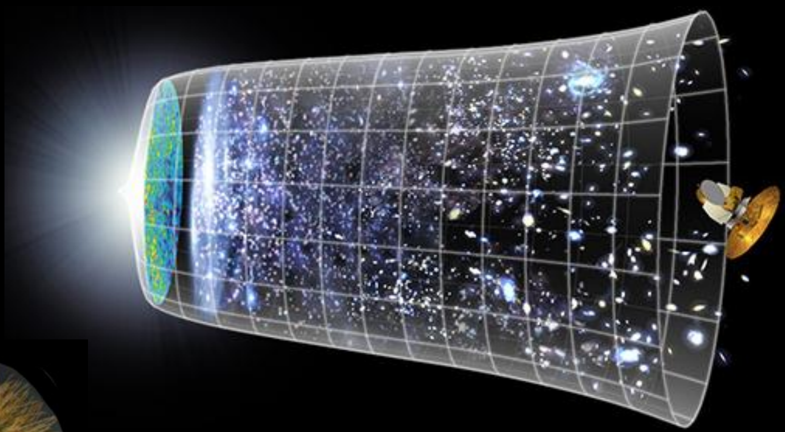
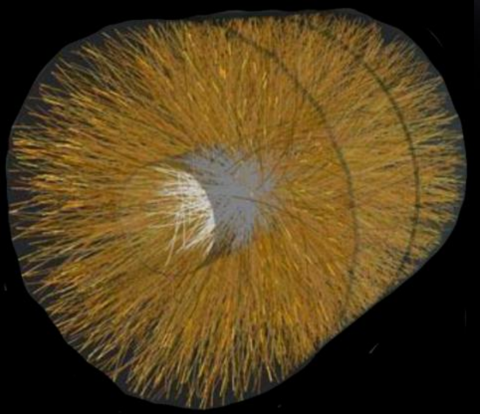
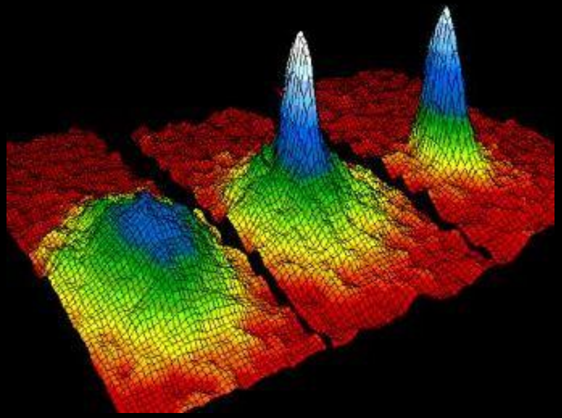
Nonthermal Fixed Points and Bose Condensation: From the Early Universe to Cold Atoms



WMAP Science Team



ALICE/CERN



JILA/NIST

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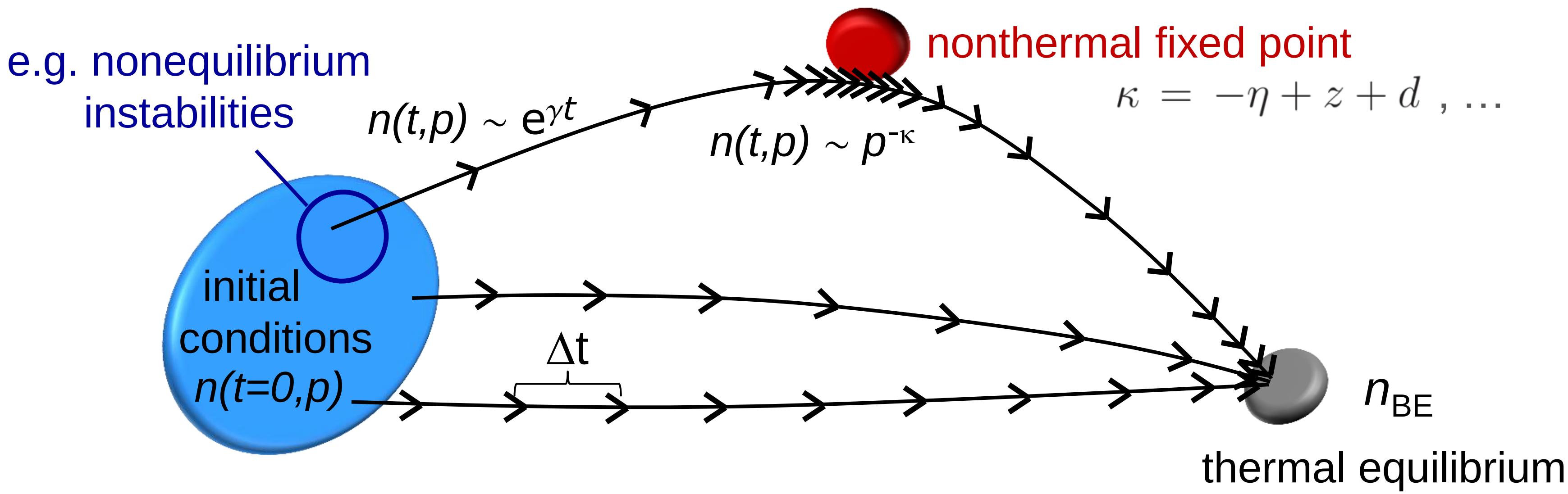
Universität Heidelberg

DYNAMICS AND THERMODYNAMICS IN ISOLATED QUANTUM SYSTEMS

KITP 2012

Thermalization in closed quantum systems



- **Characteristic nonequilibrium time scales? Relaxation? Instabilities?**

  → Prethermalization

  Berges, Borsanyi, Wetterich PRL (2004),
  Moeckel, Kehrein, PRL (2008), …

- **Diverging time scales far from equilibrium? Universality?**

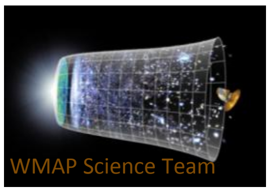
  → Nonthermal fixed points

  Berges, Rothkopf, Schmidt PRL (2008), …
  Nowak, Sexty, Gasenzer, PRB (2011), …

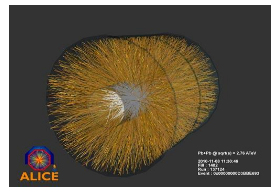
  Berges, Sexty, PRL (2012)
  Nowak, Gasenzer arXiv:1206.3181

Universality far from equilibrium

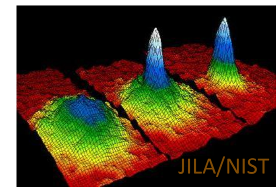
Early-universe preheating
(~10^{16} GeV)



Heavy-ion collisions
(~100 MeV)



Cold quantum gas dynamics
(~10^{-13} eV)



Instabilities, `overpopulation´, …

Nonthermal fixed points

Very different microscopic dynamics can lead to
same *macroscopic scaling phenomena*

Nonthermal RG fixed points

RG: 'microscope' with varying resolution of length scale

~ $1/k$

Fixed point: physics looks the same for 'all' resolutions (in rescaled units)

scaling form, e.g. $F_k = \frac{1}{2}\langle\{\Phi,\Phi\}\rangle_k \sim \frac{1}{k^{2+\kappa}}$ ⟵ 'occupation number' exponent

similarly, retarded propagator: $G_k^{\mathrm{R}} \sim \frac{1}{k^{2-\eta}}$ ⟵ anomalous dimension

Hierarchy of possible infrared fixed points:

$\omega(k) \sim k^z$ ⟵ dynamic exponent

- vacuum: $\kappa = -\eta$
- thermal: $\kappa = -\eta + z$
- *nonequilibrium*: $\kappa = -\eta + z + d$, ...

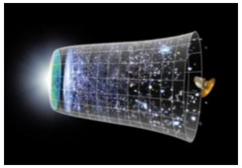
↓ *increasing complexity*

Berges, Hoffmeister, Nucl. Phys. B 813 (2009) 383

Example: heating the universe after inflation

- Chaotic inflation

Kofman, Linde, Starobinsky, PRL 73 (1994) 3195



$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\mu^2\chi^2 + \frac{\lambda}{4}\phi^4 + \frac{g}{2}\phi^2\chi^2$$

$$\phi \gg m/\sqrt{\lambda} \quad , \quad \phi_0 \sim M_{\rm P} \quad , \quad \lambda \lesssim 10^{-12} \quad , \quad g^2 \lesssim \lambda$$

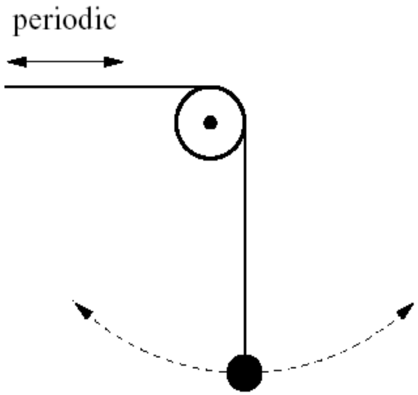
massless preheating: $m = \mu = 0$, conformally equiv. to Minkowski space

$$\frac{d^2\chi_k}{dt^2} + (k^2 + g\phi^2(t))\chi_k = 0$$

Classical oscillator analogue (parametric resonance):

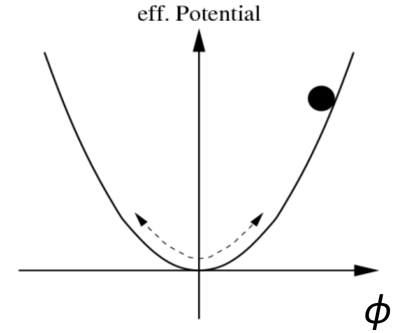
$\omega(t) \leftrightarrow \phi(t)$,
$x(t) \leftrightarrow \chi_{k=0}(t)$

$$\ddot{x} + \omega^2(t)x = 0$$



Nonequilibrium quantum field theory

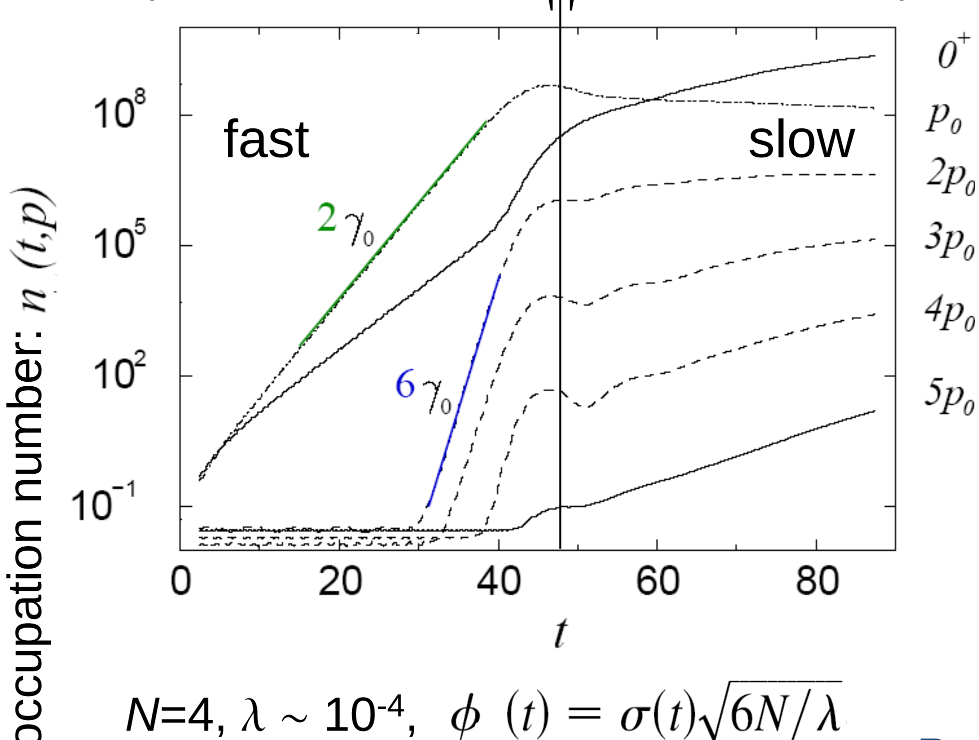
Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603

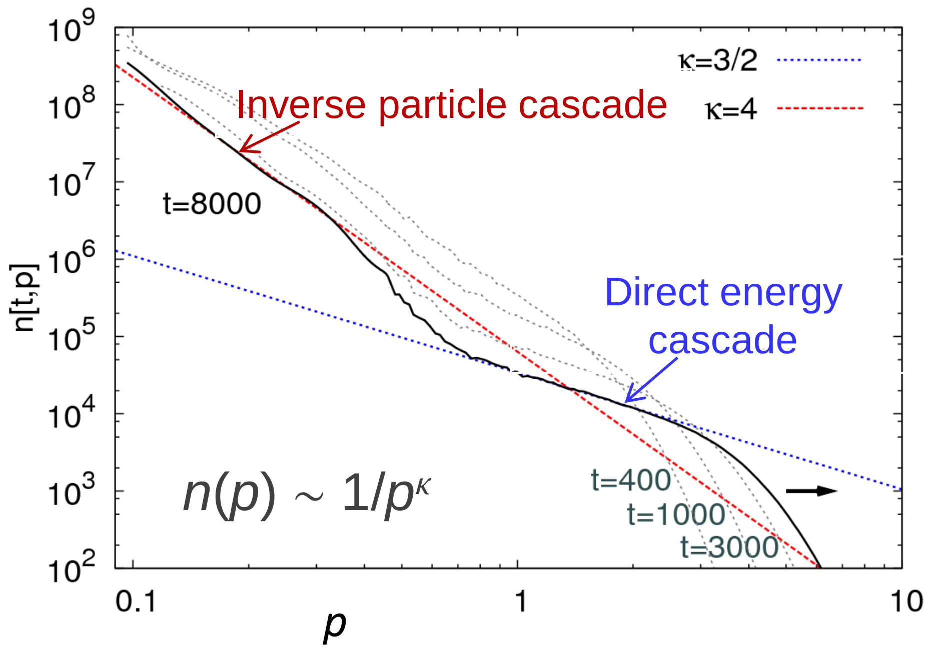


Generalize to N fields (2PI 1/N to NLO):

$$\Phi(t,p) = (\ \phi(t,p),\ \chi_1(t,p),\ \chi_2(t,p),\ldots,\ \chi_{N-1}(t,p)\ )$$

parametric resonance ⬌ *nonthermal fixed point*

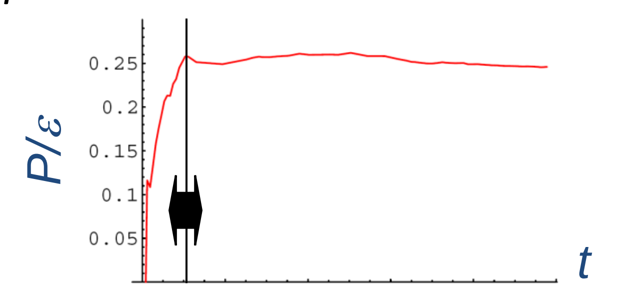




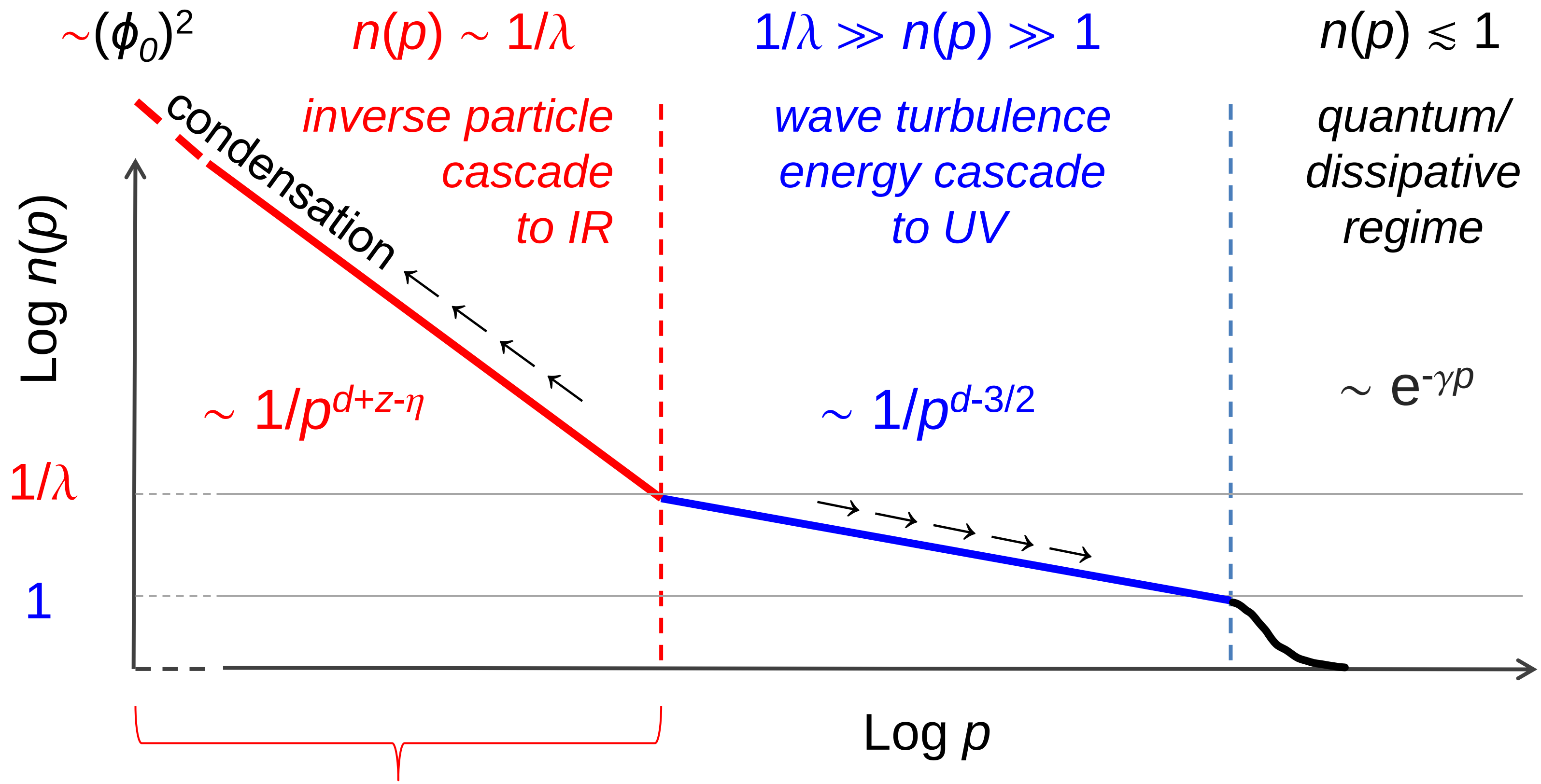
N=4, $\lambda \sim 10^{-4}$, $\phi(t) = \sigma(t)\sqrt{6N/\lambda}$
in units of $\sigma(t=0)$, method:

Berges, Serreau, PRL 91 (2003) 111601

Prethermalized equation of state:



Stationary transport of conserved charges



Dynamically generated conserved particle number! (absent in thermal equilibrium)

here relativistic, $z \approx 1$, $\eta \approx 0$

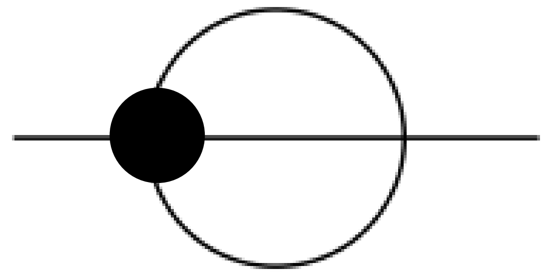
→ *'Conserved' quantities lead to universal scaling solutions far from equilibrium*

From complexity to simplicity

Complexity: many-body $n\leftrightarrow m$ processes for $n,m = 1,..,\infty$
as important as $2\leftrightarrow2$ scattering (`overpopulation´)!

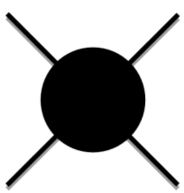
Simplicity: Resummation of the infinitely many processes leads to *effective kinetic theory* (2PI 1/*N* to NLO) dominated in the IR by

$$\frac{\partial n_{\mathbf{p}}(t)}{\partial t} = C[n](t;\mathbf{p}) \sim$$

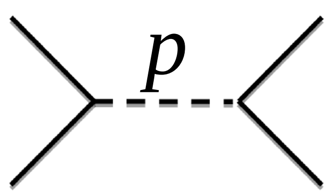


$$\overset{!}{=} 0$$

FP describes $\sharp$-*conserving* $2 \leftrightarrow 2$ scattering with *effective coupling:*



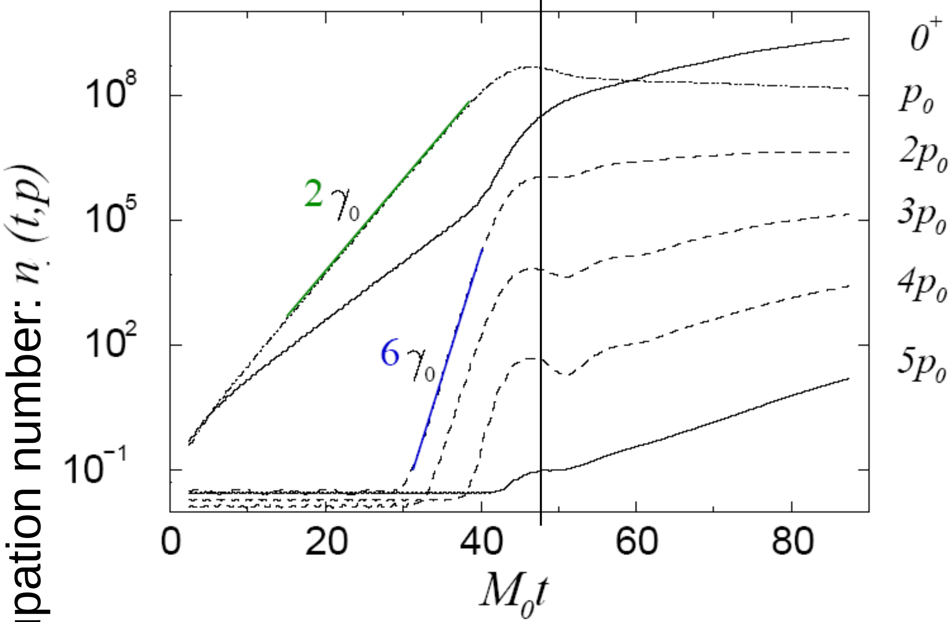
$=$

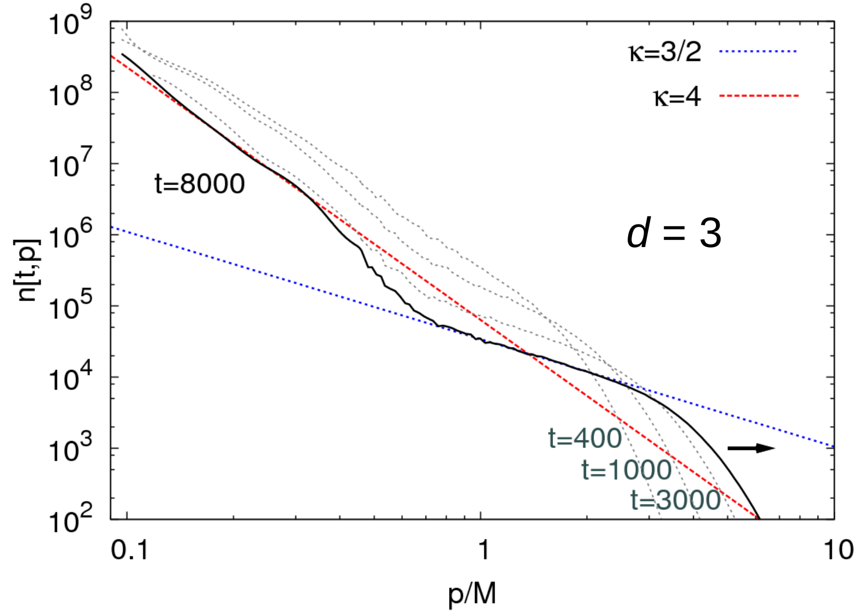


$$\sim \; p^{8\text{-}4\eta}$$

Dependence on spatial dimension *d*

parametric resonance ⟺ nonthermal fixed point:



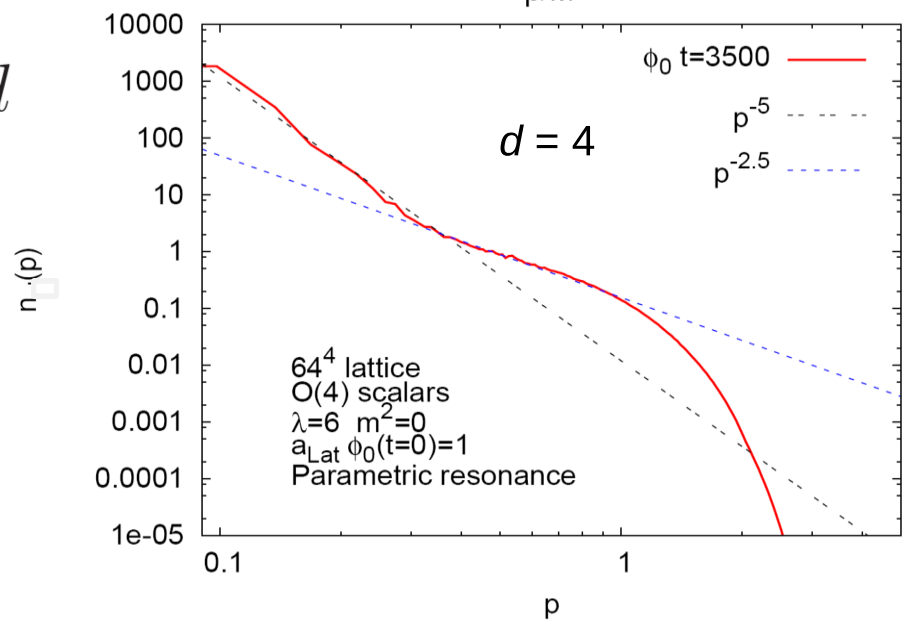


$n(t,p) \sim p^{-\kappa}$ with $\kappa = -\eta + z + d$

→ κ = 4 for *d* = 3,
κ = 5 for *d* = 4 ✓

IR

for *z* = 1 (relativistic), $\eta = 0$



Berges, Sexty, PRD 83 (2011) 085004

Bose condensation from overpopulation

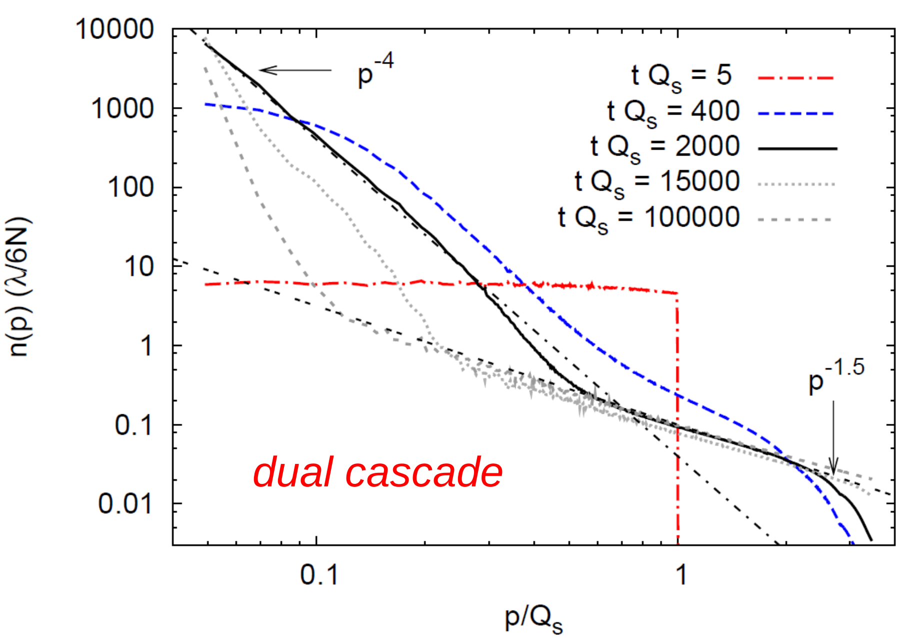
$$F(t,t';\vec{x}-\vec{y}) = \left\langle \left\{ \hat{\phi}(t,\vec{x}), \hat{\phi}(t',\vec{y}) \right\} \right\rangle$$

time-dependent condensate

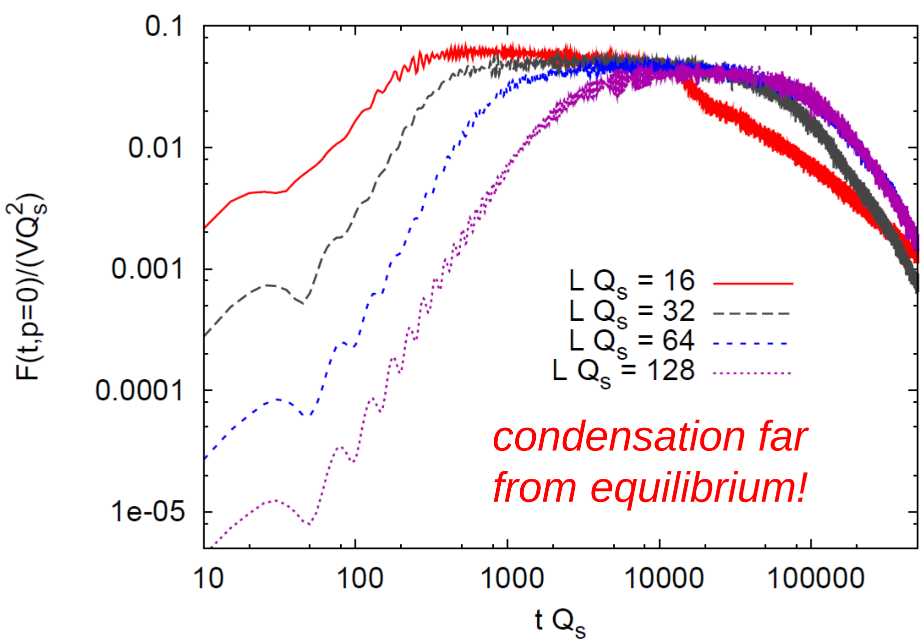
$$F(t,t;p) = \frac{1}{\omega_p(t)} \left(n_p(t) + \frac{1}{2} \right) + (2\pi)^d \delta^{(d)}(\vec{p})\, \phi_0^2(t)$$ (relativistic)

starting from initial `overpopulation´:

finite volume: $(2\pi)^d \delta^{(d)}(0) \rightarrow V$

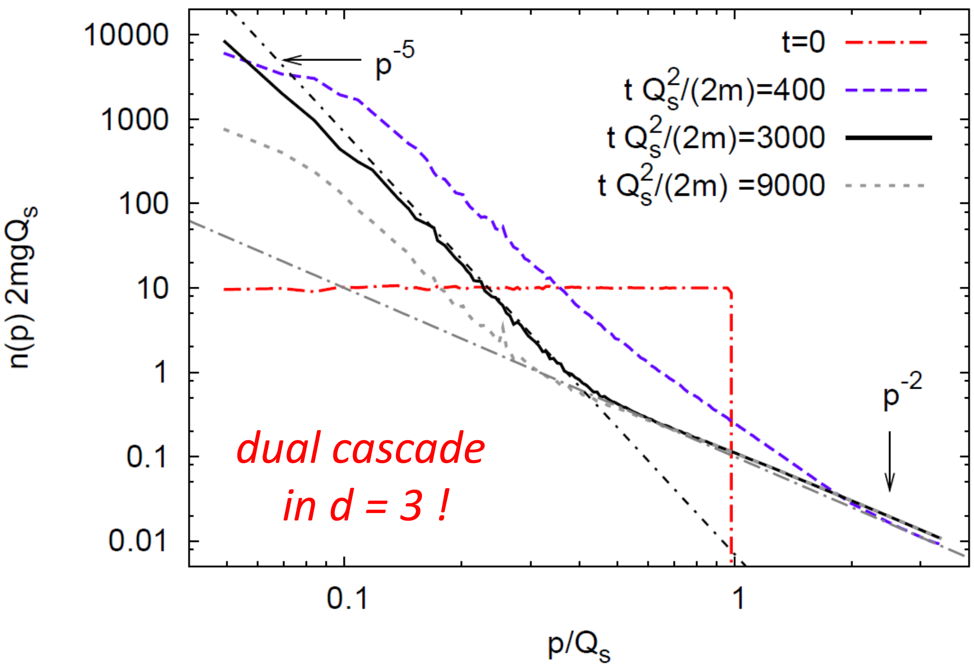


No initial condensate!



Berges, Sexty, PRL 108 (2012) 161601

Comparison to cold Bose gas (Gross-Pitaevskii)



Berges, Sexty, PRL 108 (2012) 161601

Infrared particle cascade leads to Bose condensation without subsequent decay!

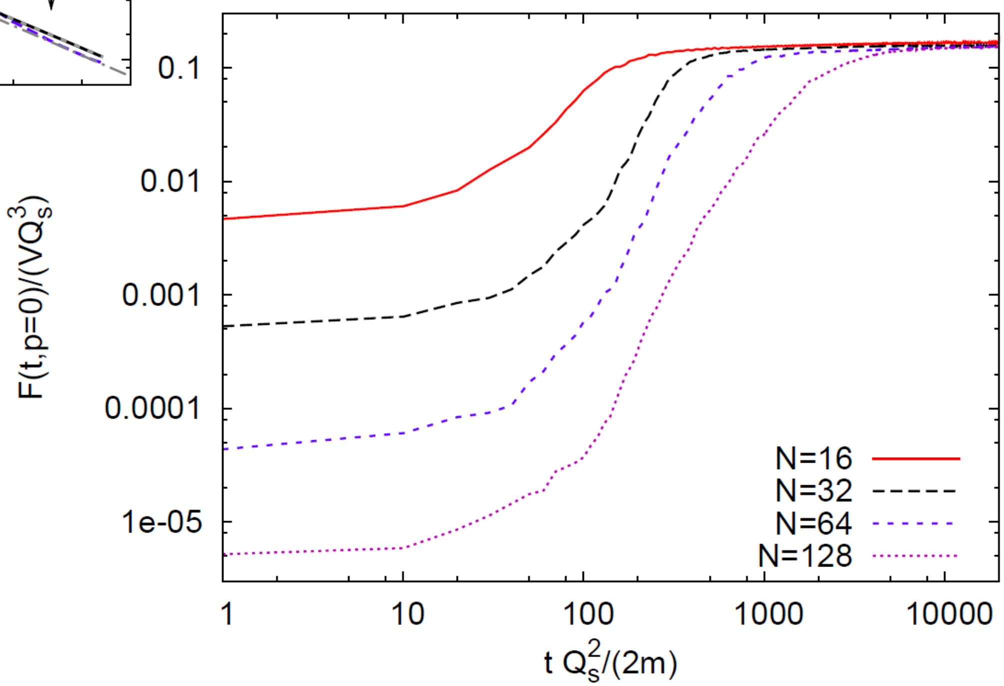
(no number changing processes)

Expected infrared cascade:

$$n(p) \sim 1/p^{d+2-\eta}$$

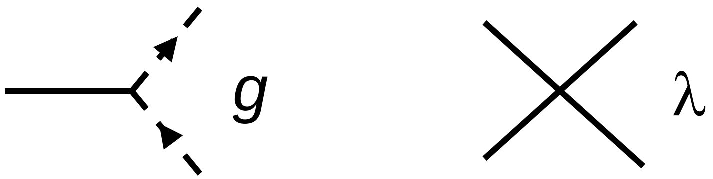
for non-relativistic dynamics

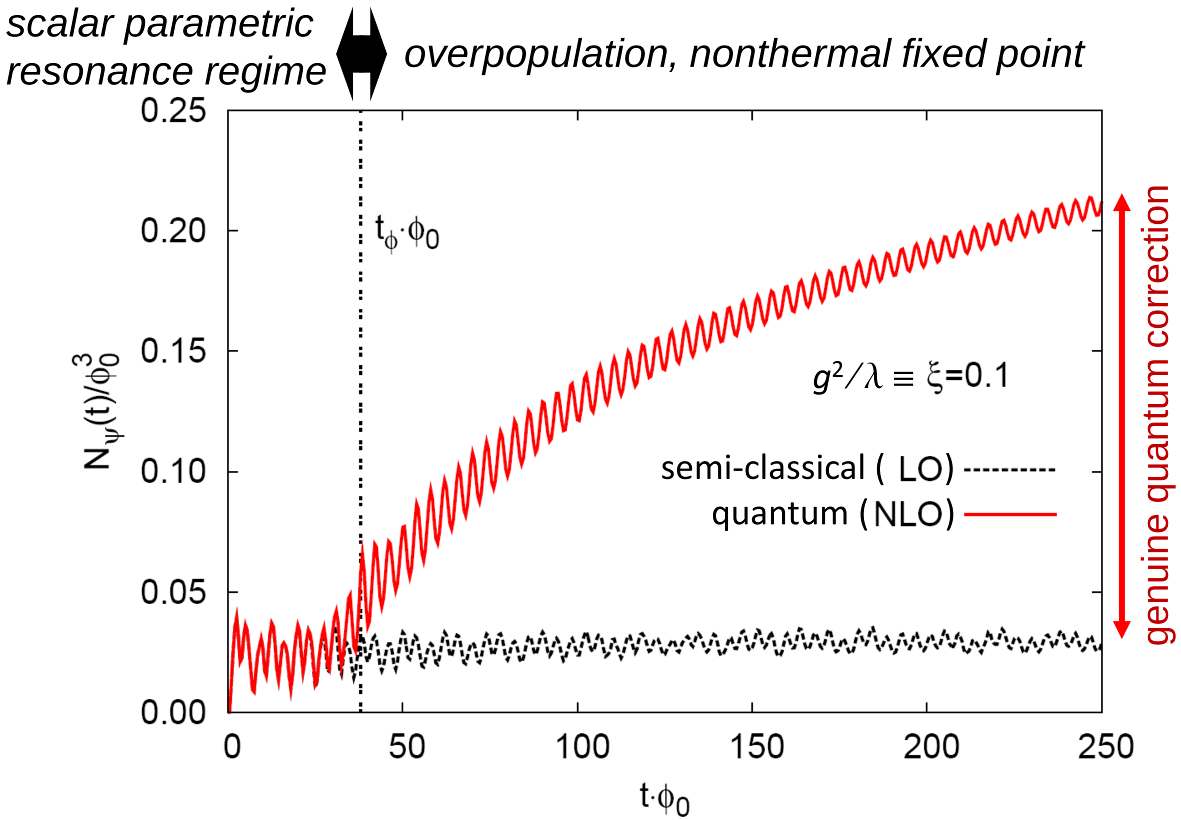
Scheppach, Berges, Gasenzer, PRA 81 (2010) 033611; Nowak, Sexty, Gasenzer, PRB 84 (2011) 020506(R); Nowak, Gasenzer arXiv:1206.3181



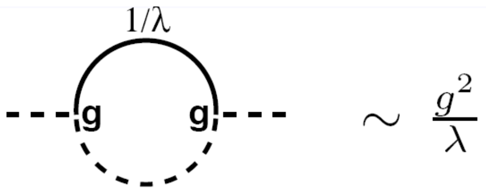
Nonthermal fixed points as quantum amplifier

Decay into (Dirac) fermions:





2PI-NLO:



Berges, Gelfand, Pruschke
PRL 107 (2011) 061301

Nonequilibrium fermion spectral function

$$\rho(x,y) = i\,\langle\{\psi(x),\bar{\psi}(y)\}\rangle$$

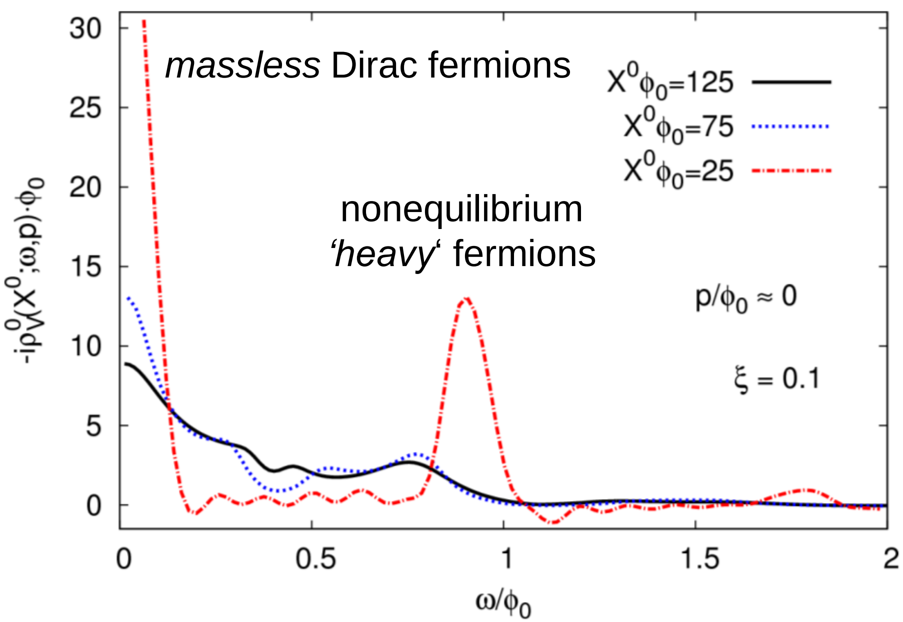
$$\rho_V^\mu = \frac{1}{4}\mathrm{tr}\,(\gamma^\mu \rho)$$ vector components

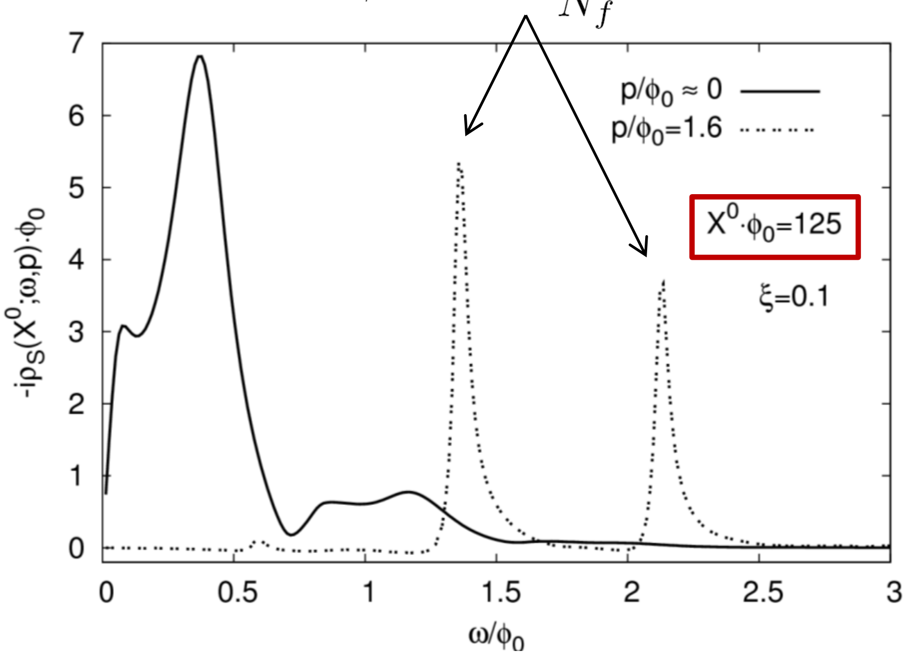
$$\rho_S = \frac{1}{4}\mathrm{tr}\,(\rho)$$ scalar component

quantum field anti-commutation relation: $-i\rho_V^0(t,t;\mathbf{p}) = 1$

Wigner transform: $(X^0 = (t + t')/2)$

$$M_\psi^{\mathrm{eff}}(t) \simeq \pm\frac{g}{N_f}|\phi(t)|$$



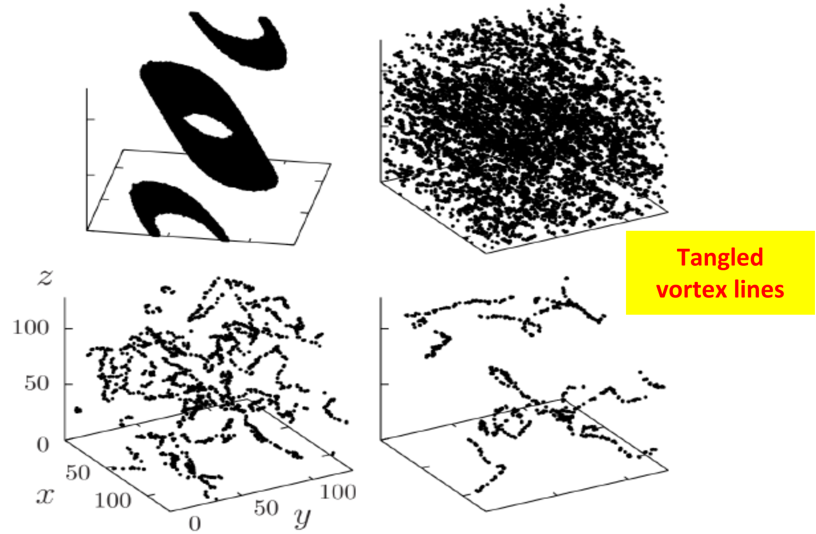


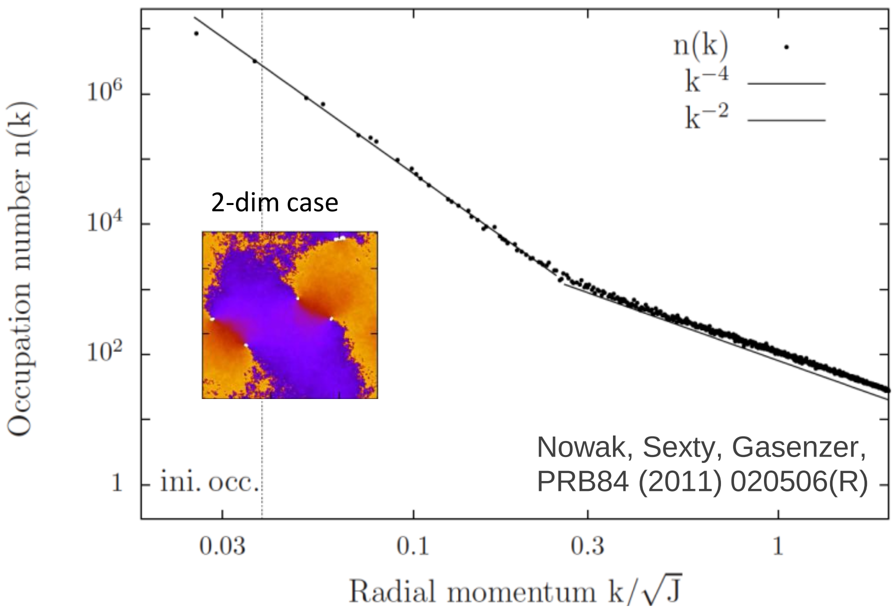
Universality far from equilibrium!

$$\lim_{p \to 0} n(p) \sim \frac{1}{p^4}$$

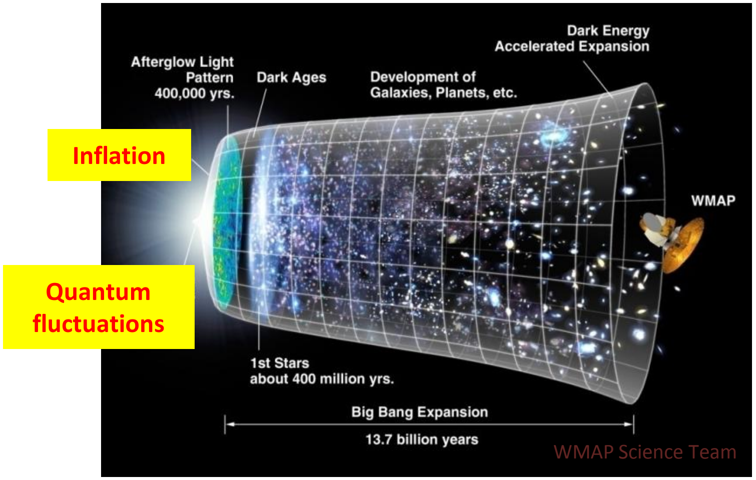
universal scaling exponents etc.

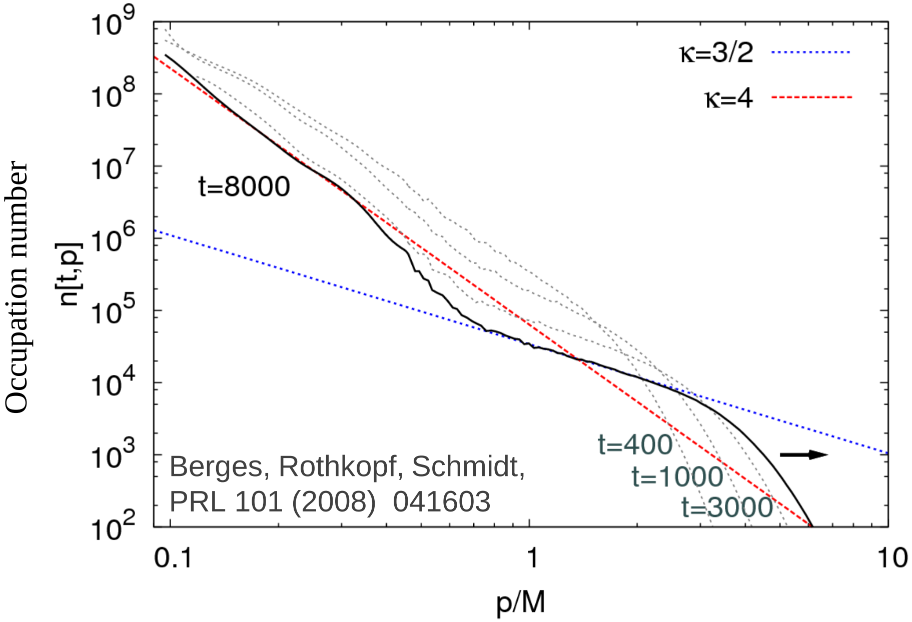
- Superfluid turbulence in a cold Bose gas





- Reheating dynamics after chaotic inflation





Conclusions

Nonthermal fixed points / universality far from equilibrium:

- crucial for thermalization process from instabilities/overpopulation!

- strongly nonlinear regime of stationary transport (*dual* cascade)!

- Bose condensation from inverse particle cascade!

- large amplification of quantum corrections for fermions!

- nonabelian gauge theory results indicate same wave turbulence exponents (Bose condensation!?) as for scalars!

Turbulence/Bose condensation for gluons?

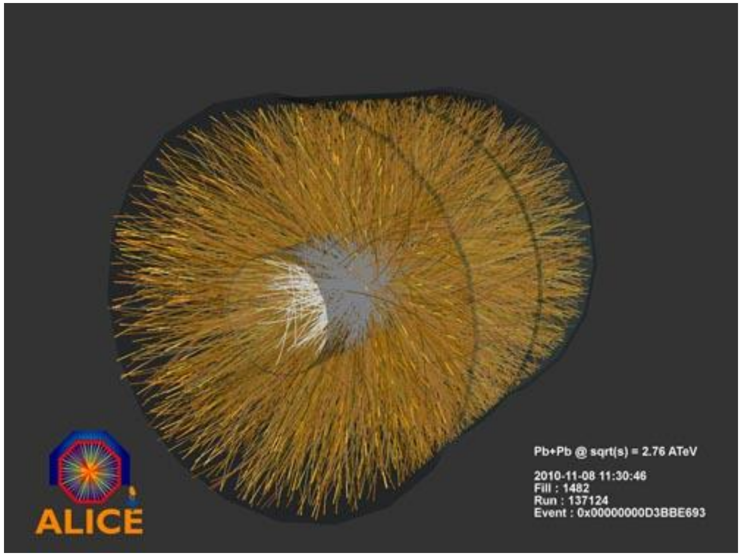
Field strength tensor, here for *SU*(2):

$$F^a_{\mu\nu}[A] \,=\, \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

Equation of motion:

$$(D_\mu[A]F^{\mu\nu}[A])^a \,=\, 0$$

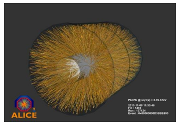
$$D^{ab}_\mu[A] \,=\, \partial_\mu \delta^{ab} + g\epsilon^{acb} A^c_\mu$$



Classical-statistical simulations accurate for sufficiently large fields/high gluon occupation numbers:

anti-commutators $\langle\{A, A\}\rangle \gg \langle[A, A]\rangle$ commutators

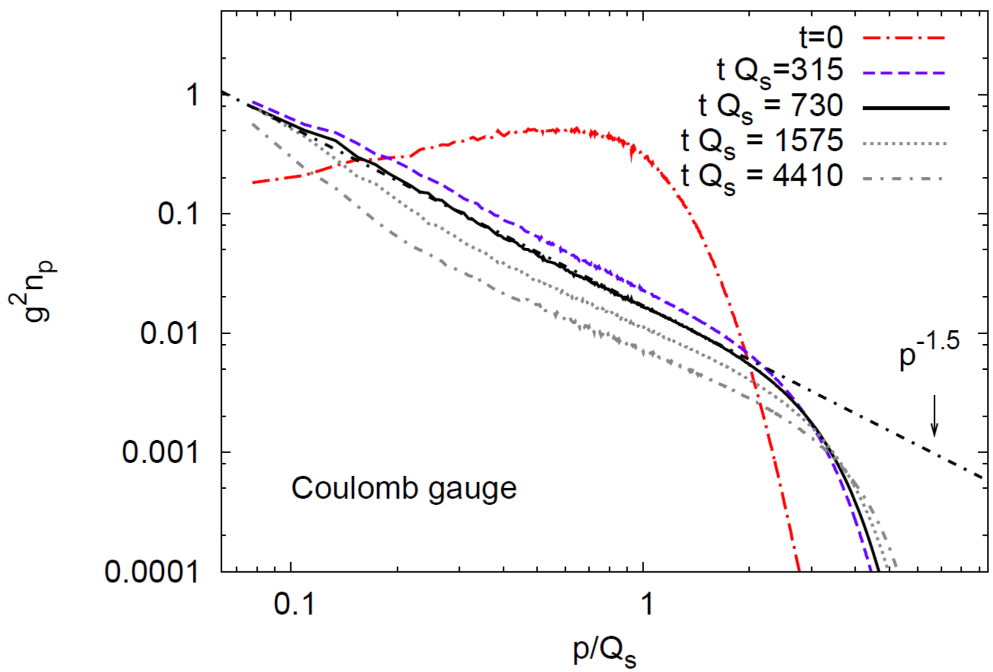
i.e. "n(p)" $\gg$ 1



Nonabelian gauge theory

Occupancy: $\sim \sqrt{\langle |A^2(p)| \rangle \langle |E^2(p)| \rangle}$

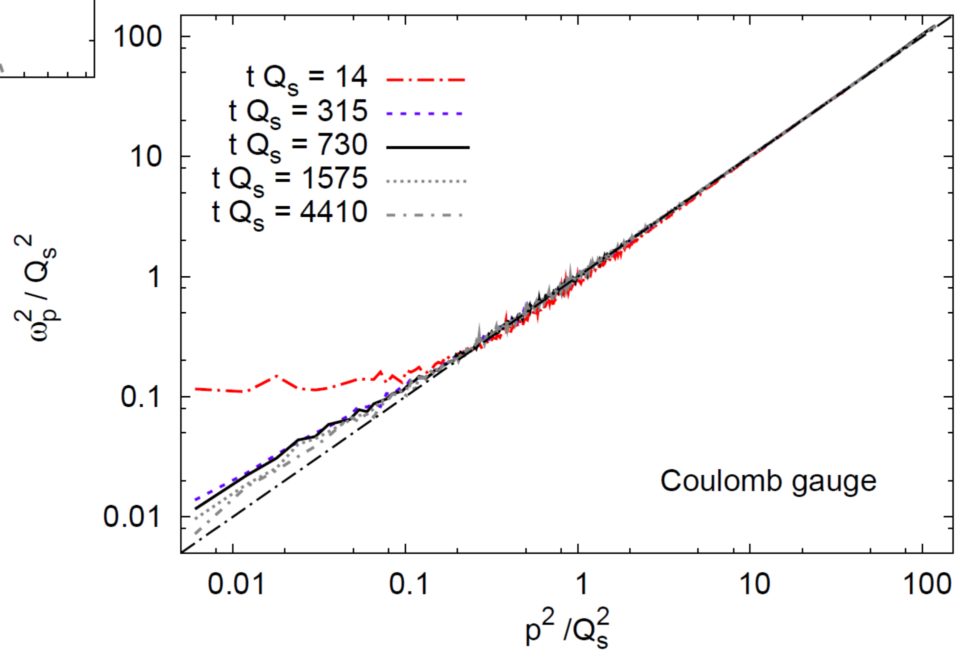
Berges, Schlichting, Sexty, arXiv:1203.4646



Initial overpopulation:

$$\epsilon \sim \frac{Q_s^4}{g^2} \quad \text{i.e.} \quad n(p \simeq Q_s) \sim \frac{1}{g^2}$$

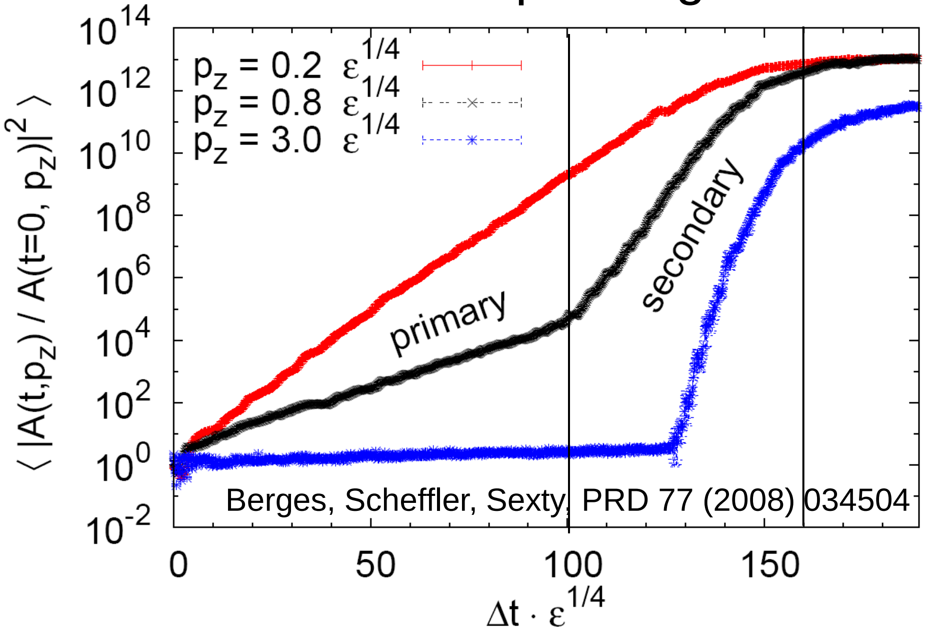
Dispersion: $\sim \sqrt{\langle |E^2(p)| \rangle / \langle |A^2(p)| \rangle}$



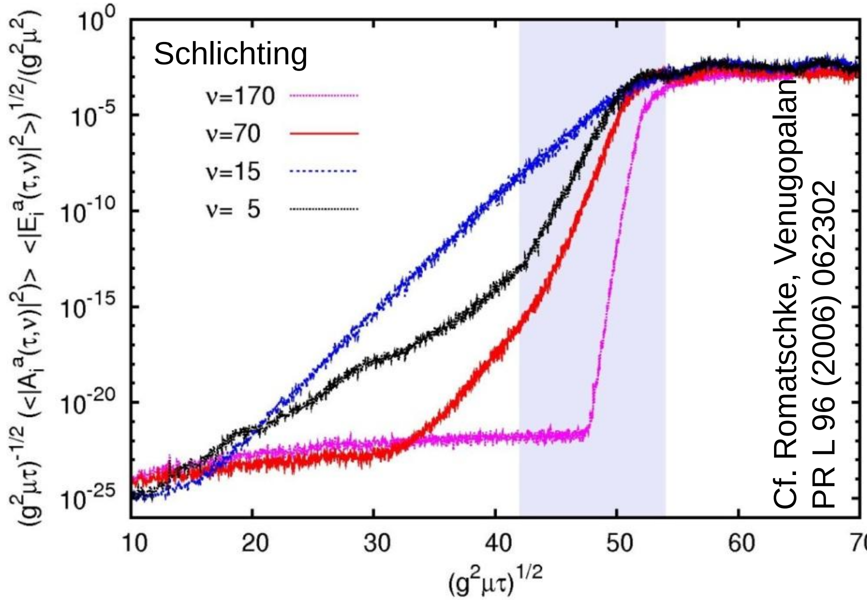
- Wave turbulence exponent 3/2
(as for scalars with condensate)!?

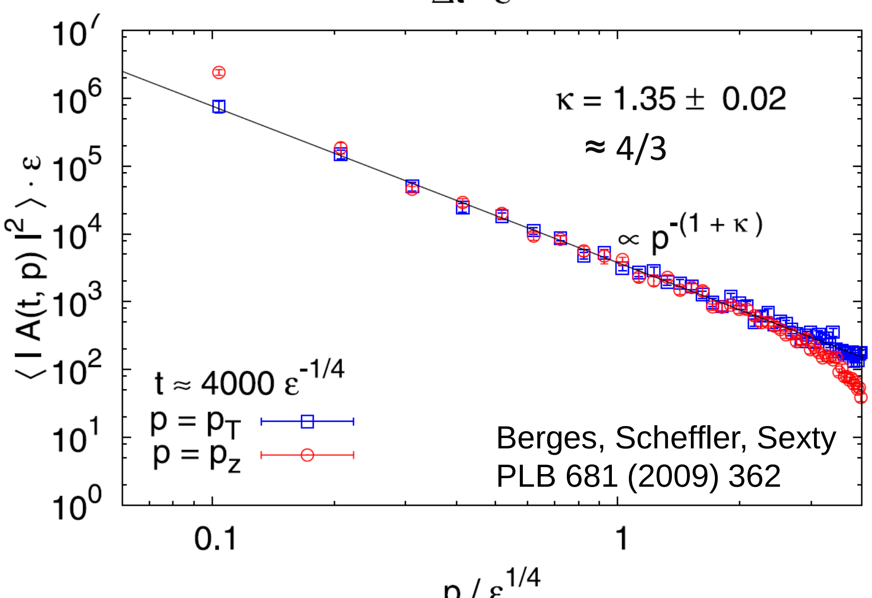
Turbulence from plasma instabilities: Lattice

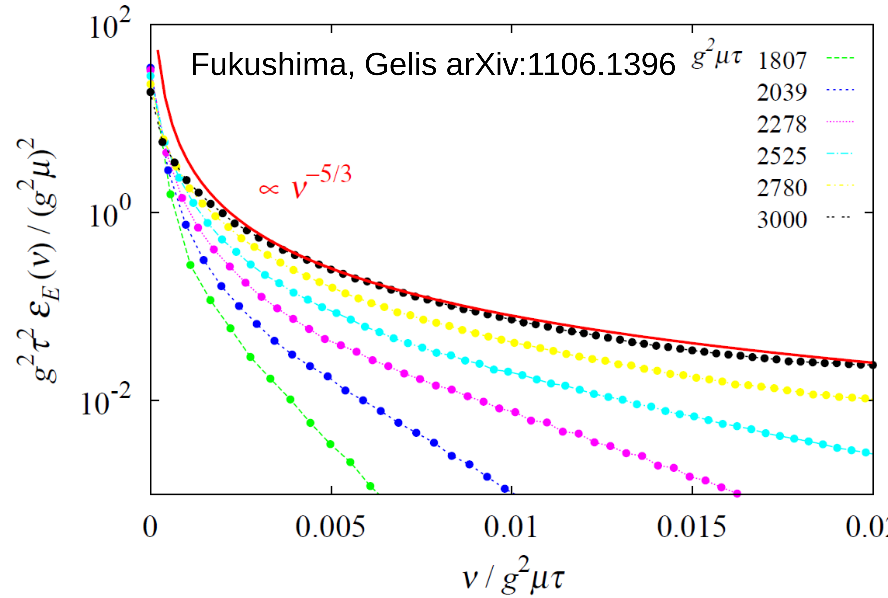
Non-expanding



With longitudinal expansion







Lattice simulations with dynamical fermions

Consider general class of models including lattice gauge theories

$$\mathcal{L} = \frac{1}{2}\partial\Phi^*\partial\Phi - V(\Phi) + \sum_k^{N_f}\left[i\bar\Psi_k\gamma^\mu\partial_\mu\Psi_k - \bar\Psi_k(MP_L + M^*P_R)\Psi_k\right]$$

$M \to m - g\Phi(x)$, $P_L = \frac{1}{2}(1-\gamma^5)$, $P_R = \frac{1}{2}(1+\gamma^5)$

$$\int\prod_k D\Psi_k^+ D\Psi_k e^{i\int\mathcal{L}(\Phi,\Psi^+,\Psi)} \Longrightarrow \boxed{\partial_x^2\Phi(x) + V'(\Phi(x)) + N_f J(x) = 0}$$

$$J(x) = J^{\mathrm{S}}(x) + J^{\mathrm{PS}}(x)$$

$$J^{\mathrm{S}}(x) = -g\left\langle\bar\Psi(x)\Psi(x)\right\rangle = g\mathrm{Tr}\,D(x,x)\,,$$
$$J^{\mathrm{PS}}(x) = -g\left\langle\bar\Psi(x)\gamma^5\Psi(x)\right\rangle = g\mathrm{Tr}\,D(x,x)\gamma^5$$

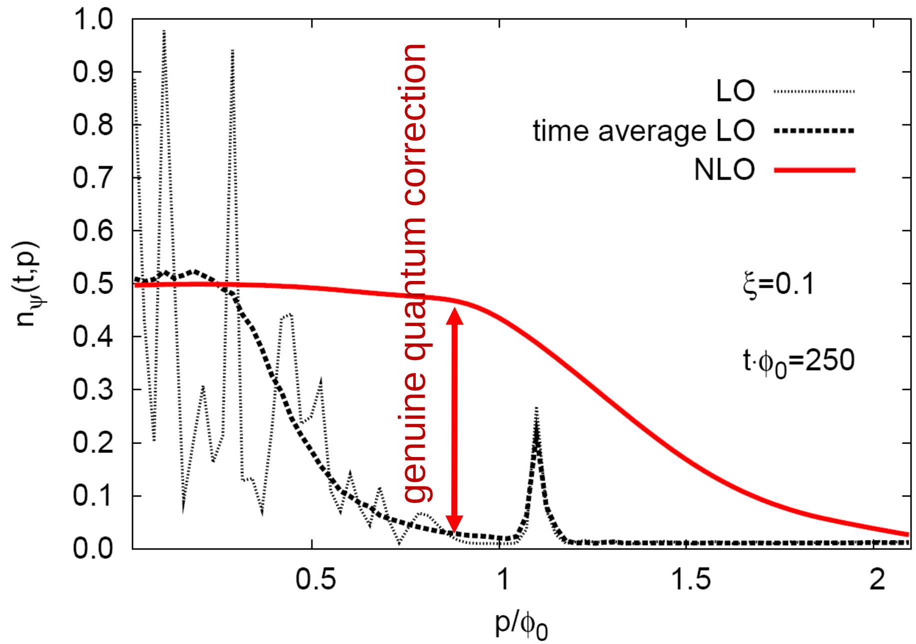
For classical $\Phi(x)$ the exact equation for the fermion $D(x,y)$ reads:

$$\boxed{(i\gamma^\mu\partial_{x,\mu} - m + g\mathrm{Re}\,\Phi(x) - ig\mathrm{Im}\,\Phi(x)\gamma^5)D(x,y) = 0}$$

Very costly (4 4 N^3 N^3)! Use low-cost fermions of Borsanyi & Hindmarsh

Aarts, Smit, NPB 555 (1999) 355 PRD 79 (2009) 065010

Occupation number distributions



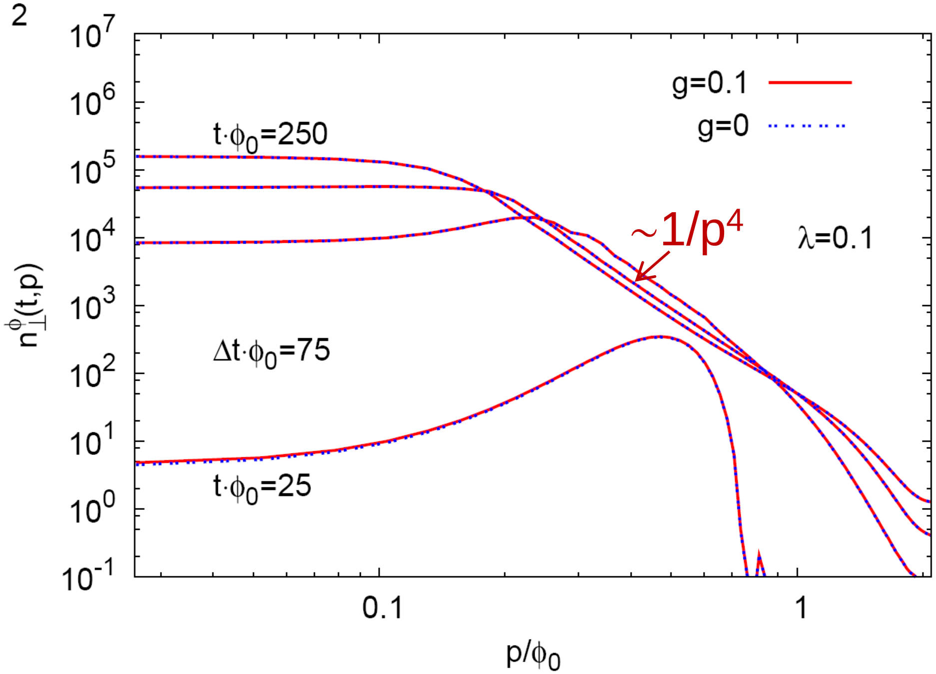
Fermions

(2PI-NLO)

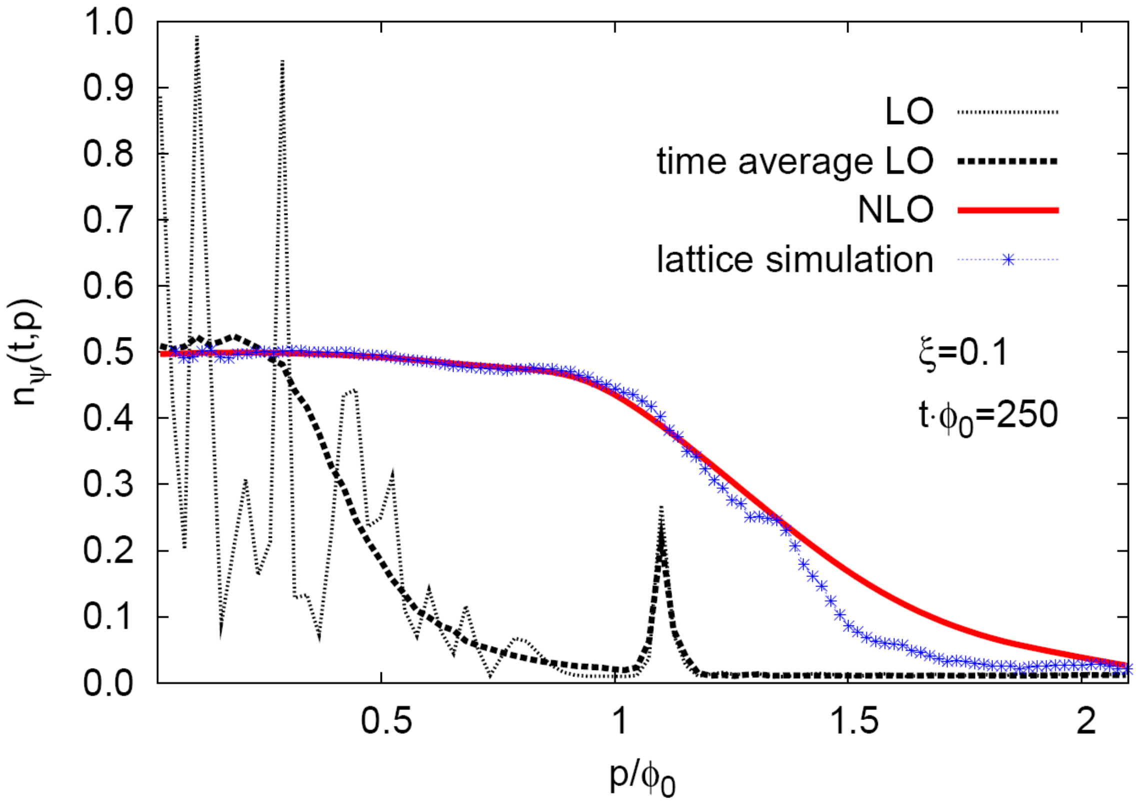
IR fermions thermally occupied

Bosons

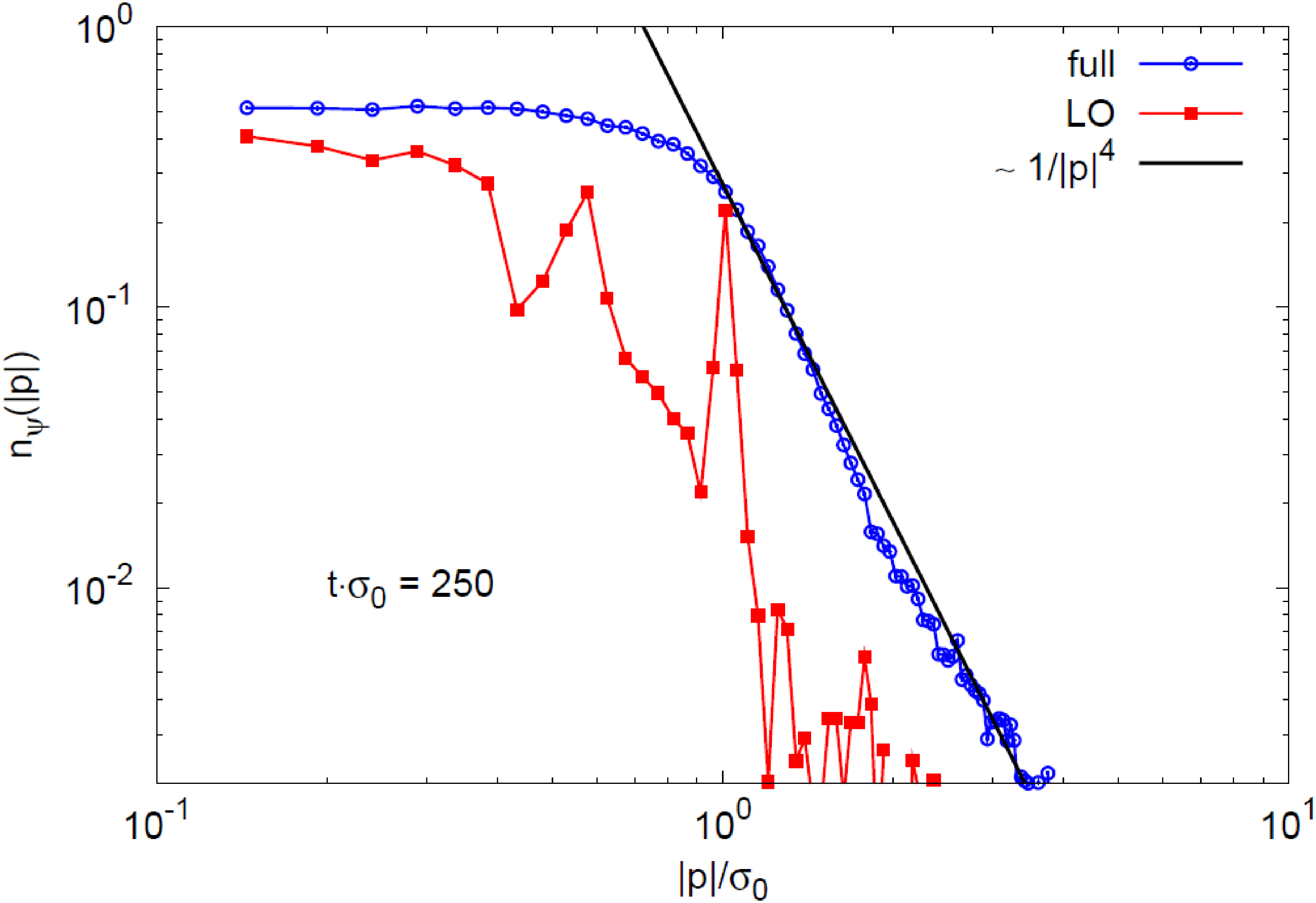
Bosons still far from equilibrium



Real-time dynamical fermions in 3+1 dimensions!



- Wilson fermions on a 64^3 lattice   Berges, Gelfand, Pruschke, PRL 107 (2011) 061301
- Very good agreement with NLO quantum result (2PI) for $\xi \ll 1$
  (differences at larger *p* depend on Wilson term → larger lattices)
- Lattice simulation can be applied to strongly correlated regime $\xi \sim 1$!



full
LO
~ 1/|p|^4
t·σ_0 = 250
n_ψ(|p|)
|p|/σ_0
10^0
10^-1
10^-2
10^1

Digression: weak wave turbulence

Boltzmann equation for *relativistic* 2↔2 scattering, $n_1 \equiv n(t,p_1)$:

http://www.aeiou.at/aeiou.encyclop.b/b638771.htm

$$\frac{\mathrm{d}n_1}{\mathrm{d}t} = \int \frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2} \int \frac{\mathrm{d}^3p_3}{(2\pi)^3 2E_3} \int \frac{\mathrm{d}^3p_4}{(2\pi)^3 2E_4}$$

$$\times \quad \delta^3(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)(2\pi)^4 |M|^2$$

momentum conservation — energy conservation — scattering

$$\times \Big(n_3 n_4 (1 + n_1)(1 + n_2) - n_1 n_2 (1 + n_3)(1 + n_4) \Big)$$

"gain" term — "loss" term

Different stationary solutions, $\mathrm{d}n_1/\mathrm{d}t=0$, in the (classical) regime $n(p) \gg 1$:

1. $n(p) = 1/(e^{\beta\omega(p)} - 1)$ thermal equilibrium
2. $n(p) \sim 1/p^{4/3}$ turbulent *particle* cascade
3. $n(p) \sim 1/p^{5/3}$ *energy* cascade

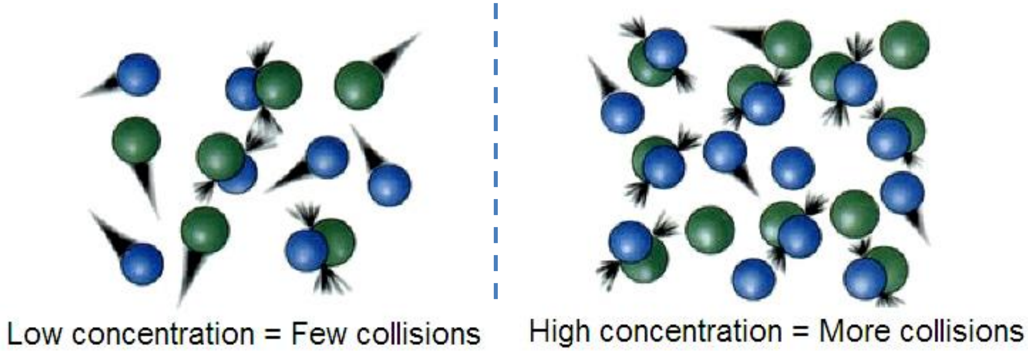
(2 and 3:) *Kolmogorov -Zakharov spectrum*

…associated to stationary transport of conserved quantities

Range of validity of Kolmogorov-Zakharov

E.g. self-interacting scalars with quartic coupling: $|M|^2 \sim \lambda^2 \ll 1$

$n(p) \lesssim 1$ | $1 \ll n(p) \ll 1/\lambda$ | $n(p) \sim 1/\lambda$



`overpopulation´
(non-perturbative)

?

analytically well described by 2PI effective action techniques!

Very high concentration = ?

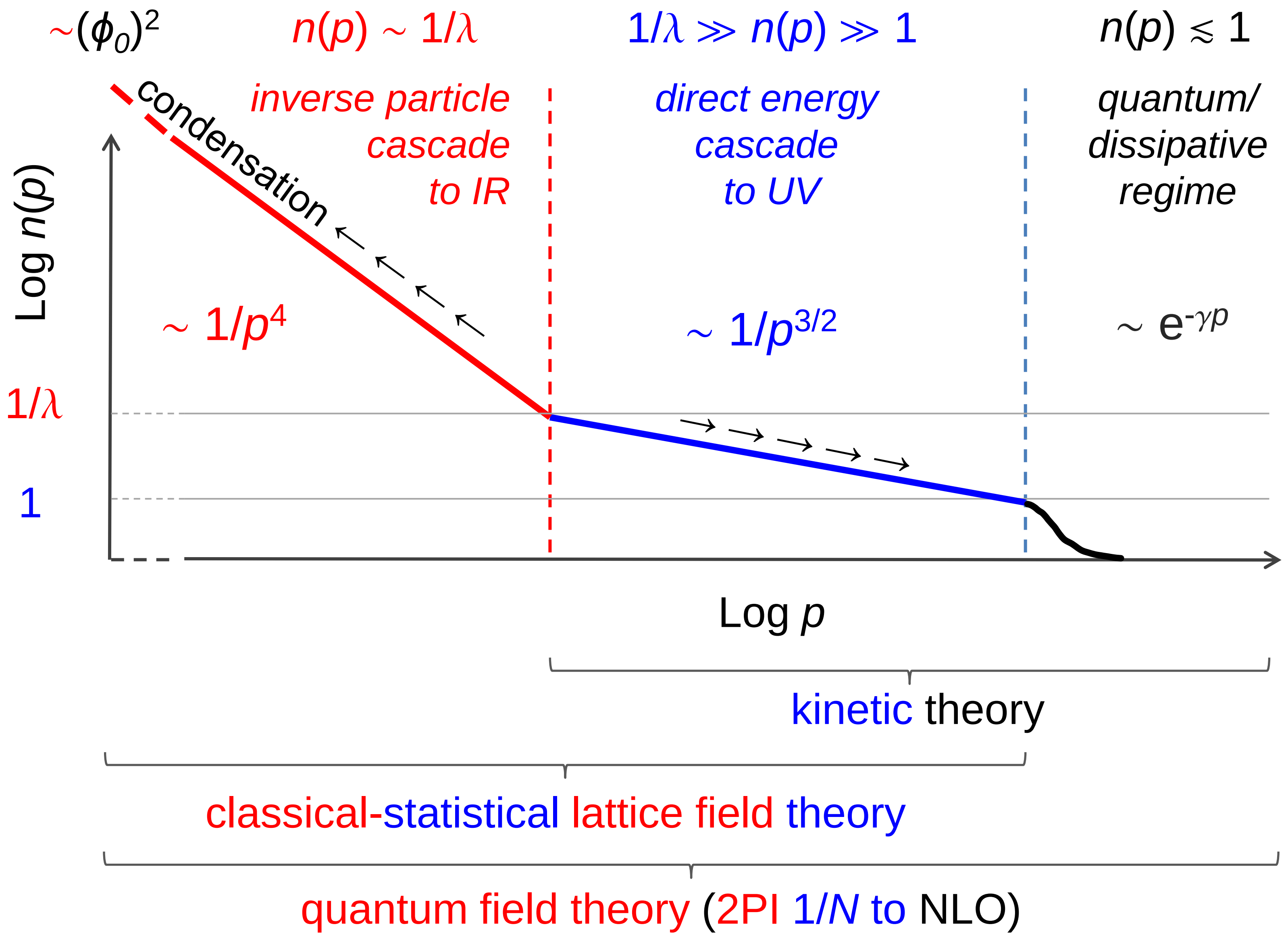
http://upload.wikimedia.org/wikipedia/commons/4/41/Molecular-collisions.jpg

Weak wave turbulence solutions are limited to the "window"

$1 \ll n(p) \ll 1/\lambda$, since for

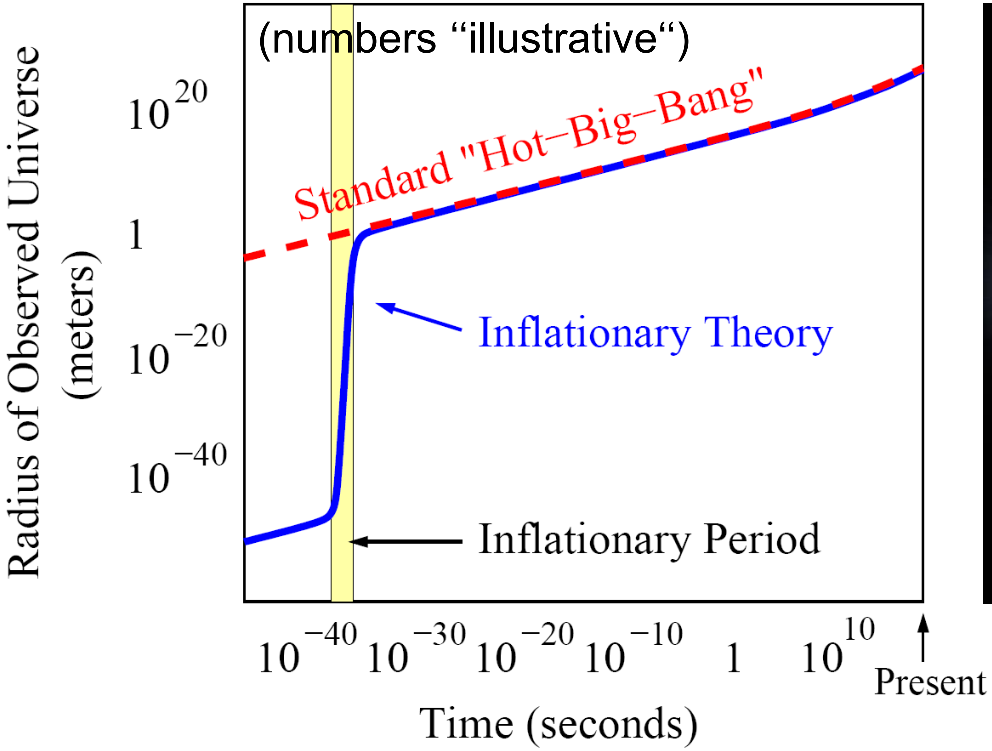
$n(p) \sim 1/\lambda$ the n↔m scatterings for n,m=1,..,∞ are as important as 2↔2 **!**

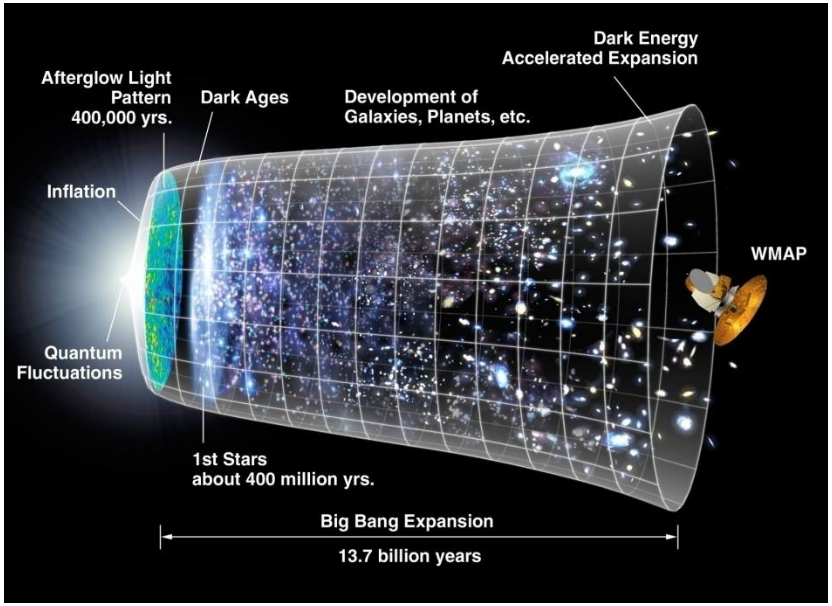
Methods


~(ϕ0)2
n(p) ~ 1/λ
1/λ ≫ n(p) ≫ 1
n(p) ≲ 1
condensation
inverse particle cascade to IR
direct energy cascade to UV
quantum/ dissipative regime
Log n(p)
~ 1/p4
~ 1/p3/2
~ e-γp
1/λ
1
Log p
kinetic theory
classical-statistical lattice field theory
quantum field theory (2PI 1/N to NLO)

Heating the Universe after inflation: a quantum example

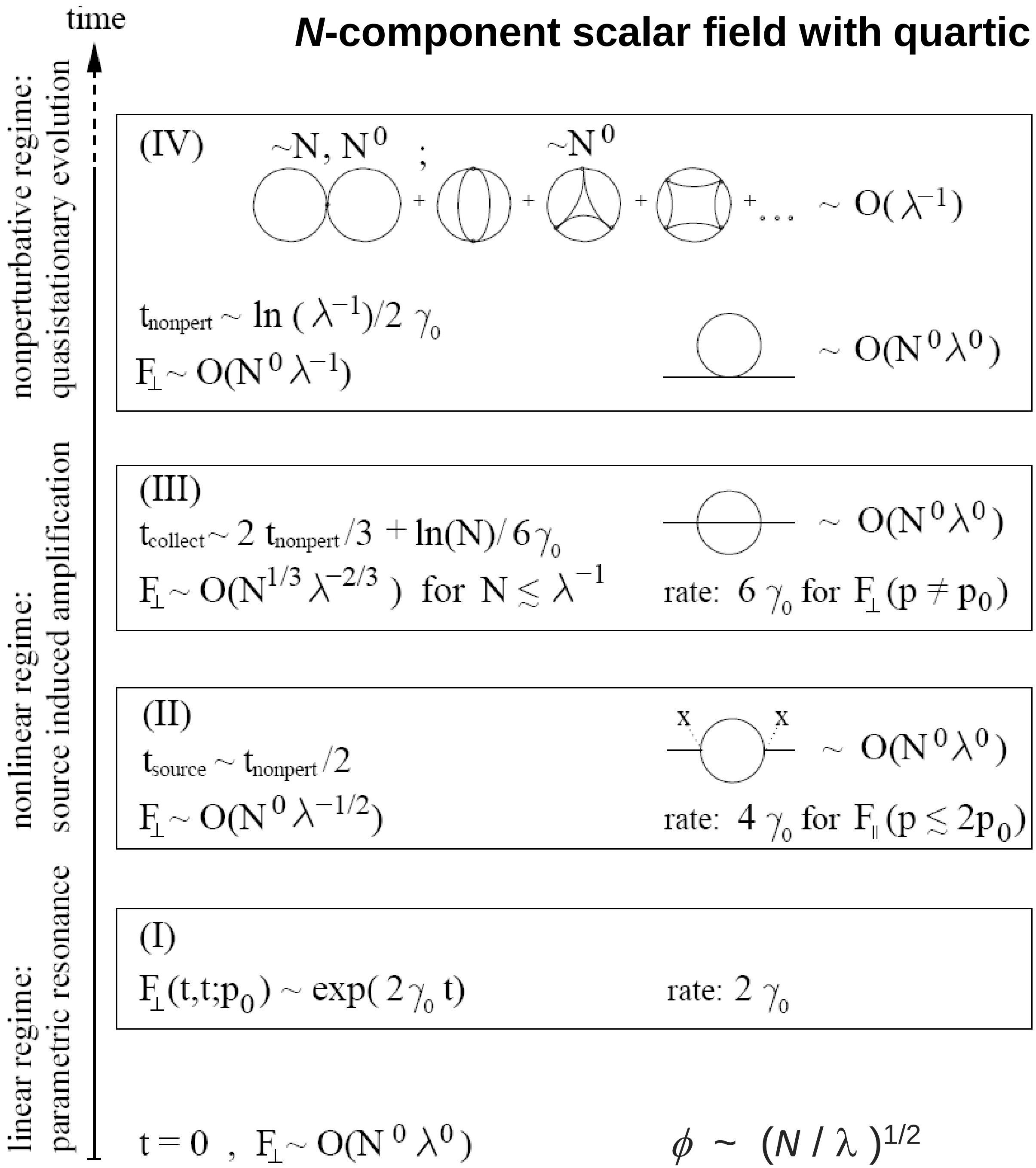
Schematic evolution:





- Energy density of matter ($\sim a^{-3}$) and radiation ($\sim a^{-4}$) decreases
- Enormous heating after inflation to get 'hot-big-bang' cosmology!

N-component scalar field with quartic self-coupling λ



slow

Nonperturbative: saturated occupation numbers $\sim 1/\lambda$

$\rightarrow$ *all processes* $O(1)$

$\rightarrow$ *universal*

fast

Nonlinear – perturbative: occupation numbers $< 1/\lambda$

secondary growth rates $c\,(2\gamma_0)$ with $c = 2,3,\ldots$

Classical/linear: primary growth rate