Quantum Quench in Conformal Field Theory from a General Short-Ranged State

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(Global) Quantum Quench

- prepare an extended system (in thermodynamic limit) at time $t = 0$ in a (translationally invariant) pure state $|\psi_0\rangle$ – e.g. the ground state of some hamiltonian H_0
- evolve unitarily with a hamiltonian H for which $|\psi_0\rangle$ is not an eigenstate and has extensive energy above the ground state of H
- how do correlation functions of local observables, and quantum entanglement of subsystems, evolve as a function of t?
- for a compact subsystem do they become stationary?
- if so, what is the stationary state?
- is the reduced density matrix thermal?

Quantum quench in a 1+1-dimensional CFT

- P. Calabrese + JC [2006,2007] studied this problem in 1+1 dimensions when $H = H_{\mathrm{CFT}}$ and $|\psi_0\rangle$ is a state with short-range correlations and entanglement
- H_{CFT} describes the universal low-energy, large-distance properties of many gapless 1d systems
- 1+1-dimensional CFT is exactly solvable, so we can get analytic results for interacting systems – however, these turn out to depend only on general properties of any CFT

Results

- there is a parameter τ_0 characterising $|\psi_0\rangle$ such that:
- one-point functions of local quantities in general decay towards their ground state values:

  $\langle\Phi(x,t)\rangle \sim e^{-\pi\Delta_\Phi t/2\tau_0}$ where Δ_Φ is the scaling dimension of Φ

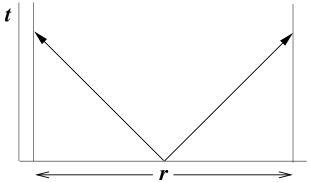
- for times $t > |x_1 - x_2|/2\text{v}$, the correlation functions become stationary and decay exponentially:

  $$\langle\Phi(x_1,t_1)\Phi(x_2,t_2)\rangle \sim e^{-\pi\Delta_\Phi|x_1-x_2|/2\text{v}\tau_0}$$

  for $t_1 = t_2$, and $\sim e^{-\pi\Delta_\Phi|t_1-t_2|/2\tau_0}$ for $x_1 = x_2$
- the (conserved) energy density is $\pi c/6(4\tau_0)^2$
- the von Neumann entropy of a region of length ℓ saturates for $t > \ell/2\text{v}$ at

  $$S \sim (\pi c/3(4\tau_0))\ell$$

- all these results are *precisely* those expected for the same CFT at temperature $T = (4\tau_0)^{-1}$
- an extreme example of thermalisation!
  - no time average necessary
  - it happens after a finite time $t \sim \ell/2\mathrm{v}$
- results accord with a simple physical picture of entangled pairs of quasiparticles emitted from correlated regions



- this picture has more general applicability (Lieb-Robinson)

Quantum quenches in integrable models

- however studies of quenches in integrable models [(Rigol,Dunjko,Yurovsky,Olshanii),...,(Calabrese,Essler,Fagotti)] have led to the conclusion that the steady state should be a 'generalised Gibbs ensemble' (GGE) with a separate 'temperature' conjugate to each local conserved quantity
- 1+1-dimensional CFT is super-integrable: e.g. all powers $T(z)^j$ and $\overline{T}(\bar{z})^{\bar{j}}$ of the stress tensor (and its derivatives) correspond to local conserved currents, leading to conserved charges
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- so why did CC find a simple Gibbs ensemble?
- this can be traced to a simplifying assumption about the form of the initial state
- what is the effect of relaxing this assumption?

Review of CC [2006,2007]

- we want to compute

$$\langle\psi_0|e^{itH_{\rm CFT}}\,\mathcal{O}\,e^{-itH_{\rm CFT}}|\psi_0\rangle$$

- we could get this from imaginary time by considering

$$\langle\psi_0|e^{-\tau_2 H_{\rm CFT}}\,\mathcal{O}\,e^{-\tau_1 H_{\rm CFT}}|\psi_0\rangle$$

  and continuing $\tau_1 \to it$, $\tau_2 \to -it$
- 'slab' geometry with boundary condition $\equiv \psi_0$, but thickness $\tau_1 + \tau_2 = 0$ ☹

Resolution, updated

- in general we can write any translationally invariant state with short-range correlations and entanglement in the form

$$|\psi_0\rangle \propto e^{-\sum_j \lambda_j \int \phi_j^{(b)}(x)dx}\,|B\rangle$$

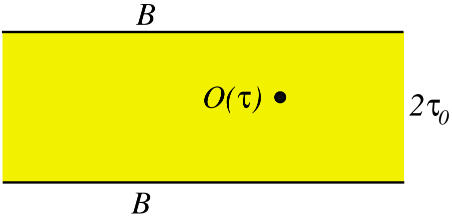
  where $|B\rangle$ is an 'ideal' state (e.g. a product state) which corresponds to a fixed point of the boundary RG, and $\phi_j^{(b)}$ are all possible (irrelevant) boundary operators
- one of the most important is the stress tensor $T_{\tau\tau}$ with RG eigenvalue $1-2=-1$: note that $\int T_{\tau\tau}(x)dx = H_{\rm CFT}$
- CC's assumption was equivalent to the assertion that this is the *only* one:

$$|\psi_0\rangle \propto e^{-\tau_0 H_{\rm CFT}}\,|B\rangle$$

‘Moving the goalposts’

$$\langle\psi_0|\mathcal{O}(\tau)|\psi_0\rangle = \langle B|\, e^{-\tau_0 H}\, \mathcal{O}(\tau)\, e^{-\tau_0 H}\, |B\rangle$$

- to compute $\langle\psi_0|\mathcal{O}(\tau)|\psi_0\rangle$ we therefore consider a slab $-\tau_0 < \tau < \tau_0$ with boundary conditions corresponding to the ideal state $|B\rangle$:

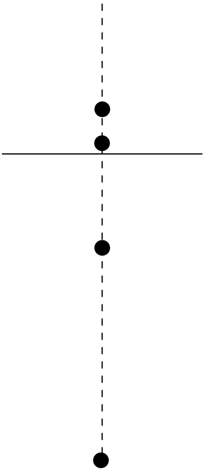


- and continue the result to $\tau \to it$

- because $|B\rangle$ is conformally invariant, the correlations in the slab are related to those in the upper half z-plane by $z = ie^{\pi w/2\tau_0}$
- power-law behaviour in the z-plane $\Rightarrow$ exponential behaviour in t and x

- in particular, $x + i(\tau \to it)$ is mapped to

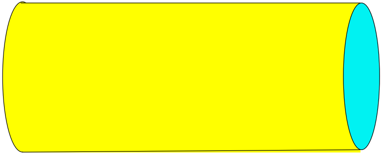
$$z = i\,e^{\pi(x-t)/2\tau_0}\,; \quad \bar{z} = -i\,e^{\pi(x+t)/2\tau_0}\ (\neq z^*!)$$



- except for narrow regions $O(\tau_0)$ near the light cone, points are exponentially ordered along imaginary z-axis: correlators can be computed by successive OPEs
- for $t \gg \tau_0$ the $\bar{z}$'s move off to $-i\infty$ and the boundary effectively plays no role

This implies:

- invariance under rotations in the z-plane, and since $z = ie^{\pi(x+i\tau)/2\tau_0}$, $\Rightarrow$ stationarity in τ and therefore t
- periodicity of correlators under $\tau \to \tau + 4\tau_0 \Rightarrow$ the slab effectively becomes a cylinder: finite temperature!



All the other conclusions of CC then follow straightforwardly.

Relaxing CC's assumption

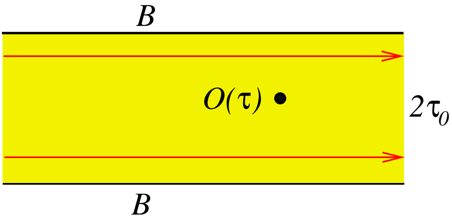
- more generally let us suppose

$$|\psi_0\rangle \propto e^{-\tau_0 H_{\rm CFT}}\, e^{-\sum_j{}'\lambda_j \int \phi_j^{(b)}(x)dx}|B\rangle$$

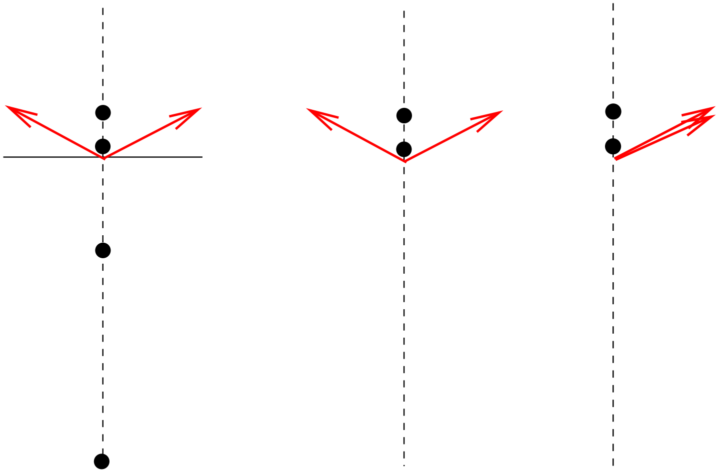
- first consider the case where $\phi_j^{(b)}(x) = T(x)^j \; \left(= \overline{T}(x)^j \right)$ (or more generally combinations of derivatives and powers of T)
- since $T = \overline{T}$ on B, we can write

$$\int_B T(x)^j dx = \tfrac{1}{2}\int_B T(z)^j dz + \tfrac{1}{2}\int_B \overline{T}(\bar z)^j d\bar z$$

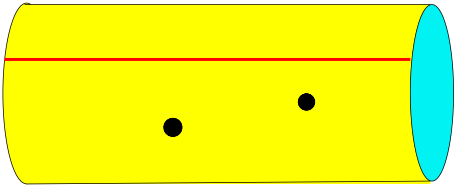
- since the operators are (anti-)holomorphic, we can distort the contours away from the boundary:



- in the half-plane



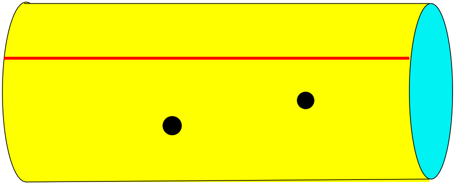
- on the cylinder



$$\mathrm{Tr}\,\mathcal{O}\,e^{-\beta H-\sum_j'\lambda_j H_j}$$

where $H_j = \int [T(x)^j + \overline{T}(x)^j]dx$

- on the cylinder



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Generalised Gibbs Ensemble (GGE)

Observable consequences of GGE in CFT

$$H_j \propto \frac{1}{\beta^{2j-1}} \sum_{n_1+\cdots+n_p=0} :L_{n_1} L_{n_2} \cdots L_{n_p}: + \text{c.c.}$$

$$\propto L_0^j + \text{terms with } n_p \geq 1 + \text{c.c.}$$

so acting on a primary operator $H_j \propto \Delta_\Phi^j$

- so in the stationary regime, a 2-point function decays as

$$\langle \Phi(x_1, t) \Phi(x_2, t) \rangle \propto e^{-|x_1 - x_2|/\xi_\Phi}$$

where $$\xi_\Phi^{-1} = \frac{2\pi\Delta_\Phi}{\beta} \left[1 + \sum_j \lambda_j \left(\frac{2\pi\Delta_\Phi}{\beta^2} \right)^{j-1} \right]$$

- effective temperature, as determined by decay of correlations, depends on the observable
- similar consequences for entropy, etc

More general boundary perturbations

- more general boundary perturbations $\phi_j^{(b)}$ with scaling dimensions $\Delta_j \neq$ integer are consistent with a GGE only if we posit the existence of bulk *parafermionic* holomorphic currents $\phi_j(z)$ with dimension Δ_j and include the corresponding non-local conserved charges $H_j = \int \phi_j(x,t)dx$ in the GGE
- example: quench to the critical point in a transverse-field Ising model from disordered state with a small longitudinal field ($\Delta = \frac{1}{2}$) gives rise to a fermionic charge $\int [\psi(x) + \overline{\psi}(x)]dx$ (??)

Universality?

- in general, by dimensional analysis,

$$\xi_\Phi^{-1} = \frac{2\pi\Delta_\Phi}{\beta}\left[1 + O\left(\frac{\lambda_j}{\beta^{\Delta_j - 1}}\right)\right]$$

- so if $\Delta_j > 1$ (irrelevant initial perturbation) the stationary state is more Gibbsian if β large (shallow quench)
- one can also add irrelevant terms to $H_{\rm CFT}$: e.g.
  - $T\overline{T}$, corresponding to left-right scattering
  - $T^j + \overline{T}^j$, corresponding to curvature of dispersion relation
- these are irrelevant for a *shallow* quench
- do they drive crossover to a different behaviour for a deep quench?

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- these are irrelevant for a *shallow* quench
- do they drive crossover to a different behaviour for a deep quench?
- if so, what? True thermalisation?

Conclusions

- a quantum quench in 1+1-dimensional CFT from a more general state leads to results consistent with a GGE
- the conserved quantities are in 1-1 correspondence with possible boundary perturbations - some of these may be non-local in the bulk
- the effects of GGE include observable-dependent effective temperatures
- they should be less important for shallow quenches