Coarse translation symmetry and exotic renormalization groups in fracton phases

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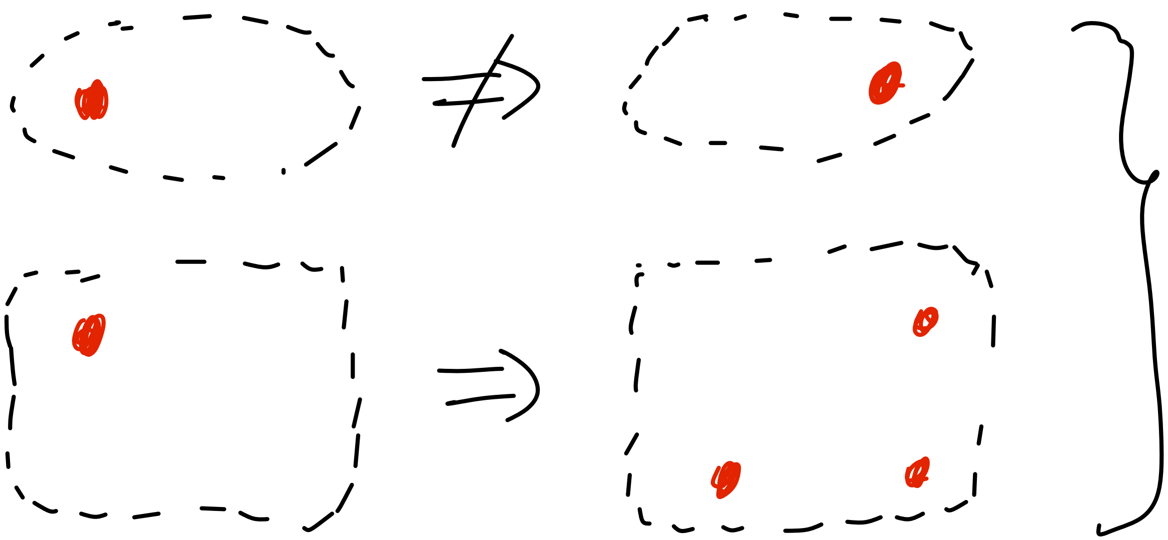
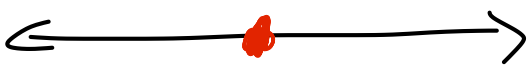
Fractons: what are they?

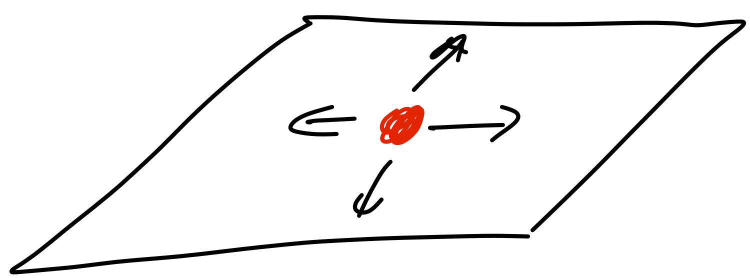
- Gapped excitations of restricted mobility



fractons:
single excitations
immobile



lineon



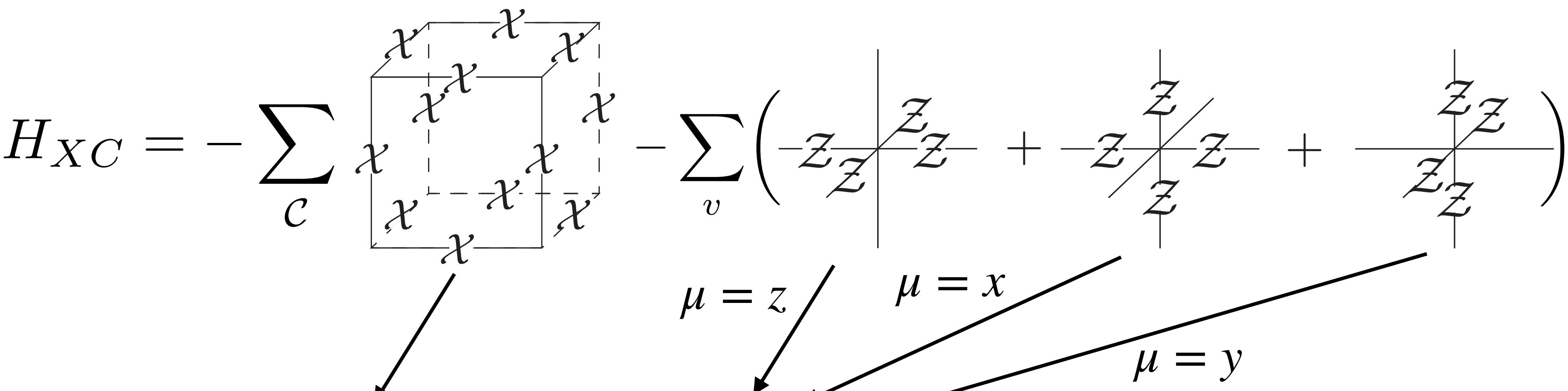
planon

Example: X-cube fracton model

Vijay, Haah & Fu

- Place qubits on the links of a *d=3* simple cubic lattice

- Hamiltonian defined in terms of *X* and *Z* Pauli operators:



$$H_{XC} = -\sum_c B_c - \sum_v \sum_{\mu=x,y,z} A_v^\mu$$

- Sum of commuting terms (like toric code model)

- X-cube model has fracton, lineon and planon excitations

- Haah's cubic code is a similar model with only fracton excitations

- By now there is a vast landscape of examples (which we do not understand well!)

Why are fracton systems interesting?

- Quantum information properties/applications
  (*e.g.* progress toward self-correcting quantum memory)
- Interesting constrained quantum dynamics
- New class of quantum phases of matter
- Related to exotic quantum field theories ("UV/IR mixing")

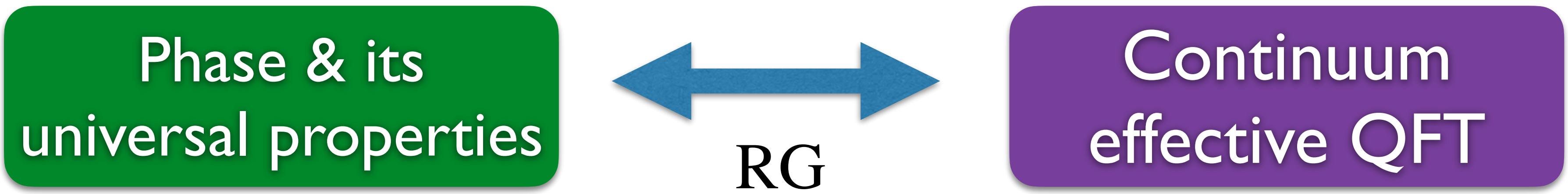
Fracton phases are challenging theoretically

- Rough early timeline of fracton theory
  - First identification of fractons (Chamon 2004)
  - Toward self-correcting quantum memories (Haah 2011)
  - Simpler models and condensed matter viewpoint (Vijay, Haah & Fu 2015-16; Pretko 2016; followed by many others)
- After ~7 years, a general theoretical framework for fracton phases still seems far off

Why are fracton phases challenging/interesting/important?

Fracton phases don’t obey usual relationships among phases, renormalization group (RG), and effective quantum field theories (QFTs)

<u>Conventional situation</u>

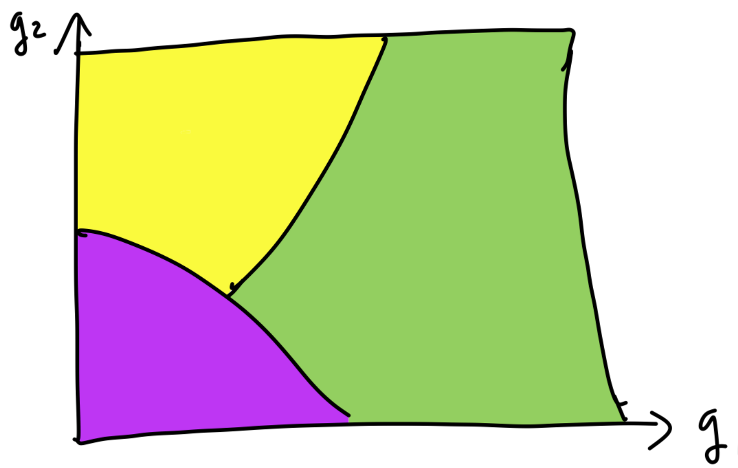


- Universal properties of the phase are invariant under RG: long-wavelength, low-energy properties
- These properties encoded in effective QFT
- Believed to hold for all gapped phases with only fully mobile excitations, opens up powerful tools, *e.g.* topological quantum field theory (TQFT)

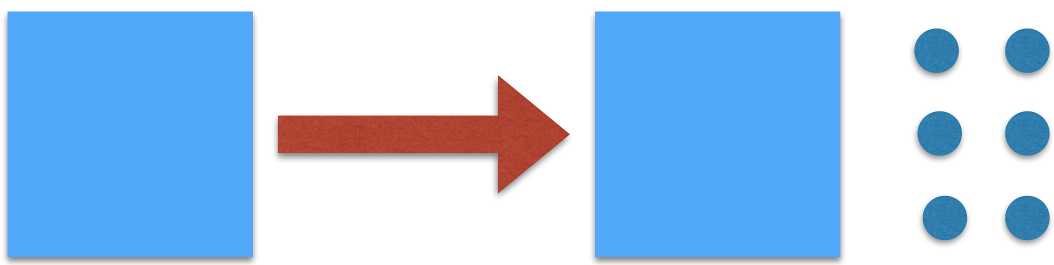
Phases of quantum matter

"standard phases" ≡ equivalence classes of quantum systems, generated by two operations

1. Deformation



2. Stacking with trivial systems (product states)

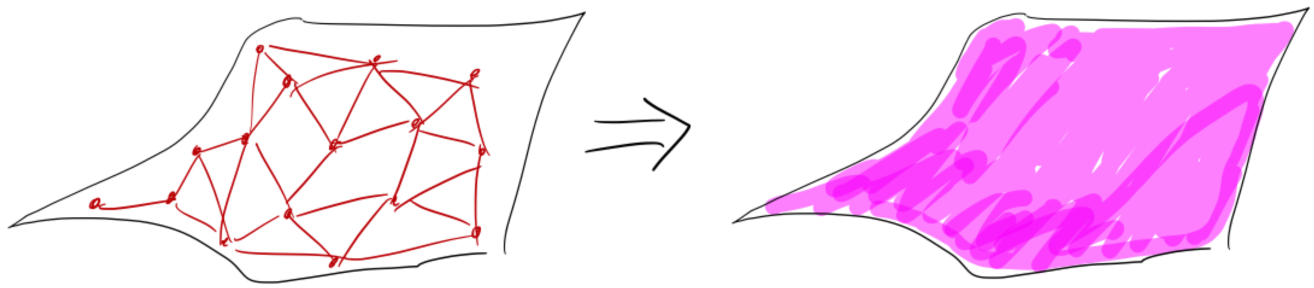


$|\psi\rangle$ $\qquad\qquad$ $|\psi\rangle \otimes |\uparrow\uparrow\uparrow\cdots\rangle$

Stacking physically motivated from lattice models as idealizations of continuum systems

Renormalization group

- Renormalization group (RG): focus on physics at longer distances (times), eliminate short-distance degrees of freedom

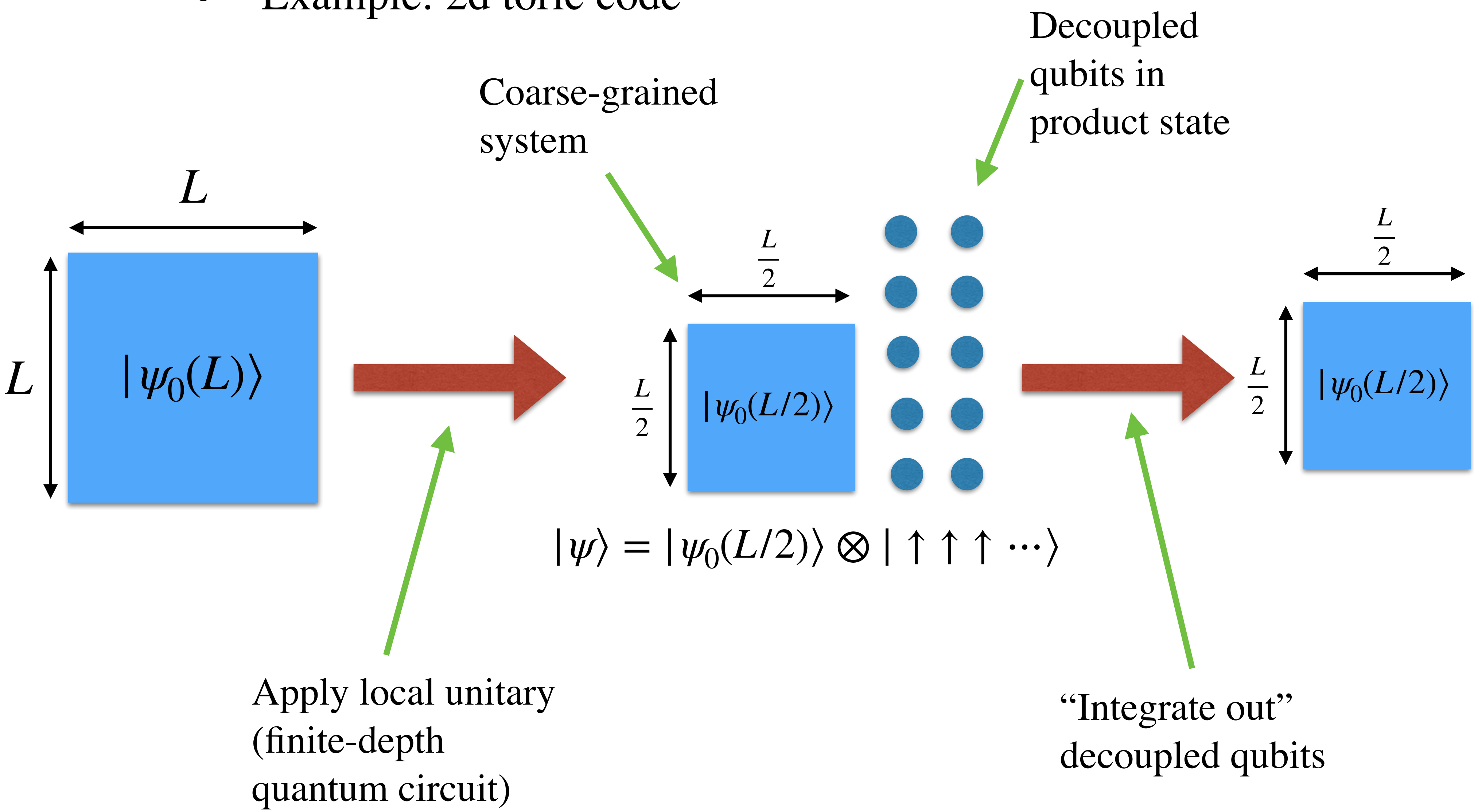


- RG provides a link between a lattice model and an effective continuum quantum field theory (QFT) description

Entanglement RG

Aguado & Vidal

- Some gapped models are RG fixed points
- Example: 2d toric code



Why are fracton phases challenging/interesting/important?

Developing picture for (some) fracton phases

Phase & its universal properties

Continuum effective QFT?

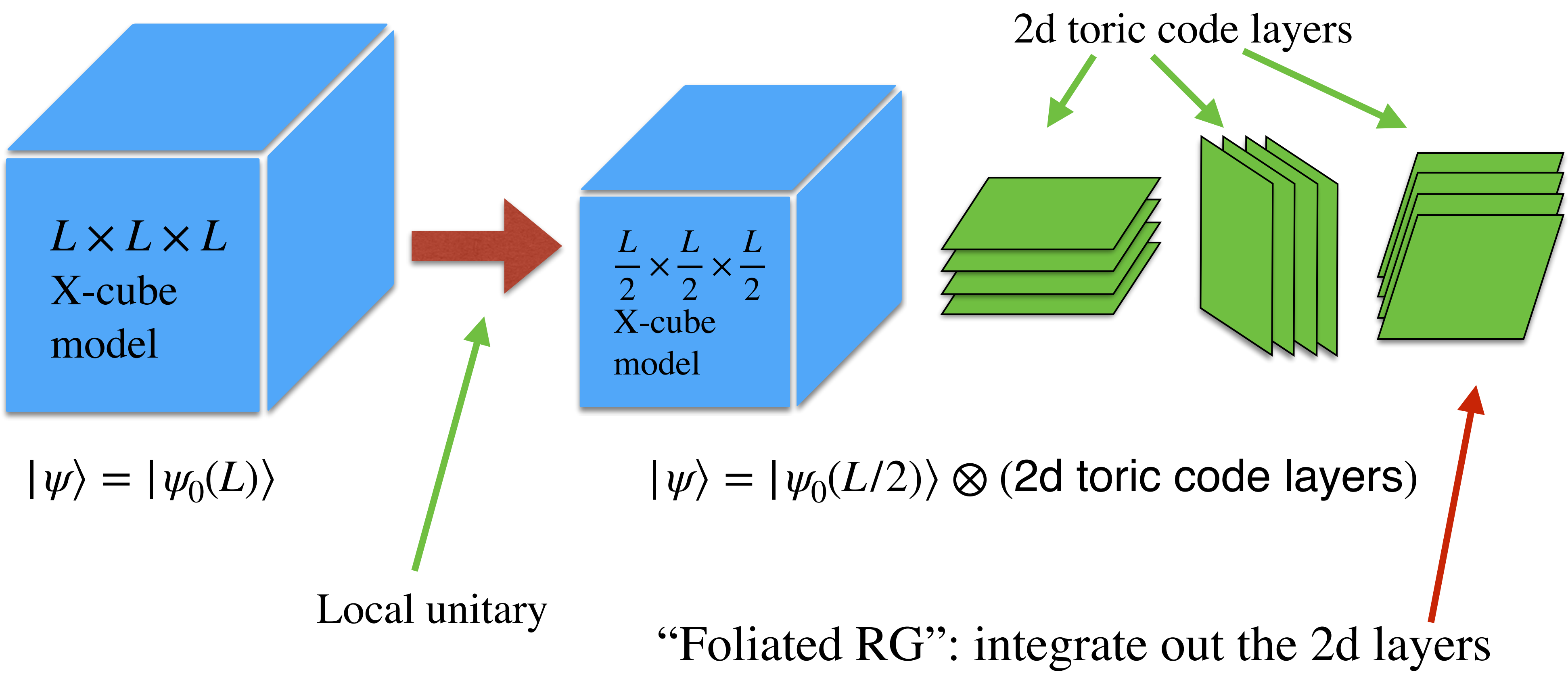
"Exotic" RG

How to define phases?

- Exotic RG is needed to make fracton models into RG fixed points

Exotic RG for X-cube model

Shirley, Slagle, Wang & Chen



- Foliated RG makes the X-cube model into a fixed point
- Leads to notion of foliated fracton phases:
  Treat 2d layers as trivial, *i.e.* $A \simeq A \otimes (2d\ layers)$

Exotic RG for Haah's cubic code

Haah

$$\text{Cubic code}(L) \cong \text{Cubic-code}(L/2) \otimes \text{Model-B}(L/2)$$

$$\text{Model-B}(L) \cong \text{Model-B}(L/2) \otimes \text{Model-B}(L/2)$$

- Exotic RG for Haah's code: integrate out model-B
  (Dua, Sarkar, Williamson & Cheng)
- But model-B is a fracton model with similar properties to Haah's code … does it make any sense to integrate it out? Is Haah's code an example of a phase with no useful continuum limit?

Summary of current state of the art

<u>Foliated fracton models (X-cube + many cousins)</u>

Foliated fracton phase

Continuum effective QFT?

Foliated RG (integrate out 2d layers)

Coarser equivalence than standard phases!

- Belief: standard fracton phases not associated with RG fixed points
- Some recent improvements/generalizations to foliated RG
  (Zongyuan Wang, Xiuqi Ma, David T. Stephen, MH & Xie Chen)
- Not clear whether corresponding picture for Haah's code (and other "type II" fracton models) makes sense

Rest of this talk: revise this picture

~~Foliated~~ fracton phase

Continuum effective QFT?

~~Foliated RG (integrate out 2d layers)~~

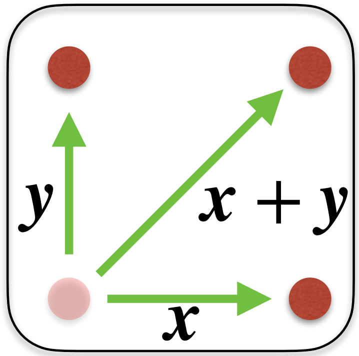
~~Coarser equivalence than standard phases!~~

Exotic RG (includes foliated RG)

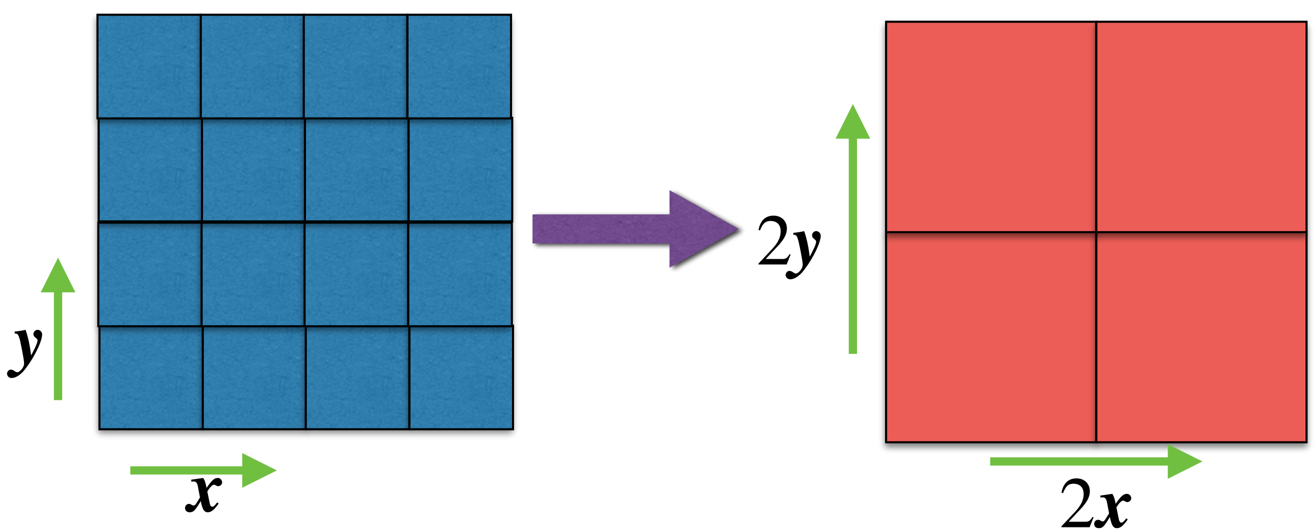
- With a minor twist on standard definition of phases, universal properties of fracton phases are RG-invariant (under exotic RG)
- Association between foliated fracton phases and foliated RG is actually not clear (but foliated phases still important as a higher level of structure)
- Revised picture applies to a large class of fracton phases, including Haah's code

Coarse translation symmetry

- Key property: mobility of excitations
- Assume and use translation symmetry as a tool to describe mobility (Haah; Pai & MH)



- Disadvantage: unwanted extra information having nothing to do with mobility (c.f. symmetry enriched topological phases)
- "Coarse translation:" compromise by allowing *limited* breaking of translation symmetry



Enlarge unit cell only by a finite amount

Coarse translation invariant (CTI) phases

- Equivalence relation on infinite, translation-invariant gapped quantum systems generated by certain operations

- Operations (glossing over many details):

    1. Continuous deformation (keeping gap open)

    2. Stacking with trivial product states

    3. "Forgetting" limited amount of translation symmetry

    4. "Rescaling": two systems differing only by a choice of lattice constant are considered equivalent

This operation is crucial and differs from some earlier work. Can be justified by lattice homotopy of Po, Watanabe, Jian & Zaletel.

- More technically: put degrees of freedom on sites of simple cubic lattice with its full translation symmetry, combine #3 and #4 into a single operation (group d.o.f. together within enlarged unit cell)

Apply to X-cube model

unit cell size

- Take the X-cube model and double the unit cell, then

$$\text{X-cube}(a) \simeq \text{X-cube}(a) \otimes \text{2d toric code layers}(a)$$

(means the systems on the left and right are in the same CTI phase)

- Under foliated RG… apply local unitary

$$\text{X-cube}(a) \cong \text{X-cube}(2a) \otimes \text{2d toric code layers}(2a)$$
$$\to \text{X-cube}(2a) \to \text{X-cube}(a)$$

integrate out rescale

- These systems are all in the same CTI phase, so all universal properties of the CTI X-cube phase are also RG-invariant

More general application to bifurcating models

- Suppose that upon enlarging the unit cell, a system A satisfies

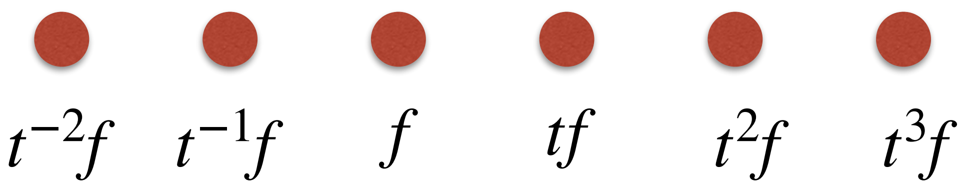
$$A \simeq A \otimes B$$

- RG procedure: integrate out B. A is a fixed point, and universal properties of its CTI phase are RG-invariant
- Applies to Haah's code, where B is model-B!
- Surprising — I didn't expect this RG do have anything to do with the universal properties of Haah's code.
- Question: can B be non-unique? Would imply distinct RG procedures. Universal properties of A invariant under all possible such RG's.

Properties of CTI phases

- What properties of CTI phases are universal? Can use translation symmetry, but must be robust to enlarging unit cell.

- Example: an excitation f has restricted mobility along some translation vector t if all the translates $t^n f$ are distinct excitations (not related by any local process)



- Precise definition of a fracton phase: at least one restricted mobility excitation

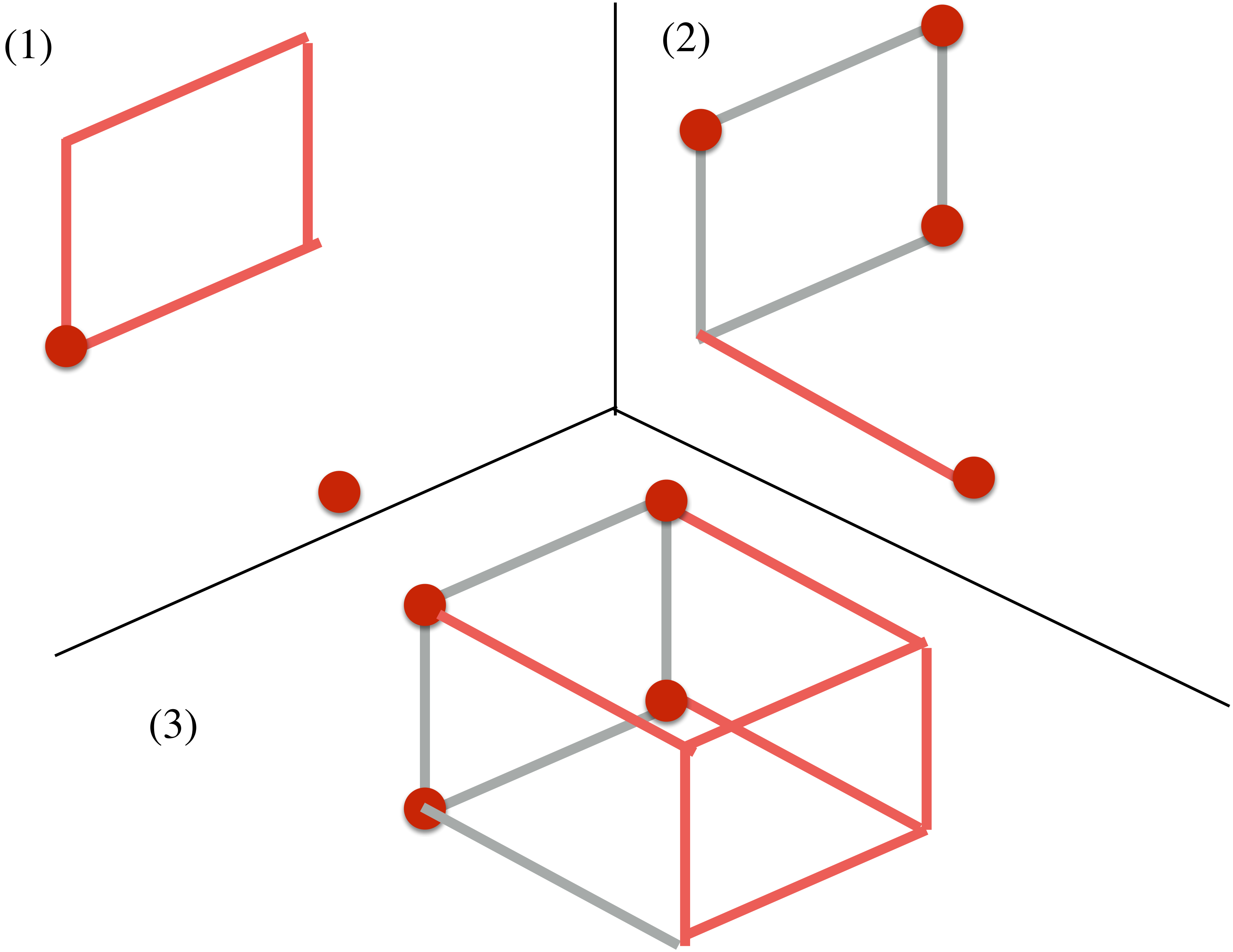
- Other examples include finer information about the mobility of excitations, and also their statistics (as studied by Pai & MH)

- Note: fractons can have self-exchange statistics (preprint to appear tonight with Hao Song, Nat Tantivasadakarn, Wilbur Shirley & MH)

Application to X-cube and semionic X-cube models

- This application actually dates to 2019! Seed of idea to use coarse translation invariance to characterize fracton phases comes from earlier work of Shriya Pai & MH.
- X-cube model can be constructed from coupled toric code layers (Ma, Lake, Chen, MH; Vijay)
- There is a variant “semionic X-cube model” constructed from doubled semion layers (Ma, Lake, Chen, MH)
- The X-cube and semionic X-cube models are in the same foliated fracton phase (Shirley, Slagle, Chen)
- However, they are in distinct CTI phases, distinguished by lineon-lineon exchange statistics (Shriya Pai, MH)

Lineon-lineon exchange process



Summary / outlook

- In fracton systems, universal properties of (essentially) standard phases are associated with fixed points of exotic renormalization groups
- I am optimistic that these universal properties can be captured nicely by continuum theories (coarse translations $\approx$ continuum translations?)
- Lots of work to do to understand coarse translation invariant universal properties and characterize fracton phases in terms of these
- Better understanding model-B seems to be crucial to understand universal properties of Haah's code