\delta(t) = [\omega_1(t), \; \omega_2(t), \cdots]^{\mathrm{T}}$

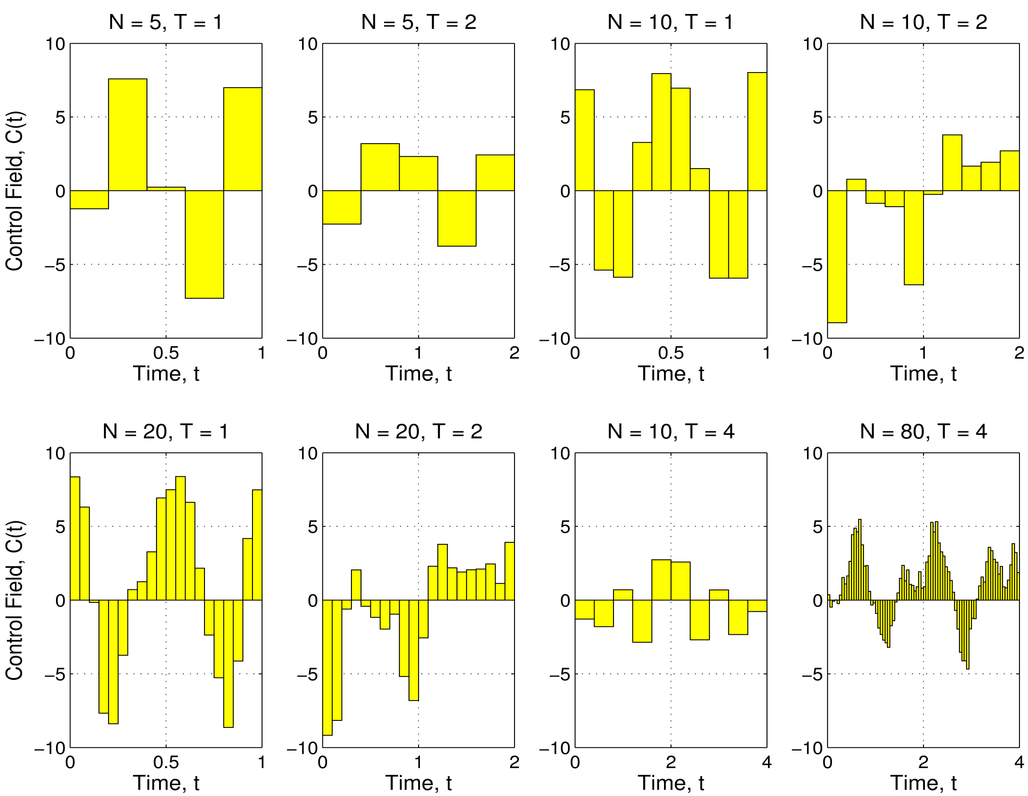
Robust Quantum Gates

$$H(t) = \omega_x \sigma_x + C(t)\omega_z \sigma_z \longrightarrow \dot{U}(t) = -\mathrm{i}H(t)U(t)$$

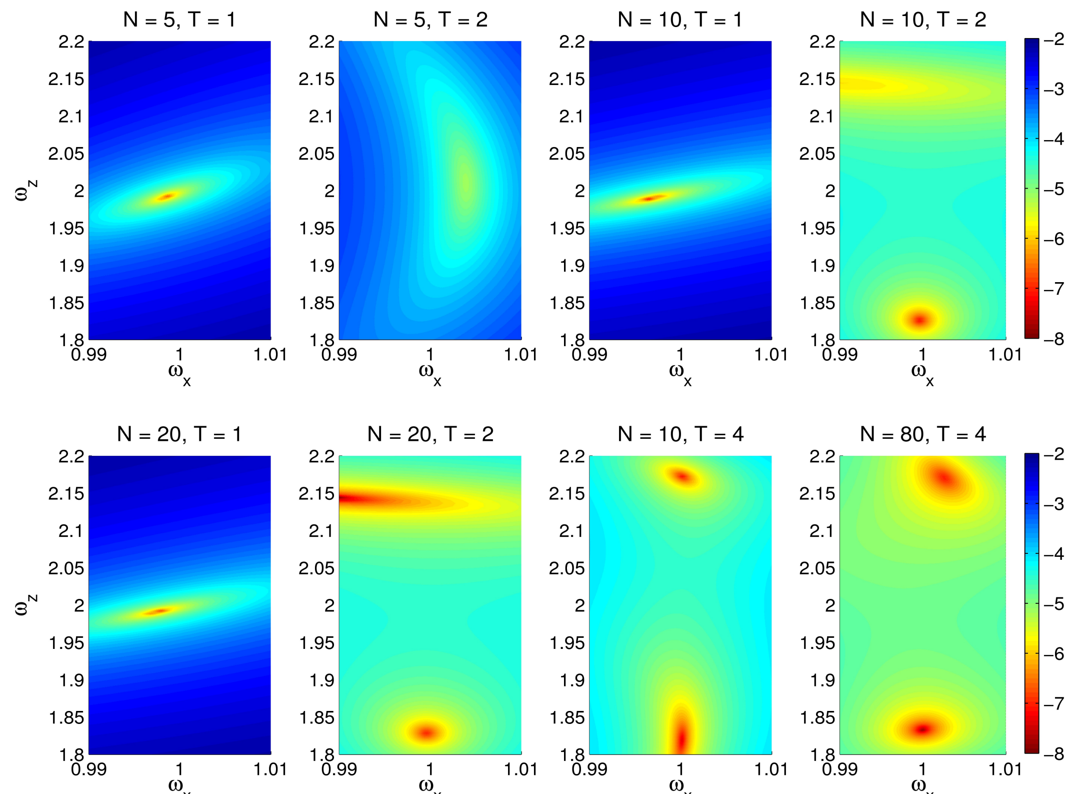
10% drift uncertainty (ω_x)

1% control amplitude uncertainty (ω_z)

$$U(T) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



Control Fields



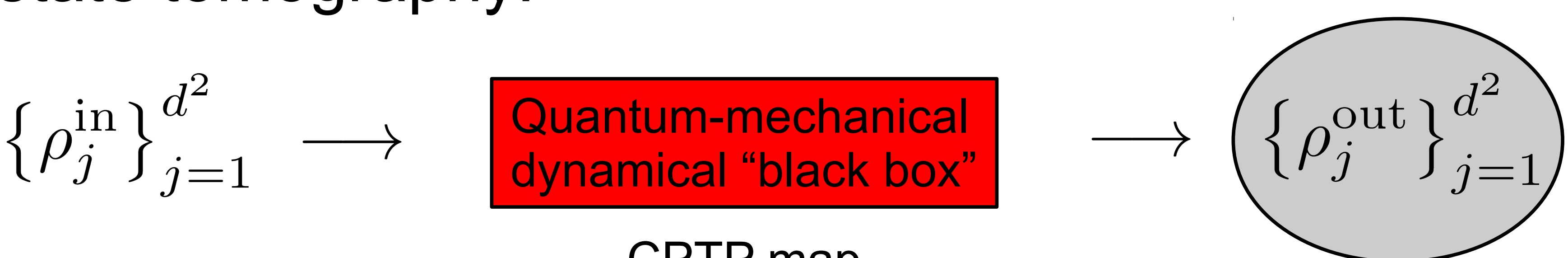
$\log_{10}(1 - \text{fidelity})$

V&V of Controlled QI Systems

- Characterization: Determining dynamics, qualities, properties, etc. of devices, models, etc.
- Fault detection, isolation, and recovery (e.g., QEC)
- Model calibration *versus* prediction
- Verification: “Did you build ***it*** correctly?”
- Validation: “Did you build the correct ***thing***?”
- State/process *determination* versus *validation*

State and Process Tomography

- Quantum state tomography
  - One qubit: $\rho = \left(\mathcal{I} + \vec{r}\cdot\vec{\sigma}\right)/2$
  - Bloch sphere measurements: $\langle\sigma_j\rangle = \mathrm{Tr}(\rho\sigma_j) = r_j$
- Quantum process tomography: identify the process via state tomography:



CPTP map

$$\rho_j^{\mathrm{out}} = \sum_k A_k \rho_j^{\mathrm{in}} A_k^\dagger$$

Perform state tomography on each output state: d^2-1 measurements!

I. Chuang & M. Nielsen, JMO, **44** (1997)

State to Process Tomography

1. Perform tomography on states $\left\{\rho_j^{\text{out}}\right\}_{j=1}^{d^2}$
2. Expand output density matrices:

$$\rho_j^{\text{out}} = \sum_k A_k \rho_j^{\text{in}} A_k^\dagger = \sum_{\alpha,\beta} \chi_{\alpha\beta} \sigma_\alpha \rho_j^{\text{in}} \sigma_\beta^\dagger = \sum_k r_{jk} \sigma_k$$

3. Expand basis operators

$$\sigma_\alpha \rho_j^{\text{in}} \sigma_\beta^\dagger = \sum_k \xi_{jk}^{\alpha\beta} \sigma_k$$

4. Combine expressions 2 and 3

$$\rho_j^{\text{out}} = \sum_k r_{jk} \sigma_k = \sum_k \sum_{\alpha,\beta} \chi_{\alpha\beta} \xi_{jk}^{\alpha\beta} \sigma_k \implies R = X\Xi$$

State Tomography Estimators

$$\rho = (\mathcal{I} + \vec{r} \cdot \vec{\sigma})/2 \longrightarrow \rho = \left(\mathcal{I} + \vec{s} \cdot \vec{\lambda}\right)/d$$

Multiqubit state

- Least-squares approach: $\mathcal{O}(d^2)$

$$\min_{\rho} \sum_j \left[m_j - \mathrm{Tr}(\mathcal{O}_j \rho)\right]^2$$, where $\vec{M}$: measurements

subject to $\mathrm{Tr}(\rho) = 1$ and $\rho \geq 0$

- Compressed-sensing approach: $\mathcal{O}(R_\rho d \log(d))$

$$\min_{\tilde{\rho}} \mathrm{Tr}(\tilde{\rho})$$, where $\rho = \tilde{\rho}/\mathrm{Tr}(\tilde{\rho})$

subject to $\min_{\tilde{\rho}} \sum_j \left[m_j - \mathrm{Tr}(\mathcal{O}_j \tilde{\rho})\right]^2 \leq \varepsilon$ and $\tilde{\rho} \geq 0$

P. Gross et al., PRL, **105** (2010)

Limitations of Tomography

- Scaling of measurements required is exponential in the number of qubits:

$$d^2 \Rightarrow 4^{n_\mathrm{q}} \text{ (QST); } d^4 - d^2 \Rightarrow 16^{n_\mathrm{q}} - 4^{n_\mathrm{q}} \text{ (QPT)}$$

- Conventional state and process tomography assumes high-fidelity measurements and preparations.
- Most technologies (except optics) do not have a full reference frame (e.g., independent calibrated X, Y, Z axes on the Bloch sphere). There may only be 1 preparation and 1 measurement operation.

QPT via Parameter Estimation

- Nonlinear mapping of Hamiltonian to process matrix in general, so certain conditions must be satisfied:
  - Use some knowledge of the underlying dynamics (Hamiltonian form: couplings, commutativity, etc.).
  - Sparsity: Number of Hamiltonian parameters may be much smaller than the elements of the process matrix.

  M. Branderhorst et al., NJP, **11** (2009)   A. Shabani et al., PRA, **84** (2011)

- Optimal Hamiltonian identification: Combining optimal control data and efficient data inversion

J. Geremia & H. Rabitz, PRL, **89** (2002)   J. Geremia & H. Rabitz, PRA, **70** (2004)

Randomized Benchmarking

"Twirling", i.e., $\int_{U(d)} U\Lambda\left(U^\dagger \rho U\right)U^\dagger dU$, transforms Λ uniquely to

a depolarizing channel Λ_d with the same average fidelity as Λ:

$$\Lambda_\mathrm{d}(\rho) = p\rho + (1-p)\mathcal{I}/d,$$

where $\bar{\mathcal{F}}\left(\Lambda, \mathcal{I}\right) = \bar{\mathcal{F}}\left(\Lambda_\mathrm{d}, \mathcal{I}\right) = p + (1-p)/d,$

and $\mathcal{F}_\rho\left[\mathcal{E}_1, \mathcal{E}_2\right] = \left[\mathrm{Tr}\sqrt{\sqrt{\mathcal{E}_1(\rho)}\mathcal{E}_2(\rho)\sqrt{\mathcal{E}_1(\rho)}}\right]^2.$

Objective: Estimate p while addressing the limitations of QPT.

Randomized Benchmarking

Via a 2-design, $\frac{1}{|\mathcal{C}|}\sum_{j=1}^{|\mathcal{C}|} C_j \Lambda\left(C_j^\dagger \rho C_j\right) C_j^\dagger = \int_{U(d)} U \Lambda\left(U^\dagger \rho U\right) U^\dagger dU.$

However, $|\mathcal{C}|$ grows exponentially with the number of qubits!

Protocol

1. For $m \leq M$, generate K_m random Clifford gate sequences:

$$S_{km}(\rho) = \frac{1}{m}\sum_{j=1}^{m} C_j \Lambda\left(C_j^\dagger \rho C_j\right) C_j^\dagger$$

2. Measure the "survival probability" (sequence fidelity):

$$F_{km} = \mathrm{Tr}\left[E_\psi S_{km}(\rho)\right],\ \text{where } E_\psi = \rho = |\psi\rangle\langle\psi| \text{ ideally}$$

3. Calculate average sequence fidelity F_m

4. Repeat for different values of m and fit results to fidelity model

E. Magesan et al., PRA, **85** (2012)

V&V of Controlled QI Systems

- Improved tomography via machine learning
- What are the tools for QMU, UQ, and V&V of quantum systems?
- Development of methods for efficient simulation of quantum systems, e.g., surrogates
- What are the roles of device and model/software V&V for controlled QI systems?
- What are the relevant validation metrics?
- What is the role of Hamiltonian estimation?