

# Adilet Imambekov (1982 - 2012)



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## Bethe Ansatz

### Exactly solvable case of a one-dimensional Bose-Fermi mixture

Adilet Imambekov and Eugene Demler  
*Department of Physics, Harvard University, Cambridge MA 02138*  
(Dated: February 2, 2008)

We consider a one dimensional interacting bose-fermi mixture with equal masses of bosons and fermions, and with equal and repulsive interactions between bose-fermi and bose-bose particles. Such a system can be realized in experiments with ultracold boson and fermion isotopes in optical lattices. We use the Bethe-ansatz technique to find the ground state energy at zero temperature for any value of interaction strength and density ratio between bosons and fermions. We prove that the mixture is always stable against demixing. Combining exact solution with the local density approximation we calculate density profiles and collective oscillation modes in a harmonic trap. In the strongly interacting regime we use exact wavefunctions to calculate correlation functions for bosons and fermions under periodic boundary conditions.

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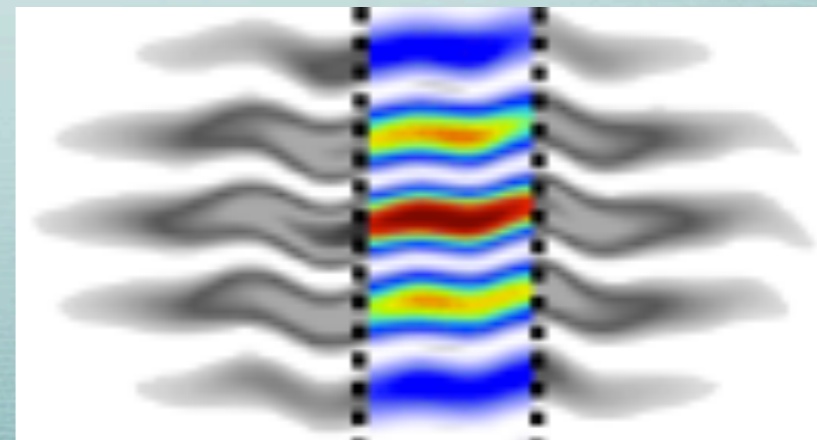
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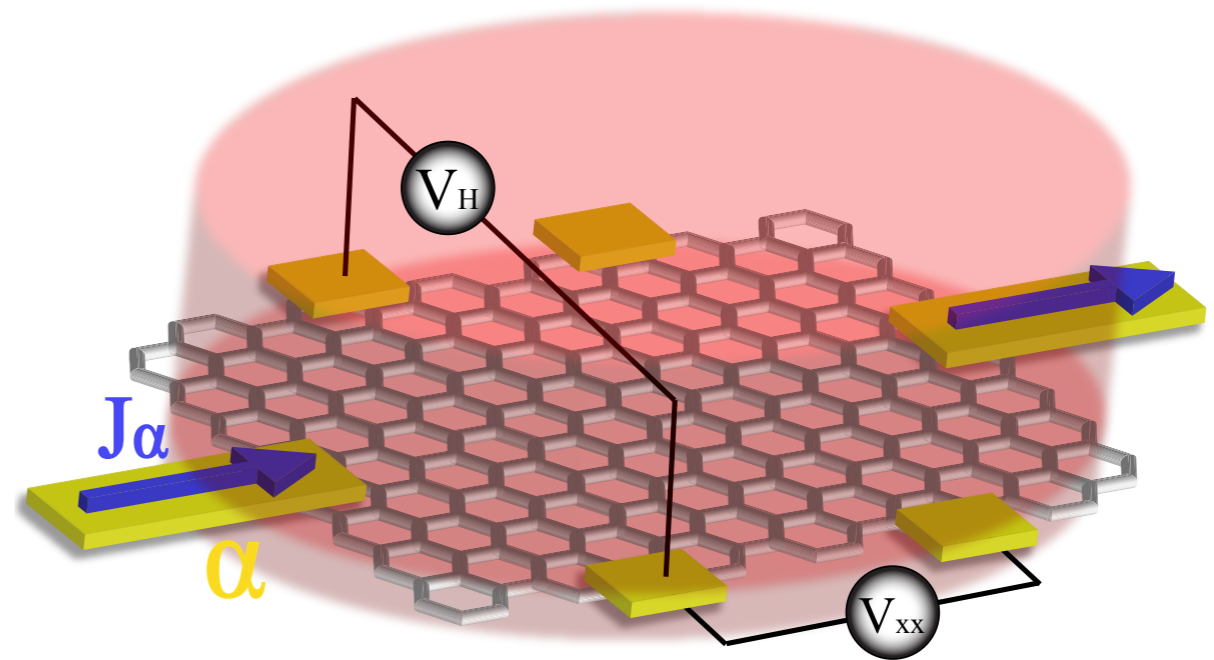
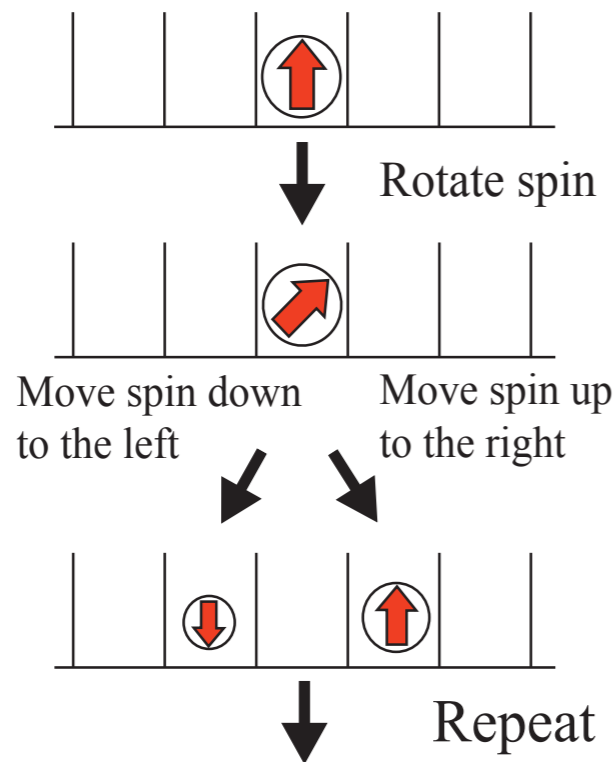
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## Distribution of Interference patterns



# Novel topological phenomena in non-equilibrium systems



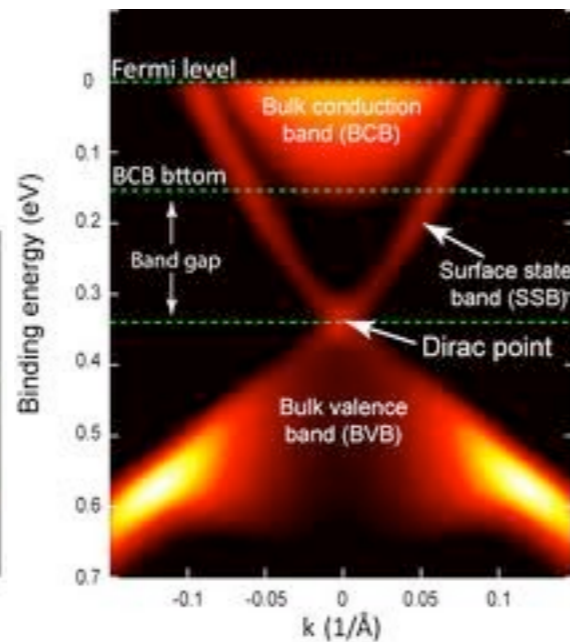
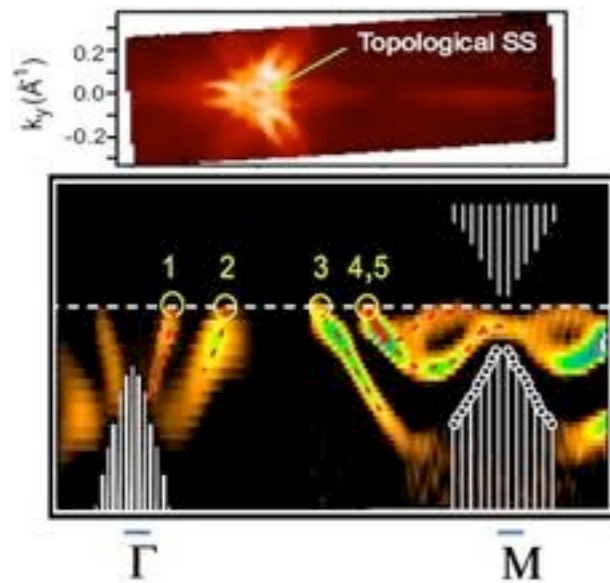
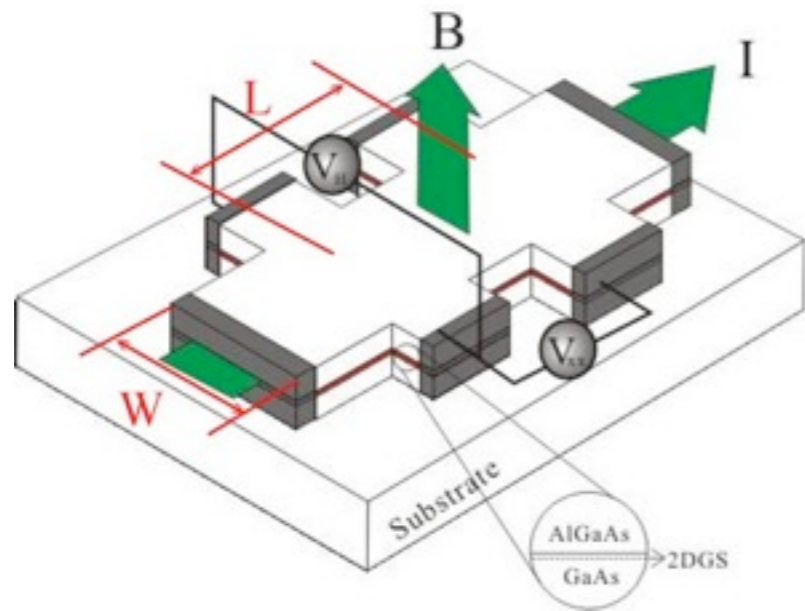
**Takuya Kitagawa ( Harvard University)**

in collaboration with Mark Rudner, Erez Berg , Takashi Oka,  
Liang Fu, Eugene Demler (Theory)  
Andrew White's group, Immanuel Bloch's group (Experiment)

# New Topological phenomena in driven systems

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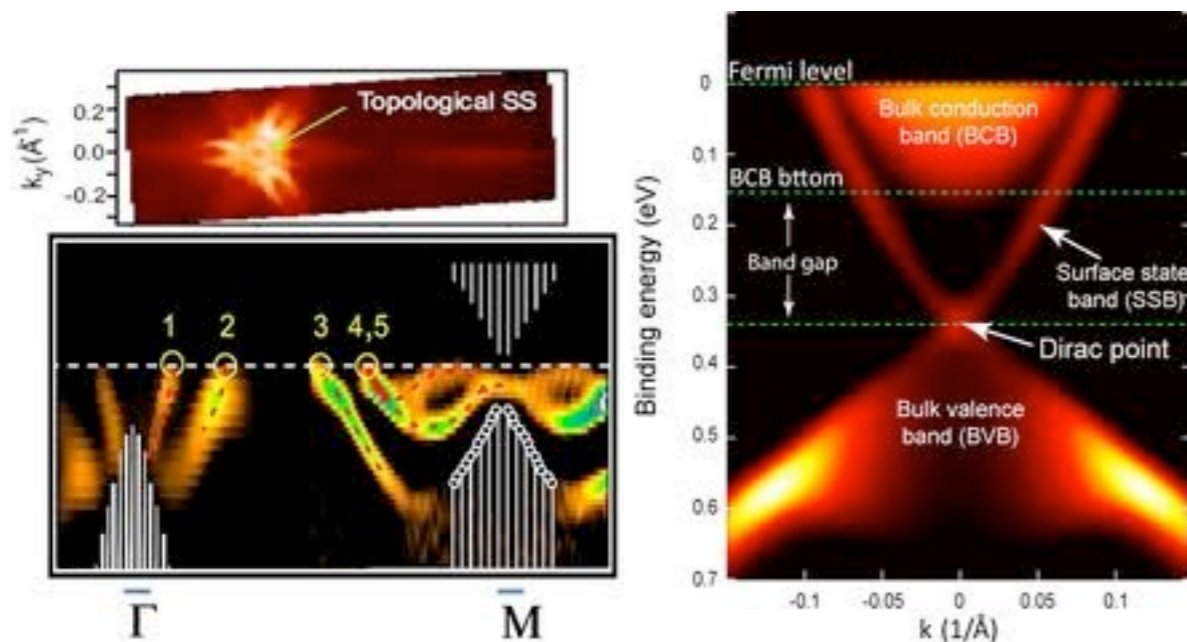
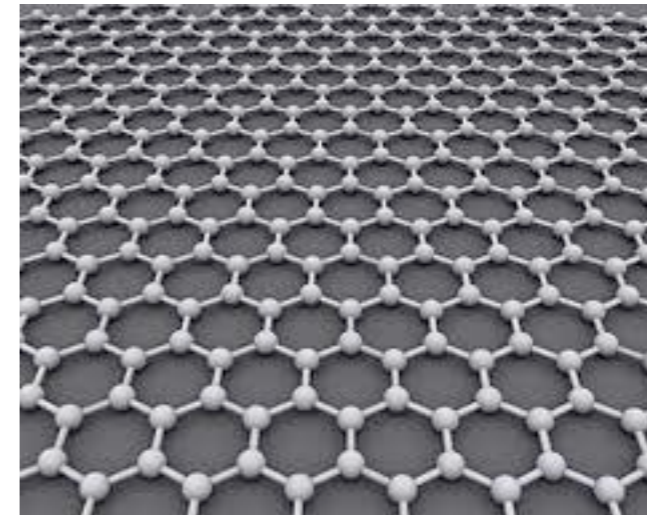
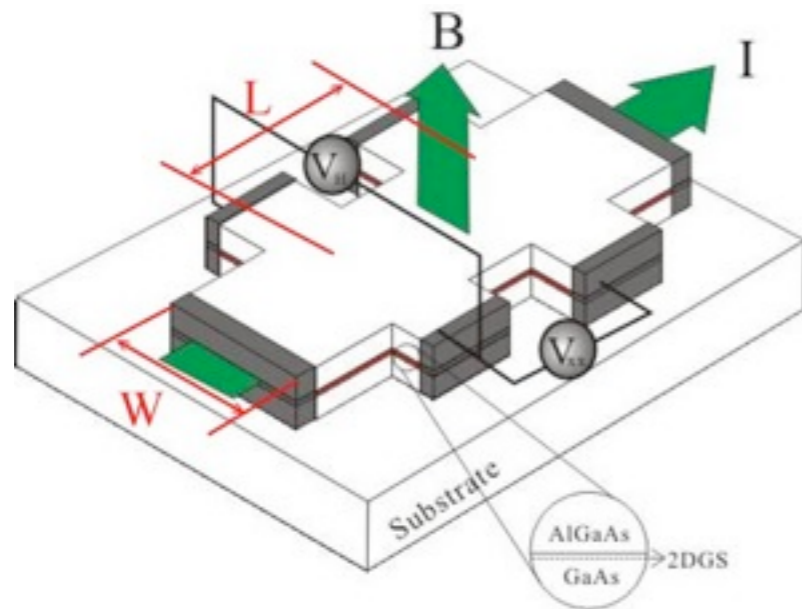
## Topological phases in equilibrium



# New Topological phenomena in driven systems

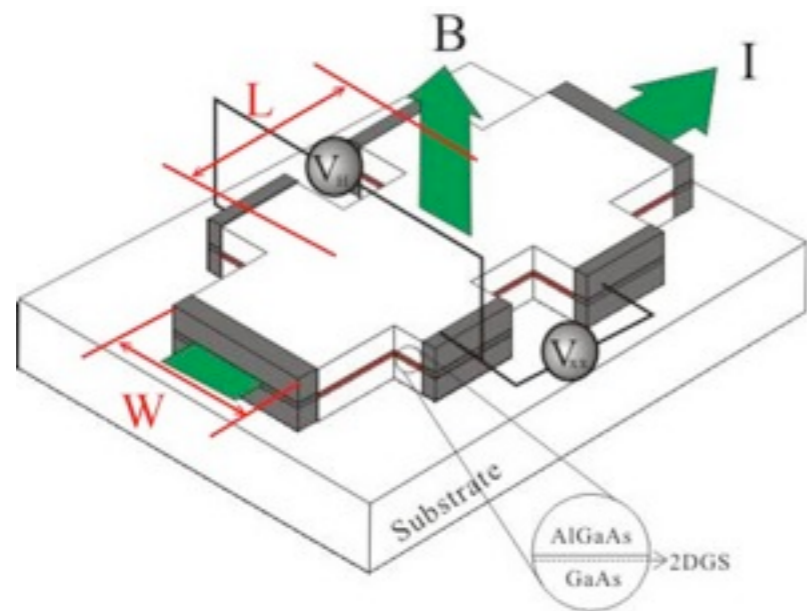
Topological phases in equilibrium

Switchable properties?

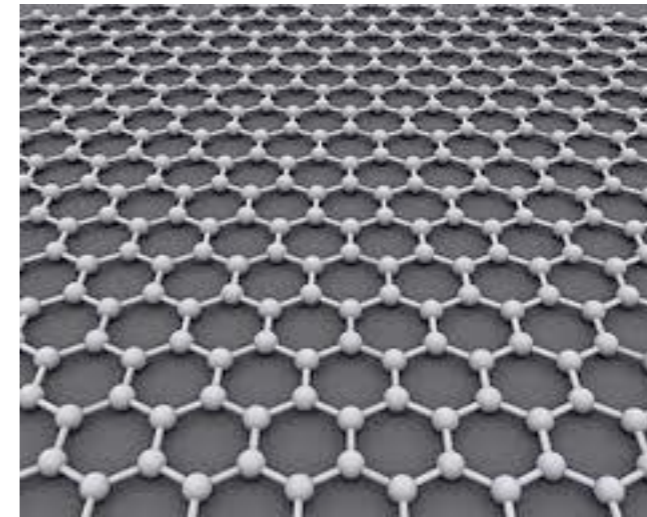


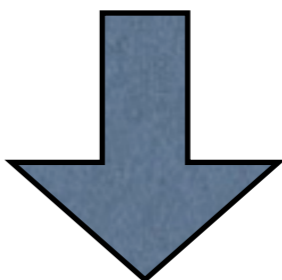
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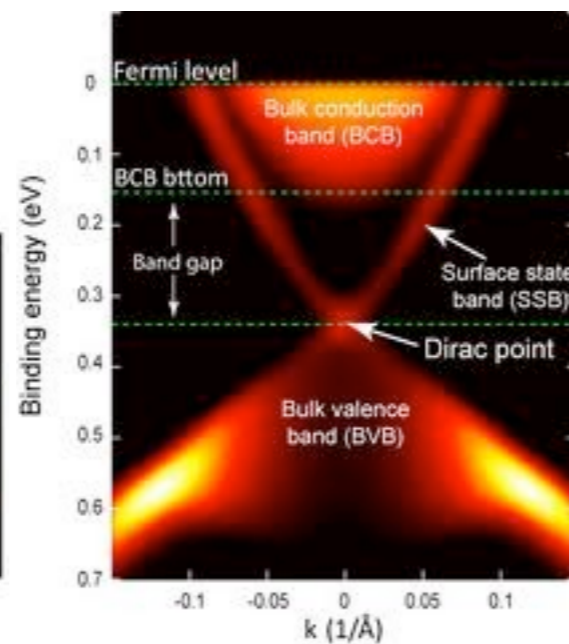
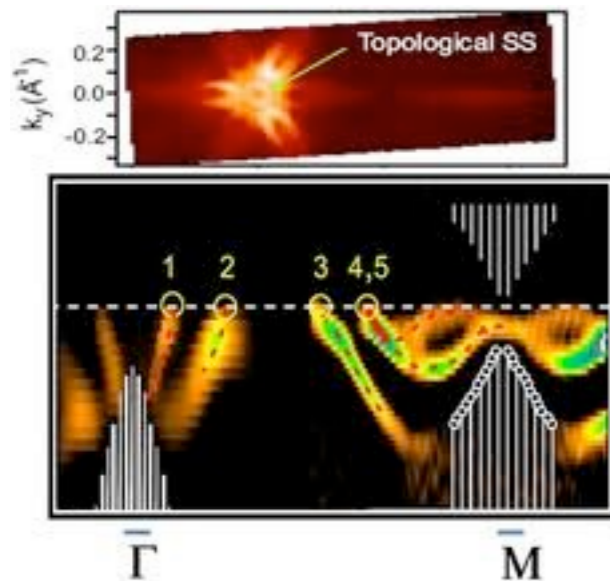
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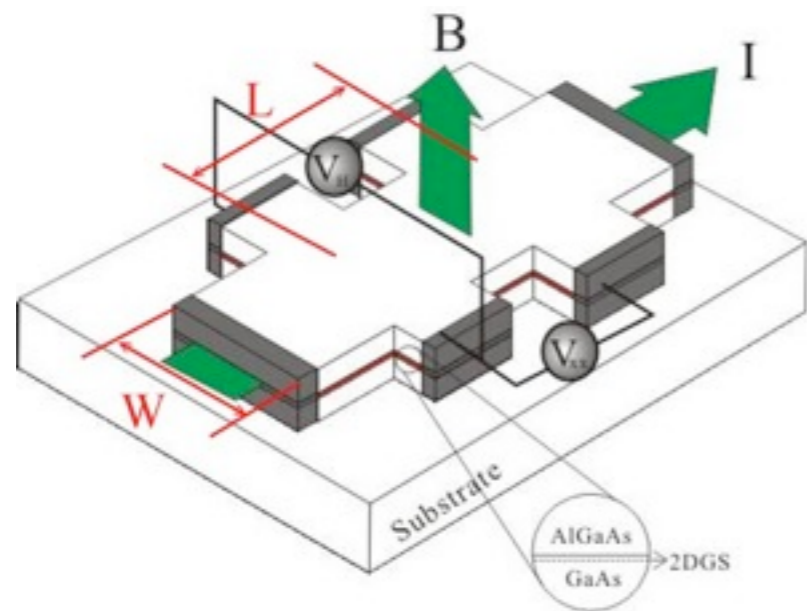
Light  ?



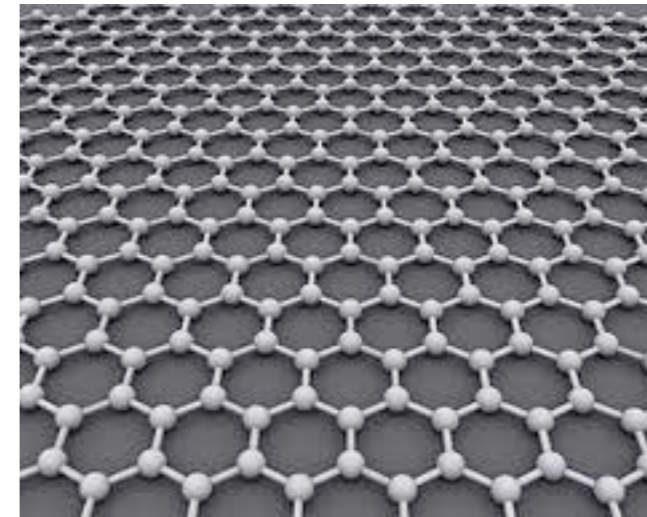


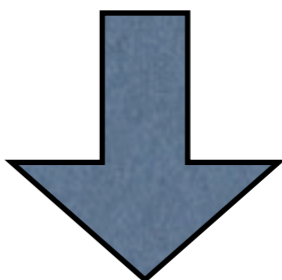
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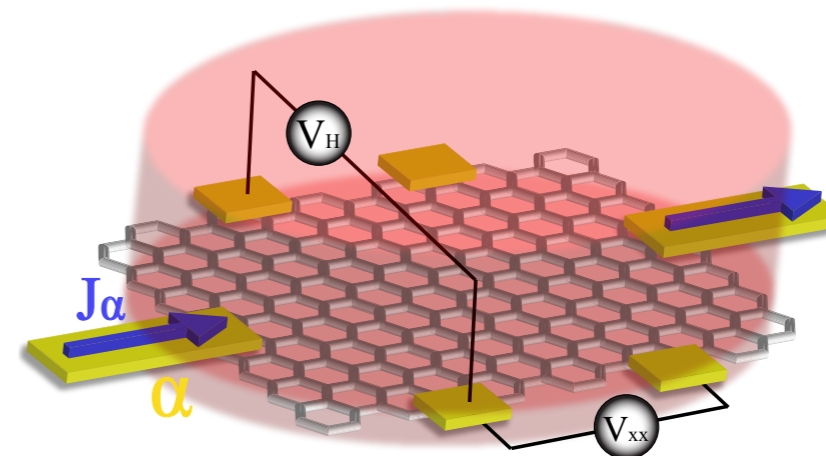
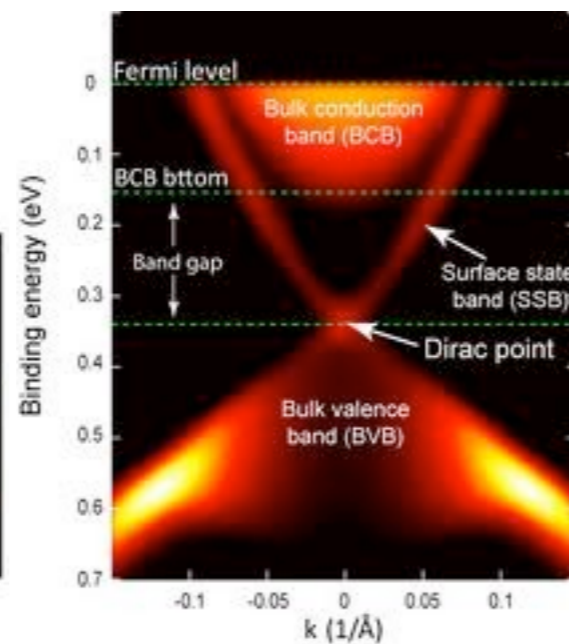
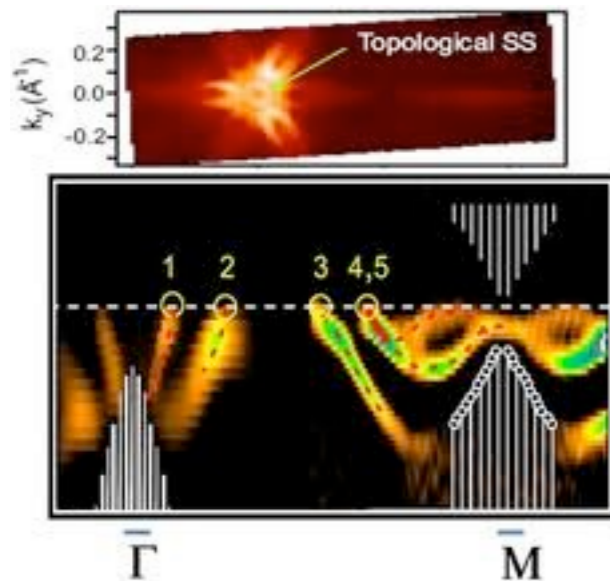
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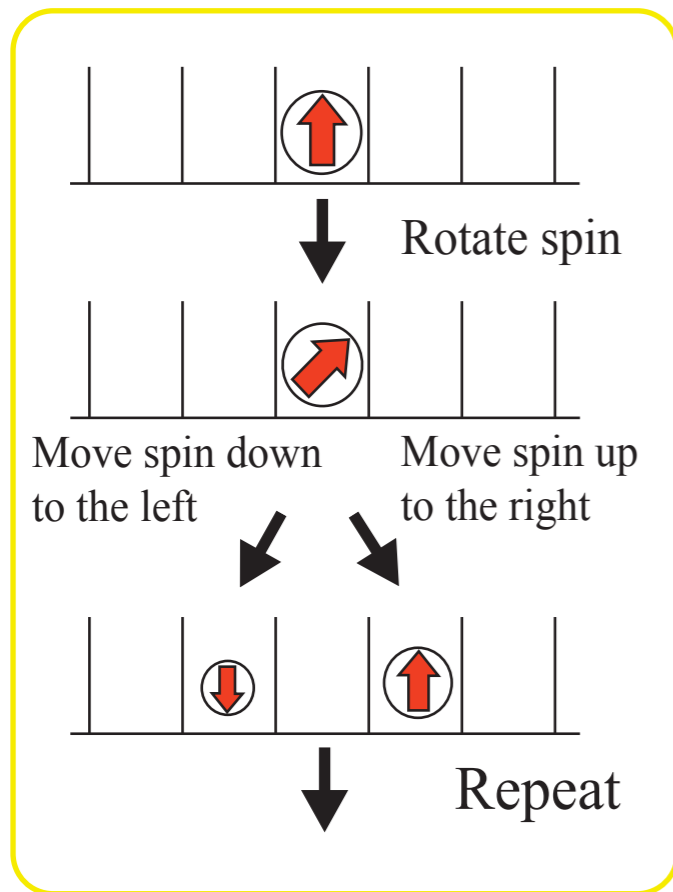


T. Kitagawa, T. Oka, et al,  
[Phys. Rev. B 84, 235108 \(2011\)](https://doi.org/10.1103/PhysRevB.84.235108)

# New Topological phenomena in driven systems

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## Quantum optics

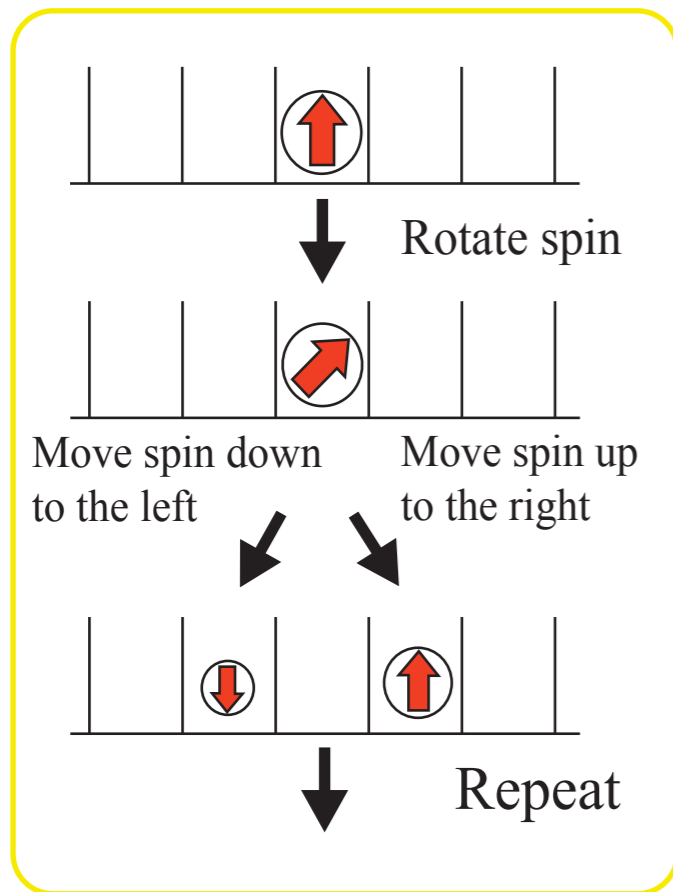


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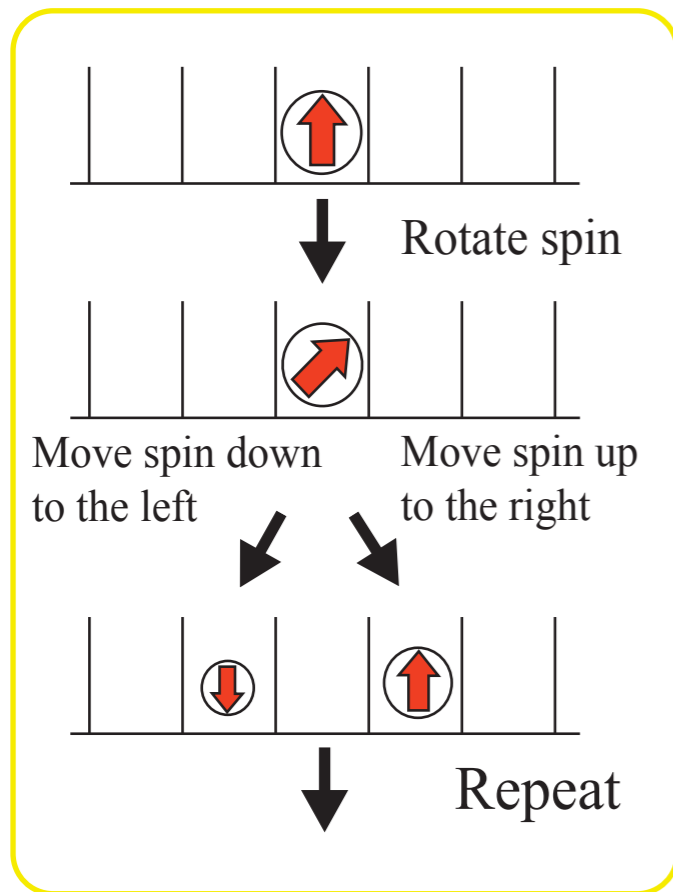
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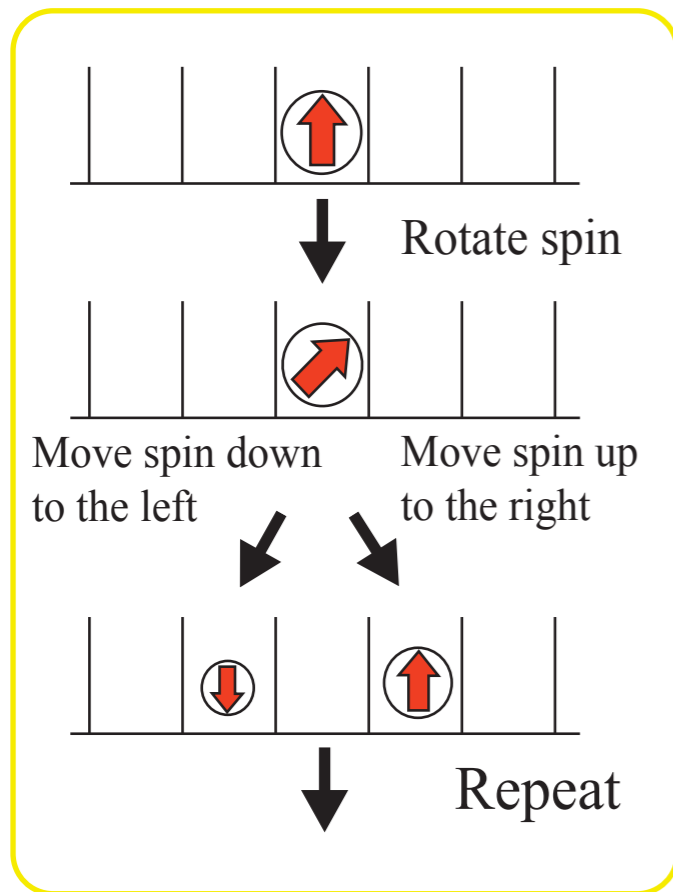
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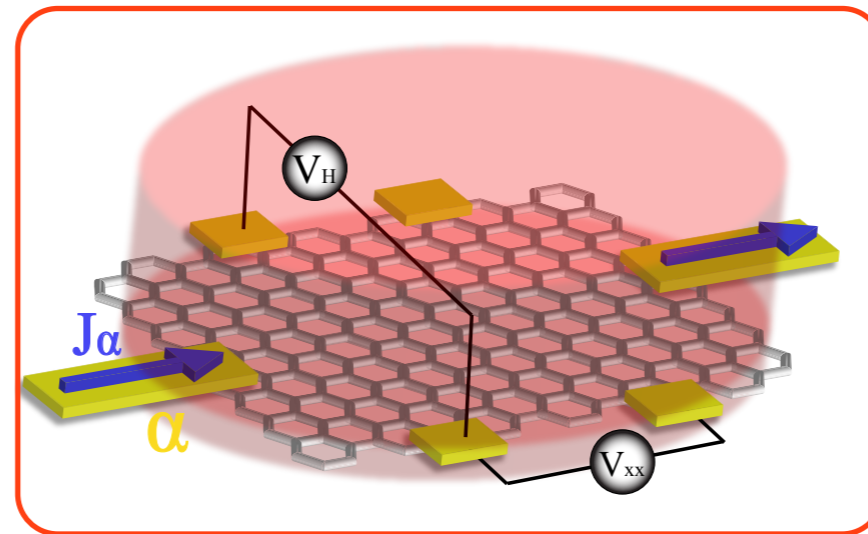
**Experimental realizations!**

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## condensed matter



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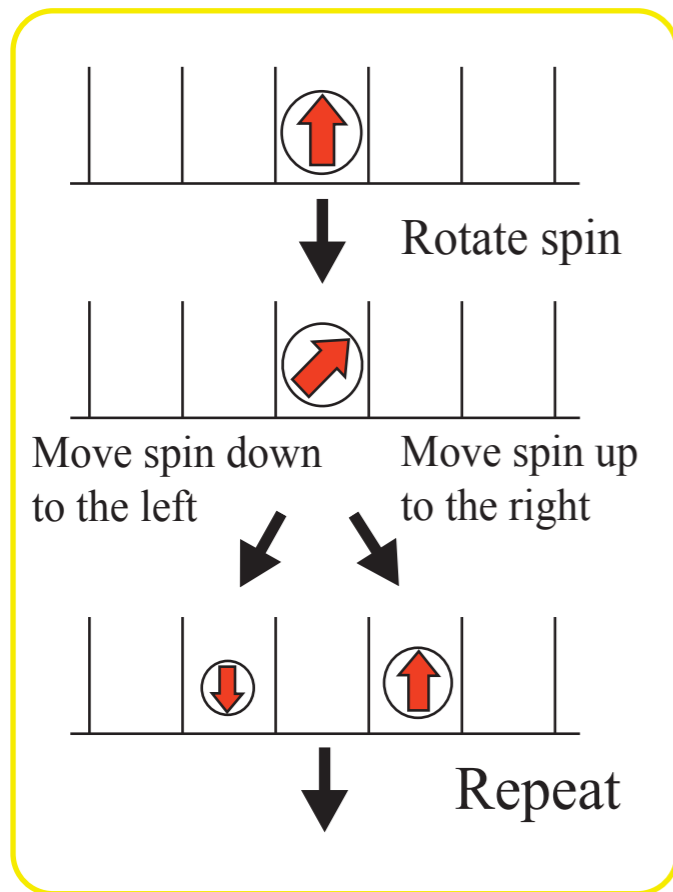
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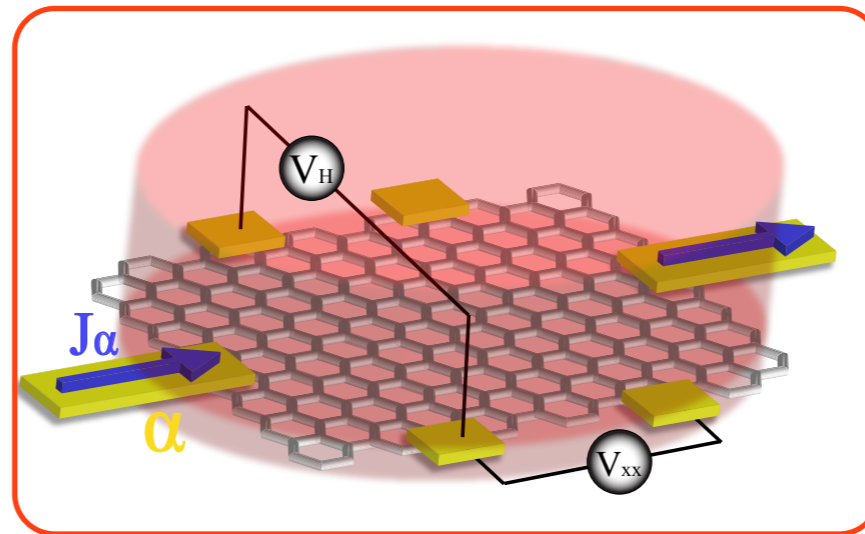
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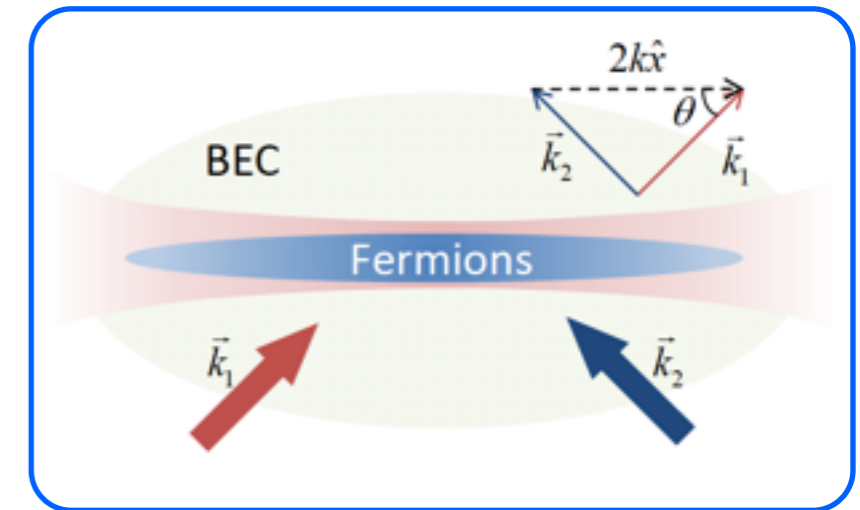
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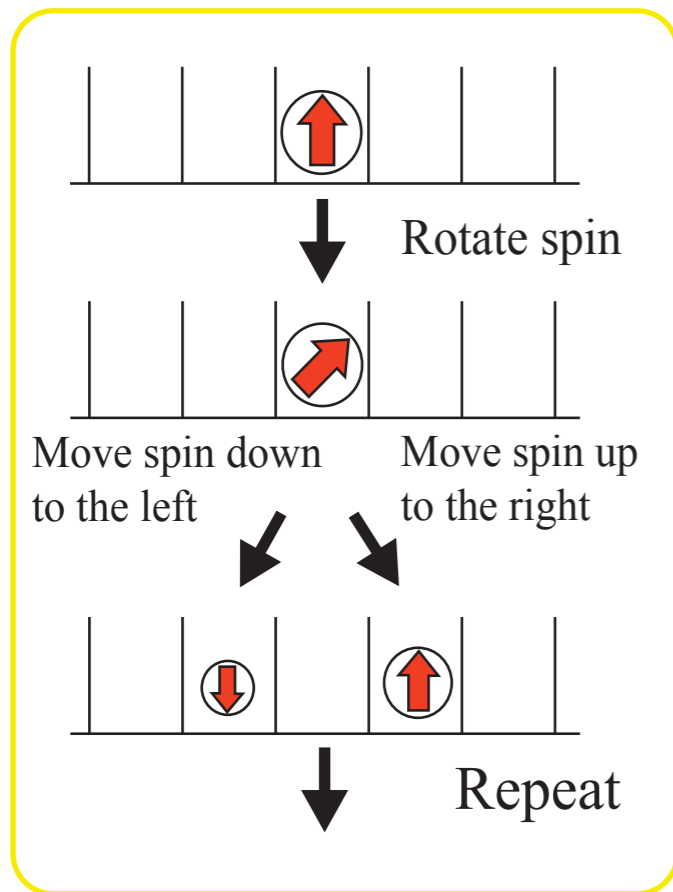
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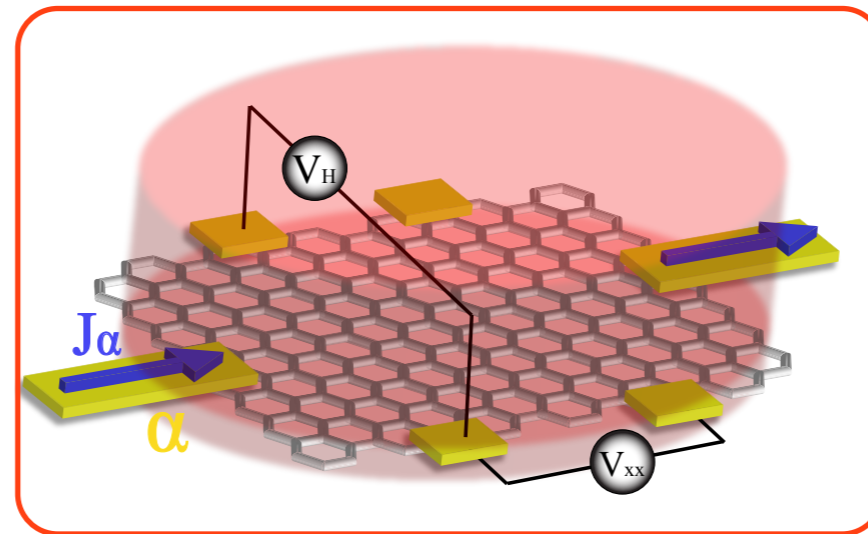
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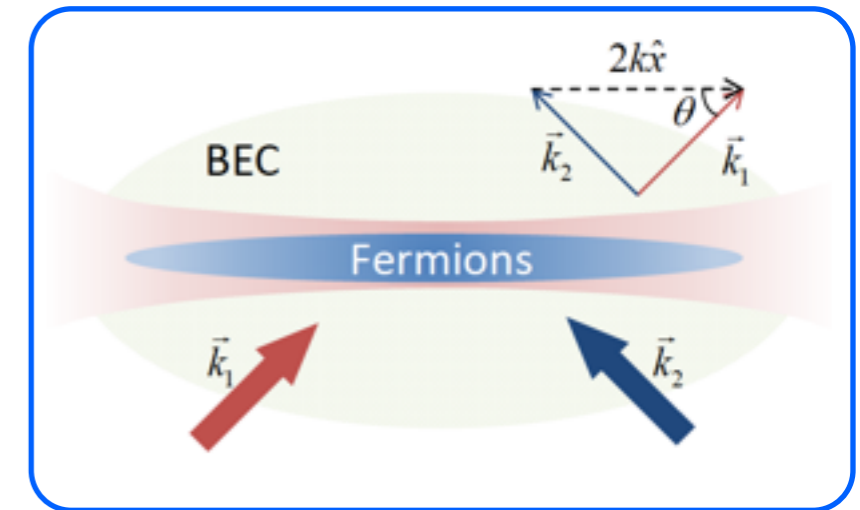
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- Topological phases are rare in static systems-**new platforms**

**Experimental realizations!**

- Novel phenomena unique to non-equilibrium systems



# Outline

1. Overview of topological phases
2. Topology of periodically driven systems
  - a. Design of static topology through driving
3. Examples from driven hexagonal lattice system;  
*Graphene under circularly polarized light*
4. Detection scheme of topological band
5. Conclusion

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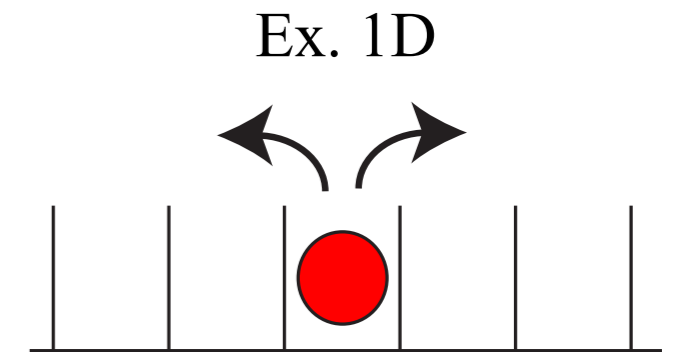
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# Floquet approach to periodically driven systems

Consider *non-interacting*, single particle Hamiltonian in a lattice which is periodically driven

$$H(t) = H(t + T)$$

where  $T$  is one period of dynamics

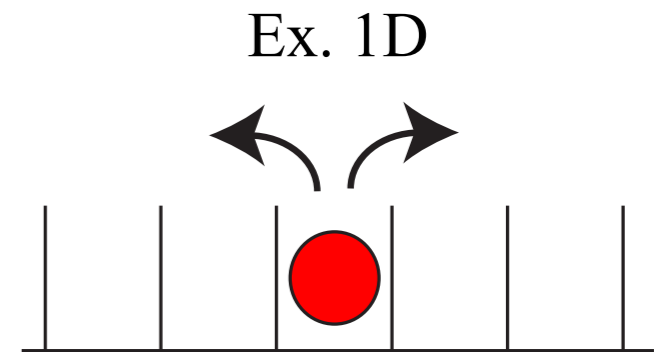


# Floquet approach to periodically driven systems

Consider *non-interacting*, single particle Hamiltonian in a lattice which is periodically driven

$$H(t) = H(t + T)$$

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We characterize such dynamics through the evolution operator after one period called Floquet operator:

$$U(T) = \mathcal{T} \left( e^{-i \int_0^T H(t') dt'} \right) \quad \mathcal{T} \text{ is time ordering}$$

Topological behavior is captured in the topology of Floquet operator

# Effective Hamiltonian

Evolution operator of one period

$$U(T) = \mathcal{T} \left( e^{-i \int_0^T H(t') dt'} \right) \quad \mathcal{T} \text{ is time ordering}$$

We can define a local, static Hamiltonian  $H_{\text{eff}}$  through

$$U(T) = \mathcal{T} e^{-i \int_0^T H(t) dt} \equiv e^{-i H_{\text{eff}} T}$$

“Stroboscopic” simulation of  $H_{\text{eff}}$

see, T.Kitagawa et al (2010) PRB

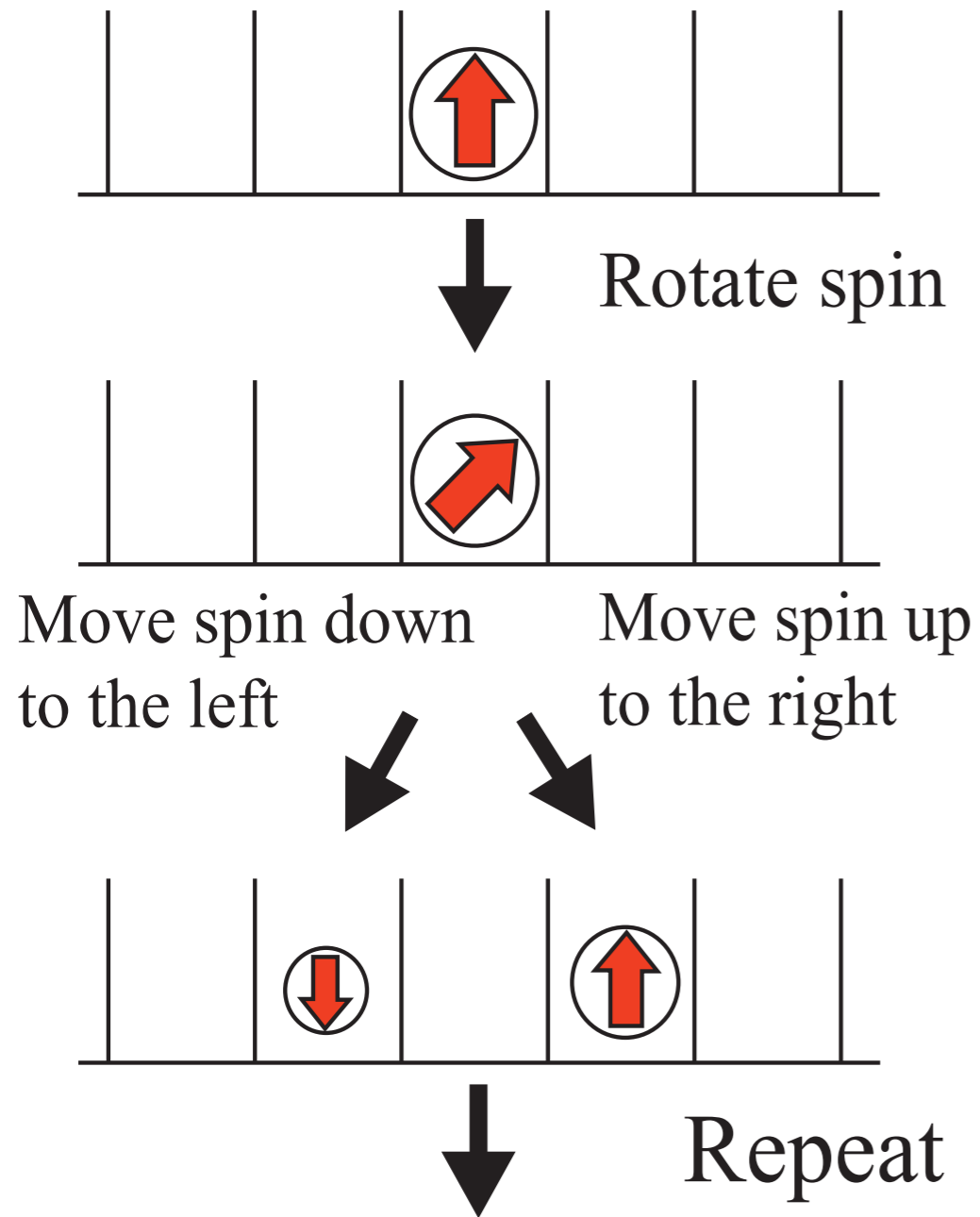
# Topology of Effective Hamiltonian

		TRS	PHS	SLS	$d = 1$	$d = 2$	$d = 3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
chiral (sublattice)	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	-1	0	-	$\mathbb{Z}$	-
	DIII	-1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	-1	1	-	-	$\mathbb{Z}$

“Periodic table” of topological phases

(Qi et al PRB 2008, Schnyder, Ryu et al PRB 2008,  
Kitaev AIP Conference proceeding 2009)

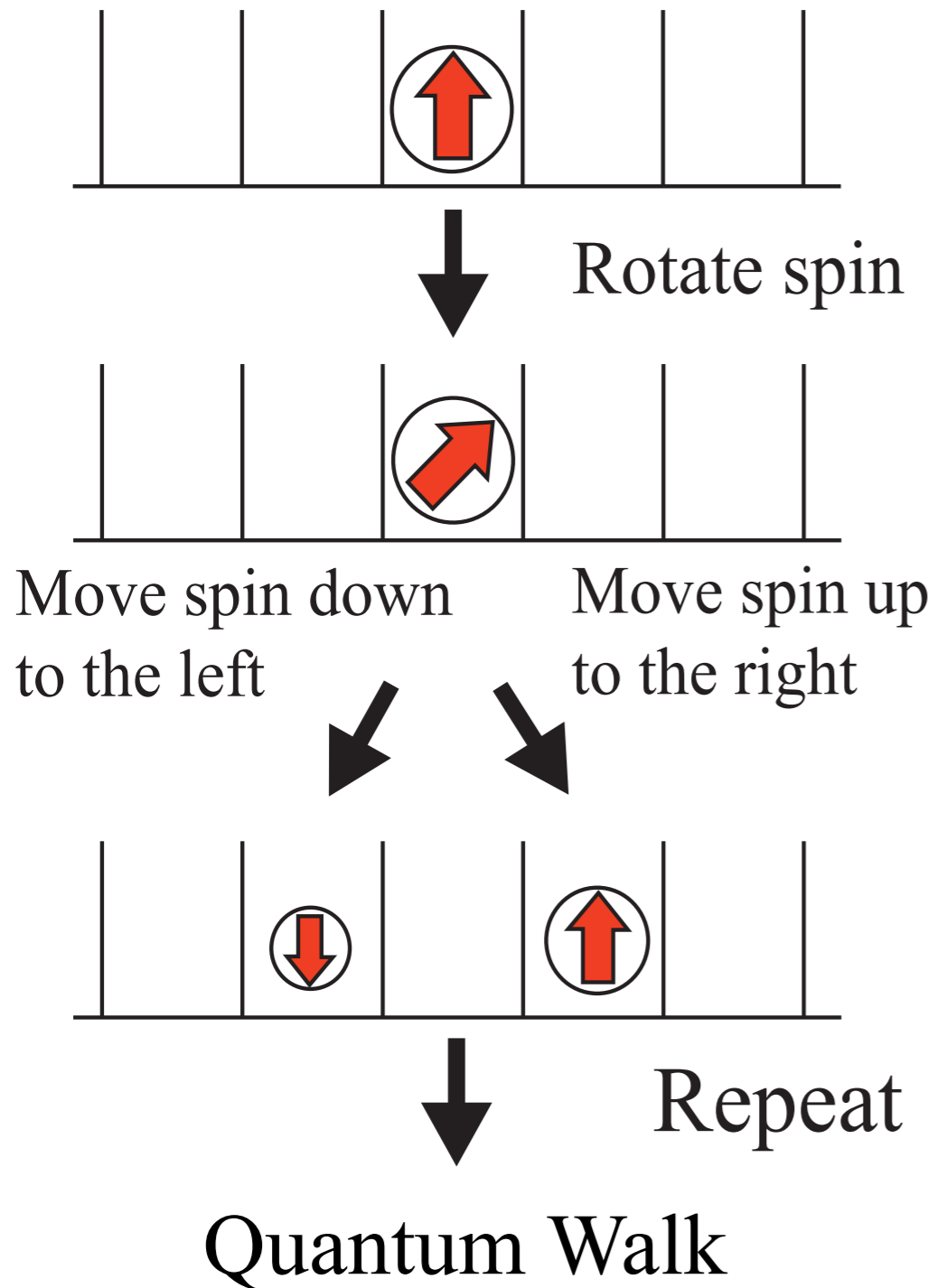
# Example: quantum walk



Quantum Walk

T. Kitagawa, et al Phys. Rev. A 82 033429 (2010)

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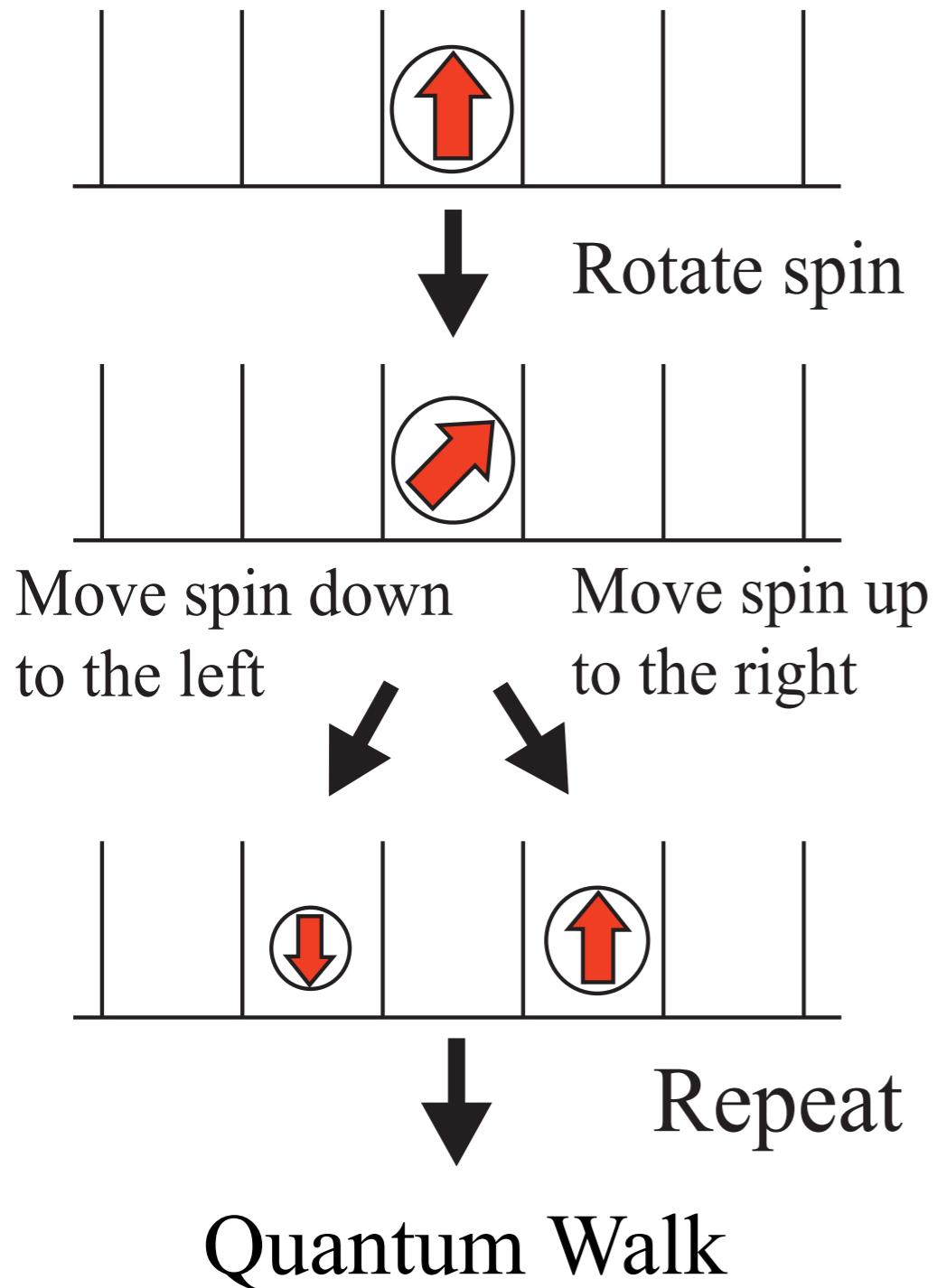


$$R(\theta) = e^{-i\theta\sigma_y}$$

$$= \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$



# Example: quantum walk



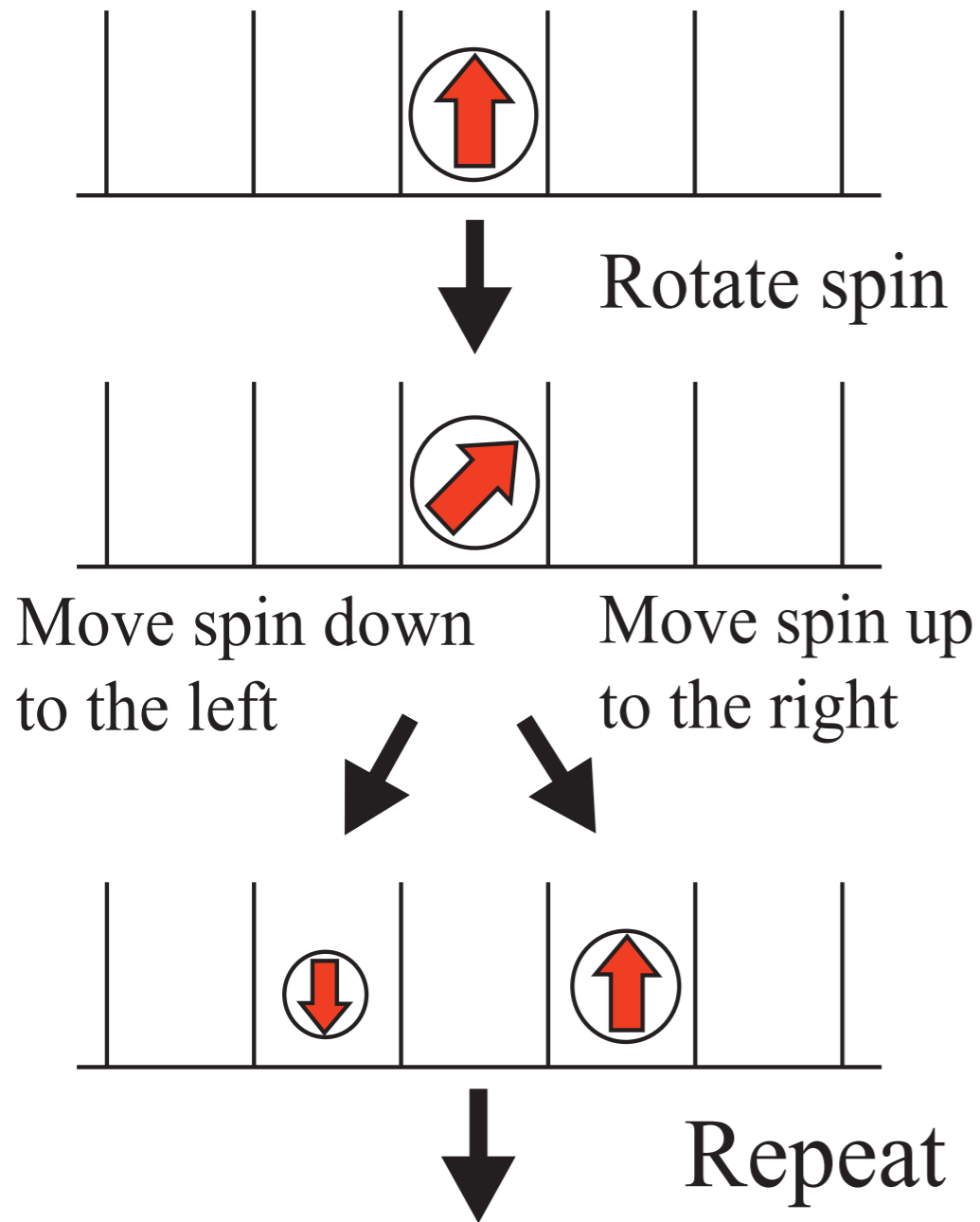
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$$T = \sum_x |x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow| + |x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow|$$

$$= \sum_k e^{ik\sigma_z} \otimes |k\rangle\langle k|$$

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$$= \sum_k e^{ik\sigma_z} \otimes |k\rangle\langle k|$$

$$U_{onestep} = TR(\theta)$$

$$= \sum_k e^{ik\sigma_z} e^{-i\theta\sigma_y} \otimes |k\rangle\langle k|$$

# Hamiltonian formulation

$$U_{onestep} \equiv e^{-iH_{eff}(\theta)\delta t}$$

Effective Hamiltonian is  $2 \times 2$  matrix

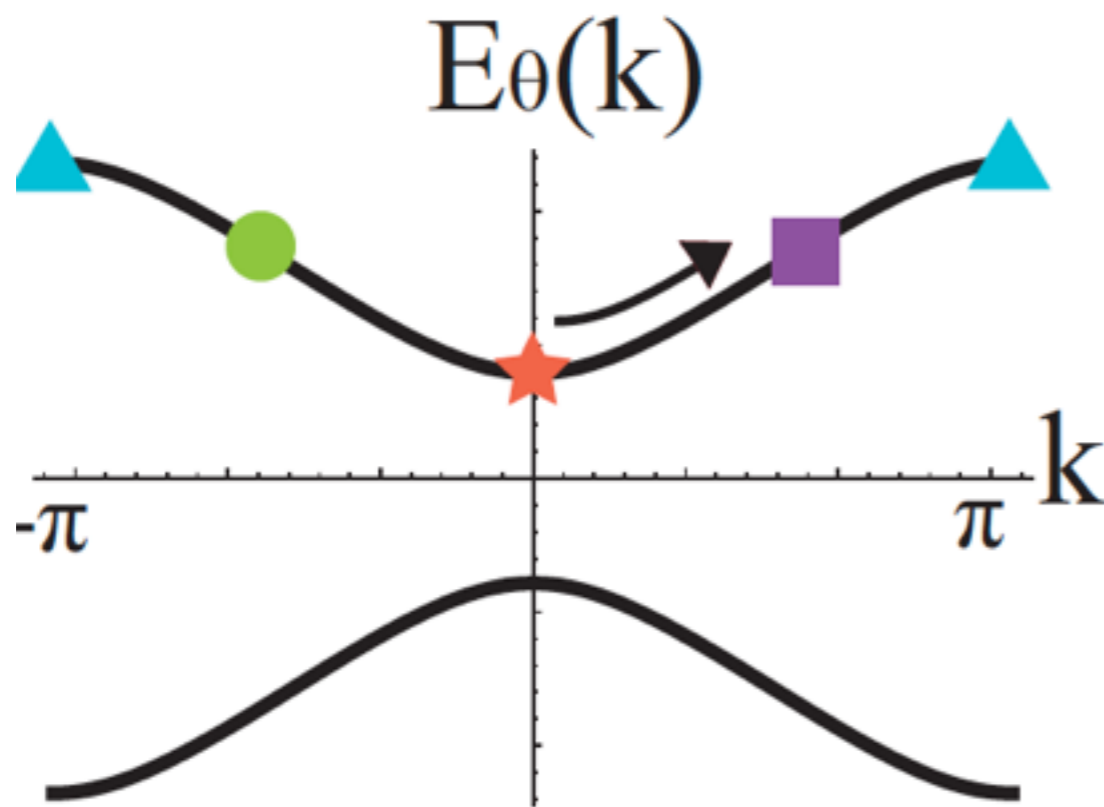
$$H(\theta) = \int_{-\pi}^{\pi} dk [E_{\theta}(k) \mathbf{n}_{\theta}(k) \cdot \boldsymbol{\sigma}] \otimes |k\rangle\langle k|,$$

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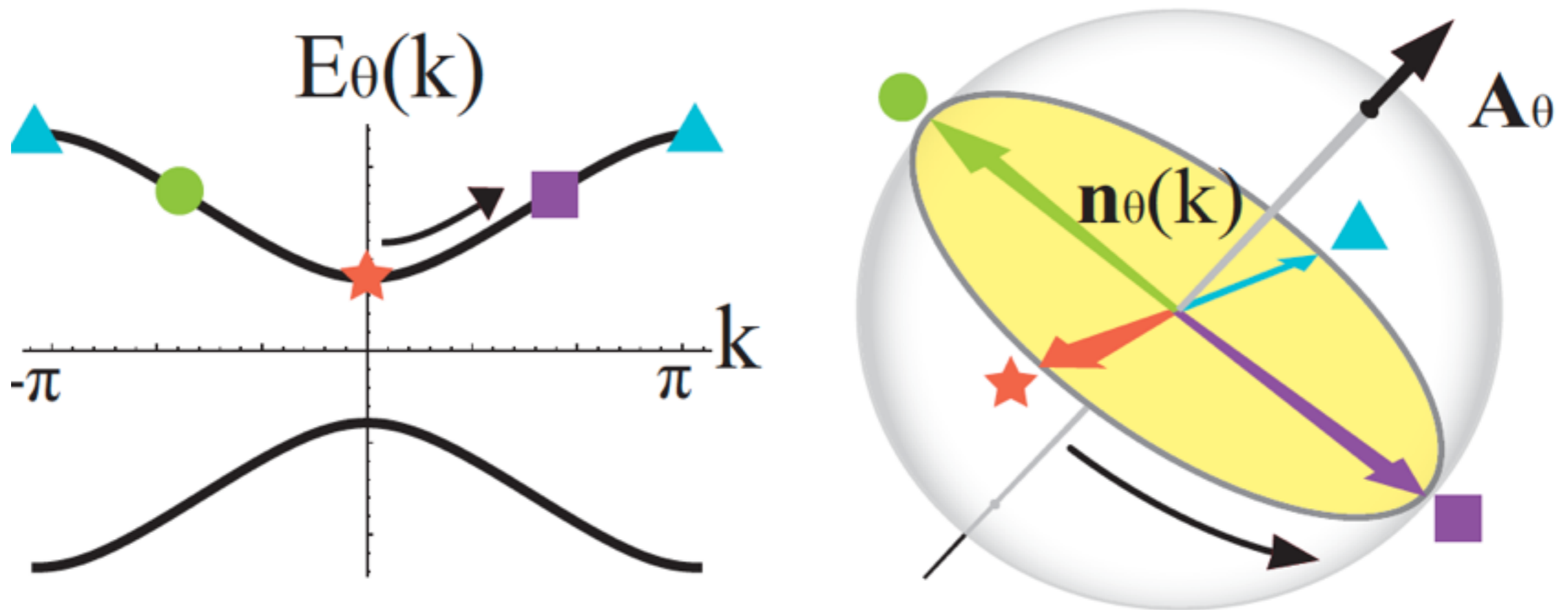


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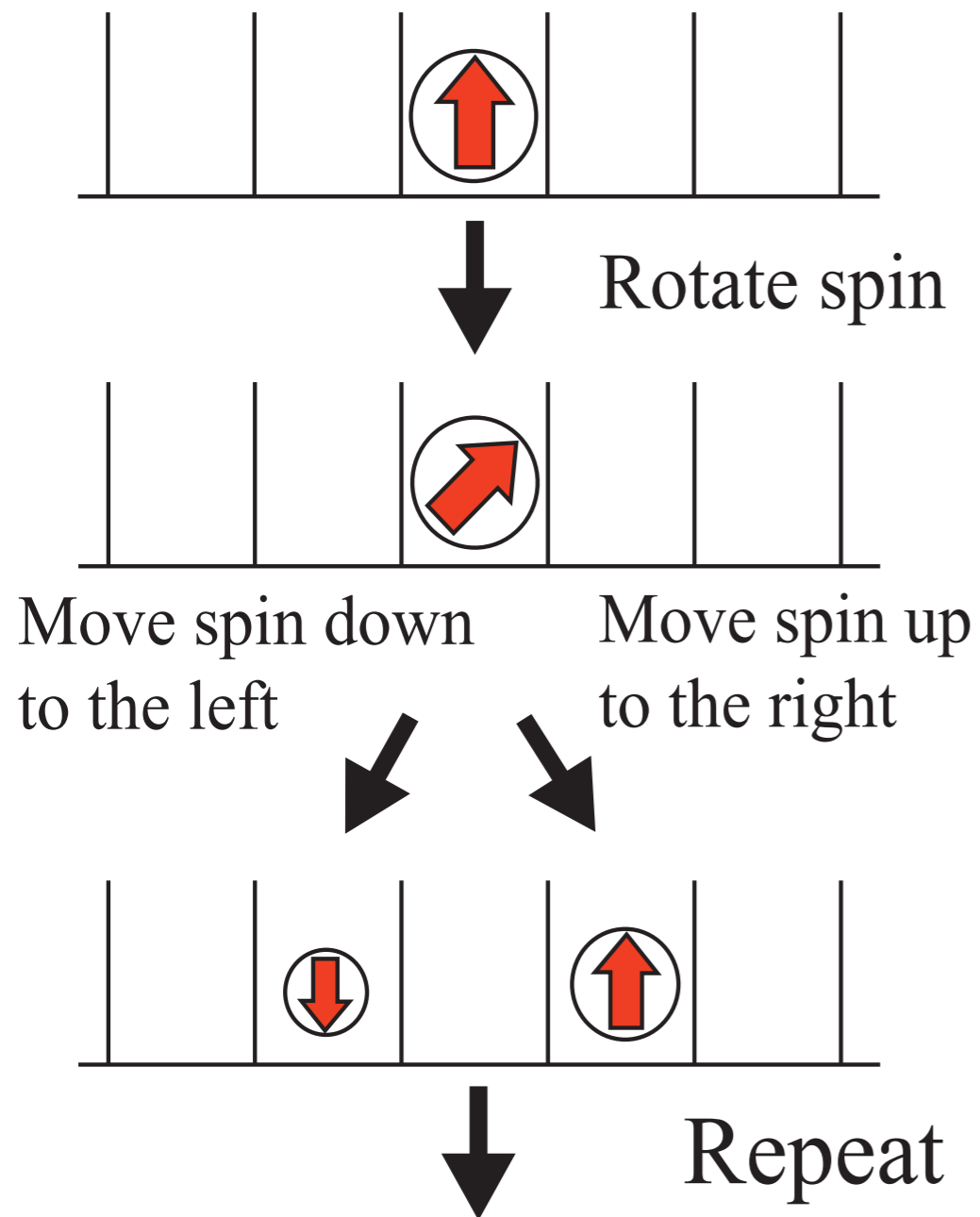
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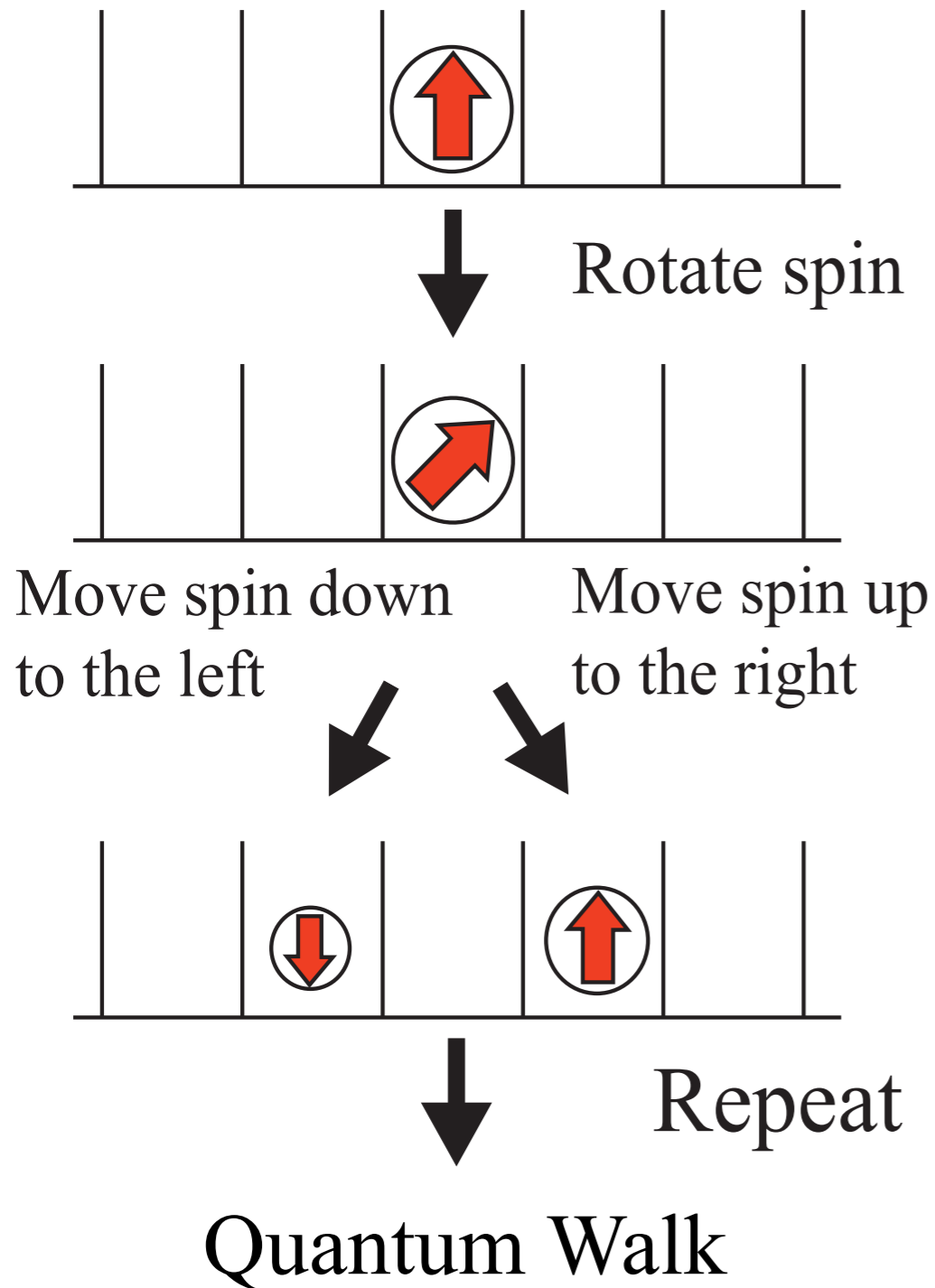
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# Example: quantum walk

Realizes 1D topological phase!

Su-Schrieffer-Heeger model

Jackiw-Rebbi model



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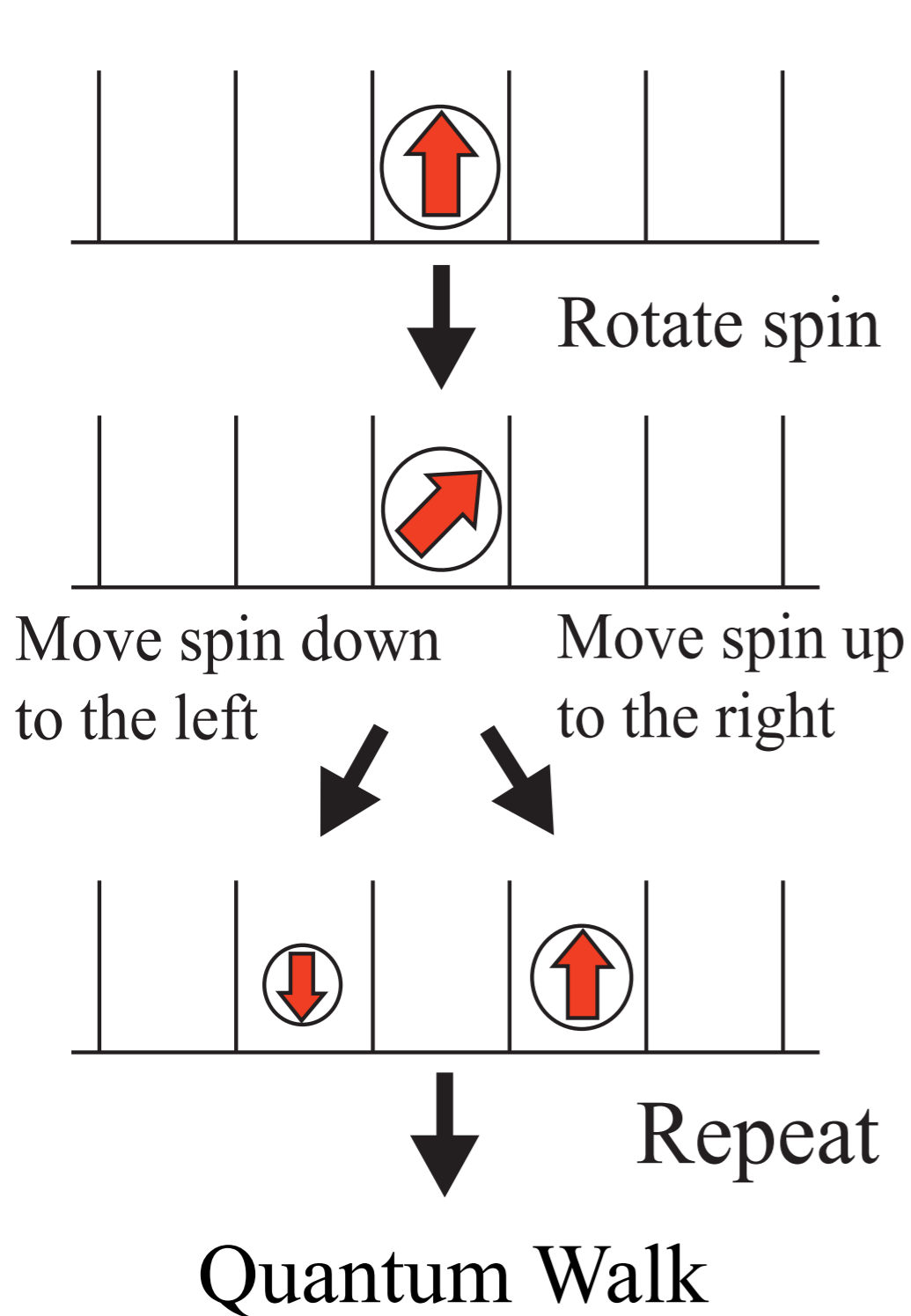
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Topologically protected  
zero energy bound state



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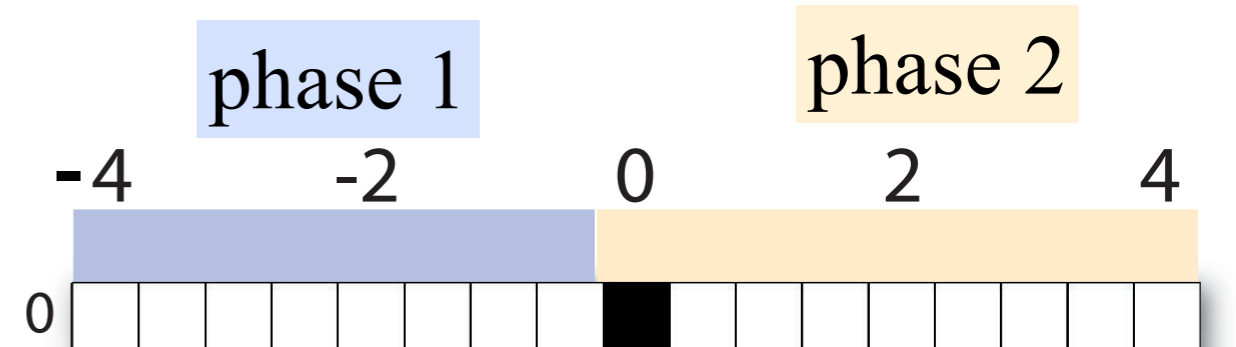
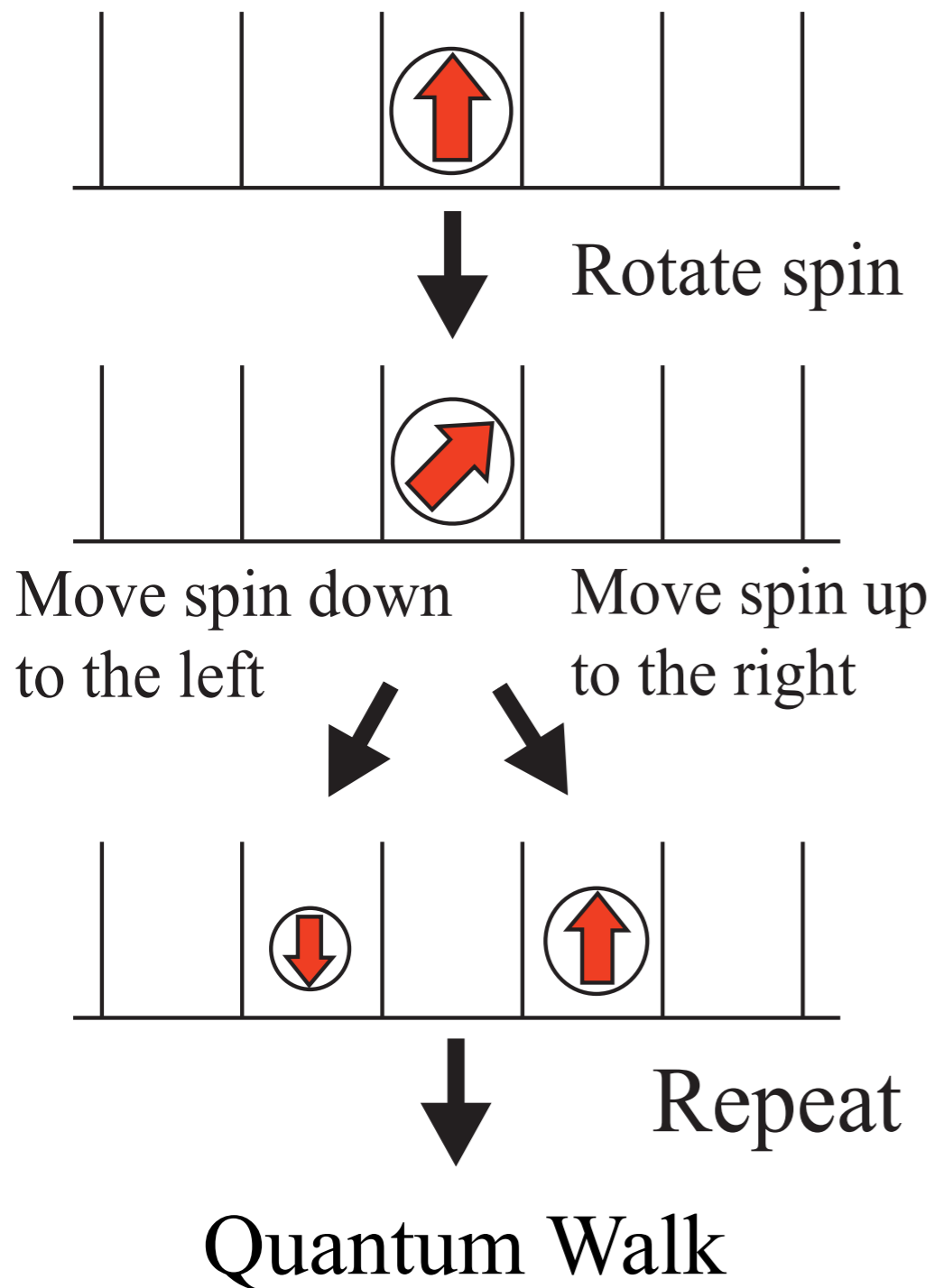
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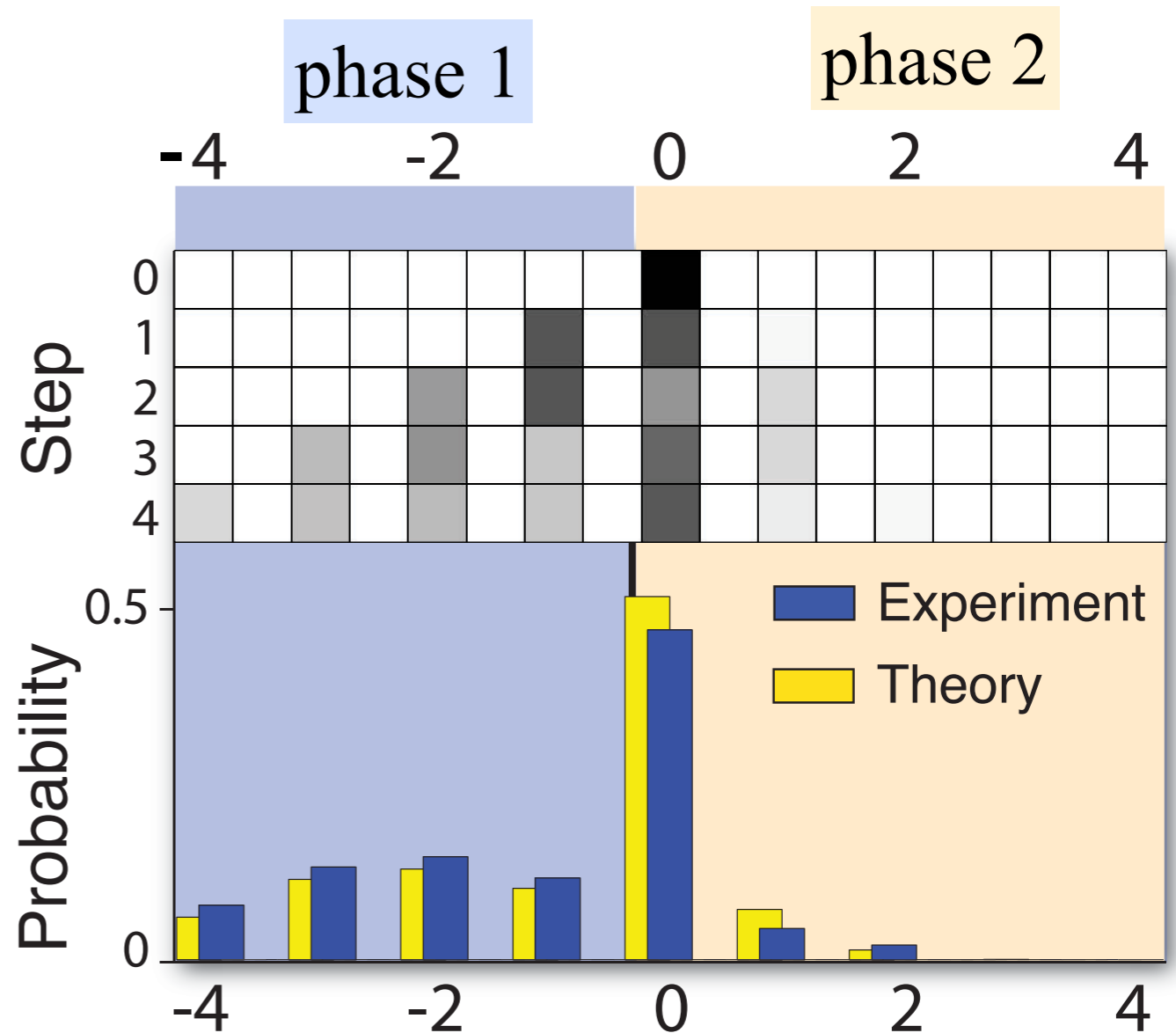
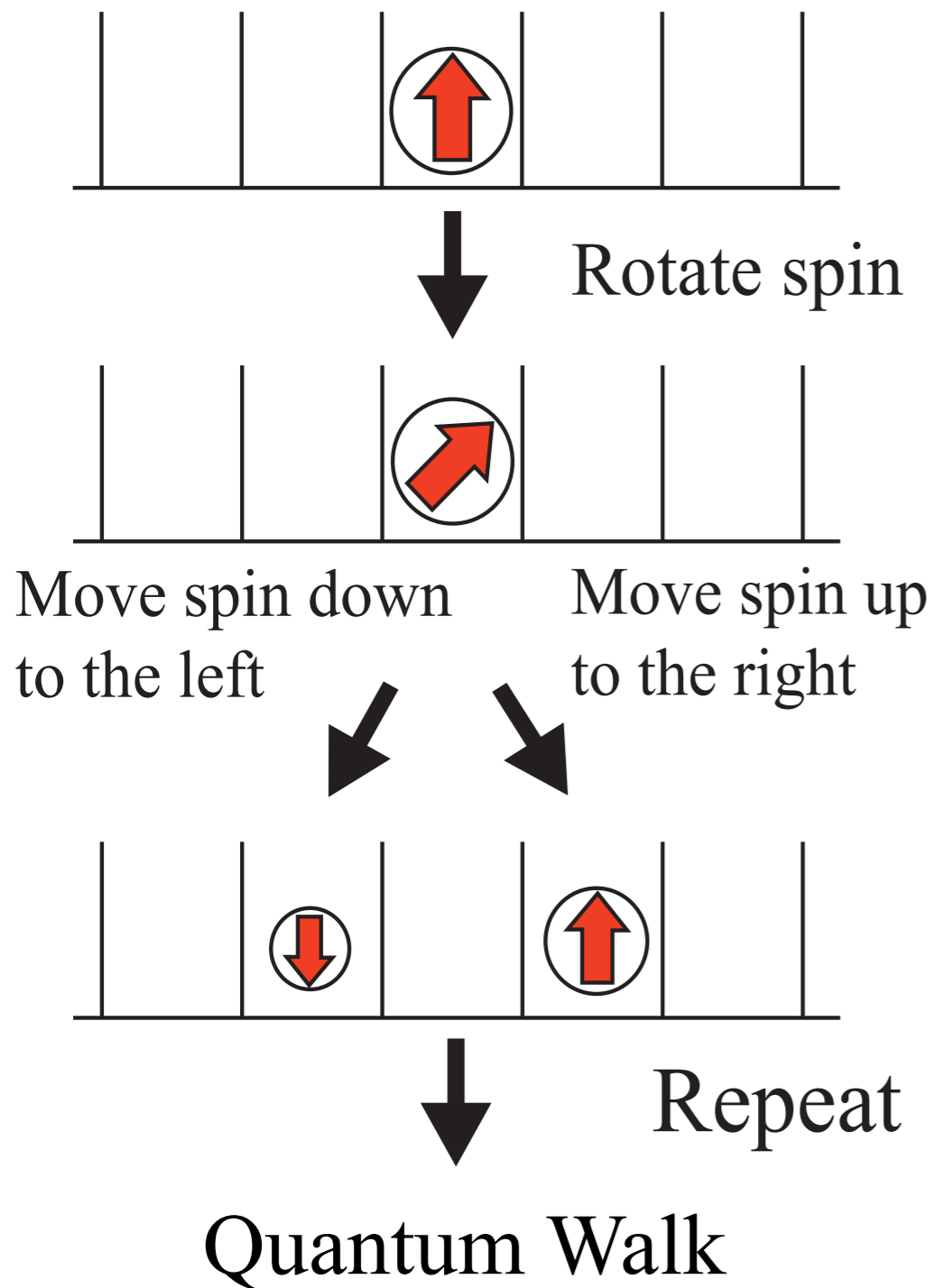
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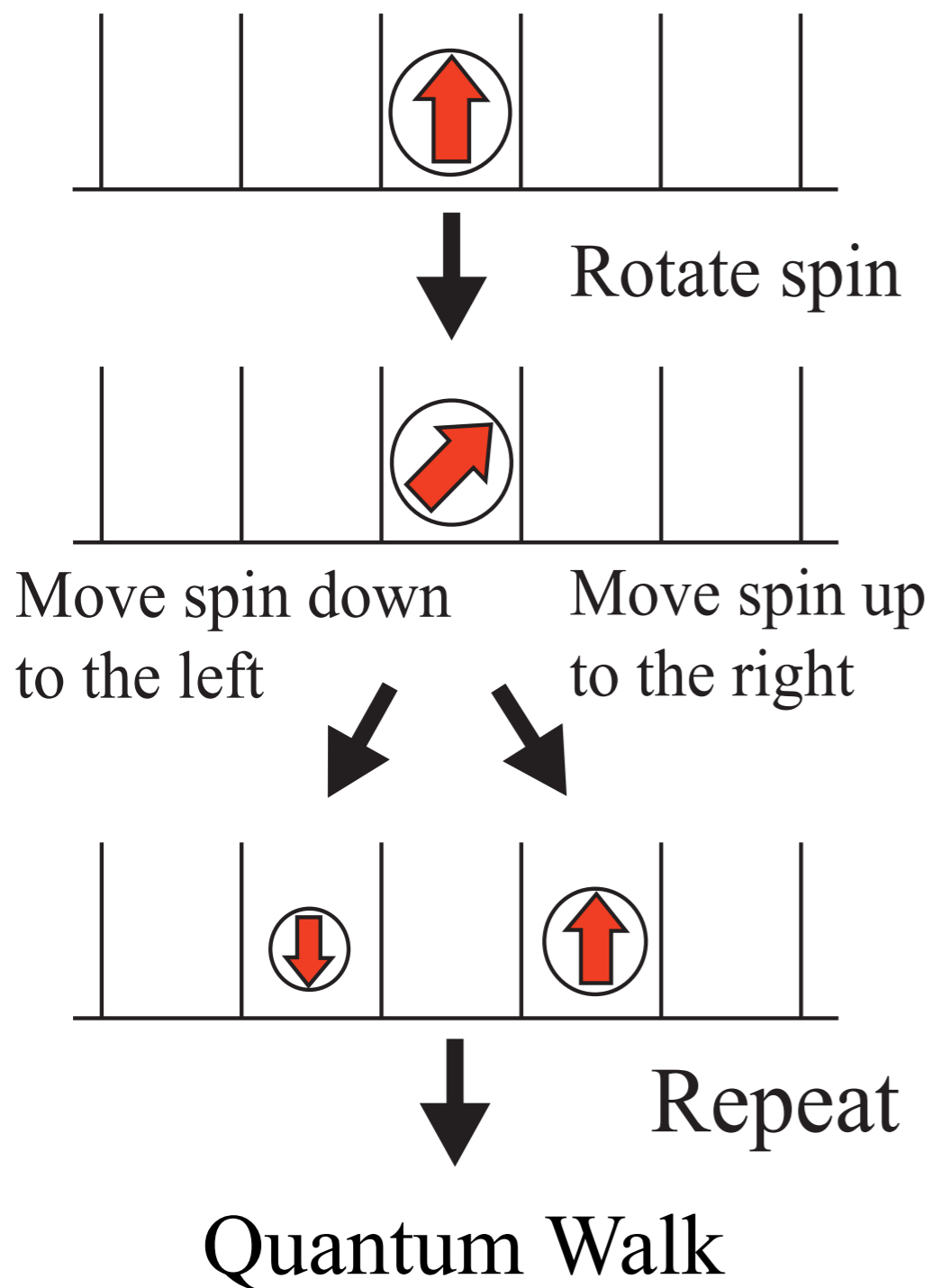
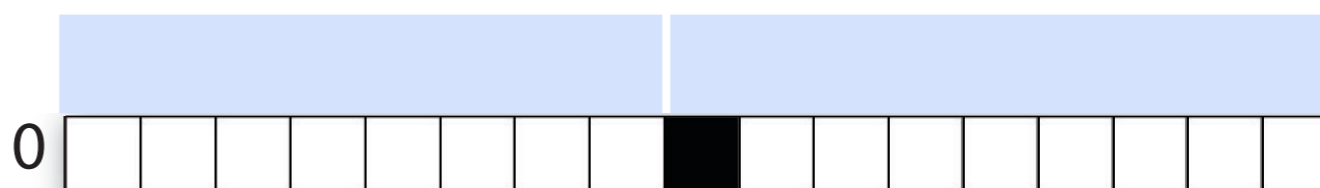
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Su-Schrieffer-Heeger model  
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Topologically protected  
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phase 1

phase 1



Step

Probability

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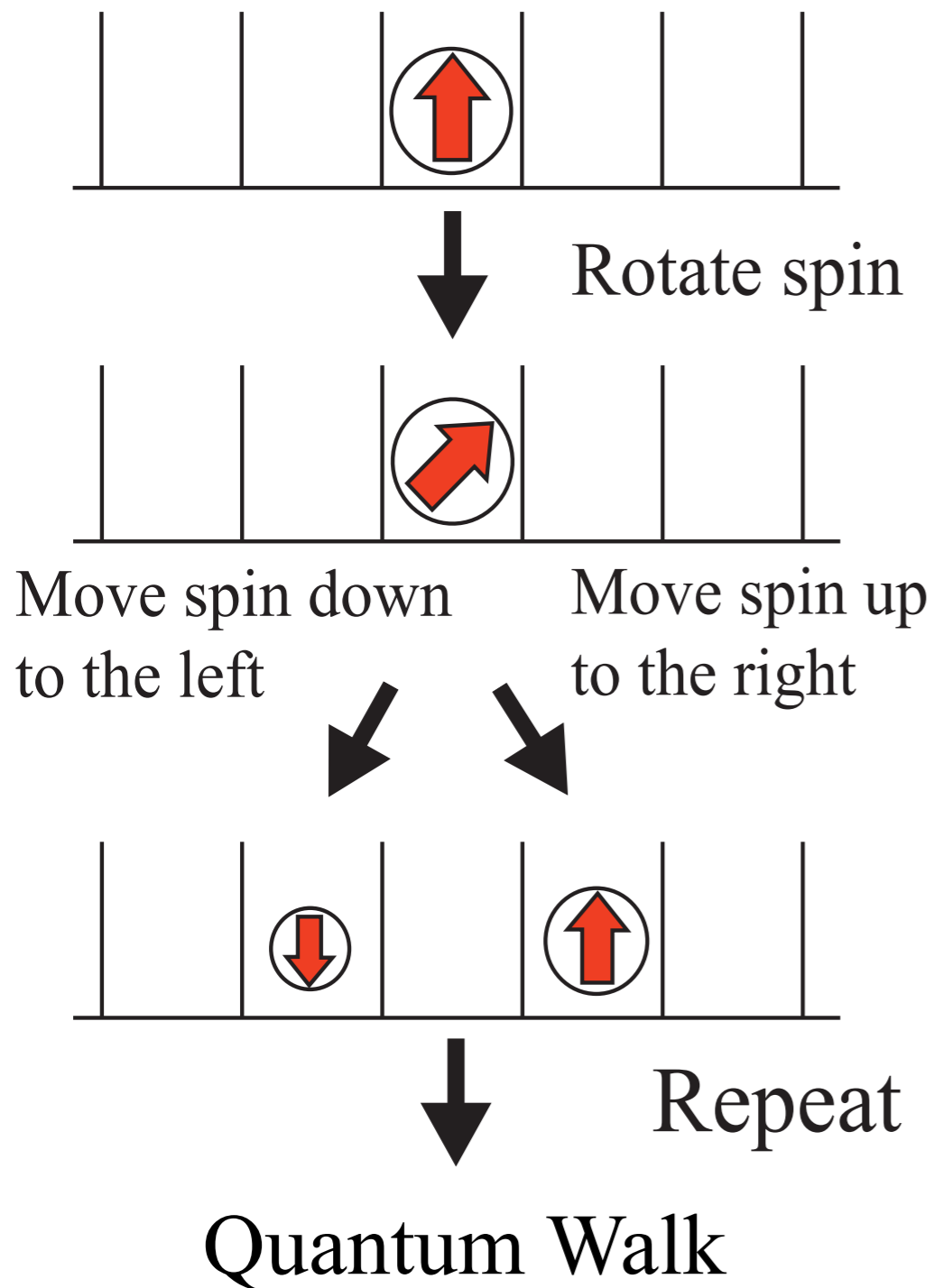
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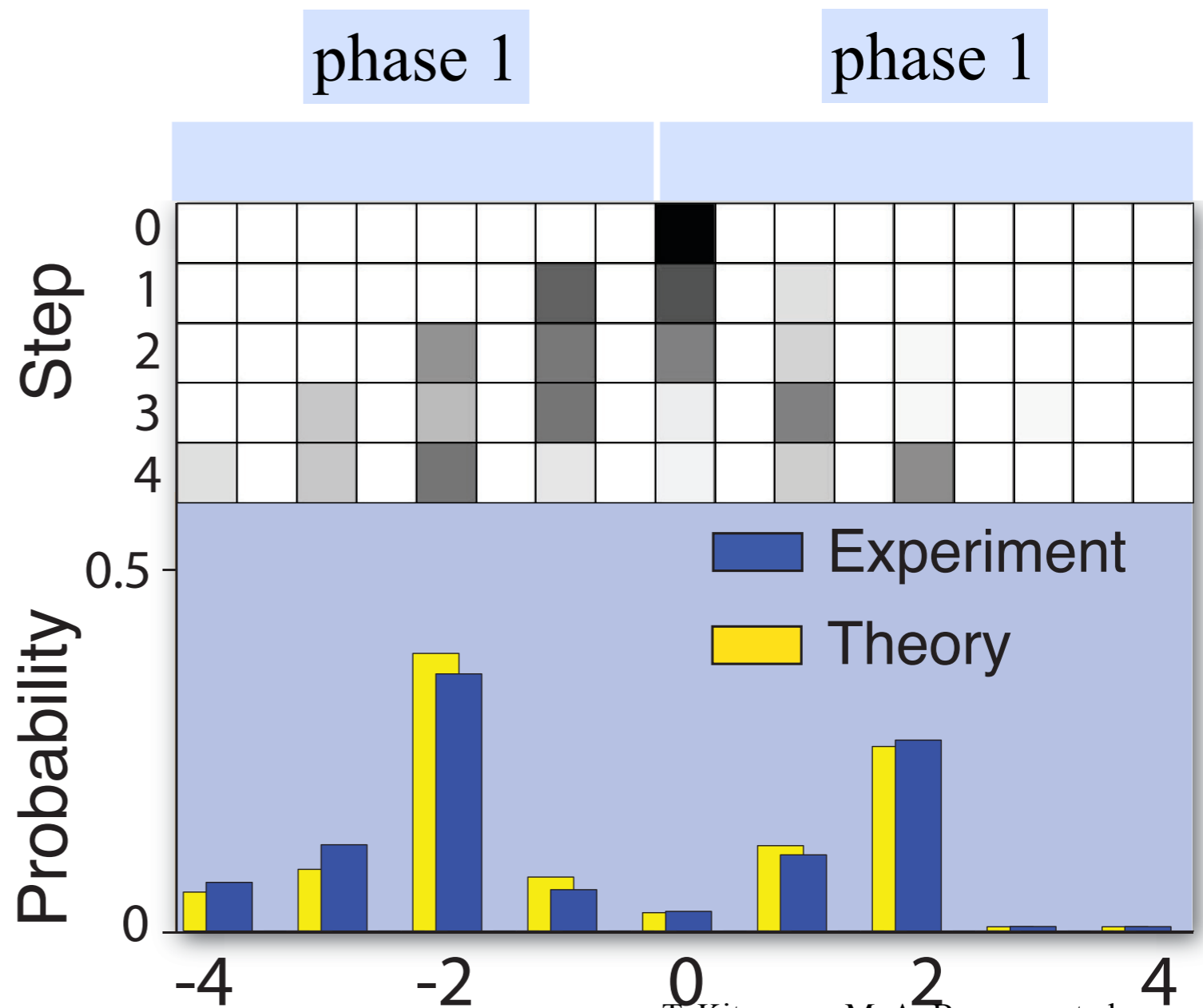
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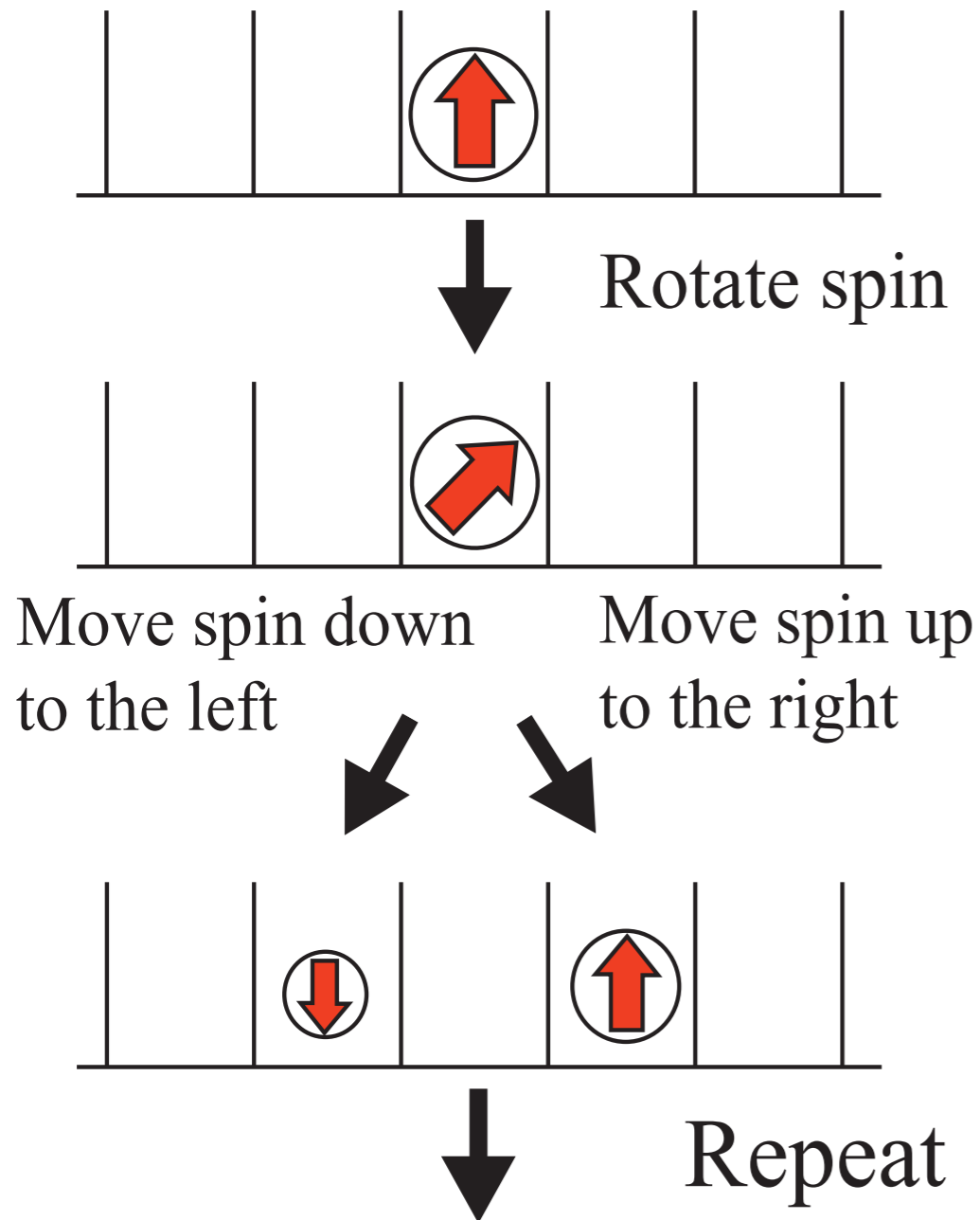
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## Quantum Walk

quantum walks allows versatile realizations of topological phases

		Particle-Hole Symmetry			Particle-Hole Symmetry		
		+1	-1	$\times$	+1	-1	$\times$
Time-Reversal Symmetry	+1	$Z$ SSH					
	-1	$Z_2$	$Z$		$Z_2$		$Z_2$ QSH
	$\times$	$Z_2$			$Z$ Chiral	$Z$	$Z$ IQH
		1D			2D		

# Outline

1. Overview of topological phases
2. Topology of periodically driven systems
  - a. Design of static topology through driving
- 3. Examples from driven hexagonal lattice system;  
*Graphene under circularly polarized light***
4. Detection scheme of topological band
5. Conclusion

# Photo-induced integer quantum Hall effect without Landau levels

Non-equilibrium realization of Haldane model

Condensed matter realization

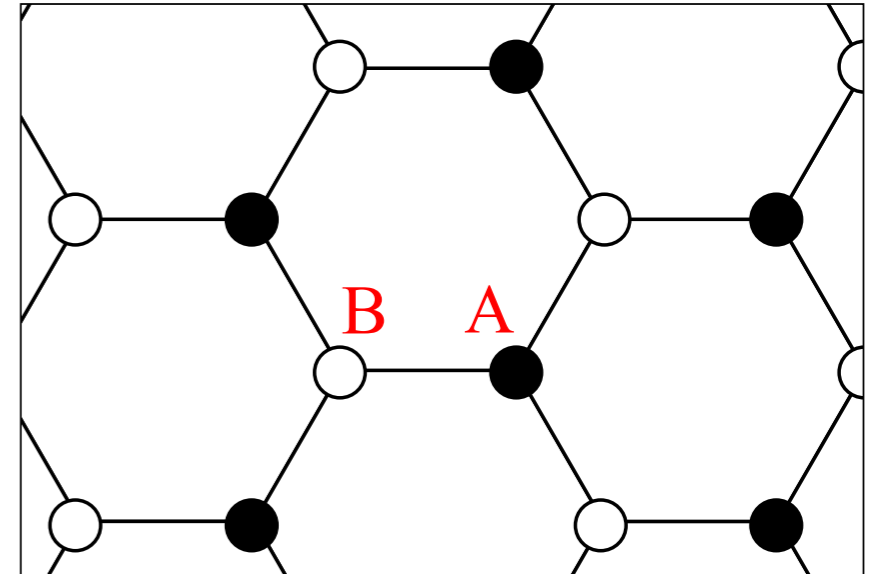
T. Kitagawa, et al, [Phys. Rev. B 84, 235108 \(2011\)](#)

# Driven graphene

$$H(t) = -J \sum_{\langle ij \rangle} e^{iA_{ij}(t)} c_i^\dagger c_j,$$

$$A_{ij}(t) = A \vec{r}_{ij} \cdot (\cos(t\Omega), \sin(t\Omega))$$

Circularly polarized light

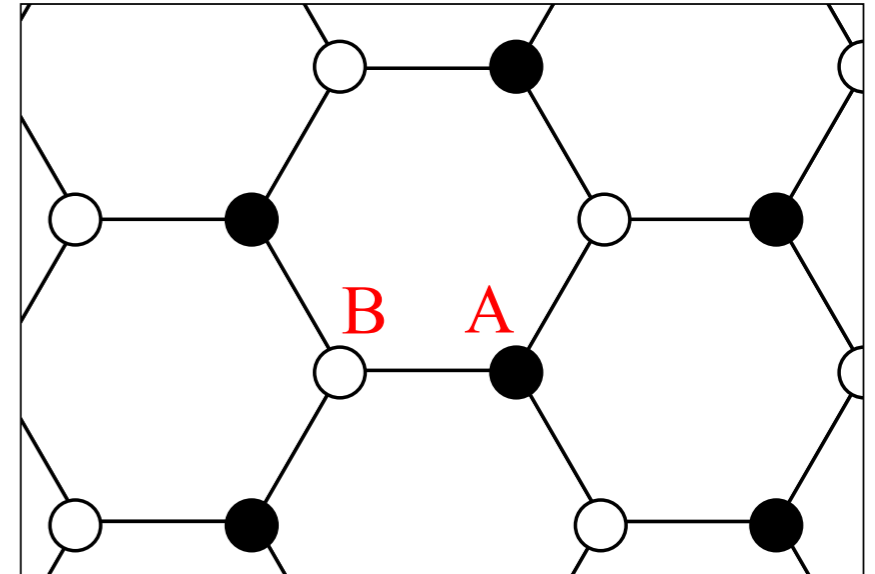




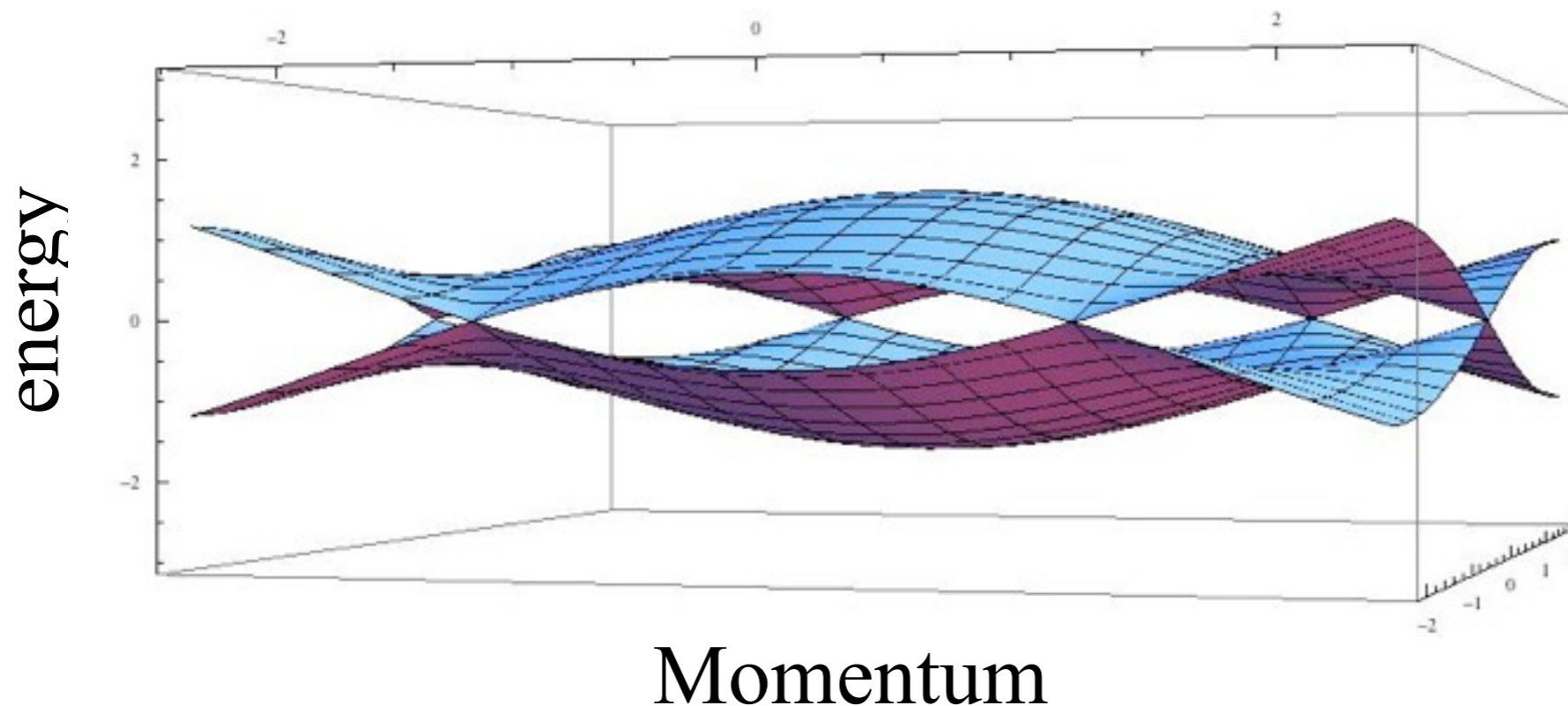
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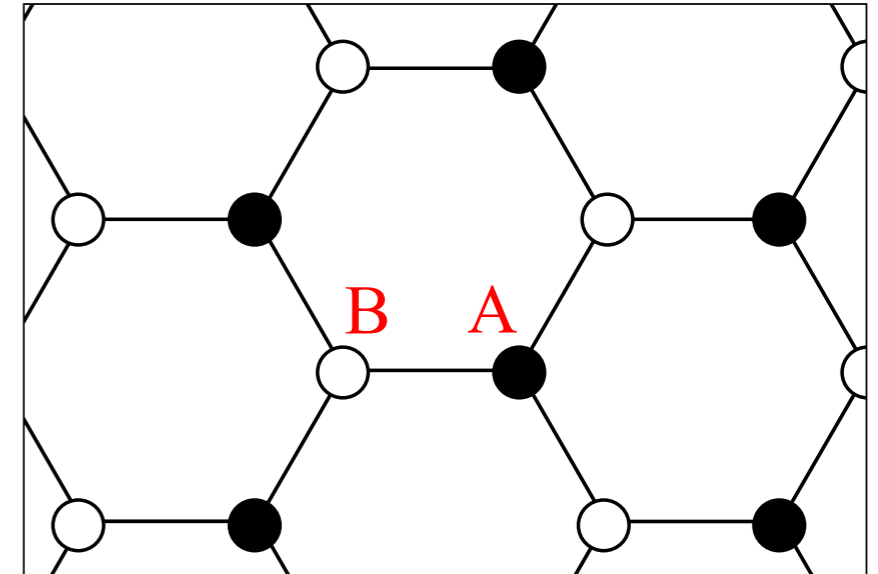
In the absence of light



# Driven graphene

$$H(t) = -J \sum_{\langle ij \rangle} e^{iA_{ij}(t)} c_i^\dagger c_j,$$

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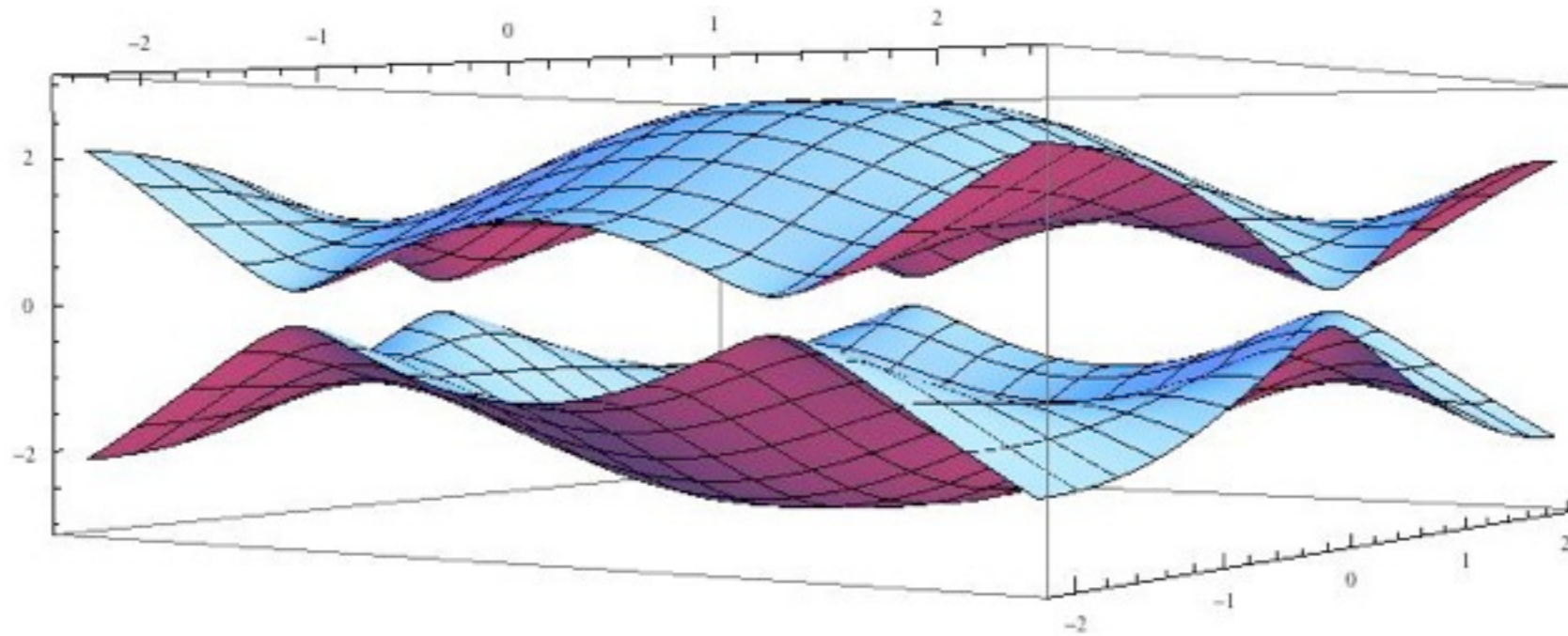


Circularly polarized light

Assume the driving frequency  $\Omega \gg J$

$$A \neq 0$$

Quasi-energy

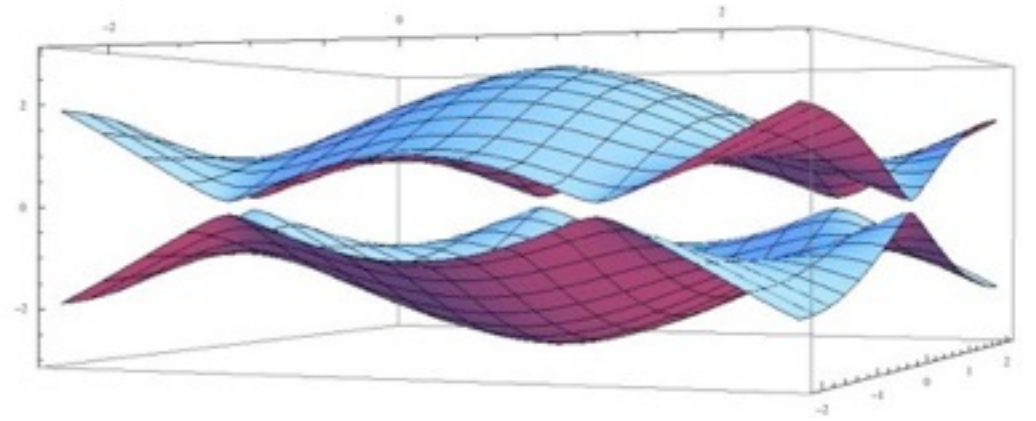


Momentum

c.f. Oka and Aoki, Phys. Rev. B 79, 081406 (R) (2009), see also Erratum

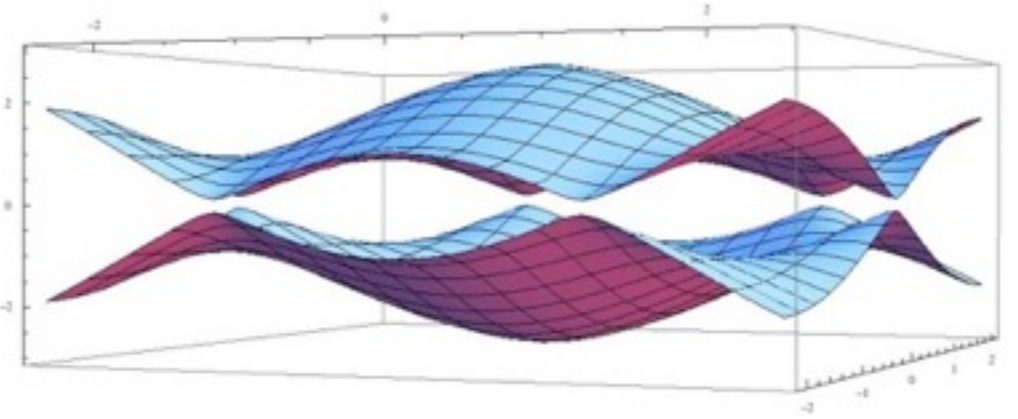
# Effective Hamiltonian and Chern numbers

$$U(T) = \mathcal{T} e^{-i \int_0^T H(t) dt} \equiv e^{-i H_{\text{eff}} T}$$



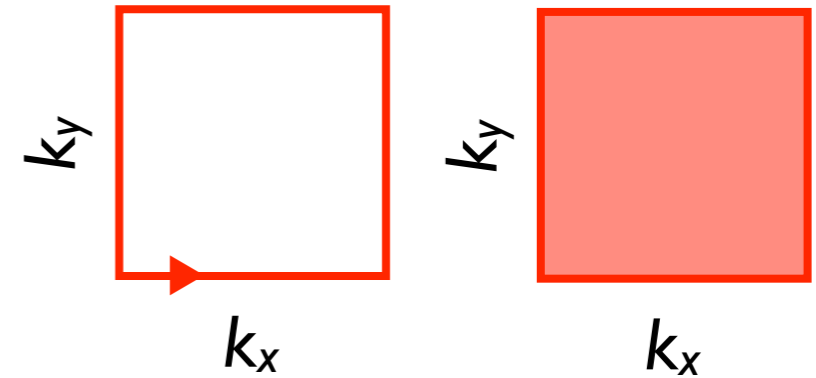
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Chern number

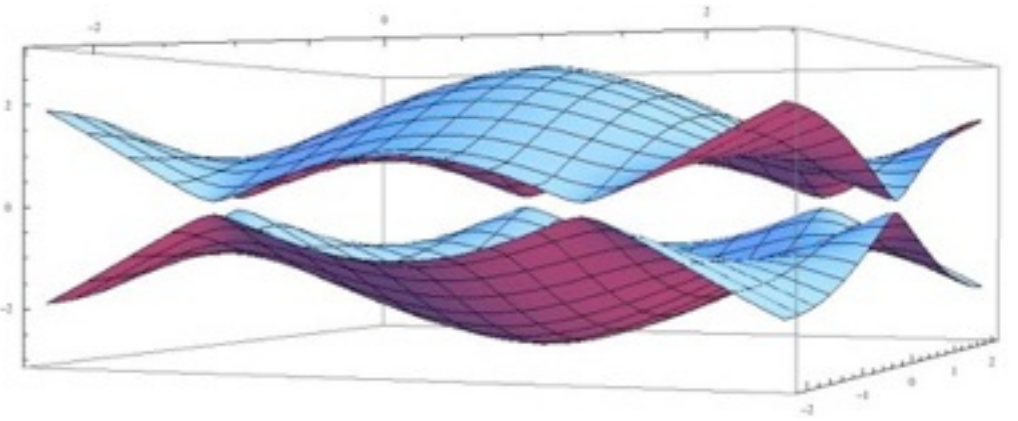
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Phase accumulation of adiabatic move

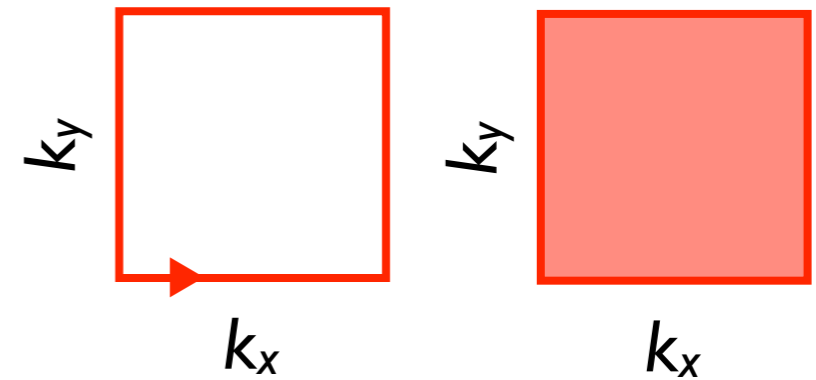
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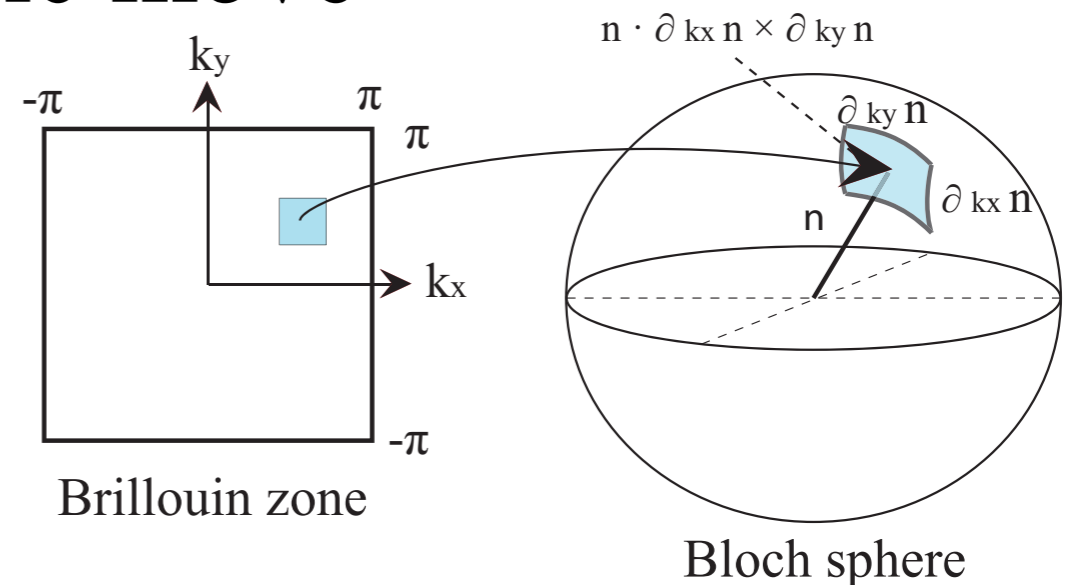
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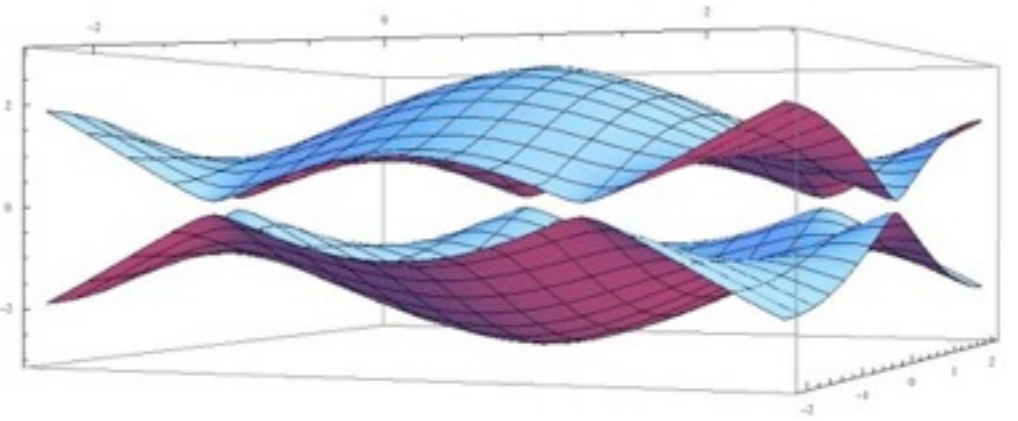
$$H_{\text{eff}}(\mathbf{k}) = \varepsilon(\mathbf{k}) \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

$$C_{\pm} = \frac{\pm 1}{4\pi} \int_{\text{FBZ}} \mathbf{n} \cdot (\partial_{k_x} \mathbf{n} \times \partial_{k_y} \mathbf{n}) d^2 \mathbf{k}$$



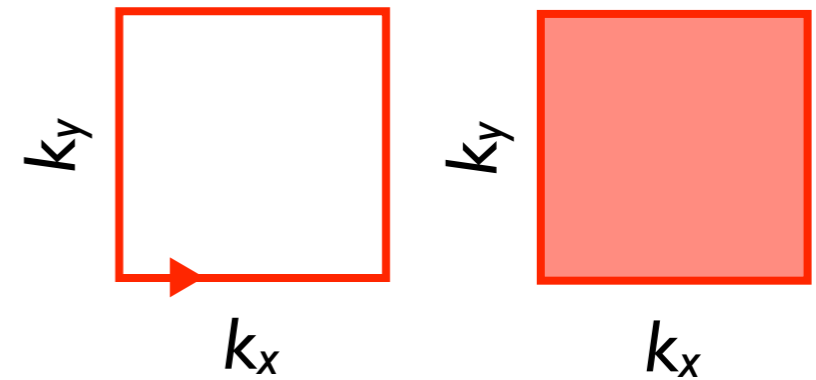
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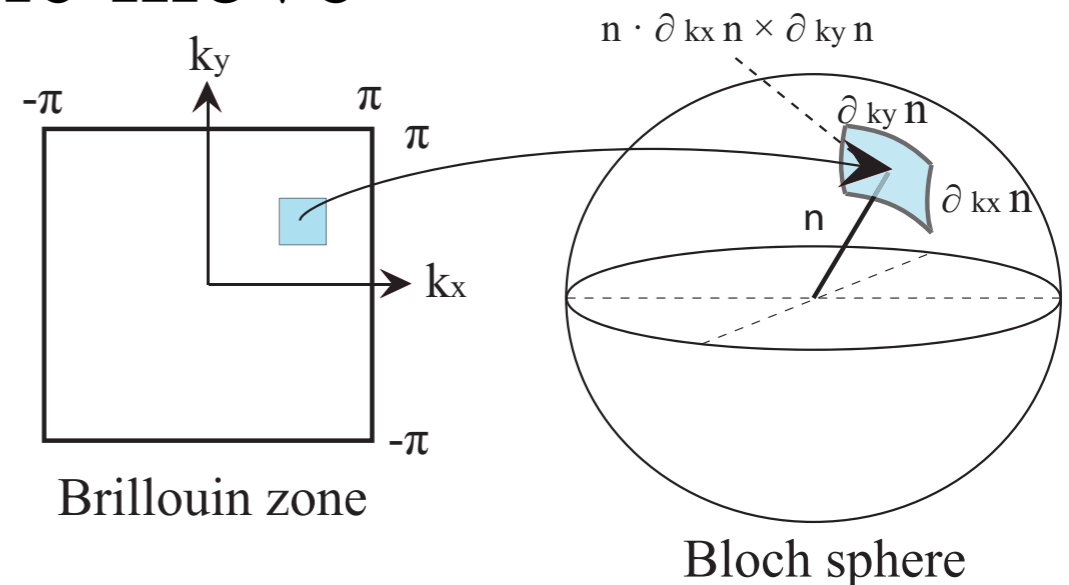
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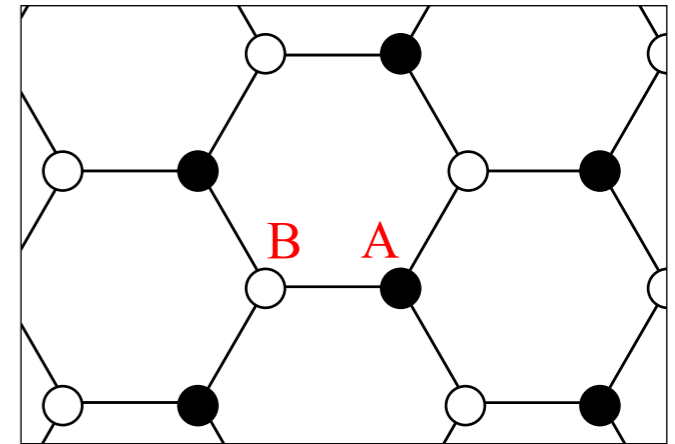
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**± 1 !**

# Effective Hamiltonian from perturbation theory

$$H(t) = -J \sum_{\langle ij \rangle} e^{iA_{ij}(t)} c_i^\dagger c_j,$$
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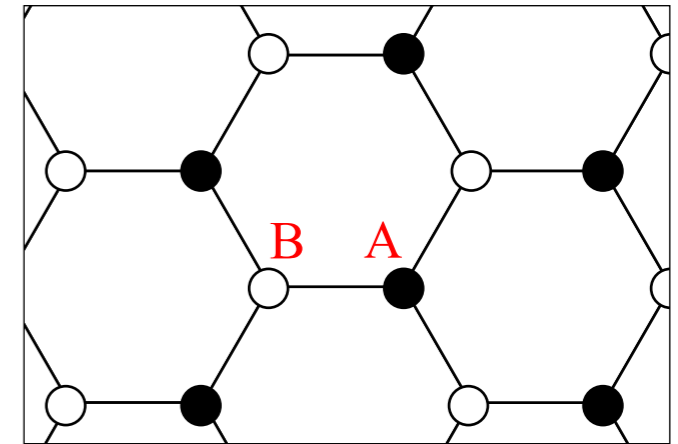


Assume high frequency  $\Omega \gg J$  and weak intensity  $\mathcal{A} = eAa/\hbar \ll 1$

# Effective Hamiltonian from perturbation theory

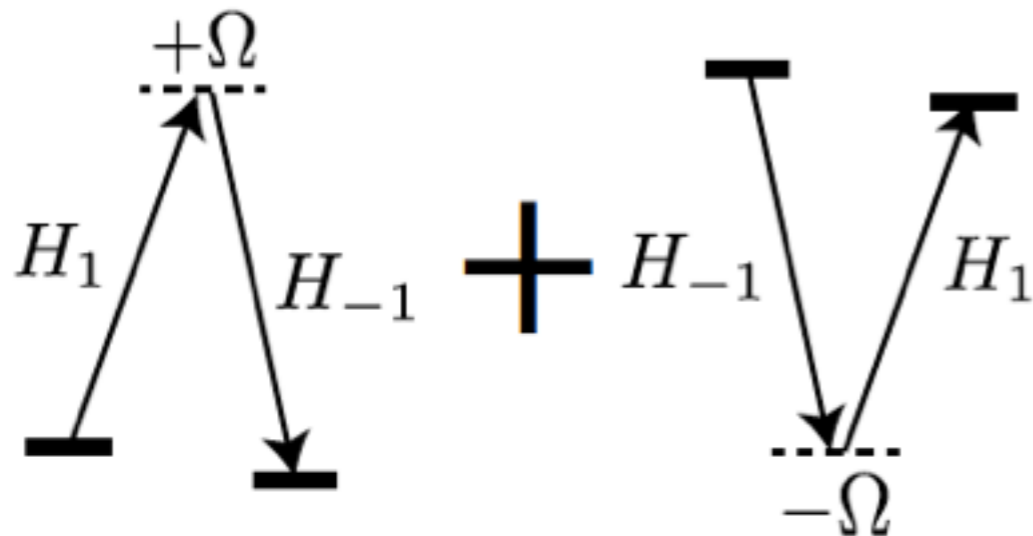
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$$H_{\pm 1} = \frac{1}{T} \int_0^T H(t) e^{\pm it\Omega} dt$$

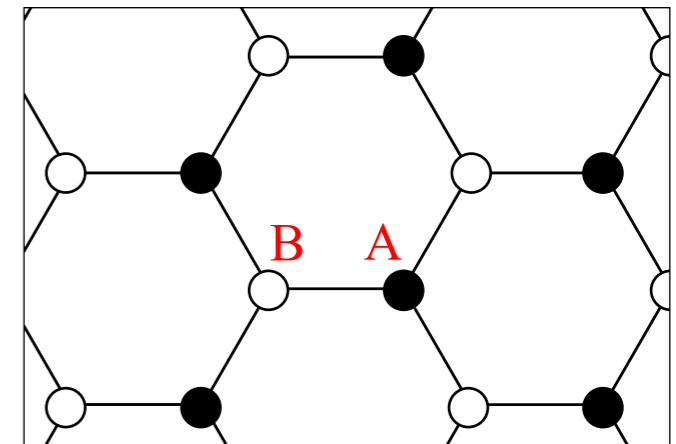




# Effective Hamiltonian from perturbation theory

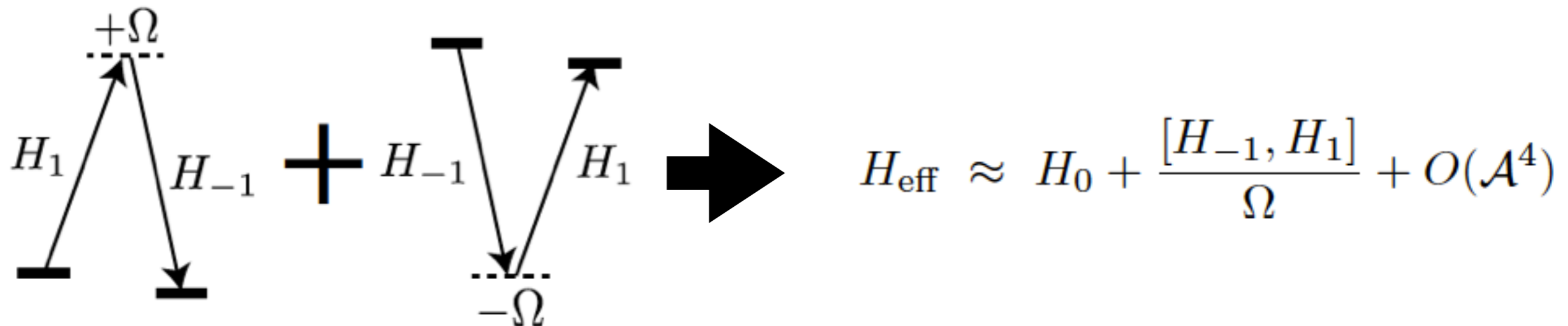
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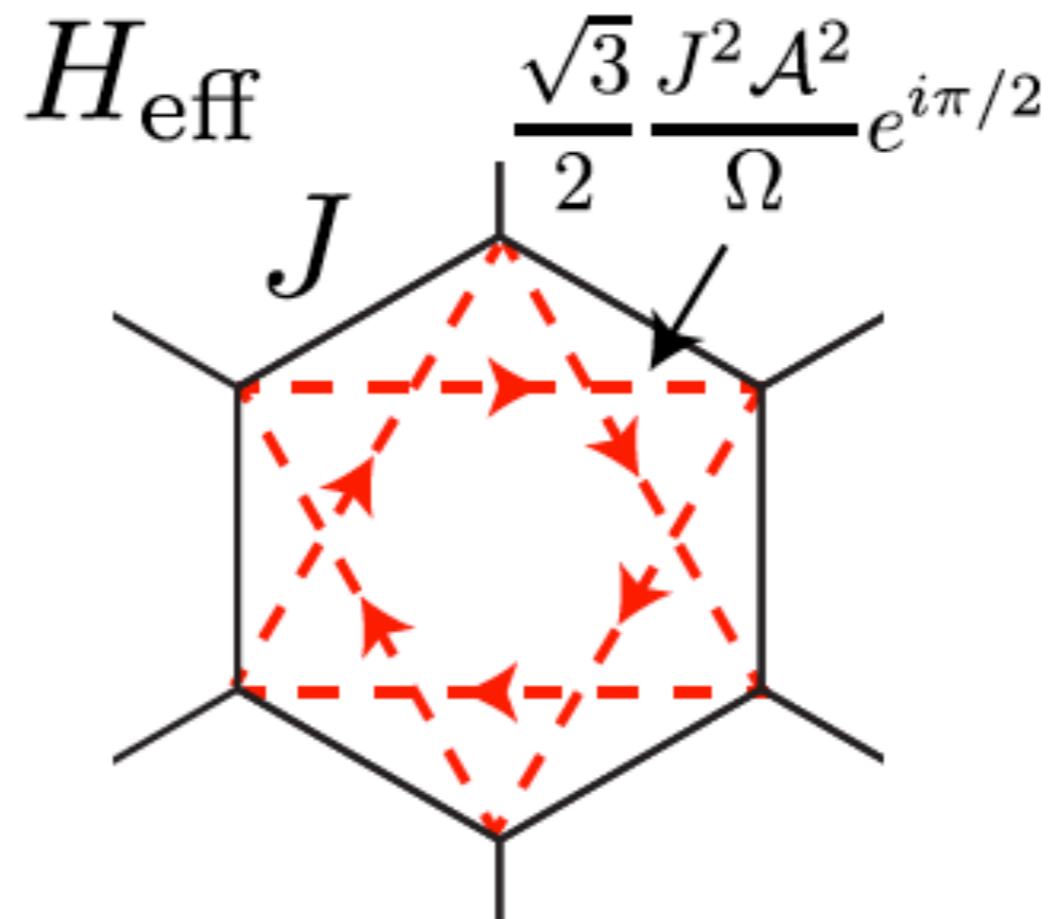


# Real space picture of effective Hamiltonian

$$H(t) = -J \sum_{\langle ij \rangle} e^{iA_{ij}(t)} c_i^\dagger c_j, \quad H_{\text{eff}} \approx H_0 + \frac{[H_{-1}, H_1]}{\Omega}$$

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Non-equilibrium realization of Haldane model!

F.D.M. Haldane, Phys. Rev. Lett. 61, 2015(1998)

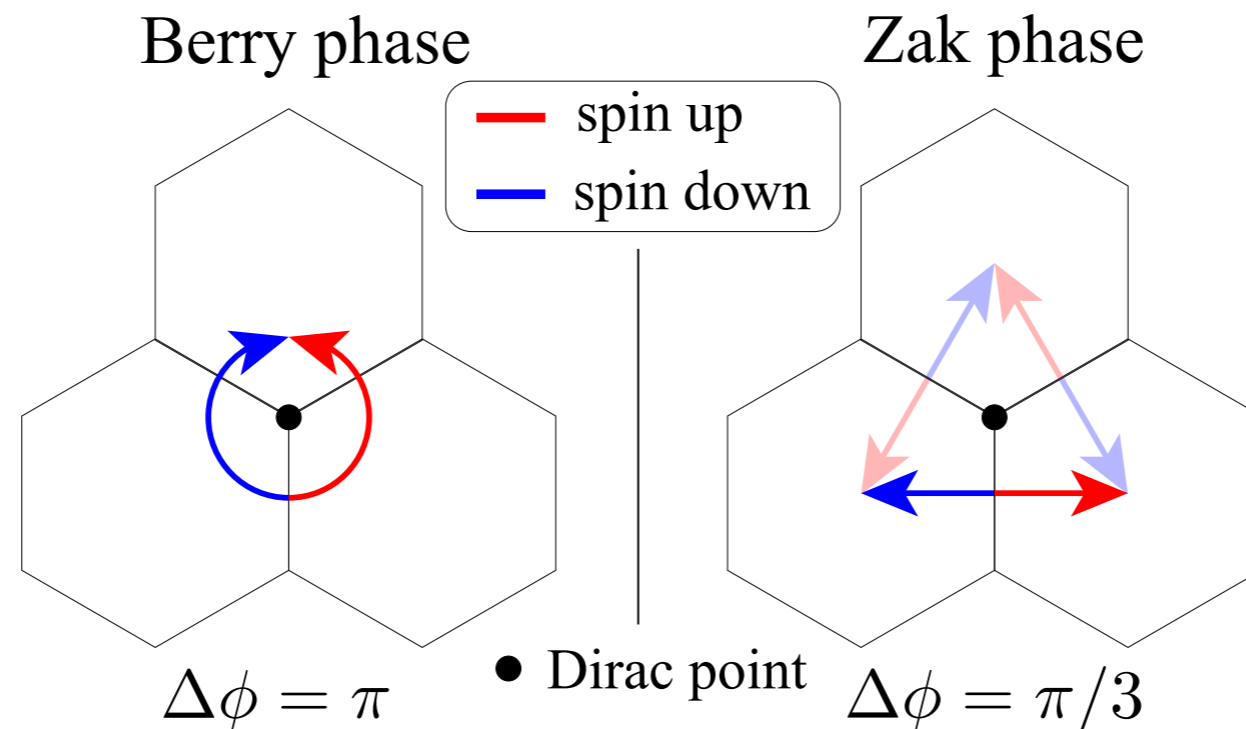
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# Measurement of topology of bands

Theory : D. Abanin, T. Kitagawa, and E. Demler

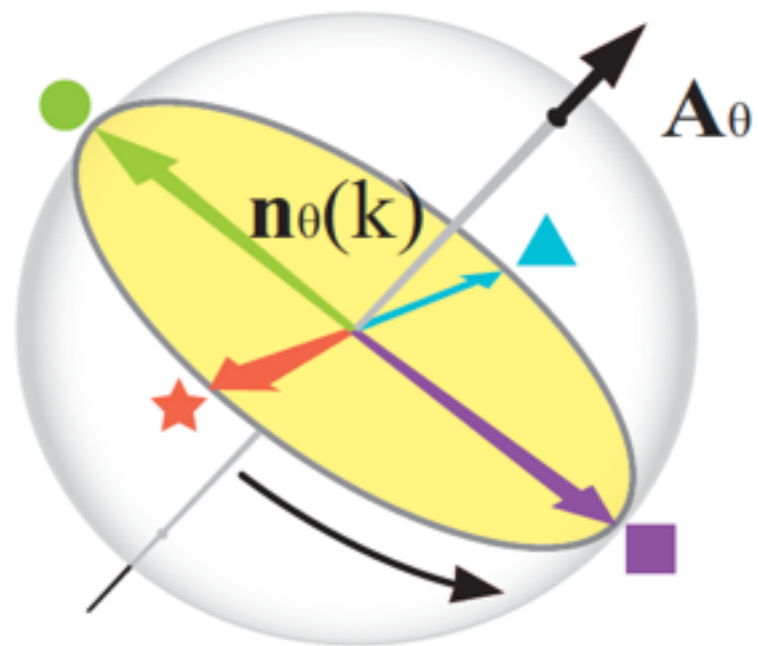
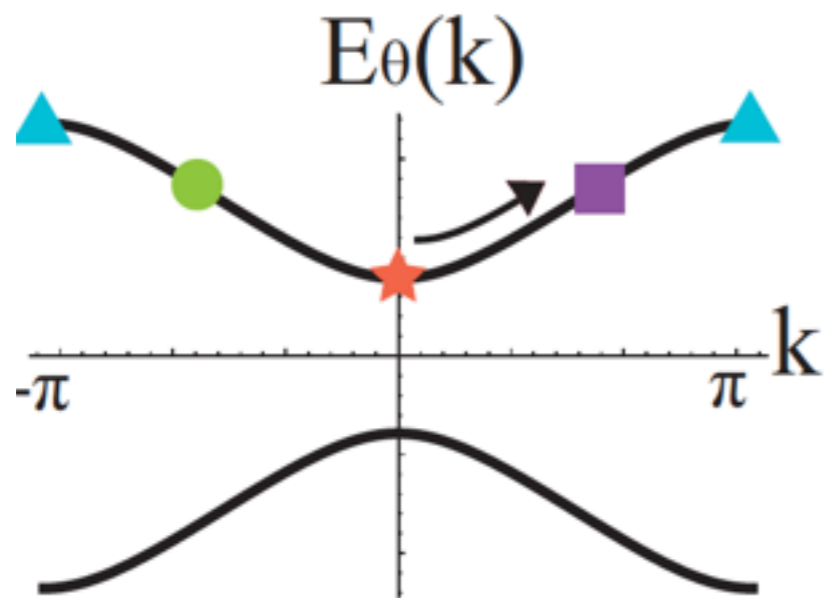
Experiment: M. Aidelsburger, M. Atala, J. Barreiro, I. Bloch



# Topology of bands

# Topology of bands

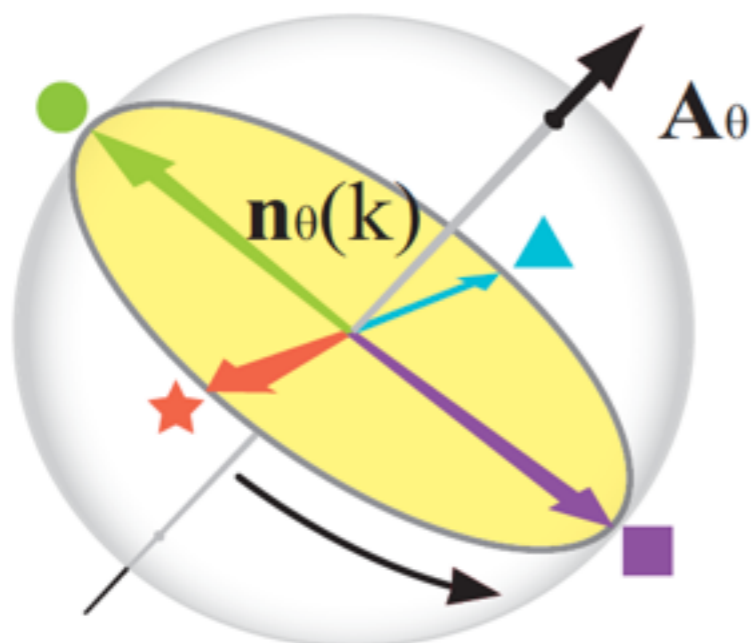
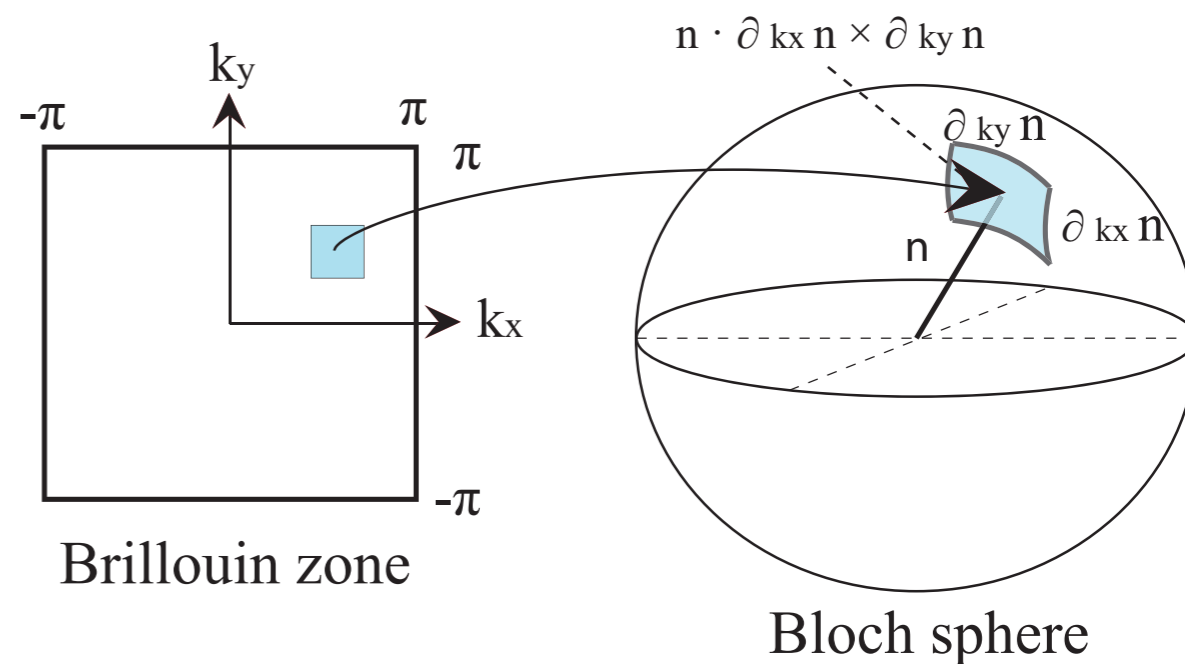
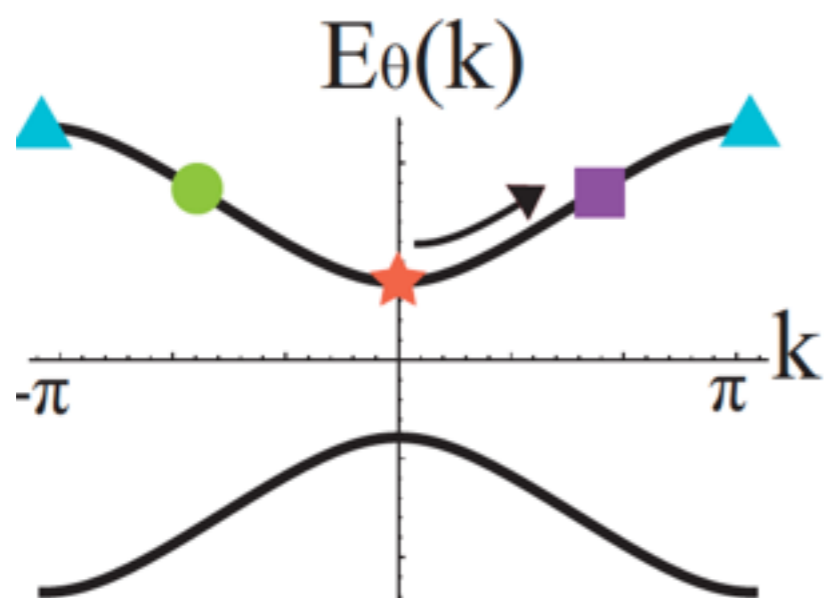
## 1D:SSH phase



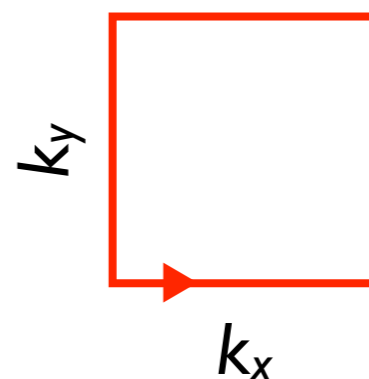
# Topology of bands

1D:SSH phase

2D: IQH

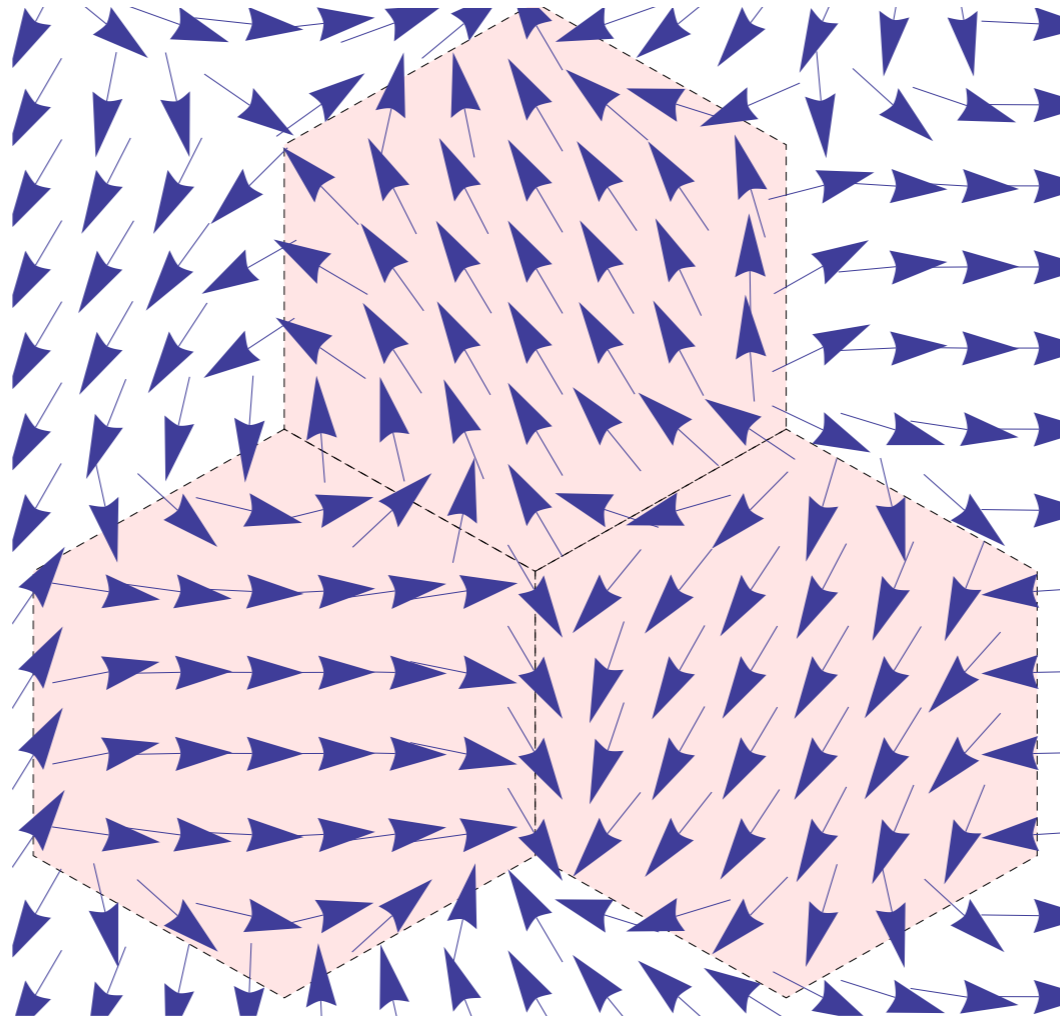


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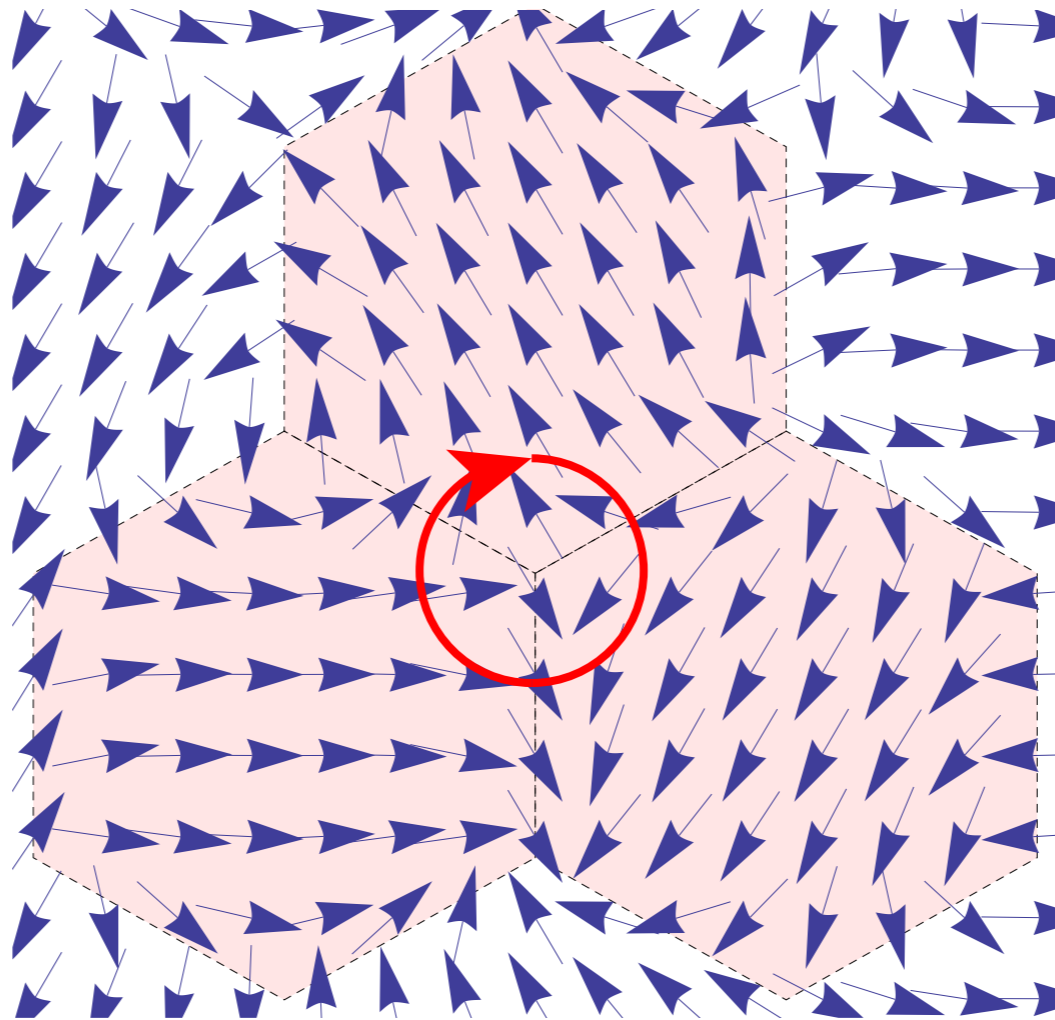




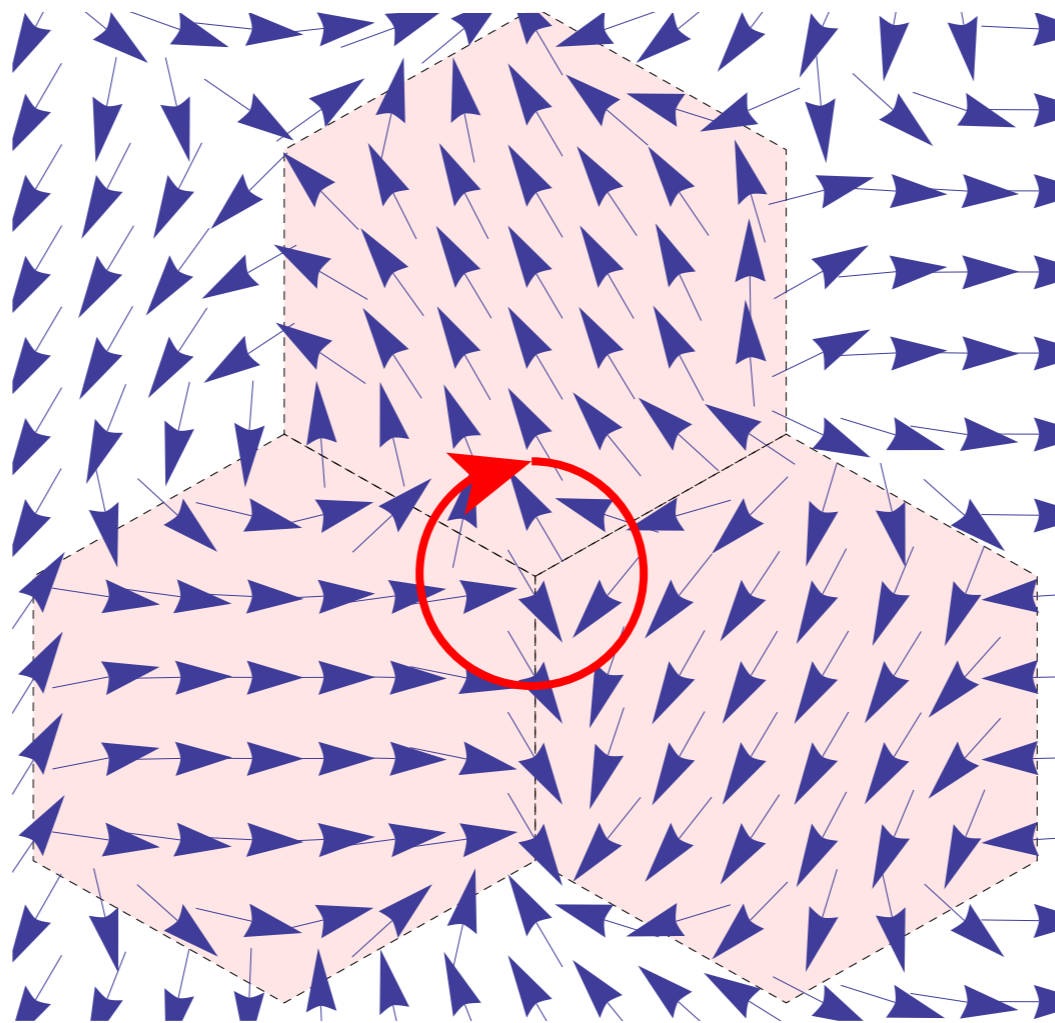
# Naive measurement



# Naive measurement

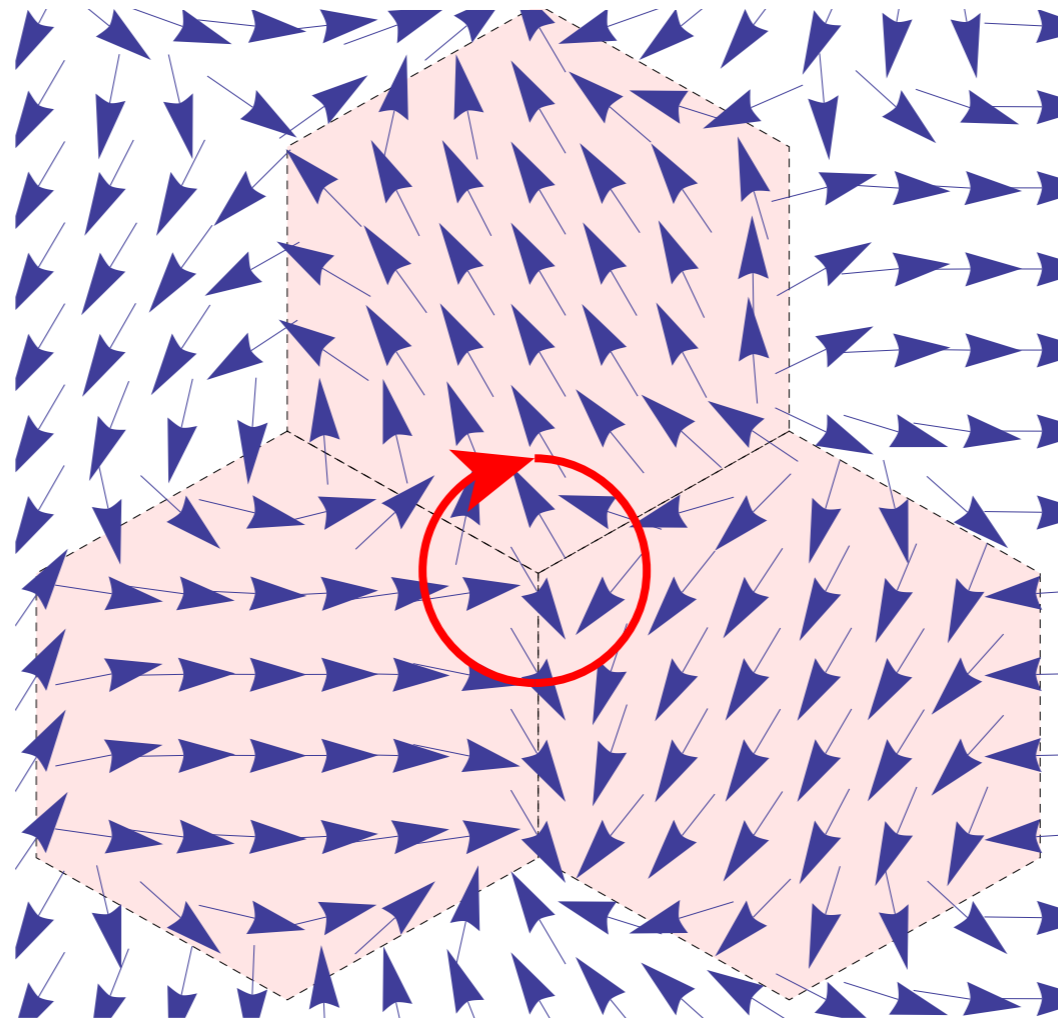


# Naive measurement



$$\Delta\phi = \pi$$

# Naive measurement

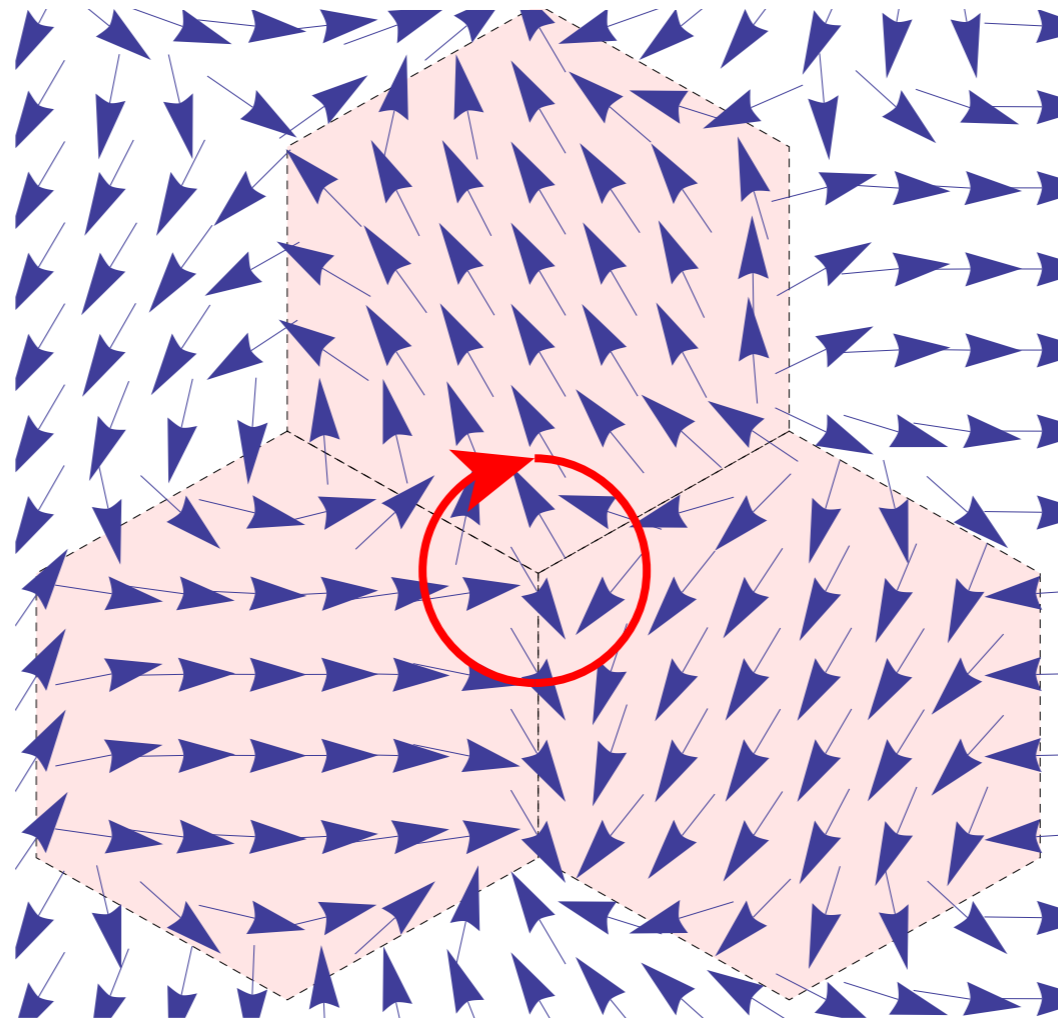


$$\Delta\phi = \pi$$

**Problem with this naive approach**

1. Trajectory is curved and complicated.

# Naive measurement

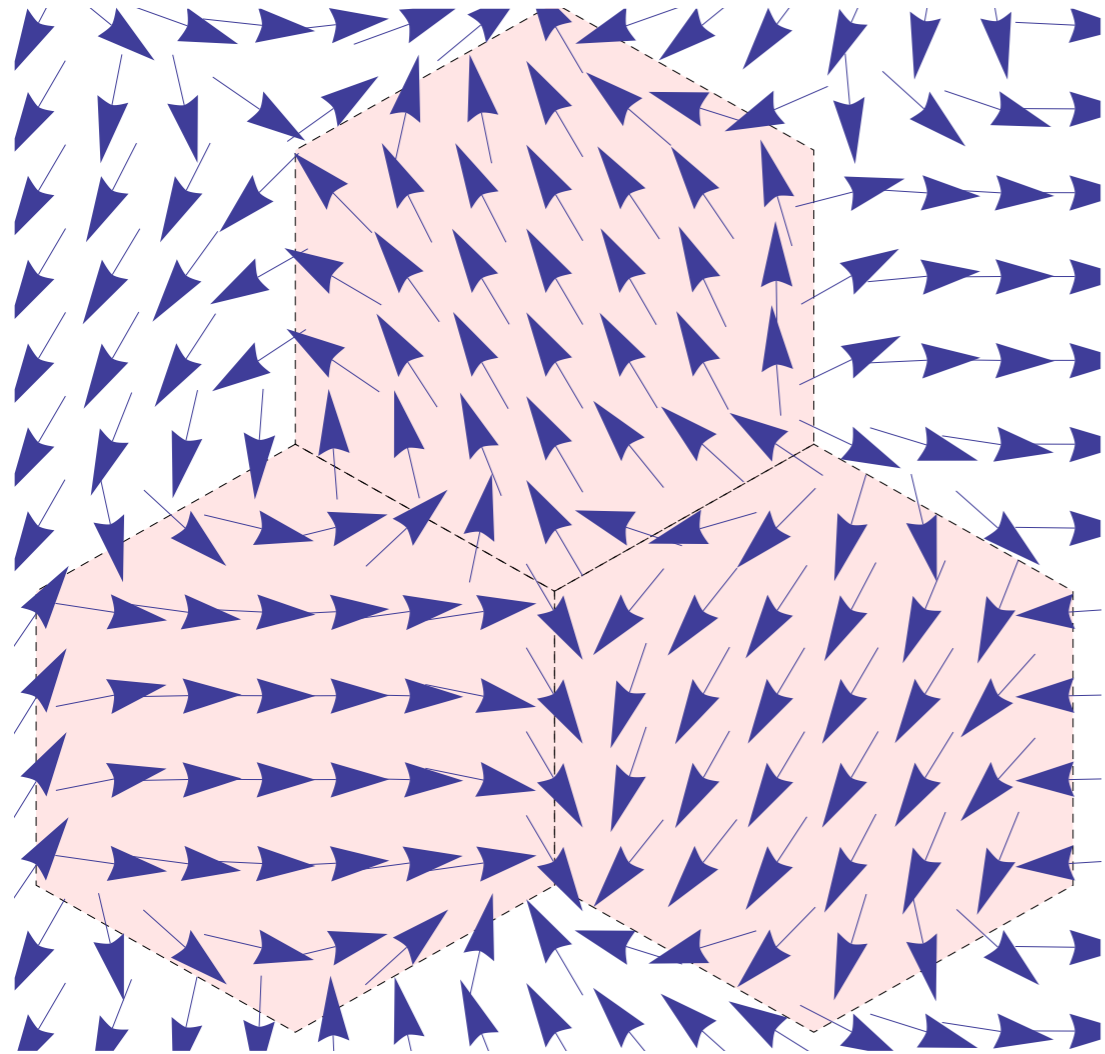


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## Problem with this naive approach

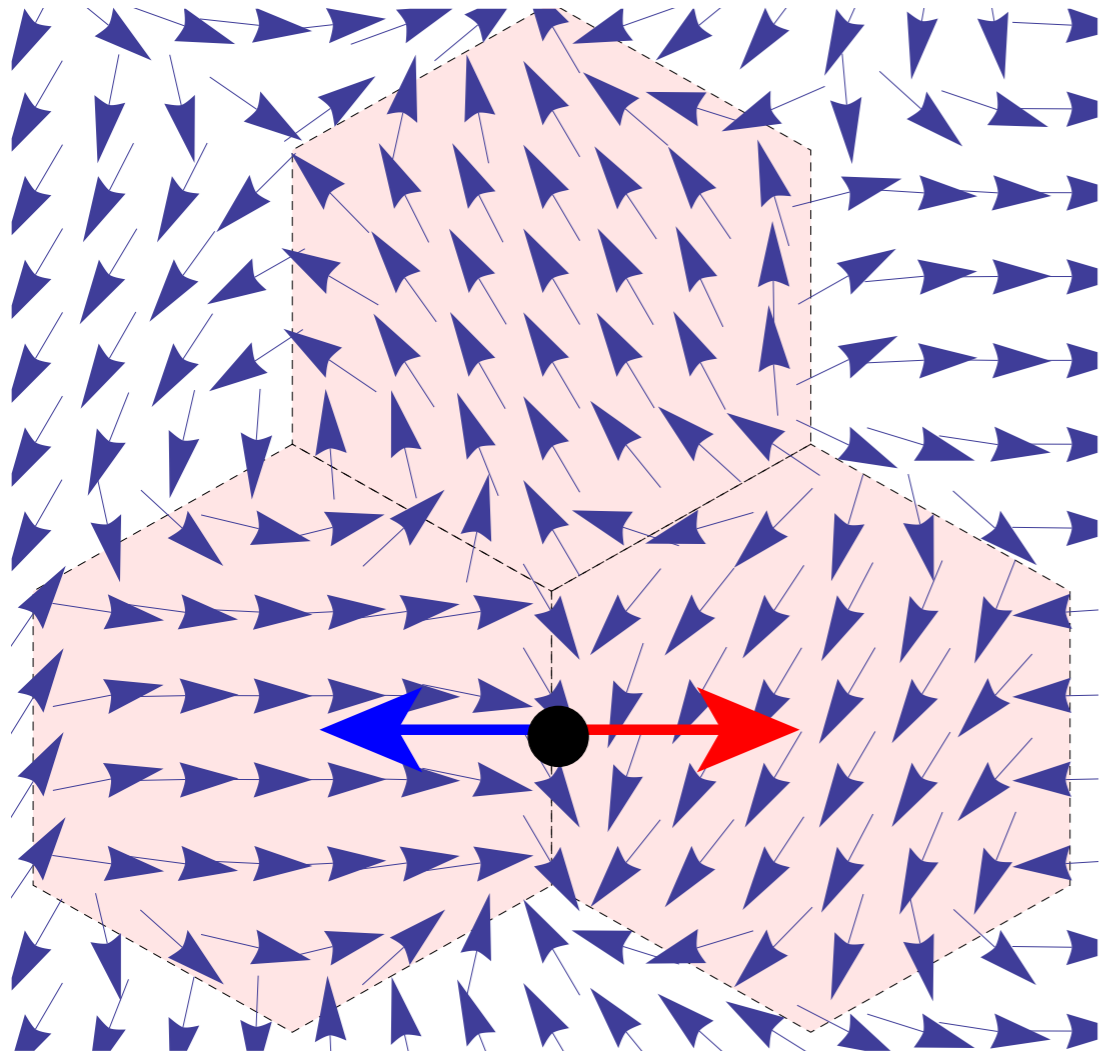
1. Trajectory is curved and complicated.
2. Need to separate dynamical phase from Berry phase

# Measurement with Ramsey-type interferometer



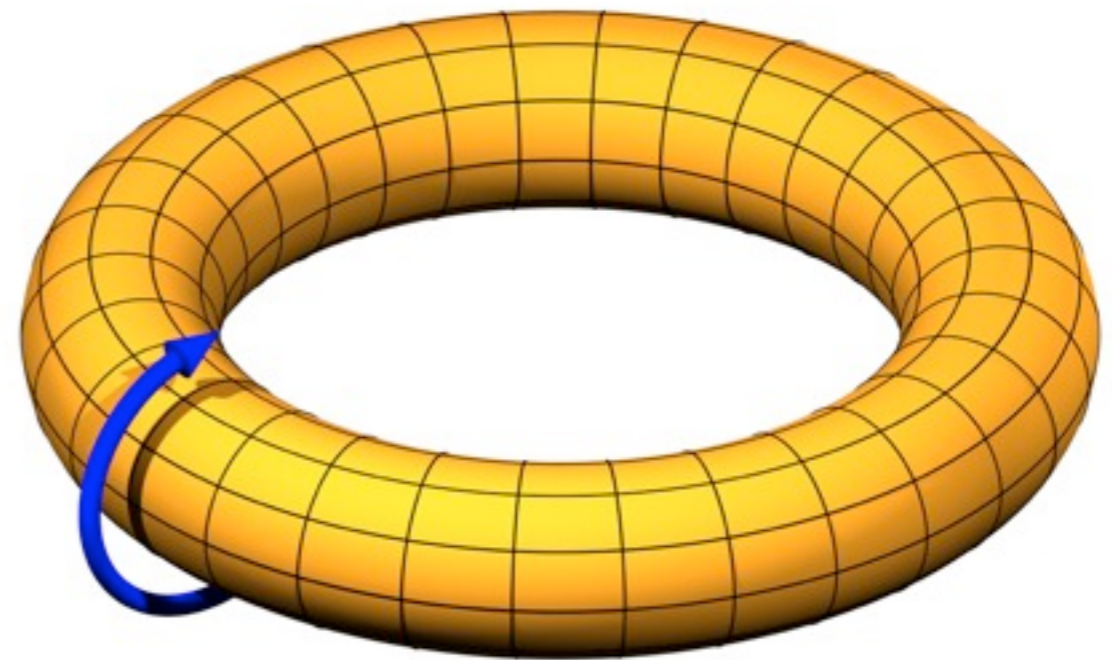
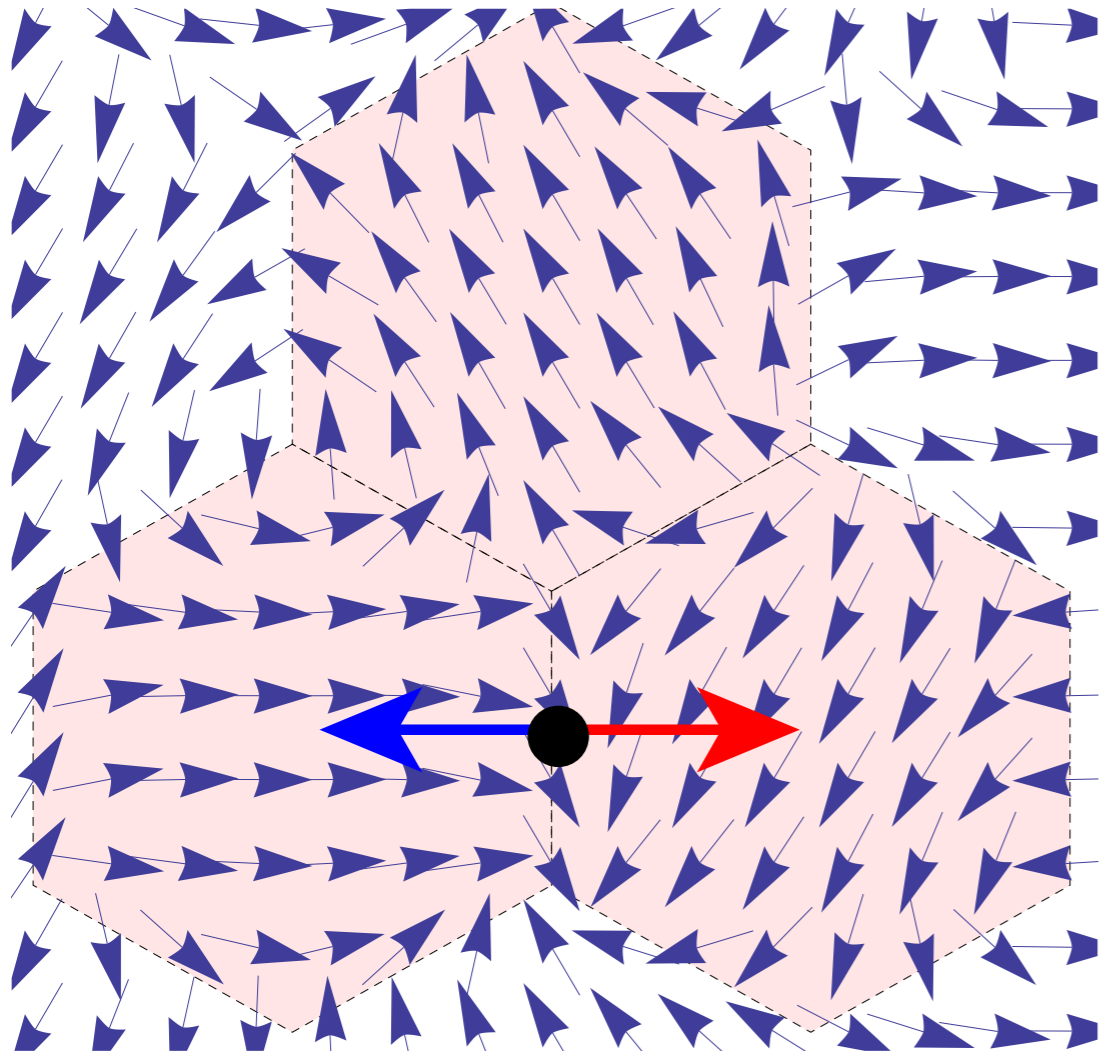
# Measurement with Ramsey-type interferometer

Use two hyperfine states



# Measurement with Ramsey-type interferometer

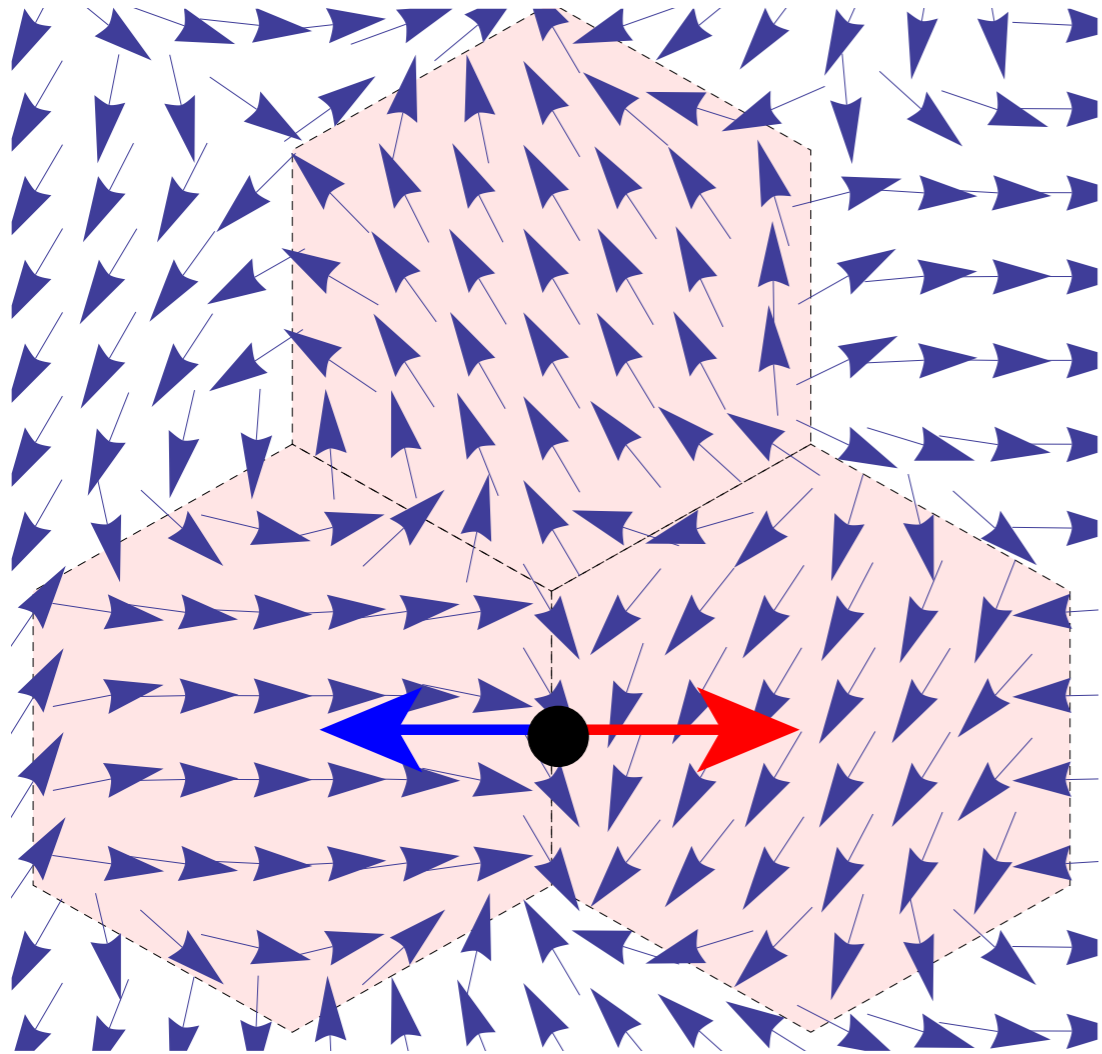
Use two hyperfine states





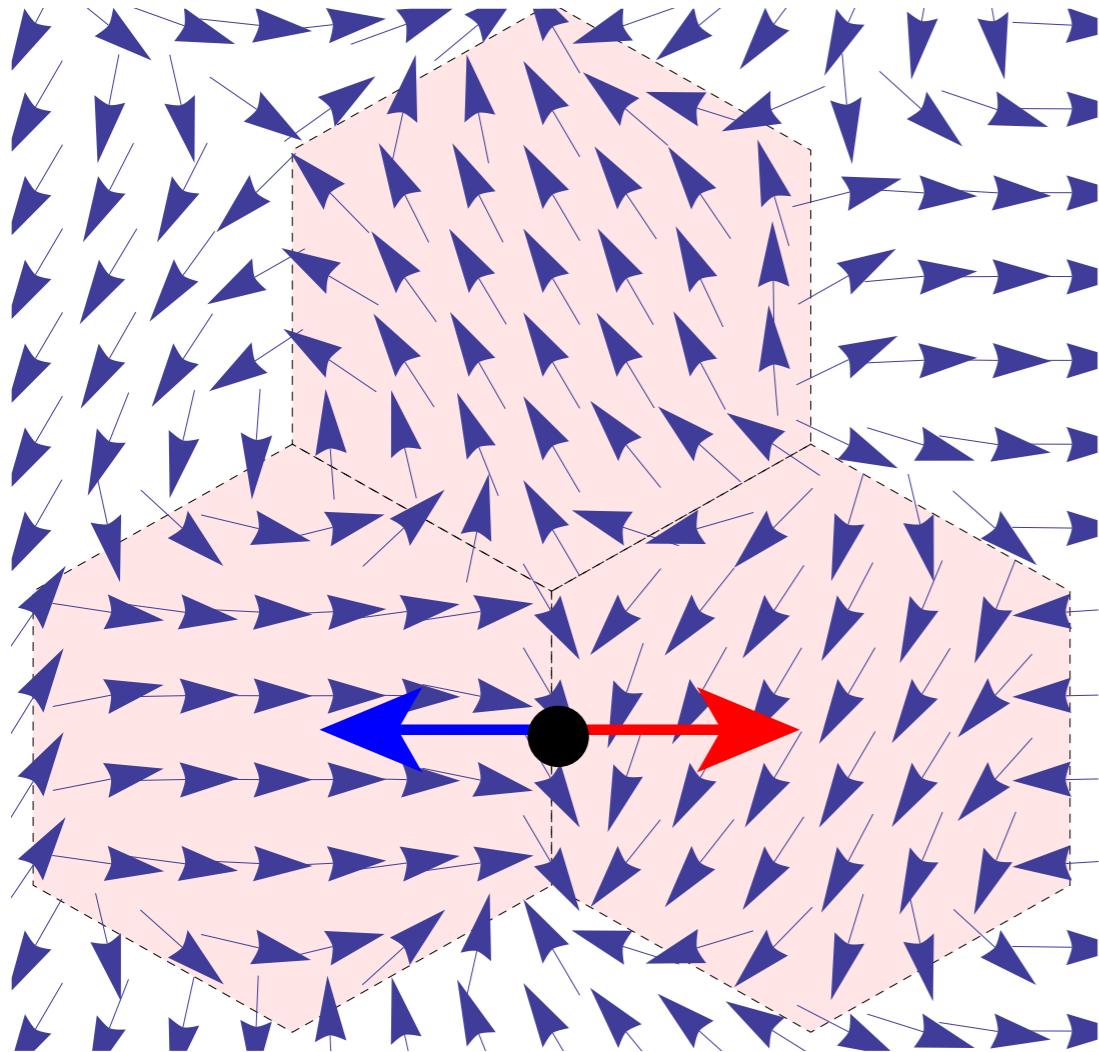
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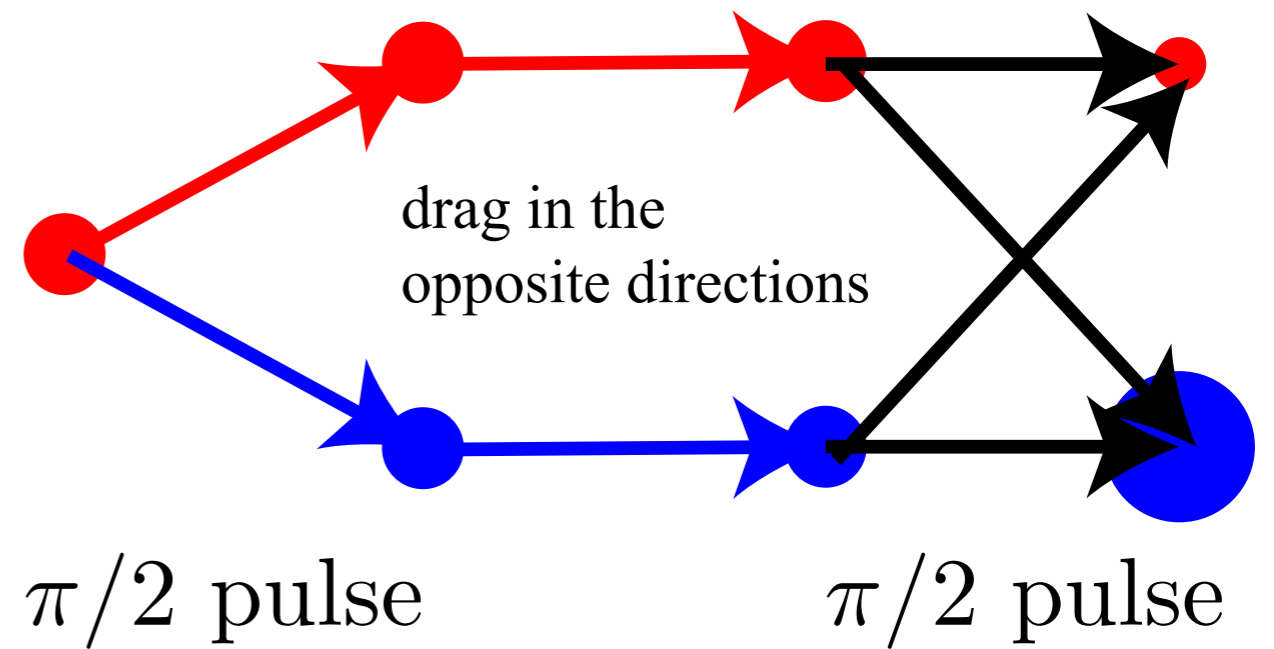


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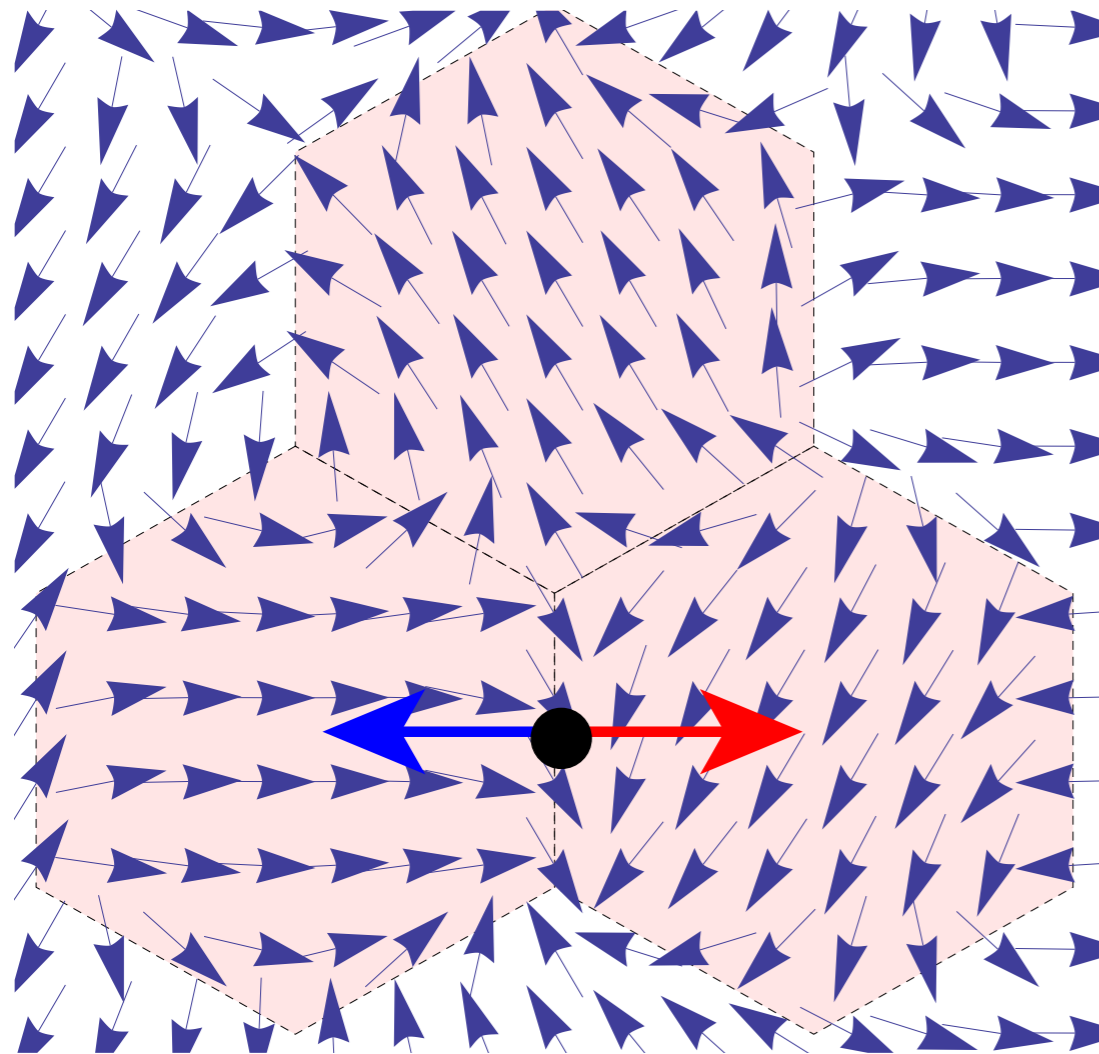


Ramsey-type interference experiment

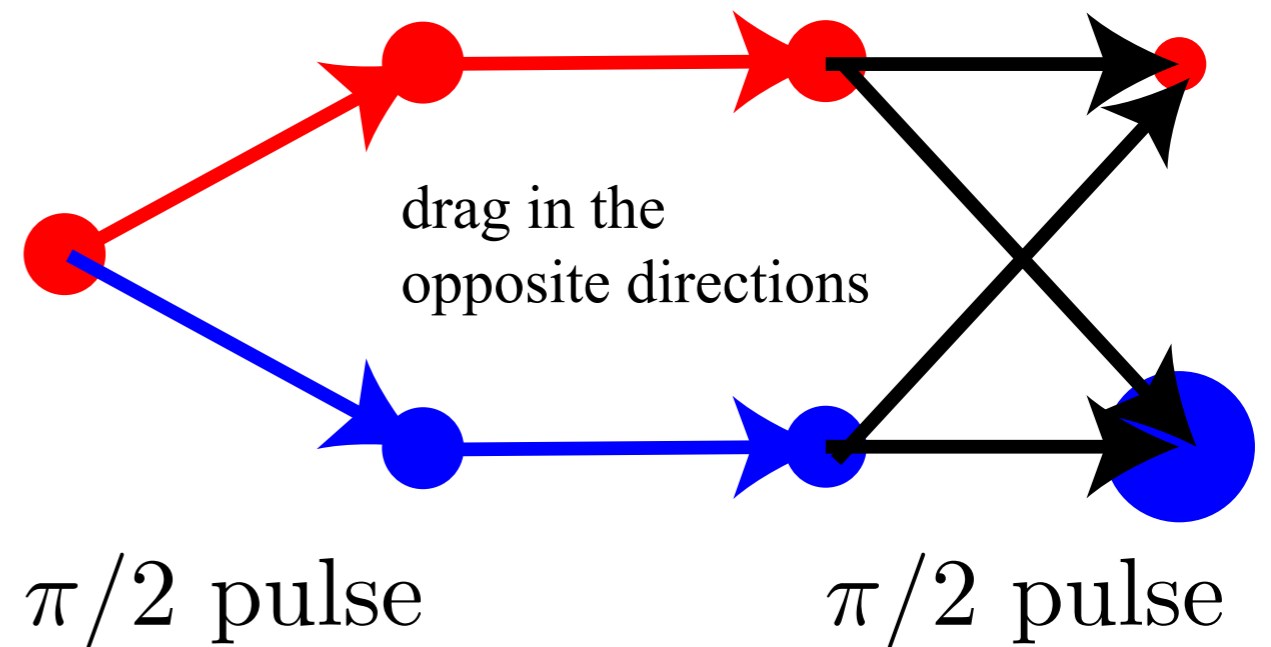


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Ramsey-type interference experiment



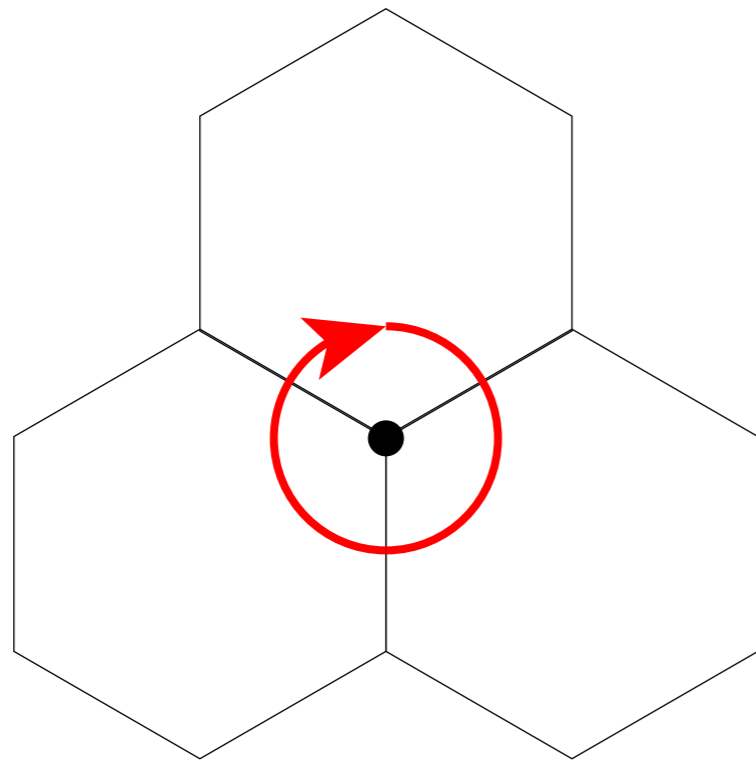
**Solves the problems**

1. Only requires straight-line trajectory
2. Dynamical phase will cancel between the two trajectory

# Zak phase under Bloch oscillations

This scheme measures the phase accumulation under Bloch oscillation

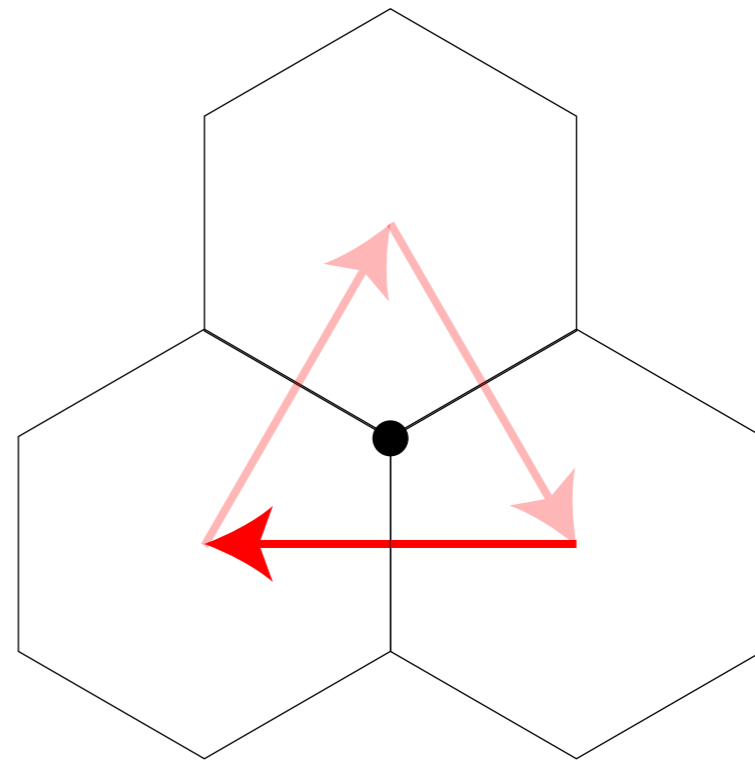
Berry phase



$$\Delta\phi = \pi$$

● Dirac point

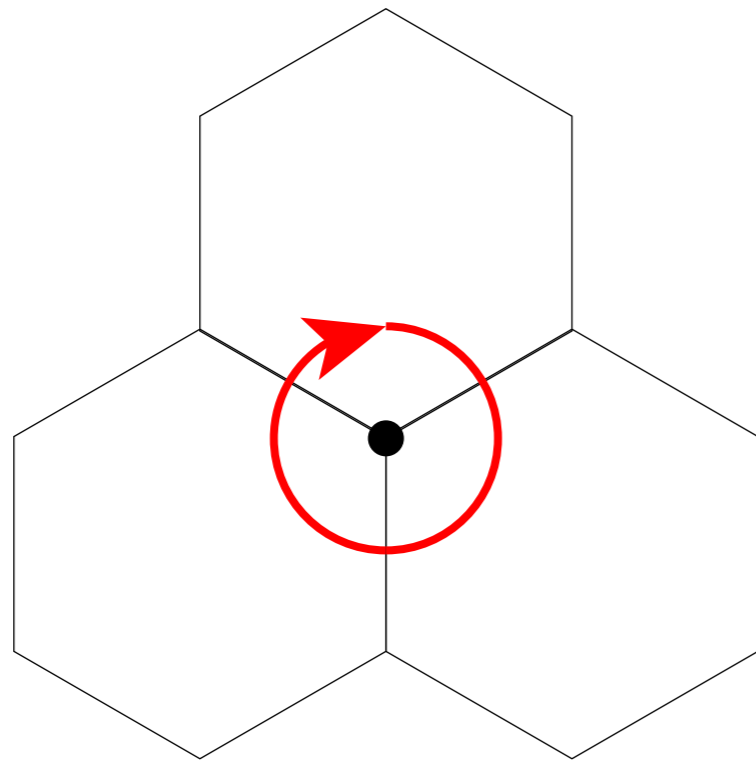
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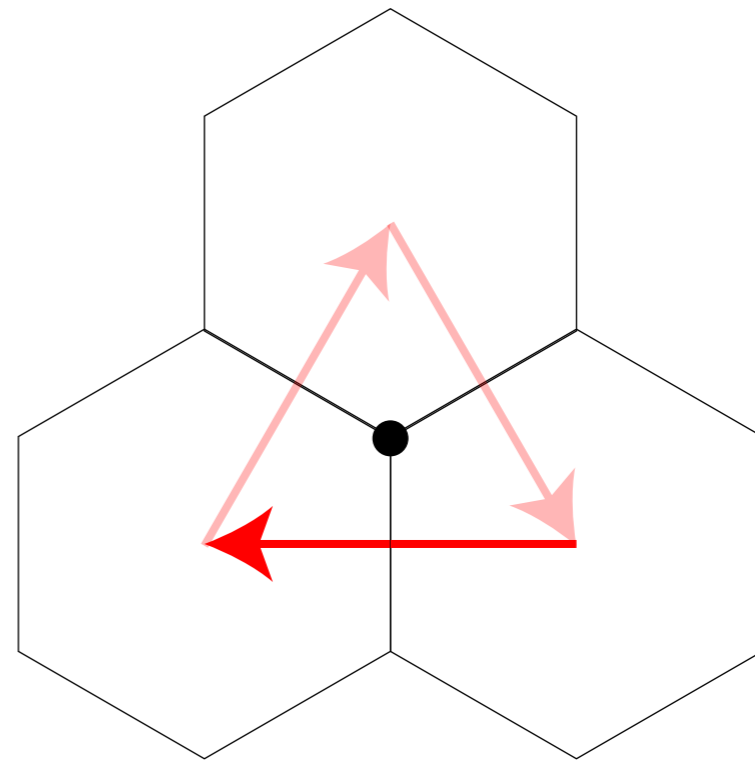
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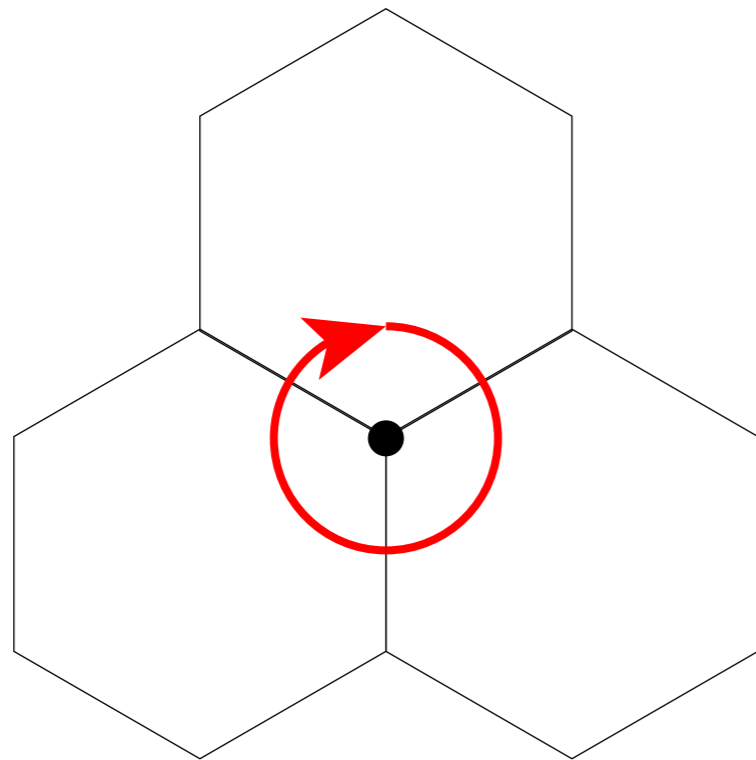


$$\Delta\phi = \pi/3$$

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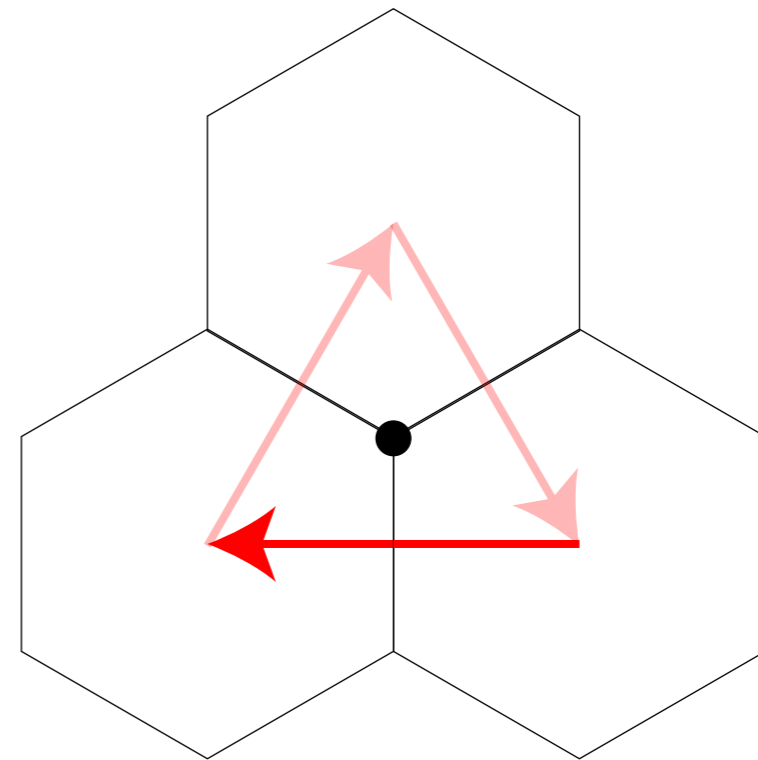
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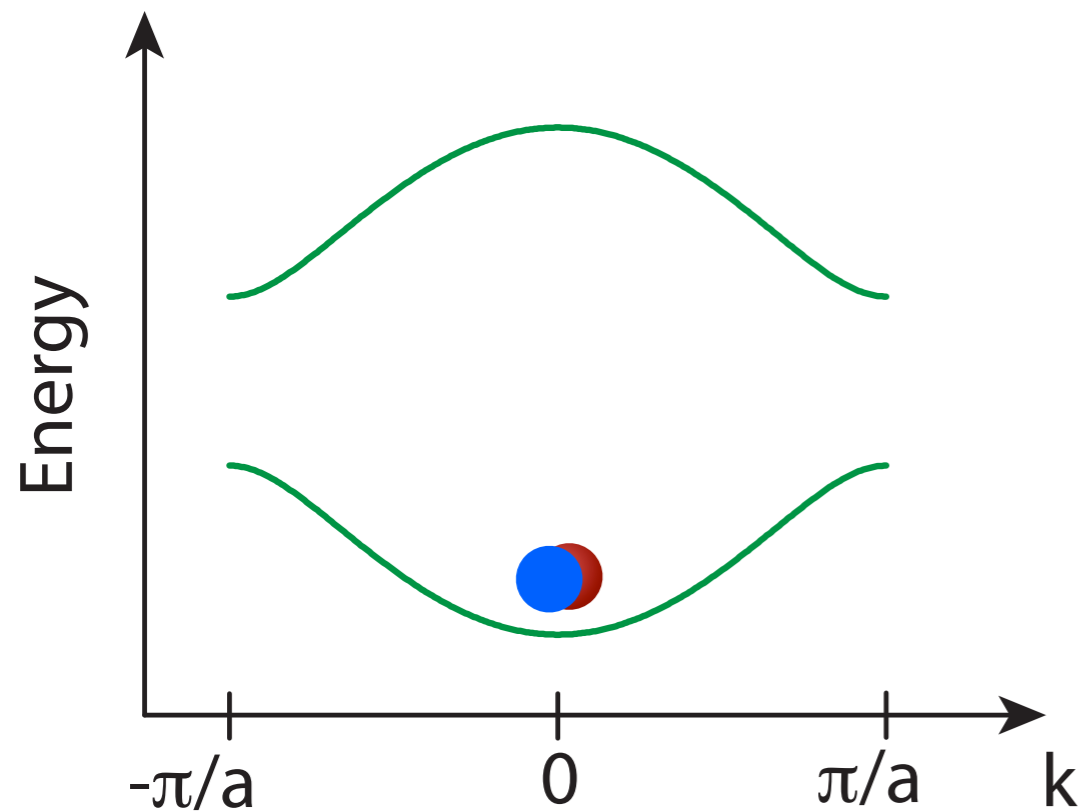
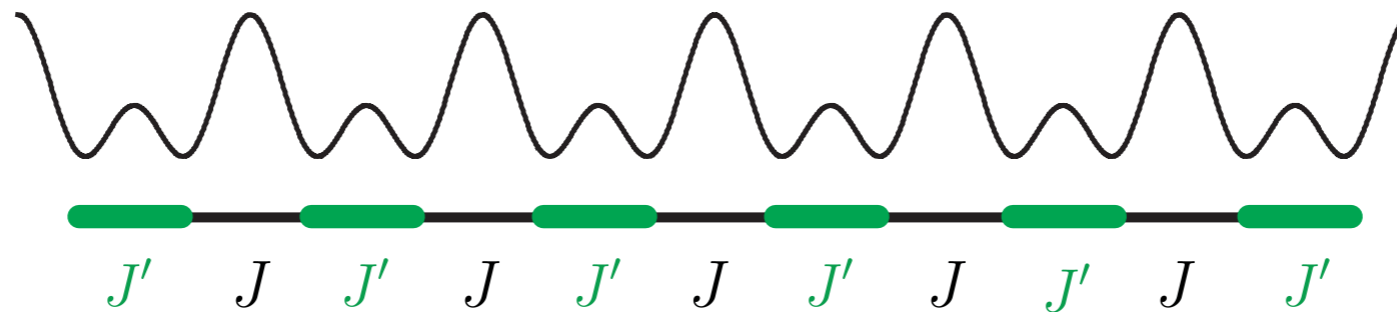


$$\Delta\phi = \pi/3$$

Zak phase is related to Berry phase through a geometrical symmetry of the lattice!

# Measurement of Zak phase for SSH phase

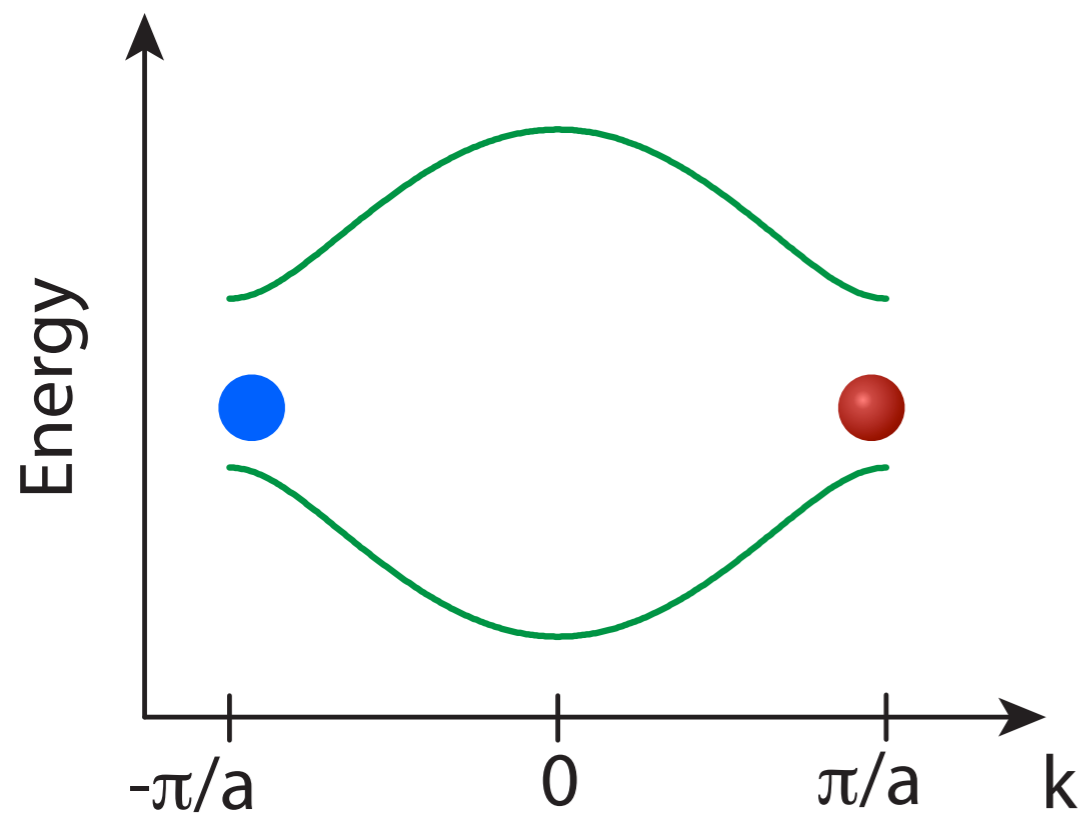
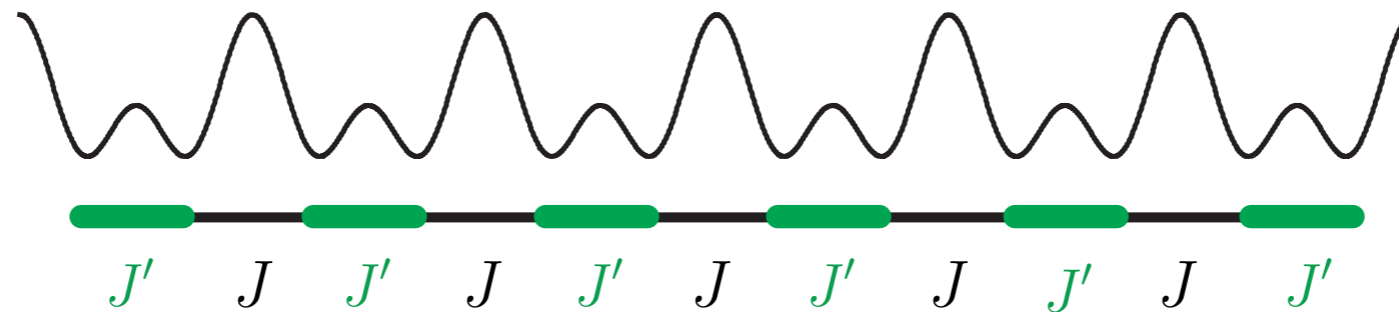
## Dimerized-Lattice Model (SSH model)



Experiment: M. Aidelsburger, M. Atala, J. Barreiro, I. Bloch

# Measurement of Zak phase for SSH phase

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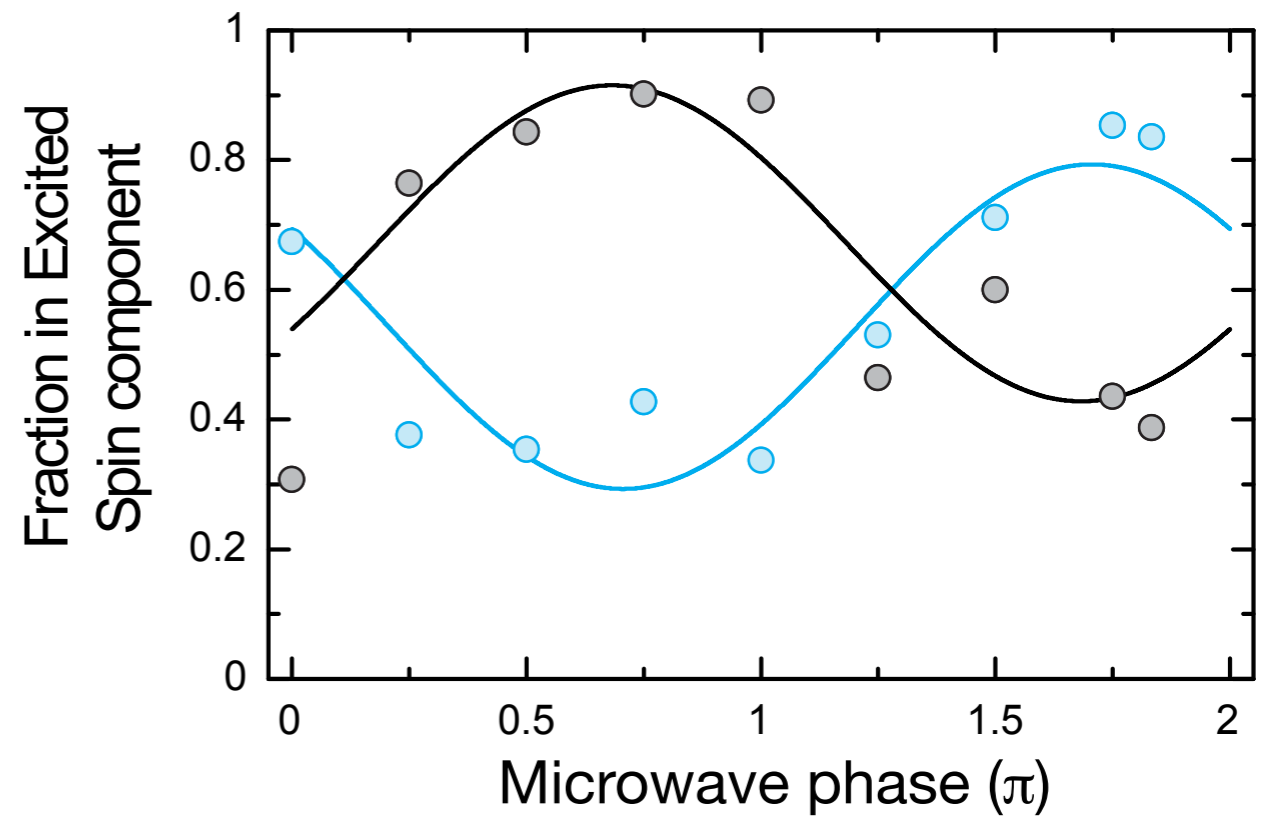
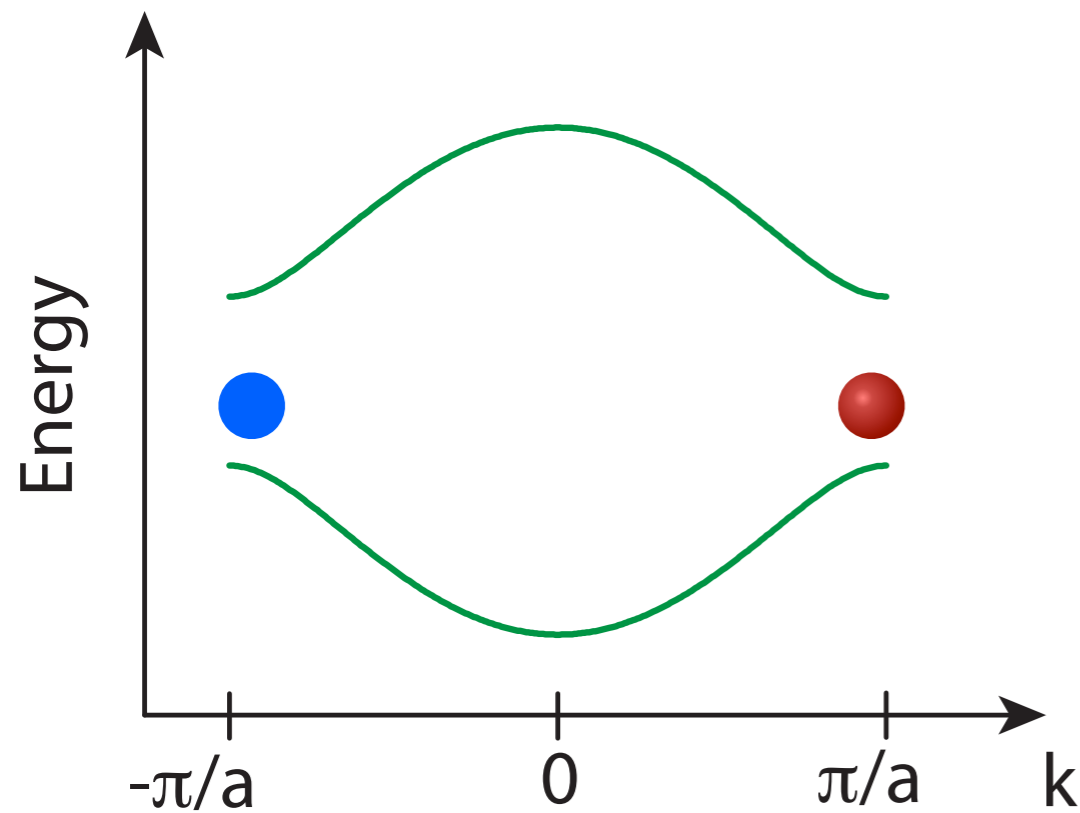
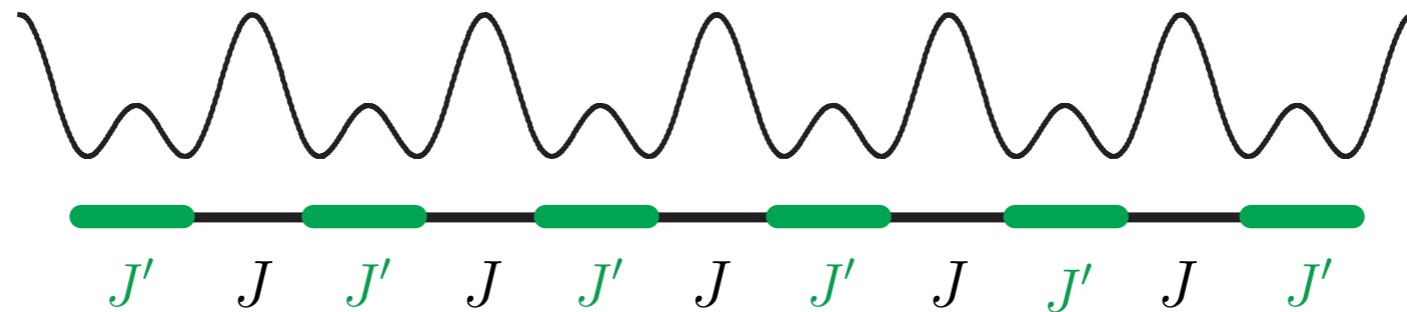


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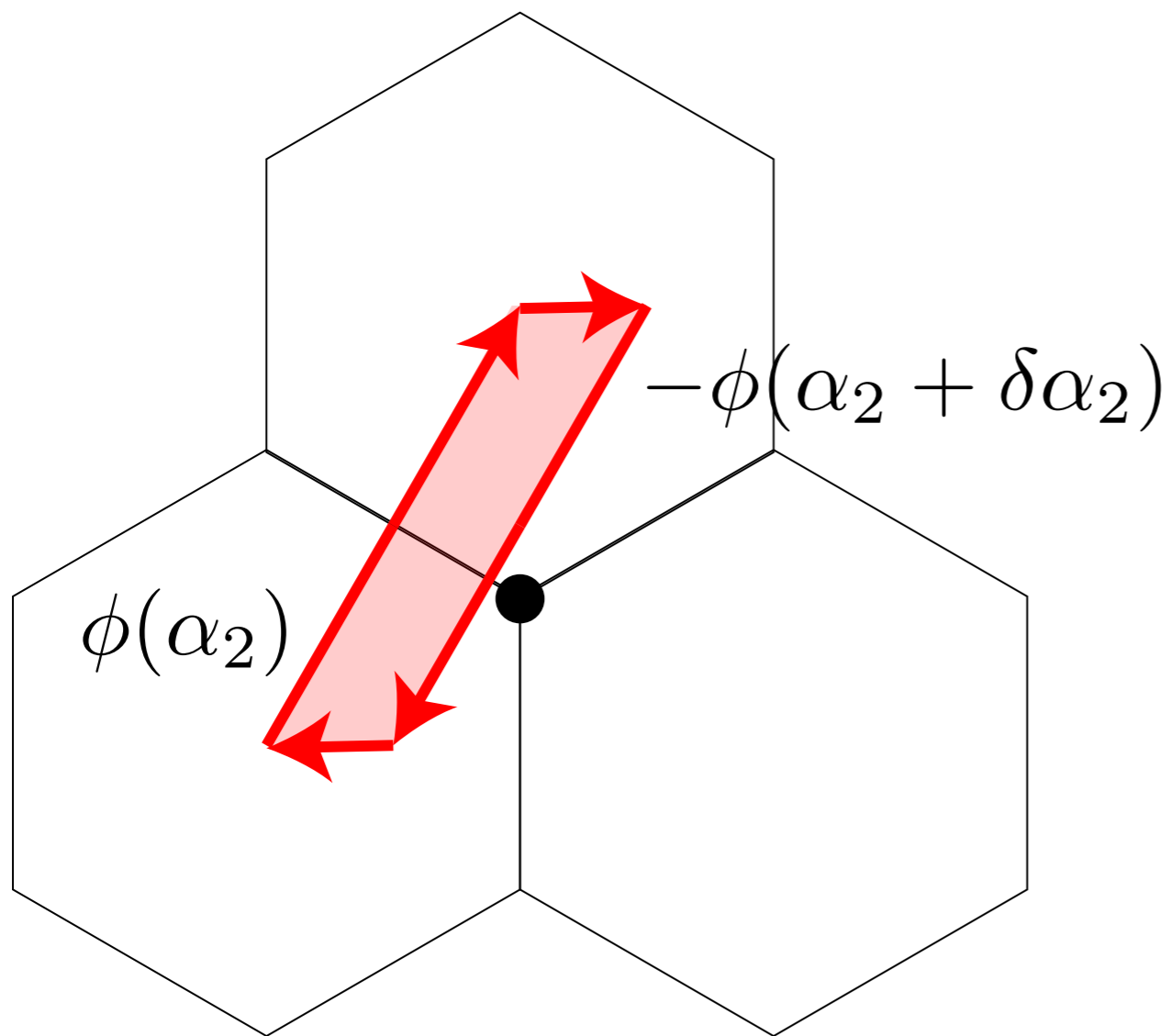
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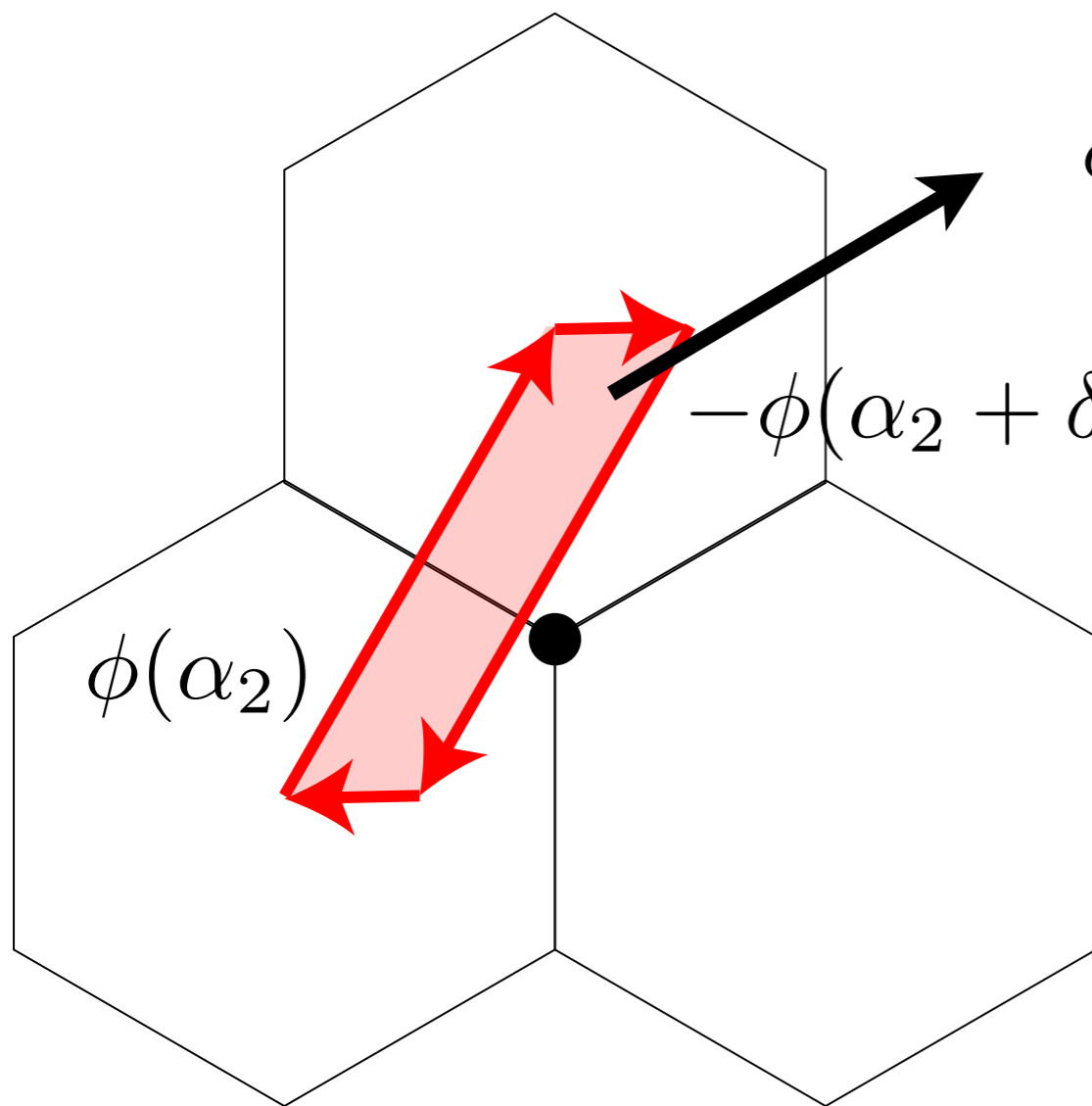
# Relation between Zak phase and Chern number

Measurement of (integrated) local  
Berry curvature from Zak phase



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Measurement of (integrated) local  
Berry curvature from Zak phase



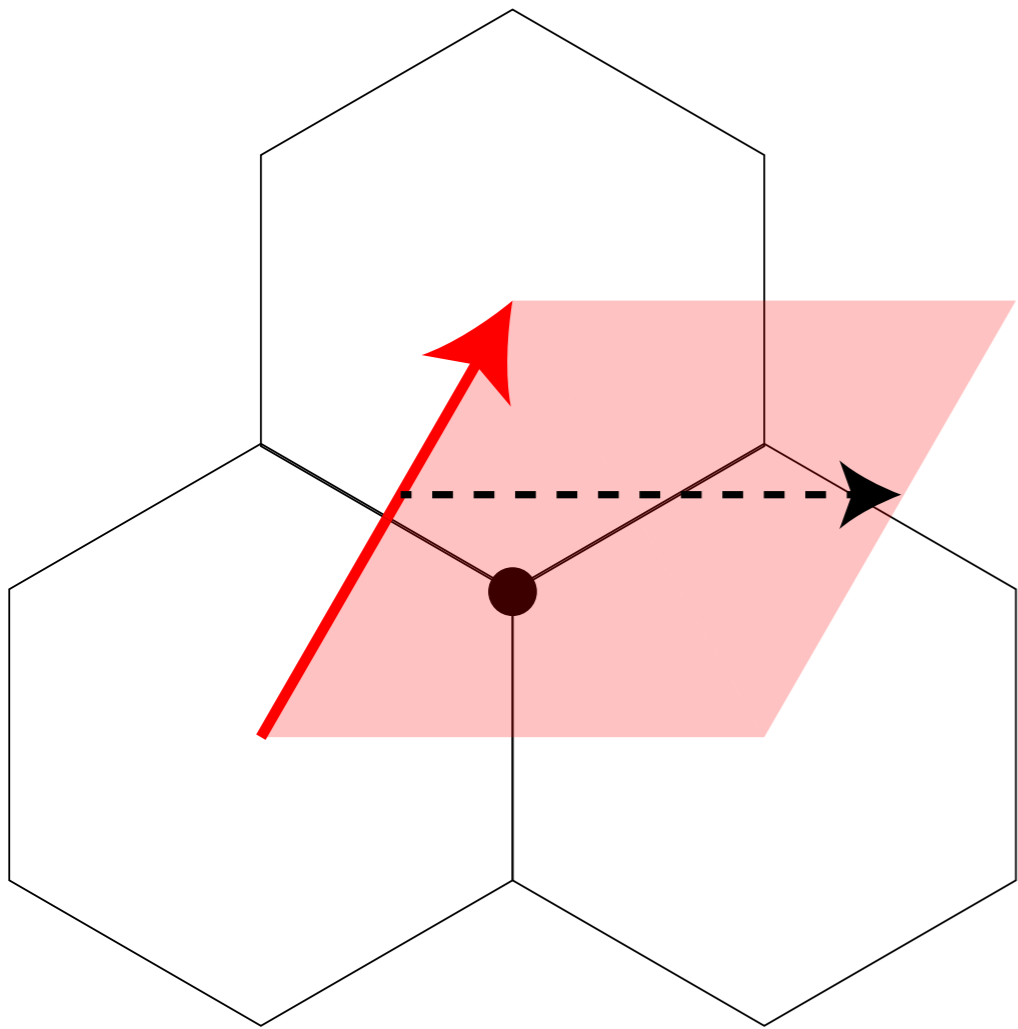
$$\phi(\alpha_2) - \phi(\alpha_2 + \delta\alpha_2) = \partial\phi(\alpha_2)\delta\alpha_2$$

$$-\phi(\alpha_2 + \delta\alpha_2)$$

$$\phi(\alpha_2)$$

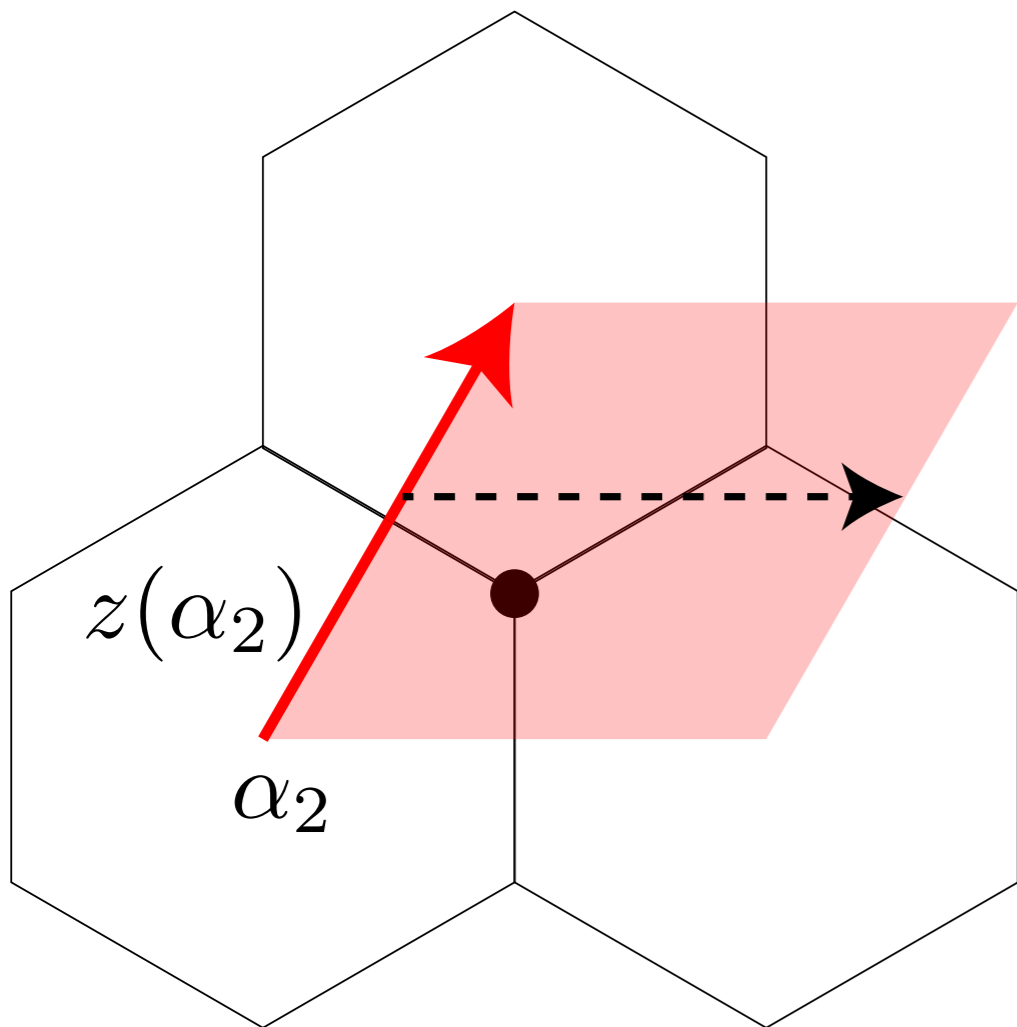
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Zak phase



# Relation between Zak phase and Chern number

## Zak phase

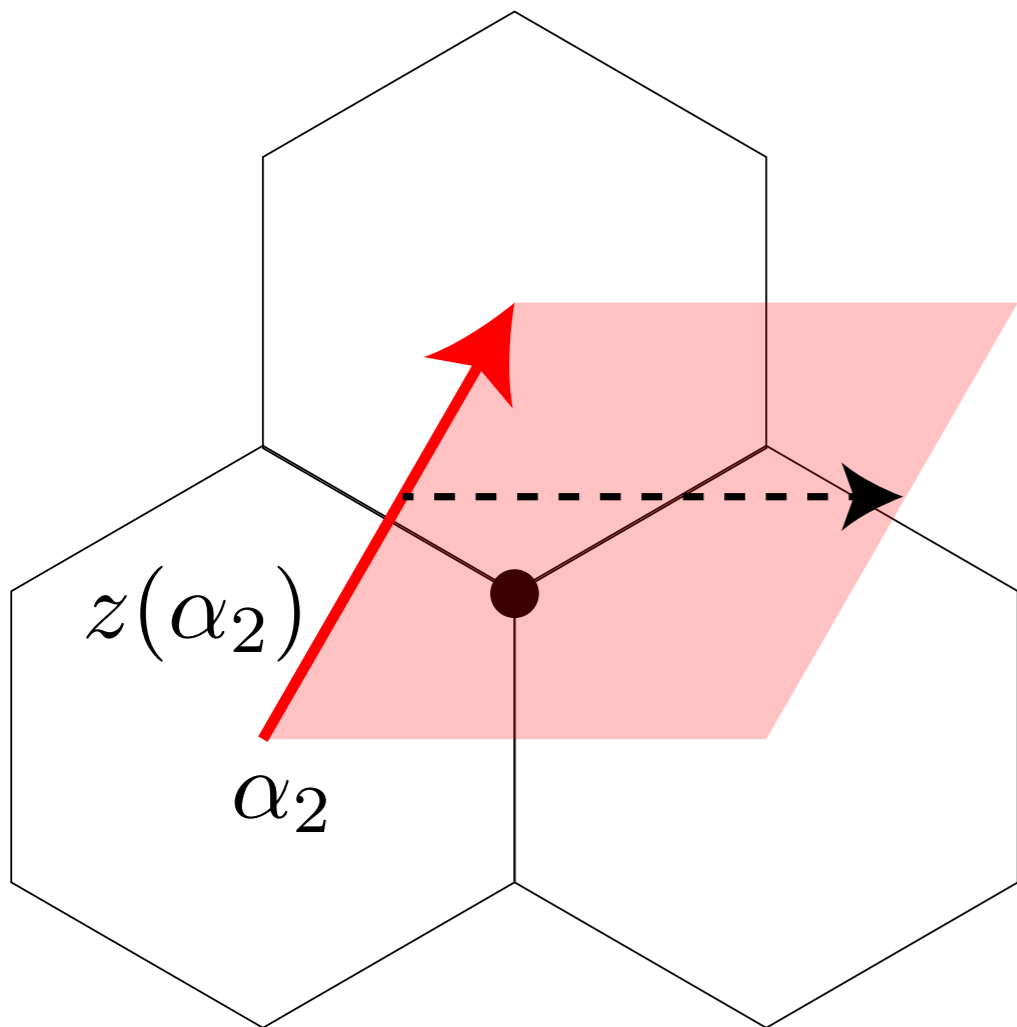


Measure Zak phase for different initial points in the primitive cell

$$z(\alpha_2) = e^{-i\varphi_B(\alpha_2)}$$

# Relation between Zak phase and Chern number

## Zak phase



Measure Zak phase for different initial points in the primitive cell

$$z(\alpha_2) = e^{-i\varphi_B(\alpha_2)}$$

## Chern number

$$c = -\frac{i}{2\pi} \int_0^1 d\alpha_2 \bar{z}(\alpha_2) \partial_{\alpha_2} z(\alpha_2)$$

# Outline

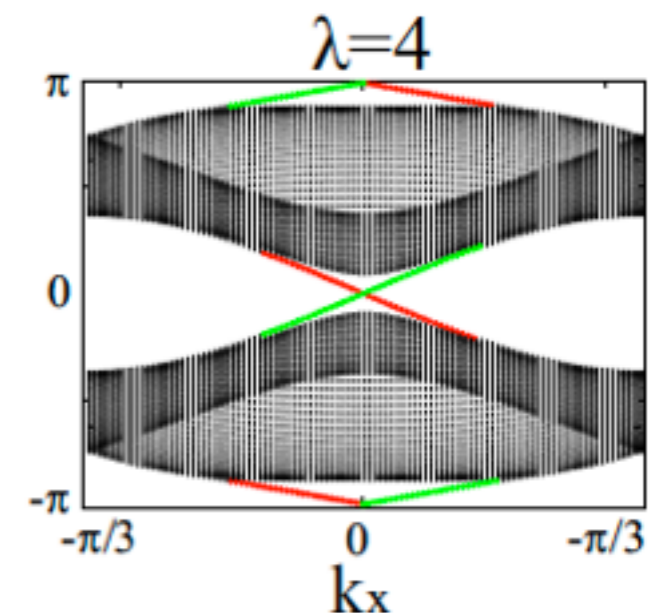
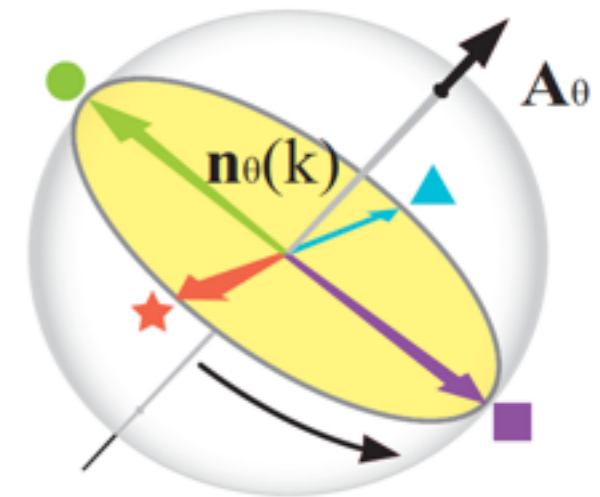
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- 5. Conclusion**

# Conclusion

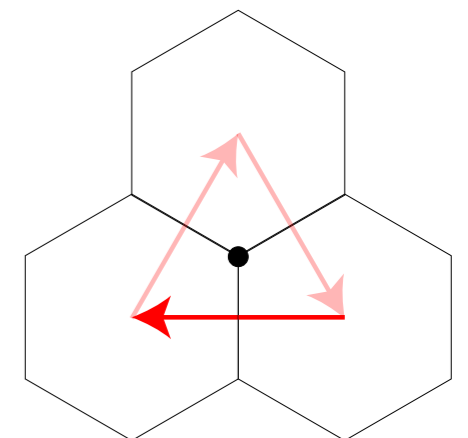
1. Rich and unique topological behaviors exist for periodically driven systems

2. Examples of such non-equilibrium topological phenomena are abundant in quantum optics, cold atoms and condensed matter systems.

3. Band Topology can be directly measured through a combination of Bloch oscillation and Ramsey interference in atomic physics.



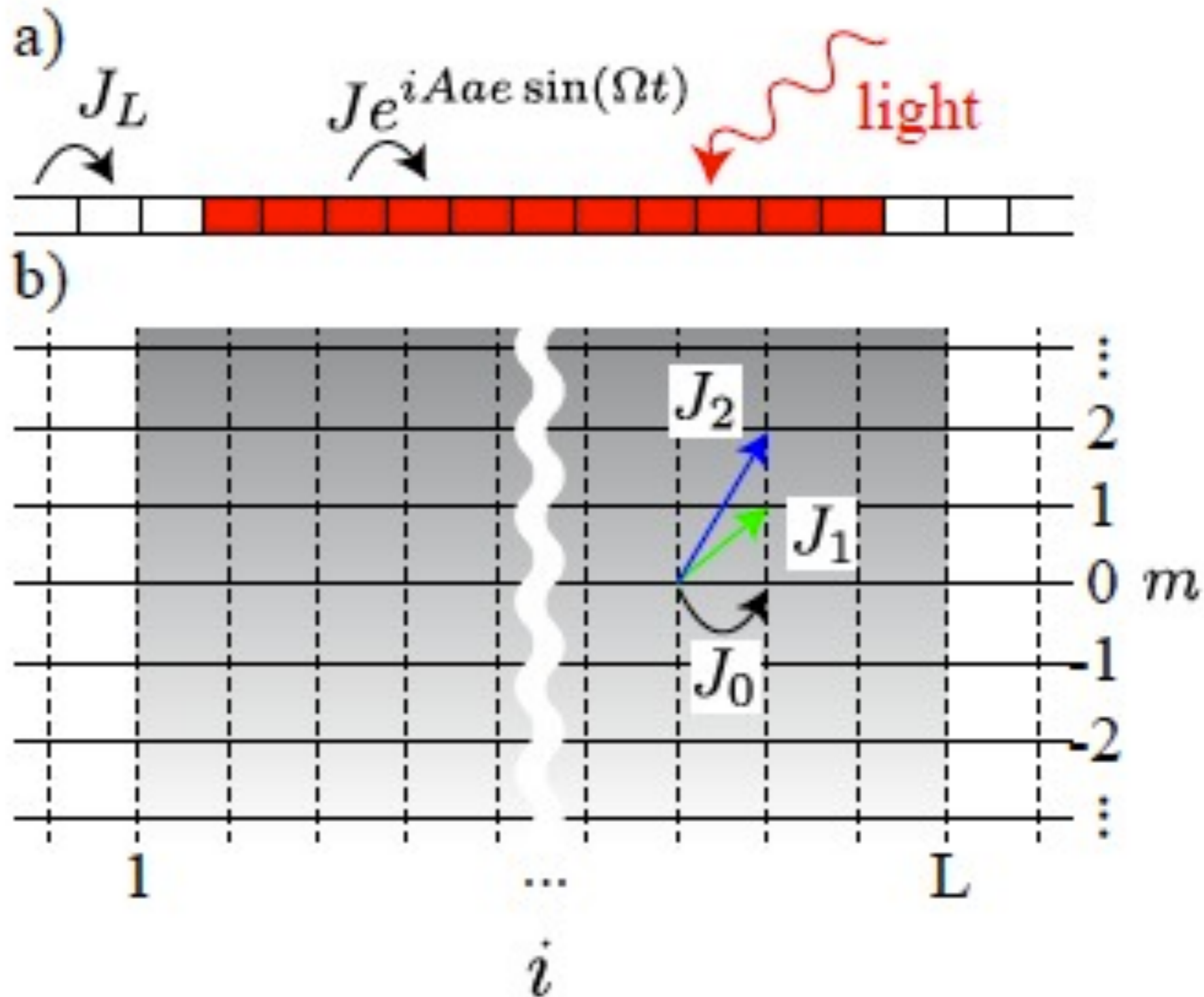
Zak phase





# Further work

Dimension increase from light application



# Further work

## Dimension increase from light application

