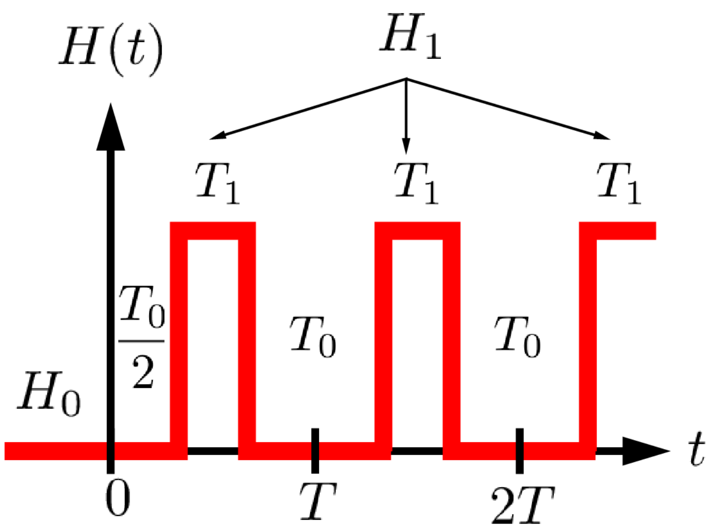
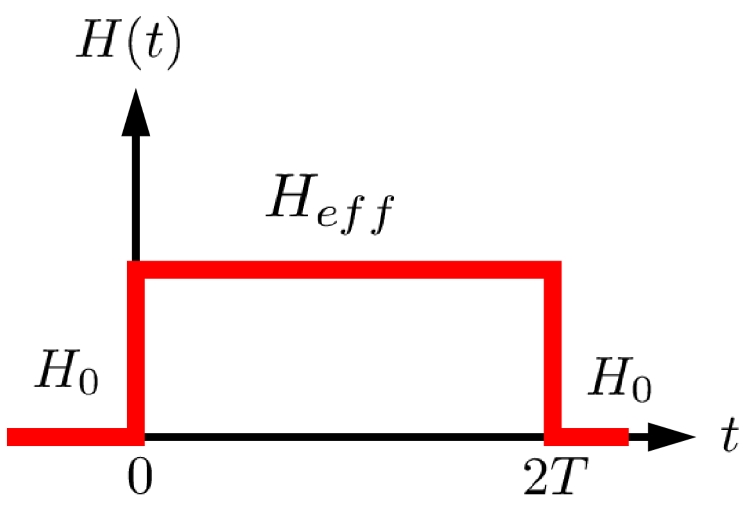
# Many-body energy localization transition in periodically driven systems

KITP Luca D'Alessio (BU) Sept 20$^{th}$ 2012

# Driving protocol




$$H_0 = B_x H_{B_x}$$

$$H_1 = J_z H_z + J_z' H_z' + J_{||} H_{||} + J_{||}' H_{||}'$$

where

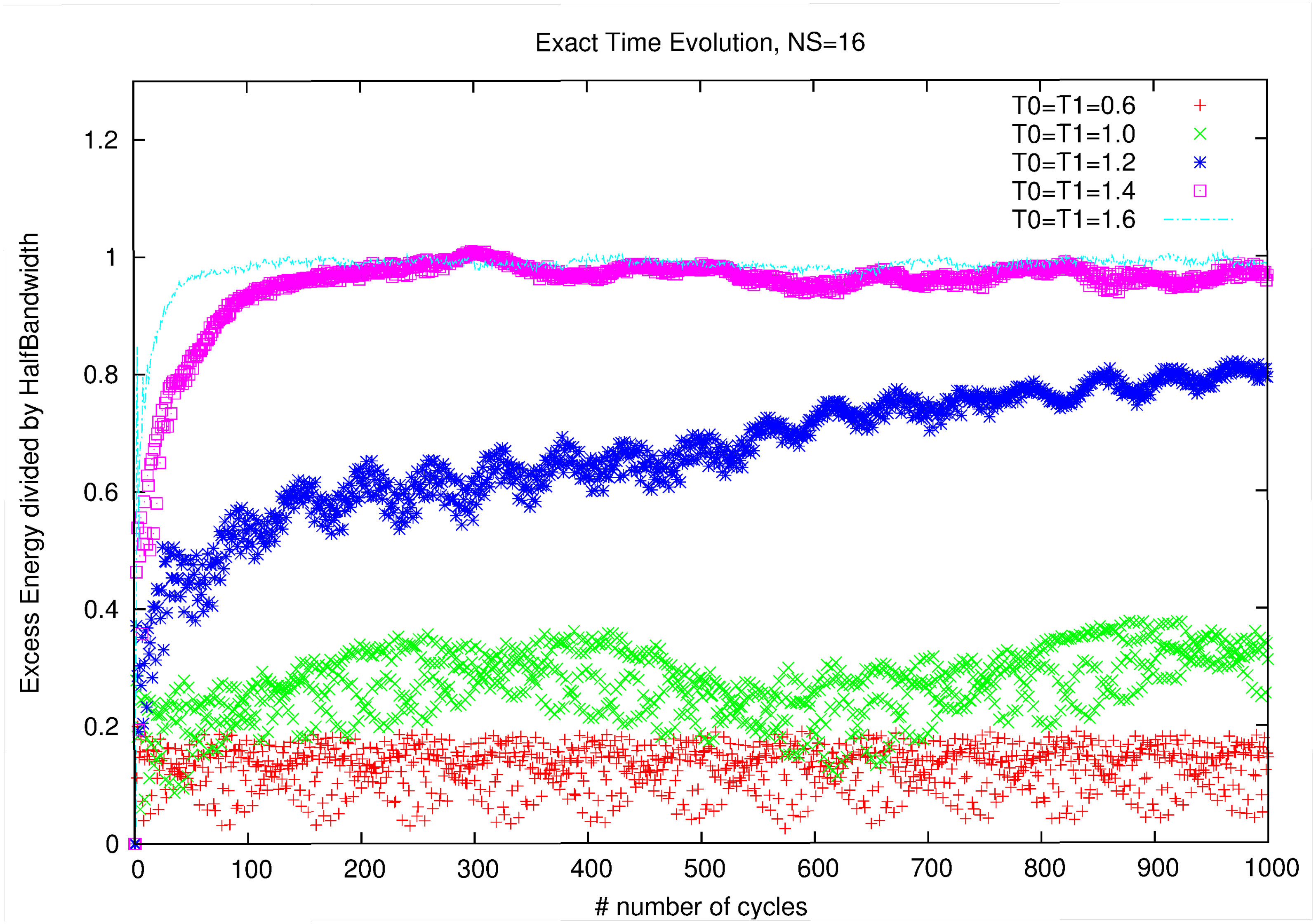
$$H_{B_x} = \sum_n s_n^x$$

$$H_z = \sum_n \left(s_n^z S_{n+1}^z\right), \; H_{||} = \sum_n \left(s_n^x s_{n+1}^x +_n^y s_{n+1}^y\right)$$

$$H_z' = \sum_n \left(s_n^z S_{n+2}^z\right), \; H_{||}' = \sum_n \left(s_n^x s_{n+2}^x +_n^y s_{n+2}^y\right)$$

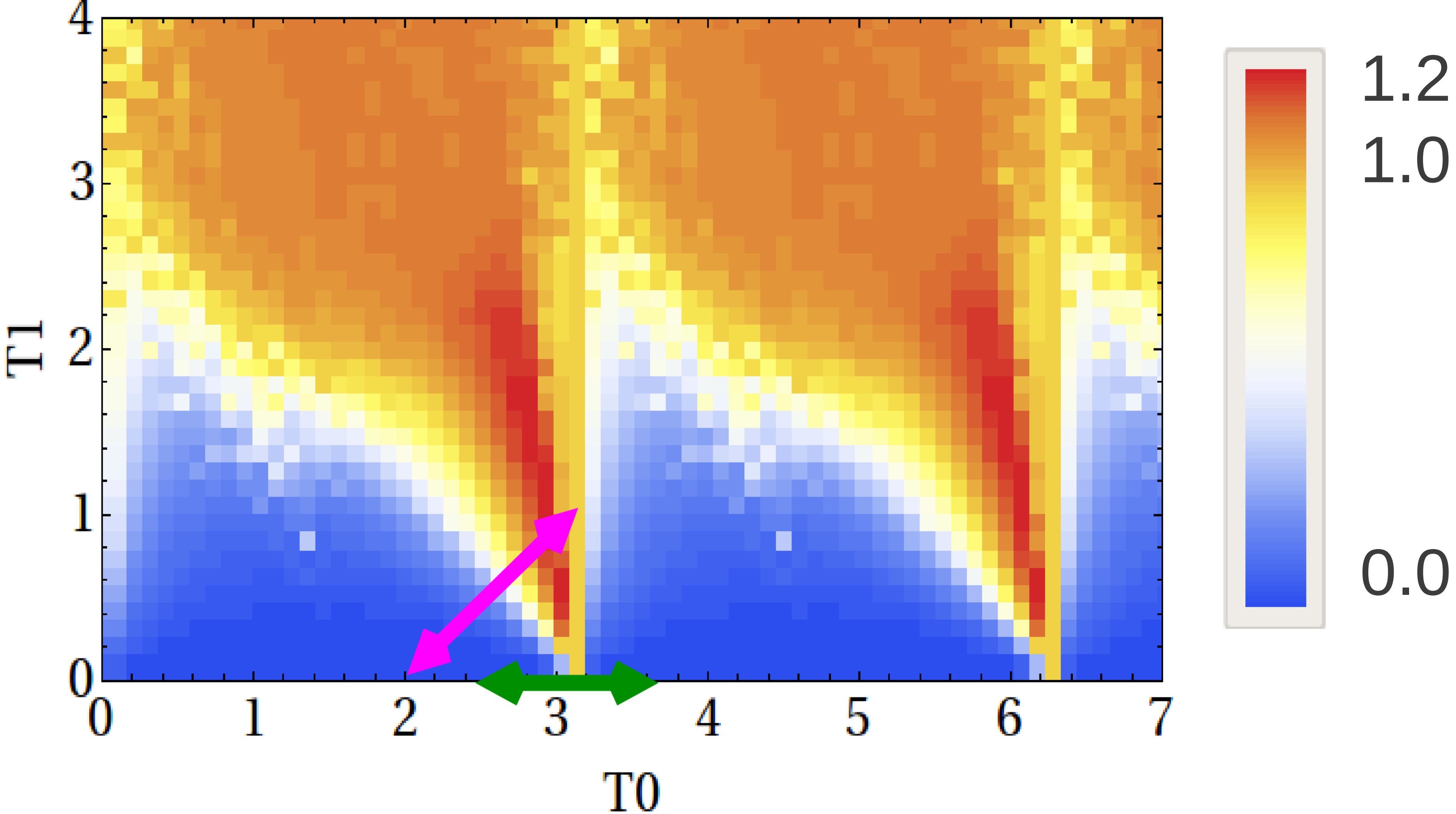
with $B_x = 1, \; J_z = -J_{||}' = \frac{1}{2}, \; J_z' = \frac{1}{40}, \; J_{||} = -\frac{1}{4}$

# Keep H0 and H1 fixed and change T0 and T1



# Keep H0 and H1 fixed and change T0 and T1

$$2\frac{\langle H_0\rangle(t\to\infty)-E_{gs}}{E_{max}-E_{gs}}$$



# Resummation along T0-axis using Hausdorf-Baker-Campbell formula

Product of exponentials of not-commuting operators:

$$Z \equiv \log\left[e^X e^Y e^X\right] = 2X + Y - \sum_{n=1}^{\infty} \frac{2(2^{2n-1}-1)}{(2n)!} B_{2n} \left\{X^{2n}; Y\right\} + \mathcal{O}(Y^2)$$

Where: $X \to \frac{H_0 T_0}{i\hbar}, Y \to \frac{H_1 T_1}{i\hbar}, Z \to \frac{H_{eff} T}{i\hbar}$

and $\{X^{2n}; Y\} \equiv [X, [X, [..., Y]]]$

Trick: each commutators with H0 flip spins,
two nested commutators are identity. Resummation is possible.

# Resummation along T0-axis using Hausdorf-Baker-Campbell formula

After resummation:

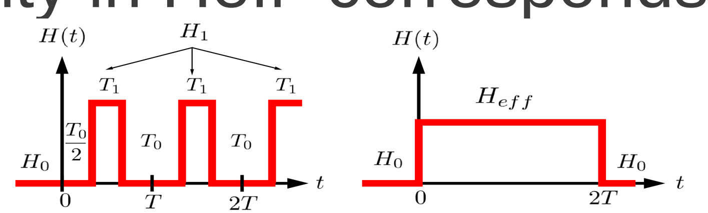
$$H_{eff} = H_{av} - \frac{T_1}{2T}\left(1 - \lambda \cot\left(\frac{\lambda}{2}\right) + \lambda \cot(\lambda)\right)\left(\Delta W_{zz-yy} + \Delta' W'_{zz-yy}\right) + \mathcal{O}(t_1^2)$$

Where: $\Delta = J_z - J_{||}, \Delta' = J'_z - J'_{||}, \lambda = \dfrac{B_x T_0}{\hbar}$

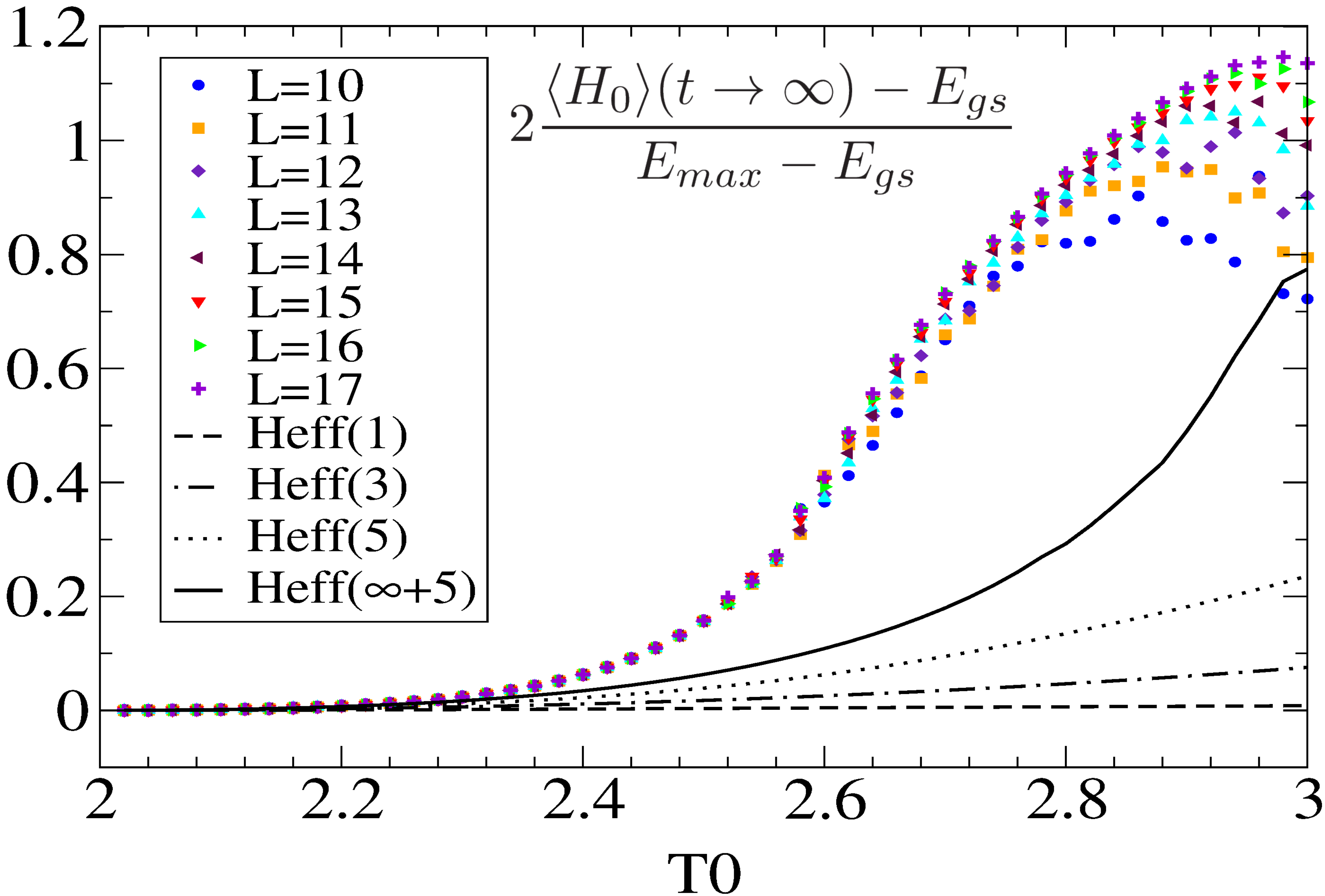
and $W_{zz-yy} = \sum_n \left(s_n^z s_{n+1}^z - s_n^y s_{n+1}^y\right)$

Message 1: Heff is singular for $\lambda = n\pi \rightarrow T_0 = n\pi$

Message 2: each singularity in Heff corresponds to singularity in dynamical behaviour.



# Away for T0-axis resummation is impossible. We use numerics



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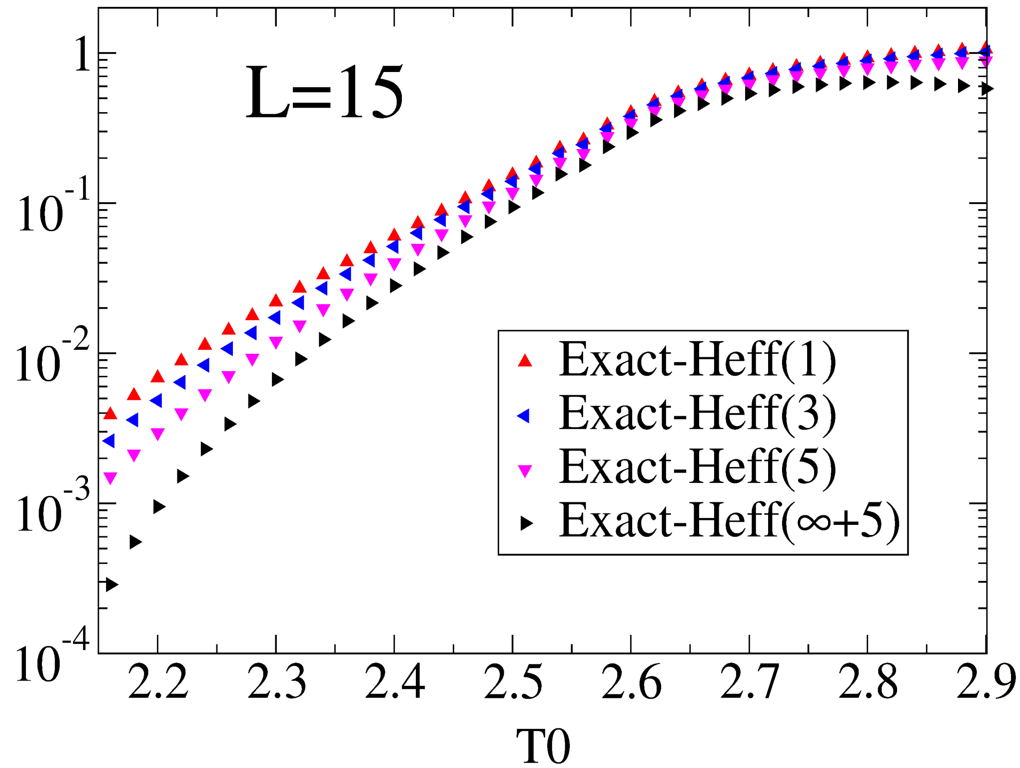


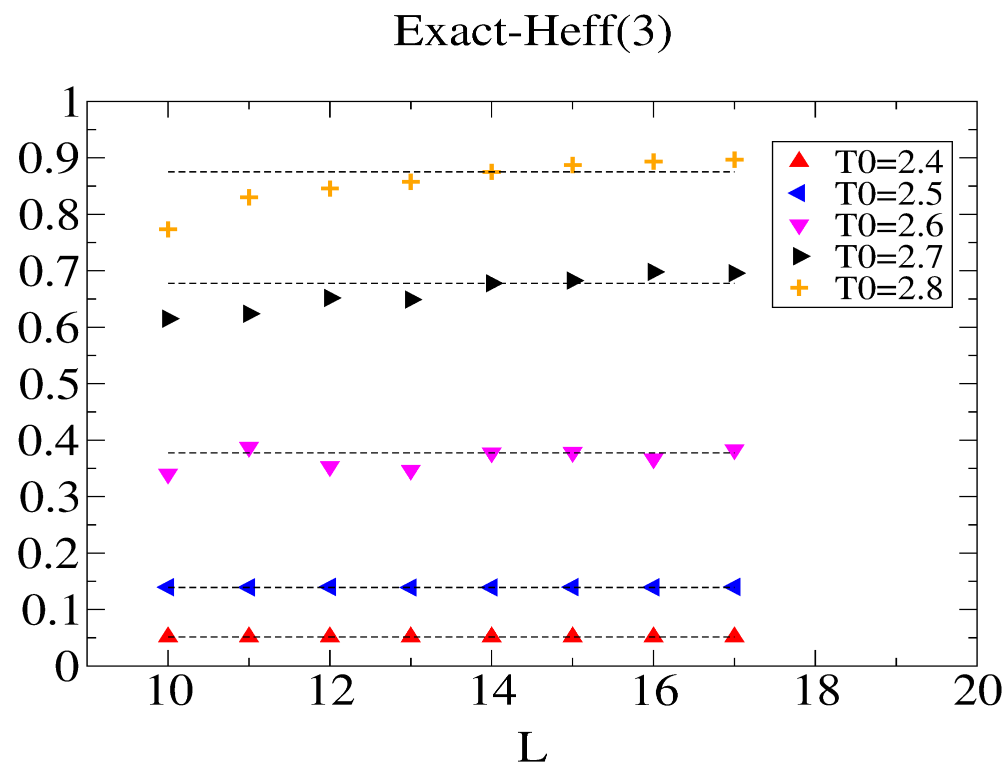


Message 3: It seems that when Magnus series breaks down
The system becomes delocalized

# Conclusion:

1) Magnus expansion provides systematic way to obtain time-independent description of periodically driven system as a power series in the period of the driving.

2) Break down of Magnus expansion signals a many-body localization transition

3) This setup might be realized in cold-atoms experiments