Quantum Strangeness, Entanglement and Quantum Information.

G J Milburn

Centre for Engineered Quantum Systems, The University of Queensland

Santa Barbara, March 2013.

Setting the scene...

Cafe Central Vienna.

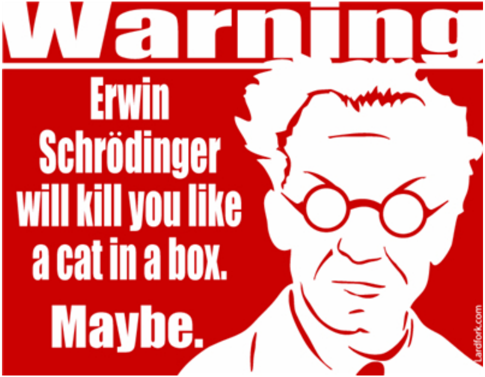
Setting the scene...

The Cafe Josephinum is a smell first, a stinging smell of roasted Turkish beans too heavy to waft on air and so waiting instead for the more powerful current of steam blown off the surface of boiling saucers fomenting to coffee.

. . . the coffee is a fuel to power ideas.

A MADMAN DREAMS OF TURING MACHINES by Janna Levin

Erwin Schrödinger: 1887-1961.


Warning
Erwin
Schrödinger
will kill you like
a cat in a box.
Maybe.
Lardfork.com

I was born and educated in Vienna with Ernest Mach's teaching and personality still pervading the atmosphere. I was devoted to his numerous writings . . . Both Boltzmann and Mach were just as much interested in philosophy . . . as they were in physics.

Boltzmann's approach had consisted in forming "pictures", mainly in order to be extremely certain of avoiding contradictory assumptions.

E. Schrödinger in a letter to Eddington, 1940 .

January 1926

$$\Psi$$

Schrödinger published in **Annalen der Physik**,

*" Quantisierung als Eigenwertproblem "**

…what is now known as the Schrödinger equation.

*tr. Quantisation as an Eigenvalue Problem.

Quantum coherence.

Functional engineered quantum systems exploit *quantum coherence*.

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Quantum coherence: the hidden lever of the physical world.

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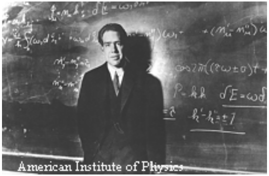
Functional engineered quantum systems exploit *quantum coherence*.

Quantum coherence: the hidden lever of the physical world.

Controlling quantum coherence enables us to *reversibly* change irreducible uncertainty into perfect certainty.

Quantum coherence.



The physical universe is irreducibly random.

Given complete knowledge of the state of a physical system, there is at least one measurement the results of which are completely random.

Quantum coherence.

Given complete knowledge of a physical state there is at least one measurement the results of which are completely certain.

Probability in QM.

QM calculus to determine the probability for measurement outcomes.

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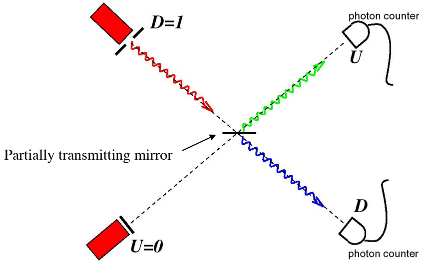
Probability amplitudes for one kind of measurement determine probability amplitudes for all kinds of measurements.

A list of probability amplitudes is a state

Example: single photon at a beam splitter.

A coin toss with photons.



Probability of reflection =1/2
Probability of transmission=1/2

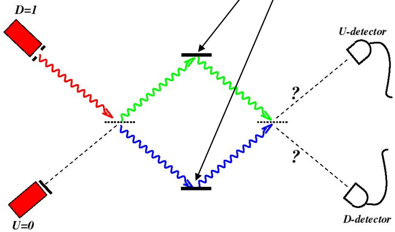
Prob. Count photon at U=1/2
Prob. Count photon at D =1/2

Is this a coin-toss ?
Does this encode one bit?

Example: single photon at a beam splitter.

- Toss a photon twice.

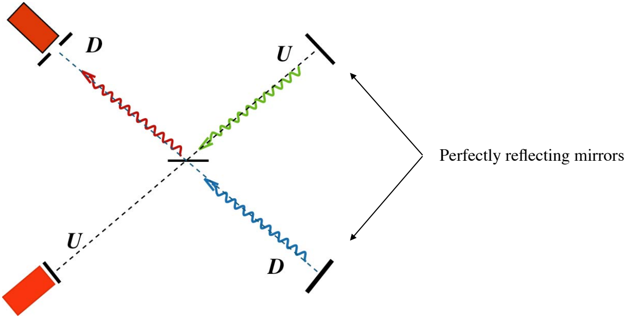
Perfectly reflecting mirrors



Is this like tossing a coin twice ?

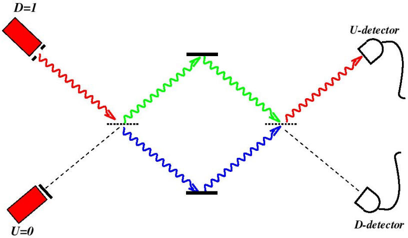
Example: single photon at a beam splitter.

- HINT: beam splitters can be *time reversed*.



Example: single photon at a beam splitter.

A one photon bit? No.



Experiment:

detection at U is certain.

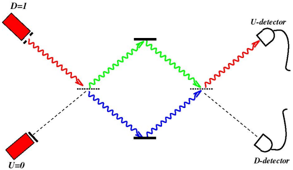
– Irreducible randomness is made certain !

Example: single photon at a beam splitter.

- Bayes' sum rule ☞ Feynman's sum rule
  - if an event can happen in two (or more) indistinguishable ways, first add the probability amplitudes, then square to get the probability.
  - probability amplitudes are not necessarily positive real numbers !

Example: single photon at a beam splitter.

- Count at U can happen in two ways:
  - Two reflections (RR) & and two transmissions (TT), indistinguishable.
  - Need probability amplitudes: $\boldsymbol{A}(\mathrm{RR})$ and $\boldsymbol{A}(\mathrm{TT})$
- Count at D can happen in two ways:
  - RT & TR, indistinguishable.
  - Need probability amplitudes: $\boldsymbol{A}(\mathrm{RT})$ and $\boldsymbol{A}(\mathrm{TR})$
- Probability for count at U or D:
  - P(count at U)= $(\boldsymbol{A}(\mathrm{RR}) + \boldsymbol{A}(\mathrm{TT}))^2$
  - P(count at D)= $(\boldsymbol{A}(\mathrm{RT}) + \boldsymbol{A}(\mathrm{TR}))^2$



Example: single photon at a beam splitter.

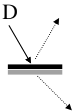
- Assignment of probabilities.
  - Choose distinct output amplitudes for each distinct input:

$$A^{(D)}(\mathbf{U}) = \frac{1}{\sqrt{2}}; \quad A^{(D)}(\mathbf{D}) = \frac{1}{\sqrt{2}}$$

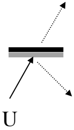
$$A^{(U)}(\mathbf{U}) = \frac{1}{\sqrt{2}}; \quad A^{(U)}(\mathbf{D}) = -\frac{1}{\sqrt{2}}$$

Represent amplitudes as an *ordered pair*:

$$(\ A(\mathrm{D}),\ A(\mathrm{U})\ )$$



$$(1,0) \rightarrow \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$



$$(0,1) \rightarrow \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

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$$A(RR) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{2}$$

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$$P(U) = |A(RR) + A(TT)|^2 = 1$$

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$$P(U) = |A(RR) + A(TT)|^2 = 1$$

$$P(D) = |A(RT) + A(TR)|^2 = 0$$

LIGO: the largest quantum machine.

LIGO Livingston Observatory
Louisiana

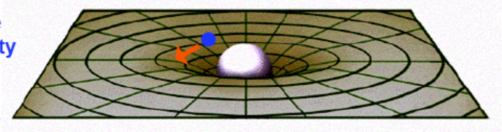
Laser Interferometer Gravitational (wave) Observatory

LIGO: the largest quantum machine.

LIGO

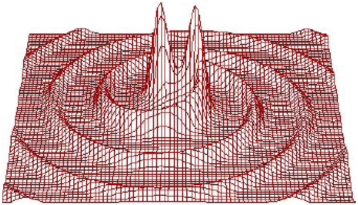
Gravitational Waves

Static gravitational fields are described in General Relativity as a curvature or warpage of space-time, changing the distance between space-time events.

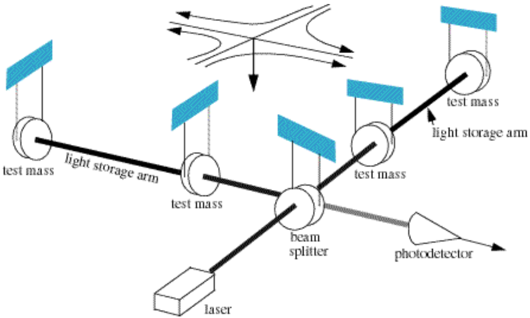


Shortest straight-line path of a nearby test-mass is a ~Keplerian orbit.

If the source is moving (at speeds close to c), *eg*, because it's orbiting a companion, the "news" of the changing gravitational field propagates outward as gravitational radiation – a wave of spacetime curvature



LIGO: the largest quantum machine.



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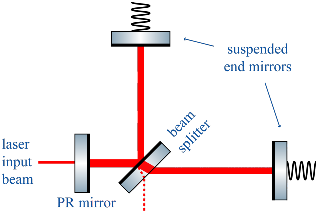


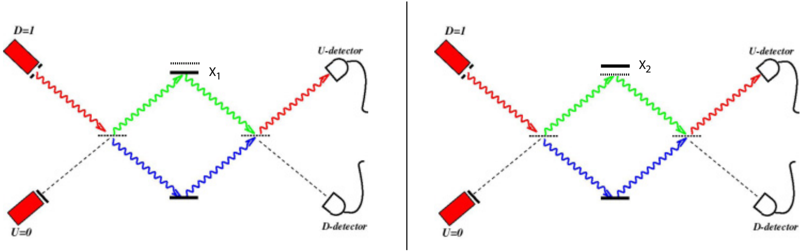
Objective: measure the change in the distance between the mirrors due to a gravitational wave.

Sensitivity required:

10^{-18} metres... one attometer.

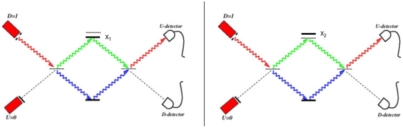
Gravitational waves are weak!

Quantum description.



Two different path lengths at two distinct times in the period of the gravitational force.

Quantum description.



$$A(RR) = \frac{1}{\sqrt{2}} e^{i\theta} \frac{1}{\sqrt{2}} = \frac{e^{i\theta}}{2}$$

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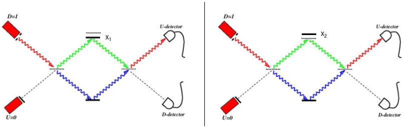

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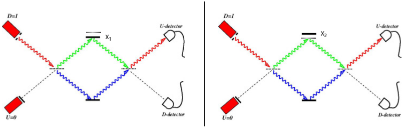

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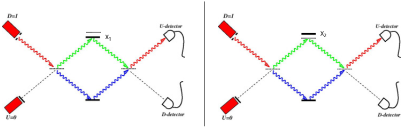

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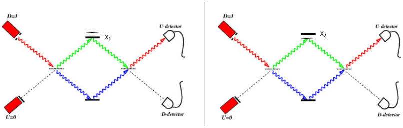

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$$P(U) = |A(RR) + A(TT)|^2 = \cos^2 \theta$$

Quantum description.

Each single photon trial provides less than a single bit of information on average.

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Repeat for N trials, varying θ, and compute relative frequency for U detections.

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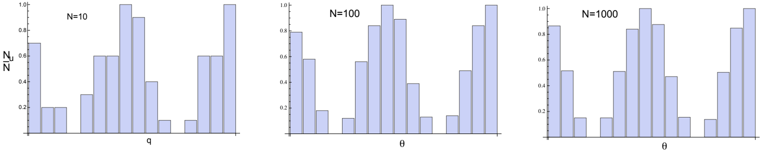

Quantum description.

Readout: $x = 0$ if detected at U

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Mean and variance of result:

$$\begin{aligned} \bar{X} &= \cos\theta \\ \Delta X^2 &= P(U)(1 - P(U)) = \frac{\sin^2(2\theta)}{4} \end{aligned}$$

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$$\delta\theta^2 \geq \frac{1}{4N}$$

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Need $N >> 1$.

perfect measurement of relative path length???

LIGO: the largest quantum machine.

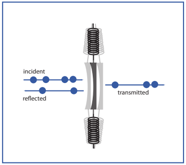
Enter quantum mechanics . . . Heisenberg uncertainty principle:

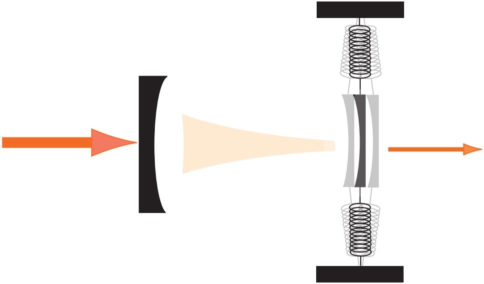
If we measure the relative position of the mirrors accurately we necessarily kick the mirrors uncontrollably . . . radiation pressure.

accurate measurements cost!

Enforcing the uncertainty principle.

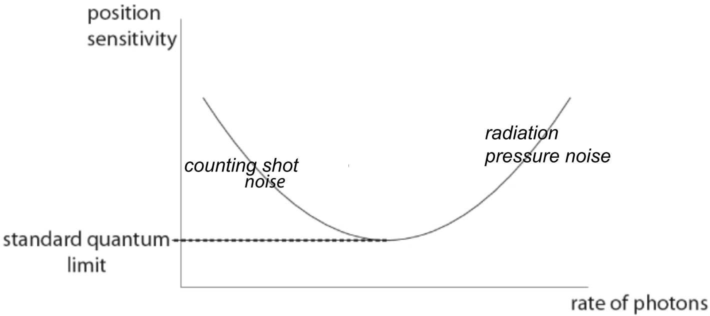
Radiation pressure shot noise...





Random reflections of photons shake the mirror.

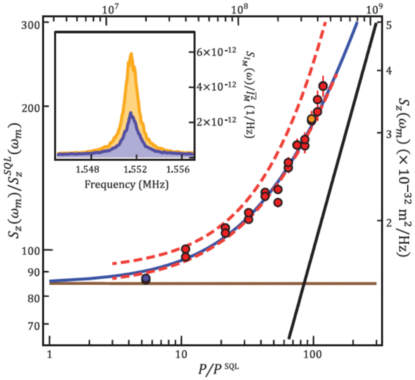
The standard quantum limit.



Radiation pressure noise enforces the Heisenberg uncertainty principle.

Measurement of radiation pressure noise.

Observation of Radiation Pressure Shot Noise on a Macroscopic Object
T. P. Purdy *et al.*
Science **339**, 801 (2013);
DOI: 10.1126/science.1231282



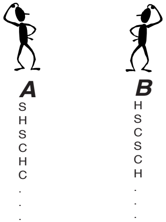
- Quantum entanglement.
  - Measurements made on the joint state of separate systems.
    - Reveal correlations that cannot arise from classical statistical theory.
  - Quantum correlation is called *entanglement*.
    - Entanglement arise when Feynman's rule is applied to the results of measurements made on two or more systems

Quantum entanglement.

- Classical correlation.
  - Bellsville, an urban allegory.
    - No more than the result of a single *yes/no* question can be recorded.
    - Loophole: Ask different questions of each of a pair of twins.
  - Choose questions from
    - H (height) + if tall, − otherwise
    - S (sex) + if female, − if male
    - C (eye color) + if blue, − otherwise.
  - Perfect correlation.
    - If same question is asked on each twin of a pair, the answers are *identical*

Quantum entanglement.

- What correlations occur for different questions ?



- In how many cases are the answers the same ?
  (Note: in each run, one trait remains unknown.)

Quantum entanglement.

- The standard analysis.
  - Answers are *determined* by hidden genes, randomly distributed.
  - Gene is an ordered triple: eg (S,H,C) ☛ (+,+, -)
  - There are $2^3 = 8$ possible genes for three traits.

- What is the probability for S = +, H = + ?

$$P(S = +, H = +) = P(+ + -) + P(+ + +) \ \text{(Bayes' rule)}$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Quantum entanglement.

- Classical correlation.
  - What is the probability that the *same* answer is found for *different* questions ?

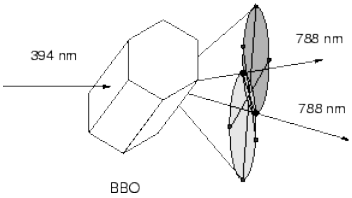
$$P(same) = P(S+, H+) + P(S-, H-)$$

$$P(S+, H+) = P(S-, H-) = \frac{1}{4}$$

$$\therefore P(same) = \frac{1}{2}$$

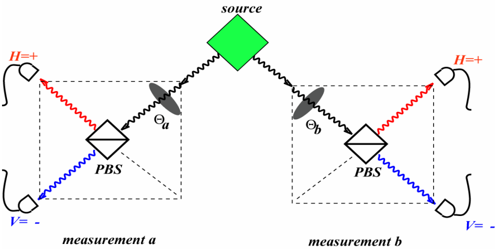
Quantum entanglement.

- Twin polarised photons.
  - Spontaneous parametric down conversion
    - A property of certain crystals to absorb a single photon and emit two photons, each with half the frequency (double the wavelength).
    - In type II down conversion, each photon has strongly correlated polarisation: can be made to have the *same* polarisation.



Quantum entanglement.

Polarised photon twins: Type II down conversion.



- When both polarisers are set to the same angle $\theta_a = \theta_b$, both measurements give the *same* result, either ++ or --. Each case (++) or (--) occurs with equal probability.

Quantum entanglement.

- What is the probability that when the polarisers are set at *different* angles the results are the same ?
- By analogy with the experiment on twins we might expect that the answer is 1/2.
- **The experiment gives 1/4 for certain choices of angles.**

Quantum entanglement.

Apply Feynman's rule:

Example: choice of angles we find

$$-\mathcal{A}(+,+,+) = \frac{1}{4\sqrt{2}} \quad ; \mathcal{A}(+,+,-) = \frac{-3}{4\sqrt{2}}$$

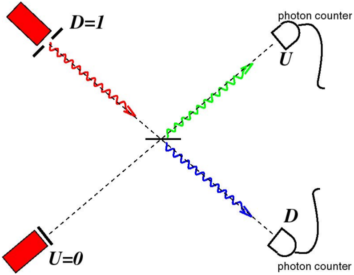
Q. Where do these come from?…
A. *Quantum electrodynamics* …which is a bit beyond the level of this subject.

$$-\ P(\ A+,B+) = \left|\frac{1}{4\sqrt{2}} - \frac{3}{4\sqrt{2}}\right|^2 = \frac{1}{8}$$

- Likewise P(A - , B -) = 1/8
- P(same) = P(A + , B +) + P(A - , B -) = 1/8 + 1/8 = 1/4
- A similar calculation shows that P(different) = 3/4.

Quantum information.

Coding bits in quantum states.



State of photon after beam splitter ?

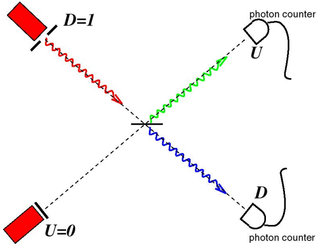
Not reflected or transmitted.

Not logical 1 or 0.

It is a *superposition* of both possibilities.

It is a ***qubit.***

Quantum information.



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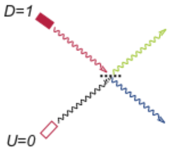
It is a *superposition* of both possibilities.

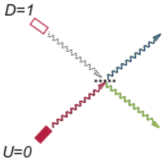
It is a ***qubit.***

Coding bits in quantum states. $D : (1, 0) \to \mathbf{1} \quad U : (0, 1) \to \mathbf{0}$

Quantum information.

- superposition of binary strings.

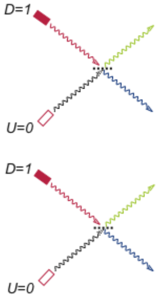




$$\mathbf{1} \rightarrow \mathbf{1} + \mathbf{0} \qquad \mathbf{0} \rightarrow \mathbf{1} - \mathbf{0}$$

Quantum information.

Superpositions of binary strings length 2



Two **physical qubits can encode *four* binary numbers simultaneously**

$$\mathbf{1\,1} \rightarrow \mathbf{1\,1} + \mathbf{1\,0} + \mathbf{0\,1} + \mathbf{0\,0}$$

Quantum information.

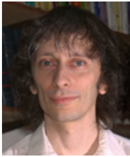
N physical qubits can encode 2^N binary numbers simultaneously

A quantum computer can *process* all 2^N numbers in parallel on a *single* machine with N physical qubits.

Quantum information.

- D. Deutsch, Oxford, 1985

  Quantum theory, the Church-Turing principle and the universal quantum computer.

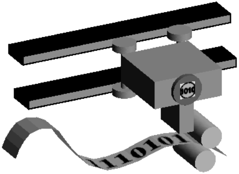


- Prepare input as a superposition of all possible inputs.
- Run computer once to give all possible values of the calculation.

Quantum information.

- A physical computer operating by quantum rules.
  - ➢ could it compute more ***efficiently*** than a conventional computer ?

Quantum information.



- Turing machines.
- Church-Turing thesis:

  A computable function is one that is computable by a universal Turing machine.

Quantum information.

- Efficiency:
  - How many steps are required to compute a function (how many operations per second)?
  - How does the number of steps depend on the *size* of the problem.

- Find the prime factors of
  - 2385269 (1001000110010101110101)

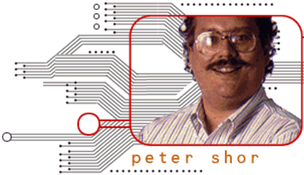
How ?…divide by 2….no
Divide by 3….no
And so on until
Divide by 541…yes... 2385269 = 541 x 4409

- In general to factor integer X, need $\sqrt{X}$ steps.

Add one digit to X, need about three times as many steps that is an exponential increase !

Quantum information.

- Peter Shor, AT&T, USA, 1993
  - a quantum algorithm to find prime factors of large composites N
  - public key cryptography no longer safe !
- Key step:
  - find the 'order' of the function:

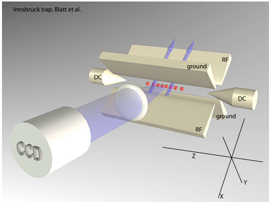


$$f(a) = x^a \bmod N$$

(x is random, but GCD(x,N)=1)

Quantum computing today.

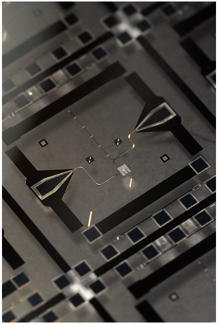
Ion traps.



Blatt group, Innsbruck.

Quantum computing today.

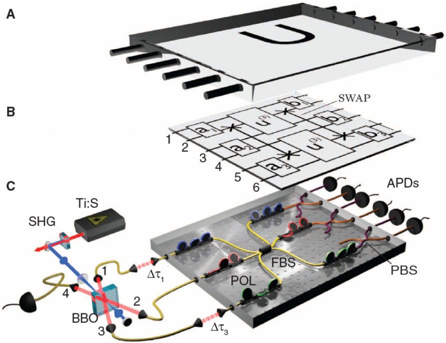
Superconducting circuits.



Martinis group UCSB.

Quantum computing today.

Photonic circuits.



White group EQuS, Brisbane.

Quantum information.

Aaronson & Arkipov:

Predicting the (probabilistic) results of a given quantum-mechanical experiment, to finite accuracy, cannot be done by a classical computer in probabilistic polynomial time, unless factoring integers can as well.

The boson sampling problem.

Finding the permananent of a $n \times n$ matrix is a **hard*** problem.

*sharp-P-complete

The boson sampling problem.

Finding the permananent of a $n \times n$ matrix is a **hard*** problem.

$$\mathrm{Perm}\begin{pmatrix} a & b & c \\ l & m & n \\ p & q & r \end{pmatrix} = a(mr + nq) + b(lr + np) + c(lq + mp)$$

* sharp-P-complete

The boson sampling problem.

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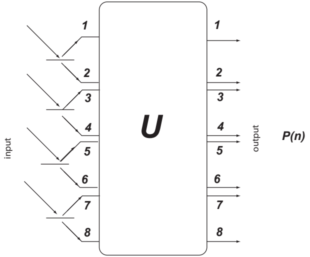
Aaronson & Arkipov: In a photonic QC with linear optics, the output probability distributions are given by permanents.

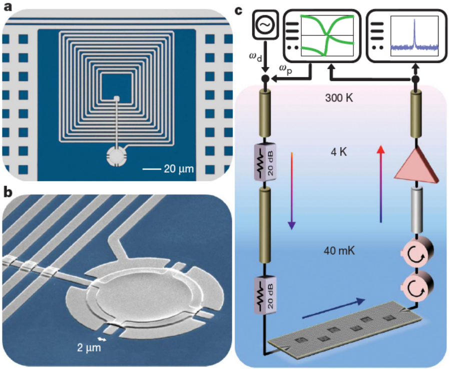


* sharp-P-complete

The boson sampling problem.

Photonic Boson Sampling in a Tunable Circuit
Matthew A. Broome *et al.*
Science **339**, 794 (2013);
DOI: 10.1126/science.1231440

Engineered quantum systems.



Teufel et al. (NIST) Nature, March (2011).

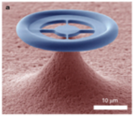
-Fabricated (artificial) devices that operate by the control of quantum coherence.

-Involves a very large number of atomic systems.

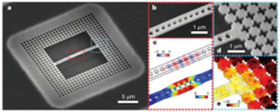
- Quantise a collective, macroscopic degree of freedom.

Optomechanics: an engineered quantum systems.

Aspelmeyer,
Physik in unserer Zeit, 2011



Kippenberg,
Nature, 2012

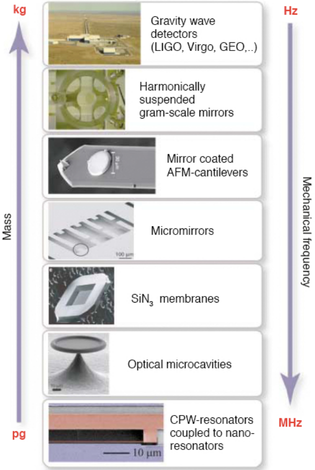


Painter Nature, 2012

Coupling photons to bulk *quantised* elastic modes.
Use laser cooling to reach the vibrational ground state.

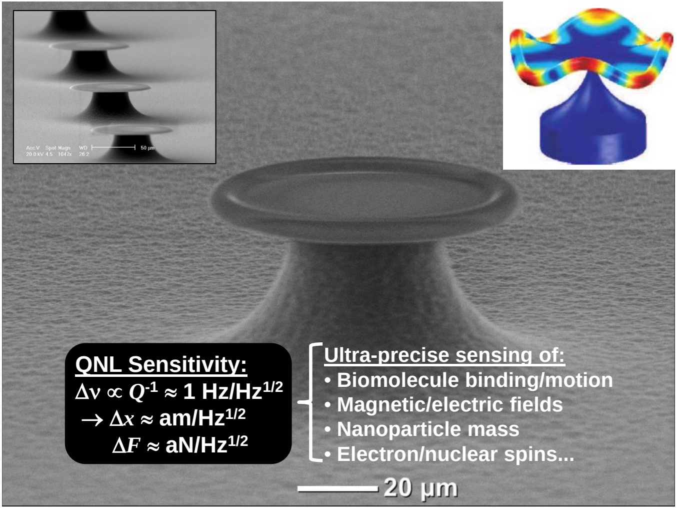
Macroscopic to microscopic.

T. J. Kippenberg[1]*† and K. J. Vahala[2]*
29 AUGUST 2008 VOL 321 **SCIENCE**



Engineered quantum systems ...
.... moving the quantum/classical border.

Applications: metrology.



Bowen lab, UQ.

The quantum world is strange ..

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and enables powerful new technologies.

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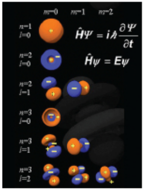

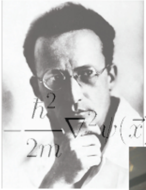

$$i\hbar\frac{\partial\psi(\vec{x})}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{x}) + V(\vec{x})\psi(\vec{x})$$