qcontrol13 Discussion
Classical control in a quantum world.

G J Milburn

Centre for Engineered Quantum Systems, The University of Queensland

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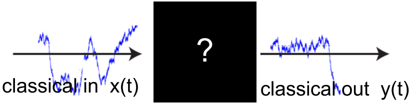
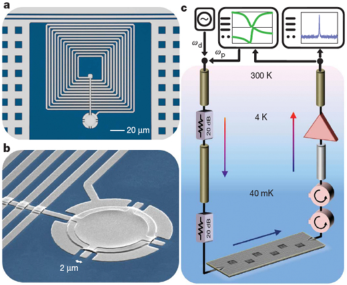
What is the problem?

Classical (stochastic process) in: Classical (stochastic process) out

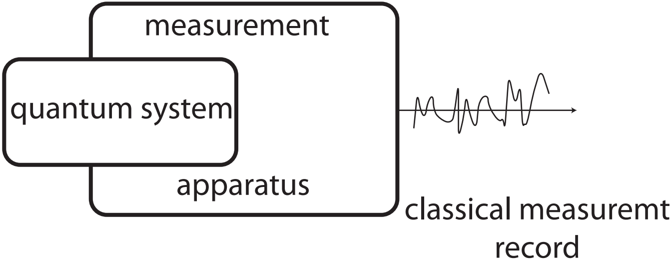




Teufel et al. (NIST) Nature, March (2011).

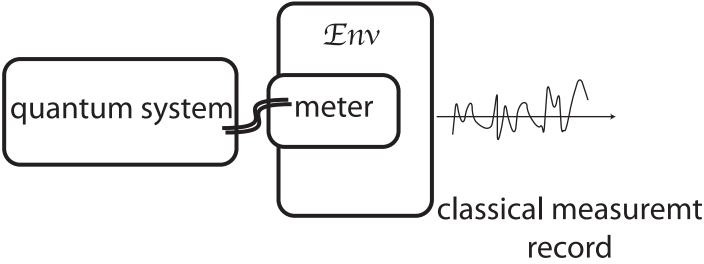
The old problem.

Classical measurement records.



The old problem.

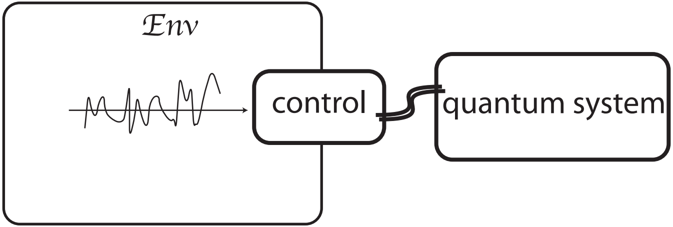
Classical measurement records.



The interaction between the meter and the environment fixes the pointer basis.

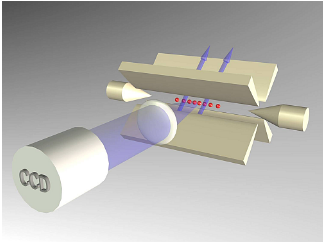
The control problem.

Classical control records.



If we quantise the controller, why doesn't the controller become entangled with the system?

Classical control in QC.



Qubit gates implemented by external control lasers, treated classically.

van Enk and Kimble, *On the classical character of control fields in quantum information processing.*
Quant. Inf. Com. (2002).

Classical control in quantum optics.

Silberfarb & Deutsch, *Entanglement generated between a single atom and a laser pulse*
PR A 69, 042308 (2004).

Nha. & Carmichael, *Decoherence of a two-state atom driven by coherent light.*
Phys. Rev. A (2005).

Noh, & Carmichael, *Disentanglement of source and target and the laser quantum state.*
Phys. Rev. Lett. (2008).

A related problem: quantum-classical hybrid dynamics.

Consistently combine classical and quantum dynamics.

$$H_h = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 + \frac{P^2}{2M} + \frac{M\Omega^2}{2}Q^2 + H_I$$

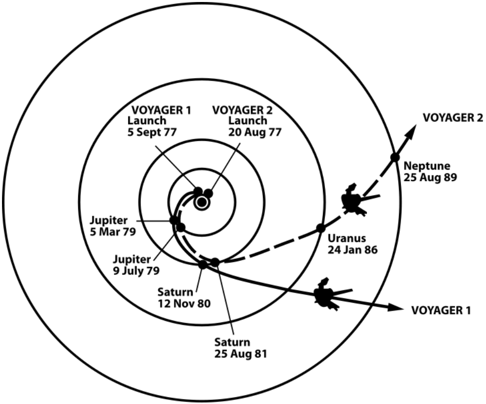
$$H_I = \epsilon Q(t)\hat{x}^2$$

Heisenberg eqns:

$$\begin{aligned} \frac{d\hat{x}}{dt} &= \frac{\hat{p}}{m} \\ \frac{d\hat{p}}{dt} &= -m\omega^2\hat{x} - 2\epsilon Q(t)\hat{x} \\ \dot{Q} &= \frac{P}{M} \\ \dot{P} &= -m\Omega^2 Q - \epsilon\hat{x}^2 \qquad ???? \end{aligned}$$

A related problem: quantum-classical hybrid dynamics.

Gravity . . . irreducibly classical control



A related problem: quantum chemistry.

Robbie Grunwald and Raymond Kapral , *Decoherence and quantum-classical master equation dynamics*
J. Chem. Phys. 126, 114109 (2007)

> *Proton and electron transfer processes in the condensed phase and in biomolecules fall into this category, as do many vibrational relaxation processes.*

A related problem: quantum-classical hybrid dynamics.

Anderson *Quantum backreaction on 'classical' variables.* Phys. Rev. Lett. 1995

Halliwell *Effective theories of coupled classical and quantum variables from decoherent histories: a new approach to the back reaction problem.* Phys. Rev. D. 1998

Peres. & Terno *Hybrid classical-quantum dynamics.* Phys. Rev. A. 2001

Diosi *The gravity-related decoherence master equation from hybrid dynamics*. J. Phys. Conf. Ser. 306, 012006. 2011

Quantum-classical hybrid dynamics: Diosi's approach

Postulate a classically parameterized quantum density operator, $\hat{\rho}(q,p)$;

$$\hat{\rho}_Q \equiv \int dqdp\hat{\rho}(q,p) \quad P(q,p) = \mathrm{tr}\hat{\rho}(q,p)$$

Hamiltonian $\hat{H} = \hat{H}(q,p)$ and Dynamics "Aleksandrov bracket"

$$\frac{d\hat{\rho}(q,p)}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}(q,p)] + \mathrm{Herm}\left\{\hat{H},\hat{\rho}(q,p)\right\}_P$$

$$\mathrm{Herm}\left\{\hat{H},\hat{\rho}\right\}_P = \frac{1}{2}\left\{\hat{H},\hat{\rho}\right\}_P - \frac{1}{2}\left\{\hat{\rho},\hat{H}\right\}_P$$

$\{f,g\}_P$ classical Poison bracket.

Does NOT preserve positivity!

Quantum-classical hybrid dynamics: Diosi's approach

The fix: add noise to BOTH the classical and the quantum system.

$$\hat{H} = \hbar\kappa\hat{q}Q \to \hbar\kappa(\hat{q} + \delta]\hat{q})(Q + \delta Q)$$

$$\langle\delta\hat{q}(t)\hat{q}(t')\rangle = D_q\delta(t - t') \quad \mathcal{E}[\delta Q(t)Q(t')] = D_c\delta(t - t')$$

Average Aleksandrov bracket over quantum and classical noise:

$$\begin{aligned}\frac{d\hat{\rho}(q,p)}{dt} \quad = \quad & -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(q,p)] + \mathrm{Herm}\left\{\hat{H}, \hat{\rho}(q,p)\right\}_P \\ & - \frac{1}{2\hbar^2}D_c[\hat{q},[\hat{q},\hat{\rho}]] + \frac{1}{2}D_q\{Q,\{Q,\hat{\rho}\}\}\end{aligned}$$

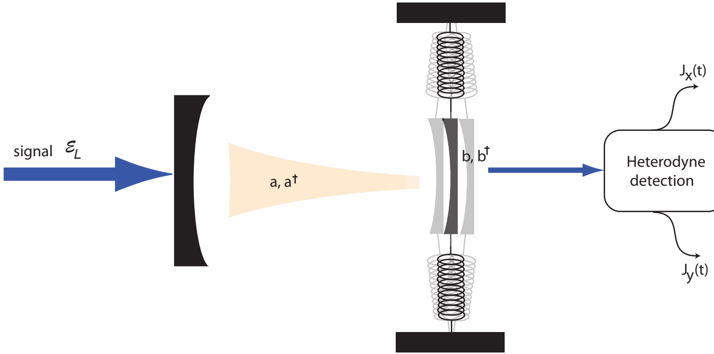
and remains positive if

$$D_q D_c \geq \frac{\hbar^2}{4}$$

.. a the cost of adding decoherence to the quantum dynamics. In the case of gravity this leads to Penrose decoherence.

The *physical* control problem.

Example: cavity optomechanics.



Objective: classical (optical) force acting on the moving mirror.

The *physical* control problem.

Radiation pressure interaction: $\hbar G_0 a^\dagger a(b + b^\dagger)$
Assume cavity is strongly damped and driven by a laser.
Heterodyne measurement of the intracavity field.

$$
\begin{aligned}
dJ_x(t) &= \kappa\langle a + a^\dagger\rangle dt + \sqrt{2\kappa}dW_1 \\
dJ_y(t)dt &= -i\kappa\langle a - a^\dagger\rangle dt + \sqrt{2\kappa}dW_2
\end{aligned}
$$

Unconditional master equation for the mirror:

$$
\frac{d\rho_m}{dt} = -\frac{i}{\hbar}[H_m, \rho_m] + \Gamma\mathcal{D}[b + b^\dagger]\rho_m
$$

$\Gamma = \frac{4G_0^2\alpha_0^2}{\kappa}$ and $\alpha_0 = \frac{-i\sqrt{\kappa}\mathcal{E}_L}{\kappa/2+i\Delta}$

The *physical* control problem.

Conditional dynamics, given the heterodyne signal, is equivalent to the Hamiltonian dynamics,

$$H(t) = \hbar\omega_m b^\dagger b + \hbar G_0 \frac{|J(t)|^2}{\kappa}(b + b^\dagger)$$

a classical noisy force.

$$J(t) = J_x(t)_i + J_y(t)$$

GJM *Decoherence and the conditions for the classical control of quantum systems*, Proc. Roy Soc. A, 370, 4469 (2012).

The *physical* control problem.

Continuous weak measurement of position and momentum:

Unconditional

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{1}{2\hbar}\Gamma_1[\hat{x},[\hat{x},\hat{\rho}]] - \frac{1}{2\hbar}\Gamma_2[\hat{p},[\hat{p},\hat{\rho}]].$$

Conditional dynamics

$$d\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}]dt - \frac{1}{2\hbar}\Gamma_1[\hat{x},[\hat{x},\hat{\rho}]]dt - \frac{1}{2\hbar}\Gamma_2[\hat{p},[\hat{p},\hat{\rho}]]dt$$
$$+\hbar^{-1/2}\Gamma_1^{1/2}\mathcal{H}[\hat{x}]\hat{\rho}\, dW_1 + \hbar^{-1/2}\Gamma_2^{1/2}\mathcal{H}[\hat{p}]\hat{\rho}\, dW_2$$

Measurement record.

$$dX_1(t) = \mathrm{tr}(\hat{x}\hat{\rho}(t))dt + \frac{1}{2}\hbar^{1/2}\Gamma_1^{-1/2}\, dW_1(t),$$

$$dX_2(t) = \mathrm{tr}(\hat{p}\hat{\rho}(t))dt + \frac{1}{2}\hbar^{1/2}\Gamma_2^{-1/2}\, dW_2(t),$$

See Scott and GJM, PRA, 042101 (2001) .

The *physical* control problem.

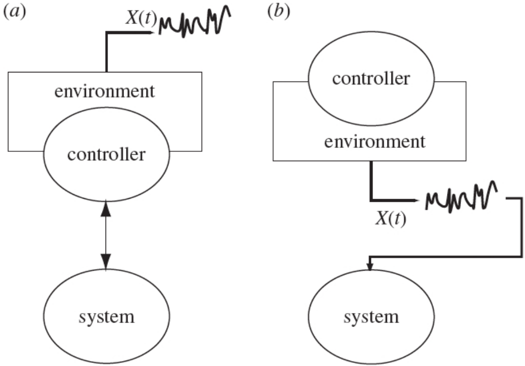


Figure 1. Equivalent ways of describing the interaction between two quantum systems as classical control. In (a), the controller and the system interact unitarily but the controller is an open-system subject to continuous measurement, with measurement record $X(t)$. In (b), the measurement record is used as a classical control signal to directly drive the target system. In both cases, the unconditional dynamics (average over the measurement record) and conditional dynamics (conditional on the measurement record) of the target system is the same.