Renormalization approach to open quantum system dynamics

Giulia Gualdi, Christiane Koch

Institüt für Theoretische Physik, Universität Kassel

KITP

Santa Barbara, 15 01 2012

the big picture I

In a realistic set up a quantum system can never be fully isolated from its surrounding environment $\Rightarrow$ it is an open quantum system

the big picture I

In a realistic set up a quantum system can never be fully isolated from its surrounding environment ⇒ it is an open quantum system

⇒decoherence must be accounted for any time one aims at a realistic understanding or manipulation of a quantum system

(e.g. quantum optimal control QOC, implementation of quantum technologies)

the big picture I

In a realistic set up a quantum system can never be fully isolated from its surrounding environment $\Rightarrow$ it is an open quantum system

$\Rightarrow$decoherence must be accounted for any time one aims at a realistic understanding or manipulation of a quantum system

(e.g. quantum optimal control QOC, implementation of quantum technologies)

$\Rightarrow$**it is fundamental to have a reliable way of simulating open system dynamics starting from a microscopical picture**

the big picture I

In a realistic set up a quantum system can never be fully isolated from its surrounding environment $\Rightarrow$ it is an open quantum system

$\Rightarrow$decoherence must be accounted for any time one aims at a realistic understanding or manipulation of a quantum system

(e.g. quantum optimal control QOC, implementation of quantum technologies)

$\Rightarrow$**it is fundamental to have a reliable way of simulating open system dynamics starting from a microscopical picture**

"traditional" approach to open system dynamics:

retain minimal info about the environment E and formulate an equation in terms of a superoperator acting on $\rho_S(t) = \mathrm{Tr}_E \rho(t)$

$\Rightarrow$ simulate the open system dynamics via a **Master Equation**

Main limits of the "traditional" approach:

Main limits of the "traditional" approach:

- in general applicable only under Markovian or close-to-Markovian noise approximation (perturbative approach)

  $\Rightarrow$ weak S-E coupling, drastic assumptions on the bath (in general as soon as it has non-trivial structure fail to be fulfilled)

the big picture II

Main limits of the "traditional" approach:

- in general applicable only under Markovian or close-to-Markovian noise approximation (perturbative approach)

  ⇒ weak S-E coupling, drastic assumptions on the bath (in general as soon as it has non-trivial structure fail to be fulfilled)

  ⇒**what to do with a spin bath?** (in general no straightforward characterization via a spectral density, no clear *a priori* characterization of Markovian/non-Markovian, no analytical correlation functions, strong S-E coupling)

Main limits of the "traditional" approach:

- in general applicable only under Markovian or close-to-Markovian noise approximation (perturbative approach)

  $\Rightarrow$ weak S-E coupling, drastic assumptions on the bath (in general as soon as it has non-trivial structure fail to be fulfilled)

  $\Rightarrow$**what to do with a spin bath?** (in general no straightforward characterization via a spectral density, no clear *a priori* characterization of Markovian/non-Markovian, no analytical correlation functions, strong S-E coupling)

- not a controllable approximation

Main limits of the "traditional" approach:

- in general applicable only under Markovian or close-to-Markovian noise approximation (perturbative approach)

  ⇒ weak S-E coupling, drastic assumptions on the bath (in general as soon as it has non-trivial structure fail to be fulfilled)

  ⇒**what to do with a spin bath?** (in general no straightforward characterization via a spectral density, no clear *a priori* characterization of Markovian/non-Markovian, no analytical correlation functions, strong S-E coupling)

- not a controllable approximation

Why do we care about non-Markovian dynamics?

the big picture II

Main limits of the "traditional" approach:

- in general applicable only under Markovian or close-to-Markovian noise approximation (perturbative approach)

  ⇒ weak S-E coupling, drastic assumptions on the bath (in general as soon as it has non-trivial structure fail to be fulfilled)

  ⇒**what to do with a spin bath?** (in general no straightforward characterization via a spectral density, no clear *a priori* characterization of Markovian/non-Markovian, no analytical correlation functions, strong S-E coupling)

- not a controllable approximation

Why do we care about non-Markovian dynamics?

- interesting non-Markovian quantum systems (e.g. strong SE interaction and/or spin bath)

  solid-state devices for quantum computation and information (G. De Lange et al., Science 330, 60 (2010); H. Bluhm et al., Nature Phys. 7, 109 (2010),...), light-harvesting complexes (J. Prior, A. W. Chin, S. F. Huelga, M. B. Plenio, PRL 105, 050404 (2010); F. Caruso, S. Huelga, M. Plenio PRL 105, 190501 (2010); M. Sarovar, A. Ishizaki, G. R. Fleming, K. B. Whaley, Nature Phys. 6, 462 (2010),...), etc...

the big picture II

Main limits of the "traditional" approach:

- in general applicable only under Markovian or close-to-Markovian noise approximation (perturbative approach)

  ⇒ weak S-E coupling, drastic assumptions on the bath (in general as soon as it has non-trivial structure fail to be fulfilled)

  ⇒**what to do with a spin bath?** (in general no straightforward characterization via a spectral density, no clear *a priori* characterization of Markovian/non-Markovian, no analytical correlation functions, strong S-E coupling)

- not a controllable approximation

Why do we care about non-Markovian dynamics?

- interesting non-Markovian quantum systems (e.g. strong SE interaction and/or spin bath)

  solid-state devices for quantum computation and information (G. De Lange et al., Science 330, 60 (2010); H. Bluhm et al., Nature Phys. 7, 109 (2010),...), light-harvesting complexes (J. Prior, A. W. Chin, S. F. Huelga, M. B. Plenio, PRL 105, 050404 (2010); F. Caruso, S. Huelga, M. Plenio PRL 105, 190501 (2010); M. Sarovar, A. Ishizaki, G. R. Fleming, K. B. Whaley, Nature Phys. 6, 462 (2010),...), etc...

- controllability is expected to be better for non-Markovian systems due to information backflow

observations and conjectures

Observation:

in very different contexts numerical simulations of non-Markovian dynamics based on truncating the number of environmental modes often show that a comparatively small number of modes is sufficient to reach converged results

(R.Baer, R. Kosloff JCP **106** 8862 (1997); C.P.Koch,T.Klüner,H.J.Freund, R.Kosloff PRL **90** 117601 (2003); H.J. Hogben, P.Hore, I.Kuprov, JCP **132** 174101 (2010), K.H.Hughes, C.D. Christ, I.Burghardt, JCP **131** 024109 (2009),....)

Observation:

in very different contexts numerical simulations of non-Markovian dynamics based on truncating the number of environmental modes often show that a comparatively small number of modes is sufficient to reach converged results

(R.Baer, R. Kosloff JCP **106** 8862 (1997); C.P.Koch,T.Klüner,H.J.Freund, R.Kosloff PRL **90** 117601 (2003); H.J. Hogben, P.Hore, I.Kuprov, JCP **132** 174101 (2010), K.H.Hughes, C.D. Christ, I.Burghardt, JCP **131** 024109 (2009),....)

Conjecture:

it takes time to establish correlations between system and environment

Observation:

in very different contexts numerical simulations of non-Markovian dynamics based on truncating the number of environmental modes often show that a comparatively small number of modes is sufficient to reach converged results

(R.Baer, R. Kosloff JCP **106** 8862 (1997); C.P.Koch,T.Klüner,H.J.Freund, R.Kosloff PRL **90** 117601 (2003); H.J. Hogben, P.Hore, I.Kuprov, JCP **132** 174101 (2010), K.H.Hughes, C.D. Christ, I.Burghardt, JCP **131** 024109 (2009),....)

Conjecture:

it takes time to establish correlations between system and environment

$\Rightarrow$ the system interacts *progressively* with the environment

observations and conjectures

Observation:

in very different contexts numerical simulations of non-Markovian dynamics based on truncating the number of environmental modes often show that a comparatively small number of modes is sufficient to reach converged results

(R.Baer, R. Kosloff JCP **106** 8862 (1997); C.P.Koch,T.Klüner,H.J.Freund, R.Kosloff PRL **90** 117601 (2003); H.J. Hogben, P.Hore, I.Kuprov, JCP **132** 174101 (2010), K.H.Hughes, C.D. Christ, I.Burghardt, JCP **131** 024109 (2009),....)

Conjecture:

it takes time to establish correlations between system and environment

$\Rightarrow$ the system interacts *progressively* with the environment

$\Rightarrow$ it is a quite general feature of quantum dynamics.

Observation:

in very different contexts numerical simulations of non-Markovian dynamics based on truncating the number of environmental modes often show that a comparatively small number of modes is sufficient to reach converged results

(R.Baer, R. Kosloff JCP **106** 8862 (1997); C.P.Koch,T.Klüner,H.J.Freund, R.Kosloff PRL **90** 117601 (2003); H.J. Hogben, P.Hore, I.Kuprov, JCP **132** 174101 (2010), K.H.Hughes, C.D. Christ, I.Burghardt, JCP **131** 024109 (2009),....)

Conjecture:

it takes time to establish correlations between system and environment

$\Rightarrow$ the system interacts *progressively* with the environment

$\Rightarrow$ it is a quite general feature of quantum dynamics.

In this talk:

prove the conjecture

identify the necessary ingredients for an accurate and efficient simulation of open quantum systems

observations and conjectures

Observation:

in very different contexts numerical simulations of non-Markovian dynamics based on truncating the number of environmental modes often show that a comparatively small number of modes is sufficient to reach converged results

(R.Baer, R. Kosloff JCP **106** 8862 (1997); C.P.Koch,T.Klüner,H.J.Freund, R.Kosloff PRL **90** 117601 (2003); H.J. Hogben, P.Hore, I.Kuprov, JCP **132** 174101 (2010), K.H.Hughes, C.D. Christ, I.Burghardt, JCP **131** 024109 (2009),....)

Conjecture:

it takes time to establish correlations between system and environment

$\Rightarrow$ the system interacts *progressively* with the environment

$\Rightarrow$ it is a quite general feature of quantum dynamics.

In this talk:

prove the conjecture

identify the necessary ingredients for an accurate and efficient simulation of open quantum systems

General philosophy: system-bath unitary dynamics + renormalization group VS system non-unitary dynamics + *a priori* assumptions on the bath

Outline

Outline

- discrete environments/local interactions $\Rightarrow$ **quasi-locality of quantum dynamics**

Outline

- discrete environments/local interactions $\Rightarrow$ **quasi-locality of quantum dynamics**
- continuous environments/non-local interactions $\Rightarrow$ **quasi-finite resolution of quantum dynamics**

Outline

- discrete environments/local interactions $\Rightarrow$ **quasi-locality of quantum dynamics**

- continuous environments/non-local interactions $\Rightarrow$ **quasi-finite resolution of quantum dynamics**

- correspondence between discrete and continuous environments

Outline

- discrete environments/local interactions $\Rightarrow$ **quasi-locality of quantum dynamics**
- continuous environments/non-local interactions $\Rightarrow$ **quasi-finite resolution of quantum dynamics**
- correspondence between discrete and continuous environments
- $\Rightarrow$ **time-induced renormalization of system-environment interaction**

Quasi-Locality of quantum dynamics

discrete environments: microscopic model

system-bath Hamiltonian

$$\hat{H} = \hat{H}_S + \sum_{i=1}^{N_S^{int}} \sum_{j=1}^{N_B^{int}} \hat{\Phi}_{ij}^{SB} + \sum_{i \leq j=1}^{N_B} \hat{\Phi}_{ij}^{B}$$

with $\hat{\Phi}_{ij} = \sum_{\mu=0}^{dim(\mathcal{B}(\mathcal{H}_i))-1} \sum_{\nu=0}^{dim(\mathcal{B}(\mathcal{H}_j))-1} J_{ij}^{\mu\nu} \hat{O}_i^{\mu} \hat{O}_j^{\nu}$ and $N_S^{int} \leq N_S; N_B^{int} \leq N_B \to \infty$.

Our goal is to truncate the sums over the environmental DOF in a well-defined manner

system-bath Hamiltonian

$$\hat{H} = \hat{H}_S + \sum_{i=1}^{N_S^{int}} \sum_{j=1}^{N_B^{int}} \hat{\Phi}_{ij}^{SB} + \sum_{i \le j=1}^{N_B} \hat{\Phi}_{ij}^{B}$$

with $\hat{\Phi}_{ij} = \sum_{\mu=0}^{dim(\mathcal{B}(\mathcal{H}_i))-1} \sum_{\nu=0}^{dim(\mathcal{B}(\mathcal{H}_j))-1} J_{ij}^{\mu\nu} \hat{O}_i^{\mu} \hat{O}_j^{\nu}$ and $N_S^{int} \le N_S; N_B^{int} \le N_B \to \infty$.

Our goal is to truncate the sums over the environmental DOF in a well-defined manner

$\Rightarrow$ we need to quantify the influence of the DOF upon each other

system-bath Hamiltonian

$$\hat{H} = \hat{H}_S + \sum_{i=1}^{N_S^{int}} \sum_{j=1}^{N_B^{int}} \hat{\Phi}_{ij}^{SB} + \sum_{i \leq j=1}^{N_B} \hat{\Phi}_{ij}^{B}$$

with $\hat{\Phi}_{ij} = \sum_{\mu=0}^{dim(\mathcal{B}(\mathcal{H}_i))-1} \sum_{\nu=0}^{dim(\mathcal{B}(\mathcal{H}_j))-1} J_{ij}^{\mu\nu} \hat{O}_i^{\mu} \hat{O}_j^{\nu}$ and $N_S^{int} \leq N_S; N_B^{int} \leq N_B \to \infty$.

Our goal is to truncate the sums over the environmental DOF in a well-defined manner

$\Rightarrow$ we need to quantify the influence of the DOF upon each other

$\Rightarrow$ we need to introduce a metric.

Hamiltonian-graph correspondence

Hamiltonian-graph correspondence

Hilbert space: $H_{tot} \iff$ graph

$$G(N, E) \begin{cases} N = \{\text{nodes} : N_S + N_B\} \\ E = \{\text{edges} : \; J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu, \nu\} \end{cases}$$

Hamiltonian-graph correspondence

Hilbert space: $H_{tot} \iff$ graph

$$G(N,E)\begin{cases} N = \{\text{nodes} : N_S + N_B\} \\ E = \{\text{edges} : \; J_{ij}^{\mu\nu} \neq 0 \quad \text{for any } \mu,\nu\} \end{cases}$$

Adjacency matrix: A with $A_{ij} = \begin{cases} 1 & \text{if} \quad J_{ij}^{\mu\nu} \neq 0 \quad \text{for any } \mu,\nu \\ 0 & \text{if} \quad J_{ij}^{\mu\nu} = 0 \end{cases}$

Hamiltonian-graph correspondence

Hilbert space: $H_{tot} \iff$ graph

$$G(N,E)\begin{cases} N = \{\text{nodes} : N_S + N_B\} \\ E = \{\text{edges} : \; J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu,\nu\}\end{cases}$$

Adjacency matrix: A with $A_{ij} = \begin{cases} 1 & \text{if} \quad J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu,\nu \\ 0 & \text{if} \quad J_{ij}^{\mu\nu} = 0\end{cases}$

Example: physical lattice = one dimensional chain

Hamiltonian-graph correspondence

Hilbert space: $H_{tot} \iff$ graph

$$G(N,E)\begin{cases} N = \{\text{nodes}: N_S + N_B\} \\ E = \{\text{edges}: \ J_{ij}^{\mu\nu} \neq 0 \quad \text{for any } \mu,\nu\} \end{cases}$$

Adjacency matrix: A with $A_{ij} = \begin{cases} 1 & \text{if} \quad J_{ij}^{\mu\nu} \neq 0 \quad \text{for any } \mu,\nu \\ 0 & \text{if} \quad J_{ij}^{\mu\nu} = 0 \end{cases}$

Example: physical lattice = one dimensional chain

$$H = \sum_i J\hat{O}_i\hat{O}_{i+1}$$

Hamiltonian-graph correspondence

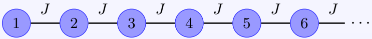
Hilbert space: $H_{tot} \iff$ graph

$$G(N,E)\begin{cases} N = \{\text{nodes} : N_S + N_B\} \\ E = \{\text{edges} : \ J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu,\nu\} \end{cases}$$

Adjacency matrix: A with $A_{ij} = \begin{cases} 1 & \text{if} \quad J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu,\nu \\ 0 & \text{if} \quad J_{ij}^{\mu\nu} = 0 \end{cases}$

Example: physical lattice = one dimensional chain

$$H = \sum_i J\hat{O}_i\hat{O}_{i+1}$$



Hamiltonian-graph correspondence

Hilbert space: $H_{tot} \iff$ graph

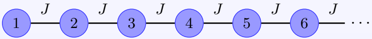
$$G(N,E)\begin{cases} N = \{\text{nodes}: N_S + N_B\} \\ E = \{\text{edges}: \; J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu,\nu\} \end{cases}$$

Adjacency matrix: A with $A_{ij} = \begin{cases} 1 & \text{if} \quad J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\, \mu,\nu \\ 0 & \text{if} \quad J_{ij}^{\mu\nu} = 0 \end{cases}$

Example: physical lattice = one dimensional chain

$H = \sum_i J\hat{O}_i\hat{O}_{i+1}$

$H = \sum_i (J_1\hat{O}_i\hat{O}_{i+1} + J_2\hat{O}_i\hat{O}_{i+2})$



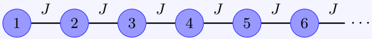
Hilbert space: $H_{tot} \iff$ graph

$$G(N,E)\begin{cases} N = \{\text{nodes}: N_S + N_B\} \\ E = \{\text{edges}: \ J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\ \mu,\nu\} \end{cases}$$

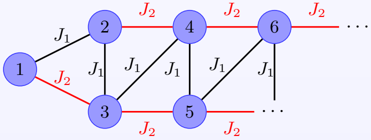
Adjacency matrix: A with $A_{ij} = \begin{cases} 1 & \text{if} \quad J_{ij}^{\mu\nu} \neq 0 \quad \text{for any}\ \mu,\nu \\ 0 & \text{if} \quad J_{ij}^{\mu\nu} = 0 \end{cases}$

Example: physical lattice = one dimensional chain

$$H = \sum_i J \hat{O}_i \hat{O}_{i+1}$$



$$H = \sum_i (J_1 \hat{O}_i \hat{O}_{i+1} + J_2 \hat{O}_i \hat{O}_{i+2})$$



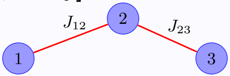
weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i, j = 1, \cdots, N_S + N_B$

(remove indices of internal degrees of freedom)

weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i, j = 1, \cdots, N_S + N_B$

(remove indices of internal degrees of freedom)

- walk of length n on **G**: $\pi_n(i,j) = [i = i_0, i_1, \cdots, i_n = j]$

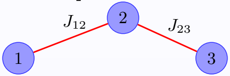


weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i, j = 1, \cdots, N_S + N_B$

(remove indices of internal degrees of freedom)

- walk of length n on **G**: $\pi_n(i,j) = [i = i_0, i_1, \cdots, i_n = j]$

  weight: $\prod_{k=0}^{n} J_{i_k, i_{k+1}}$



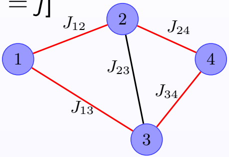
weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i, j = 1, \cdots, N_S + N_B$

(remove indices of internal degrees of freedom)

- walk of length n on **G**: $\pi_n(i,j) = [i = i_0, i_1, \cdots, i_n = j]$

  weight: $\prod_{k=0}^{n} J_{i_k, i_{k+1}}$

- all walks of length n on **G**: $\Pi_n(i,j) = \sum \pi_n(i,j)$



weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i, j = 1, \cdots, N_S + N_B$

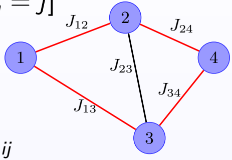
(remove indices of internal degrees of freedom)

- walk of length n on **G**: $\pi_n(i,j) = [i = i_0, i_1, \cdots, i_n = j]$

  weight: $\prod_{k=0}^{n} J_{i_k, i_{k+1}}$

- all walks of length n on **G**: $\Pi_n(i,j) = \sum \pi_n(i,j)$

  weight: $w(\Pi_n(i,j)) = [J^n]_{ij}$



weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i,j = 1,\cdots,N_S + N_B$

(remove indices of internal degrees of freedom)

- walk of length n on **G**: $\pi_n(i,j) = [i = i_0, i_1, \cdots, i_n = j]$

  weight: $\prod_{k=0}^{n} J_{i_k,i_{k+1}}$
- all walks of length n on **G**: $\Pi_n(i,j) = \sum \pi_n(i,j)$

  weight: $w(\Pi_n(i,j)) = [J^n]_{ij}$

metric d on G: shortest path between two nodes

$$\mathbf{d(i,j) := \min\{n \in \mathbb{N}_0 : [A^n]_{i,j} \neq 0\}}$$

weight matrix on G: $\mathbf{J_{ij}} = (\sum_{\mu\nu}[J_{ij}^{\mu\nu}]^2)^{1/2}$ with $i,j = 1,\cdots,N_S+N_B$

(remove indices of internal degrees of freedom)

- walk of length n on **G**: $\pi_n(i,j) = [i = i_0, i_1, \cdots, i_n = j]$

  weight: $\prod_{k=0}^{n} J_{i_k,i_{k+1}}$
- all walks of length n on **G**: $\Pi_n(i,j) = \sum \pi_n(i,j)$

  weight: $w(\Pi_n(i,j)) = [J^n]_{ij}$

metric d on G: shortest path between two nodes

$$\mathbf{d(i,j) := \min\{n \in \mathbb{N}_0 : [A^n]_{i,j} \neq 0\}}$$

$\Rightarrow$ reorder bath DOF according to their distance from the system S

$$\hat{H} = \sum_{d=0}^{\infty}\left(\hat{h}_d + \hat{h}_{d,d+1}\right) \qquad (\hat{h}_0 = \hat{H}_S, \hat{h}_{01} = \hat{H}_{SB}),$$

$\hat{h}_d$ interactions within same layer, $\hat{h}_{d,d+1}$ between two successive layers

system dynamics

Generic system operator $\hat{A}_S$ evolves as

$$\hat{A}_S(t) = e^{i\hat{H}t}\hat{A}_S e^{-i\hat{H}t} = \hat{A}_S + \sum_{d=1}^{\infty} \frac{(-it)^d}{d!} \hat{C}_d$$

with $\hat{C}_d = [\hat{H}, \hat{C}_{d-1}]$ and $\hat{C}_0 = \hat{A}_S$.

Generic system operator $\hat{A}_S$ evolves as

$$\hat{A}_S(t) = e^{i\hat{H}t}\hat{A}_S e^{-i\hat{H}t} = \hat{A}_S + \sum_{d=1}^{\infty} \frac{(-it)^d}{d!} \hat{C}_d$$

with $\hat{C}_d = [\hat{H}, \hat{C}_{d-1}]$ and $\hat{C}_0 = \hat{A}_S$.

$\hat{C}_d$ has non-vanishing commutators only with terms in $\hat{H}$ up to $\hat{h}_{d,d+1}$

system dynamics

Generic system operator $\hat{A}_S$ evolves as

$$\hat{A}_S(t) = e^{i\hat{H}t}\hat{A}_S e^{-i\hat{H}t} = \hat{A}_S + \sum_{d=1}^{\infty} \frac{(-it)^d}{d!} \hat{C}_d$$

with $\hat{C}_d = [\hat{H}, \hat{C}_{d-1}]$ and $\hat{C}_0 = \hat{A}_S$.

$\hat{C}_d$ has non-vanishing commutators only with terms in $\hat{H}$ up to $\hat{h}_{d,d+1}$

$\Rightarrow$ at n_{th} perturbative order

$$\hat{C}_n = [\hat{H}, \hat{C}_{n-1}] \equiv [\hat{H}_n, \hat{C}_{n-1}] \quad \text{with} \quad \hat{H}_n = \sum_{d=0}^{n-1} \left(\hat{h}_d + \hat{h}_{d,d+1}\right)$$

the truncation of the full generator $\hat{H}$ to the first n layers of the graph.

Generic system operator $\hat{A}_S$ evolves as

$$\hat{A}_S(t) = e^{i\hat{H}t}\hat{A}_S e^{-i\hat{H}t} = \hat{A}_S + \sum_{d=1}^{\infty} \frac{(-it)^d}{d!} \hat{C}_d$$

with $\hat{C}_d = [\hat{H}, \hat{C}_{d-1}]$ and $\hat{C}_0 = \hat{A}_S$.

$\hat{C}_d$ has non-vanishing commutators only with terms in $\hat{H}$ up to $\hat{h}_{d,d+1}$

$\Rightarrow$ at n_{th} perturbative order

$$\hat{C}_n = [\hat{H}, \hat{C}_{n-1}] \equiv [\hat{H}_n, \hat{C}_{n-1}] \quad \text{with} \quad \hat{H}_n = \sum_{d=0}^{n-1} \left(\hat{h}_d + \hat{h}_{d,d+1} \right)$$

the truncation of the full generator $\hat{H}$ to the first n layers of the graph.

truncation of $\hat{H}$ $\equiv$ truncation of perturbative expansion

system dynamics

Generic system operator $\hat{A}_S$ evolves as

$$\hat{A}_S(t) = e^{i\hat{H}t}\hat{A}_S e^{-i\hat{H}t} = \hat{A}_S + \sum_{d=1}^{\infty} \frac{(-it)^d}{d!} \hat{C}_d$$

with $\hat{C}_d = [\hat{H}, \hat{C}_{d-1}]$ and $\hat{C}_0 = \hat{A}_S$.

$\hat{C}_d$ has non-vanishing commutators only with terms in $\hat{H}$ up to $\hat{h}_{d,d+1}$

$\Rightarrow$ at n_{th} perturbative order

$$\hat{C}_n = [\hat{H}, \hat{C}_{n-1}] \equiv [\hat{H}_n, \hat{C}_{n-1}] \quad \text{with} \quad \hat{H}_n = \sum_{d=0}^{n-1} \left(\hat{h}_d + \hat{h}_{d,d+1}\right)$$

the truncation of the full generator $\hat{H}$ to the first n layers of the graph.

truncation of $\hat{H}$ $\equiv$ truncation of perturbative expansion

DOF at distance n from S contribute only from the nth perturbative order
$\Rightarrow$ system dynamics appreciably affected only when corresponding perturbative term is non-negligible.

Error made by replacing $\hat{H}$ by the truncated generator = remainder of the series

$$\left\|\hat{A}_S(t) - \hat{A}_S^n(t)\right\| \leq \left\|\hat{A}_S\right\| \sum_{d=n+1}^{\infty} \frac{(2t\mathcal{O})^d}{d!} \sum_{i,j\in\mathcal{I}_d} [J^d]_{ij}$$

where $\mathcal{O} = \max_{(i,j)\in N;\mu,\nu} \left\|\hat{O}_i^{\mu}\hat{O}_j^{\nu}\right\|$ and $\mathcal{I}_d = \{i \in N : d(s,i) \leq d\}$ the set of DOF at distance at most d from S.

Error made by replacing $\hat{H}$ by the truncated generator = remainder of the series

$$\left\|\hat{A}_S(t)-\hat{A}_S^n(t)\right\| \leq \left\|\hat{A}_S\right\| \sum_{d=n+1}^{\infty} \frac{(2t\mathcal{O})^d}{d!} \sum_{i,j\in\mathcal{I}_d} [J^d]_{ij}$$

where $\mathcal{O} = \max_{(i,j)\in N;\mu,\nu} \left\|\hat{O}_i^{\mu}\hat{O}_j^{\nu}\right\|$ and $\mathcal{I}_d = \{i \in N : d(s,i) \leq d\}$ the set of DOF at distance at most d from S.

if each DOF interacts with a finite number of other DOF (local finiteness) $\Rightarrow \sum_{i,j\in\mathcal{I}_d}[J^d]_{ij} \leq (\bar{c}^2\|J\|)^d$ ($\bar{c}$ = maximum vertex degree of G)$\Rightarrow$ the sum can be bounded

Error made by replacing $\hat{H}$ by the truncated generator = remainder of the series

$$\left\|\hat{A}_S(t) - \hat{A}_S^n(t)\right\| \leq \left\|\hat{A}_S\right\| \sum_{d=n+1}^{\infty} \frac{(2t\mathcal{O})^d}{d!} \sum_{i,j\in\mathcal{I}_d} [J^d]_{ij}$$

where $\mathcal{O} = \max_{(i,j)\in N;\mu,\nu} \left\|\hat{O}_i^{\mu}\hat{O}_j^{\nu}\right\|$ and $\mathcal{I}_d = \{i \in N : d(s,i) \leq d\}$ the set of DOF at distance at most d from S.

if each DOF interacts with a finite number of other DOF (local finiteness) $\Rightarrow \sum_{i,j\in\mathcal{I}_d}[J^d]_{ij} \leq (\bar{c}^2\|J\|)^d$ ($\bar{c}$ = maximum vertex degree of G)$\Rightarrow$ the sum can be bounded

$\Rightarrow$ **Lieb-Robinson bound**

$$\left\|\hat{A}_S(t) - \hat{A}_S^n(t)\right\| \leq \left\|\hat{A}_S\right\| e^{-(n-vt)} \quad (1)$$

with $v = 2\mathcal{O}\bar{c}^2\|J\|e$

to be more quantitative:Lieb-Robinson bound Comm. Math. Phys.**28**,251 (1972)

Error made by replacing $\hat{H}$ by the truncated generator = remainder of the series

$$\left\|\hat{A}_S(t) - \hat{A}_S^n(t)\right\| \leq \left\|\hat{A}_S\right\| \sum_{d=n+1}^{\infty} \frac{(2t\mathcal{O})^d}{d!} \sum_{i,j\in\mathcal{I}_d} [J^d]_{ij}$$

where $\mathcal{O} = \max_{(i,j)\in N;\mu,\nu} \left\|\hat{O}_i^\mu \hat{O}_j^\nu\right\|$ and $\mathcal{I}_d = \{i \in N : d(s,i) \leq d\}$ the set of DOF at distance at most d from S.

if each DOF interacts with a finite number of other DOF (local finiteness) $\Rightarrow \sum_{i,j\in\mathcal{I}_d}[J^d]_{ij} \leq (\bar{c}^2\|J\|)^d$ ($\bar{c}$ = maximum vertex degree of G)$\Rightarrow$ the sum can be bounded

$\Rightarrow$ **Lieb-Robinson bound**

$$\left\|\hat{A}_S(t) - \hat{A}_S^n(t)\right\| \leq \left\|\hat{A}_S\right\| e^{-(n-vt)} \qquad (1)$$

with $v = 2\mathcal{O}\bar{c}^2\|J\|e$

Bath DOF outside of the effective light cone give only an exponentially vanishing contribution to $\hat{A}_S(t)$. The full bath is needed only in the limit of infinite time.

Quasi-finite resolution of quantum dynamics

[1] besides for normal modes (Prior, Chin, Huelga,Plenio PRL **105**, 050404 (2010))

Hamiltonian of a central system interacting with a continuous environment

$$\begin{aligned}\hat{H} &= \hat{H}_S + \hat{O}_S^I \int_0^{x_{max}} J(x) \left(\hat{c}_x + \hat{c}_x^\dagger\right) dx \\ &+2 \int_0^{x_{max}} \int_x^{x_{max}} K(|x-x'|) \Big[c_x c_{x'}^\dagger + c_x^\dagger c_{x'} \\ &+ c_x^\dagger \hat{c}_x c_{x'}^\dagger \hat{c}_{x'} \Big] dx\, dx' + \int_0^{x_{max}} g(x) \hat{c}_x \hat{c}_x^\dagger dx\,,\end{aligned}$$

x = relevant bath variable, $x_{max} < \infty$ finite cut-off, and $\hat{O}_S^I$ = generic system operator, $\|c\| = \max_{x\in[0,x_{max}]} \|\hat{c}_x\| < \infty$

[1] besides for normal modes (Prior, Chin, Huelga,Plenio PRL **105**, 050404 (2010))

Hamiltonian of a central system interacting with a continuous environment

$$\begin{aligned}\hat{H} &= \hat{H}_S + \hat{O}_S^I \int_0^{x_{max}} J(x) \left(\hat{c}_x + \hat{c}_x^\dagger\right) dx \\ &+2 \int_0^{x_{max}} \int_x^{x_{max}} K(|x-x'|) \Big[c_x c_{x'}^\dagger + c_x^\dagger c_{x'} \\ &+ c_x^\dagger \hat{c}_x c_{x'}^\dagger \hat{c}_{x'} \Big] dx\, dx' + \int_0^{x_{max}} g(x) \hat{c}_x \hat{c}_x^\dagger dx \,,\end{aligned}$$

x = relevant bath variable, $x_{max} < \infty$ finite cut-off, and $\hat{O}_S^I$ = generic system operator, $\|c\| = \max_{x \in [0, x_{max}]} \|\hat{c}_x\| < \infty$

problem: S interacts with all bath DOF which all may interact among themselves (non-local interactions) $\Rightarrow$ graph with all nodes at distance 1 from S $\Rightarrow$ in general no LR bound [1]

[1] besides for normal modes (Prior, Chin, Huelga,Plenio PRL **105**, 050404 (2010))

Hamiltonian of a central system interacting with a continuous environment

$$
\begin{aligned}
\hat{H} &= \hat{H}_S + \hat{O}_S^I \int_0^{x_{max}} J(x) \left(\hat{c}_x + \hat{c}_x^\dagger\right) dx \\
&\quad + 2\int_0^{x_{max}} \int_x^{x_{max}} K(|x-x'|) \Big[c_x c_{x'}^\dagger + c_x^\dagger c_{x'} \\
&\quad + c_x^\dagger \hat{c}_x c_{x'}^\dagger \hat{c}_{x'} \Big] dx\, dx' + \int_0^{x_{max}} g(x) \hat{c}_x \hat{c}_x^\dagger dx \,,
\end{aligned}
$$

$x =$ relevant bath variable, $x_{max} < \infty$ finite cut-off, and $\hat{O}_S^I =$ generic system operator, $\|c\| = \max_{x \in [0, x_{max}]} \|\hat{c}_x\| < \infty$

problem: S interacts with all bath DOF which all may interact among themselves (non-local interactions) $\Rightarrow$ graph with all nodes at distance 1 from S $\Rightarrow$ in general no LR bound [1]

general applicable bound for continuous environments using the idea of the 'surrogate Hamiltonian' (Baer, Kosloff, JCP **106**,8862 (1997))

[1] besides for normal modes (Prior, Chin, Huelga,Plenio PRL **105**, 050404 (2010))

the surrogate generator

the surrogate generator

Sequence of n sampling points: $\{x_i\}_{i=0}^{n-1}$, in $[0, x_{max}]$, with $x_i < x_{i+1}$

the surrogate generator

Sequence of n sampling points: $\{x_i\}_{i=0}^{n-1}$, in $[0, x_{max}]$, with $x_i < x_{i+1}$

$\Rightarrow$ partition $P_n = \{\delta x_i\}$ with $\delta x_i = x_{i+1} - x_i$, $|P_n| = \max_{i<n}(\delta x_i)$

the surrogate generator

Sequence of n sampling points: $\{x_i\}_{i=0}^{n-1}$, in $[0, x_{max}]$, with $x_i < x_{i+1}$

$\Rightarrow$ partition $P_n = \{\delta x_i\}$ with $\delta x_i = x_{i+1} - x_i$, $|P_n| = \max_{i<n}(\delta x_i)$

$\Rightarrow$ sequence of partitions $\{P_n\}$ with $|P_{n+1}| < |P_n|$

the surrogate generator

Sequence of n sampling points: $\{x_i\}_{i=0}^{n-1}$, in $[0, x_{max}]$, with $x_i < x_{i+1}$

$\Rightarrow$ partition $P_n = \{\delta x_i\}$ with $\delta x_i = x_{i+1} - x_i$, $|P_n| = \max_{i<n}(\delta x_i)$

$\Rightarrow$ sequence of partitions $\{P_n\}$ with $|P_{n+1}| < |P_n|$

$\Rightarrow$ sequence of Hamiltonians $\{\hat{H}_{P_n}\}$ with

$$\begin{aligned} \hat{H}_{P_n} &= \hat{H}_S + \hat{O}_S^I \sum_{i=0}^{n-1} \tilde{J}_i (\hat{c}_i + \hat{c}_i^\dagger) + \\ & 2 \sum_{i<j=0}^{n-1} \tilde{K}_{ij} \left[\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i + \hat{c}_i^\dagger \hat{c}_i \hat{c}_j^\dagger \hat{c}_j \right] \\ & + \sum_{i=0}^{n-1} \tilde{g}_i \hat{c}_i^\dagger \hat{c}_i, \end{aligned}$$

$\hat{c}_i = \hat{c}_{x_i}$, $\tilde{J}_i = J(x_i)\delta x_i$, $\tilde{K}_{ij} = K(|x_i - x_j|)\delta x_i \delta x_j$, and $\tilde{g}_i = g(x_i)\delta x_i$ rescaled couplings at the n sampling points

the surrogate generator

Sequence of n sampling points: $\{x_i\}_{i=0}^{n-1}$, in $[0, x_{max}]$, with $x_i < x_{i+1}$

$\Rightarrow$ partition $P_n = \{\delta x_i\}$ with $\delta x_i = x_{i+1} - x_i$, $|P_n| = \max_{i<n}(\delta x_i)$

$\Rightarrow$ sequence of partitions $\{P_n\}$ with $|P_{n+1}| < |P_n|$

$\Rightarrow$ sequence of Hamiltonians $\{\hat{H}_{P_n}\}$ with

$$\begin{aligned}\hat{H}_{P_n} &= \hat{H}_S + \hat{O}_S^I \sum_{i=0}^{n-1} \tilde{J}_i(\hat{c}_i + \hat{c}_i^\dagger) + \\ &\quad 2\sum_{i<j=0}^{n-1} \tilde{K}_{ij}\left[\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i + \hat{c}_i^\dagger \hat{c}_i \hat{c}_j^\dagger \hat{c}_j\right] \\ &\quad + \sum_{i=0}^{n-1} \tilde{g}_i \hat{c}_i^\dagger \hat{c}_i,\end{aligned}$$

$\hat{c}_i = \hat{c}_{x_i}$, $\tilde{J}_i = J(x_i)\delta x_i$, $\tilde{K}_{ij} = K(|x_i - x_j|)\delta x_i \delta x_j$, and $\tilde{g}_i = g(x_i)\delta x_i$ rescaled couplings at the n sampling points

truncation + rescaling $\Rightarrow$ $\hat{H}_{P_n}$ = Riemann sums built on P_n approximating $\hat{H}$ $\Rightarrow$ $\lim_{n\to\infty} \hat{H}_{P_n} = \hat{H}$

finite-resolution of quantum dynamics

finite-resolution of quantum dynamics

error made by time evolving $\hat{A}_S$ using $\hat{H}_{P_n}$ instead of $\hat{H}$

$$\mathcal{R}(P_n) = \|\hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t)\| \leq R_1(P_n) + R_2(P_n)$$

finite-resolution of quantum dynamics

error made by time evolving $\hat{A}_S$ using $\hat{H}_{P_n}$ instead of $\hat{H}$

$$\mathcal{R}(P_n) = \|\hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t)\| \leq R_1(P_n) + R_2(P_n)$$

- $\mathcal{R}_1(P_n)$ = error made in assuming $\hat{H}$ and $\hat{H}_{P_n}$ to commute
  $\Rightarrow$ at finite t vanishes for $n \to \infty$ due to the convergence of Riemann sums as $t^2 \left\| [\hat{H}, \hat{H}_{P_n}] \right\|$

finite-resolution of quantum dynamics

error made by time evolving $\hat{A}_S$ using $\hat{H}_{P_n}$ instead of $\hat{H}$

$$\mathcal{R}(P_n) = \|\hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t)\| \leq R_1(P_n) + R_2(P_n)$$

- $\mathcal{R}_1(P_n)$ = error made in assuming $\hat{H}$ and $\hat{H}_{P_n}$ to commute
  $\Rightarrow$ at finite t vanishes for $n \to \infty$ due to the convergence of Riemann sums as $t^2 \left\| [\hat{H}, \hat{H}_{P_n}] \right\|$
- $R_2(P_n)$ = distance between $\hat{A}_S$ and its evolution under $\hat{H} - \hat{H}_{P_n}$.

finite-resolution of quantum dynamics

error made by time evolving $\hat{A}_S$ using $\hat{H}_{P_n}$ instead of $\hat{H}$

$$\mathcal{R}(P_n) = \|\hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t)\| \leq R_1(P_n) + R_2(P_n)$$

- $\mathcal{R}_1(P_n)$ = error made in assuming $\hat{H}$ and $\hat{H}_{P_n}$ to commute
  $\Rightarrow$ at finite t vanishes for $n \to \infty$ due to the convergence of Riemann sums as $t^2 \left\| [\hat{H}, \hat{H}_{P_n}] \right\|$
- $R_2(P_n)$ = distance between $\hat{A}_S$ and its evolution under $\hat{H} - \hat{H}_{P_n}$.

quasi-finite resolution of quantum dynamics

$$\left\| \hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t) \right\| \leq R_1(P_n) + \left\| \hat{A}_S \right\| \left(e^{2\|\hat{H} - \hat{H}_{P_n}\| t} - 1 \right) ,$$

finite-resolution of quantum dynamics

error made by time evolving $\hat{A}_S$ using $\hat{H}_{P_n}$ instead of $\hat{H}$

$$\mathcal{R}(P_n) = \|\hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t)\| \leq R_1(P_n) + R_2(P_n)$$

- $\mathcal{R}_1(P_n)$ = error made in assuming $\hat{H}$ and $\hat{H}_{P_n}$ to commute
  $\Rightarrow$ at finite t vanishes for $n \to \infty$ due to the convergence of Riemann sums as $t^2 \left\| [\hat{H}, \hat{H}_{P_n}] \right\|$
- $R_2(P_n)$ = distance between $\hat{A}_S$ and its evolution under $\hat{H} - \hat{H}_{P_n}$.

quasi-finite resolution of quantum dynamics

$$\left\| \hat{A}_S(t) - \hat{A}_S^{H_{P_n}}(t) \right\| \leq R_1(P_n) + \left\| \hat{A}_S \right\| \left(e^{2\|\hat{H} - \hat{H}_{P_n}\| t} - 1 \right) ,$$

for $t < \infty$ the system cannot resolve the full continuum of environmental modes

$\Rightarrow$ within arbitrary accuracy a surrogate description can be used and infinitely close bath modes can be dropped.

few observations

few observations

- upper bounds to the error made by replacing the full generator, $\hat{H}$, by an effective one, $\hat{H}_n$ or $\hat{H}_{P_n}$

few observations

- upper bounds to the error made by replacing the full generator, $\hat{H}$, by an effective one, $\hat{H}_n$ or $\hat{H}_{P_n}$
- The bounds are general i.e. very conservative:

few observations

- upper bounds to the error made by replacing the full generator, $\hat{H}$, by an effective one, $\hat{H}_n$ or $\hat{H}_{P_n}$
- The bounds are general i.e. very conservative:

  In some specific cases, tighter model-dependent bounds can be derived (Burrell,Osborne PRL **99**, 167201 (2007)) and for certain classes of initial states, the scaling with time can be dramatically reduced (Hastings PRB **77**, 144302 (2008); Eisert, Cramer,Plenio RMP **82** (2010))

few observations

- upper bounds to the error made by replacing the full generator, $\hat{H}$, by an effective one, $\hat{H}_n$ or $\hat{H}_{P_n}$
- The bounds are general i.e. very conservative:

  In some specific cases, tighter model-dependent bounds can be derived (Burrell,Osborne PRL **99**, 167201 (2007)) and for certain classes of initial states, the scaling with time can be dramatically reduced (Hastings PRB **77**, 144302 (2008); Eisert, Cramer,Plenio RMP **82** (2010))
- Extension of the bounds to k-linear interactions straightforward

few observations

- upper bounds to the error made by replacing the full generator, $\hat{H}$, by an effective one, $\hat{H}_n$ or $\hat{H}_{P_n}$
- The bounds are general i.e. very conservative:

  In some specific cases, tighter model-dependent bounds can be derived (Burrell,Osborne PRL **99**, 167201 (2007)) and for certain classes of initial states, the scaling with time can be dramatically reduced (Hastings PRB **77**, 144302 (2008); Eisert, Cramer,Plenio RMP **82** (2010))
- Extension of the bounds to k-linear interactions straightforward
- extension to unbounded operators, only for certain classes of operators (Cramer, Serafini, Eisert in "*Quantum information and many body quantum systems*", pp 55-72 (2008); Nachtergaele *et al*, Rev. Math.Phys. **22**, 207 (2010))

Correspondence between discrete and continuous environments

discrete bath

infinite layers $\Rightarrow$ truncation

discrete bath

infinite layers $\Rightarrow$ truncation

continuous bath

single layer $\Rightarrow$ truncation + rescaling

discrete bath

infinite layers $\Rightarrow$ truncation

continuous bath

single layer $\Rightarrow$ truncation + rescaling

system-bath coupling = **weight** $\mathcal{J}_{SB}$ **of the paths needed by the system to explore all of the environment.**

discrete bath

infinite layers $\Rightarrow$ truncation

continuous bath

single layer $\Rightarrow$ truncation $+$ rescaling

system-bath coupling $=$ **weight** $\mathcal{J}_{SB}$ **of the paths needed by the system to explore all of the environment.**

- **continuous environments:** $\mathcal{J}_{SB} = \int_0^{x_{max}} J(x)dx$ $\rightarrow$ finite because the support of the integral is finite

discrete bath

infinite layers $\Rightarrow$ truncation

continuous bath

single layer $\Rightarrow$ truncation $+$ rescaling

system-bath coupling $=$ **weight $\mathcal{J}_{SB}$ of the paths needed by the system to explore all of the environment.**

- **continuous environments:** $\mathcal{J}_{SB} = \int_0^{x_{max}} J(x)dx$ $\rightarrow$ finite because the support of the integral is finite
- **discrete environments:** $\mathcal{J}_{SB} = \sum_{n=0}^{\infty} \sum_{j:d(s,j)=n} [J^n]_{sj}$ $\rightarrow$ local finiteness $\Rightarrow$ $\mathcal{J}_{SB}$ can be made finite by rescaling the coupling matrix i.e. by penalizing longer paths

  (the dynamics in Hilbert space remains unaffected since any rescaling of the coupling matrix is cancelled out by a corresponding rescaling of time)

discrete bath

infinite layers $\Rightarrow$ truncation

continuous bath

single layer $\Rightarrow$ truncation + rescaling

system-bath coupling = **weight $\mathcal{J}_{SB}$ of the paths needed by the system to explore all of the environment.**

- **continuous environments:** $\mathcal{J}_{SB} = \int_0^{x_{max}} J(x)dx$ $\rightarrow$ finite because the support of the integral is finite
- **discrete environments:** $\mathcal{J}_{SB} = \sum_{n=0}^{\infty} \sum_{j:d(s,j)=n} [J^n]_{sj}$ $\rightarrow$ local finiteness $\Rightarrow$ $\mathcal{J}_{SB}$ can be made finite by rescaling the coupling matrix i.e. by penalizing longer paths

  (the dynamics in Hilbert space remains unaffected since any rescaling of the coupling matrix is cancelled out by a corresponding rescaling of time)

local finiteness $\Leftrightarrow$ finite cut-off

discrete bath

infinite layers $\Rightarrow$ truncation

continuous bath

single layer $\Rightarrow$ truncation + rescaling

system-bath coupling = **weight $\mathcal{J}_{SB}$ of the paths needed by the system to explore all of the environment.**

- **continuous environments:** $\mathcal{J}_{SB} = \int_0^{x_{max}} J(x)dx$ $\rightarrow$ finite because the support of the integral is finite
- **discrete environments:** $\mathcal{J}_{SB} = \sum_{n=0}^{\infty} \sum_{j:d(s,j)=n} [J^n]_{sj}$ $\rightarrow$ local finiteness $\Rightarrow$ $\mathcal{J}_{SB}$ can be made finite by rescaling the coupling matrix i.e. by penalizing longer paths

  (the dynamics in Hilbert space remains unaffected since any rescaling of the coupling matrix is cancelled out by a corresponding rescaling of time)

local finiteness $\Leftrightarrow$ finite cut-off

infinitely long paths $\Leftrightarrow$ infinitely close modes

Dynamical renormalization

time naturally induces a dynamical renormalization over the system-bath interaction

$\Rightarrow$ the bounds provide a recursive update rule for the effective generators

time naturally induces a dynamical renormalization over the system-bath interaction

$\Rightarrow$ the bounds provide a recursive update rule for the effective generators

discrete environment:

time naturally induces a dynamical renormalization over the system-bath interaction

$\Rightarrow$ the bounds provide a recursive update rule for the effective generators

discrete environment:

bound + accuracy = number of bath modes as function of time $n = n(t)$

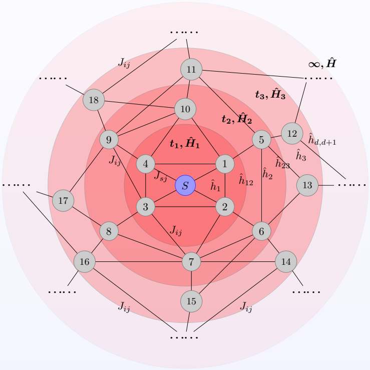
running coupling $=$

$$\tilde{\mathcal{J}}(n(t)) = \sum_{d=0}^{n(t)} \sum_{j:d(s,j)=d} \left[\tilde{J}^d\right]_{sj}$$

(effective system-bath coupling)

renormalization flow $=$

$$\lim_{t\to\infty} \tilde{\mathcal{J}}(n(t)) = \tilde{\mathcal{J}}_{SB}$$



time naturally induces a dynamical renormalization over the system-bath interaction

$\Rightarrow$ the bounds provide a recursive update rule for the effective generators

continuous environment:

time naturally induces a dynamical renormalization over the system-bath interaction

$\Rightarrow$ the bounds provide a recursive update rule for the effective generators

continuous environment:

bound + accuracy = number of bath modes/partition mesh as function of time $|P_n| = |P_{n(t)}|$

running coupling $=$

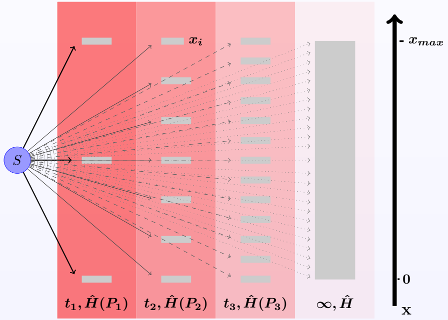
$\mathcal{J}(P_{n(t)}) = \sum_{i \in P_{n(t)}} J(x_i)\delta x_i$

(effective system-bath coupling $=$ Riemann sums)

renormalization flow $=$

$\lim_{t\to\infty} \mathcal{J}(P_{n(t)}) = \mathcal{J}_{SB}$

(convergence of Riemann sums)



computational cost & co.

- **from infinite to finite Hilbert space:**

  $\Rightarrow$ Suzuki-Trotter

  $\Rightarrow$ efficient simulation on quantum computer (polynomial in t and number of effective DOF)

computational cost & co.

- **from infinite to finite Hilbert space:**

  ⇒ Suzuki-Trotter

  ⇒ efficient simulation on quantum computer (polynomial in t and number of effective DOF)

- **classical computer:**

  still in principle exponential resources in number of effective DOF (state needs to be stored)

  ⇒ in general need further controlled restrictions of the size of the effective Hilbert (e.g. t-DMRG, restriction of excitation subspaces,etc)

- **from infinite to finite Hilbert space:**

  $\Rightarrow$ Suzuki-Trotter

  $\Rightarrow$ efficient simulation on quantum computer (polynomial in t and number of effective DOF)

- **classical computer:**

  still in principle exponential resources in number of effective DOF (state needs to be stored)

  $\Rightarrow$ in general need further controlled restrictions of the size of the effective Hilbert (e.g. t-DMRG, restriction of excitation subspaces,etc)

- **controllable approx** $\Rightarrow$ exponential scaling (in # of effective DOF)

  uncontrollable approx $\Rightarrow$ constant scaling (no bath DOF)

summary & outlook

summary & outlook

- **time-induced dynamical renormalization of S-E interaction**

  ⇒ the reduced dynamics of an arbitrary open quantum system can be obtained reliably and accurately, employing a finite-dimensional effective Hamiltonian

summary & outlook

- **time-induced dynamical renormalization of S-E interaction**

  $\Rightarrow$ the reduced dynamics of an arbitrary open quantum system can be obtained reliably and accurately, employing a finite-dimensional effective Hamiltonian

- The required **renormalizability condition**, locality of the interactions for discrete environments and finite support of the interactions for continuous environments, is **generally fulfilled**

- **time-induced dynamical renormalization of S-E interaction**

  $\Rightarrow$ the reduced dynamics of an arbitrary open quantum system can be obtained reliably and accurately, employing a finite-dimensional effective Hamiltonian

- The required **renormalizability condition**, locality of the interactions for discrete environments and finite support of the interactions for continuous environments, is **generally fulfilled**

- **(quasi-locality + quasi-finite resolution)** of quantum dynamics

  = worst case computational cost of truncation-based algorithms for non-Markovian dynamics +**a priori certification** of accuracy vs computational complexity.

summary & outlook

- **time-induced dynamical renormalization of S-E interaction**

  $\Rightarrow$ the reduced dynamics of an arbitrary open quantum system can be obtained reliably and accurately, employing a finite-dimensional effective Hamiltonian

- The required **renormalizability condition**, locality of the interactions for discrete environments and finite support of the interactions for continuous environments, is **generally fulfilled**

- **(quasi-locality + quasi-finite resolution)** of quantum dynamics

  $=$ worst case computational cost of truncation-based algorithms for non-Markovian dynamics +**a priori certification** of accuracy vs computational complexity.

- (many-body remark) **a generalized notion of approximate locality holds also for non-local interactions:**

  renormalizability seems a more general concept than locality

summary & outlook

- **time-induced dynamical renormalization of S-E interaction**

  $\Rightarrow$ the reduced dynamics of an arbitrary open quantum system can be obtained reliably and accurately, employing a finite-dimensional effective Hamiltonian

- The required **renormalizability condition**, locality of the interactions for discrete environments and finite support of the interactions for continuous environments, is **generally fulfilled**

- **(quasi-locality + quasi-finite resolution)** of quantum dynamics

  = worst case computational cost of truncation-based algorithms for non-Markovian dynamics +**a priori certification** of accuracy vs computational complexity.

- (many-body remark) **a generalized notion of approximate locality holds also for non-local interactions:**

question: general strategies to prolong convergence times?

(e.g. more general effective Hilbert space truncation schemes, characterization of the portion of Hilbert space explored by the open system dynamics, embedding in a secondary Markovian bath, etc...)

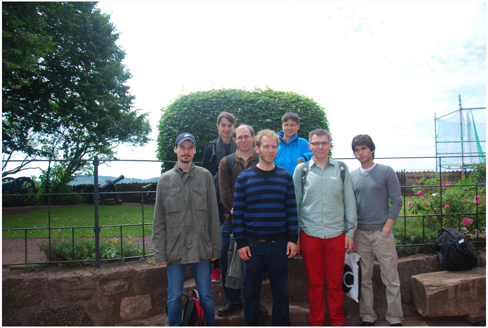
summary & outlook

what I did not talk about (but I'd be happy to discuss):

efficient characterization of quasi-unitary quantum operations

- QOC functionals for open system dynamics
- quantum process tomography/quantum device characterization

Acknowledgements



UNI KASSEL VERSITÄT

Natur Technik Kultur Gesellschaft

Group members

- Prof. Dr. Christiane Koch
- Martin Berglund
- Michael Goerz
- Esteban Goetz
- Daniel Reich
- Michal Tomza

FP7-People IEF
Marie Curie Action QOC4QIP

MARIE CURIE ACTIONS

Acknowledgements