Integer quantum Hall edge states far from equilibrium

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Work with Dmitry Kovrizhin

Phys. Rev. B 84, 085105 (2011) and Phys. Rev. Lett. 109, 106403 (2012)

Related papers: D. Kovrizhin + JTC: PRB 81 (2010), PRB 80 (2009)

JTC + Y. Gefen and M. Veillette, PRB 76 (2007)

Outline

Motivation

Experiments on evolution of non-equilibrium

electron distribution in QHE edge states

Theoretical Approaches

Physical picture: relaxation from mode dispersion

Quantum quench as approximate description

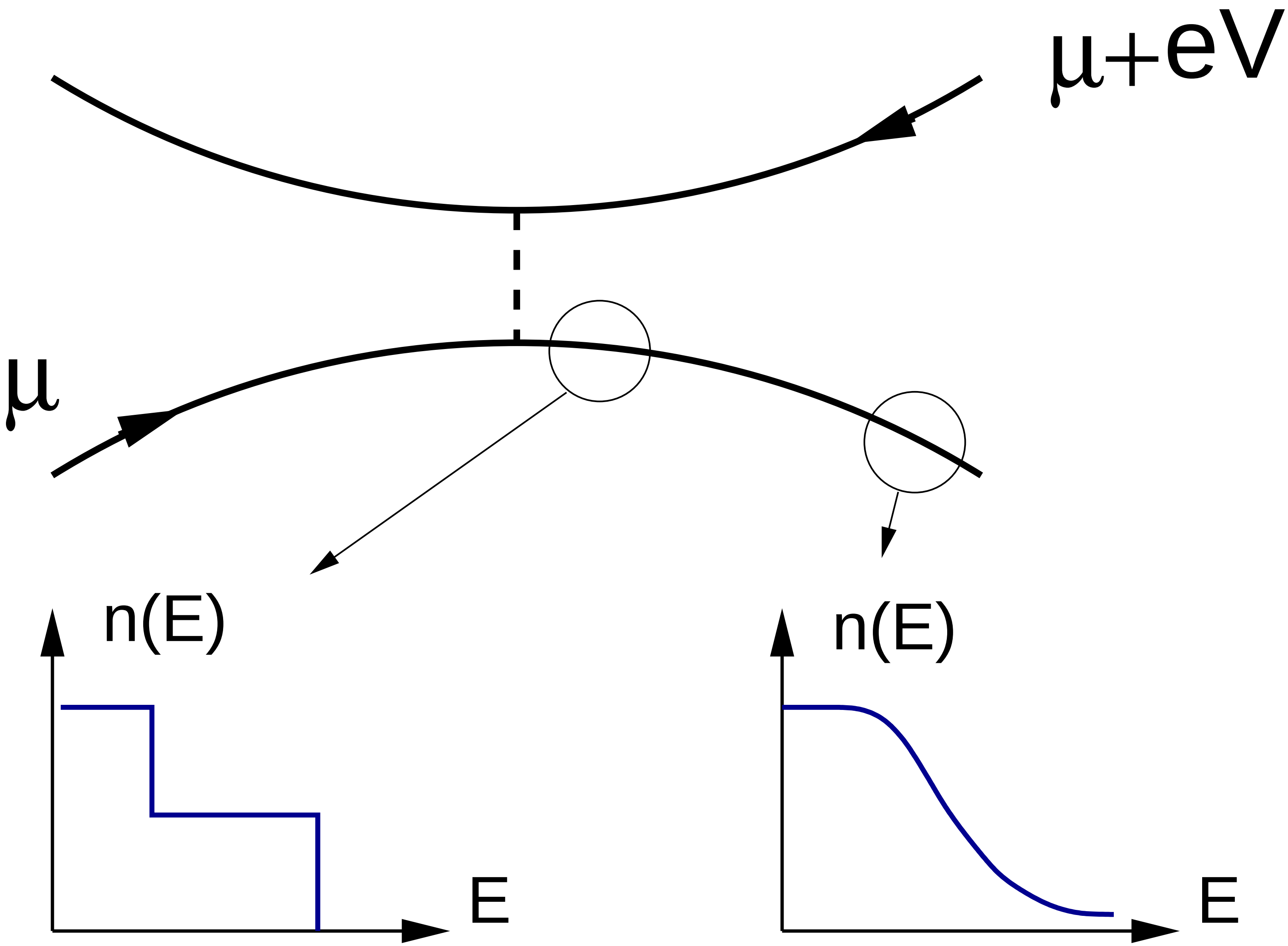
Exact treatment via bosonization + refermionisation

Results

Relaxation in an integrable system

Non-thermal steady state

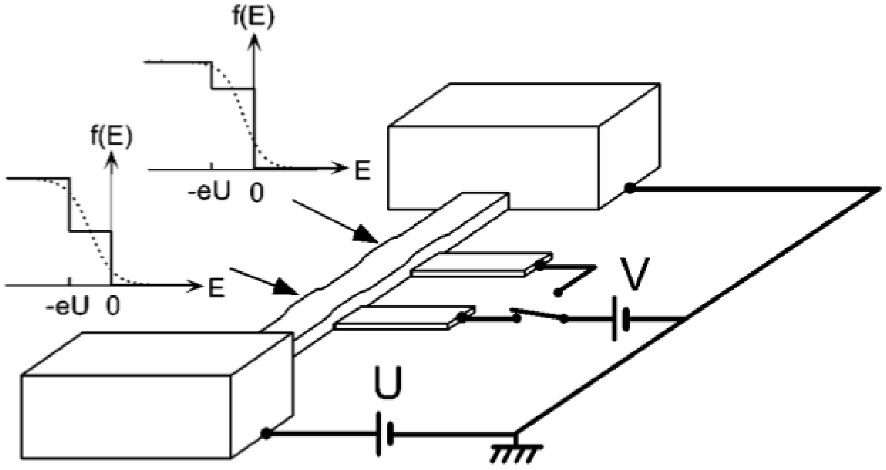
Schematic view of experiment

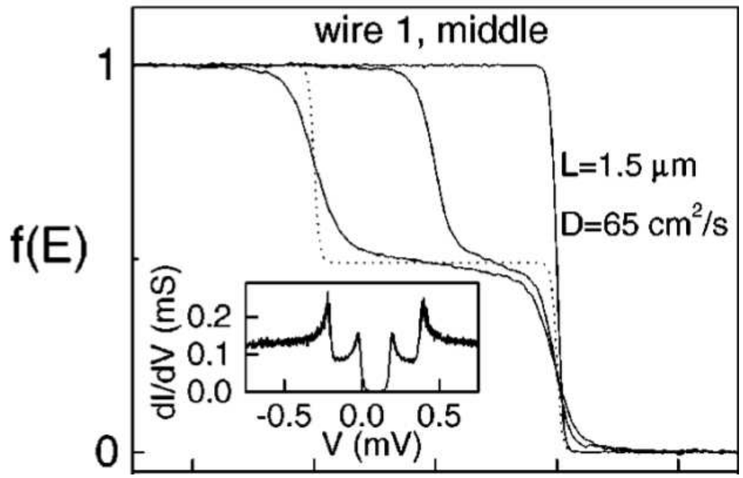


le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Analogue of earlier experiment in wires

Electron distribution in biased diffusive wires

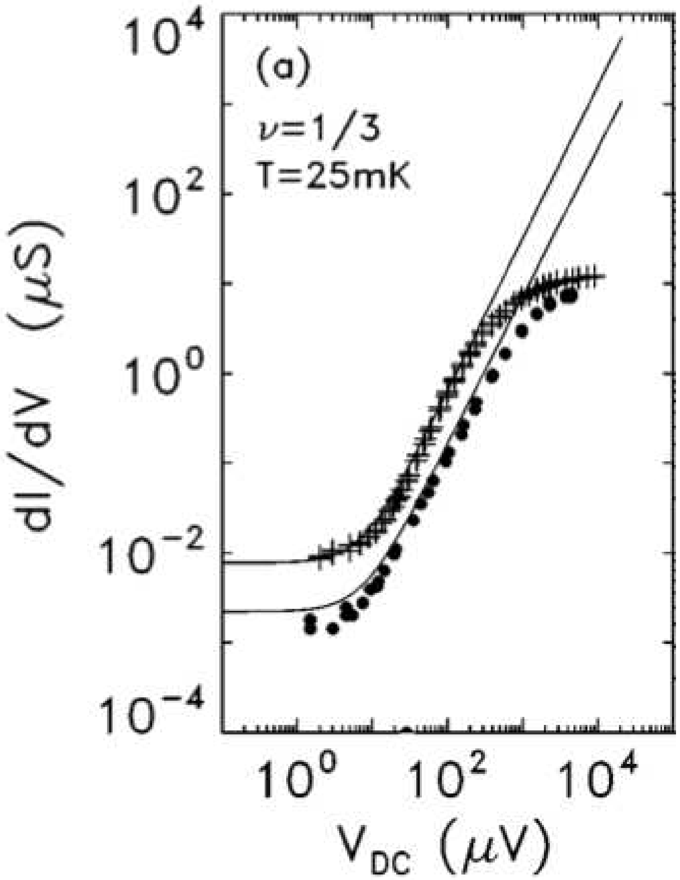




Poithier, Guzron, Birge, Esteve, & Devoret, PRL (1997)

Contrast with tunnelling probe of fractional quantum Hall edge states

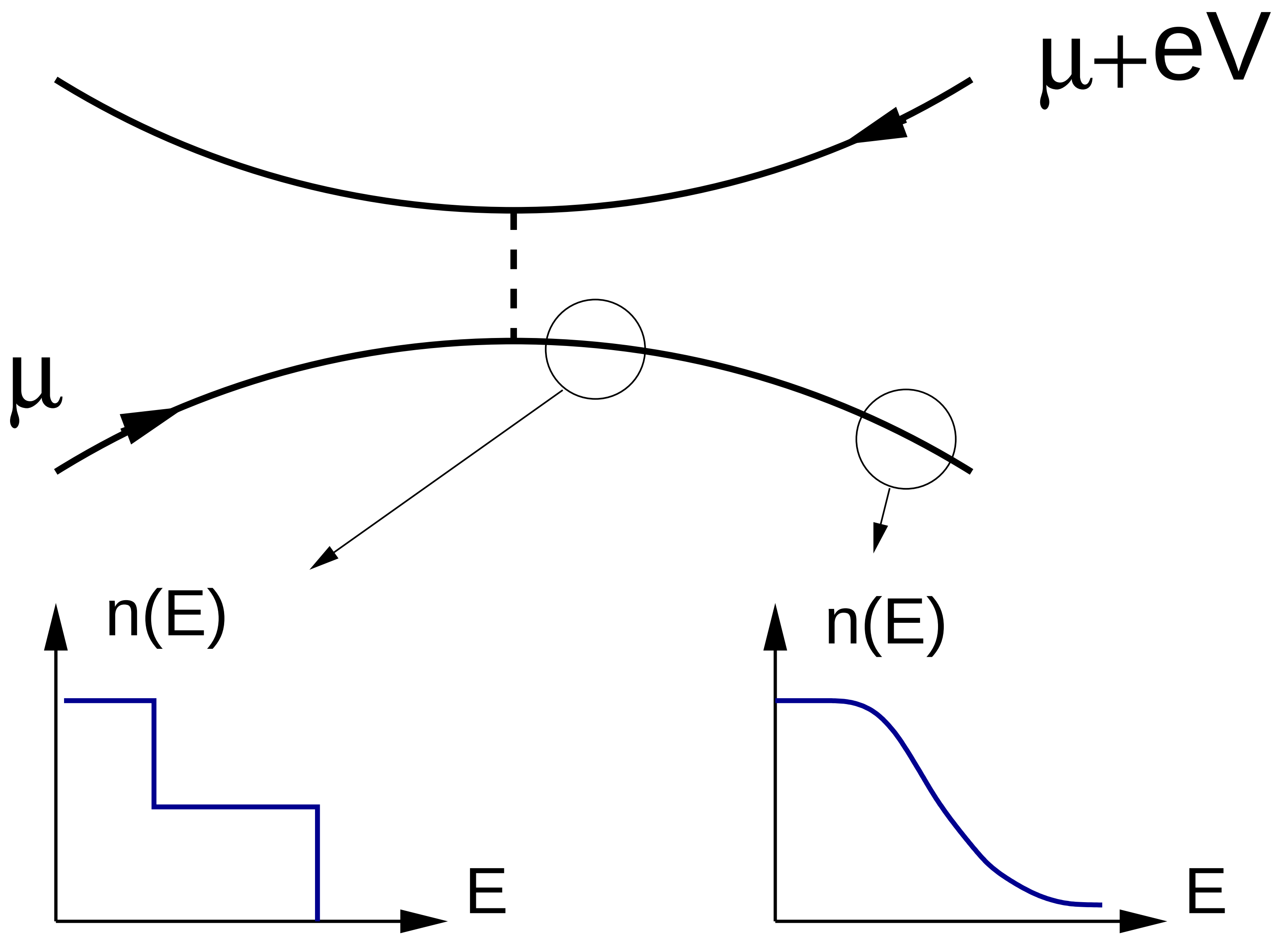
Voltage-dependent differential conductance from electron correlations



Theory: Kane & Fisher (1992)

Expt: Chang, Pfeiffer & West (1996)

Analogy with quantum quench

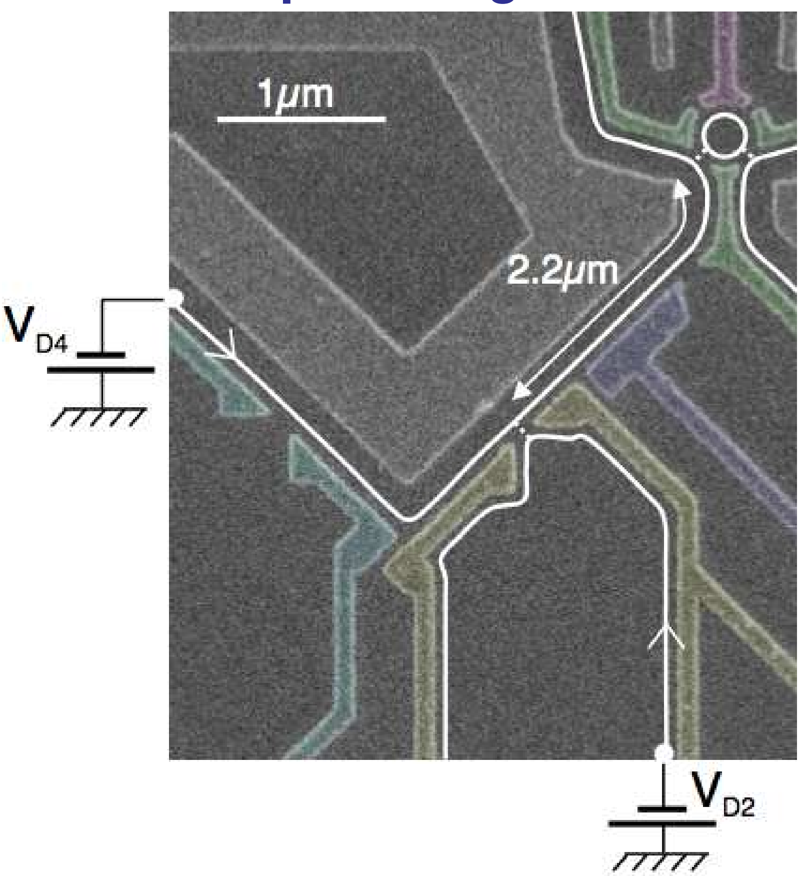


Distance from contact $\propto$ time after 'quench'

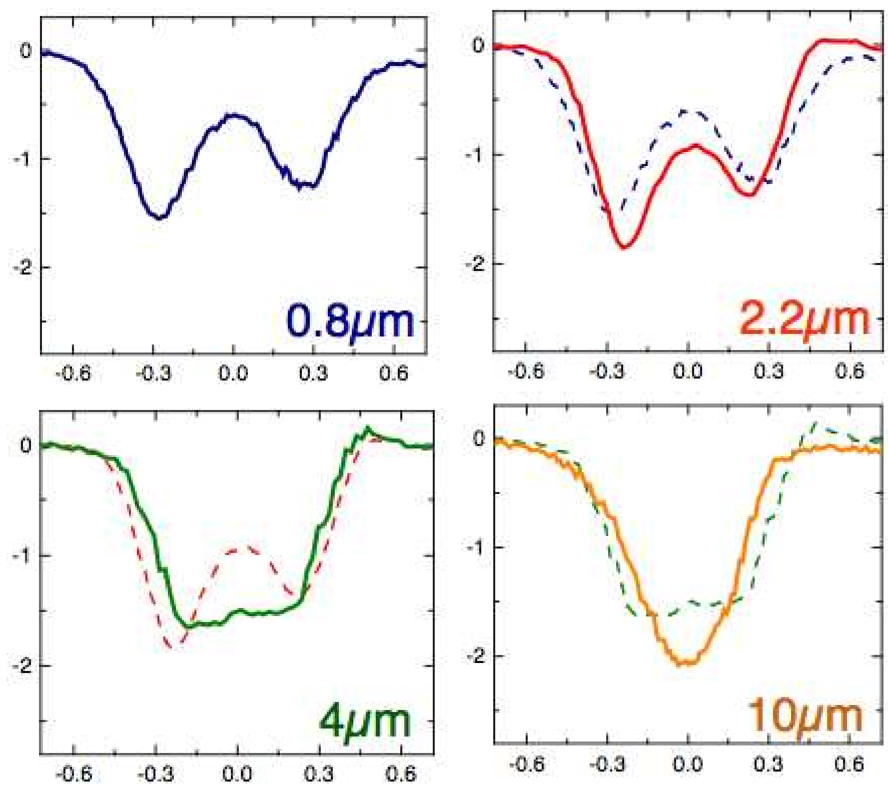
Experiment – Actual

le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Sample Design



Evolution of Distribution



$\partial n(E)/\partial E$ **vs.** E

Theoretical description of edge states

As electrons:

$$H = -i\hbar v \int dx \psi^\dagger(x)\partial_x\psi(x) + \int dx \int dx'\ U(x-x')\rho(x)\rho(x')$$

$$\rho(x) = \psi^\dagger(x)\psi(x)$$

As collective modes:

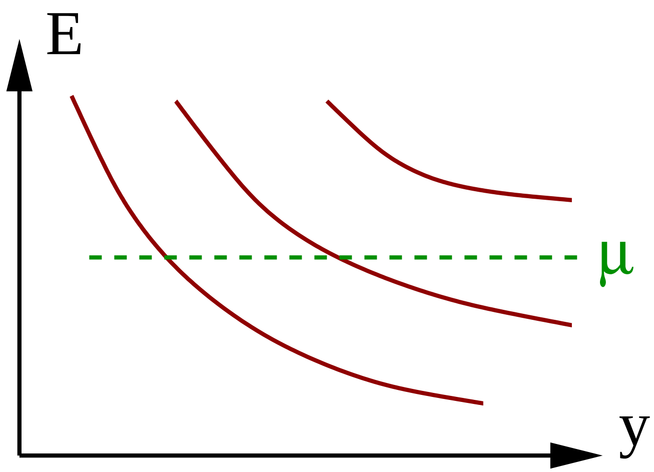
$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q$$

$$\omega(q) = [v + u(q)]\, q \qquad\qquad u(q) = (2\pi\hbar)^{-1} \int dx\ e^{iqx} U(x)$$

— related via bosonization

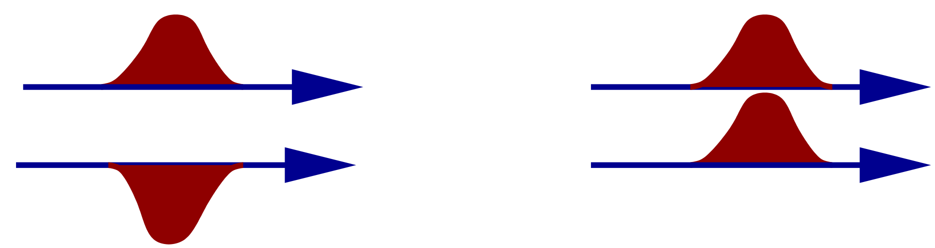
Fractionalisation at $\nu = 2$

Two filled Landau levels



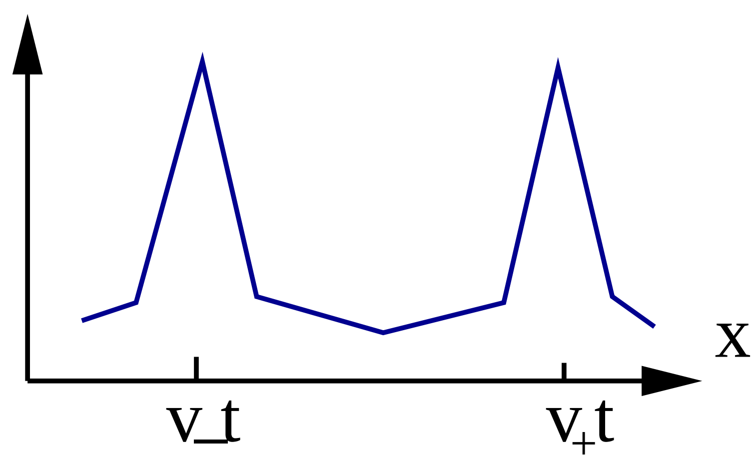
Modes mixed by interaction g

$$v_{\pm} = v \pm g$$



Injected electron fractionalises

$$|\langle \psi_1^\dagger(x,t)\psi_1(0,0)\rangle|^2 \sim$$



Description of edge state tunnelling experiment

Difficulties

Interactions treated most simply via bosonization

Tunneling at QPC simplest in fermionic language

Previous work on theory for relaxation expt

Boltzmann Eqn: Lunde ***et al*** **(2010)**

QPC as source of plasmon noise: Degiovanni ***et al*** **(2010)**

Approx theory of tunnelling: Levkivskyi & Sukhorukov (2012)

Approaches here

Physical picture

Quantum quench as toy problem

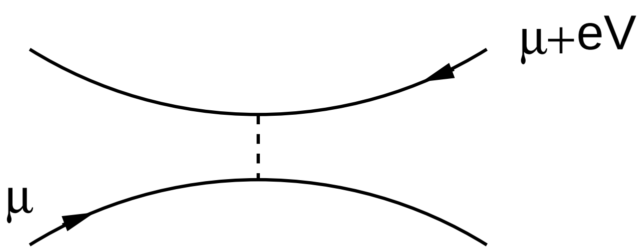
Exact treatment via bosonization + refermionisation

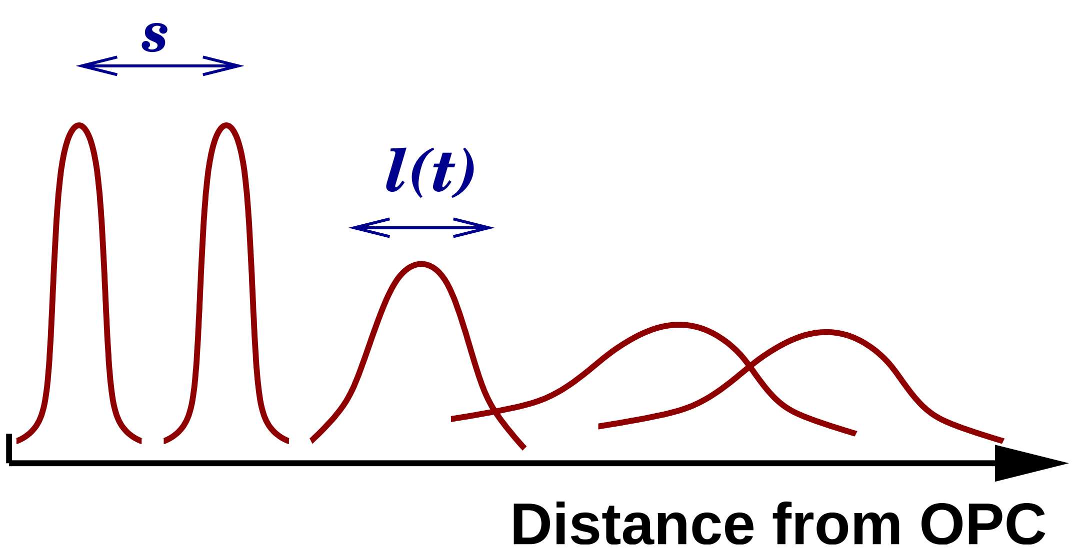
Physical picture of relaxation

Collective mode Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^{\dagger}_{nq} b_{nq}$

Edge magnetoplasmon dispersion $\rightarrow$ electron equilibration?

Initial quasi-particle separation $s \propto \hbar v / eV$





Relaxation when wavepacket spread $l(t) \gtrsim s$

Relaxation from two mode velocities

Two edge modes with short-range interactions

Two linearly dispersing modes $\omega_1(q) = v_+ q$ **&** $\omega_2(q) = v_- q$

Initial quasi-particle separation $s = \hbar v / eV$

Relaxation when wavepacket spread $l(t) \gtrsim s$

Spread $l(t) = [v_+ - v_-]t$

Relaxation time: $t_{\mathrm{eq}} \sim \frac{\hbar}{eV} \cdot \frac{v}{v_+ - v_-}$

Relaxation distance: $v\, t_{\mathrm{eq}}$

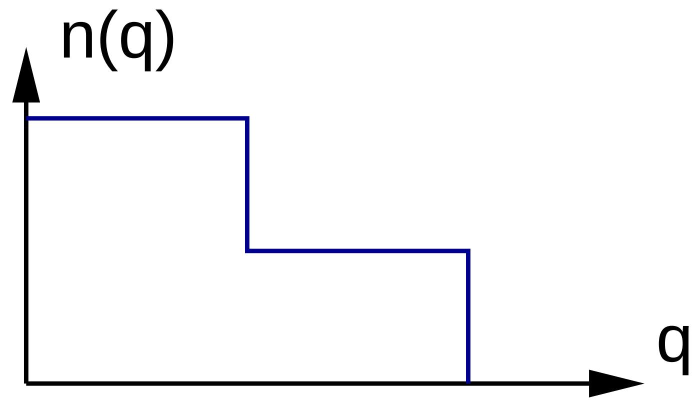
Theoretical Idealisation: Quantum quench

Evade treatment of point contact

– study time evolution in translationally-invariant edge

Initial state

$|\Psi_0\rangle$ **with**



Time evolution

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$?

Energies of collective modes conserved

— consequences for equilibration?

What is the equilibrium state?

Characterise via one-electron correlations

Calculate $G(x,t) = \langle \psi^\dagger(x,t)\psi(0,t)\rangle$

in thermal state $G(x,t) = [-2i\beta\hbar v \sinh(\pi[x+i0]/\beta\hbar v)]^{-1}$

Approach to calculations:

Alternate between fermionic and bosonic descriptions

$$\Psi(x) \sim \mathrm{e}^{-\mathrm{i}\varphi(x)}$$

$$\varphi(x) \propto \sum q^{-1/2}\left[b_q^\dagger \mathrm{e}^{-\mathrm{i}qx} + \mathrm{h.c}\right]$$

$$b_q^\dagger \propto \sum c_{k+q}^\dagger c_k$$

Initial state

simple for fermions

Time evolution

simple for bosons

Observables

fermion fields

Comparison with thermal state

Short-distance correlations

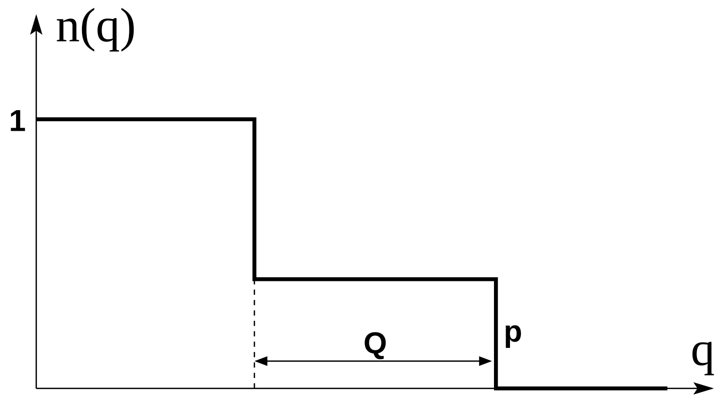
As in thermal state at same energy density

Long-distance correlations

$$G(x,t) \sim \exp(-\alpha|x|)$$ **with** α **not fixed by energy density**

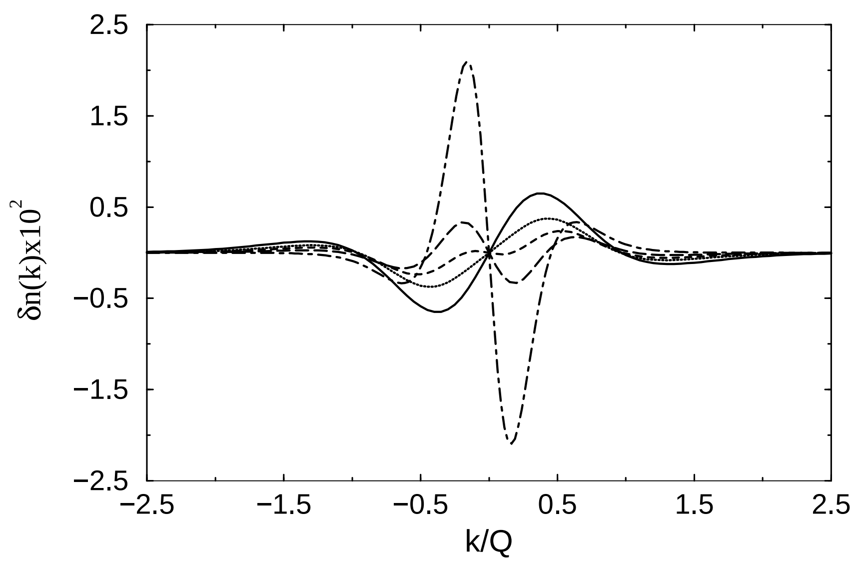
Example

Initial momentum distribution

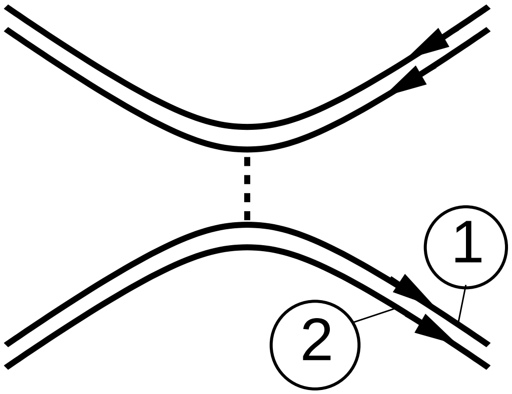


Difference from thermal in steady state

$$p = 0.1, 0.2, 0.25, 0.3, 0.5$$



Solvable model



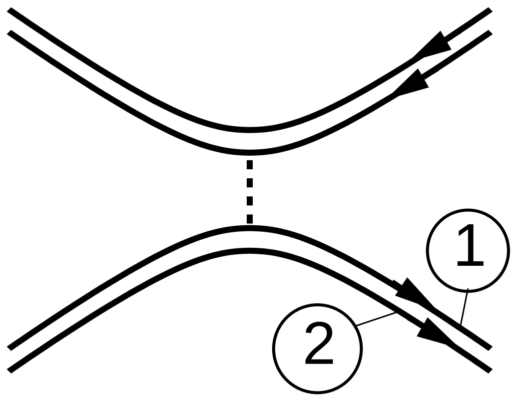
Each edge

$$\mathcal{H} = \mathcal{H}_{\mathrm{kin}} + \mathcal{H}_{\mathrm{int}}$$

$$\mathcal{H}_{\mathrm{kin}} = -\mathrm{i}\hbar v \int \Big(\Psi_1^\dagger(x)\partial_x\Psi_1(x) + \Psi_2^\dagger(x)\partial_x\Psi_2(x) \Big)\, \mathrm{d}x$$

$$\mathcal{H}_{\mathrm{int}} = 2\pi\hbar g \int \rho_1(x)\rho_2(x)\mathrm{d}x \,, \qquad \rho_n(x) = \Psi_n^\dagger(x)\Psi_n(x)$$

Solvable model



Each edge

$$\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{kin}} = -\mathrm{i}\hbar v \int \left(\Psi_1^\dagger(x)\partial_x\Psi_1(x) + \Psi_2^\dagger(x)\partial_x\Psi_2(x) \right) \mathrm{d}x$$

$$\mathcal{H}_{\text{int}} = 2\pi\hbar g \int \rho_1(x)\rho_2(x)\mathrm{d}x \,, \qquad \rho_n(x) = \Psi_n^\dagger(x)\Psi_n(x)$$

Bosonize and diagonalise

$$\Psi_{1,2}(x) \sim \mathrm{e}^{-\mathrm{i}\varphi_{1,2}(x)}$$

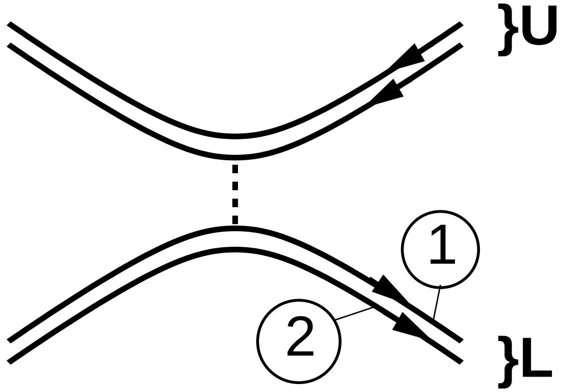
$$\mathcal{H} = \frac{\hbar v_+}{2} \int [\partial_x \varphi_+(x)]^2 \frac{\mathrm{d}x}{2\pi} + \frac{\hbar v_-}{2} \int [\partial_x \varphi_-(x)]^2 \frac{\mathrm{d}x}{2\pi}$$

Modes

$$v_\pm = v \pm g \qquad \varphi_\pm(x) = \frac{1}{\sqrt{2}}[\varphi_1(x) \pm \varphi_2(x)]$$

Refermionize

Combine bosons from opposite edges



$$\varphi_{S\pm} = \frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) + \varphi_{L\pm}(x)]$$

$$\varphi_{A\pm} = \pm\frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) - \varphi_{L\pm}(x)]$$

Tunneling

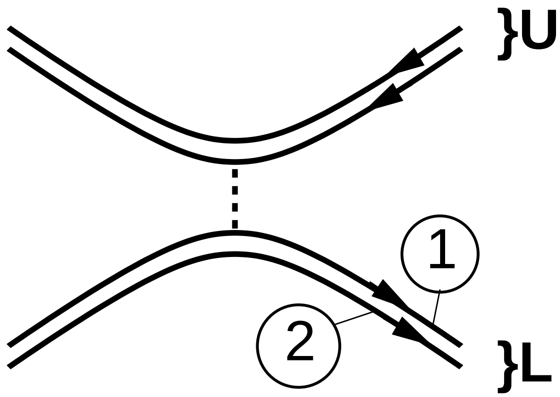
$$\mathcal{H}_{\rm tun} = t_{\rm QPC}[\Psi^\dagger_{U1}(0)\Psi_{L1}(0) + \Psi^\dagger_{L1}(0)\Psi_{U1}(0)]$$

$$\Psi^\dagger_1(0)\Psi_2(0) \sim \mathrm{e}^{\mathrm{i}[\varphi_{U1}(0)-\varphi_{L1}(0)]} \sim \mathrm{e}^{\mathrm{i}[\varphi_{A+}(0)-\varphi_{A-}(0)]} \sim \Psi^\dagger_{A+}(0)\Psi_{A-}(0)$$

Edges

$$\mathcal{H}_{\rm kin}+\mathcal{H}_{\rm int} = -\mathrm{i}\hbar\int[v_+\Psi^\dagger_{A+}(x)\partial_x\Psi_{A+}(x)+v_-\Psi^\dagger_{A-}(x)\partial_x\Psi_{A-}(x)]\mathrm{d}x$$
$$+[\mathrm{A}\leftrightarrow\mathrm{S}]$$

Observables



Electron energy distribution at x from

$$\langle \Psi^{\dagger}_{L1}(x,t)\Psi_{L1}(x,0)\rangle$$

or $$\langle \Psi^{\dagger}_{L2}(x,t)\Psi_{L2}(x,0)\rangle$$

Transforms to $\langle \mathrm{e}^{\mathrm{i}\pi(n_- - n_+)}\rangle$ **or** $\langle \mathrm{e}^{\mathrm{i}\pi(n_- + n_+)}\rangle$

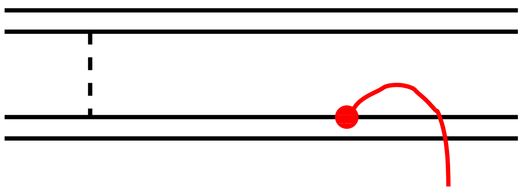
with $$n_{\pm} = \int_x^{x+v_{\pm}t} :\Psi^{\dagger}_{A\pm}(y)\Psi_{A\pm}(y): \,\mathrm{d}y$$

Energy exchange between channels — analytic result

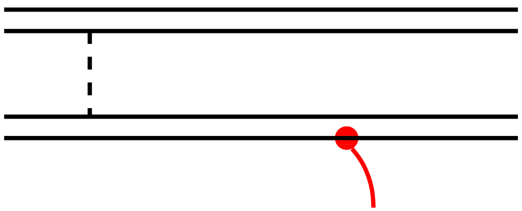
Electron distribution — evaluate free fermion averages numerically

Results: energy exchange between channels

Measurement in channel coupled at QPC

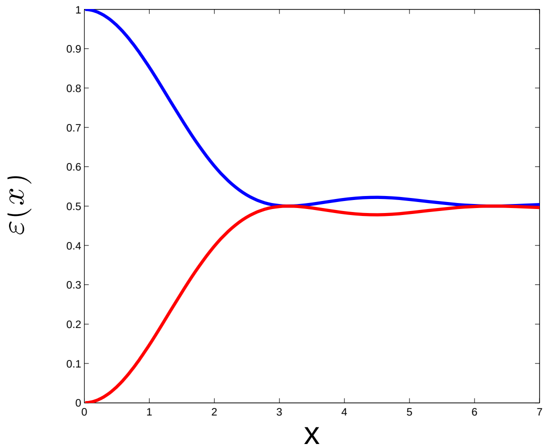


Measurement in channel coupled by interactions



Energy density

$$\varepsilon(x) = \varepsilon_{\mathrm{T}} + \frac{\varepsilon_{\mathrm{V}}}{2}\left[1 \pm \frac{\ell_{\mathrm{V}}^2 \sin^2(x/2\ell_{\mathrm{V}})}{\ell_{\mathrm{T}}^2 \sinh^2(x/2\ell_{\mathrm{T}})}\right]$$



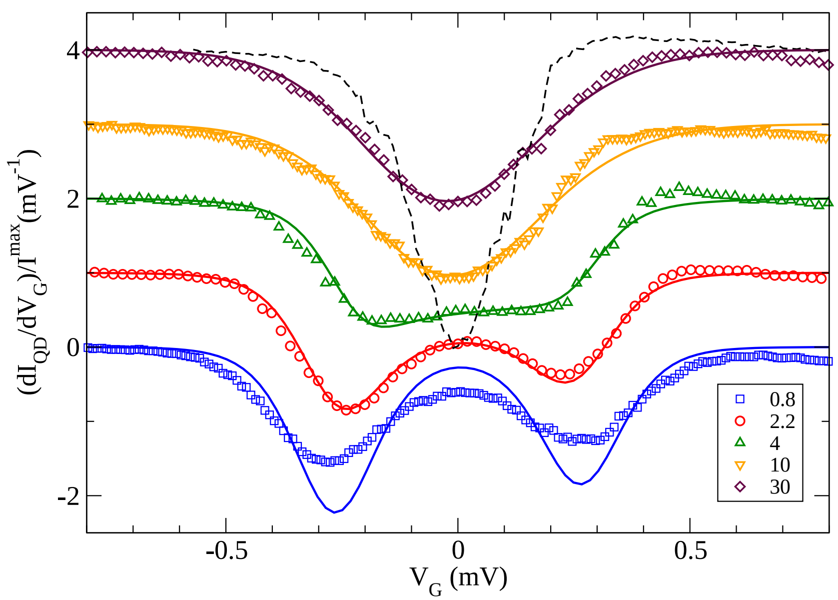
Thermal contribution ε_{T} **Contribution from bias voltage** ε_{V}

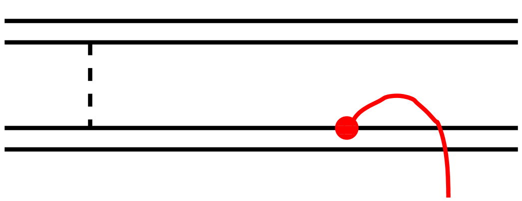
Lengthscales $\ell_{\mathrm{V}} = \frac{\hbar}{eV}\frac{v_+ v_-}{v_+ - v_-}$ **and** $\ell_{\mathrm{T}} = \frac{\hbar}{2\pi k_{\mathrm{B}} T}\frac{v_+ v_-}{v_+ - v_-}$

see also Degiovanni *et al.* **2010**

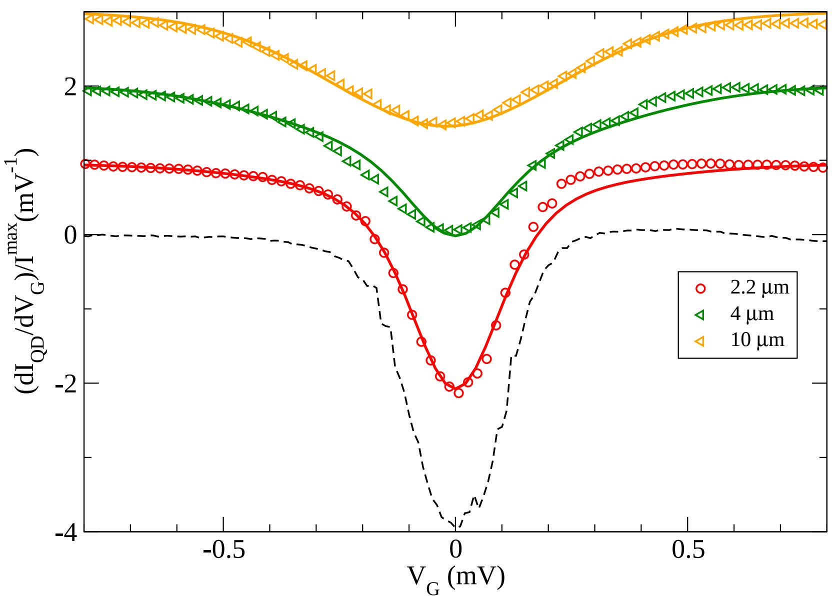
Comparison with experiment

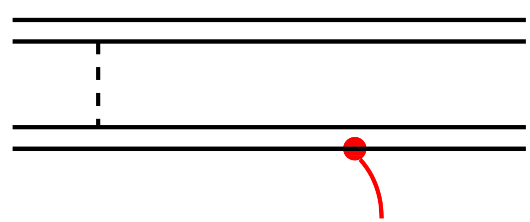
Measurement in channel coupled at QPC





Measurement in channel coupled by interactions

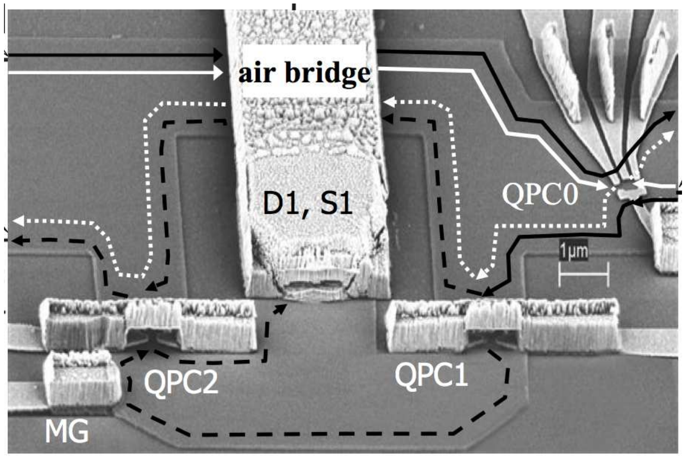




Fitting parameter: interaction strength $(v^2 + g^2)/2g = 6.5 \times 10^4 \mathrm{ms}^{-1}$

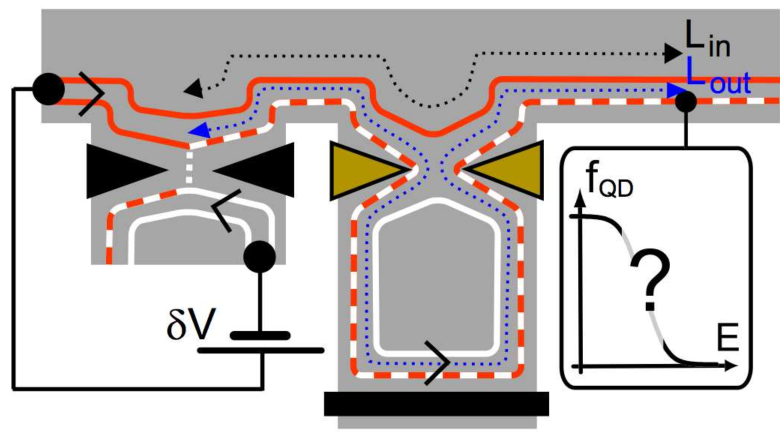
Related experiments

Interferometers out of equilibrium



Heiblum group (2005)

Controlling relaxation



Altimiras et al (2011)

Summary

Experiment probes relaxation in an integrable system

- Interactions bring system into steady state

Full problem with QPC solvable at $\nu = 2$

- Asymptotic state is non-thermal
- Calculated evolution of tunneling density of state with distance

  matches experiment well