Quantum metrology: why entanglement?

Lorenzo Maccone

Dip. Fisica, INFN Sez. Pavia,
Universita' di Pavia

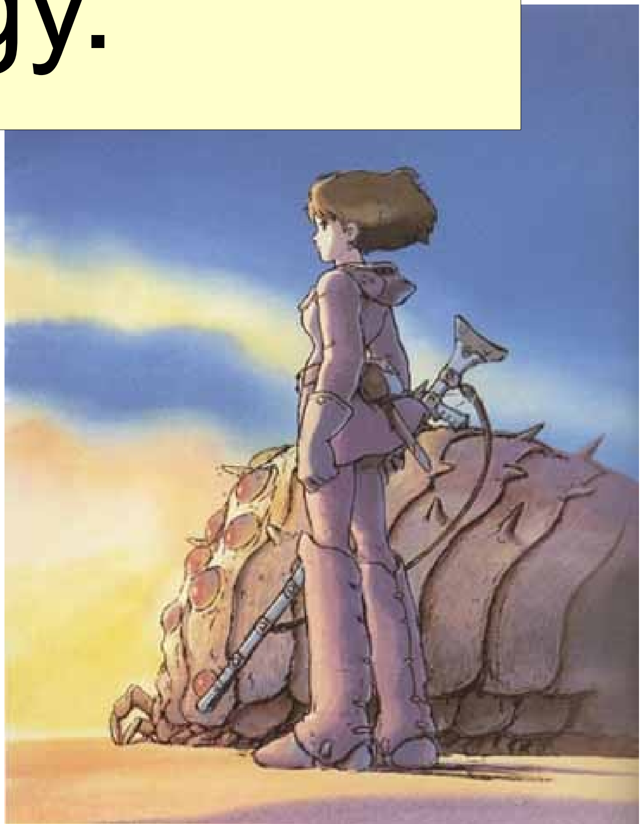
maccone@unipv.it

QUit quantum information theory group

www.qubit.it

What I'm going to talk about

I'll (try to) give an intuition of why entanglement is necessary in quantum metrology.



Intuition?!? About *entanglement*?!?

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What is entanglement anyway?

An “intuitive” easy-to-understand definition...

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same 0,1,+,- properties of the two qubits, even though the 0,1,+,- property is locally undefined for each (and complementarity **forbids** them to be jointly defined).

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that cannot exist at the same time!

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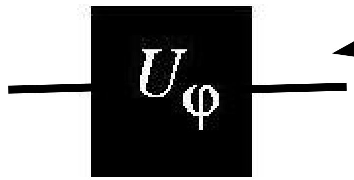
GIVEN: 

black box that implements a transformation $U_{\varphi} = e^{i\varphi H}$

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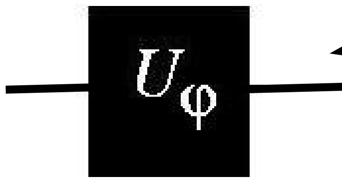

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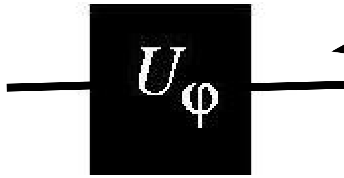
GIVEN: 

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RESULT: quantum strategies: $\Delta\varphi \sim \frac{1}{\sqrt{\nu} N}$

Heisenberg bound

number of times the N-experiment is repeated

What is quantum metrology?

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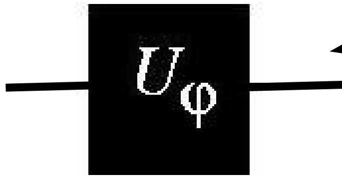

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$\sqrt{N}$ **gain of quantum metrology**

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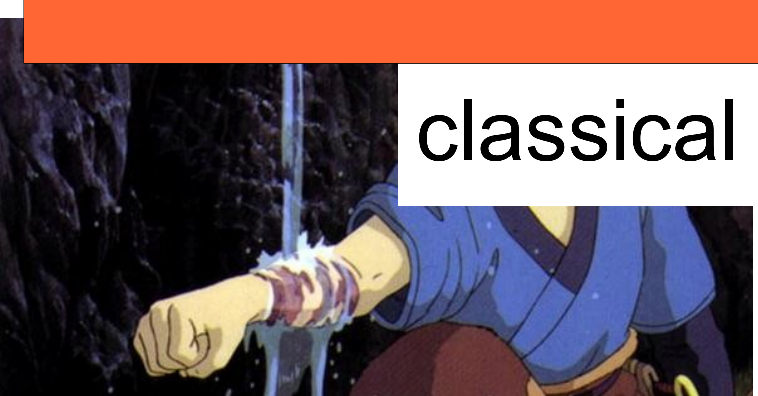


Parallel strategies

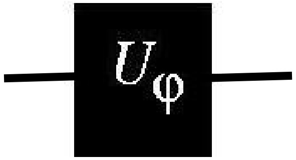
PRL 96,010401

Parallel strategies

use U_φ in parallel:

PRL 96,010401

Parallel strategies

use  in parallel:

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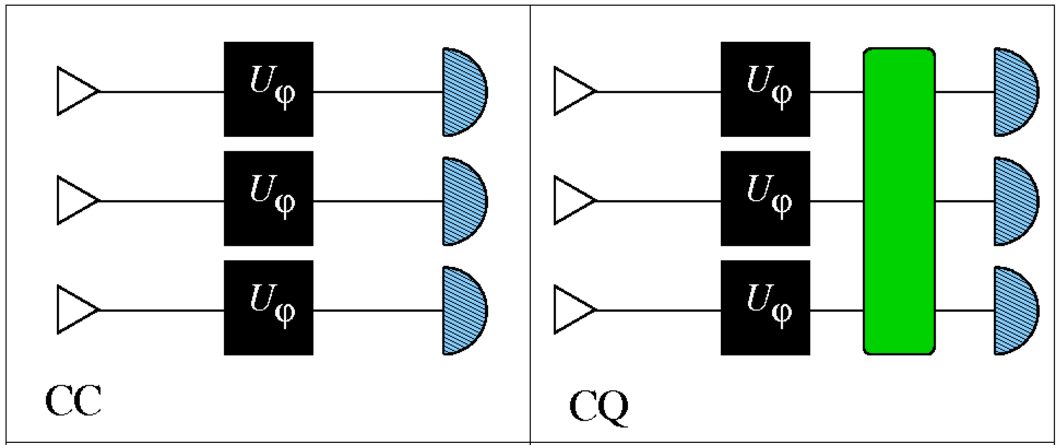

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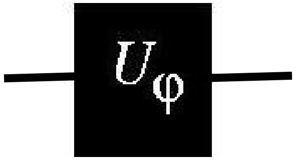
use 

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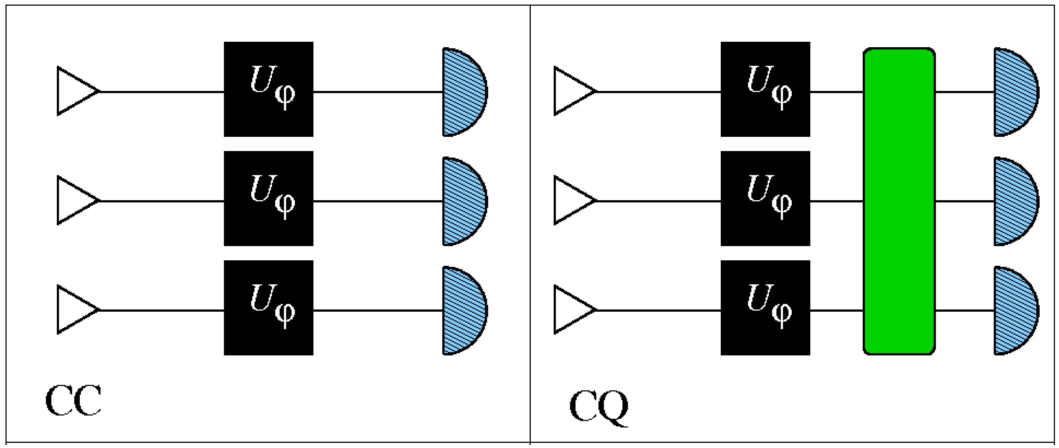

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(shot noise)

PRL 96,010401

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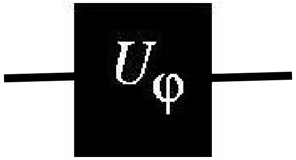
use  in parallel:

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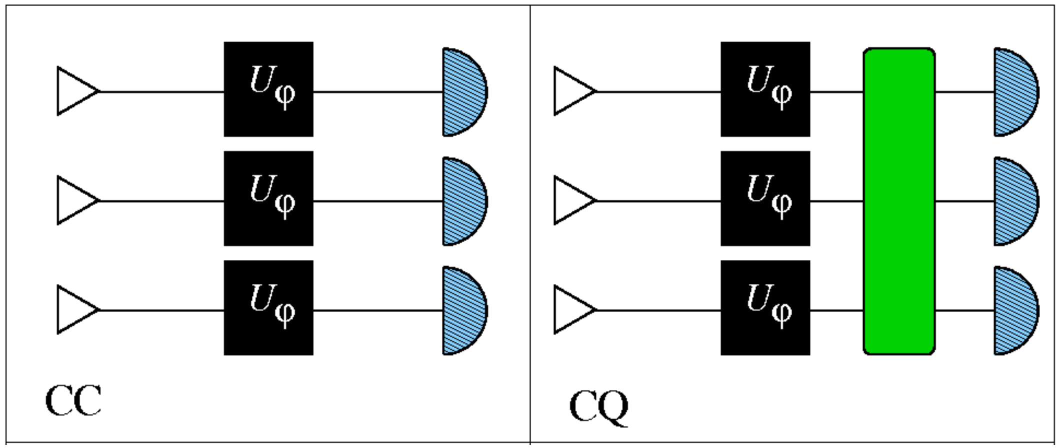

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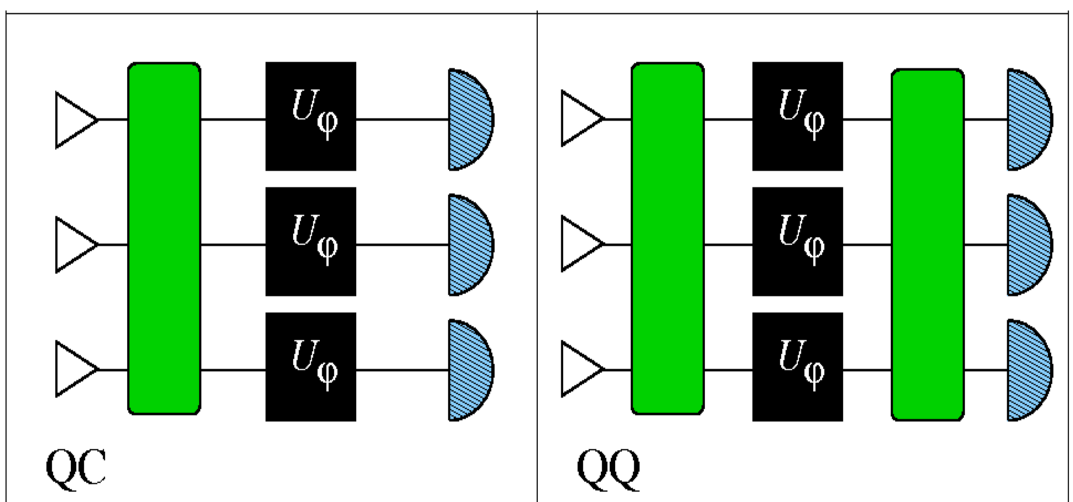

the *N* transformations act on an **entangled state**

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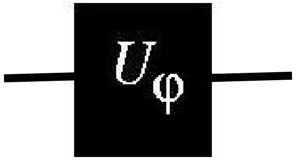
Parallel strategies

use  in parallel:

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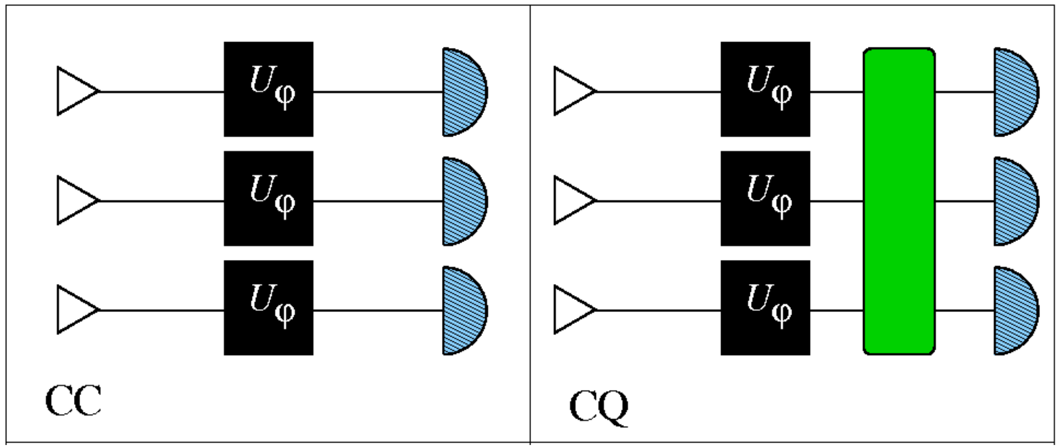

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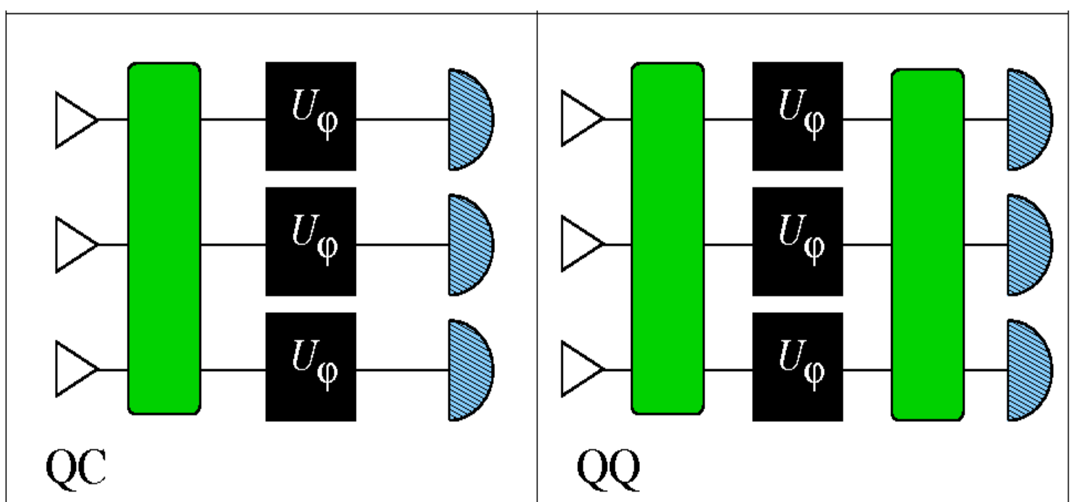

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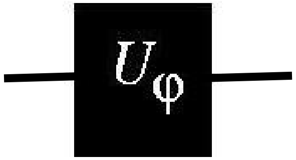
use 

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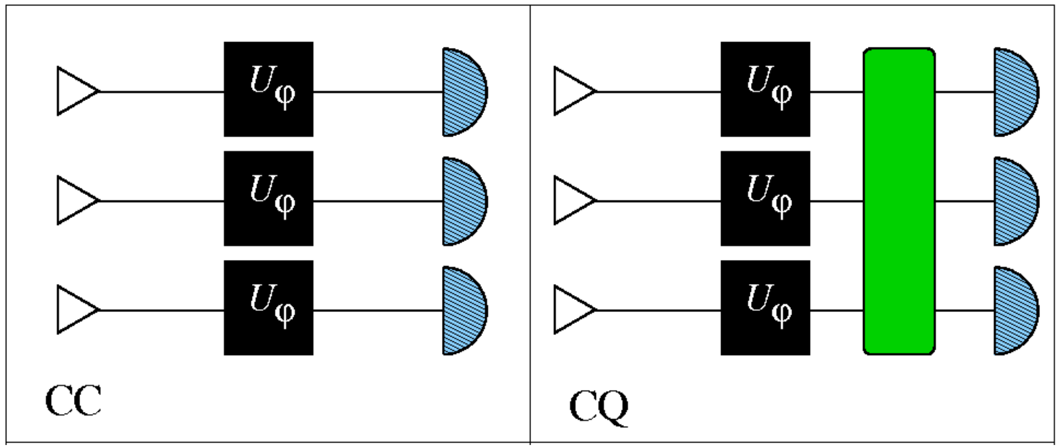

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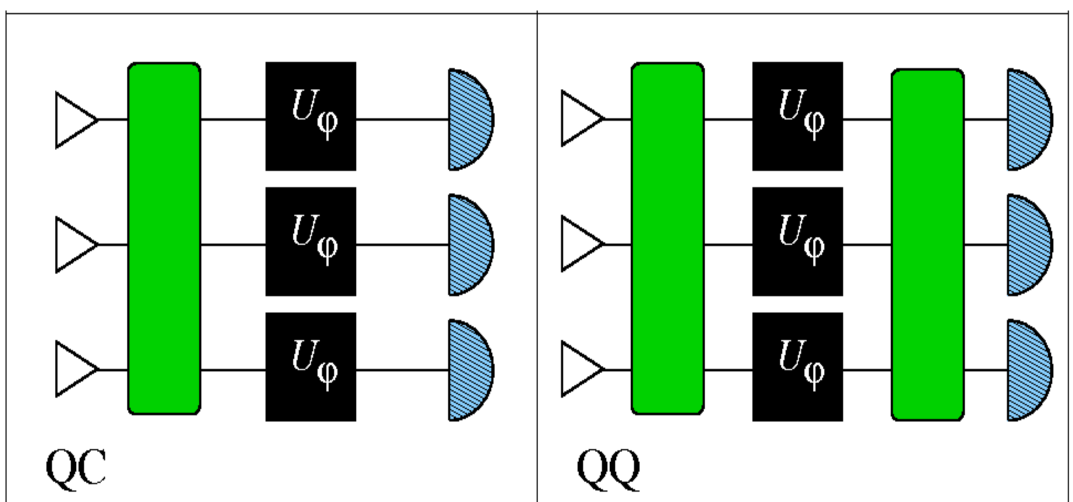

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PRL 96,010401

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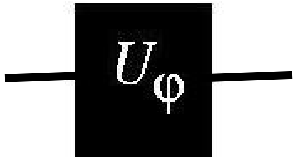
use  in series and start from $|+\rangle$ or $|-\rangle$ states:

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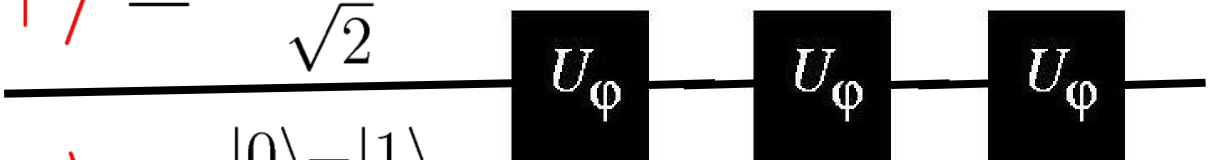

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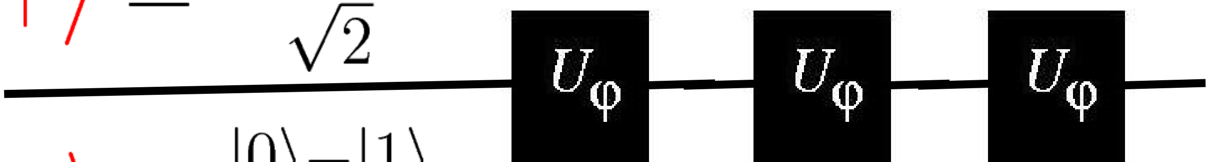

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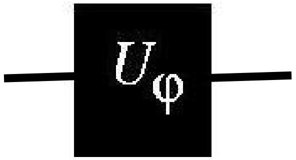
use 

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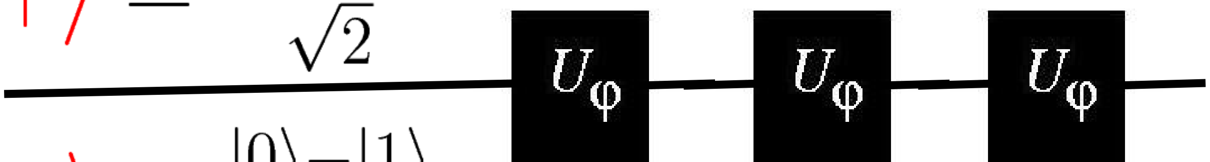

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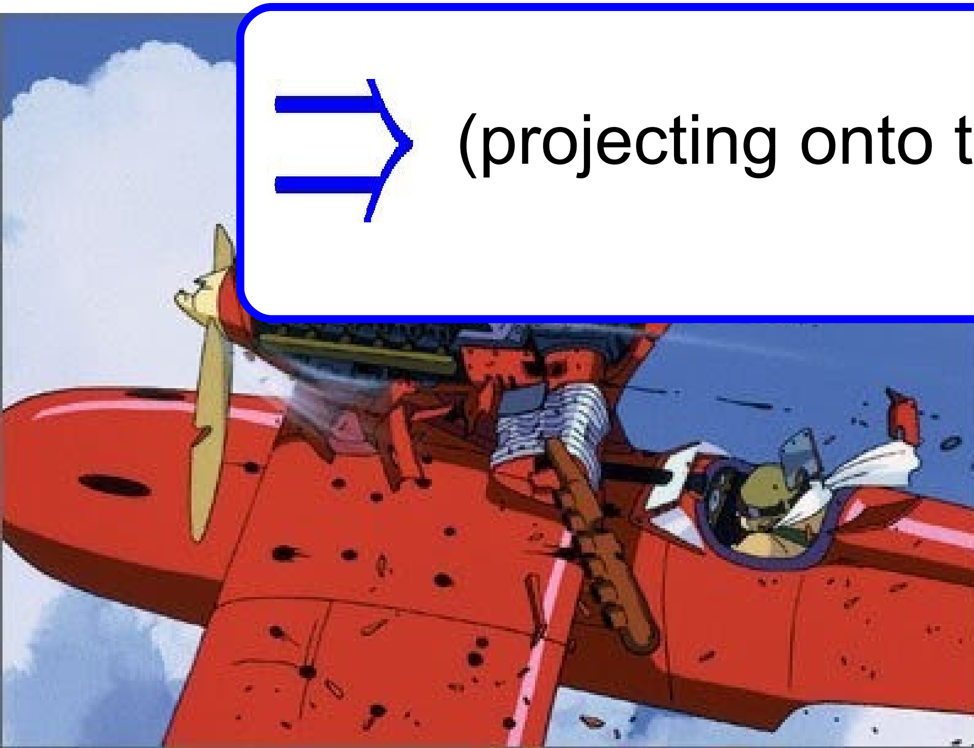

$\Rightarrow$ (projecting onto the initial state) $\Delta\varphi \propto \frac{1}{N}$

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—U_φ—U_φ—U_φ—

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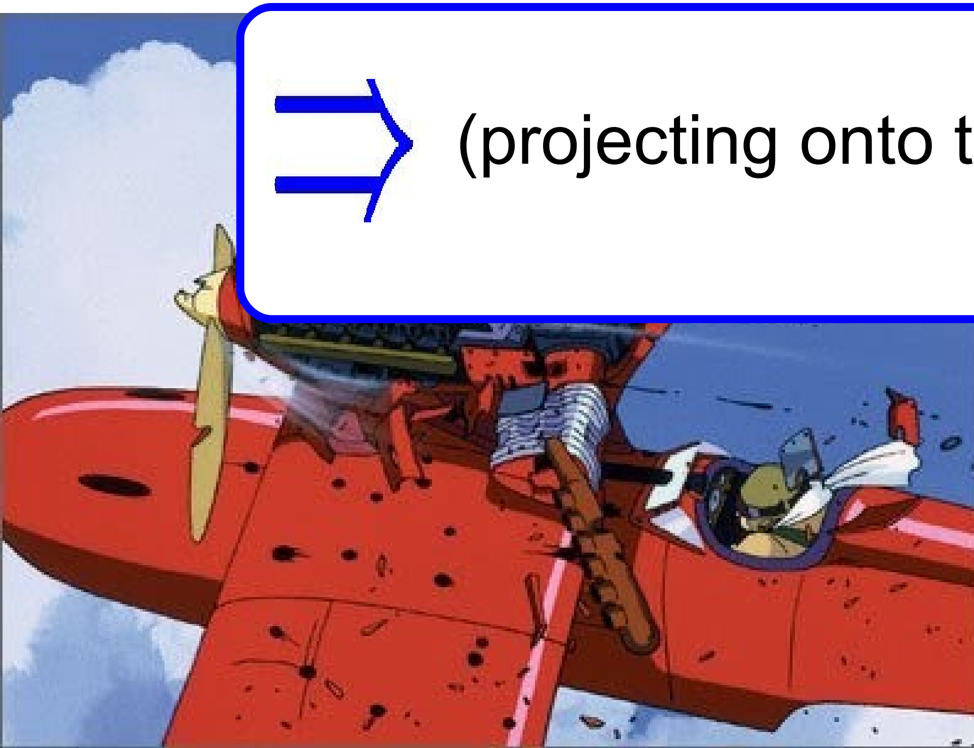

“Heisenberg”-like scaling

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So... Why entanglement?

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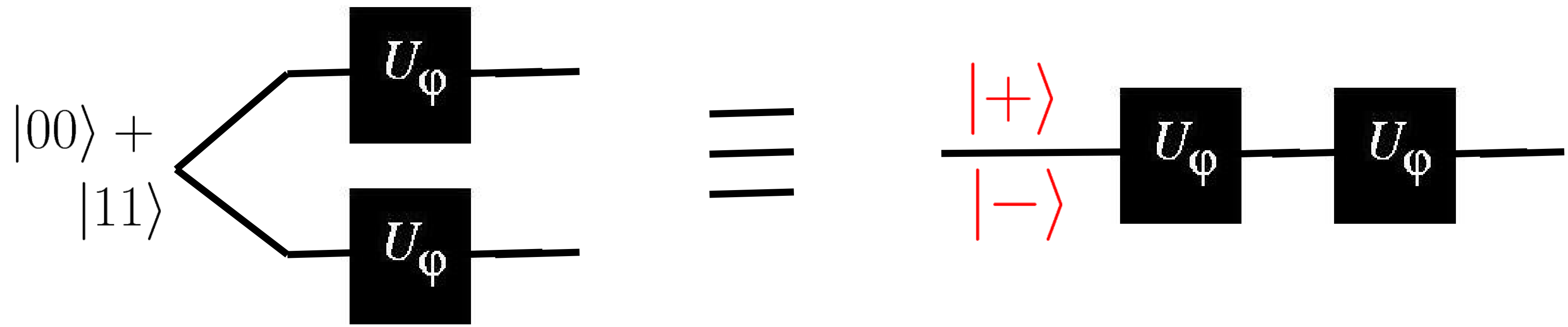
BECAUSE



i.e. entanglement turns a **parallel** strategy **into** a **sequential** one.

Simple proof

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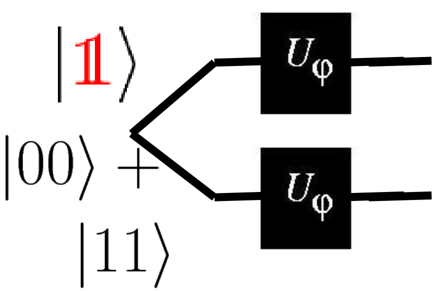

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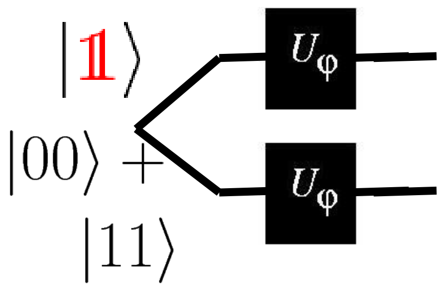

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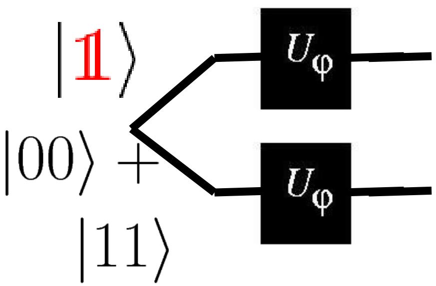

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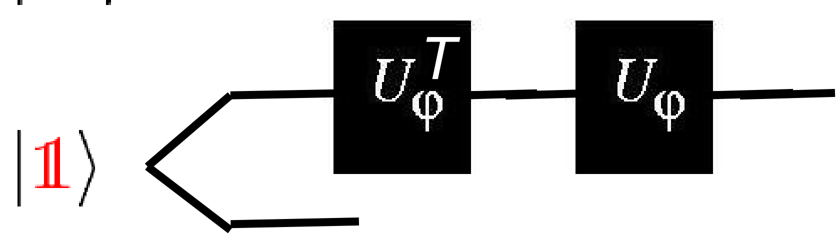

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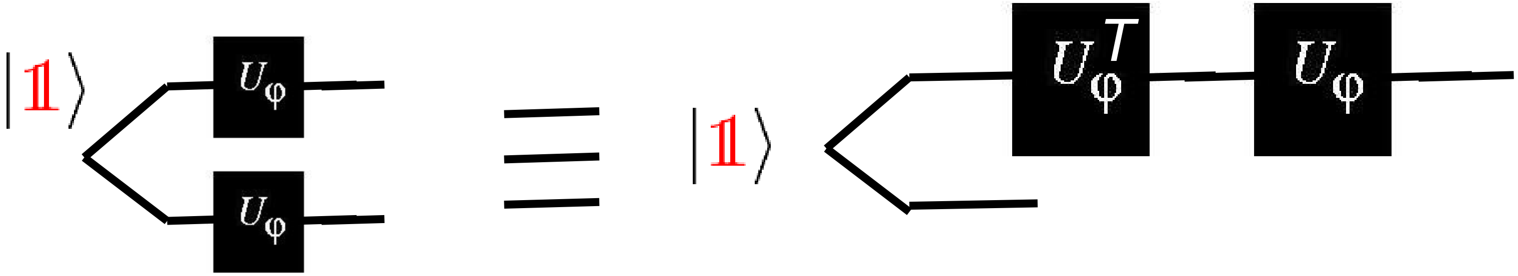
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$\equiv$
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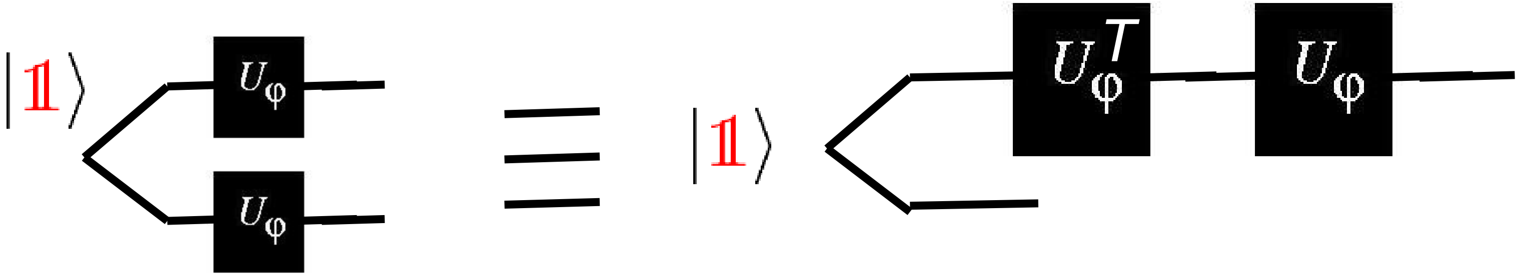

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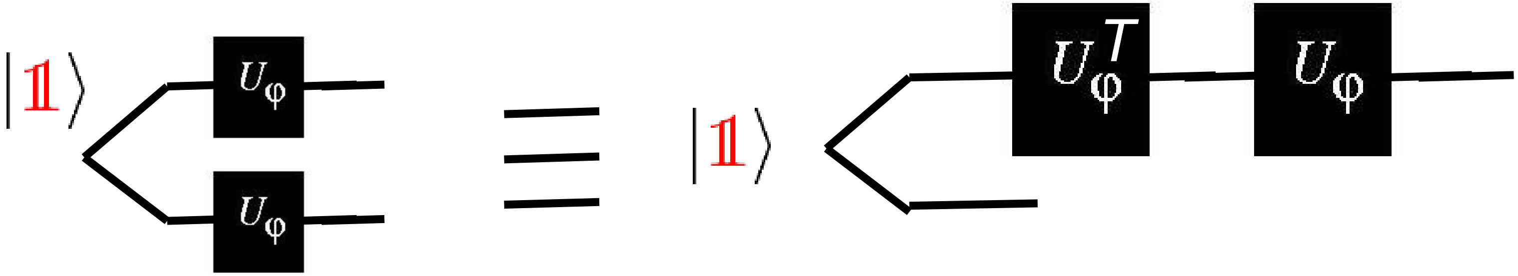

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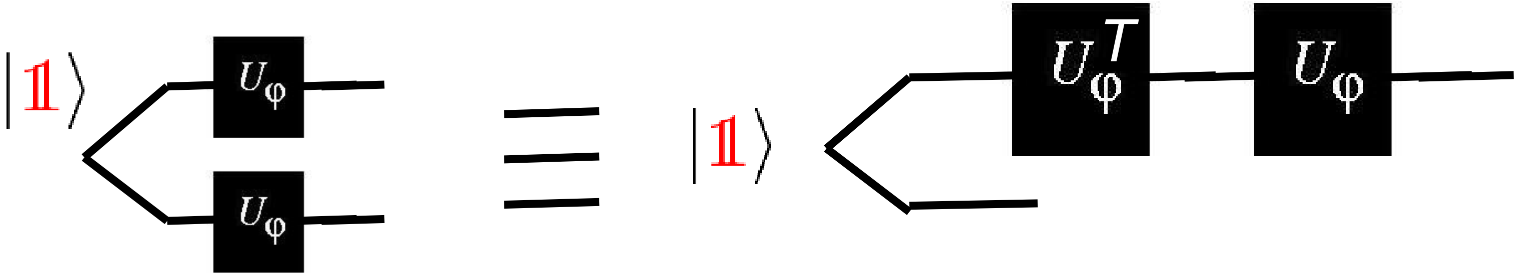

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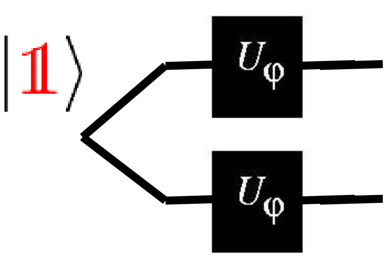

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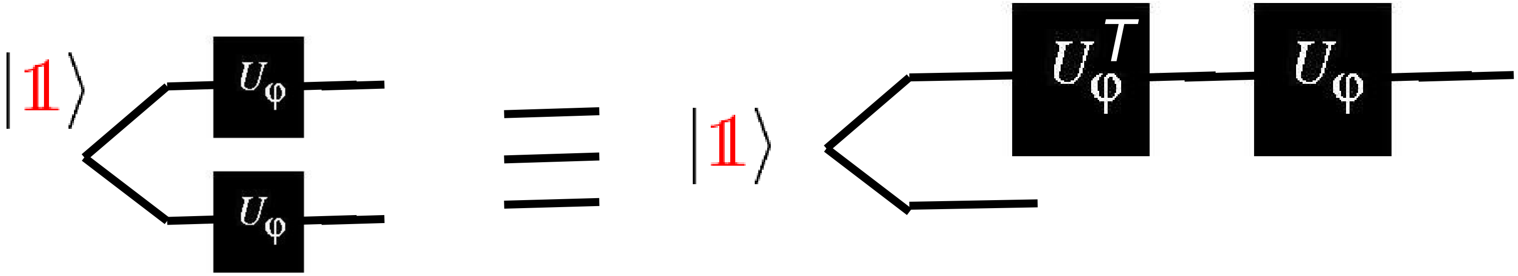

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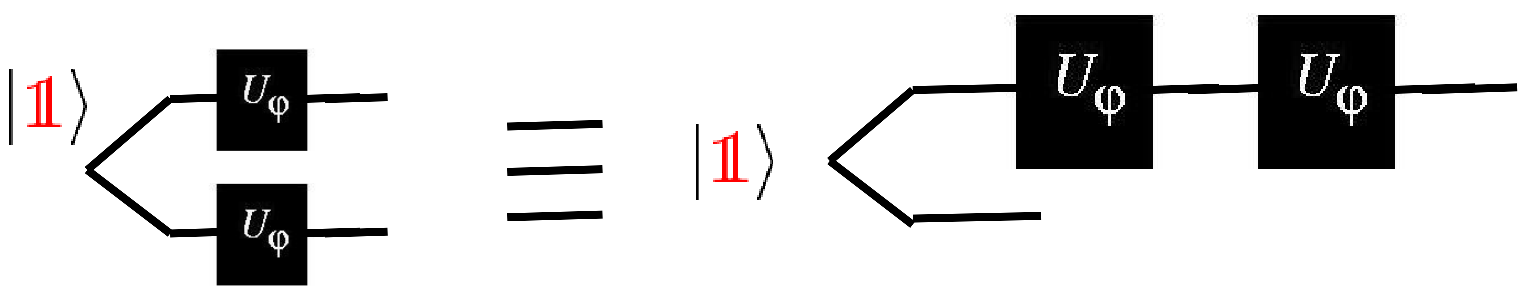

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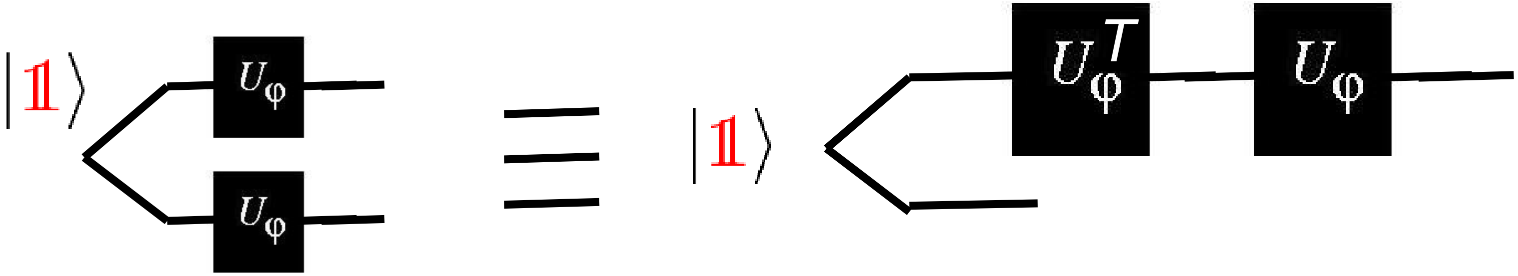

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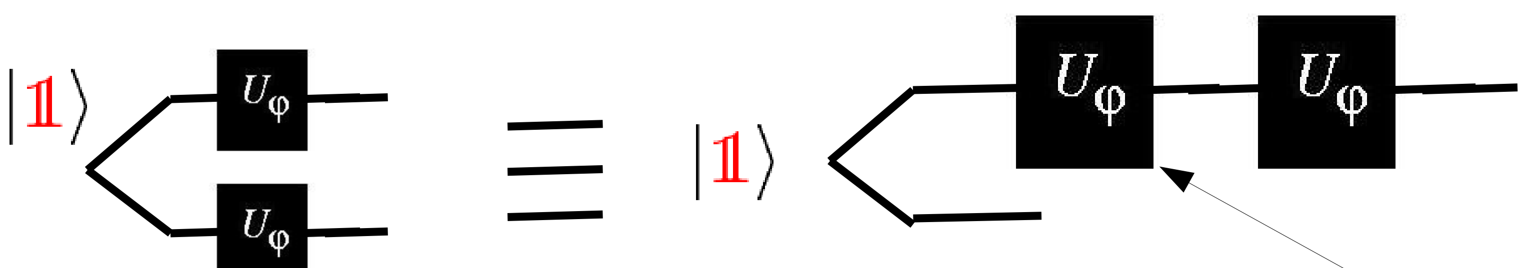

dropped the *T*

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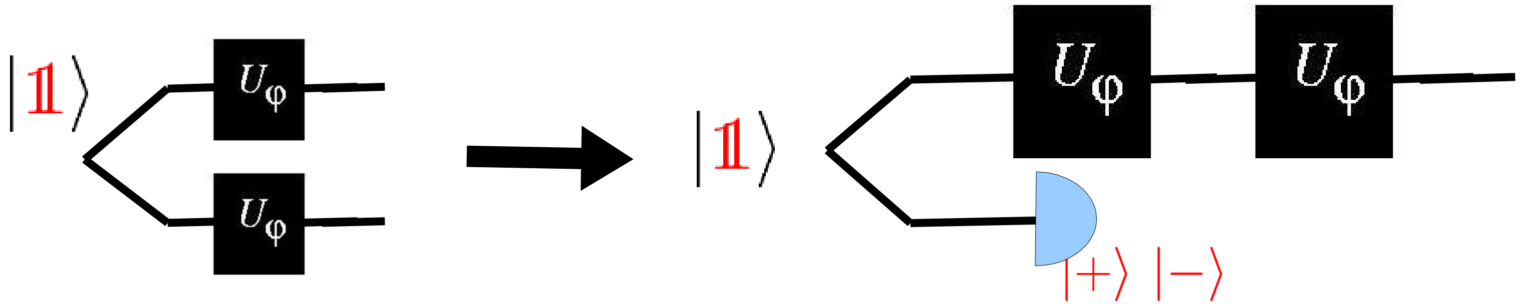

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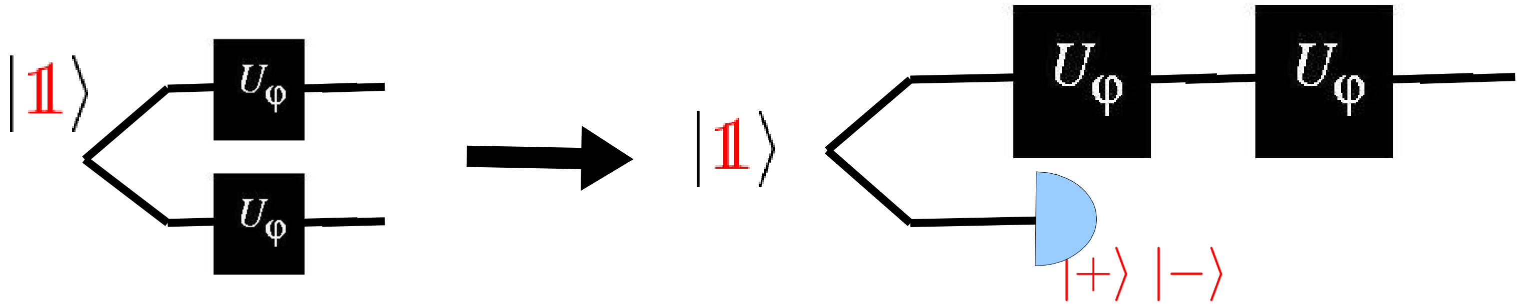
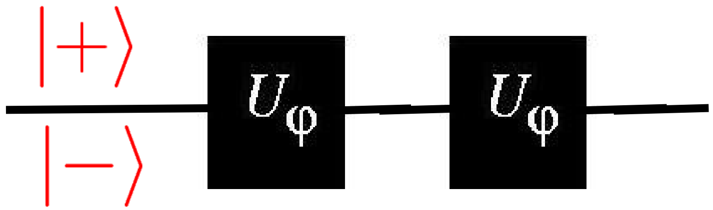


→ The other qubit is collapsed on the **same** state

(Klyshko mechanism)



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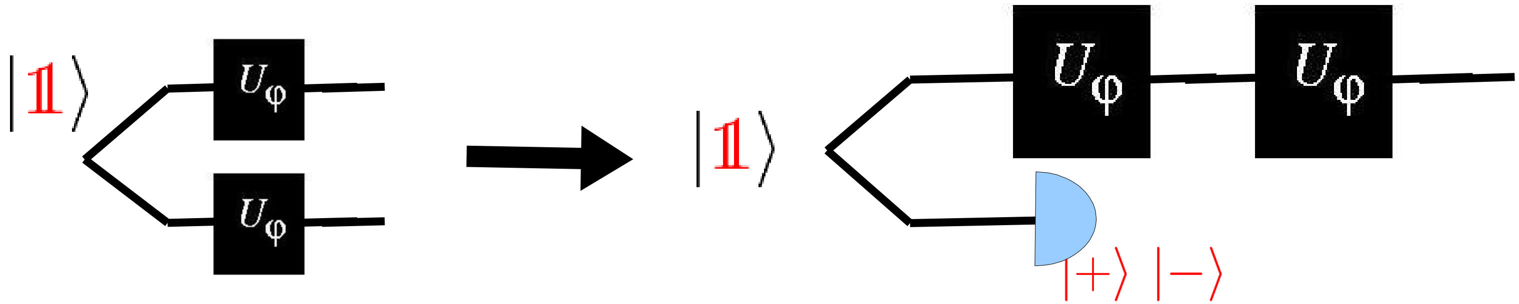
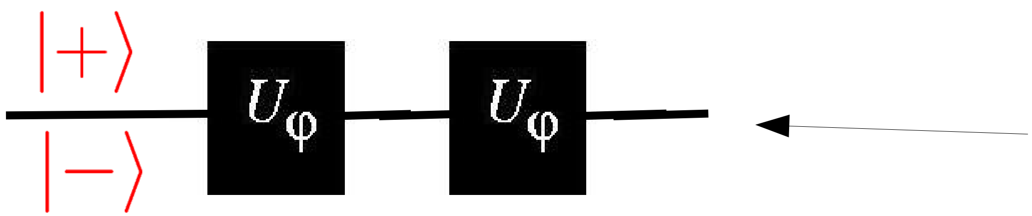


→ The other qubit is collapsed on the **same** state

(Klyshko mechanism)



sequential strategy

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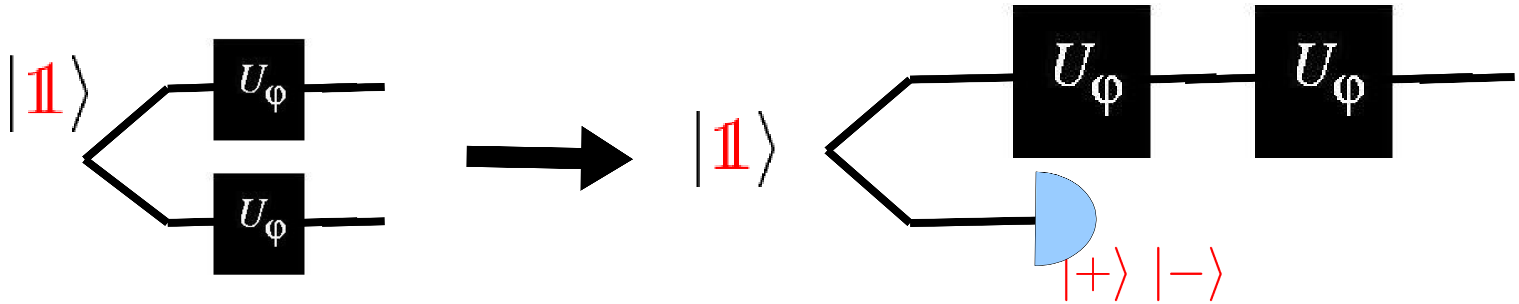

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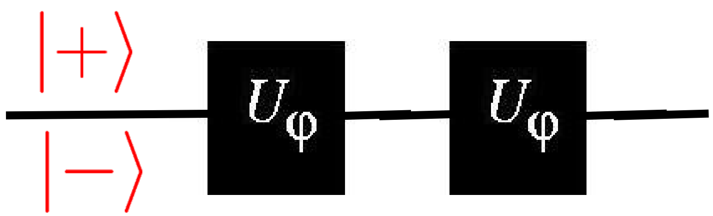
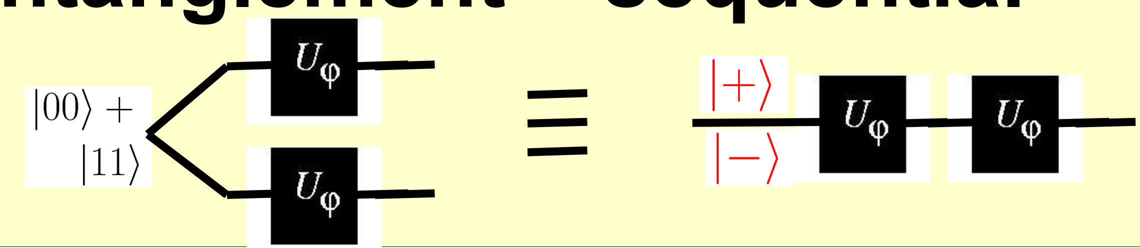


← sequential strategy

We have shown that

parallel+entanglement = sequential



So... Why entanglement?

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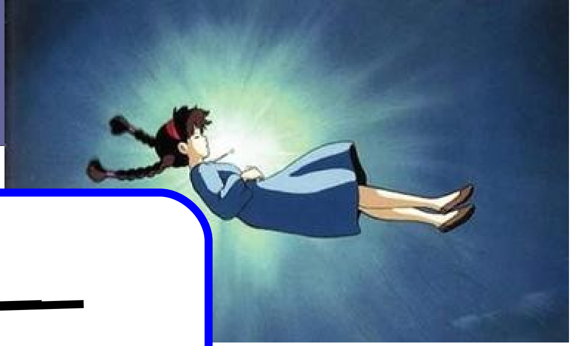

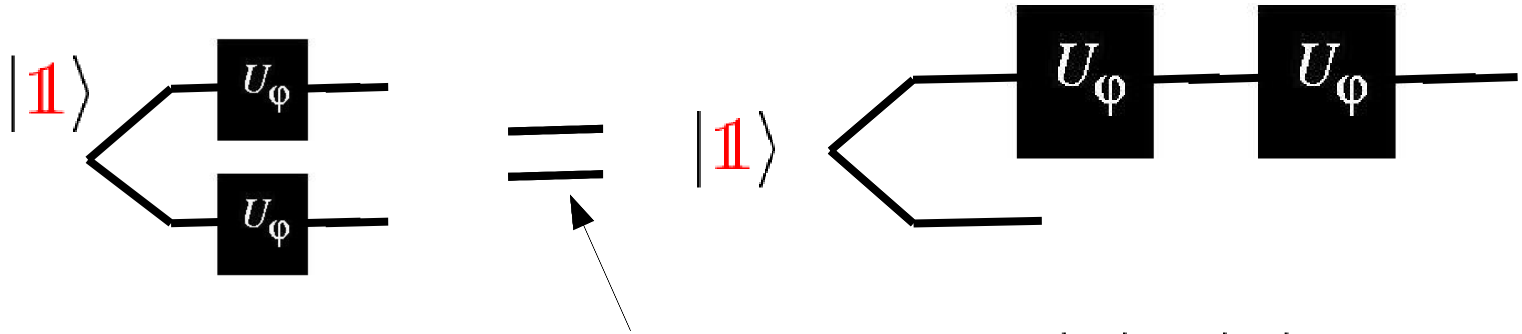

requires correlation in the $|0\rangle,\ |1\rangle$ basis

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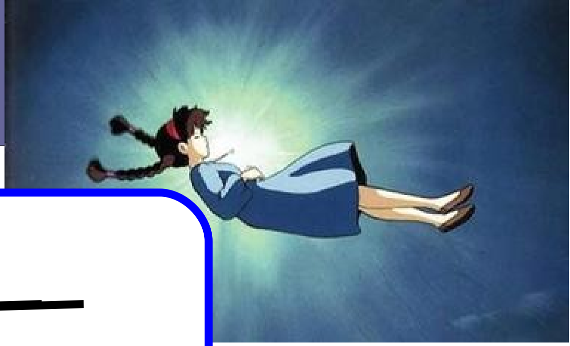

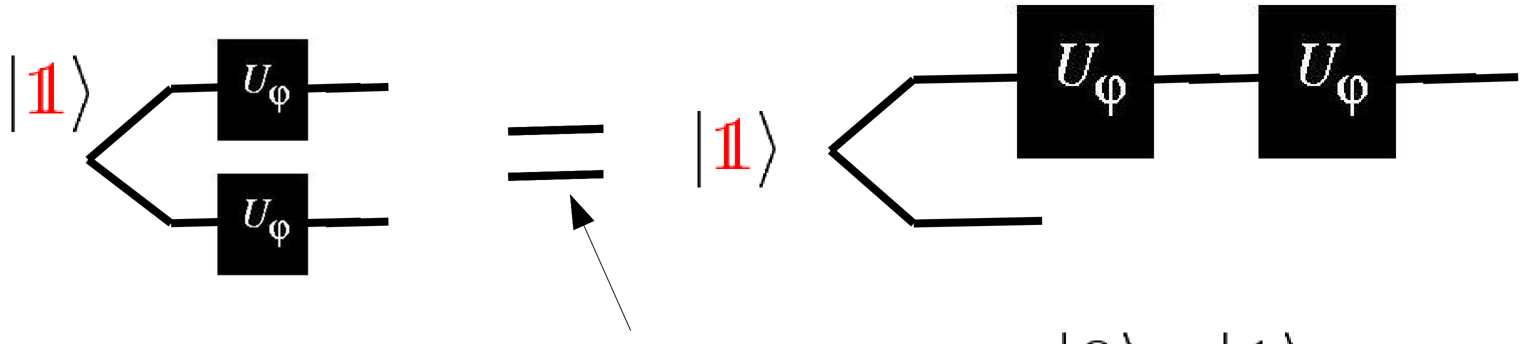

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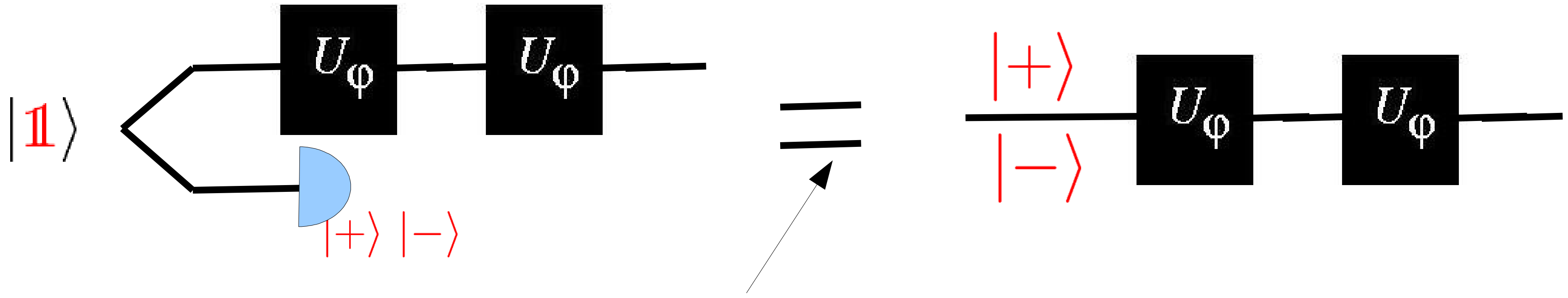

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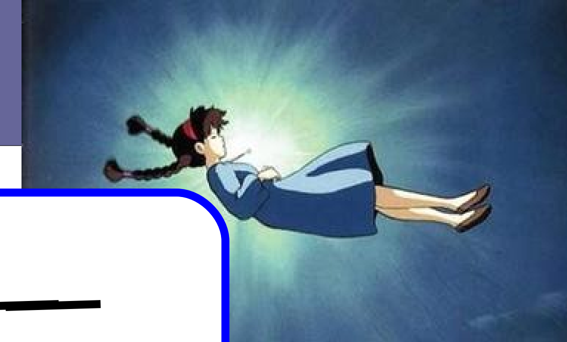

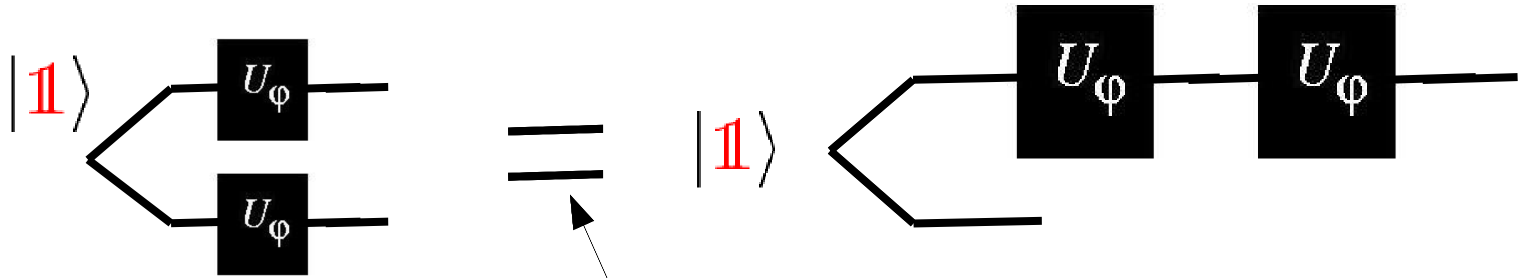

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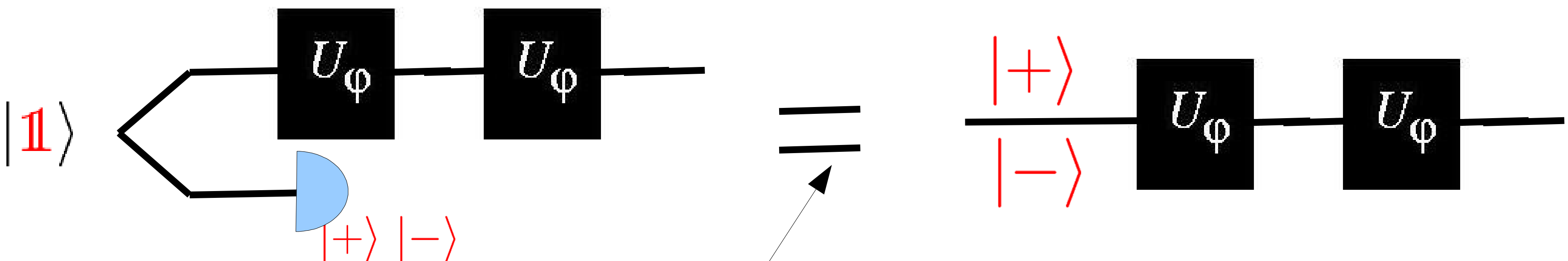
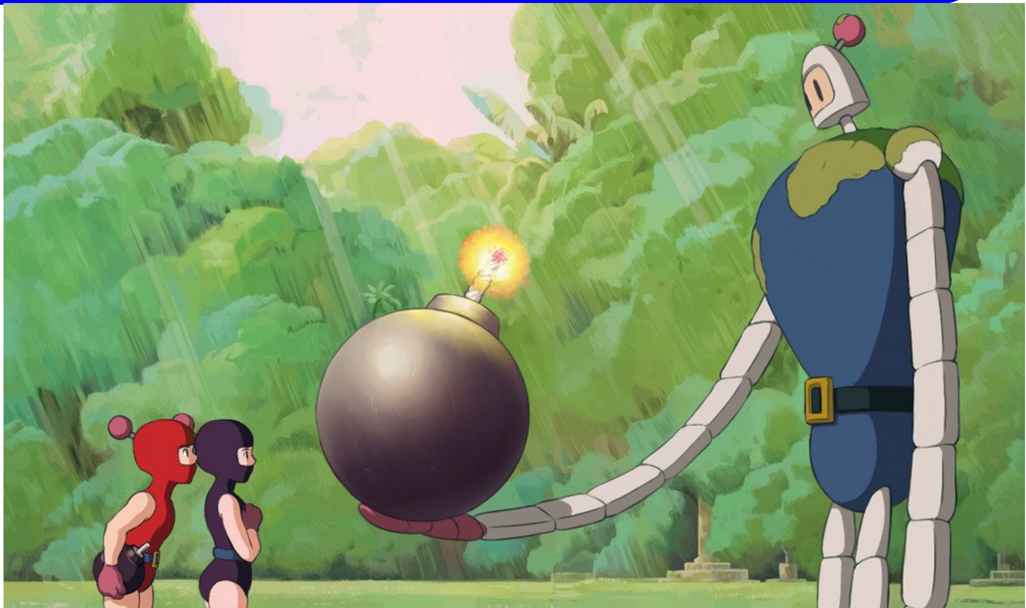


- requires correlation in the $|+\rangle \ |-\rangle$ basis

$\Rightarrow$ (they are **complementary** basis) we **need** entanglement!!

Without entanglement...

$$(|00\rangle\langle 00| + |11\rangle\langle 11|)/2$$



Without entanglement...

$$(|00\rangle\langle 00| + |11\rangle\langle 11|)/2$$

$$p = \frac{1}{2} : |00\rangle + |11\rangle;$$

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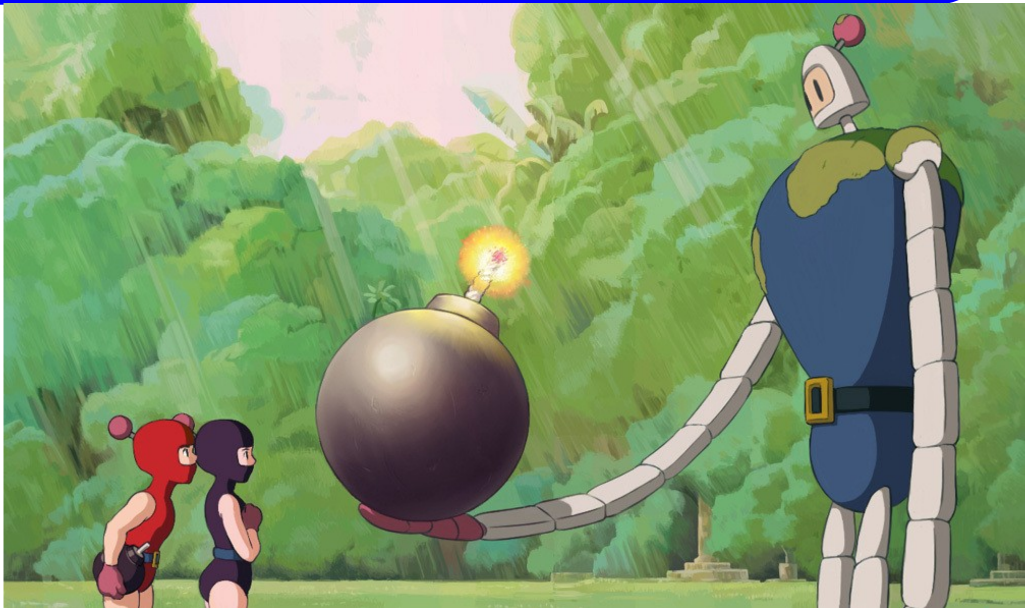

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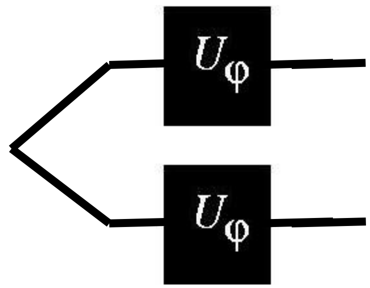
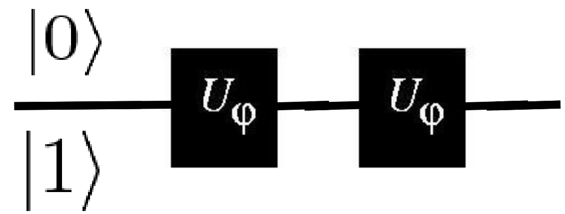
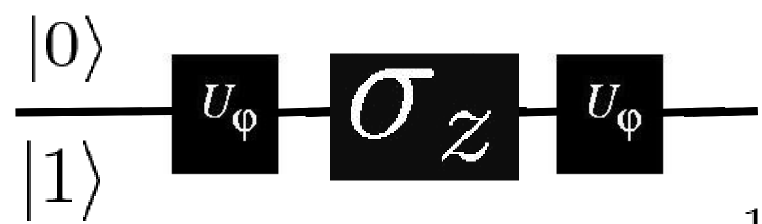


$\equiv$



$p = \frac{1}{2}$



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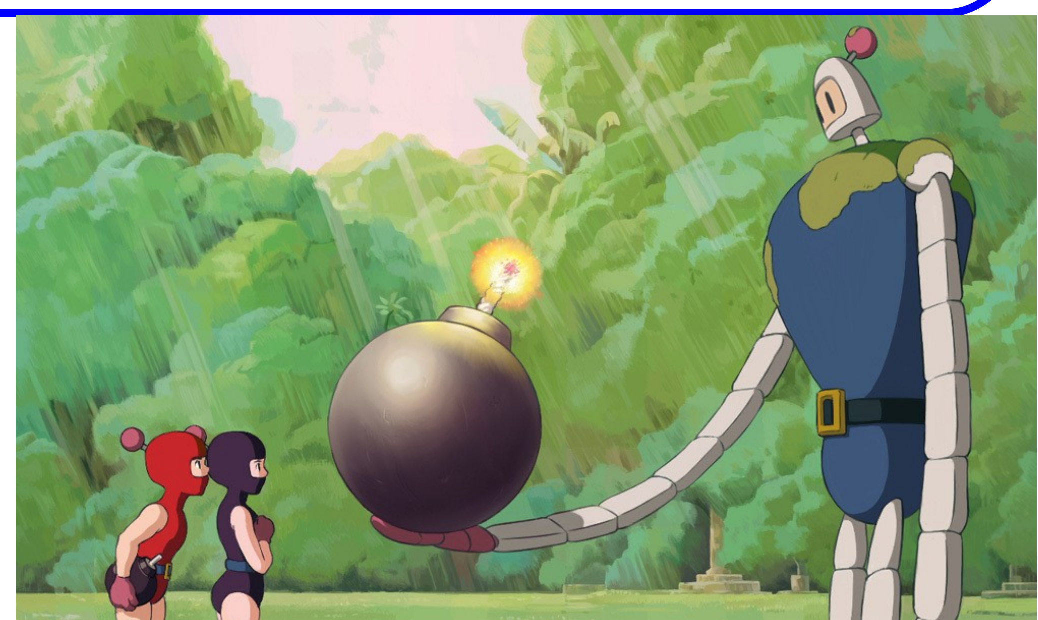

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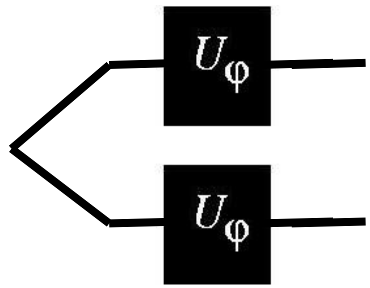

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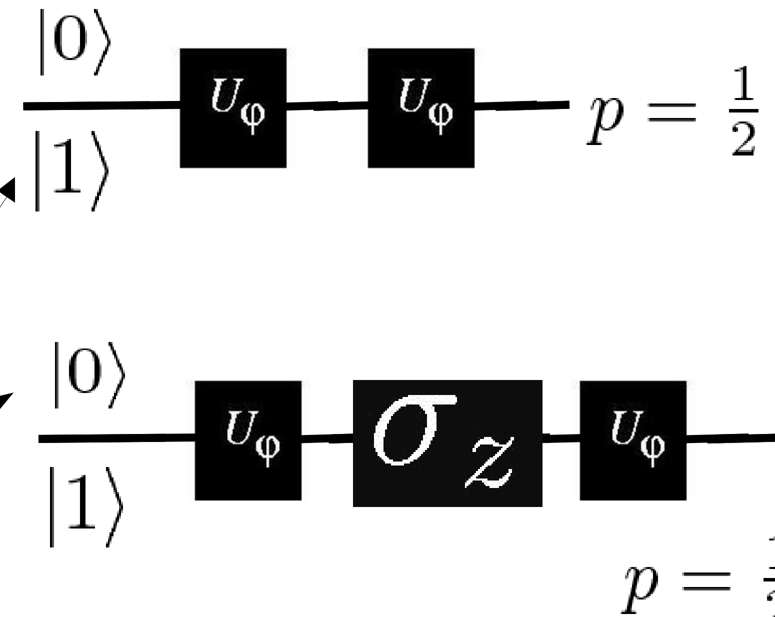
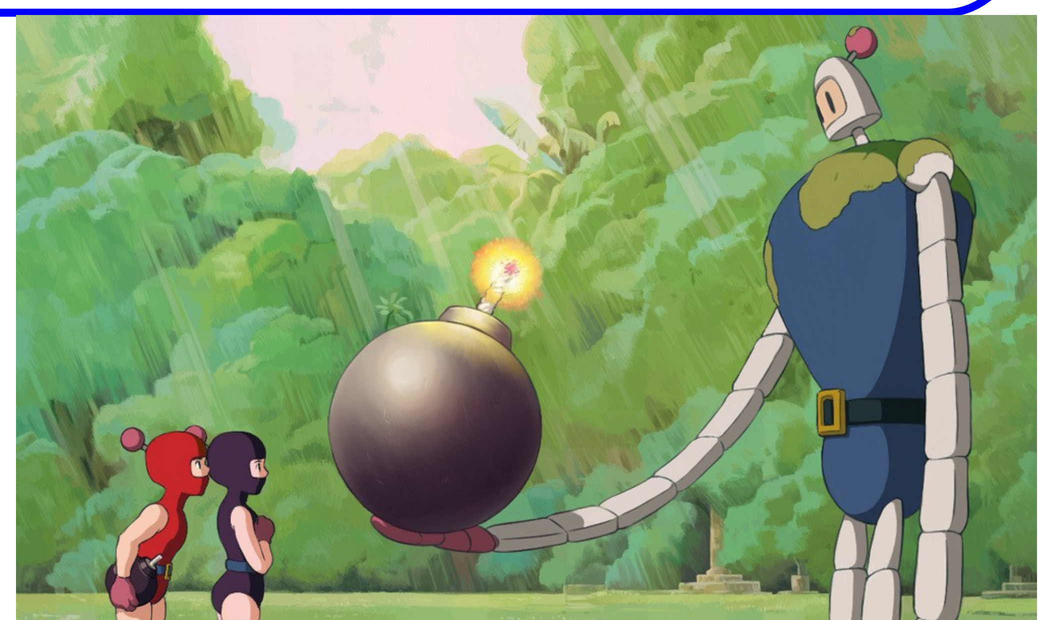


nothing happens: eigenstates!



Without entanglement...

Analogously, if we start from $(|++\rangle\langle++| + |--\rangle\langle--|)/2$

i.e. $p = \frac{1}{2} : |++\rangle + |--\rangle;$

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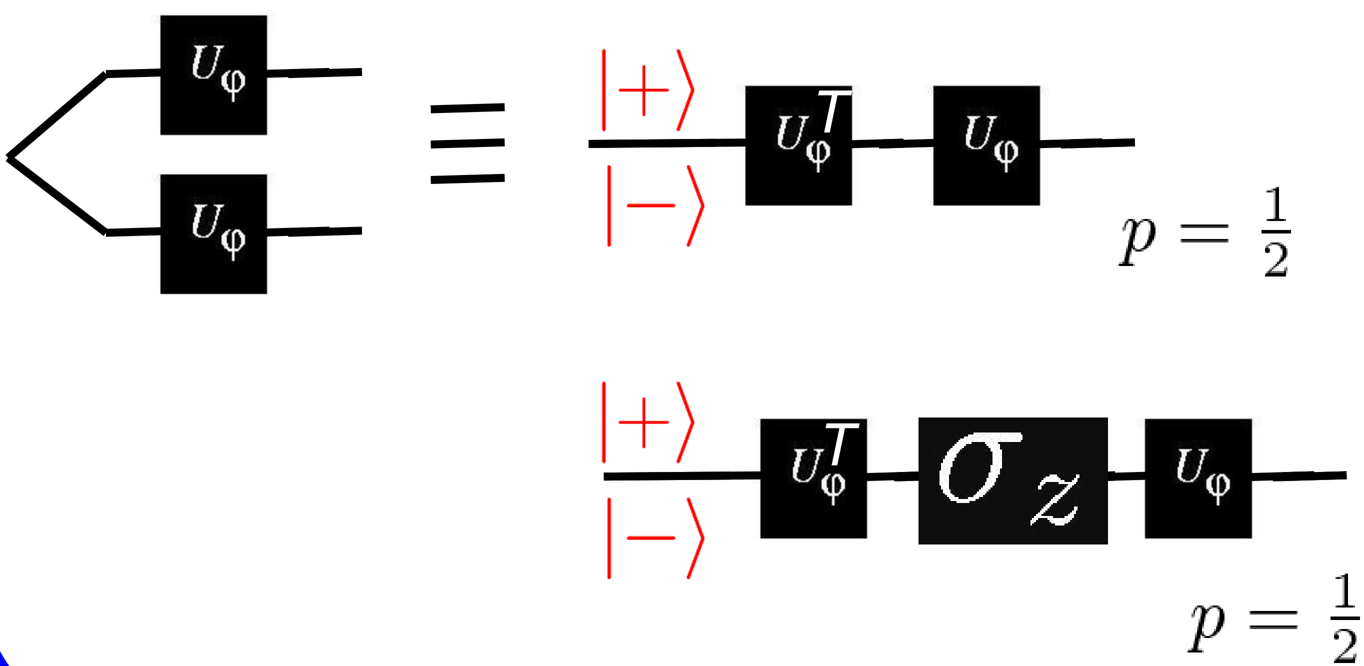

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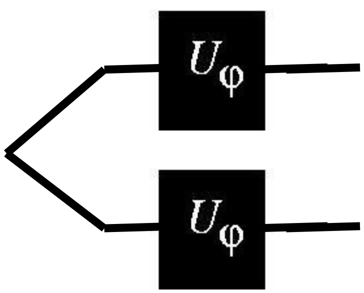

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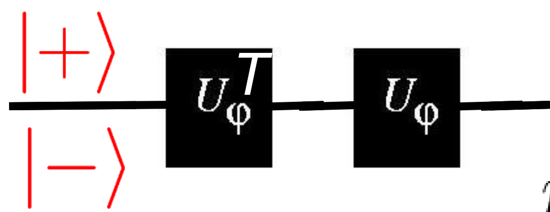
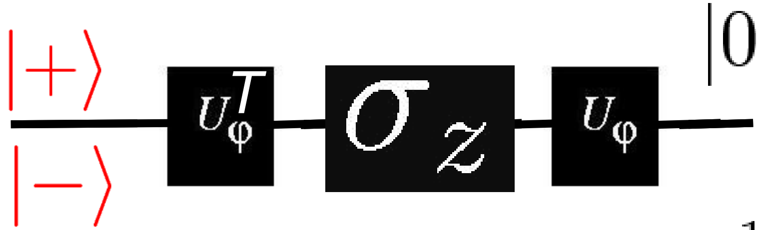


$|0\rangle + e^{i\varphi}|1\rangle$

$p=\frac{1}{2}$



$|0\rangle + e^{i(\varphi+\pi)}|1\rangle$

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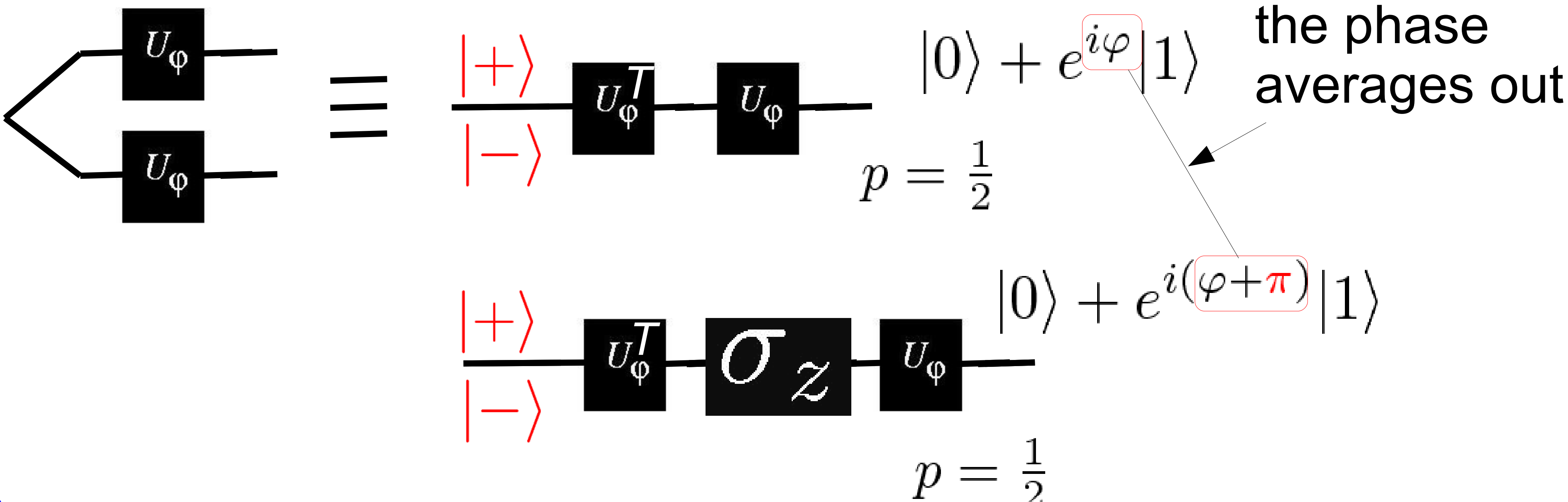

Technicalities: how to go to *N*>2

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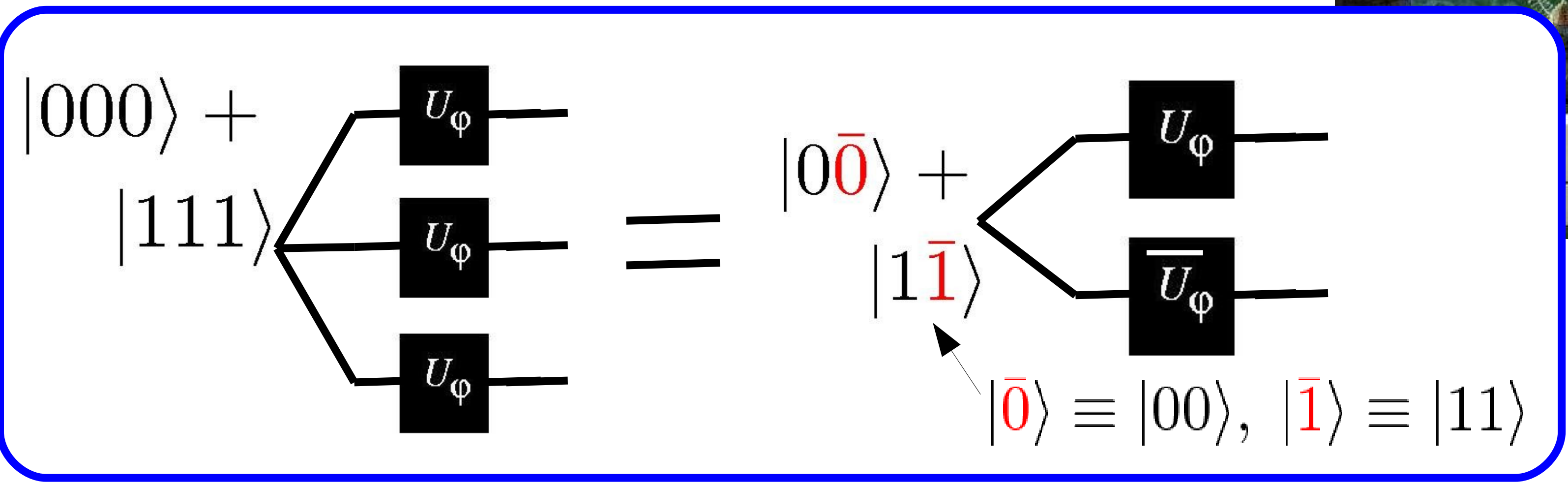

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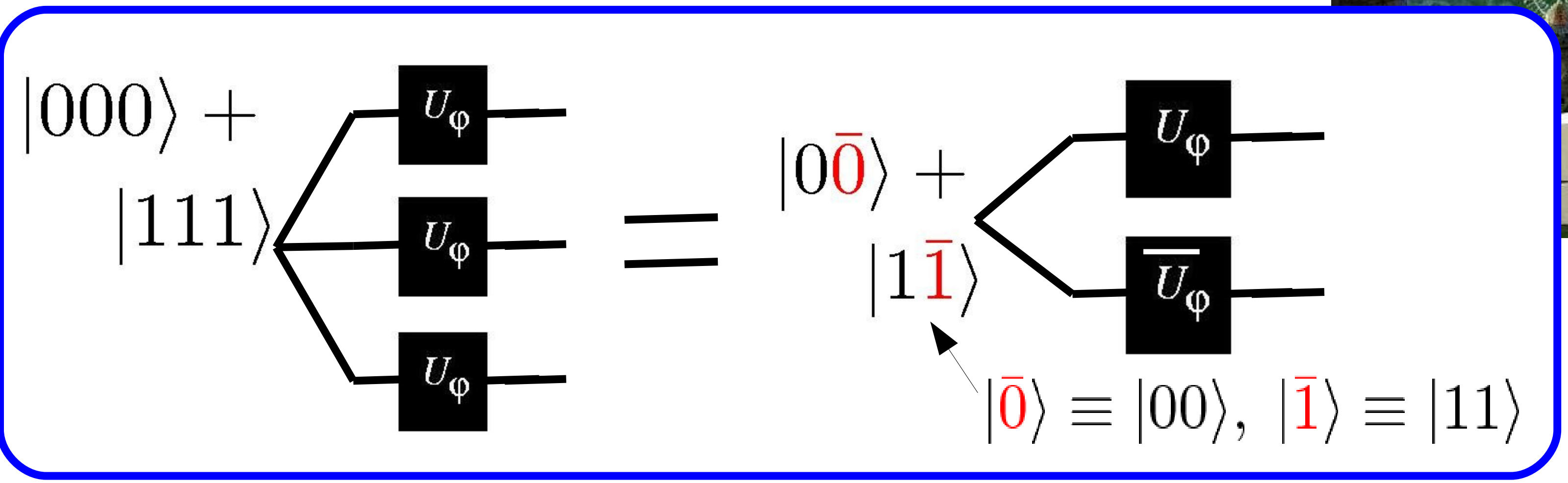

where

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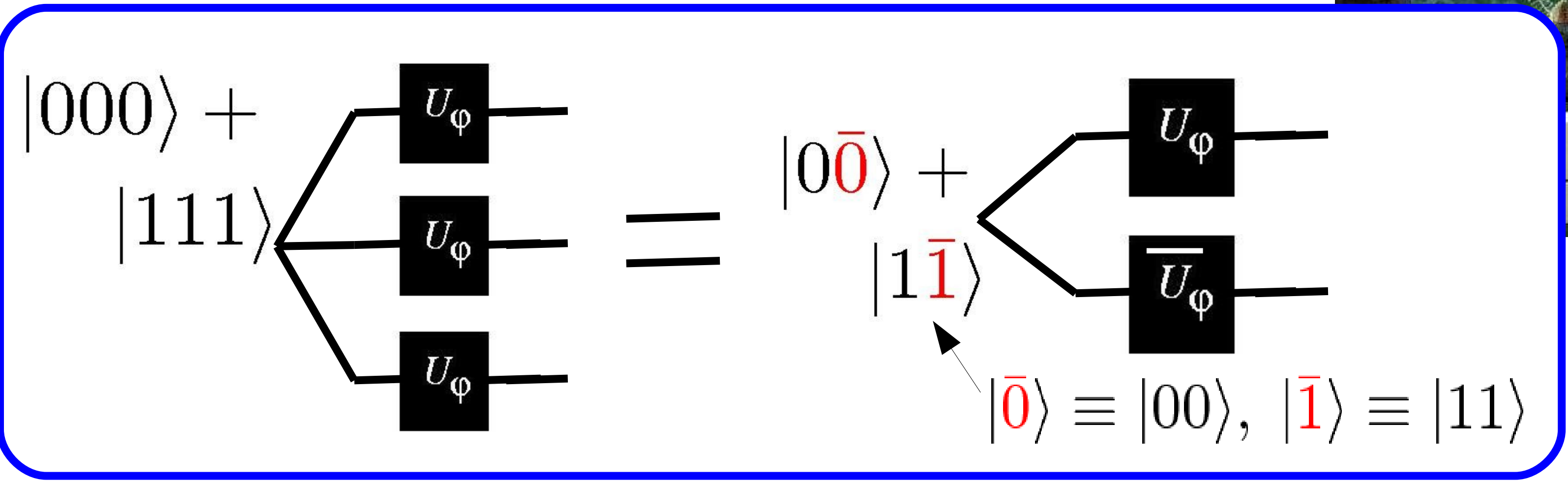

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in the $|\bar{0}\rangle,\ |\bar{1}\rangle$ (invariant) subspace

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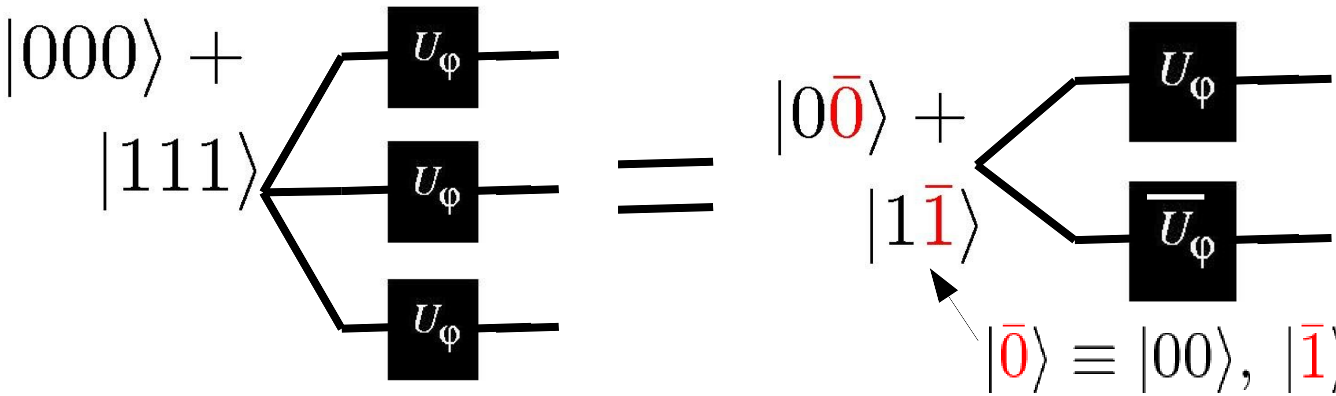

$|\bar{0}\rangle \equiv |00\rangle, \ |\bar{1}\rangle \equiv |11\rangle$

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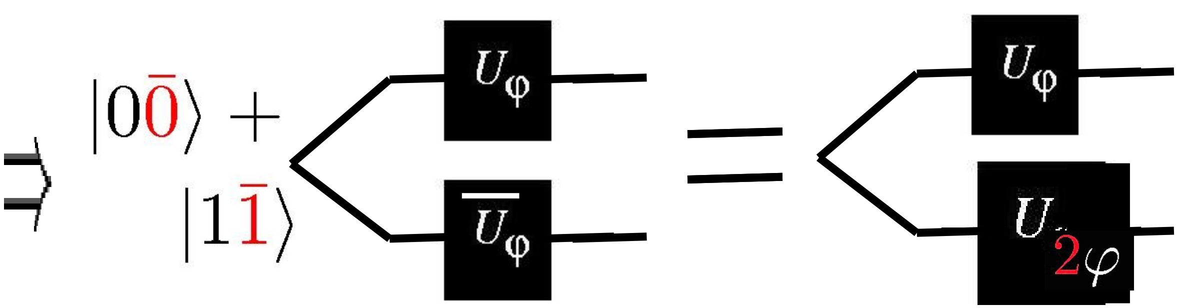

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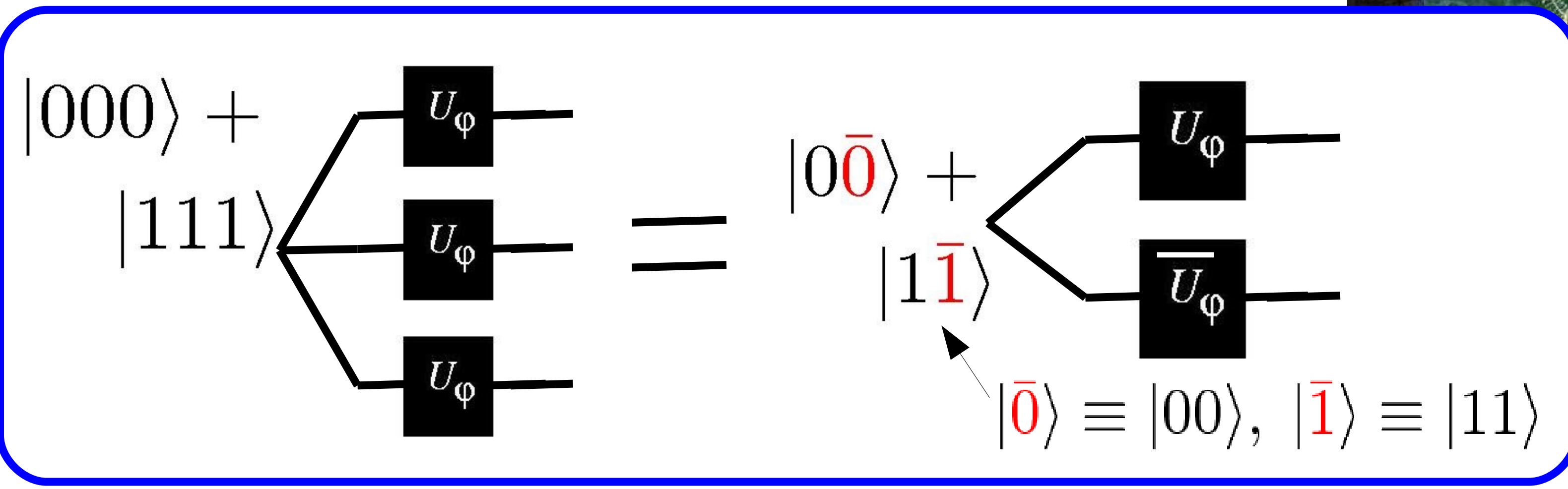

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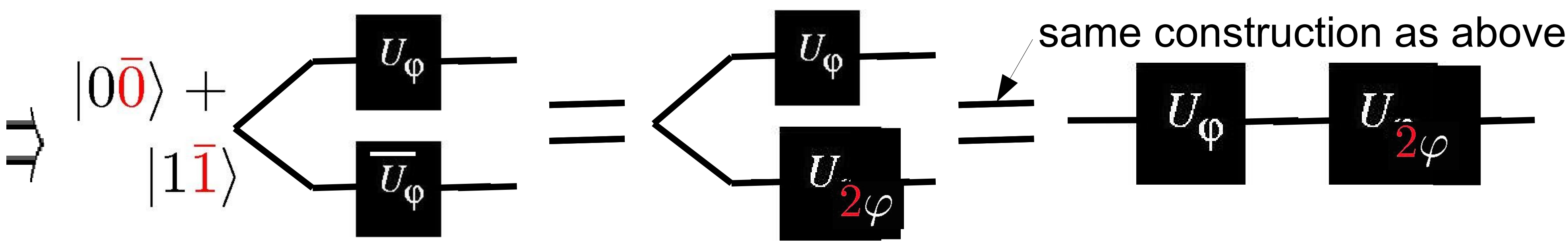

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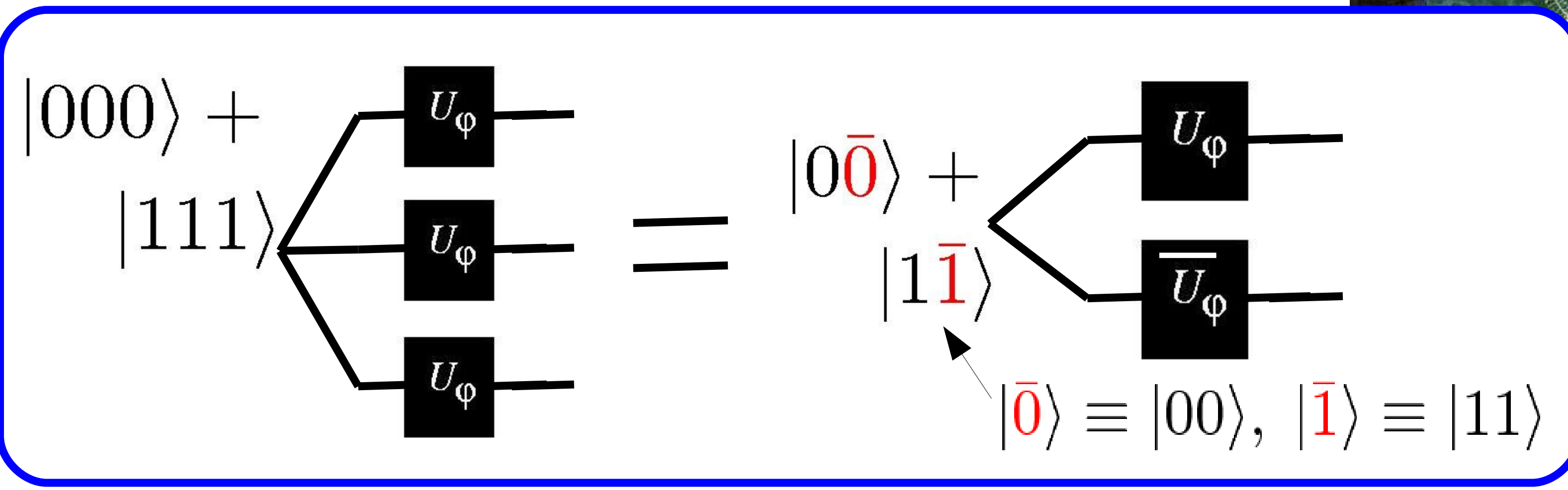

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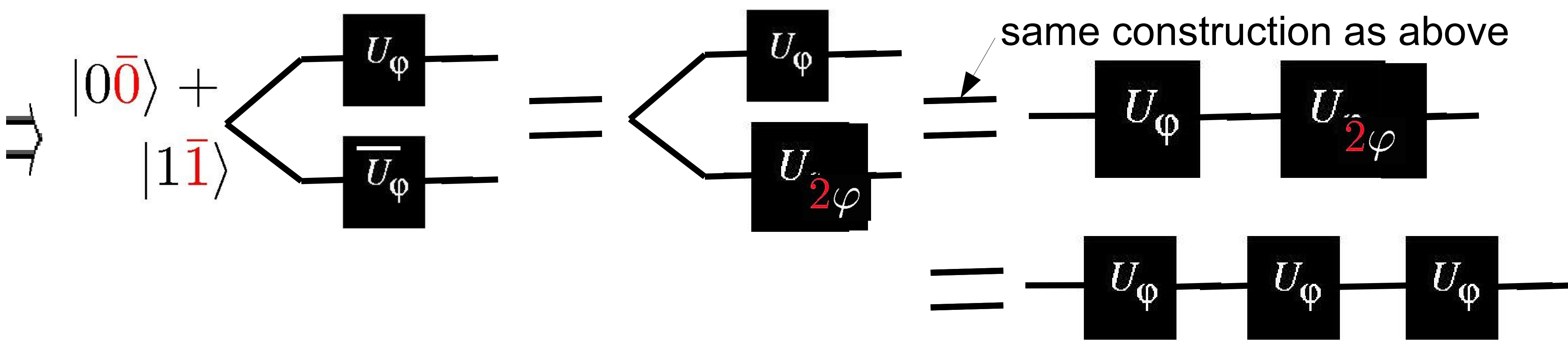

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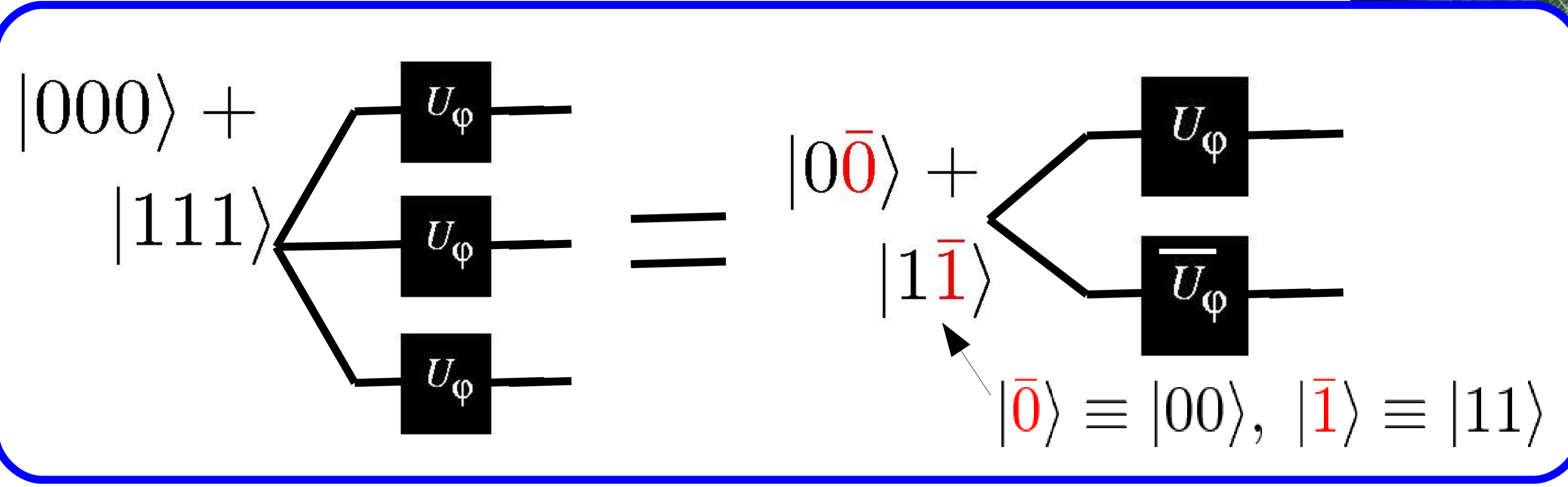

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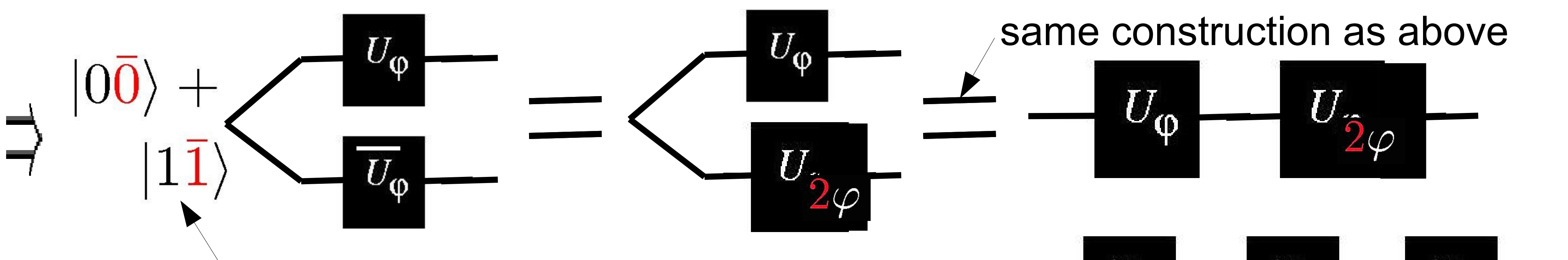
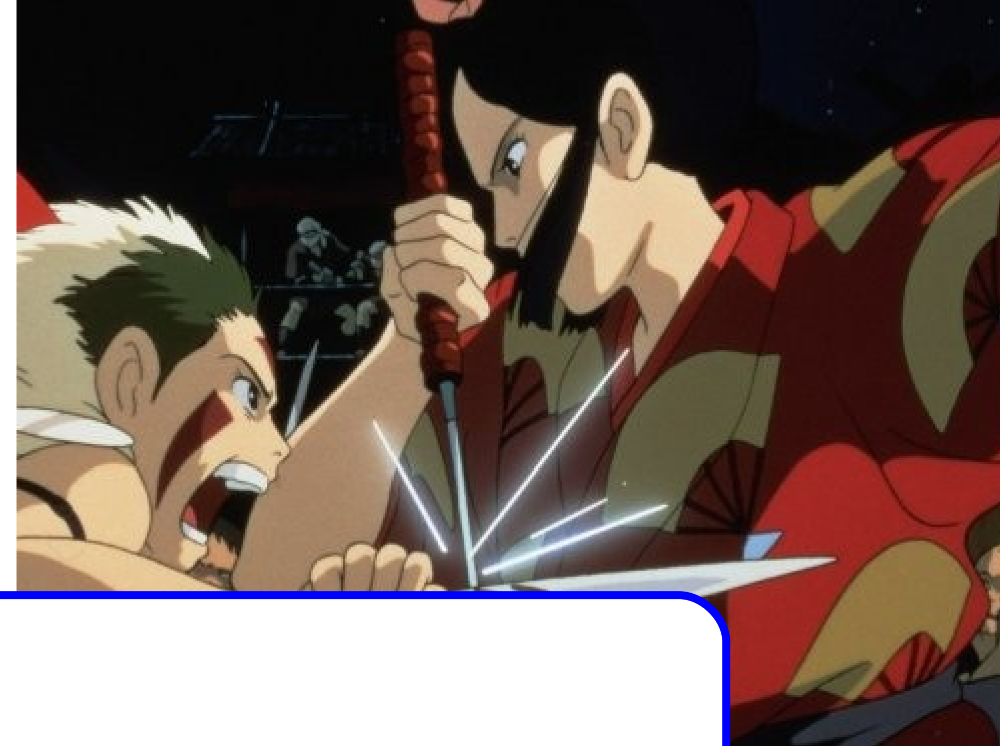
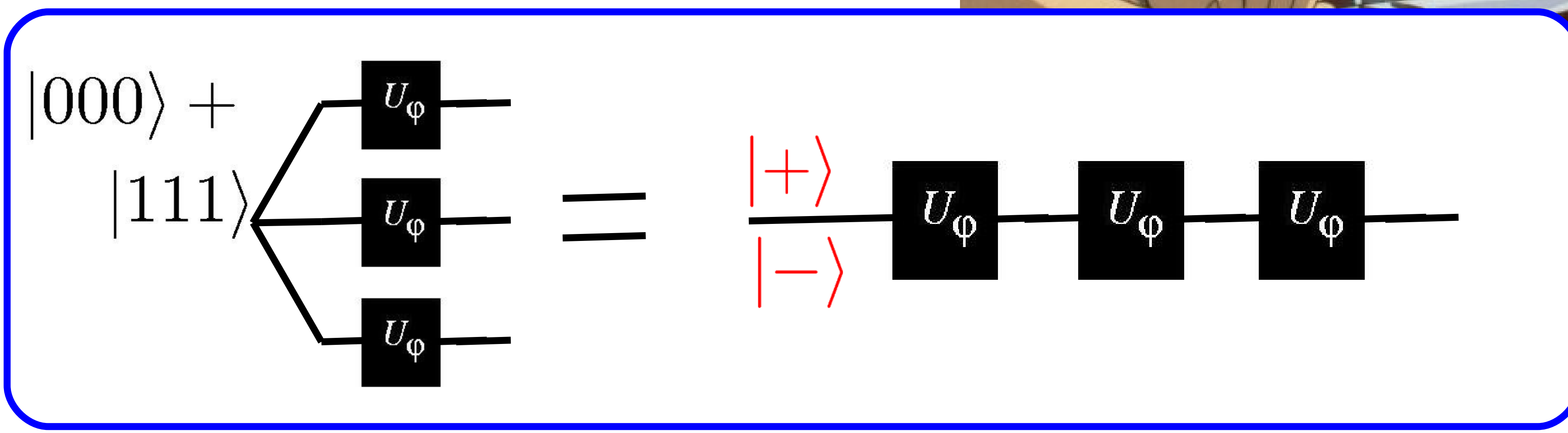


Note: this is NOT $|\mathbb{1}\rangle$ in the 3x3 space!

In other words....





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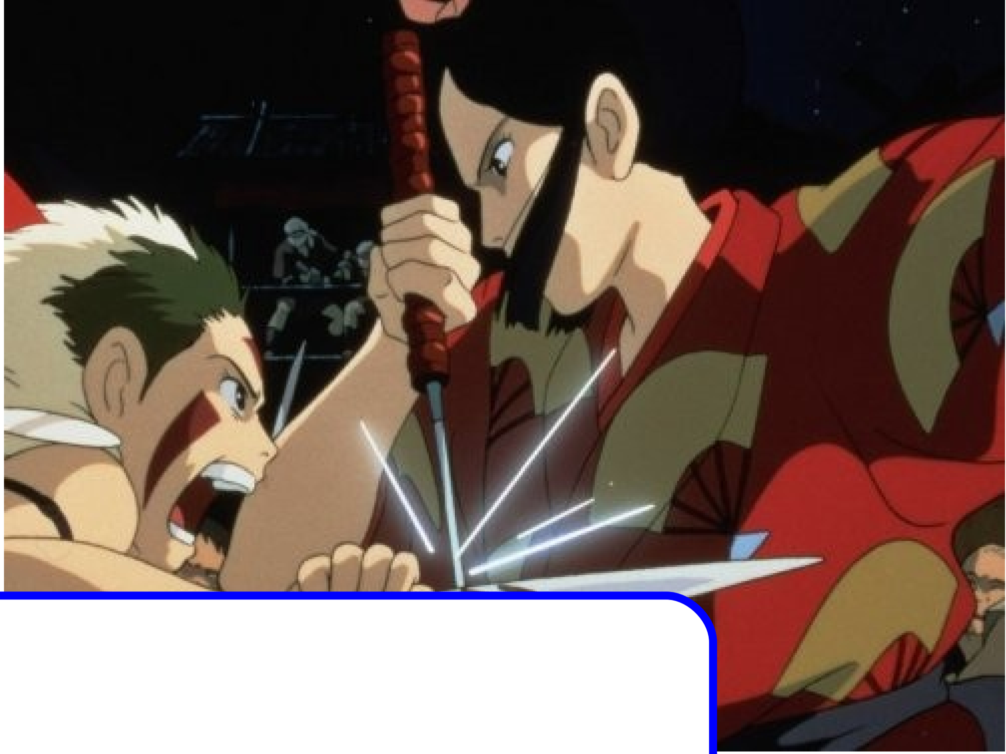

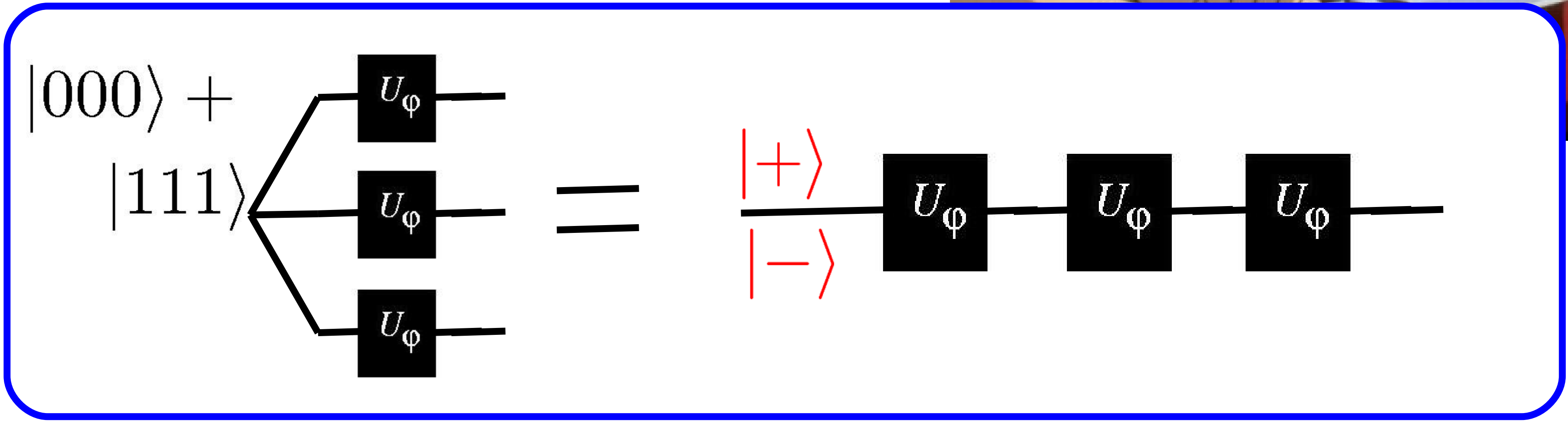

... and the same construction can be iterated for all *N*

N00N state interferometry

... possibly the most “famous” q metrology protocol

N00N state interferometry

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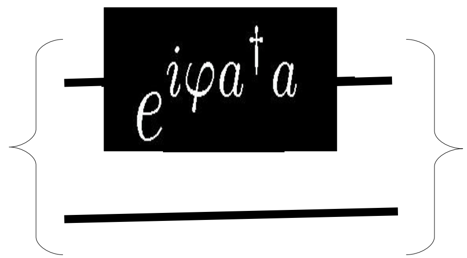
$$\frac{|N0\rangle + |0N\rangle}{\sqrt{2}} \rightarrow$$



$$\rightarrow \frac{|N0\rangle + e^{iN\varphi}|0N\rangle}{\sqrt{2}}$$

N00N state interferometry

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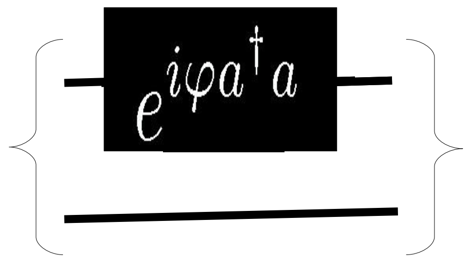

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...then project onto initial state: get $\Delta\varphi \simeq 1/N$

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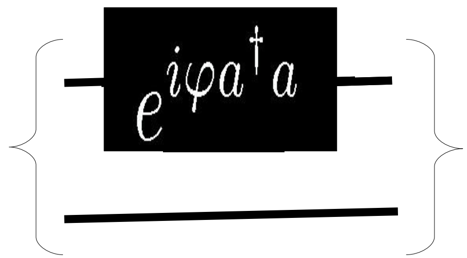

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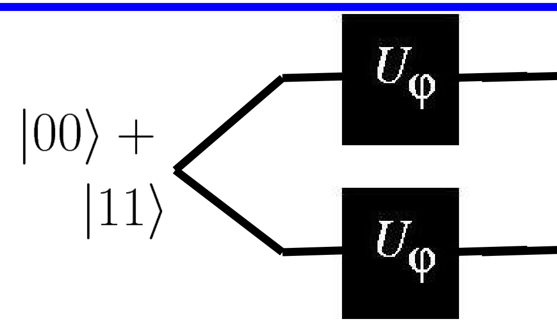
it can be cast in the form



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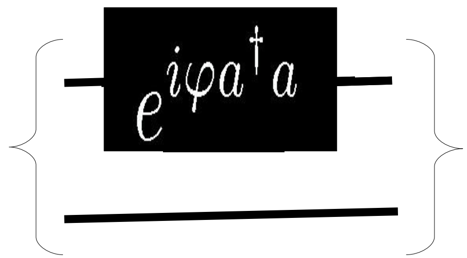

$$\rightarrow \frac{|N0\rangle + e^{iN\varphi}|0N\rangle}{\sqrt{2}}$$

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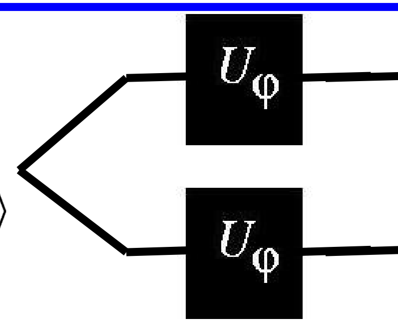

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whereas $|0N\rangle$ accumulates a phase 0, so:

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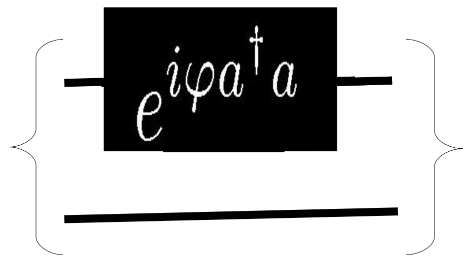

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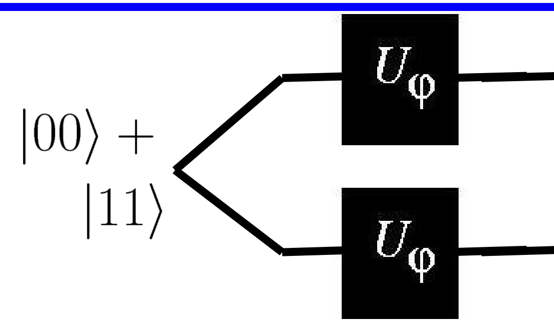

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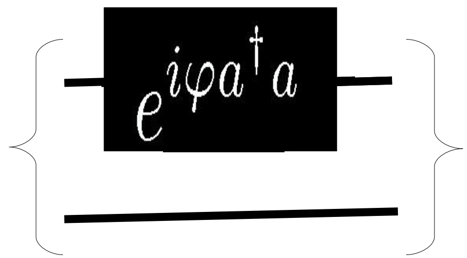
$$|N0\rangle \overset{\text{acts like}}{\equiv} |11\rangle$$

$$|0N\rangle \overset{\text{acts like}}{\equiv} |00\rangle$$

N00N state interferometry

... possibly the most "famous" q metrology protocol

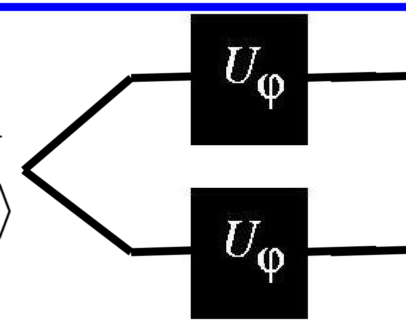
$$\frac{|N0\rangle + |0N\rangle}{\sqrt{2}} \rightarrow$$



$$\rightarrow |1\rangle$$

...then project onto initial state: get $\Delta\varphi \simeq 1/N$

it can be cast in the form $|00\rangle + |11\rangle$



by noting that $|N0\rangle$ accumulates a phase $N\varphi$ whereas $|0N\rangle$ accumulates a phase 0, so:

$$|N0\rangle \overset{\text{acts like}}{\equiv} |\bar{1}\bar{1}\rangle$$

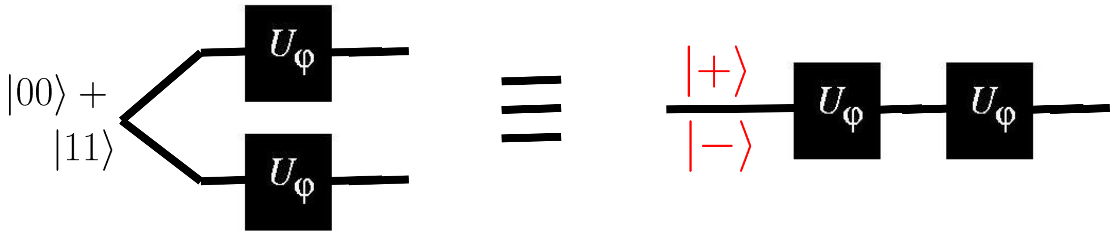
$$|0N\rangle \overset{\text{acts like}}{\equiv} |\bar{0}\bar{0}\rangle$$

$$\Rightarrow \frac{|N0\rangle + |0N\rangle}{\sqrt{2}} \overset{\text{acts like}}{\equiv} \frac{|\bar{0}\bar{0}\rangle + |\bar{1}\bar{1}\rangle}{\sqrt{2}} = |\bar{1}\rangle$$

What did I say?!?

- Intuitive definition of entanglement

- Q metrology: use entanglement to turn a parallel strategy into a sequential one.



- How to extend the construction to arbitrary *N*

- A case study: N00N state interferometry

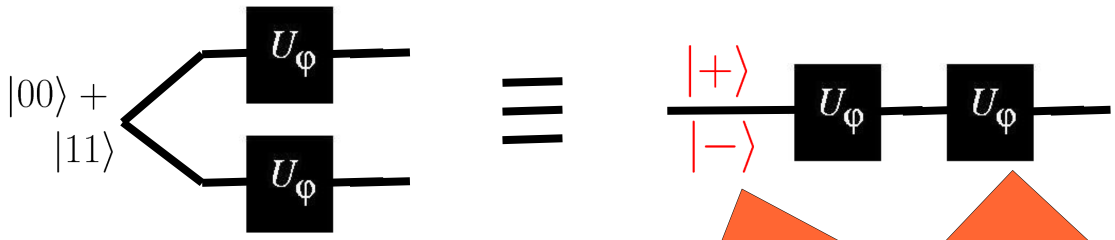
Take home message

Entanglement is necessary
in q metrology because...

Q metrology theory:PRL **96**,010401 (2006)
Recent review: Nature Phot. **5**,222 (2011)

Lorenzo Maccone
maccone@unipv.it

Take home message



Entanglement is necessary
in q metrology because...
... we need correlations
in complementary basis
to go from a parallel to a sequential strategy

Q metrology theory:PRL **96**,010401 (2006)
Recent review: Nature Phot. **5**,222 (2011)

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maccone@unipv.it

Recent result in q metrology: A new bound

Heisenberg / Cramer-Rao

$$\Delta X \geqslant \frac{1}{\sqrt{\nu}\Delta H}$$

$$e^{-ixH}$$

precision bounded by the variance $\Delta^2 H$ (**second** moment) of the generator *H*

New bound

$$\Delta X \geqslant \frac{\kappa}{\sqrt{\nu}\mathcal{H}}$$

$$\mathcal{H} \equiv \langle H \rangle - E_0$$

κ constant O(1)

ground state (minimum eigenvalue of *H*)

precision bounded by the expectation value $\langle H \rangle$ (**first** moment) of the generator *H*

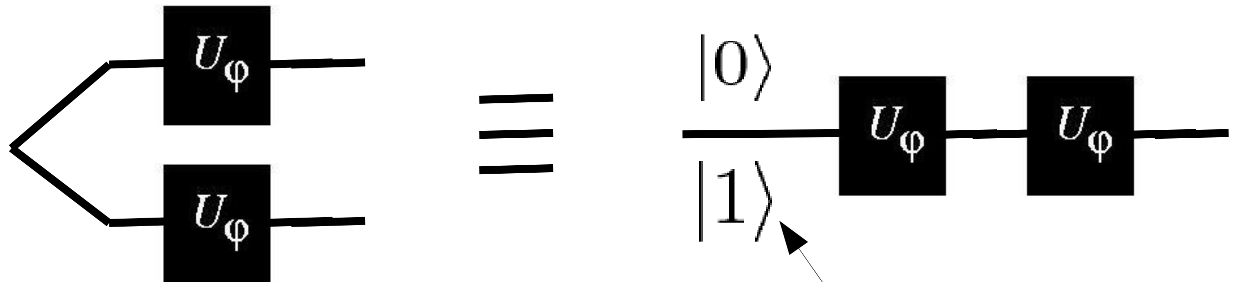
A new uncertainty relation with EXPECTATION VALUE instead of the variance

$$\Delta X \left(\langle H \rangle - E_0 \right) \geqslant \kappa$$

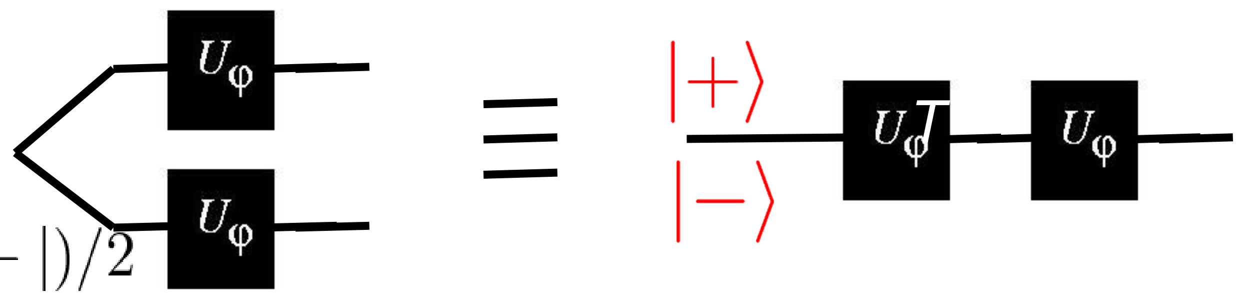
Bound: PRL **108**, 260405 (2012)
Prior info: PRL **108**, 210404 (2012)

slide sbagliata..Without entanglement...

$(|00\rangle\langle 00| + |11\rangle\langle 11|)/2$



nothing happens: eigenstates!



$(|++\rangle\langle ++| + |--\rangle\langle --|)/2$

U_φ^{T} in the $|+\rangle, |-\rangle$ basis is $U_\varphi^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$U_\varphi U_\varphi^{\dagger}|\pm\rangle = |\pm\rangle$ ← again, nothing happens

... and what about the $|+i\rangle, |-i\rangle$ basis?