Quantum quenches, dynamical transitions and off-equilibrium quantum criticality

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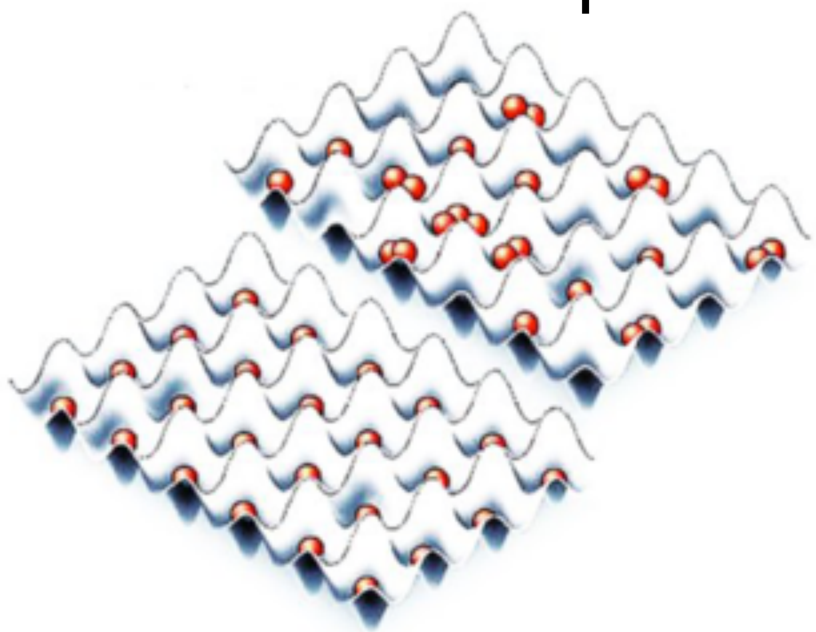
In collaboration with Bruno Sciolla (IPhT CEA Saclay):
PRL 2010, JSTAT 2011 and on condmat (very) soon

Dynamical transition & quantum quenches starting from symmetry broken phase

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$

Quantum Quench: $U_i \to U_f$



Limit of infinite dimensions: $d \to \infty$

$$H = -\frac{J}{V} \sum_{i \neq j} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$

Fisher et al '89

Quantum Quench in infinite dimensions

Site-permutation symmetry of H and $|\psi_{GS}\rangle$: the system remains in the symmetric subspace.

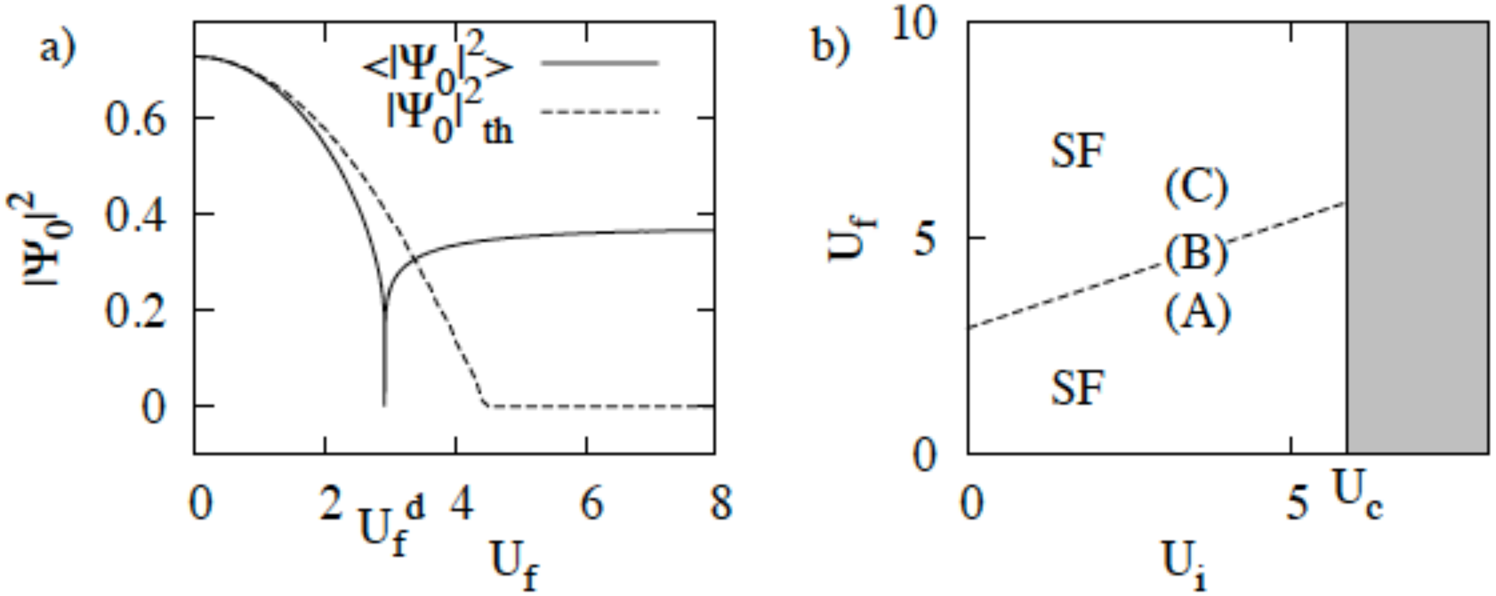
Site-permutation symmetric states:

$$|x_0, x_1, x_2, ...\rangle = \mathcal{N} \sum{}' |\{n_i\}\rangle$$

Degrees of freedom: x_i fraction of sites with i bosons

Classical dynamics for x_i in the thermodynamic limit $\frac{1}{V} \sim \hbar$

Dynamical Transition



- Logarithmic singularity of time averages (no equilibration and no damping)
- Critical U: initial energy equal to the one of the Mott (metastable) state.
- Different from the equilibrium phase transition.

“Generality” of the transition
(within Mean-Field approaches)

- Found in the Hubbard Model by Dynamical Mean Field Theory (Eckstein, Kollar, Werner ’09) and Gutzwiller (Schiro’, Fabrizio ‘10,’11)
- Transverse Field Ising in infinite D (Sciolla, GB ’11)
- Quartic quantum field theory by mean-field approximation (Gambassi, Calabrese ’10)

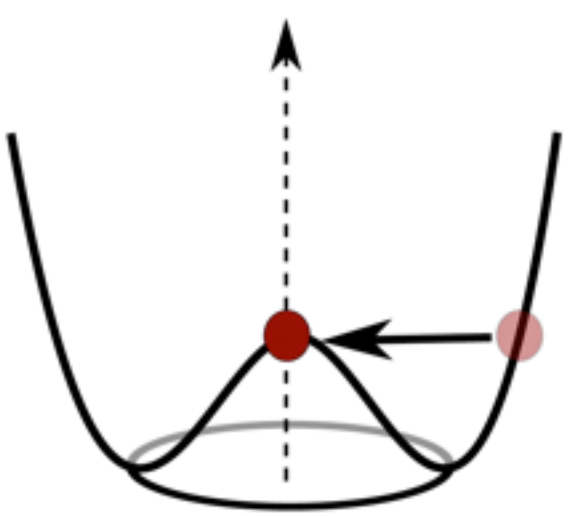
Absence of thermalization, no dynamics starting from the Mott state, no damping, no spatial correlations: need to go beyond and take into account fluctuations

Large-N quantum field theory

- Hamiltonian:

$$H = \int d^3x \frac{1}{2} \sum_a \left((\vec{\nabla}\phi_a(x))^2 + m_0^2 \phi_a(x)^2 \right) + \frac{\lambda}{4!N} \left(\sum_a \phi_a(x)^2 \right)^2$$

- Quantum phase transition: $\langle \phi_a \rangle \neq 0 \quad a = 1, ..., N$
- Mean-field theory of quantum quenches (Calabrese, Gambassi '10): $m_i \rightarrow m_f$ dynamical transition.



Large-N approximation for quantum quench

$$\psi_a(t) = \langle \hat{\phi}_a(x,t) \rangle$$

$$G_a(x-x';t,t') = \Big(\langle \{ \hat{\phi}_a(x,t), \hat{\phi}_a(x',t') \} \rangle - \psi_a(t)\psi_a(t') \Big)$$

- Using 2PI formalism (Baym-Kadanoff)

$$\partial_t^2 \phi_t = -\left(m_t^2[\phi] + \frac{\lambda}{6N} \int_p G^{\parallel}_{ptt} \right) \phi_t = -\frac{\partial V(\phi)}{\partial \phi}$$

$$\partial_t^2 G^{\perp}_{ptt'} = -\left(p^2 + m_t^2 \right) G^{\perp}_{ptt'}$$

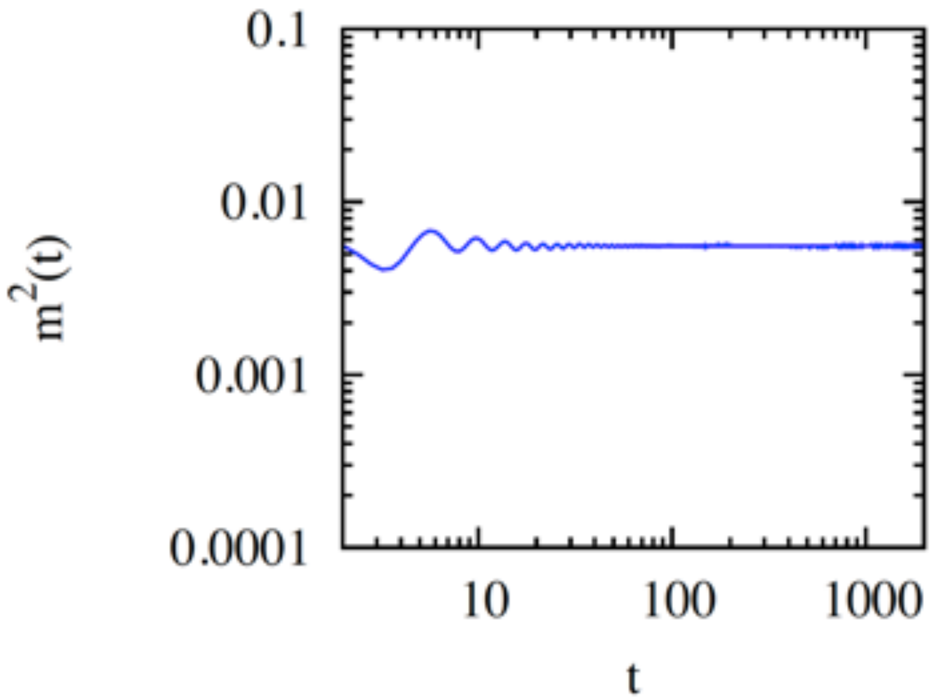
$$\partial_t^2 G^{\parallel}_{ptt'} = -\left(p^2 + m_t^2 + \frac{\lambda}{3N} \phi_t^2 \right) G^{\parallel}_{ptt'}$$

$$m_t^2 = (m_0^f)^2 + \frac{\lambda}{6N} \left(\phi_t^2 + \frac{1}{2} \int_p G^{\parallel}_{ptt} + \frac{N-1}{2} \int_p G^{\perp}_{ptt} \right)$$

See also Sotiriadis, Cardy ’09; Schiro’, Fabrizio ’11

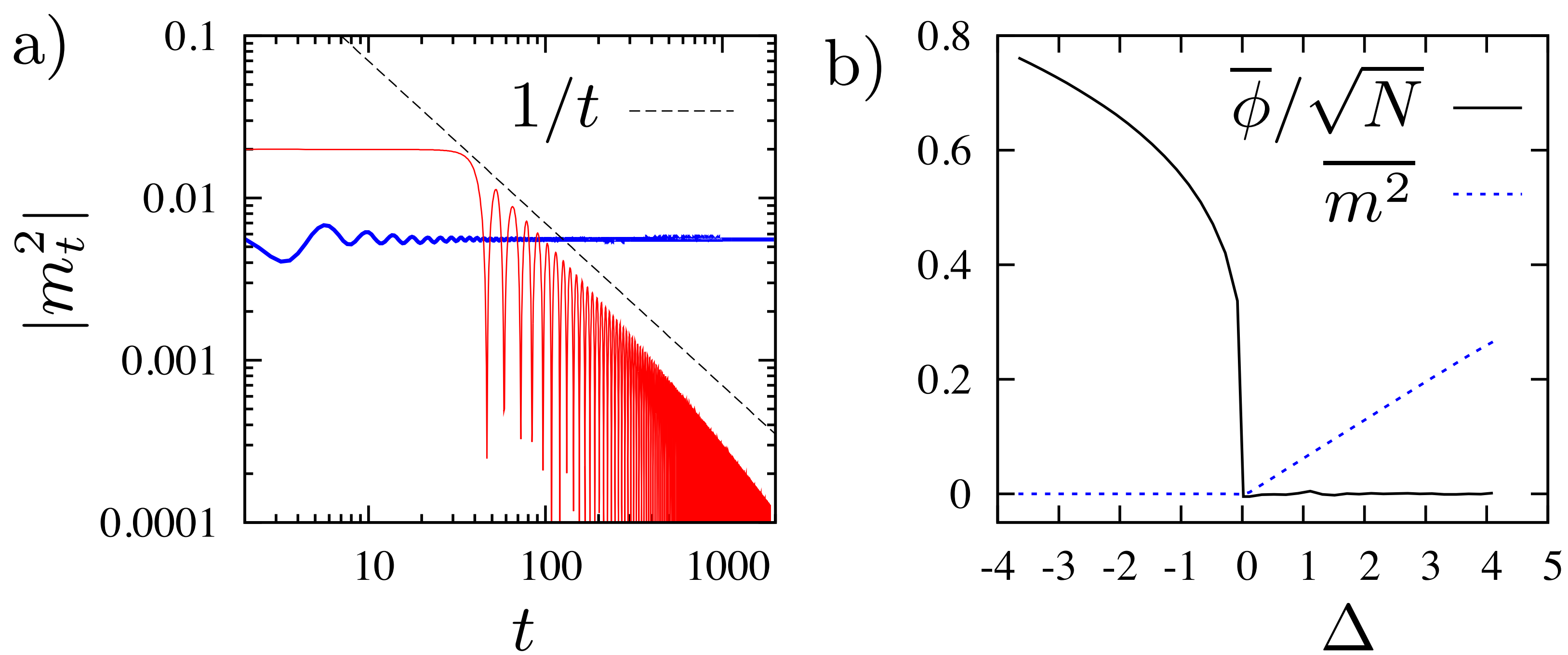
Quantum Quench: $m_i^2 < 0 \to m_f^2$

Steady state reached at long times (damping, relaxation but not to thermal equilibrium)



Strong quench

Dynamical transition (steady state)



$$\Delta = \left[(m_0^{f(d)})^2 - (m_0^f)^2\right]/(m_0^{f(d)})^2$$

Critical behavior

$$\overline{\phi} \sim |\Delta|^{1/4}$$

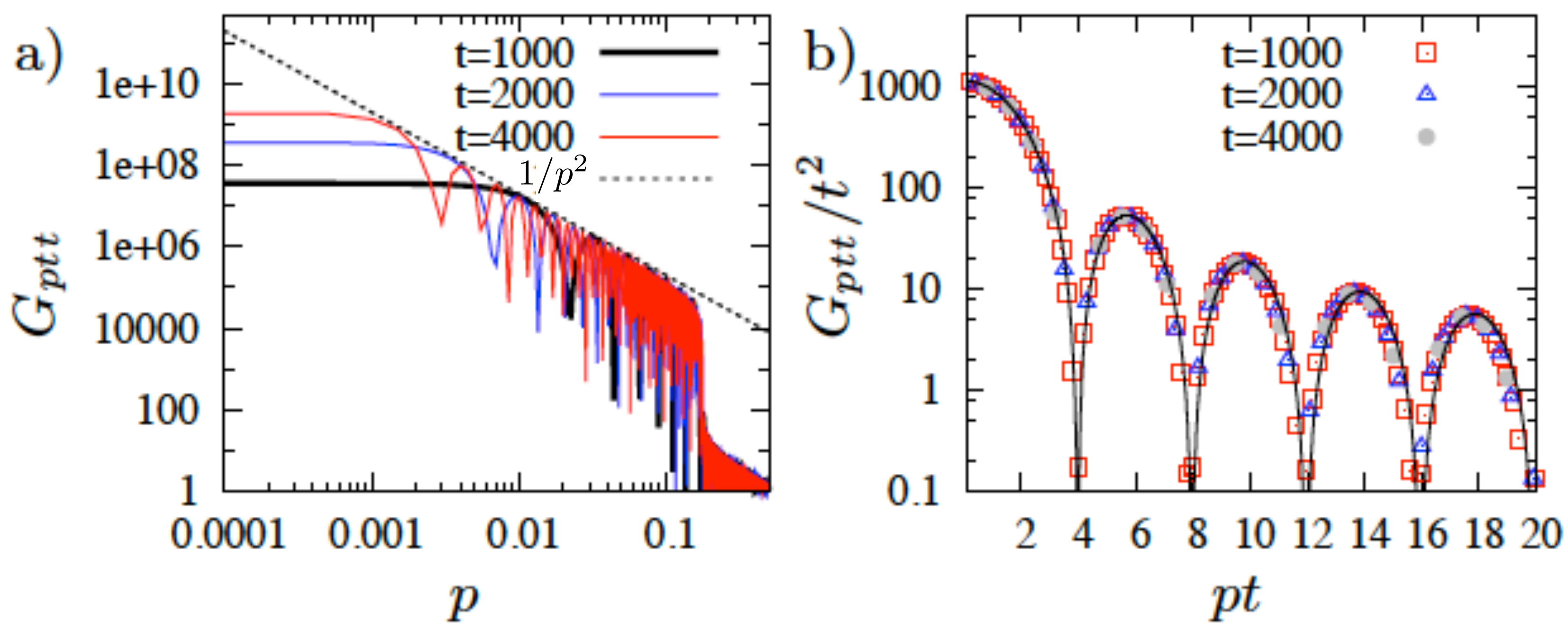
$$\overline{m^2} \sim \Delta$$

At the transition dynamic scaling and aging

The system never reaches the steady state

$$G^{\perp}(p;t,t') = \frac{1}{p^2} F\left(p\xi(t), \frac{t}{t'}\right) \qquad \xi(t) \propto t$$

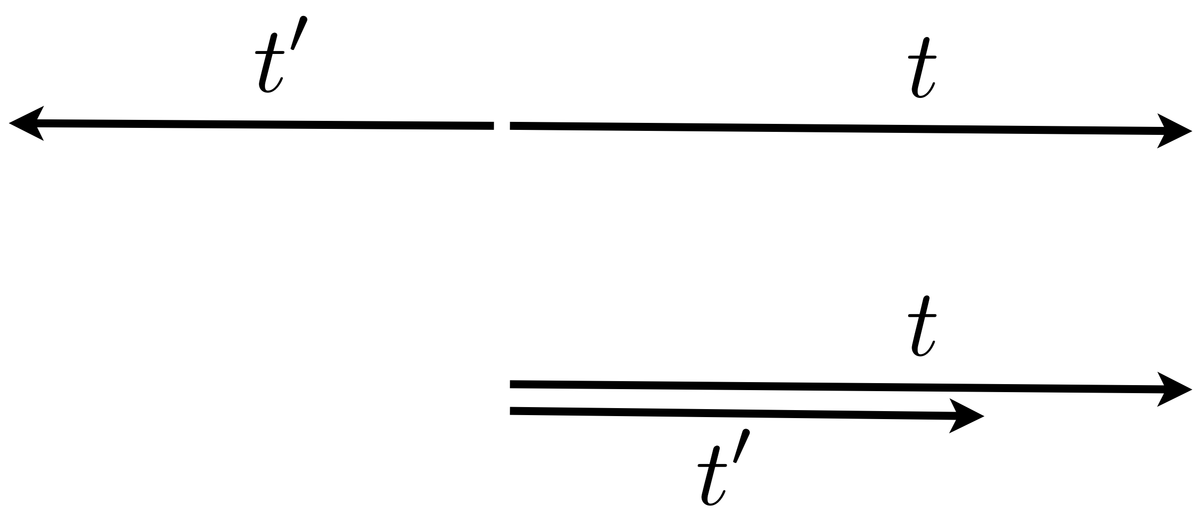
$$F(x,y) = C\left[\cos(x - xy) - \cos(x + xy)\right]$$



Quasi-particles interpretation

$$G(r;t,t') \propto \frac{1}{r}\theta(|r|-(t-t'))\theta(t+t'-|r|)$$

Quasi-particles propagation $\quad r = t \ (v=1)$



Calabrese, Cardy '07

Critical behavior

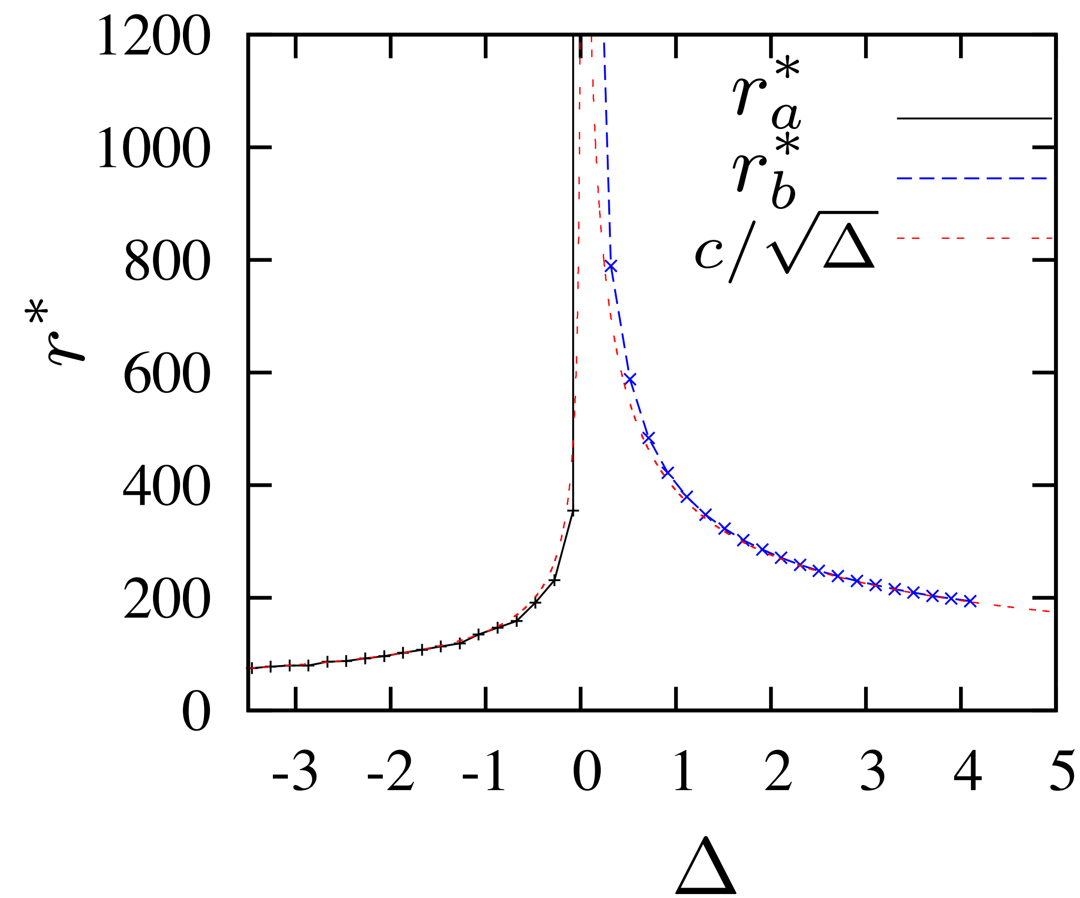
Diverging time to reach the steady state $\tau_{relax} \sim \frac{1}{\sqrt{|\Delta|}}$

$O(1) \ll t \ll \tau_{relax}$ Dynamic scaling & Aging

$t \sim \tau_{relax}$ Critical steady state

$$G^{\perp}_{ptt'} = \frac{1}{p^2} F\left(p\xi^*, \frac{t-t'}{\tau_{relax}}\right)$$

$$\xi^* \sim \frac{1}{\sqrt{|\Delta|}} \sim \tau_{relax}$$



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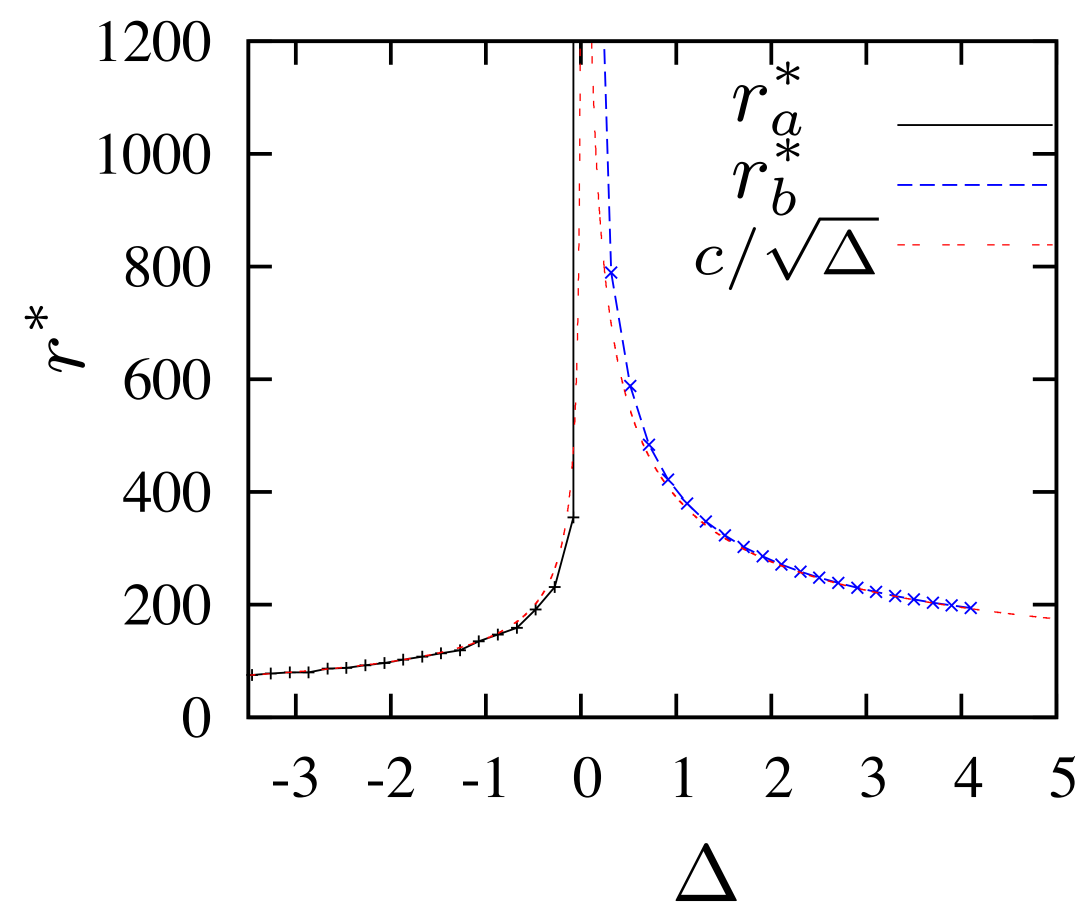

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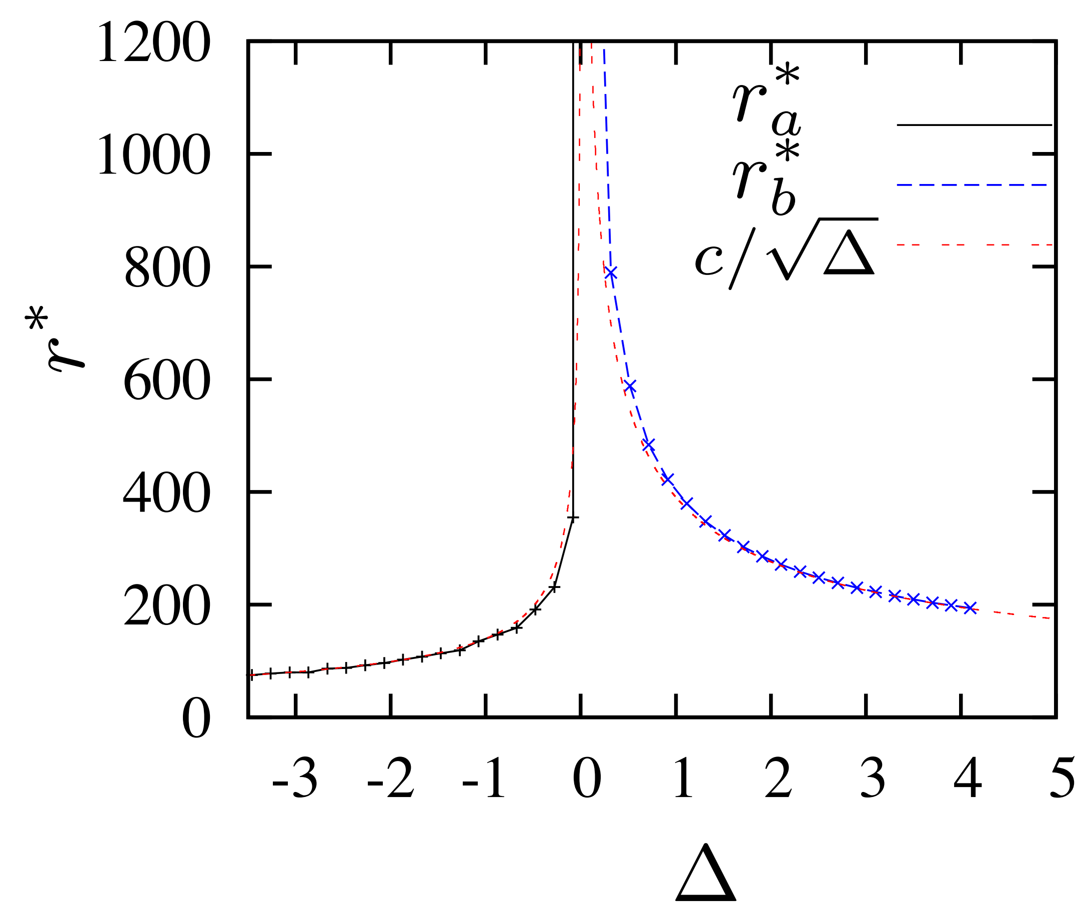
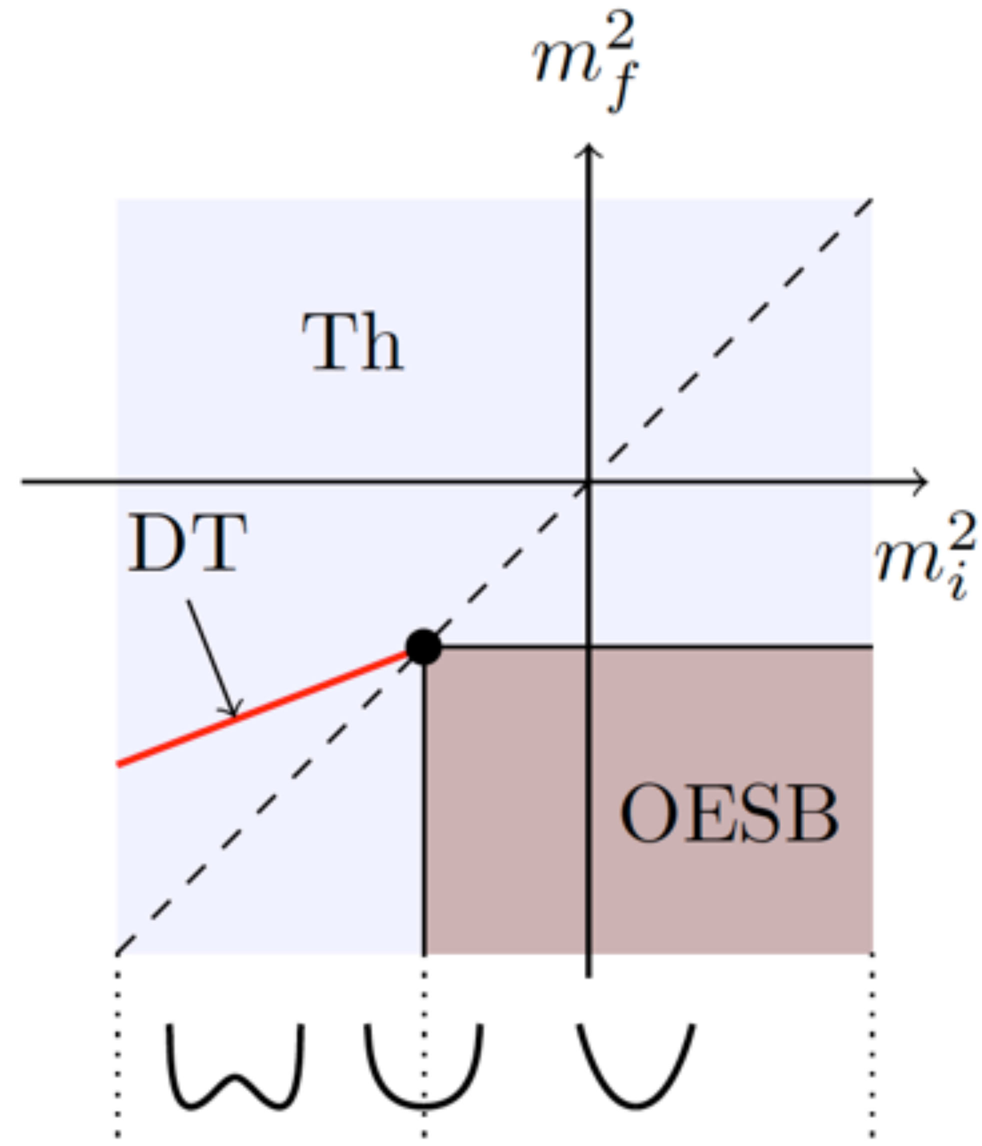


Quantum quenches from the symmetric phase

- Same dynamic scaling in the whole OESB region.
- Similarity with coarsening after thermal quenches.



Boyanovsky, De Vega et al '95, '96...

Open questions on the dynamical transition

- Connection with the quantum/thermal phase transition? (Sciolla, GB ’11, Schiro’, Fabrizio ’11)
- Does it remain once all dynamical fluctuations are included? (Calabrese, Gambassi ’10)
- Connection with coarsening?
- Evidences from numerics? (Kollath Lauchli ’10; Eckstein et al ’09)

Conclusion and Perspectives

- Dynamical transition in mean-field theory of quantum quenches.
- Beyond mean-field theory by 1/N expansion: relaxation to steady state, (different) critical exponents, diverging time and length-scales and aging for the critical quench.
- Study next-leading order and thermalization.
- Full analysis of the dynamical transition
- Effects of driving on the dynamical transition