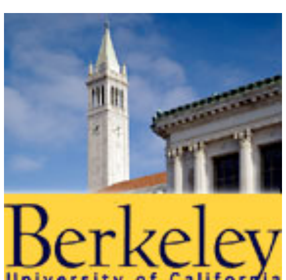
Wave function Video Camera:
Cavity-assisted monitoring of a complex quantum system dynamics

Alireza Shabani, Jan Roden, Birgitta Whaley
Chemistry Department, UC-Berkeley



KITP - Jan 2013

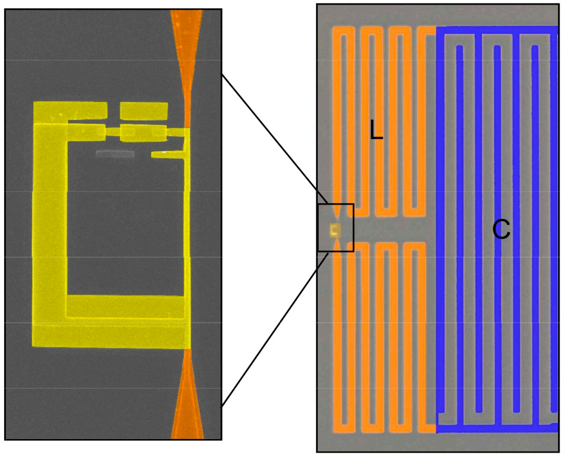
OUTLINE:

- Motivation: circuit QED and nonlinear spectroscopy
- Dispersive measurement in a cavity
- Strong and non-Markovian decoherence dynamics
- Continuous monitoring of a complex system inside a cavity
- Cavity as a wave function video camera
- A simple test to detect non-Markovian decoherence

OUTLINE:

- ❑ Motivation: circuit QED and nonlinear spectroscopy
- ❑ Dispersive measurement in a cavity
- ❑ Strong and non-Markovian decoherence dynamics
- ❑ Continuous monitoring of a complex system inside a cavity
- ❑ Cavity as a wave function video camera
- ❑ A simple test to detect non-Markovian decoherence

Quantum Nanoelectronics Lab, Irfan Siddiqi

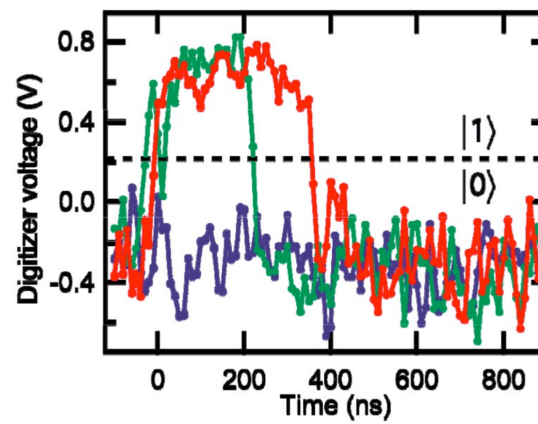


Flux qubit coupled to a microwave LC oscillator:
Circuit QED

- Continuous measurement of a solid-state qubit

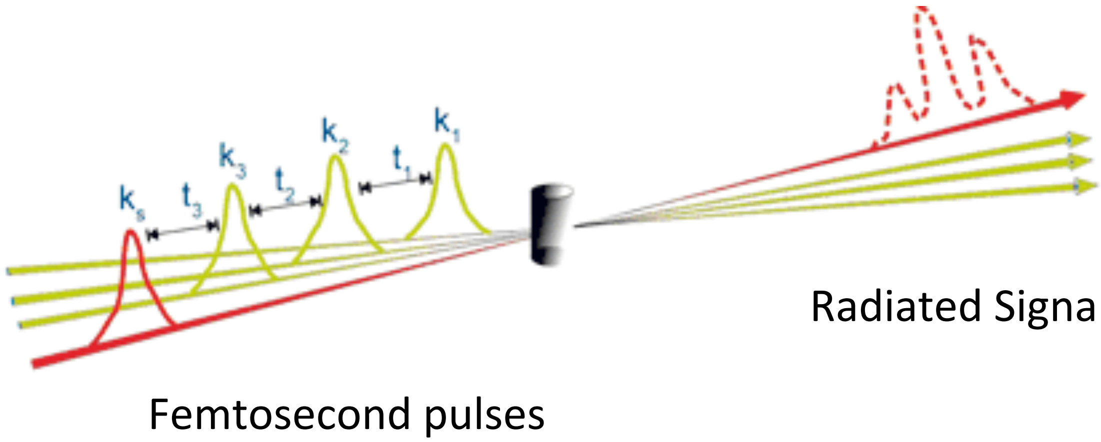
  PRL 106, 110502 (2011)

- Quantum feedback control, Nature 490, 77 (2012)



Theory for a system of interacting qubits?
Non-Markovian decoherence?

Nonlinear Spectroscopy Lab, Graham Fleming



Optical spectroscopy is limited to measuring a fixed observable:
Total dipole moment of the molecule μ

Perturbative response: $Tr(\rho(t)\mu) = Tr(\rho^1(t)\mu) + Tr(\rho^3(t)\mu) + \ldots$

linear response   nonlinear response

Reveals fluctuations between monomers ← $\langle[\mu(t_1+t_2+t_3),[\mu(t_1+t_2),[\mu(t_1),\rho(0)]]]\mu\rangle$

Quantum Process tomography can be done for a dimer only:
Yuen-Zhou et al., PNAS 108, 17615 (2011).

Can we engineer a different observable?

OUTLINE:

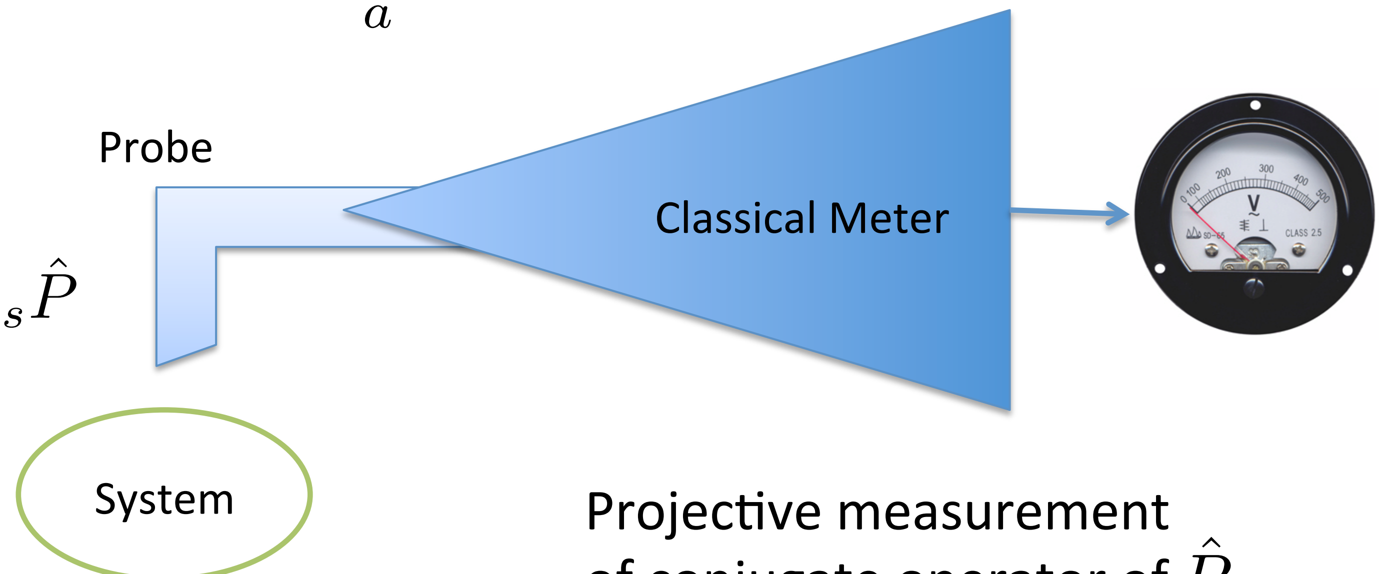
- ❑ Motivation: circuit QED and nonlinear spectroscopy
- ❑ Dispersive measurement in a cavity
- ❑ Strong and non-Markovian decoherence dynamics
- ❑ Continuous monitoring of a complex system inside a cavity
- ❑ Cavity as a wave function video camera
- ❑ A simple test to detect non-Markovian decoherence

Generalized Quantum Measurement

The measurement observable can be engineered by manipulating
The system-probe interaction.

Observable of interest $O_s = \sum_a a|a\rangle\langle a|$

$$H_{Sys-Probe} = g\hat{O}_s\hat{P}$$



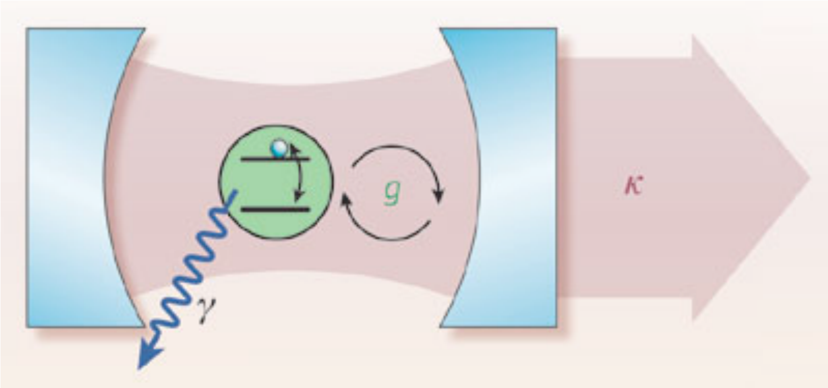
Projective measurement
of conjugate operator of $\hat{P}$

Weak measurement is achieved in the limit of small “ g ”

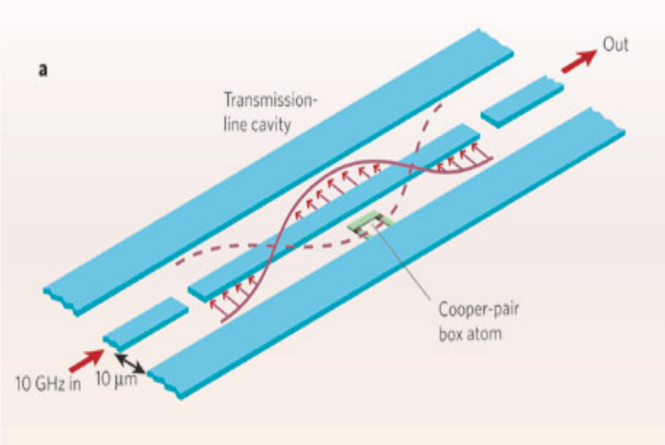
Cavity Quantum Electrodynamics

Cavity: single or multi-mode quantum harmonic oscillator.

Optical Regime: It can be realized by high finesse mirrors, but not that high that the cavity becomes a black box: some photon leakage is desired.



Microwave Regime: It can be realized by a 1D transmission line, or a 3D resonator.



Much stronger couplings can be achieved in microwave regime.

R. Schoelkopf & S. Girvin, Nature 451, 664 (2008)

Dispersive Readout of a Single Qubit

Jaynes-Cumming Hamiltonian:

$$\omega_c a^\dagger a + \Omega\sigma_z + g(a^\dagger\sigma^- + \sigma^+ a)$$

Dispersive regime (far off-resonance): $g \ll |\omega_c - \Omega|$

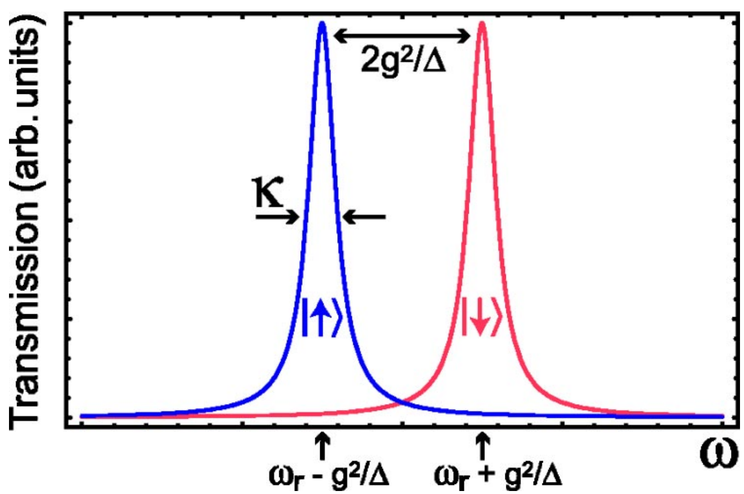
Dispersive Frame rotating with unitary:

$$\exp[\frac{g}{\omega_c - \Omega}(a\sigma^+ - a^\dagger\sigma^-)]$$

Effective Hamiltonian:

$$[\omega_c + \frac{g^2}{\Delta}\sigma_z]a^\dagger a + [\Omega + \frac{g^2}{\Delta}]\sigma_z$$

Measured Observable: σ_z

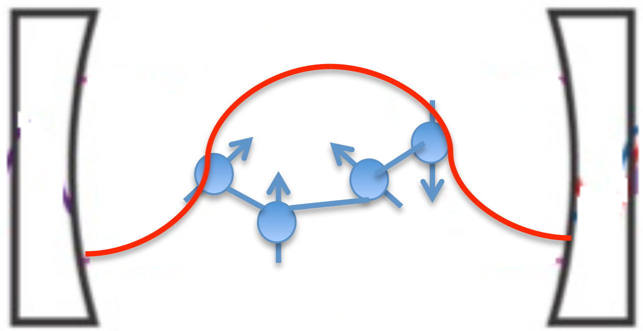


A. Blais et al., PRA 69, 062320 (2004)

Complex system coupled to a cavity

System-Cavity coupling:

$$\vec{\mu}.\vec{E}(x)$$



Multi-qubit system: $$\sum_k g_k \sigma_k^x E_c(x_k) \overset{\text{RWA}}{\approx} g_k(\sigma_k^+ a e^{-ilx_k} + \sigma_k^- a^\dagger e^{ilx_k})$$

Jaynes-Tavis-Cummings Hamiltonian: $$\pi^- = \sum_k g_k \sigma_k^- e^{ilx_k}$$

$$H_{SC} = H_S + \omega_c a^\dagger a + \pi^+ a + \pi^- a^\dagger$$

System-Cavity in Dispersive Regime

The cavity acts as a probe to indirectly measure the system.

$$H_{SC} = H_S + \omega_c a^\dagger a + \pi^+ a + \pi^- a^\dagger$$

Desired form of the interaction: $H_{Sys-Cavity} = g\hat{O}_s \hat{P}_c$

System Hamiltonian:

$$H_S = \sum_j \Omega_j |j\rangle\langle j|$$

Far off-resonance system and cavity:

$$\langle j|\pi^-|k\rangle = \pi^-_{jk} \ll |\omega_c + \Omega_j - \Omega_k|$$

Generalized unitary dispersive transformation:

$$U_D = exp[Xa^\dagger - X^\dagger a]$$

Perturbative Parameter

$$X = \sum_{jk} \frac{\pi^-_{jk}}{\omega_c + \Omega_j - \Omega_k} |j\rangle\langle k|$$

System-Cavity in Dispersive Regime

Keeping terms up to the second order of X

$$H_{SC}^{D} = U_D H_{SC} U_D^{\dagger} \approx H_S^D + \omega_c a^{\dagger} a + O_S a^{\dagger} a$$

Induces frequency-shift proportional to $\langle O_s \rangle$

Slightly modified Hamiltonian:

$$H_S^D = U_D H_S U_D^{\dagger} \approx H_S - \frac{1}{2}(X^{\dagger}\pi^{-} + \pi^{+} X)$$

Measurement Observable:

$$O_S = \frac{1}{2}([\pi^{-}, X^{\dagger}] + [X, \pi^{+}])$$

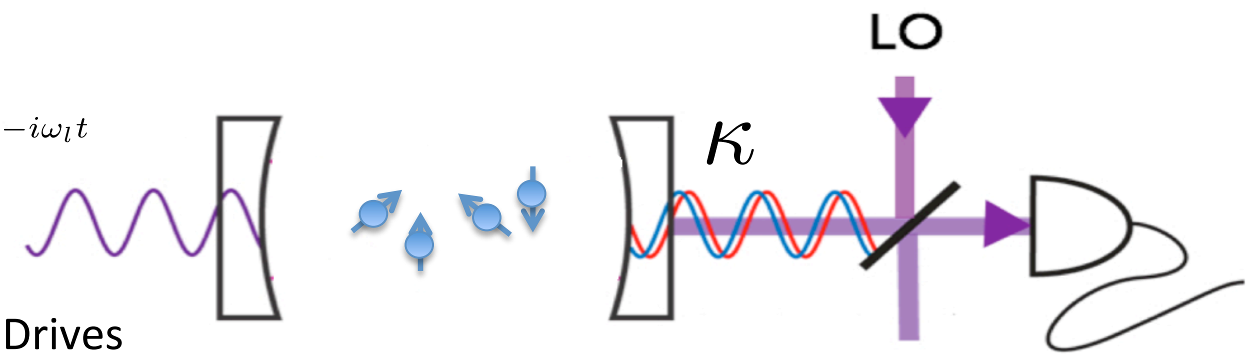
Continuous Quantum Measurement

Information about the dynamical evolution of the system can be obtained by continuously measuring phase quadrature of the leaking photons.

$$\mathcal{E}_c e^{-i\omega_c t} + \sum_l \mathcal{E}_l e^{-i\omega_l t}$$

System Drives

Cavity Drive



Homodyne Detection

Leakage: $$\mathcal{L}_{leakage}(\rho_c) = \kappa(2a\rho_c a^\dagger - a^\dagger a \rho_c - \rho_c a^\dagger a)$$

Detector Current:

$$J(t) = \beta[\eta\kappa\langle e^{-i\phi}a + e^{i\phi}a^\dagger\rangle + \sqrt{\eta\kappa}\xi(t)]$$

Detector efficiency

Gaussian White Noise

OUTLINE:

- ❑ Motivation: circuit QED and nonlinear spectroscopy
- ❑ Dispersive measurement in a cavity
- ❑ Strong and non-Markovian decoherence dynamics
- ❑ Continuous monitoring of a complex system inside a cavity
- ❑ Cavity as a wave function video camera
- ❑ A simple test to detect non-Markovian decoherence

Non-Markovian Decoherence

A Bloch-Redfield equation is commonly used in modeling system and cavity decoherence processes: <u>system is weakly coupled to a broadband environment in thermal equilibrium.</u>

$$\frac{d\rho(t)}{dt} = r_{decay}\Big[(N+1)\mathcal{D}(c)\rho(t) + N\mathcal{D}(c^\dagger)\rho(t)\Big]$$

$$\mathcal{D}[c]\rho = 2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c$$

$$N = 1/[\exp(\hbar\omega_c/kT) - 1]$$

Examples of systems with non-Markovian decoherence:

- A double quantum dot coupled to a microwave resonator,
  T. Frey *et al.* PRL 108, 046807 (2012)

- Superconducting: loss of visibility – coherence dies faster than expected.
  Simmonds et. al., Phys. Rev. Lett. 93, 077003 (2004).
  Vion et. al., Science 296, 286 (2002).

- Photosynthetic Molecules: Chromophores as exciton carriers are strongly coupled to their protein backbone. G. Fleming, Faraday Discuss. 27 (2011).

Decoherence Dynamics beyond Born-Markov

General Model: System is linearly coupled to a bosonic or fermionic bath near thermal equilibrium with Gaussian fluctuations.

$$H_{SB} = H_S + H_B + S \otimes B$$

$$H_B = \int_0^\infty d\omega \omega d(\omega) b^\dagger(\omega) b(\omega)$$

$d(\omega)$ → Bath modes density

$$B = \int_0^\infty d\omega \sqrt{d(\omega)} \eta(\omega) (b^\dagger(\omega) + b(\omega))$$

$\eta(\omega)$ → coupling strength

Quantum Fluctuation-Dissipation theorem: Near equilibrium bath fluctuations are proportional to the perturbations induced by coupling to the system.

$$C(t) = \langle \tilde{B}(t+\tau)\tilde{B}(\tau)\rangle_B = \frac{1}{\pi}\int_0^\infty d\omega J(\omega) \frac{e^{-i\omega t}}{1-e^{-\beta\omega}}$$

$J(\omega)$ → bath spectral distribution

Hierarchical Equations of Motion

Using path integral formalism, an exact master equation for a bosonic bath with colored noise (non-Markovian) was developed by Kubo and Tanimura for an ohmic spectral density with Drude-Lorentzian cut-off profile

R. Kubo, Adv. Chem. Phys. 15, 101 (1969).
Y. Tanimura, R. Kubo, J. Phys. Soc. Jpn. 58, 101 (1989).
Y. Tanimura, Phys. Rev. A 41, 6676 (1990).

$$J(\omega) = \frac{\lambda\gamma\omega}{\omega^2 + \gamma^2}$$

Recently It has be generalized to a bosonic or fermionic bath with an arbitrary parameterized bath spectral density

J. Jin *et al*, J. Chem Phys. 126, 134113 (2007).

$$\frac{\lambda\gamma\omega}{\omega^2 + \gamma^2} + \sum_k \left[\frac{\lambda_k\gamma_k + i\eta_k\omega}{(\omega - \Omega_k)^2 + \gamma_k^2} - \frac{\lambda_k\gamma_k + i\eta_k\omega}{(\omega + \Omega_k)^2 + \gamma_k^2}\right]$$

HEOM for a single qubit at High-T $(T \gg \gamma)$

$$J(\omega) = \frac{\lambda\gamma\omega}{\omega^2 + \gamma^2} \longrightarrow C(t) = \lambda(\frac{2}{\beta} - i\gamma)e^{-\gamma t}$$

HEOM can handle strong coupling and highly non-Markovian dynamics.

$$H_{int} = S \otimes B$$

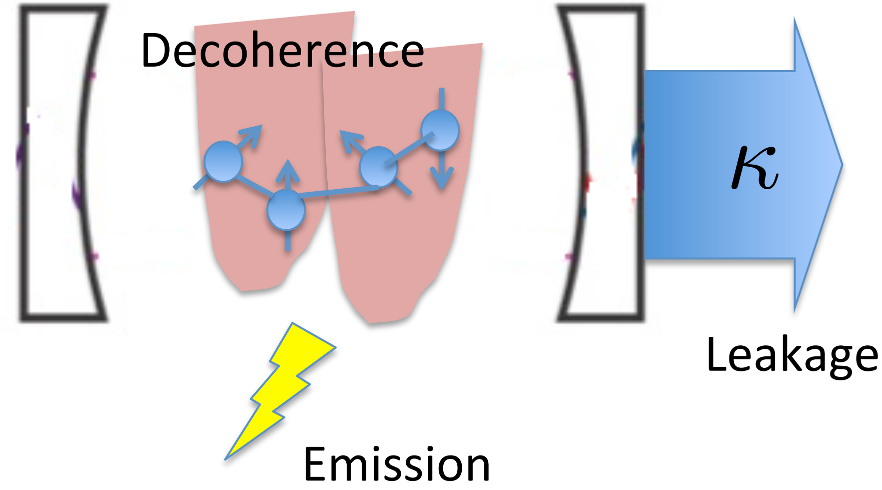
$$\sigma^0 = \rho$$

$$\begin{cases} \frac{\partial}{\partial t}\rho(t) = -i[H, \rho(t)] + i[S, \sigma^1] \\ \frac{\partial}{\partial t}\sigma^n(t) = -i[H, \sigma^n(t)] - n\gamma\sigma^n(t) + i[S, \sigma^{n+1}] \\ \qquad + n\frac{2i\lambda}{\beta}[S, \sigma^{n-1}(t)] + n\lambda\gamma\{S, \sigma^{n-1}(t)\} \end{cases}$$

auxiliary operators

Truncation Level $n\gamma \gg \omega_S$ ⟹ Higher tiers are required for slower baths (smaller γ)

Back inside our cavity



Decoherence processes:

- Cavity energy damping (photon leakage)
- System energy loss due to coupling to electromagnetic reservoir
- System decoherence due to coupling to its surrounding bath inside cavity

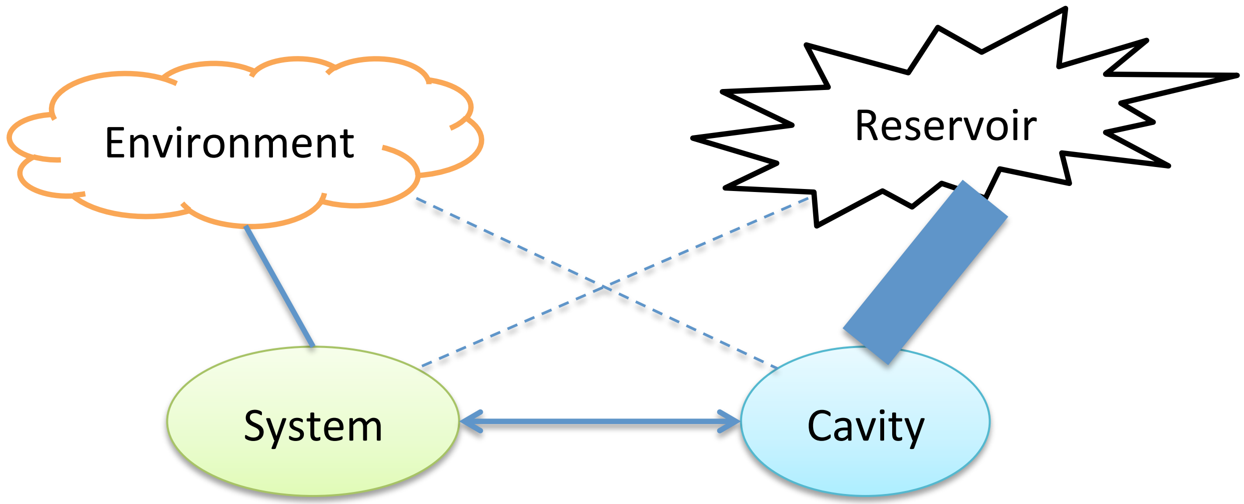
The decoherence processes are modified in dispersive regime.

OUTLINE:

- ❑ Motivation: circuit QED and nonlinear spectroscopy
- ❑ Dispersive measurement in a cavity
- ❑ Strong and non-Markovian decoherence dynamics
- ❑ Continuous monitoring of a complex system inside a cavity
- ❑ Cavity as a wave function video camera
- ❑ A simple test to detect non-Markovian decoherence

Combining Decoherence Processes

Merging two decoherence dynamics: system decoherence and leakage



In the limit of high bandwidth reservoir we just need to some up the super-operators: A Markovian plus an extended non-Markovian from HEOM

$$\mathcal{L}_{total} = \mathcal{L}_{leakage} + \mathcal{L}_{additional} + \mathcal{L}_{decoherence}$$

Next we eliminate the cavity by going to the bad cavity limit:
Strong Leakage κ

Continuous Measurement with Markovian decoherence

Detector Current:

$$dQ = 2\beta\eta|\alpha|\langle\sigma_z\rangle dt + \beta\sqrt{2\eta\kappa}dW$$

Wiener process: $dW = \xi_t dt \qquad \langle\xi_t\rangle = 0 \qquad \langle\xi_t\xi_{t'}\rangle = \delta(t-t')$

Conditional evolution of a single qubit:

Includes measurement back-action

↓

$$d\rho_t = -i\Omega_R[\sigma_x, \rho]dt + \Gamma_\phi\mathcal{D}[\sigma_z]\rho_t dt + r_d\mathcal{D}[\sigma_-]\rho_t dt + \sqrt{\Gamma}\mathcal{H}[\sigma_z]\rho_t dW$$

Stochastic Hierarchical Equations of Motion

For a single qubit:

$$d\rho = \mathcal{L}^D[\rho]dt + \frac{|\alpha|^2}{\kappa}\mathcal{D}[O_S]\rho dt + \Phi_D\sigma^1 dt - \mathcal{H}[e^{-i\phi}\frac{i\alpha}{\kappa}O_S]\rho dW$$

$$d\sigma^n = (\mathcal{L}^D - n\gamma)\sigma^n dt + \frac{|\alpha|^2}{\kappa}\mathcal{D}[O_S]\sigma^n dt + \Theta_D\sigma^{n-1}dt + \Phi_D\sigma^{n+1}dt$$
$$-\mathcal{H}[e^{-i\phi}\frac{i\alpha}{\kappa}O_S]\sigma^n dW$$

In the regime of $\gamma \ll T \ll \omega_c$

$$\mathcal{L}[\rho] = -i[H_S,\rho] + \mathcal{L}_{add}[\rho] \qquad \Phi. = i[S,.] \qquad \Theta. = n\frac{2i\lambda}{\beta}[S,.] + n\lambda\gamma\{S,.\}$$

Detector Current: $dQ = 2\beta\eta|\alpha|\langle O_S\rangle dt + \beta\sqrt{2\eta\kappa}dW$

OUTLINE:

- ❑ Motivation: circuit QED and nonlinear spectroscopy
- ❑ Dispersive measurement in a cavity
- ❑ Strong and non-Markovian decoherence dynamics
- ❑ Continuous monitoring of a complex system inside a cavity
- ❑ Cavity as a wave function video camera
- ❑ A simple test to detect non-Markovian decoherence

Continuous State Tomography

Requirement for full state tomography:
For a d-dim system, we need $d^2 - 1$ independent realizable observables.

State tomography in solid state systems is usually done with a fixed observable.
A complete set of observables is generated by applying unitary transformations before the measurement at any instant of time. $O_\alpha = U_\alpha O_S U_\alpha^\dagger$

This introduces a large overhead. Continuous weak measurement with tunable observables can be a solution to this problem.

The dynamics is revealed by averaging over many trajectories $\frac{d\bar{\rho}}{dt} = \mathcal{L}_{total}[\bar{\rho}]$

The expectation value of the observable O_S over time: $J(t) \sim Tr(O_S \bar{\rho})$

Trade-off: A weaker back-action needs a weaker measurement and therefore more trajectories.

Tunable Observable

We can change the observable by changing the cavity frequency.

$$O_S = \frac{1}{2}([\pi^-, X^\dagger] + [X, \pi^+])$$

$$X = \sum_{jk} \frac{\pi^-_{jk}}{\omega_c + \Omega_j - \Omega_k} |j\rangle\langle k|$$

$$\pi^- = \sum_k g_k \mu_k^- e^{ilx_k}$$

Tunable

How many independent observables are available?

In the absence of any symmetry in intra-qubit and qubit-cavity couplings (maximum possible):

Without spatial resolution: $d^2 - d + 1$

With spatial resolution: $d^2 - d + 1 + \log_2 d(\log_2 d - 1)$

We can measure all energy level populations and most of the coherence components.

Tunable Observable

The observable can also be tuned by changing the phase of the LO in the homodyne detection

Lalumiere, Gambetta, and Blais, PRA, 81, 040301 (2010): For a system of two non-interacting qubits, one can tune the phase of LO to measure Z_1, Z_2 or $Z_1 Z_2$
This becomes possible if the cavity is relatively good $\kappa \sim \frac{g^2}{\omega_q - \omega_c}$

<u>Measured observable in our scheme</u>

main component　　　low resolution component

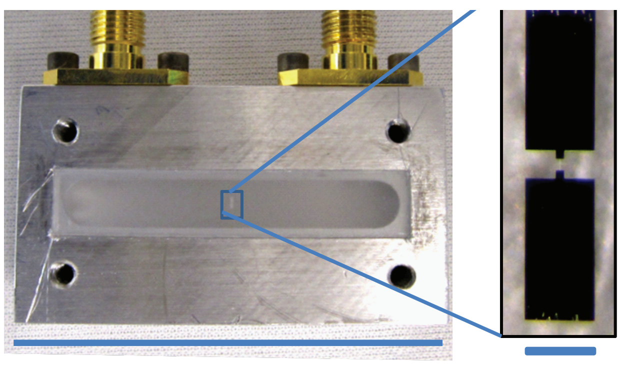
$\kappa \ll |\omega_c + \Omega_j - \Omega_k|$

$$\sin(\phi - \arg(\alpha))O_S + \boxed{\kappa \cos(\phi - \arg(\alpha))\Lambda}$$

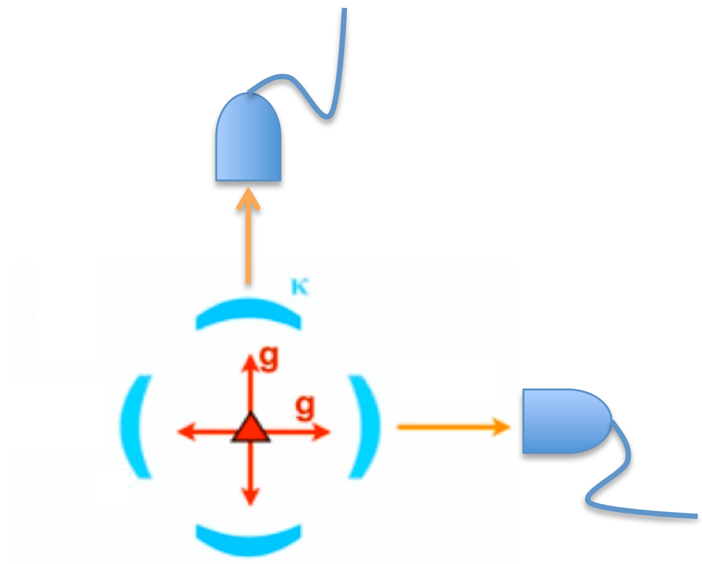
$$X = \sum_{jk} \frac{\pi^{-}_{jk}}{\omega_c + \Omega_j - \Omega_k} |j\rangle\langle k| \qquad \Lambda = \frac{1}{2}[X^\dagger, X]$$

Simultaneous multi-Observable Measurement

Next: Using a multi-mode optical cavity or microwave resonator for simultaneous measurement of complimentary observables.



Transmon qubit inside a **3D resonator**, Schoelkopf's group at Yale, PRL 107, 240501 (2011).

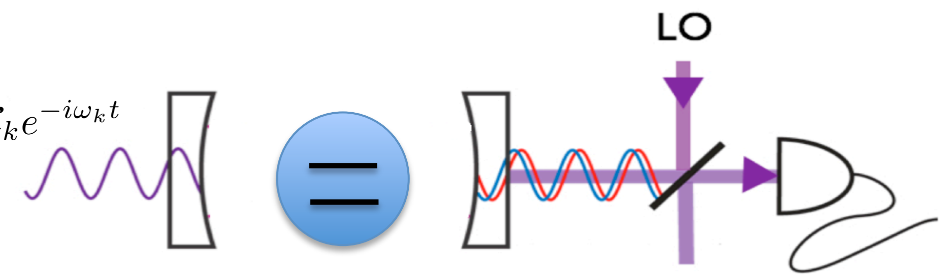


OUTLINE:

- ❑ Motivation: circuit QED and nonlinear spectroscopy
- ❑ Dispersive measurement in a cavity
- ❑ Strong and non-Markovian decoherence dynamics
- ❑ Continuous monitoring of a complex system inside a cavity
- ❑ Cavity as a wave function video camera
- ❑ A simple test to detect non-Markovian decoherence

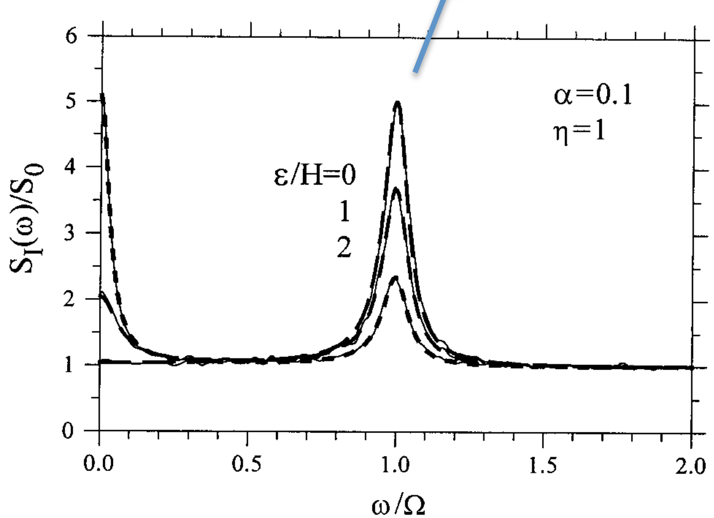
Steady-State spectrum of the measurement current

$$\mathcal{E}_c e^{-i\omega_c t} + \sum_k \mathcal{E}_k e^{-i\omega_k t}$$



$$d\rho_t = -i\Omega_R[\sigma_x, \rho]dt + \Gamma_\phi \mathcal{D}[\sigma_z]\rho_t dt + r_d \mathcal{D}[\sigma_-]\rho_t dt + \sqrt{\Gamma}\mathcal{H}[\sigma_z]\rho_t dW$$

Rabi Frequency



A. Korotkov, PRB 63, 085312 (2001).

$$S(\omega) = \int d\tau \langle O_S(t+\tau) O_S(t) \rangle e^{-i\omega\tau}$$

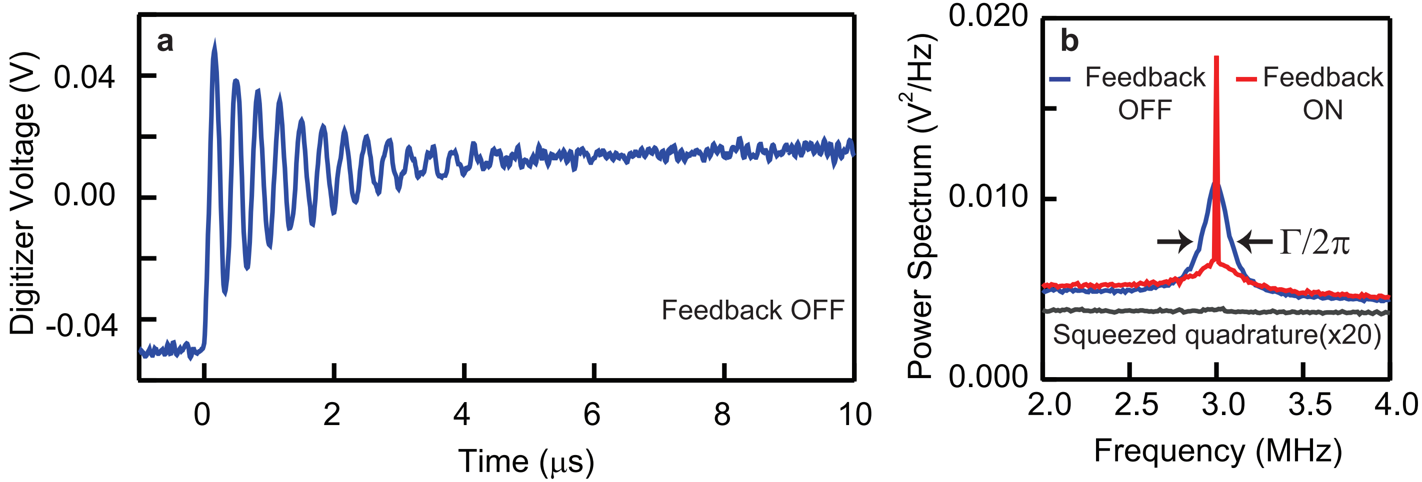
Molecular Spectroscopy: The nature of the decoherence dynamics is revealed in the molecule linear spectrum, which is nothing but averaged projective measurements.

$$S(\omega) = \int d\tau \langle \mu(\tau)\mu(0) \rangle e^{-i\omega\tau}$$

Dipole moment

Quantum feedback control of a superconducting qubit: Persistent Rabi oscillations

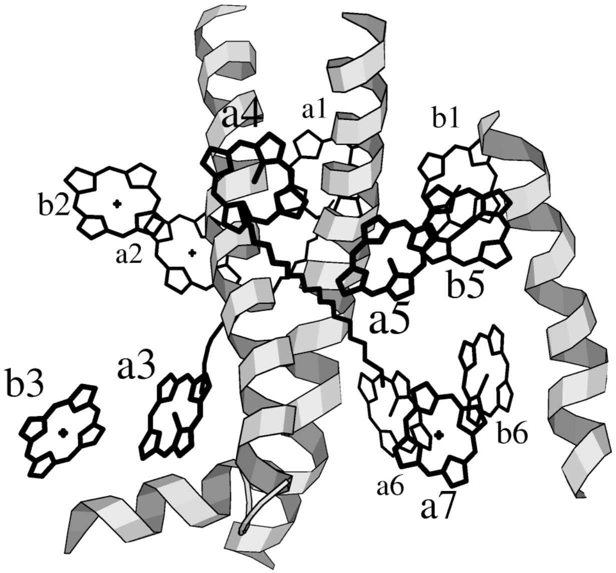
R. Vijay[1], C. Macklin[1], D. H. Slichter[1], S. J. Weber[1], K. W. Murch[1], R. Naik[1],
A. N. Korotkov[2], I. Siddiqi[1]
[1]*Quantum Nanoelectronics Laboratory, Department of Physics,*
University of California, Berkeley CA 94720 and
[2]*Department of Electrical Engineering, University of California, Riverside, CA 92521*
(Dated: May 28, 2012)



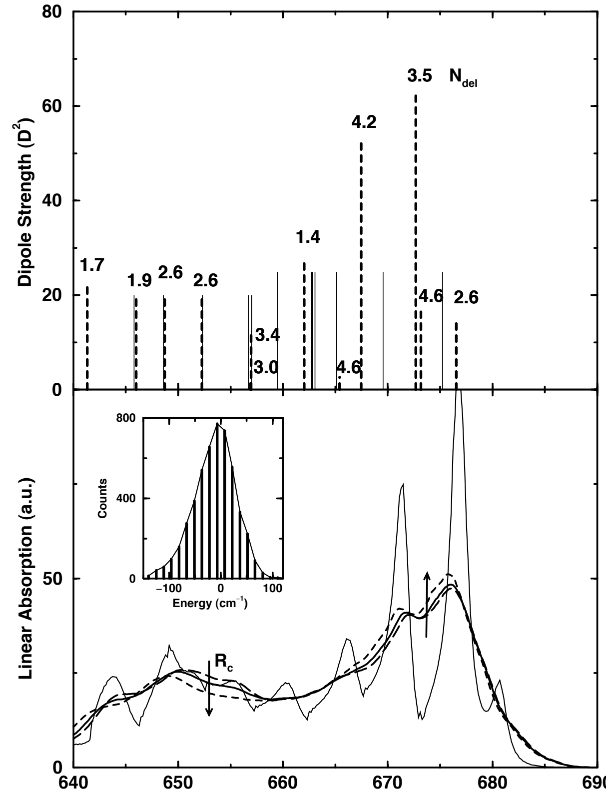
VOLUME 84, NUMBER 22 PHYSICAL REVIEW LETTERS 29 MAY 2000

Simulations of Frequency-Domain Spectra: Structure-Function Relationships in Photosynthetic Pigment-Protein Complexes

Thomas Renger[1] and Volkhard May[2]



LHC-II of green



$$\frac{\partial}{\partial t}\,\sigma_M(t) = -\imath\omega_M\sigma_M(t) - \sum_K \int_0^t d\tau\, C_{MK}(\tau)\sigma_K(t-\tau)\,.$$

$$I(\omega) \sim \left\langle \sum_M \frac{|\mu_M|^2 C_M^{\mathrm{Re}}(\omega)}{[\omega - \omega_M - C_M^{\mathrm{Im}}(\omega)]^2 + [C_M^{\mathrm{Re}}(\omega)]^2} \right\rangle_{\mathrm{conf}}.$$

Steady-State spectrum of the measurement current

Non-Markovian decoherence: Quantum dots coupled to a microwave resonator in T. Frey *et al.* PRL 108, 046807 (2012)

Bath time-correlation properties are mapped into the detector current correlations.

$\Omega_R = 3$

$\Gamma_\phi = 0.07$

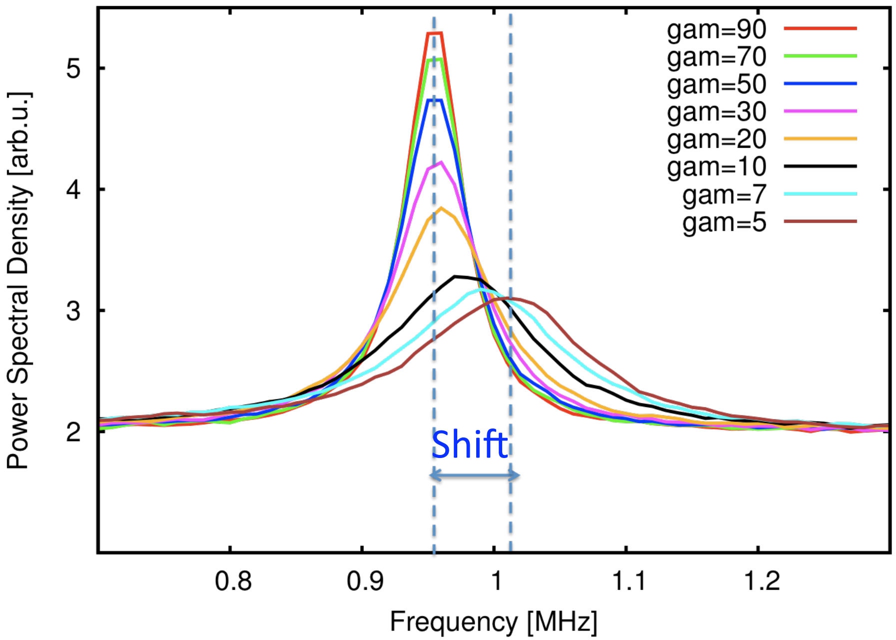
$r_d = 0.05$

$\Gamma = 0.27$

$\lambda = 0.05$

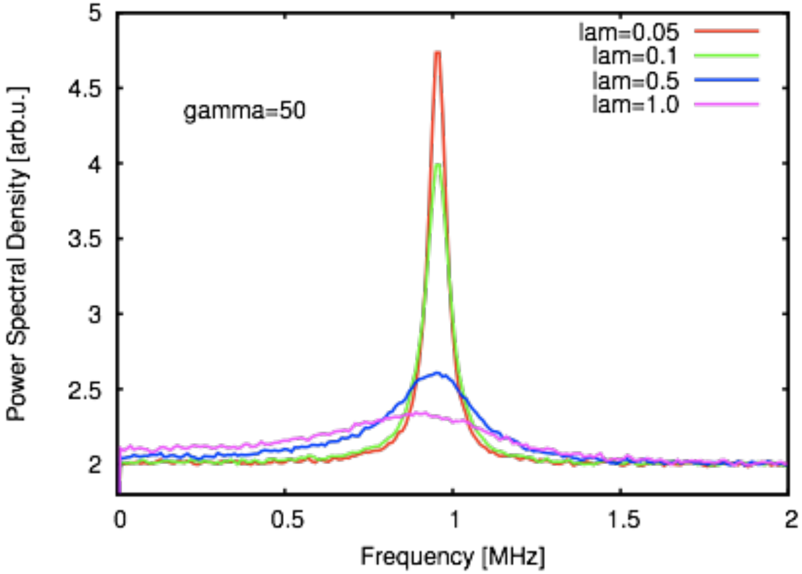
$$J(\omega) = \frac{\lambda\gamma\omega}{\omega^2 + \gamma^2}$$

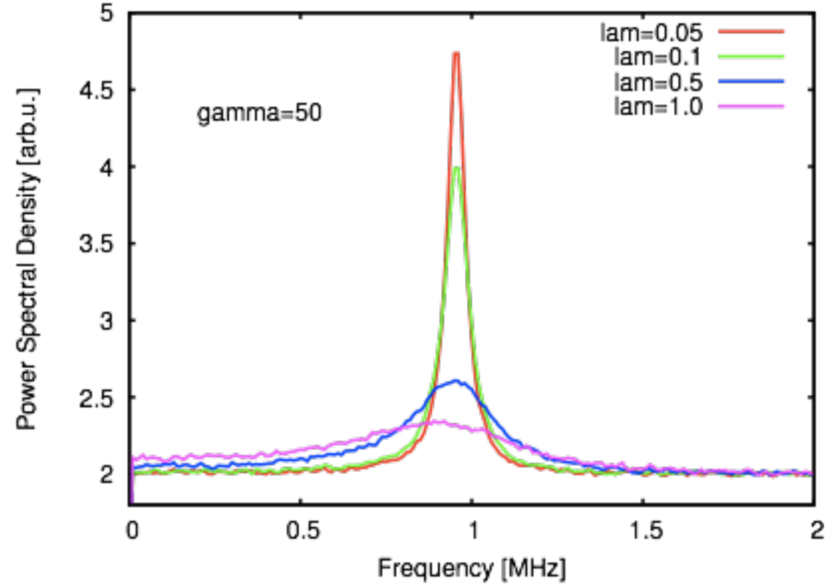
Spectra averaged over 50,000 trajectories



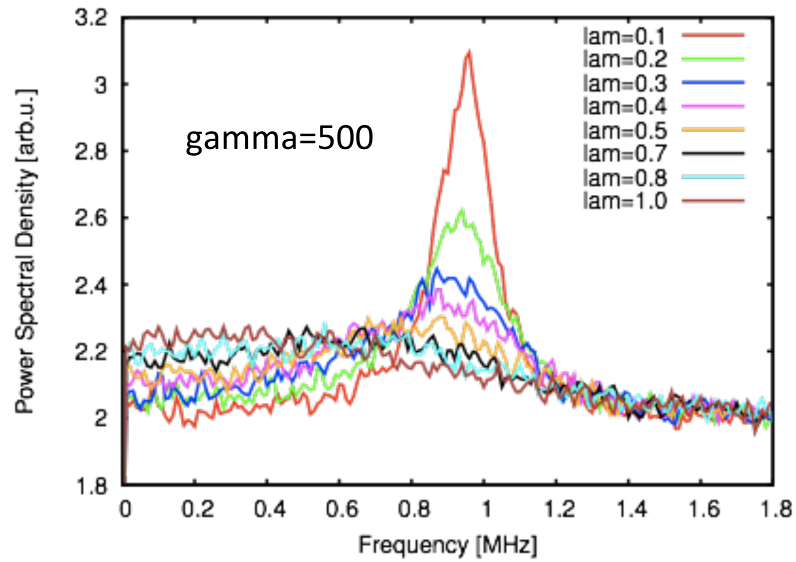
A non-Markovian or strong decoherence dynamics can induce a shift in the spectrum.

Effect of Coupling Strength





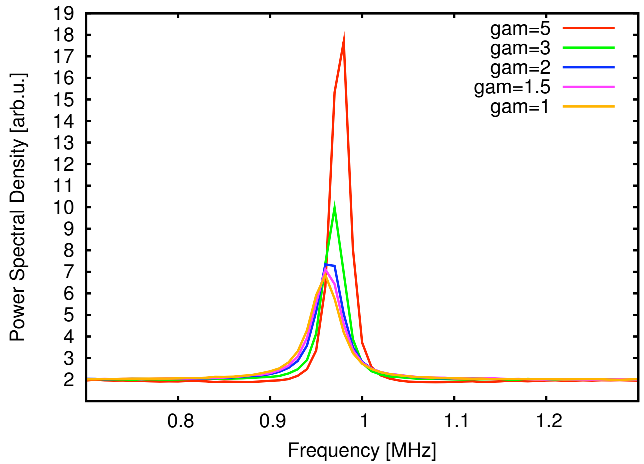
$$J(\omega) = \frac{\lambda\gamma\omega}{\omega^2 + \gamma^2}$$



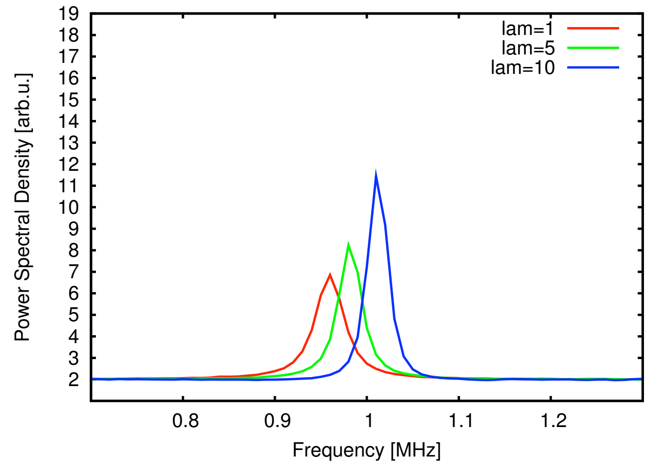
Steady-State spectrum of the measurement current

$$J(\omega) = \frac{\lambda\gamma}{(\omega-\Omega)^2+\gamma^2} - \frac{\lambda\gamma}{(\omega+\Omega)^2+\gamma^2}$$

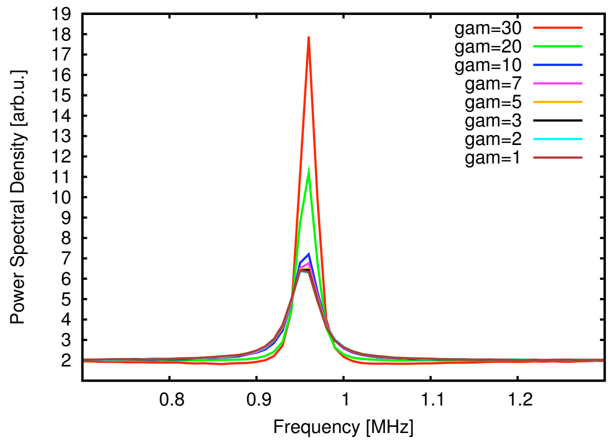
$\Omega = 20MHz$



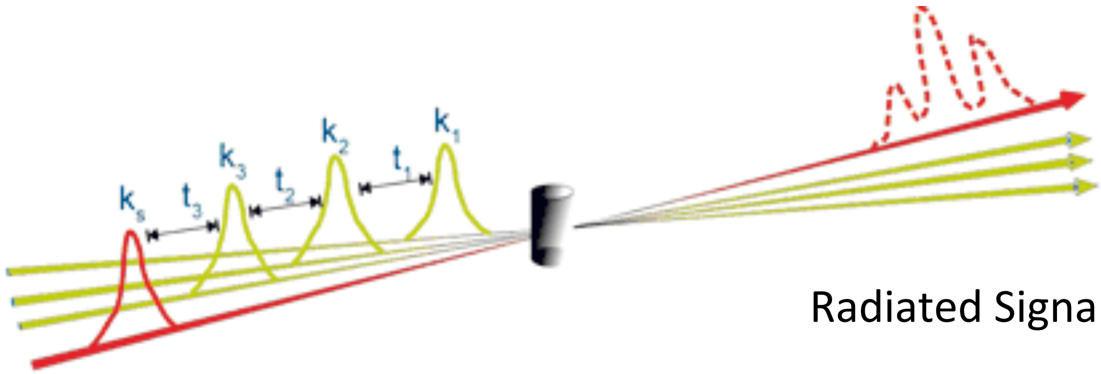
$\Omega = 20MHz$



$\Omega = 100MHz$



Nonlinear Spectroscopy Lab, Graham Fleming



Optical spectroscopy is limited to measuring a fixed observable:
Total dipole moment of the molecule μ

Perturbative response: $Tr(\rho(t)\mu) = Tr(\rho^1(t)\mu) + Tr(\rho^3(t)\mu) + \ldots$

linear response nonlinear response

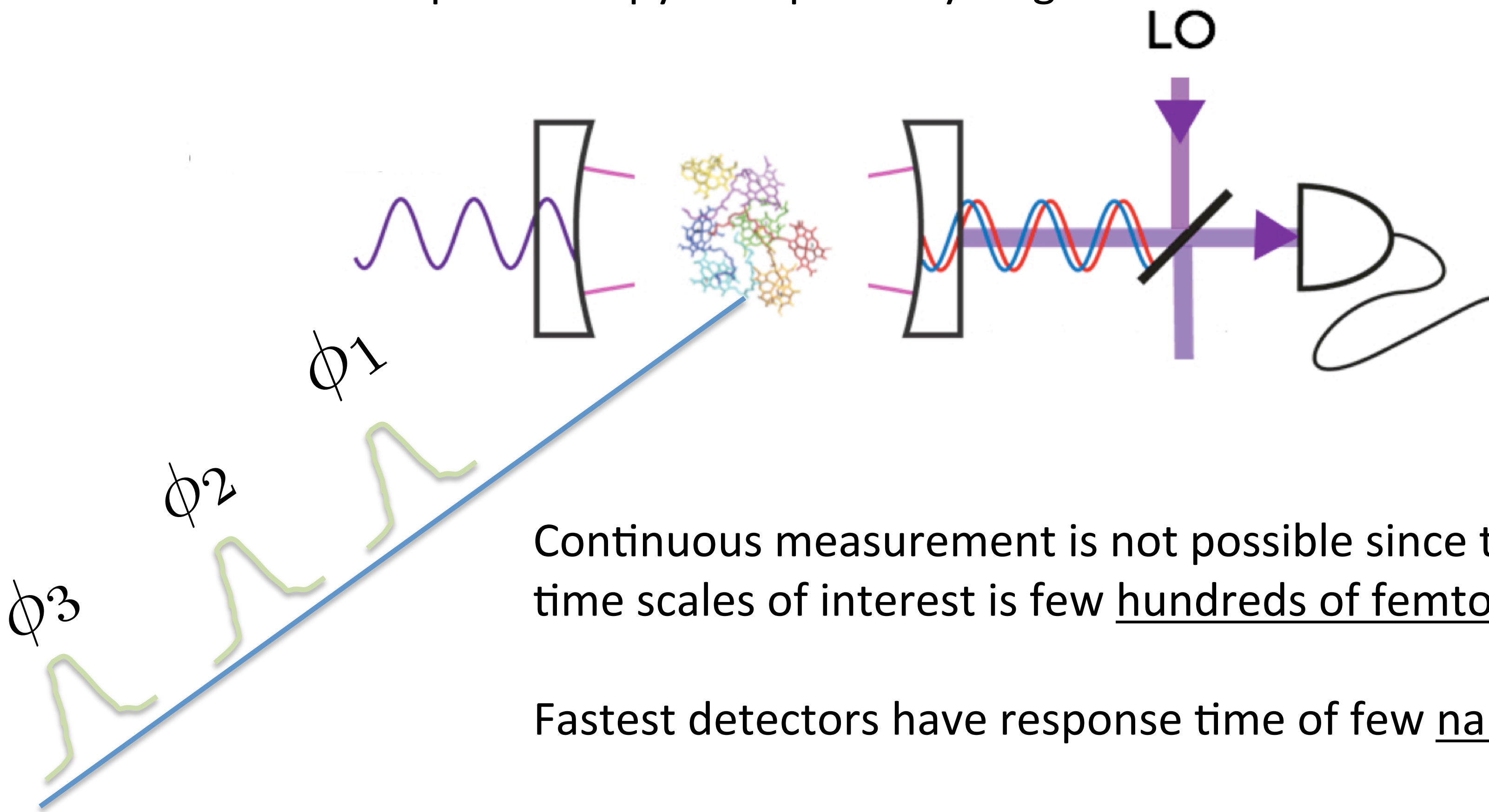
Reveals fluctuations between monomers ← $\langle[\mu(t_1+t_2+t_3),[\mu(t_1+t_2),[\mu(t_1),\rho(0)]]]\mu\rangle$

Quantum Process tomography can be done <u>for a dimer only</u>:
Yuen-Zhou et al., PNAS 108, 17615 (2011).

Can we engineer a different observable?

Toward electronic spectroscopy with tunable observable

Collinear electronic spectroscopy with phase cycling:



Continuous measurement is not possible since the time scales of interest is few <u>hundreds of femto-seconds</u>.

Fastest detectors have response time of few <u>nano-seconds</u> !

Can we detect phase shifts in leaking photons?

Conclusion and Future Work

- ❑ A general formalism is developed for monitoring a non-perturbative and non-Markovian decoherence dynamics of a complex system inside a cavity.

-- Detecting the non-Markovian nature of dynamics by measuring the steady-state spectrum of the detector current

-- Application for spin systems, molecular rotational spectroscopy, NMR (D. I. Schuster et al., 83, 012311 (2011)).

- ❑ Tunable measurement observable

-- Continuous state tomography for a complex system (Video Camera)

-- A compressed sensing algorithm for continuous state tomography- PRL 106, 100401 (2011) – PRA 84, 012107 (2011)

- ❑ Feedback control is presence on non-Markovian decoherence (ongoing project – Hanhan Li)

- ❑ Toward electronic spectroscopy with tunable measurement