上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Control Theory and Engineering: From Classical to Quantum

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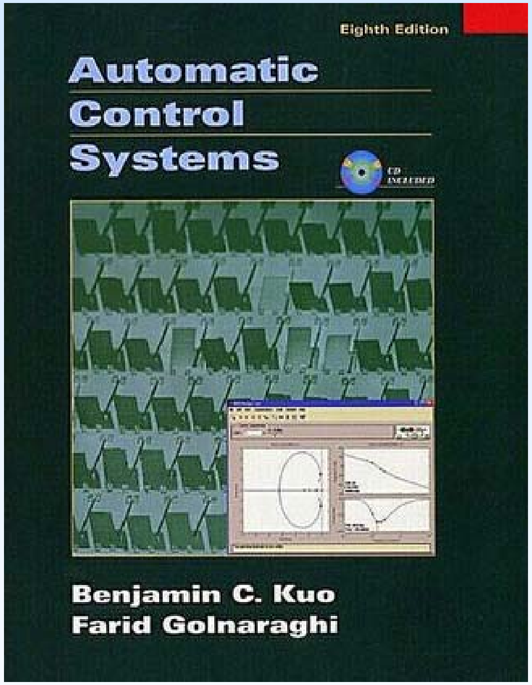
Feb 7, 2013

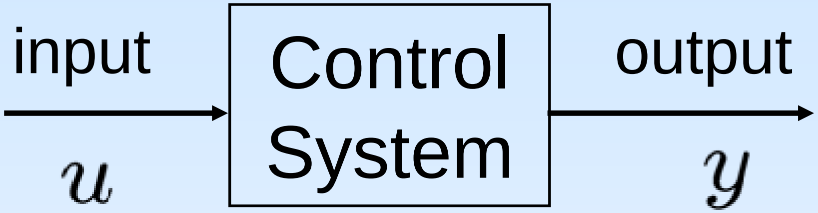
KITP, UCSB

Outline

1. Control basics
2. Engineering control designs
3. My work in quantum control

What is objective of control?





Objective of control system

Control the outputs in some prescribed manner by the inputs through the elements of the control system

Automatic control systems, Benjamin C. Kuo, Farid Golnaraghi, 8th ed, 2002

Field Overviews

TECHNICAL AREAS: IFAC 19TH WORLD CONGRESS, CAPE TOWN 24-29 AUGUST 2014

Systems and Signals: Modelling, Identification and Signal Processing; Adaptive and Learning Systems; Discrete Event and Hybrid Systems; Stochastic Systems; Networked Systems

Design Methods: Control Design; Linear Control Systems; Non-Linear Control Systems; Optimal Control; Robust Control; Distributed Parameter Systems

Computers, Cognition and Communication: Computers for Control; Computational Intelligence in Control; Telematics: Control via Communication Networks

Mechatronics, Robotics and Components: Components and Technologies for Control; Mechatronic Systems; Robotics; Human Machine Systems

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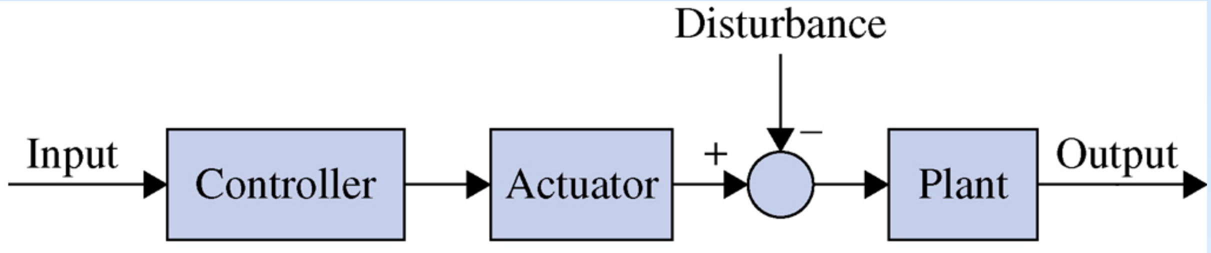
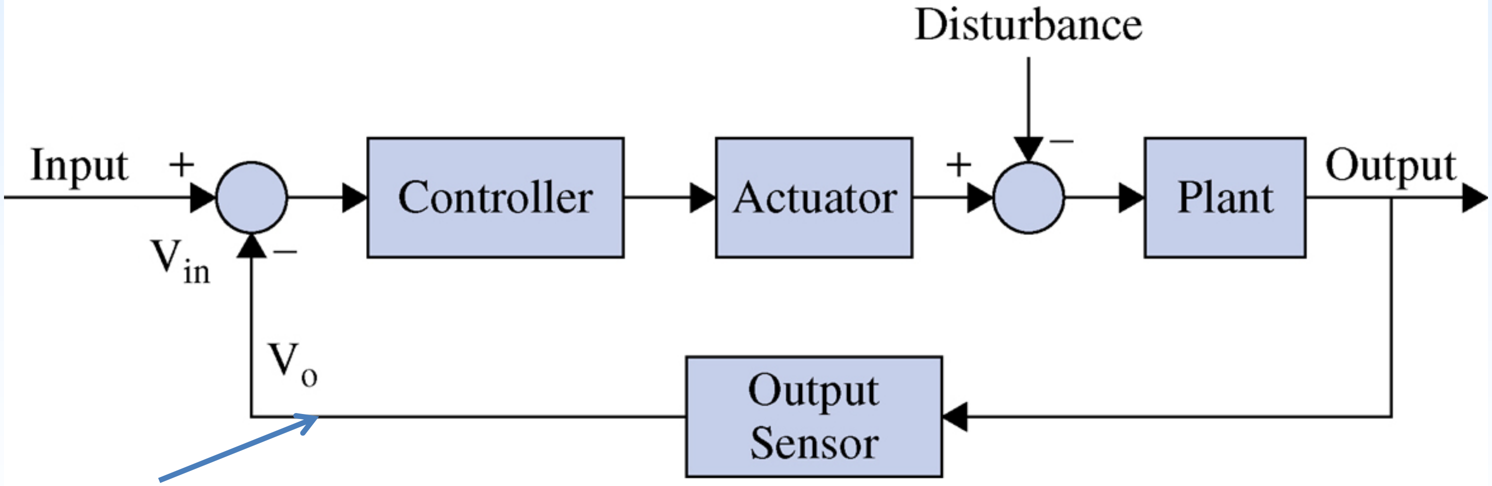
Open-loop vs Closed-loop



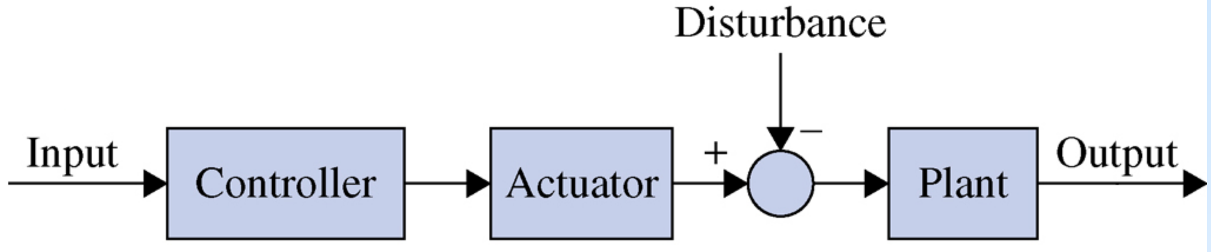
An open-loop control system



(Negative) Feedback

A closed-loop control system.

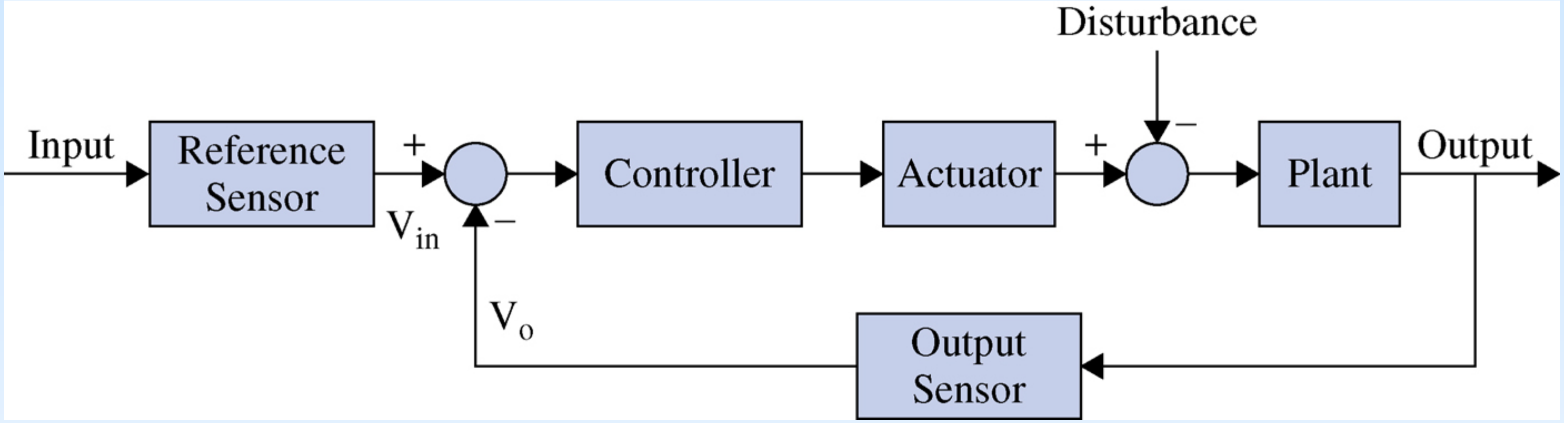
Open-loop system



- Simple
- Fast response

- May not be able to eliminate *static error*
- Sensitive to *disturbance, model mismatch,* and *parameter variations*

Closed-loop system

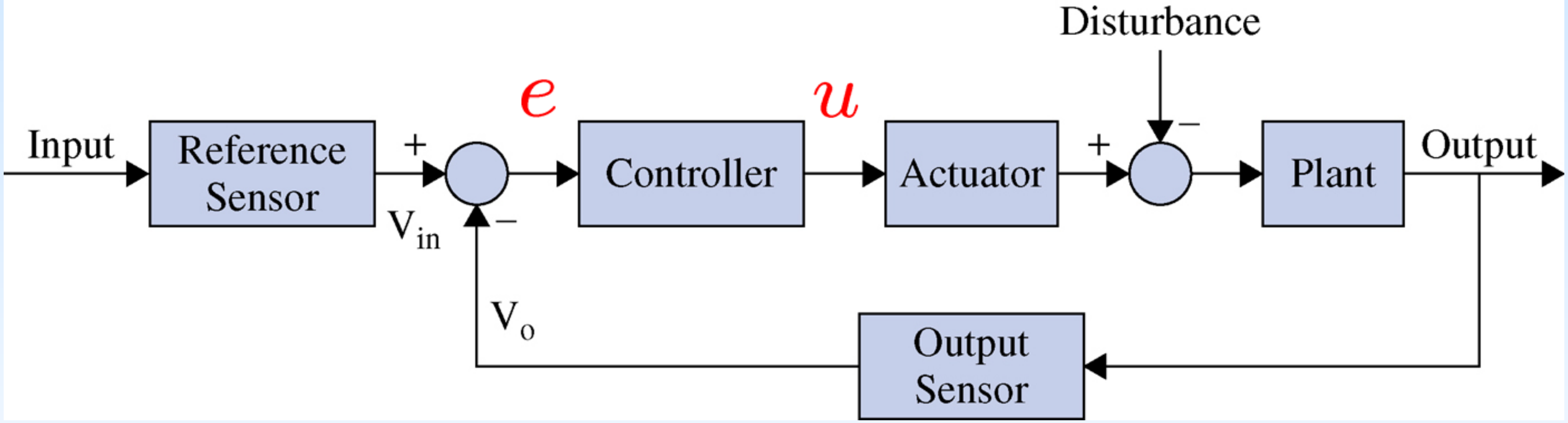


- Reduce static error
- Increase stability margin
- Increase robustness
- Other changes in bandwidth and gain

- May not be as fast as open-loop

Control design: model or non-model?

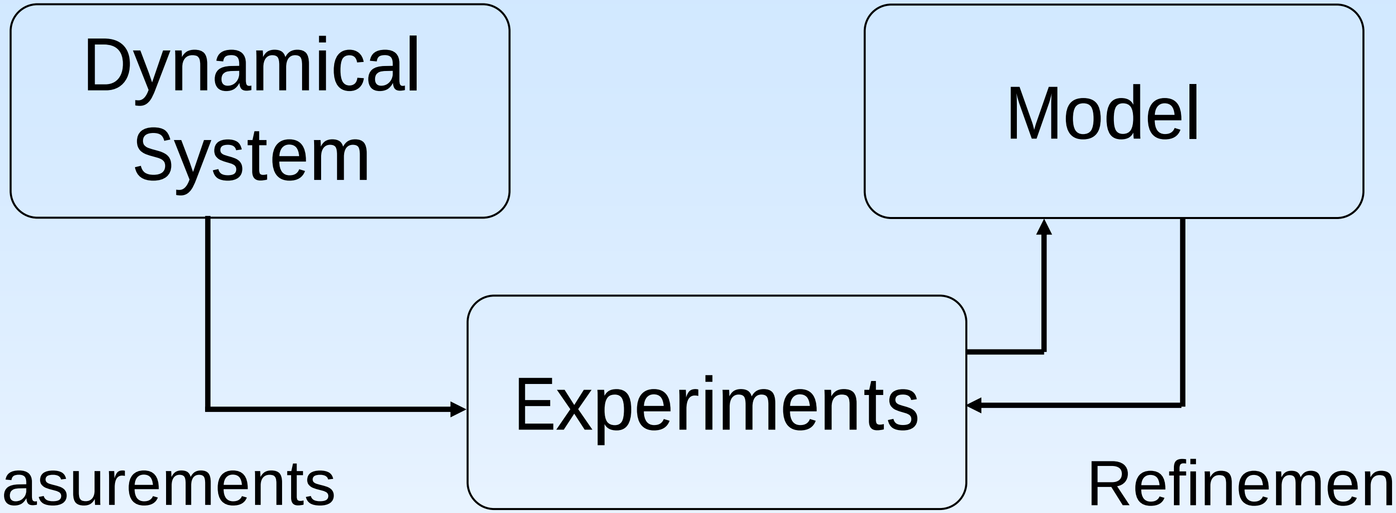
Model is not always necessary: PID control



$$u(t) = K_p e(t) + K_i \int_0^t e(t) d\tau + K_d \frac{d}{dt} e(t)$$

- Easy to implement, low cost
- May be difficult to tune, especially for MIMO case

Model-Based Control System Design



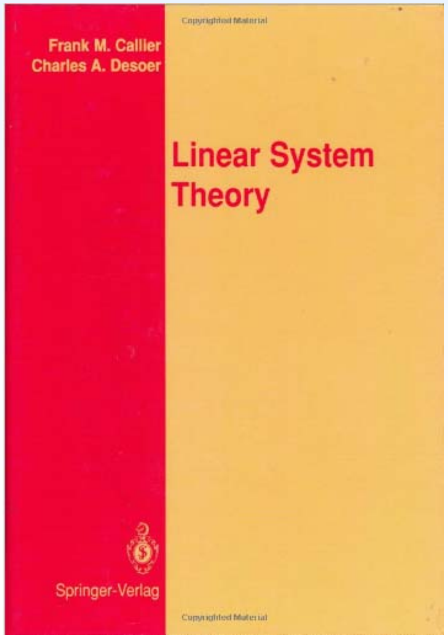
Dynamical System

Model

Experiments

Measurements

Refinements
Updates
Validation
Understanding

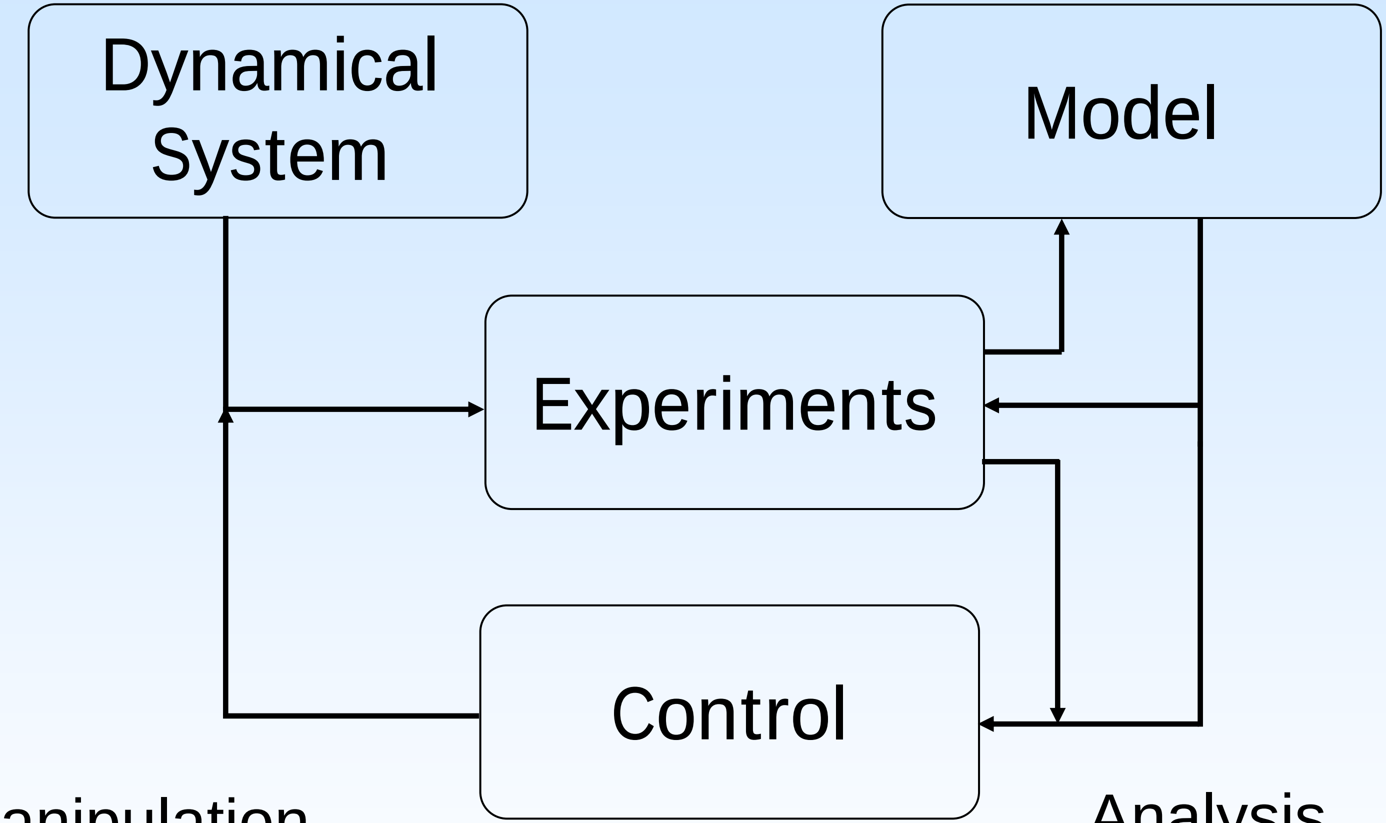


What do we expect a model:

- Compact
- Predictive

Linear System Theory, Frank M. Callier, Charles A. Desoer, 1991

Model-Based Control System Design



Dynamical System

Model

Experiments

Control

Manipulation
Testing

Analysis
Design optimization
Updates
Simulation
Validation

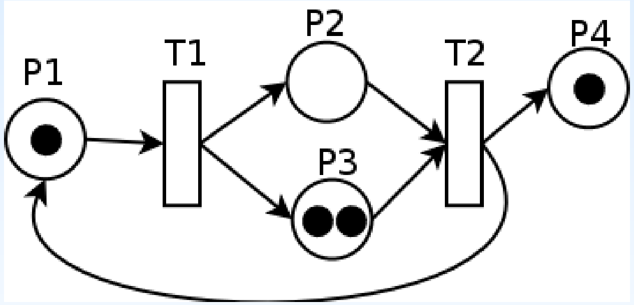
Meet performance specifications

Many different models

- Rule-based model:

  If A1, then B1; …; If An, then Bn

- Finite Automata, Petri Net;



- Differential or difference equations

$$
\begin{aligned}
\dot{x} &= f(x, u, t) \\
y &= g(x, u, t)
\end{aligned}
\qquad
\begin{aligned}
x_{k+1} &= f_k(x_k, u_k) \\
y_k &= g_k(x_k, u_k)
\end{aligned}
$$

Modeling related techniques

Where to get the model: physical principle, I/O data, …

- System Identification: model structure
- Parameter estimation

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx + du \end{aligned} \quad \Longleftrightarrow \quad T(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{a_n s^n + \cdots + a_1 s + a_0}$$

Linearization

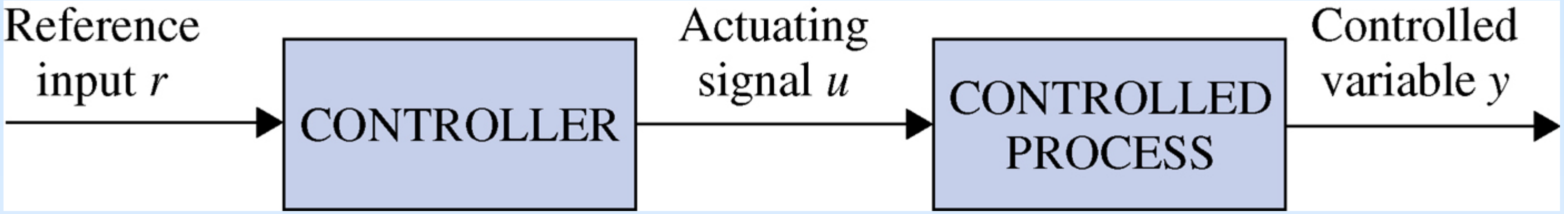
Methods for linear systems often work unreasonably well, in practice, for nonlinear systems

Model reduction

Necessary for control design

- Match time domain response
- Match frequency response

Inverse control: the best controller?



Controller $\mathcal{M}^{-1} : \mathcal{Y} \rightarrow \mathcal{U}$ Model $\mathcal{M} : \mathcal{U} \rightarrow \mathcal{Y}$

$$\mathcal{M} \circ \mathcal{M}^{-1} = \mathtt{Id}$$

Not feasible:

- Inverse system may be *non-causal* or *unstable*:

  original: $\dot{y} = -y + \dot{u} - u$

  Inverse: $\dot{u} = u + \dot{r} + r$

- Sensitive to *disturbance, model mismatch* and *parameter uncertainty*

Control design methods

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Working Domain

- Freq domain (TF based): root locus, Bode plot, H_∞
- Time domain: Least Quadratic Regulation (LQR), Predictive control, adaptive control

Application oriented methods:

- Chemical process control
- Aeronautics and Astronautics
- Robotics & mechatronics
- Social systems
- …
- Quantum systems!

Control: some fundamental questions

Controllability and Observability

- Transfer from one state to the other in *finite* time?

LTI
$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx + du \end{aligned}$$

Check:

$$\mathrm{Rank}[b, Ab, \cdots, A^{n-1}b] = n$$

Nonlinear
$$\dot{\psi} = i(\sigma_x + u\sigma_z)\psi$$

Check:

$$\mathrm{Lie}\{\sigma_x, \sigma_z\} = \mathfrak{su}(2)$$

- Constructive controllability, Reachability space (constrained inputs)

Stability: BIBO, state space, exponential, asympotic

Optimal, Robustness, …

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Engineering system designs

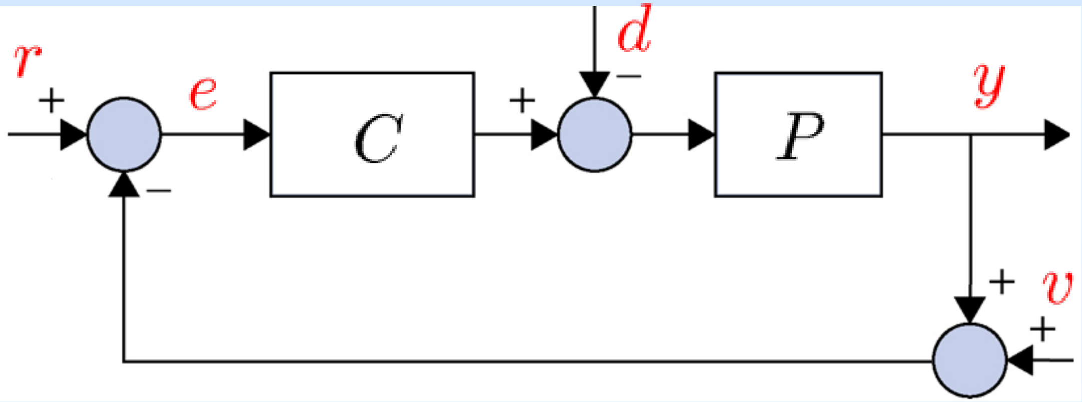
Hard disk drive servo

- Performance
- Budget & reliability for products in mass production

Rapid Thermal Processing (RTP)

- Requisite procedure in wafer processing
- Temperature control
- Extremely demanding performance specifications

Control design: all about tradeoff



$$\begin{aligned} e &= r - y - v \\ y &= P(Ce - d) \\ &= PCe - Pd \end{aligned}$$

$$\begin{aligned} y &= PCr - PCy - PCv - Pd \\ y &= \frac{PC}{1+PC}r - \frac{PC}{1+PC}v - \frac{P}{1+PC}d \end{aligned}$$

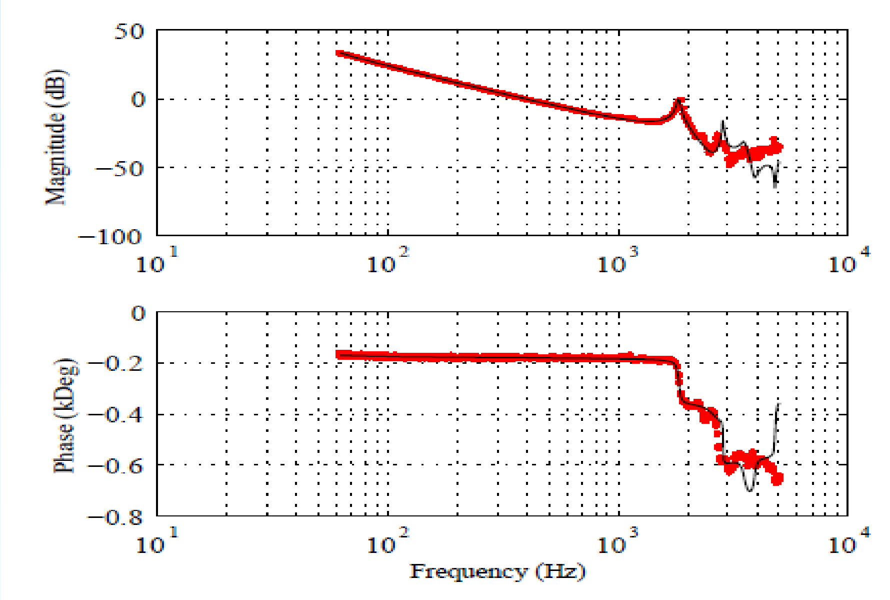
$$E_{\mathrm{cl}} = r - y = \underbrace{\frac{1}{1+PC}}_{\mathcal{S}} r + \underbrace{\frac{PC}{1+PC}}_{\mathcal{T}} v + \frac{P}{1+PC} d$$

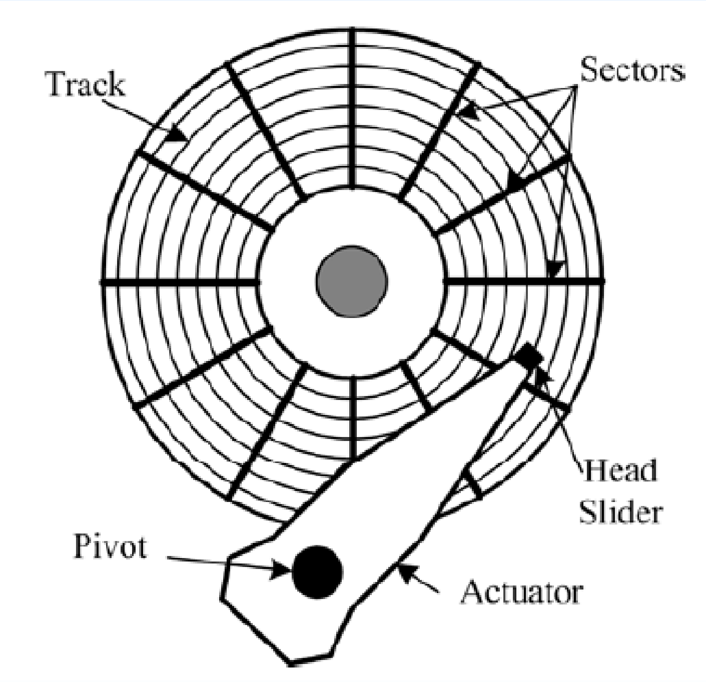
Cannot reduce $\mathcal{S}$ and $\mathcal{T}$ at the same time: $\mathcal{S}+\mathcal{T}=1$

Hard disk drive servo

Frequency domain design

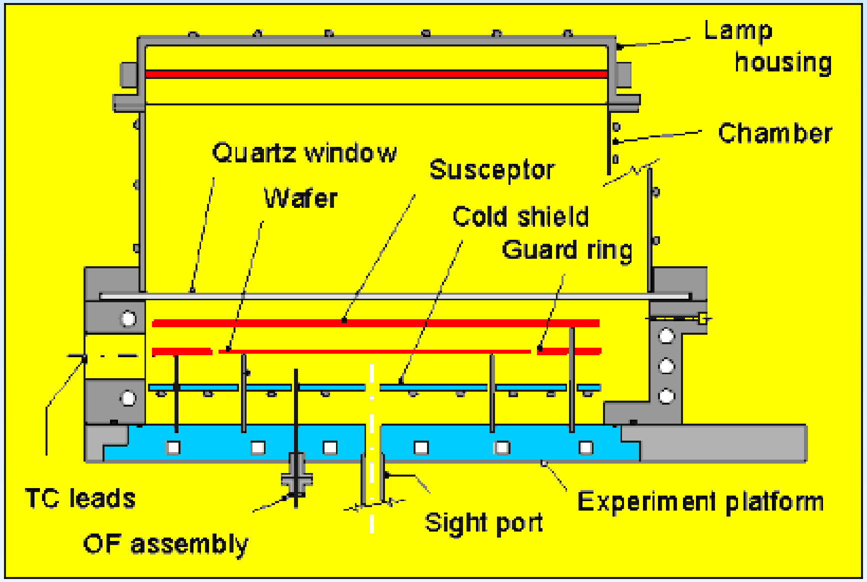
$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= u \end{aligned}$$





- Head positioning servo: track following & track seeking
- Actuator servo
- Spindle motor control
- Servo track writer

RTP systems



Demanding Performance specs

- Fast ramping rate
- Low overshoot
- Tight uniformity

How to design?

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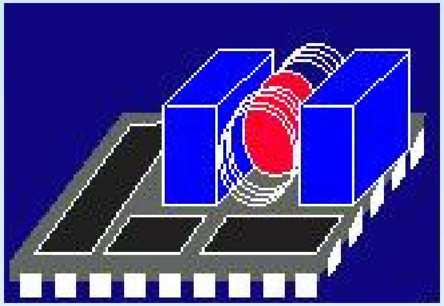
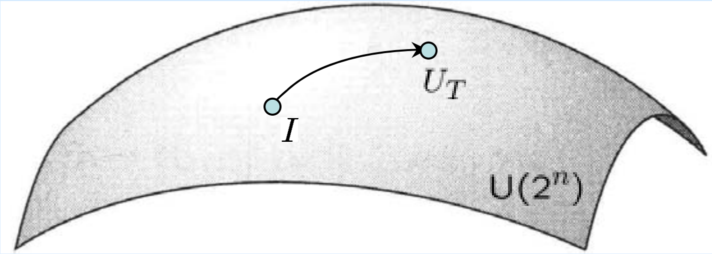
Optimal steering on Lie group



Quantum system



Control system on Lie group

State space: all the n-qubit quantum operations

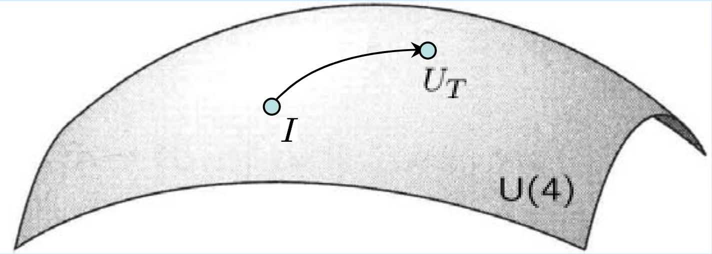
$$\mathrm{U}(2^n) = \{U \in \mathbb{C}^{2^n \times 2^n} : UU^\dagger = I\}$$

Dynamics: Schrödinger equation

$$i\hbar\dot{U} = H({\color{red}v})U, \quad U(0) = I$$

Implementation of quantum operations → Control problem on Lie group $\mathrm{U}(2^n)$

Steering in the tetrahedron

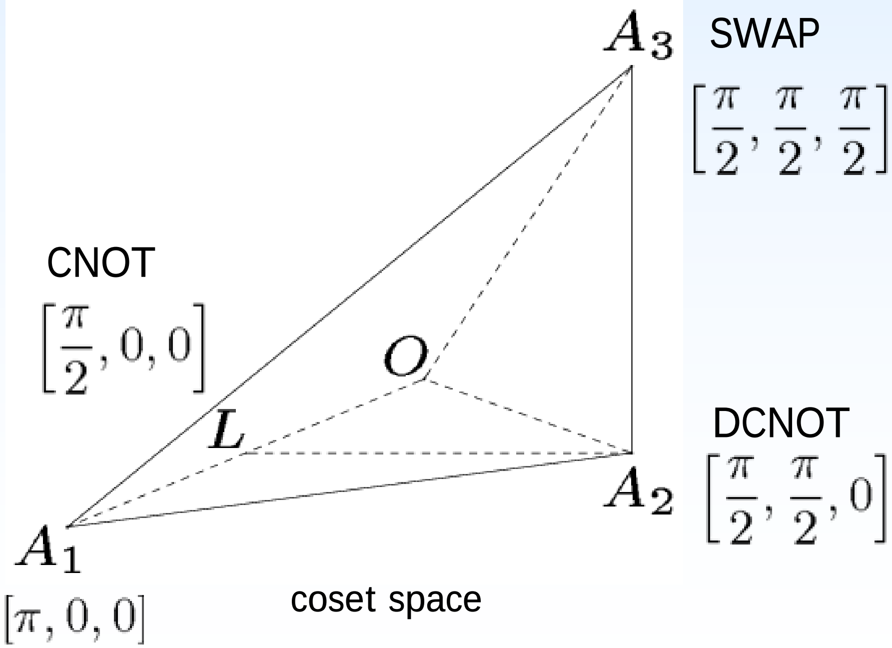


For 2-qubit system:

15 dimensional control problem on U(4)

$$i\hbar\dot{U} = H(v)U, \quad U(0) = I$$

simplified

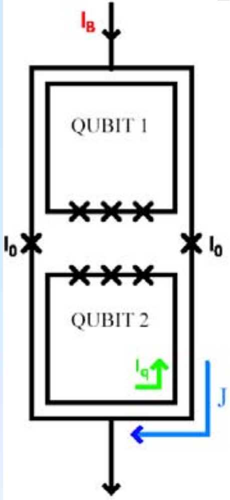


coset space

3 dimensional steering problem in Weyl chamber

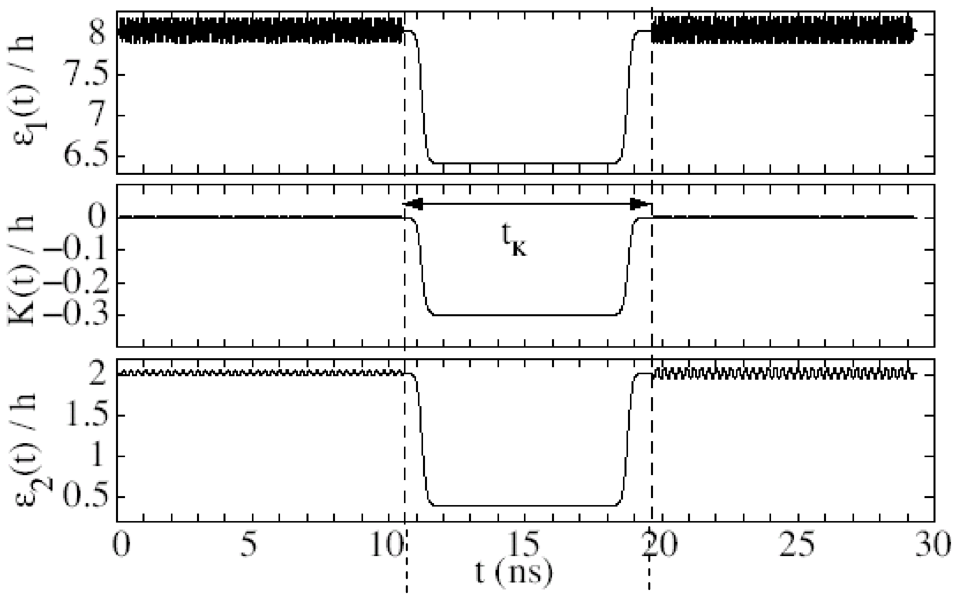
J. Zhang et al, PRA, 67, 042313 (2003)

Example: superconducting qubits



Two-qubit Hamiltonian

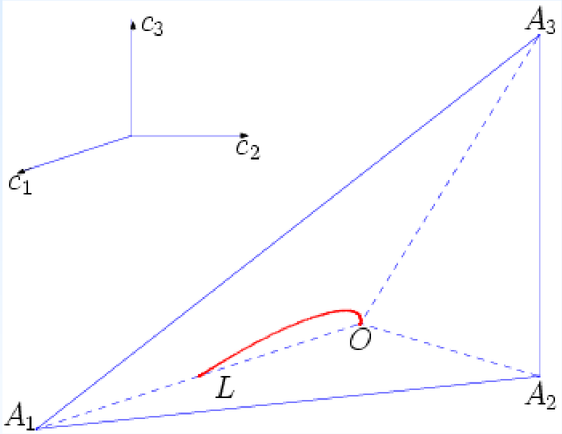
$$H = \sum_{i=1,2} \left(\frac{\epsilon_i}{2} \sigma_z^i + \frac{\Delta_i}{2} \sigma_x^i \right) + K \sigma_z^1 \sigma_z^2$$



Single qubit rotations

Two qubit manipulations

Single qubit rotations



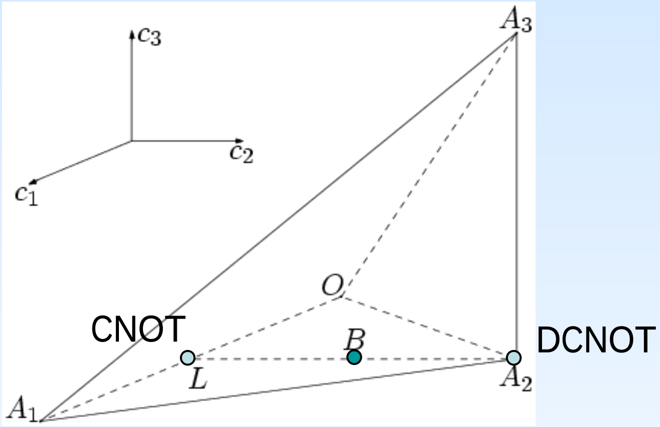
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

CNOT

B. Plourde et al, PRB, 70, 140501(R), 2004

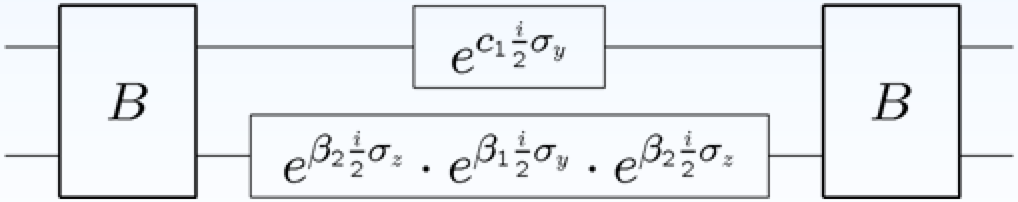
Quantum circuit construction

B-Gate: Two Suffices!



$$= \exp\{\frac{i}{2}(\frac{\pi}{2}\sigma_x^1\sigma_x^2 + \frac{\pi}{4}\sigma_y^1\sigma_y^2)\}$$

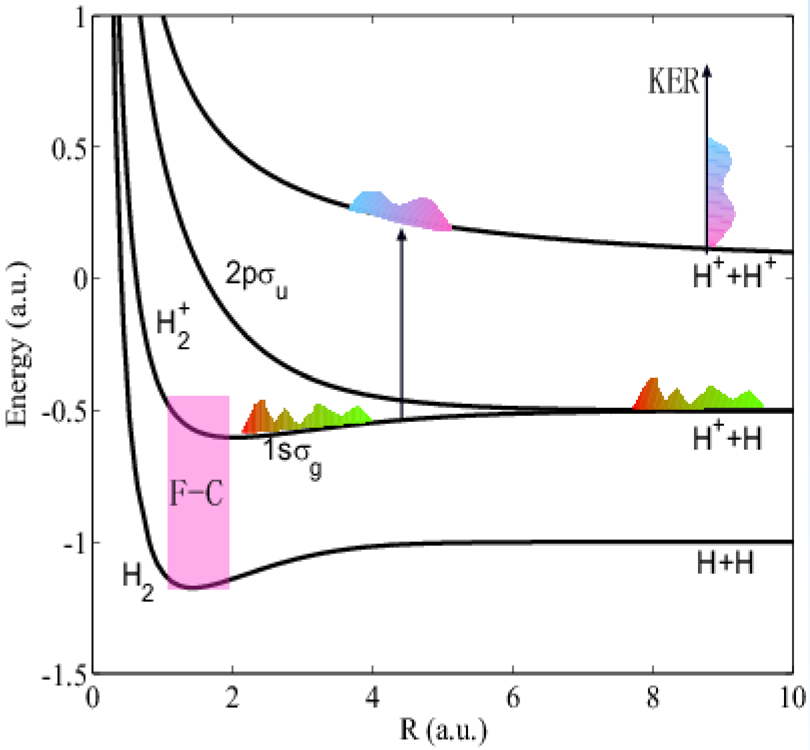
$$[c_1, c_2, c_3] \sim$$



where β_1 and β_2 are determined by c_2 and c_3.

J. Zhang et al, PRL 93, 020502 (2004)

H_2^+ nuclear wave packets transfer



- Shine a strong laser pulse to H_2
- An H_2^+ ion in 1sσ_g
- From Franck-Condon approx., initial NWP of H_2^+ is in the ground state of H_2
- A time-delayed probe pulse E(t) is applied to transfer NWP to the ground state of H_2^+

J. Zhang and F. He, In preparation (2013)

H_2^+ nuclear wave packets transfer

Dynamics

$$i\frac{\partial}{\partial t}\begin{bmatrix}\psi_g(x,t)\\ \psi_u(x,t)\end{bmatrix}=\begin{bmatrix}T_x+V_g(x) & d_{gu}(x)E(t)\\ d_{gu}(x)E(t) & T_x+V_u(x)\end{bmatrix}\begin{bmatrix}\psi_g(x,t)\\ \psi_u(x,t)\end{bmatrix},$$

$\psi_g(x,t)$ and $\psi_u(x,t)$: NWP of the electron in $1s\sigma_g$ and $2p\sigma_u$ states;

d_{gu} : the dipole coupling;

$T_x=-\frac{1}{2M}\frac{\partial^2}{\partial x^2}$, $M=918$

$V_g(x)$: potential for $1s\sigma_g$

$V_u(x)$: potential for $2p\sigma_u$

Initial NWP $\psi_g(x,0)=\psi^0_{\mathrm{Gr}}(x),\quad \psi_u(x,0)=0,$

Target NWP $\psi_g(x,T_f)=\psi^0_g(x)$

Transfer from the ground state of H_2 to the ground state of H_2^+

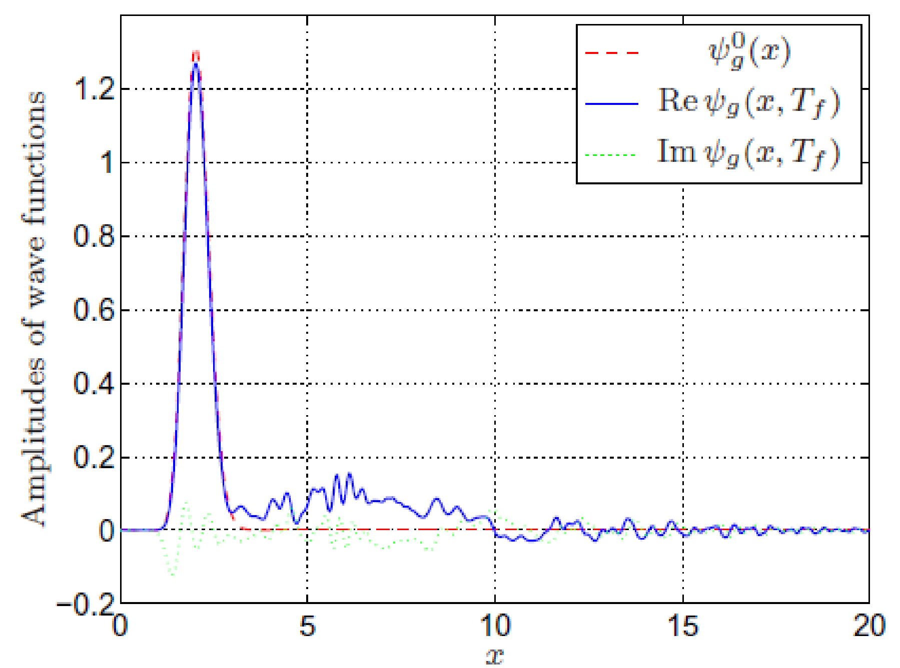
J. Zhang and F. He, In preparation (2013)

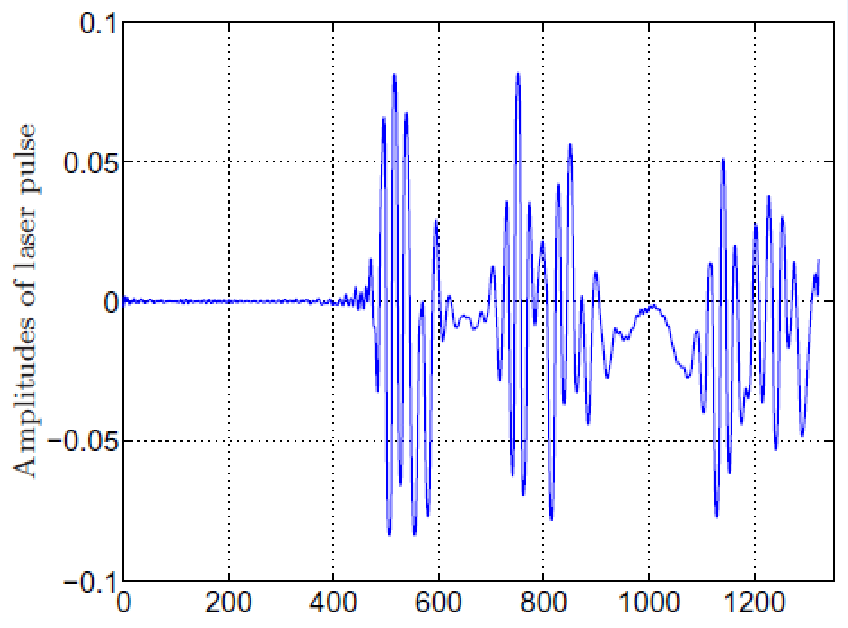
H_2^+ nuclear wave packets transfer

Difficulties:

- To avoid ionization of H_2^+, E(t) is bounded within 0.1;
- Computational intensive procedure to solve Schrodinger's equation

Use sequential linear programming to solve it





Thank you!