Universal “rephasing” dynamics

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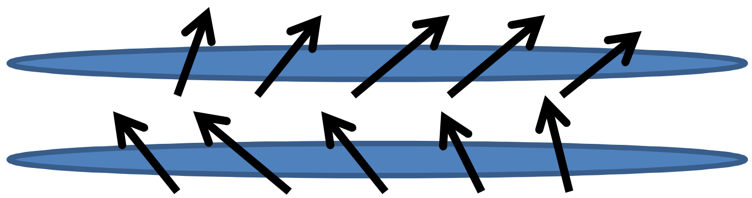
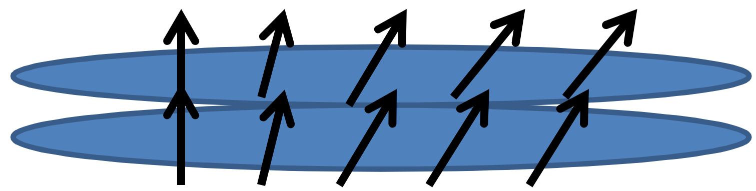
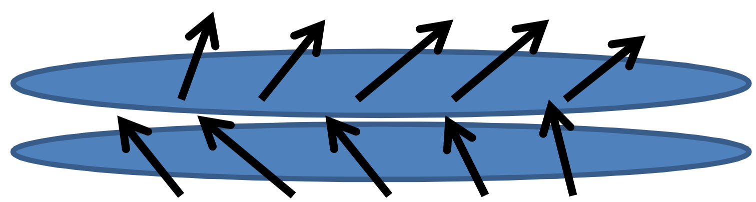
What happens when we couple two independent (quasi)condensates?



Tunneling → rephasing



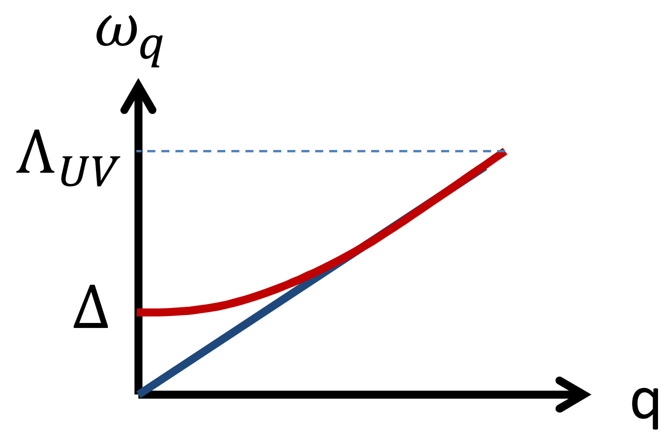
Heating → dephasing



Who wins? Is the answer universal?

Universality – handwaving argument

t < 0	No tunneling: gapless spectrum (Goldstone mode of the relative phase)
t > 0	Finite tunneling : excitation gap Δ



How much heating?

$$\frac{\delta E}{N} = \frac{\text{Total Energy}}{\text{Number of modes}} \approx \Delta \frac{\Delta}{\Lambda_{UV}}$$

Small coupling: $\Delta \ll \Lambda_{UV}$

$$\frac{\delta E}{N} \ll \Delta$$

→ universal quantum dynamics protected by the gap

De Grandi, Gritsev, Polkovnikov (PRB, 2010)

Outlook

(a) Mean field

(b) One dimension

(c) Q&A, Cookies

Mean-field model (Josephson junction)

$$H(t) = \frac{\mu}{N}[(\delta n_1)^2 + (\delta n_2)^2] + j_\perp(t)(\psi_1^\dagger \psi_2 + H.c.)$$

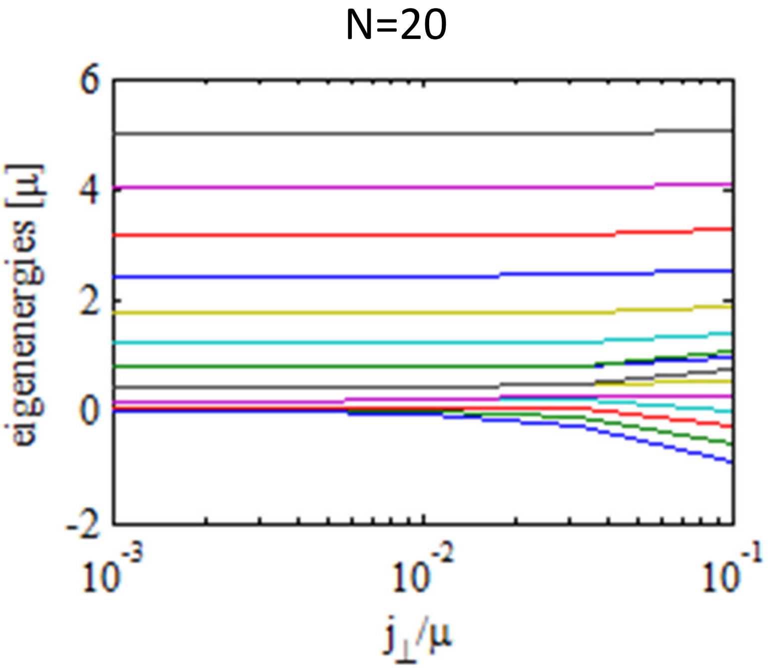
$$\delta n_1 = \psi_1^\dagger \psi_1 - \frac{N}{2}$$

✓ Initial Hamiltonian is gapless:

$$\Delta = \frac{2\mu}{N} \to 0$$

✓ Final Hamiltonian is gapped:

$$\Delta = \omega_p = \sqrt{4 j_\perp \mu}$$

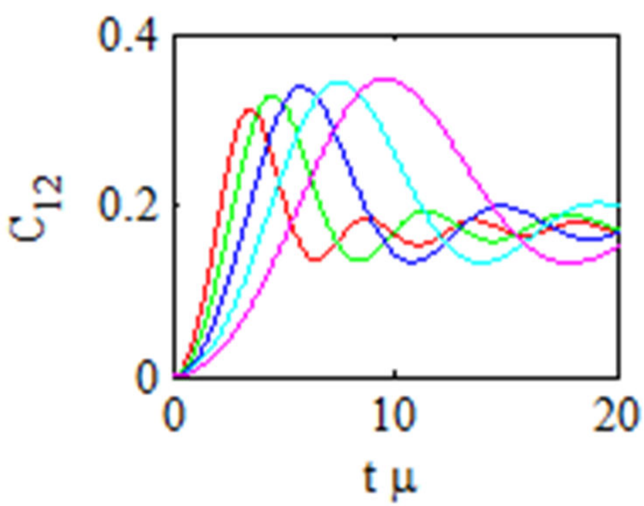


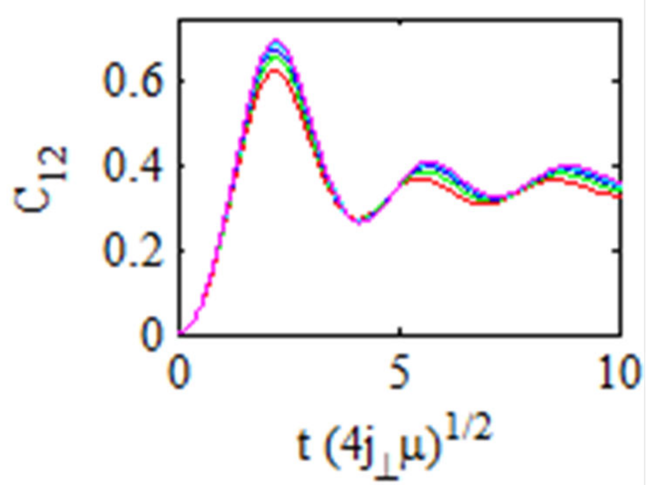
Mean field – observing universality

$$C_{12}(t) = \langle \psi_1^\dagger \psi_2 + H.c.\ \rangle$$

$\frac{j_\perp}{\mu} = 0.1, 0.06, 0.037, 0.022, 0.013$

Rescaling





What is the universal function?

Mean field – universal curves

In the thermodynamic limit $N \to \infty$ we can map the problem to classical equation of motion

$$H(t) = \frac{\mu}{N}[\delta n_1^2 + \delta n_2^2] + j_\perp(t)\left(\psi_1^\dagger \psi_2 + H.c.\right)$$

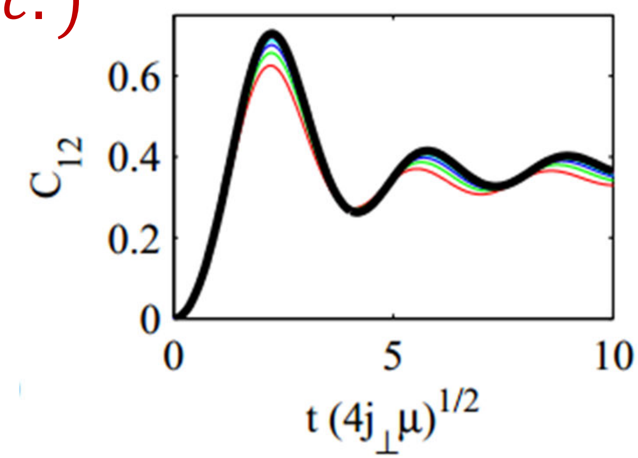
$$\psi_1 \to \sqrt{N} e^{i\phi_1}$$

$$\delta n_1 \to \frac{N}{2\mu}\partial_t \phi_1$$

$$\phi = \phi_1 - \phi_2$$

$$\hat{\phi} = \phi_1 + \phi_2$$

$$\frac{1}{2\mu}\partial_t^2 \phi = 2 j_\perp(t) \cos(\phi)$$



Non-linear simple pendulum with period $\Delta = \sqrt{4 j_\perp \mu}$: analytic solution via elliptic functions **(black curve)**

One dimension

The intial state does not have a well defined phase

$$H(t) = H_{LL}[\phi_1] + H_{LL}\left[\phi_2\right] + g\,\Theta(t)\int dx\,\cos(\phi_1 - \phi_2)$$

Relative and COM coordinates:

$$\phi = \phi_1 - \phi_2$$
$$\hat{\phi} = \phi_1 + \phi_2$$

$$H(t) = H_{LL}\left[\hat{\phi}\right] + H_{LL}[\phi] + g\,\Theta(t)\int dx\,\cos(\phi)$$

Quench in a Sine-Gordon – many different cases!

Iucci & Cazalilla (2010)

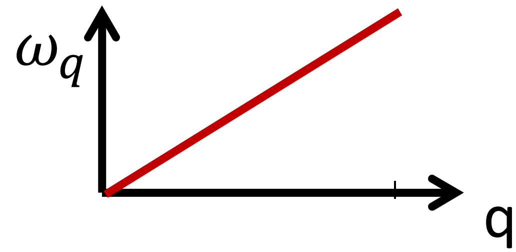
Mitra (2012)

Quench in a Sine-Gordon - review

(1) Turning **off** the coupling:

Long time dynamics is **classical** ("Dephasing")

Calabrese&Cardy, Bistritzer&Altman (2006)

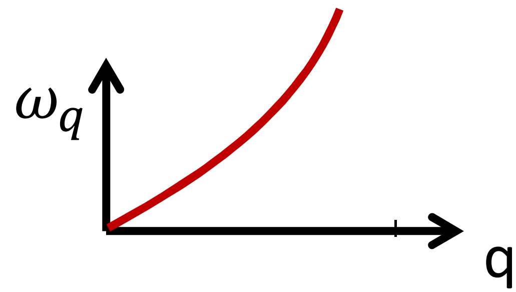


$$\Delta = 0$$

(2) Turning **on** an irrelevant coupling: (K>2)

Long time dynamics is **not-universal**

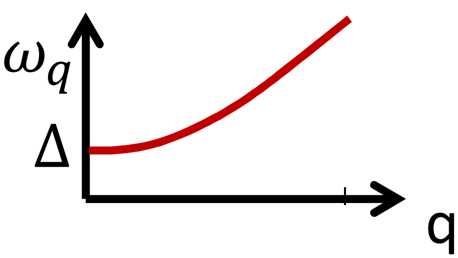
Mitra and Giamarchi (2011)



$$\frac{\delta E}{N} \sim \Lambda_{UV}$$

(3) Turning **on** a relevant coupling: (K<2)

Long time dynamics is **quantum** ("Rephasing")



$$\frac{\delta E}{N} \ll \Delta$$

Independent modes approximation

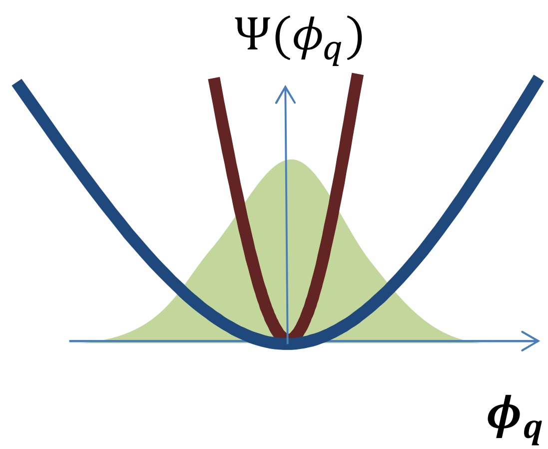
Iucci & Cazalilla (2010)

Fluctuations of the *q* mode oscillate

$$\langle |\phi_q(t)|^2 \rangle \approx \frac{K}{|q|}\cos^2(\Delta t) \; \rightarrow \; \frac{K}{2|q|}$$

Total fluctuations $\langle \phi^2(x,t) \rangle$ diverge

$$\langle \cos(\phi(t)) \rangle = e^{-\frac{1}{2}\langle \phi^2 \rangle} \; \rightarrow \; 0$$



→ The system does not rephase!

Why is this wrong?

Non-perturbative series expansion

$$\langle \cos\phi(x,t)\rangle = \int \mathcal{D}\phi \, \cos\phi \; e^{iS_0 + g\int_0^t dt'dx' \cos(\phi')}$$

$$= \sum_{N=1}^{\infty} \frac{1}{N!} (ig)^N \left\langle \cos\phi \left(\int_0^t dt'dx' \cos\phi' \right)^N \right\rangle_0$$

$$\sim \sum_{N=1}^{\infty} c_N \, g^N (\mu t)^{2N-(N+1)K/2}$$

$$\Delta = \mu \left(\frac{g}{\mu} \right)^{\frac{1}{2-K}}$$

$$\sim \left(\frac{\Delta}{\mu} \right)^K \underbrace{\sum_{N=1}^{\infty} c_N \, (\Delta t)^{2N-(N+1)\frac{K}{2}}}_{R(\Delta t)}$$

Expectation value - scaling result

$$\langle \cos(\phi(t)) \rangle = \left(\frac{\Delta}{\Lambda_{UV}}\right)^{K} \mathrm{R}(\,\Delta\, t\,)$$

Same scaling exponent as in the ground state

Lukyanov & Zamolodchikov (1997)

Universal dynamics – connecting short times to the steady state

Theory: Janssen etal (1989)

Numerical: Zheng (1998)

Review: Albano etal (RPP 2011)

Non universal contributions

From the UV cutoff ←→ integral over short times

$$\langle \cos(\phi(t)) \rangle = \left(\frac{\mathrm{g}}{\Lambda_{UV}}\right)^{\frac{K}{2-K}} \mathrm{R}(\Delta t) + A\frac{\mathrm{g}}{\Lambda_{UV}} + o(\mathrm{g}^2)$$

Must be an analytic function of "g"

⇒ The universal part is dominant for $\frac{K}{2-K} < 1 \Leftrightarrow K < 1$

At K=1 Exact solution (refermionization)

$$\langle \cos(\phi(t)) \rangle = \frac{\mathrm{g}}{\Lambda_{UV}}[\mathrm{CosIntegral}(2\mathrm{g}t) + A]$$

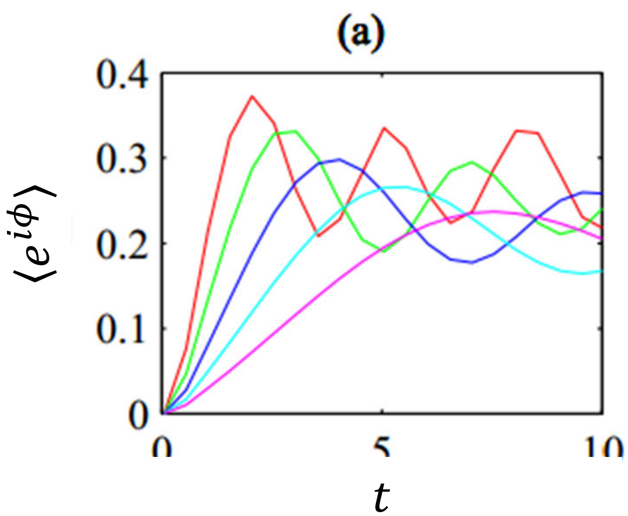
both terms scale like g

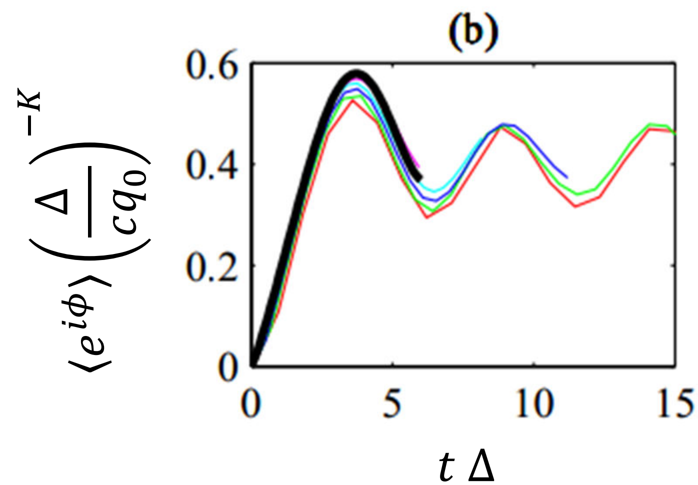
Iucci & Cazalilla (2010)

Lattice model (TEBD)

Hard core bosons ladder:

K = 0.4





Black curve: short time perturbative result

More about universality

How sudden is "sudden"?

$$\tau_{\mathrm{ramp}} \ll \frac{1}{\Delta}$$

How cold is "cold"?

$$T_0 \ll \Delta$$

Spatially integrated coherence?

$$Z = \int_{-L/2}^{L/2} dx\, e^{i\phi(x)}$$

$$P(|Z|) = f(K, \Delta L, \Delta t)$$

Equilibrium case: V. Gritsev, E. Altman, E. Demler, A. Polkovnikov, Nat.Phys. (2006)

A. Imambekov, V. Gritsev, E. Demler, PRA (2008)

Summary

1. The "rephasing" process is dominated by the low-frequency modes and is thus **universal**.

$$\langle \cos(\phi(t)) \rangle = \left(\frac{\Delta}{\mathrm{c}q_0}\right)^K \mathrm{R}(\Delta t)$$

2. The **scaling** ansatz relates the perturbative short-time dynamics to the non-perturbative steady states

3. The full distribution of the **intereference** phase and contrast show respectively mean-field and beyond mean-field dynamics

Non-equal time correlations?

Analytic expression for the one dimensional case?

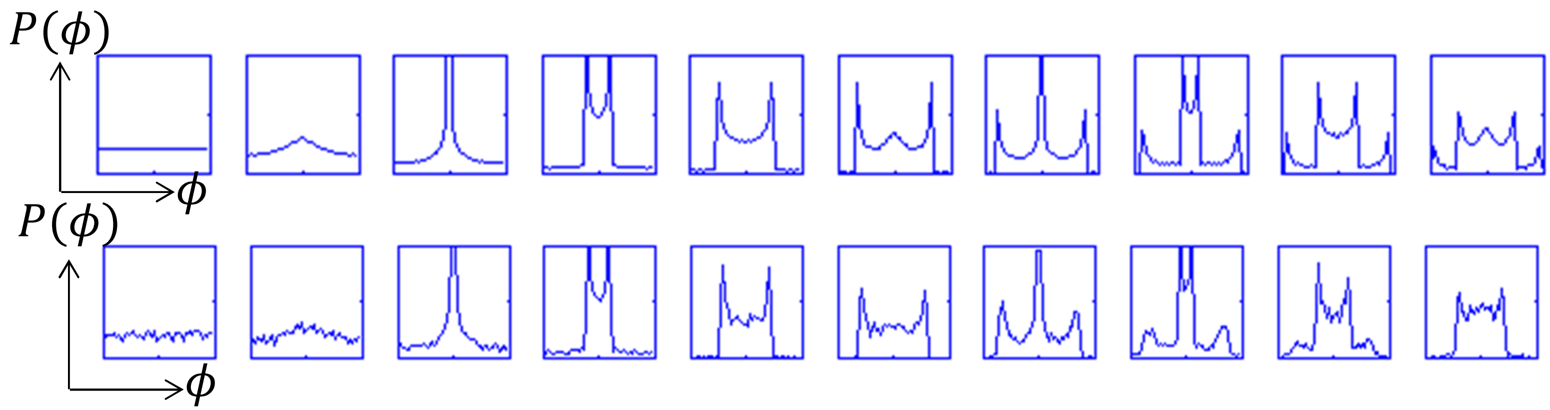
Extension to other systems: Kondo physics from short-time QMC?

Q.: Why integrated coherence?

Problem: In atom-on-a-chip $K \ll 1$ → the dynamics is **very** mean field

(a) Mean field:

(b) 1d (K=0.02):



Solution: use |Z| to observe 1d physics

$$Z = \int_{-L/2}^{L/2} dx\, e^{i\phi(x)}$$

(a) Mean field:

(b) 1d (K=0.02):