Quantum Simulation: Dream or Nightmare?

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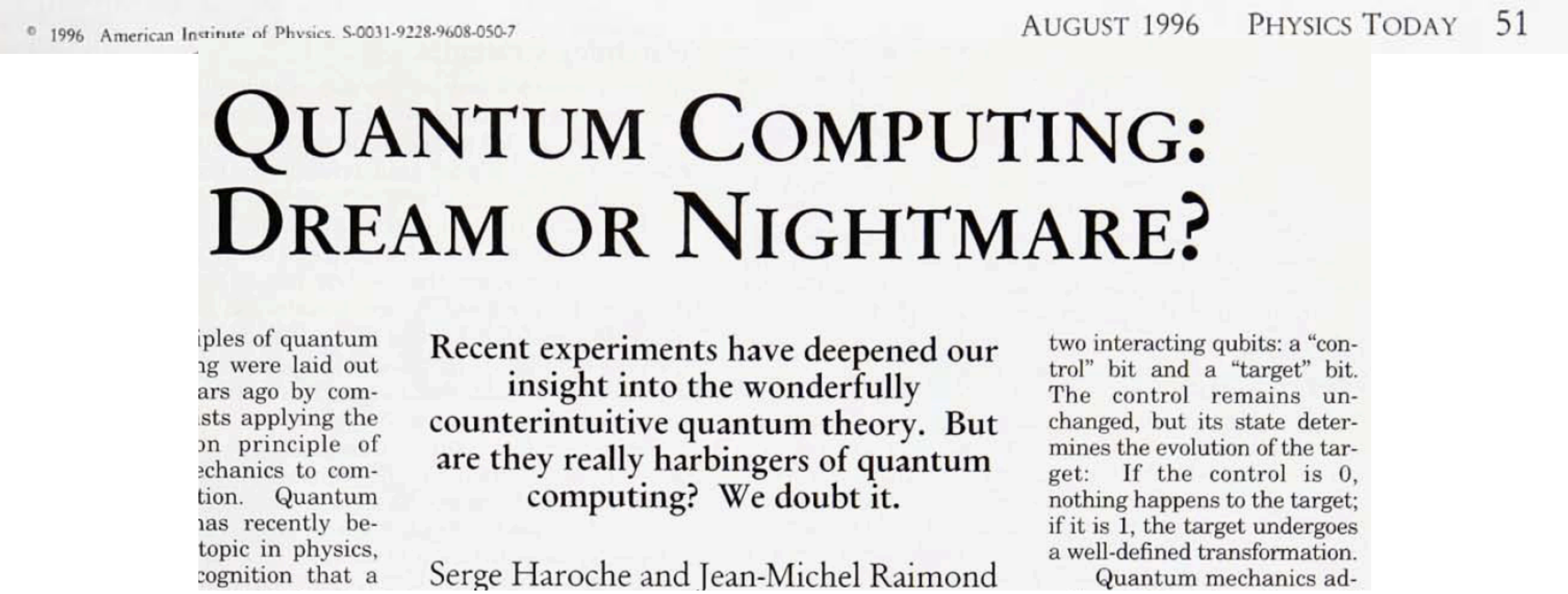
Quantum Computer: Dream or Nightmare?



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QUANTUM COMPUTING: DREAM OR NIGHTMARE?

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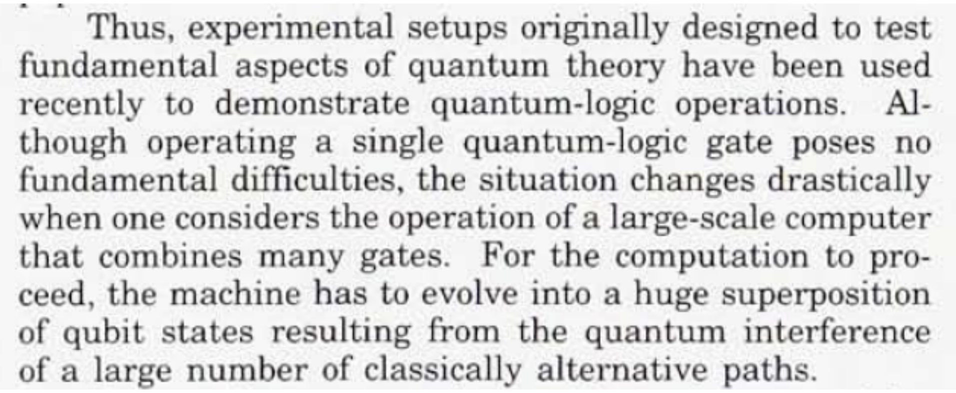
Recent experiments have deepened our insight into the wonderfully counterintuitive quantum theory. But are they really harbingers of quantum computing? We doubt it.

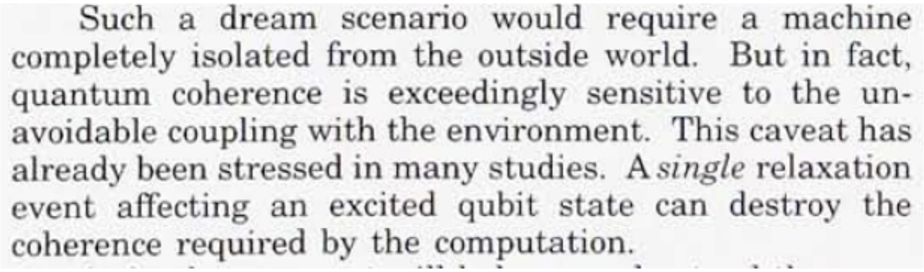
Serge Haroche and Jean-Michel Raimond

two interacting qubits: a "control" bit and a "target" bit. The control remains unchanged, but its state determines the evolution of the target: If the control is 0, nothing happens to the target; if it is 1, the target undergoes a well-defined transformation.

Quantum mechanics ad-

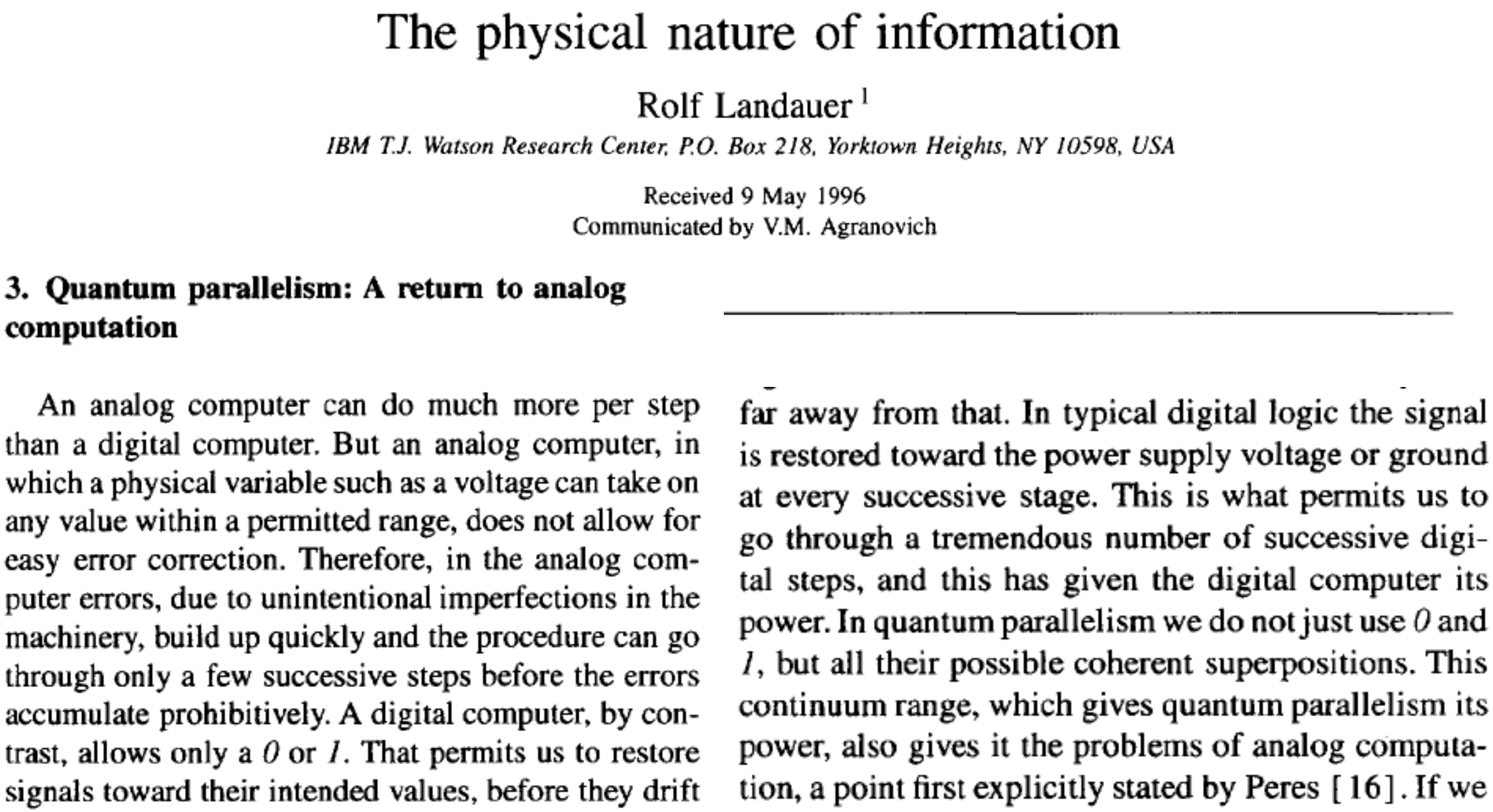


Thus, experimental setups originally designed to test fundamental aspects of quantum theory have been used recently to demonstrate quantum-logic operations. Although operating a single quantum-logic gate poses no fundamental difficulties, the situation changes drastically when one considers the operation of a large-scale computer that combines many gates. For the computation to proceed, the machine has to evolve into a huge superposition of qubit states resulting from the quantum interference of a large number of classically alternative paths.



Such a dream scenario would require a machine completely isolated from the outside world. But in fact, quantum coherence is exceedingly sensitive to the unavoidable coupling with the environment. This caveat has already been stressed in many studies. A *single* relaxation event affecting an excited qubit state can destroy the coherence required by the computation.

Quantum Computer: Analog or Digital?



The physical nature of information

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3. Quantum parallelism: A return to analog computation

An analog computer can do much more per step than a digital computer. But an analog computer, in which a physical variable such as a voltage can take on any value within a permitted range, does not allow for easy error correction. Therefore, in the analog computer errors, due to unintentional imperfections in the machinery, build up quickly and the procedure can go through only a few successive steps before the errors accumulate prohibitively. A digital computer, by contrast, allows only a *0* or *1*. That permits us to restore signals toward their intended values, before they drift far away from that. In typical digital logic the signal is restored toward the power supply voltage or ground at every successive stage. This is what permits us to go through a tremendous number of successive digital steps, and this has given the digital computer its power. In quantum parallelism we do not just use *0* and *1*, but all their possible coherent superpositions. This continuum range, which gives quantum parallelism its power, also gives it the problems of analog computation, a point first explicitly stated by Peres [16]. If we

But then I meet distinguished physicists who say things like:

"A quantum computer is *obviously* just a souped-up analog computer: continuous voltages, continuous amplitudes, what's the difference?"

"A quantum computer with 400 qubits would have ~2^{400} classical bits, so it would violate a cosmological entropy bound"

"My classical cellular automaton model can explain everything about quantum mechanics!
(How to account for, e.g., Schor's algorithm for factoring prime numbers is a detail left for specialists)"

"Who cares if my theory requires Nature to solve the Traveling Salesman Problem in an instant? Nature solves hard problems all the time—like the Schrödinger equation!"

Slide Stolen from Scott Aaronson – "NIST Talk"

Analog Computers and Complexity

- An ideal analog computer can solve NP-complete problems

THE COMPLEXITY OF ANALOG COMPUTATION †

Mathematics & Computers in Simulation 28 (1986) 91-113.

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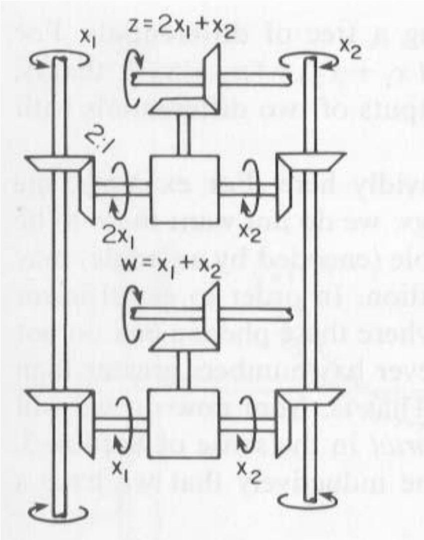


3-SAT Solver

If a strongly NP-complete problem can be solved by an analog computer, and if P ≠ NP, and if Strong Church's Thesis is true, then the analog computer cannot operate successfully with polynomial resources.

Analog Computers and Complexity

- The real-word analog computers vs. the ideal model
  - **Physical quantities in the ideal equations have infinite precision → infinite resources.**
  - **Any finite imperfection in initializing, control, signal/noise can lead to uncontrollable errors.**
  - **Shannon developed digital encoding to make information *ROBUST to physical imperfections.***

Quantum Computing Is Not Analog

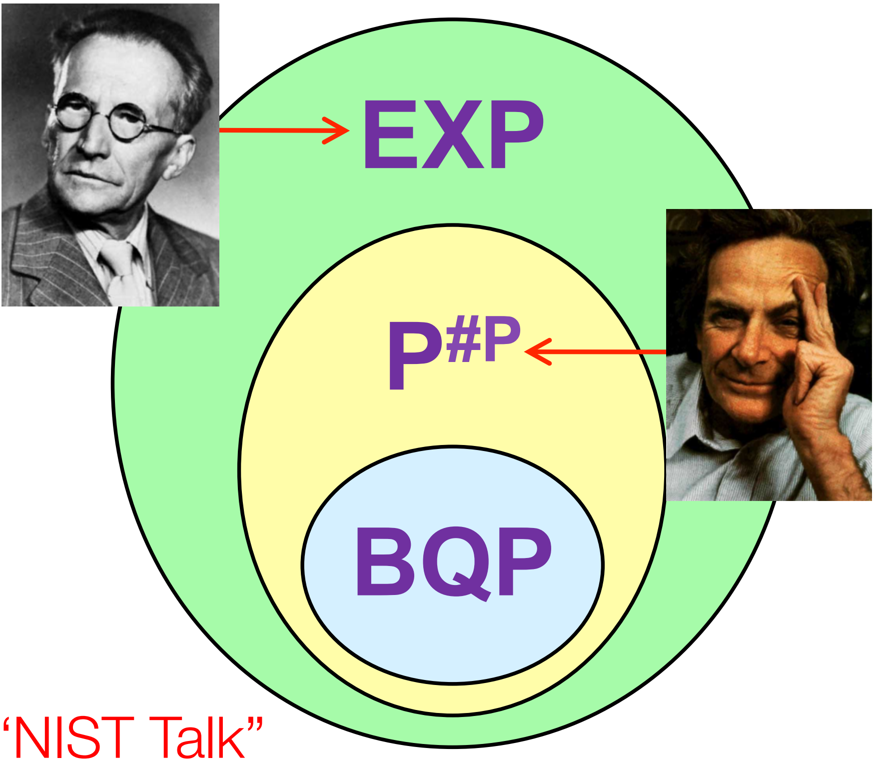
$$i\frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

is a linear equation, governing quantities (amplitudes) that are not directly observable

This fact has many profound implications, such as…

The Fault-Tolerance Theorem

Absurd precision in amplitudes is *not* necessary for scalable quantum computing



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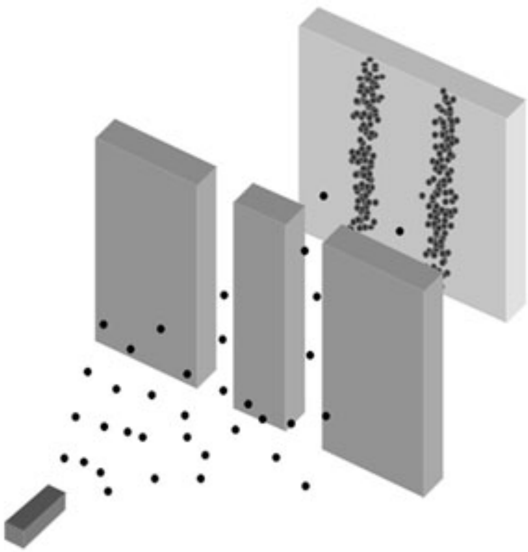
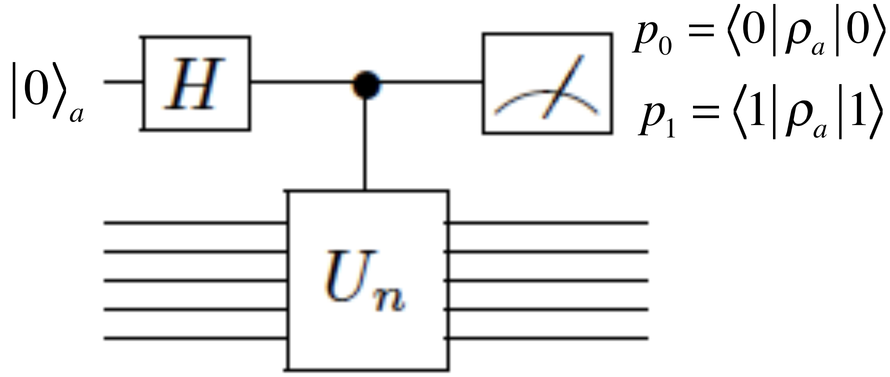
Quantum Computer: Analog or Digital?

Wave-particle duality



Analog-digital duality



$$p_0 = \langle 0|\rho_a|0\rangle$$

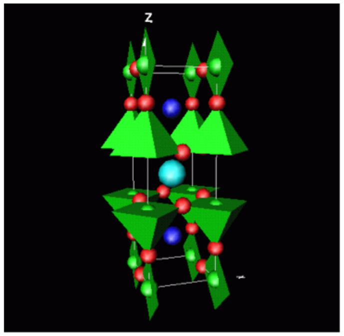
$$p_1 = \langle 1|\rho_a|1\rangle$$

$$|\psi\rangle = \sum_{x=0}^{2^n-1} c_x |x\rangle$$

Can digitize (and efficiently approximate) a class of continuous unitary transformations into a fault-tolerant finite set of universal gates

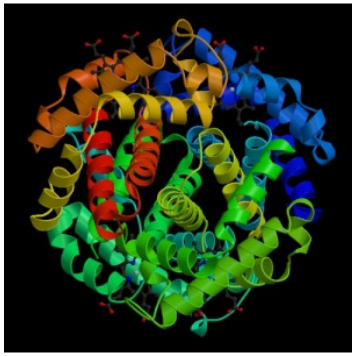
Goal of a Quantum Simulator: Special purpose “quantum computer”: Explore Quantum Complexity

- Emergent properties of many-body quantum sytems



Quantum Magnetism

High Tc Superconductivity

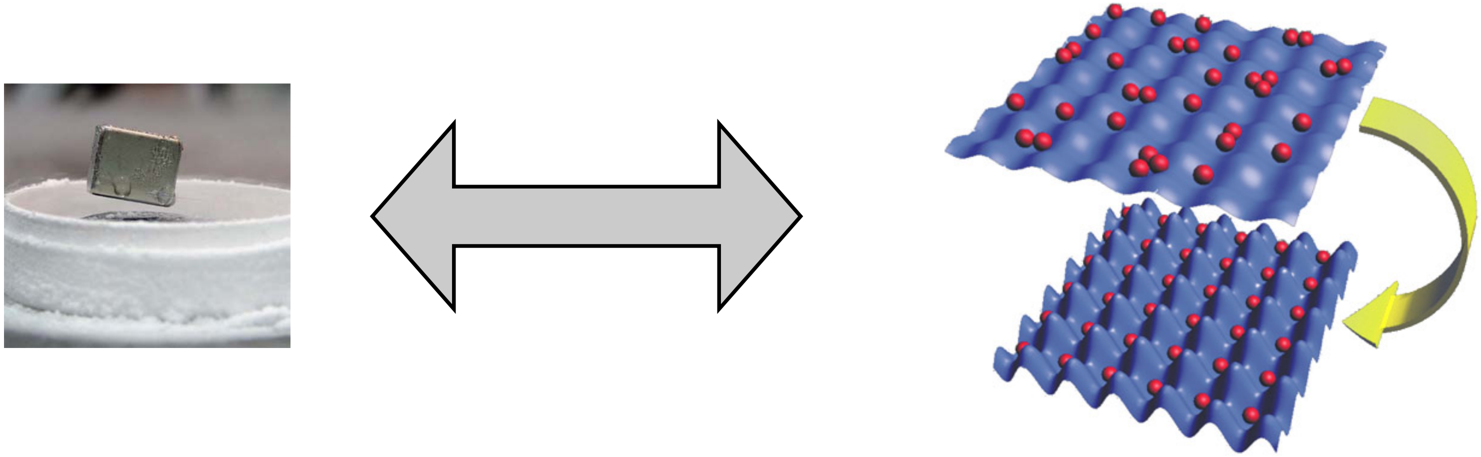


Complex molecular structure

Feynman: Quantum Simulation (Emulation)

- R. P. Feynman, “Simulating Physics with Computers”, Int. J. Theor. Phys. 467 (1982).

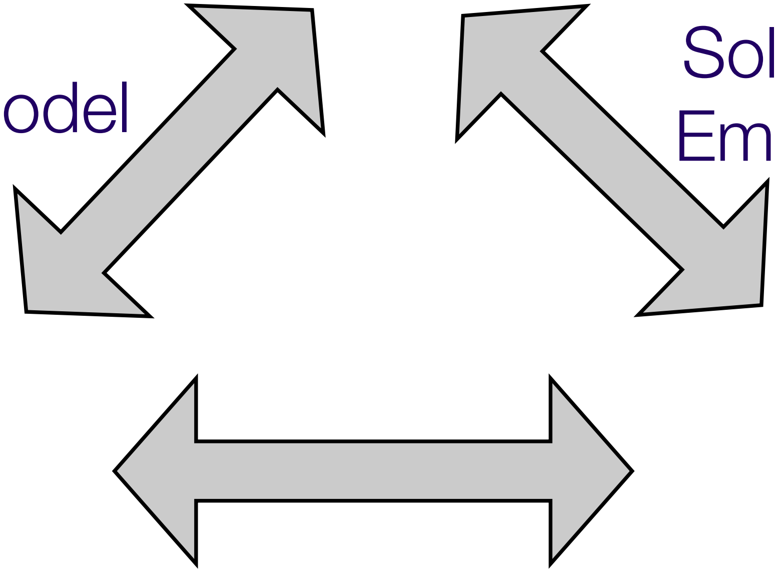
Emulate Nature’s many-body quantum mechanics with *engineered* many-body systems governed by the same quantum mechanical laws.



Simulation vs. Model Solver

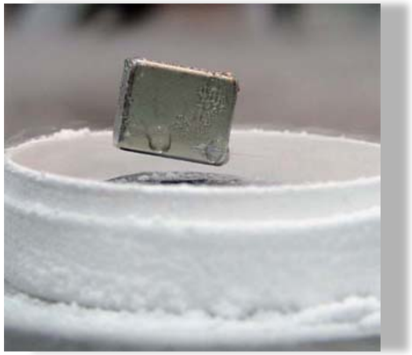
$$H = \sum_{lattice} J_{ij} c_i^{\dagger} c_j + \sum_{neighbors} U c_i^{\dagger} c_j^{\dagger} c_i c_j$$

Fermi-Hubbard Model



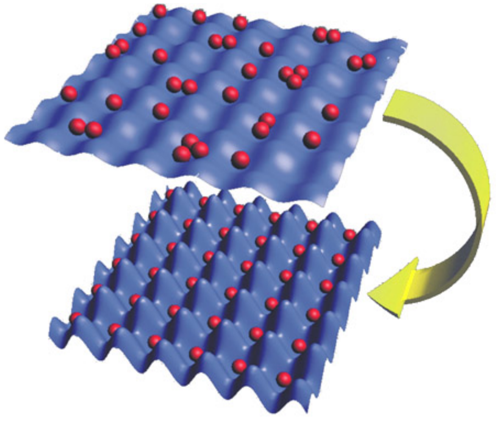
Model

Solver by Emulation



High Tc Cuprate

Simulation by Emulation

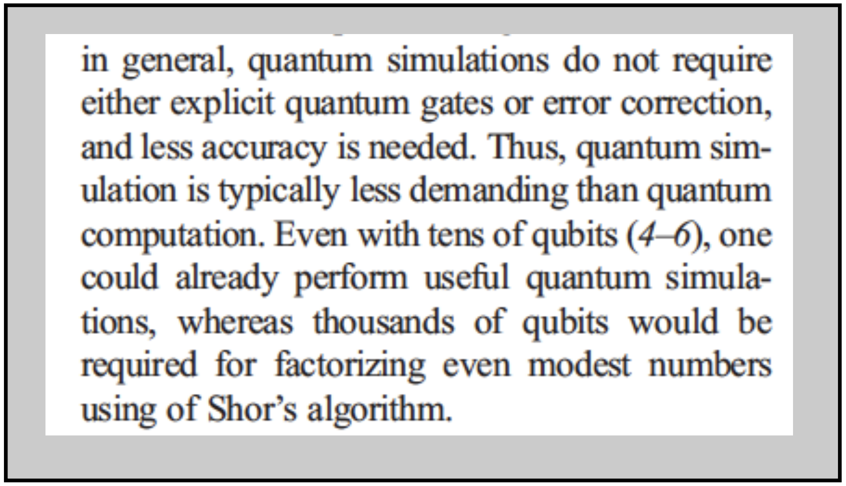


The Common Lore of Quantum Simulators


108 2 OCTOBER 2009 VOL 326 **SCIENCE** www.sciencemag.org


Quantum Simulators

Iulia Buluta[1] and Franco Nori[1,2]*


in general, quantum simulations do not require either explicit quantum gates or error correction, and less accuracy is needed. Thus, quantum simulation is typically less demanding than quantum computation. Even with tens of qubits (*4*–*6*), one could already perform useful quantum simulations, whereas thousands of qubits would be required for factorizing even modest numbers using of Shor's algorithm.

REALLY??

Robustness to Errors

- Quantum emulator is *analog.*
- Imperfect lattice, finite temperature, measurement signal-to-noise.

Fundamental Question

How robust is the analog quantum emulator for information we seek to extract, and is the robust information simulatable (poly efficiently) on a classical digital computer?

Robustness of information: What do we measure?

- Typical quantum algorithm (e.g. Shor): Measure P_x in computation basis to due answer. Requires robustness of 2^n probabilities.

$$P_x = \left|\langle x|\Psi\rangle\right|^2$$

- Typical quantum simulation: Measure local correlation function to determine the order parameter, e.g., quantum magnetism:

$$C = \sum_{neighbors} \left\langle \sigma_z^i \sigma_z^j \right\rangle$$

Question

When is *C* not efficiently calculable on a classical computer, and when it is not, how sensitive is it to errors in the quantum many-body state?

Efficient representations of many-body states

- Matrix-product states: Choose the basis according entanglement.

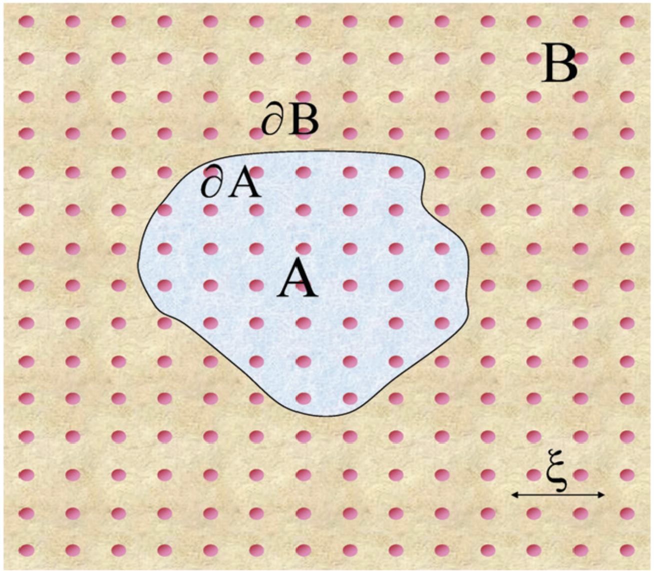


Figure 1. A quantum spin system on a lattice: the lattice is divided into two regions, A and B, with borders ∂A and ∂B, respectively. In the case of ground states of local Hamiltonians, the entanglement entropy between the two regions A and B scales as the area ∂A as opposed to the volume.

Question

Does nature make use of exponential (in number of particles) amounts of entanglement especially at finite temperature and with finite imperfection?

Example:

Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins

Joseph W. Britton[1], Brian C. Sawyer[1], Adam C. Keith[2,3], C.-C. Joseph Wang[2], James K. Freericks[2], Hermann Uys[4],
Michael J. Biercuk[5] & John J. Bollinger[1]

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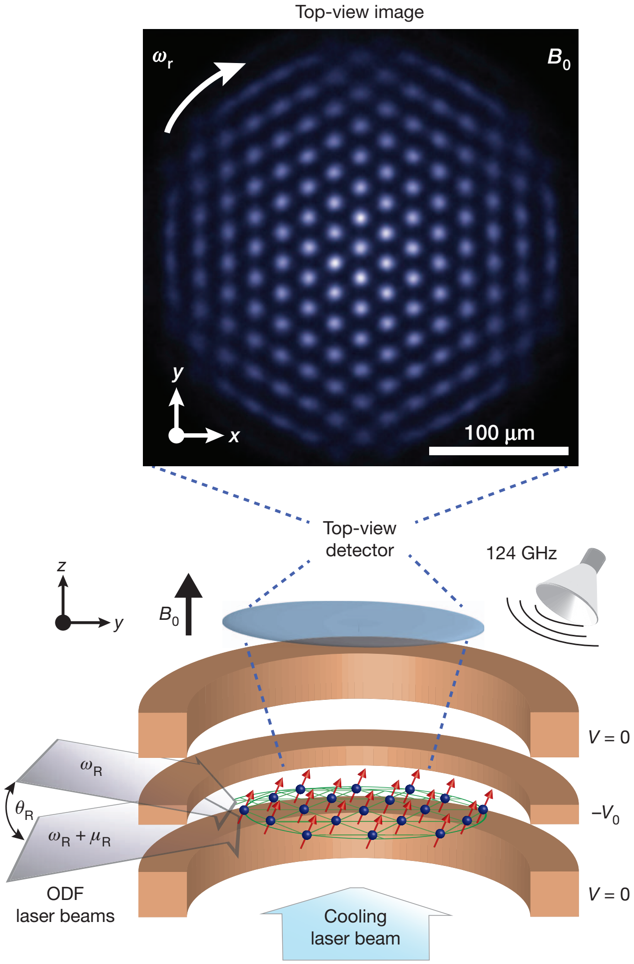
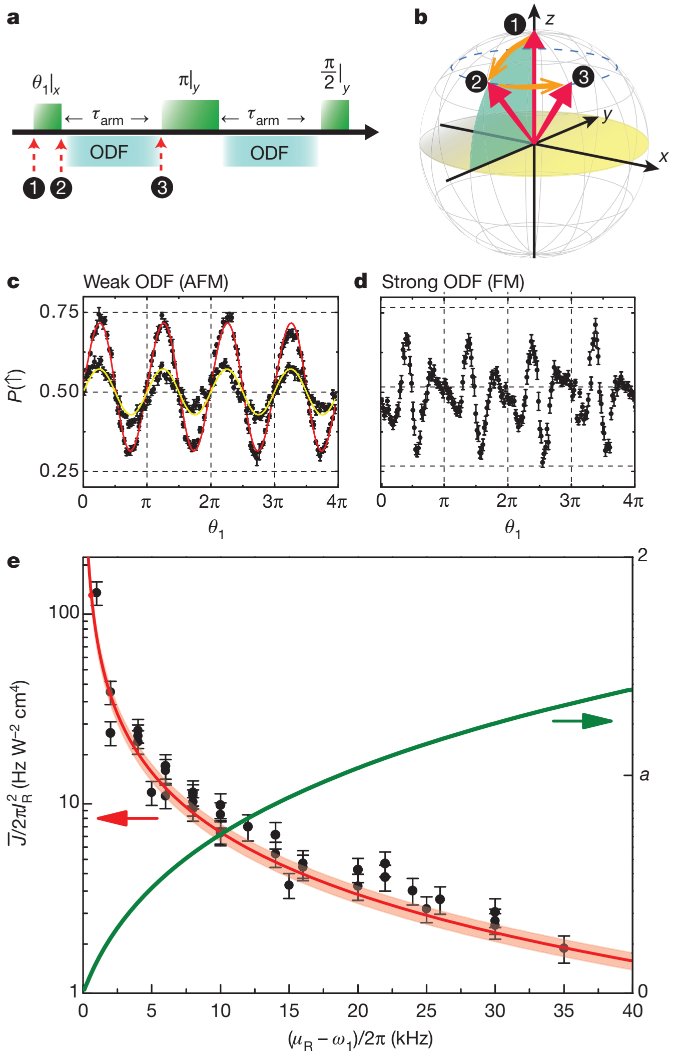


$$\hat{H}_B = \sum_i \mathbf{B}_\mu \cdot \hat{\boldsymbol{\sigma}}_i$$

$$\hat{H}_\mathrm{I} = \frac{1}{N}\sum_{i<j} J_{i,j}\hat{\sigma}_i^z\hat{\sigma}_j^z$$



Questions for discussion

- How *robust* is an analog quantum emulator for information we seek to extract, and is the robust information simulatable (poly efficiently) on a classical digital computer?

- How do we *verify* that a quantum emulator is reliable?

- Even if we don't emulate the exact model to a known precision, do we learn something important different from a *physics experiment* on a complex many-body system?

- Real materials are imperfect. What class of complex states does *nature* access? Is this different from a digital QC?

- Do we have sufficient *algorithms* on to perform digital simulation on a fault-tolerant quantum computer?