Shortcuts to adiabaticity in many-body systems

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- Motivation
- Self-similar dynamics
- Inhomogeneous phase transitions & KZM
- Excitation-free quantum phase transitions

Why speeding up things?

Quantum quenches

Probing correlations in many-body systems E. Cornell'09 finite time-of-flight in ultracold gases

• Quantum thermodynamics Adiabatic expansions are the bottleneck in quantum refrigerators & engines

• Quantum simulation Preparation of novel-quantum phases Crossing critical points



- Quantum Information Processing & Quantum Optics
 Faster Quantum gates, STIRAP, RAP, ion-transport
- Prevent decoherence and role of perturbations



STA beyond this talk

Chen, AdC, Guery-Odelin, Modugno, Muga, Ruschhaupt, Torrontegui adiabatic invariants

Calarco, Fazio, Li, Stefanatos Optimal control theory

> Barankov, De Chiara, Polkovnikov, Sengupta Nonlinear quantum quenches

Masuda, Nakamura Fast forward technique

Berry, Demirplak, Morsch, Rice Transitionless quantum driving & counterdiabatic fields



Fast expansions



A. del Campo & M. G. Boshier, Sci. Rep. 2, 648 (2012)
A. del Campo, Phys. Rev. A 84, 031606(R) (2011)
A. del Campo, EPL 96, 60005 (2011)
X. Chen, A. Ruschhaupt, Schmidt, A. del Campo, D. Guery-Odelin, J. G. Muga, PRL104, 063002 (2010)

Example: Standard Quench



Excitation: breathing mode in the time-evolution of the width of the cloud

Fast expansion without vibrational heating



Self-similar dynamics

1. Take a somewhat general many-body time-dependent Hamiltonian

$$\hat{\mathcal{H}} = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m} \Delta_i^{(D)} + \frac{1}{2} m \omega^2(t) \mathbf{x}_i^2 \right] + \epsilon \sum_{i < j} \mathcal{V}(\mathbf{x}_{ij}) \qquad \mathbf{x}_i \in \mathbb{R}^D, \ \mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$$

With a potential satisfying

$$\mathbf{V}(\lambda \mathbf{x}) = \lambda^{\alpha} \mathbf{V}(\mathbf{x})$$

2. Impose a self-similar dynamical ansatz

$$\Phi\left(\{\mathbf{x}_i\},t\right) = \frac{1}{b^{D/2}} e^{i\sum_{i=1}^{N} \frac{m\mathbf{x}_i^2 \dot{b}}{2b\hbar} - i\mu\tau(t)/\hbar} \Phi\left(\{\frac{\mathbf{x}_i}{b}\},0\right)$$

3. Get the consistency equations, i.e.

$$\ddot{b} + \omega^2(t)b = \omega_0^2/b^3 \qquad \epsilon(t) = b^{\alpha-2}$$

V. Gritsev, P. Barmettler, E. Demler, New J.Phys.**12**, 113005 (2010) A. del Campo, Phys. Rev. A **84**, 031606(R) (2011)

Exact dynamics of correlations

reduced

Under self-similar dynamics

$$g_{1}(\mathbf{x}, \mathbf{y}; t) = \frac{1}{b^{D}} g_{1}\left(\frac{\mathbf{x}}{b}, \frac{\mathbf{y}}{b}; 0\right) \exp\left(-\frac{i}{b}\frac{\dot{b}}{\omega_{0}}\frac{\mathbf{x}^{2} - \mathbf{y}^{2}}{2l_{0}^{2}}\right) \qquad \begin{array}{l} \text{One-body reduced} \\ \text{density matrix} \\ n(\mathbf{k}, t) &= b^{D} \int d\mathbf{x} d\mathbf{y} \ g_{1}(\mathbf{x}, \mathbf{y}; 0) \\ \times & \exp\left[-ib\left(\frac{\dot{b}}{\omega_{0}}\frac{\mathbf{x}^{2} - \mathbf{y}^{2}}{2l_{0}^{2}} - \mathbf{k} \cdot (\mathbf{x} - \mathbf{y})\right)\right] \end{array} \qquad \begin{array}{l} \text{Momentum} \\ \text{distribution} \end{array}$$

Adiabatic limit:
$$g_1(\mathbf{x}, \mathbf{y}; t) = \frac{1}{b^D(t)} g_1\left(\frac{\mathbf{x}}{b(t)}, \frac{\mathbf{y}}{b(t)}; 0\right),$$

 $n(\mathbf{k}, t) = b^D(t) \int d\mathbf{x} d\mathbf{y} \ g_1(\mathbf{x}, \mathbf{y}; 0) \exp\left[i\gamma \mathbf{k} \cdot (\mathbf{x} - \mathbf{y})\right]$
 $= b^D(t) n(b(t)\mathbf{k}, 0),$

Slow, unstable against losses, decoherence, perturbations

Exact dynamics of correlations

Under self-similar dynamics

$$g_{1}(\mathbf{x}, \mathbf{y}; t) = \frac{1}{b^{D}} g_{1}\left(\frac{\mathbf{x}}{b}, \frac{\mathbf{y}}{b}; 0\right) \exp\left(-\frac{i}{b}\frac{\dot{b}}{\omega_{0}} \frac{\mathbf{x}^{2} - \mathbf{y}^{2}}{2l_{0}^{2}}\right)$$
$$n(\mathbf{k}, t) = b^{D} \int d\mathbf{x} d\mathbf{y} \ g_{1}(\mathbf{x}, \mathbf{y}; 0)$$
$$\times \exp\left[-ib\left(\frac{\dot{b}}{\omega_{0}} \frac{\mathbf{x}^{2} - \mathbf{y}^{2}}{2l_{0}^{2}} - \mathbf{k} \cdot (\mathbf{x} - \mathbf{y})\right)\right]$$

Sudden quench: $(\omega(t > 0) = \omega_f = 0)$ $b(t) = \sqrt{1 + \omega_0^2 t^2}$ $b(t) \sim \omega_0 t, \dot{b} = \omega_0$

$$n(\mathbf{k},t) \sim |2\pi\omega_0 l_0^2/\dot{b}|^D g_1(\omega_0 \mathbf{k} l_0^2/\dot{b}, \omega_0 \mathbf{k} l_0^2/\dot{b})$$

Loss of off-diagonal elements, distortion of correlations (dynamical fermionization)

Design of a shortcut to adiabaticity

1. Force the scaling ansatz to reduce to the initial and final states considered Boundary conditions:

$$b(0) = 1, \quad \dot{b}(0) = 0, \quad \ddot{b}(0) = 0$$

$$b(\tau) = \sqrt{\frac{\omega_f}{\omega_0}}, \quad \dot{b}(\tau) = 0, \quad \ddot{b}(\tau) = 0$$

2. Determine an ansatz for the scaling factor (e.g. a polynomial)

$$b(t) = \sum_{j=0}^{5} a_j t^j \qquad s = t/\tau$$

$$b(t) = 6 (\gamma - 1) s^5 - 15 (\gamma - 1) s^4 + 10 (\gamma - 1) s^3 + 1$$

3. Find the driving time-dependent frequency and coupling strength from the consistency equations

$$\ddot{b} + \omega^2(t)b = \omega_0^2/b^3 \qquad \epsilon(t) = b^{\alpha-2}$$

Example

Time Evolution:



Shortcut in a Tonks-Girardeau gas



Preserves correlations

Robust against perturbations

A quantum dynamical microscope!

AdC, PRA 84, 031606(R) (2011)

Experiment with a thermal cloud

Labeyrie's group: Phys. Rev. A 82, 033430 (2010) 87 Rb in loffe-Pritchard trap



FIG. 2: Optimal trap frequency trajectory for a 35 ms vertical decompression. We plot (line) $\nu_z(t)$ for a 35 ms vertical decompression from $\nu_{0z} = 235.8$ Hz to $\nu_{fz} = 15.7$ Hz, obtained with the invariant method (see text). The symbols correspond to measured values of the vertical trap frequency during the decompression process.



FIG. 3: (Color online) Vertical trap decompression in 35 ms. We report on (a) and (b) respectively the cloud's vertical center-of-mass position z_{cm} and size σ_z versus time after decompression, for four different sequences. Open circles (green online): abrupt decompression; solid circles (black online): linear decompression in 35 ms; stars (red online): shortcut decompression in 35 ms; squares (blue online): linear decompression in 6 s.

Experiment with an interacting BEC

Labeyrie's group: EPL 93, 23001 (2011)



FIG. 1: Linear versus shortcut BEC decompression. We compare the time evolution of the BEC after two different decompression schemes: (\mathbf{A}) a 30-ms-long linear ramp and (\mathbf{B}) the shortcut trajectory (see text). The center of mass motion has been subtracted from these time of flight images for clarity.



FIG. 2: Optimal BEC decompression in 30 ms. We plot the optimal trajectories $\omega_{\perp}(t)/2\pi$ (solid) and $\omega_{\parallel}(t)/2\pi$ (×5, dashed). The insert compares the subsequent evolution of the BEC's center of mass (open symbols) and radial size (solid symbols) for the optimal (stars) and linear (squares) decompressions (GPE simulation).

Self-similar dynamics: applicability

Easy and very general applicability to classical and quantum fluids In other traps: AdC & Boshier, Sci. Rep 2, 648 (2012)

It does not require diagonalization of the Hamiltonian

Limited to processes associated with self-similar dynamics

Expansions

Transport Theory @ Muga's group: Torrontegui et al. PRA '11 Exp @ Wineland's group: Bowler et al. PRL '12

Interaction tuning in BEC: AdC EPL '11

Splitting, Interferometry,

2nd recipe: Inhomogeneous phase transitions

Idea:

When a inhomogeneous system faces a symmetry breaking scenario, let different parts of the system talk to each other, so that the same ground state in the low symmetry phase is chosen everywhere (no vortices, kinks, solitons, etc.).

Theory: Inhomogeneous Kibble-Zurek mechanism.



Classical phase transitions: T. W. B. Kibble, G. E. Volovik, JETP 1997 W. H. Zurek, *PRL*102,105702 (2009) A. del Campo et al. *PRL*105, 075701 (2010) A. del Campo et al. *NJP 13, 083022 (2011)* Quantum phase transitions: W. H. Zurek, U. Dorner, PTRSA 2008 J. Dziarmaga, M. M. Rams, NJP 2010 B. Damski, W. H. Zurek, NJP 2009

Cosmology in the lab

- Cosmology : symmetry breaking during expansion and cooling of the early universe
- Condensed matter:
 - Vortices in Helium
 - Liquid crystals
 - Superconductors
 - Superfluids

Similar free-energy landscape

near a critical point

T. W. B. Kibble, JPA 9, 1387 (1976); Phys. Rep. 67, 183 (1980) W. H. Zurek, Nature (London) 317, 505 (1985); Acta Phys. Pol. B. 1301 (1993)

Second order phase transitions

Landau theory: Free energy landscape, changes across a 2PT from single to double well potential, parameterized by a relative temperature



$$V(\varphi) = \alpha (T - T_{\rm c}) |\varphi|^2 + \frac{1}{2} \beta |\varphi|^4$$

Second order phase transitions

Universal behaviour of the order parameter: divergence of

Correlation/healing length $\xi=\xi_0/|\epsilon|^{
u}$

Dynamical relaxation time $\, au = \, au_0 / | oldsymbol{\epsilon} |^{\mu} \,$



The Kibble-Zurek mechanism



Linear quench

$$\epsilon = t/\tau_Q$$

$$au= au_0/|m{\epsilon}|^{\mu}$$

The Kibble-Zurek mechanism



Linear quench

$$\epsilon = t/\tau_Q$$

$$au= au_0/|m{\epsilon}|^{\mu}$$

The average domain size is given by the equilibrium correlation length at the freeze-out time

$$\hat{\xi} = \xi(\hat{t}) = \xi_0 / |\epsilon(\hat{t})|^{\nu}$$

The Kibble-Zurek mechanism



Linear quench

$$\epsilon = t/\tau_Q$$

$$au= au_0/|m{\epsilon}|^{\mu}$$

The average domain size is given by the equilibrium correlation length at the freeze-out time

$$\hat{\xi}=\xi_0(au_Q/\eta)^{1/4}$$

Structural phases in trapped ions

N ions on a ring trap with harmonic transverse confinement

$$H = \frac{1}{2}m\sum_{n}\dot{r}_{n}^{2} + \frac{1}{2}m\sum_{n}(\nu_{t}^{2}z_{n}^{2}) + \frac{Q^{2}}{2}\sum_{n\neq n'}\frac{1}{|r_{n} - r_{n}'|}$$

Critical transverse frequency

Linear chain

Degenerated zig-zag chains







$$\hat{\xi}=\xi_0(au_Q/\eta)^{1/4}$$

$$\label{eq:relation} \begin{split} \nu_t^{(c)2} &= 4 \frac{Q^2}{ma(0)^3} \\ \text{Fishman PRB '08} \end{split}$$

Testing KZM in the lab



MD numerics: Langevin dynamics including laser cooling N=50, 2000 realizations, quench of the transverse trapping frequency

Testing KZM in the lab



Axial and transverse harmonic potential (instead of a ring trap)



. (within LDA)

Inhomogeneous density, spatially dependent critical frequency Linear guench: $\delta(x,t) = \nu_t(t)^2 - \nu_t^c(x)^2$ $= \nu_t^{(c)}(0)^2 - \nu_t^{(c)}(x)^2 - \delta_0 \frac{t}{\tau_0}$ Causality restricts the effective size of the chain Front satisfying $\delta(x_F, t_F) = 0$ moves at velocity $v_F \sim \frac{\partial_t \delta(x, t)}{\partial_r \delta(x, t)}$ Sound velocity $\hat{v}_x = \hat{\xi}_x / \hat{\tau}_x$ ······

IKZM for solitons in BEC: Zurek *PRL* 102, 105702 (2009) AdC et al. *PRL*105, 075701 (2010)

Adiabatic dynamics is possible even in the thermodynamic limit when

 $v_F < \hat{v}_x$

in contrast with the (homogeneous) KZM

Inhomogeneous density, spatially dependent critical frequency Linear quench: $\delta(x,t) = \nu_t(t)^2 - \nu_t^c(x)^2$ $= \nu_t^{(c)}(0)^2 - \nu_t^{(c)}(x)^2 - \delta_0 \frac{t}{\tau_Q}$ Causality restricts the effective size of the chain Front satisfying $\delta(x_F, t_F) = 0$ moves at velocity $v_F \sim \frac{\partial_t \delta(x,t)}{\partial_x \delta(x,t)}$ Sound velocity $\hat{v}_x = \hat{\xi}_x / \hat{\tau}_x$

Defects appear if $v_F > \hat{v}_x$

effective size of the chain $\left.2 | \hat{X_*} |
ight.$

Inhomogeneous density, spatially dependent critical frequency Linear quench: $\delta(x,t) = \nu_t(t)^2 - \nu_t^c(x)^2$ $= \nu_t^{(c)}(0)^2 - \nu_t^{(c)}(x)^2 - \delta_0 \frac{t}{\tau_0}$ Causality restricts the effective size of the chain Front satisfying $\delta(x_F, t_F) = 0$ moves at velocity $v_F \sim \frac{\partial_t \delta(x, t)}{\partial \delta(x, t)}$ Sound velocity $\hat{v}_x = \hat{\xi}_x / \hat{\tau}_x$ Defects appear if $v_F > \hat{v}_x$ represented the chain $2|\hat{X}_*|$ where *homogeneous* KZM theory applies



$$d_{\rm o} \sim \frac{2|\hat{X}_*|}{\hat{\xi}} = \frac{L}{3\nu_t^{(c)}(0)^2 a^2 \omega_0^2} \frac{\eta \delta_0}{\tau_Q}$$

Theory vs "experiment"!





Nice agreement!

Inhomogeneous KZM: applicability

Any second-order phase transition or classical/quantum quench in the presence of critical slowing down

No experiments yet (ongoing) Inhomogeneous ion chains (Mehlstäubler's group @ PTB)

It does NOT require diagonalization of the Hamiltonian

Partial applicability to adiabatic quantum computation



Single discrete-level system: Demirplak & Rice '03, 2005; M. V. Berry '09 Experiment for TLS: Bason et al. Nature Phys. '12

Many-body: A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Transitionless quantum driving

Take a time-dependent Hamiltonian with instantaneous eigenstates:

 $\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$

Write the adiabatic approximation including the geometric phase

$$|\psi_n(t)\rangle = \exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t' E_n(t') - \int_0^t \mathrm{d}t' \langle n(t')|\partial_{t'}n(t')\rangle\right\} |n(t)\rangle$$

Look for the Hamiltonian for which these are the exact evolving states

$$i\hbar\partial_t |\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

It follows that

$$\hat{H}(t) = \sum_{n} |n\rangle E_{n} \langle n| + i\hbar \sum_{n} (|\partial_{t}n\rangle \langle n| - \langle n|\partial_{t}n\rangle |n\rangle \langle n|) \equiv \hat{H}_{0}(t) + H_{1}(t)$$

Single discrete-level system: Demirplak & Rice 2003, 2005; M. V. Berry 2009 Experiment for TLS: Bason et al. Nature Phys. (2012)

Quantum critical systems

Family of quasi-free fermion models

$$\begin{aligned} \mathscr{H}_{0} &= \sum_{\mathbf{k}} \psi_{\mathbf{k}}^{\dagger} \left[\vec{a}_{\mathbf{k}}(\lambda(t)) \cdot \vec{\sigma}_{\mathbf{k}} \right] \psi_{\mathbf{k}} \\ \vec{\sigma}_{\mathbf{k}} &= (\sigma_{\mathbf{k}}^{x}, \sigma_{\mathbf{k}}^{y}, \sigma_{\mathbf{k}}^{z}) \\ \psi_{\mathbf{k}}^{\dagger} &= (c_{\mathbf{k},1}^{\dagger}, c_{\mathbf{k},2}^{\dagger}) \end{aligned}$$

Model dependent vector $\vec{a}_{\mathbf{k}}(\lambda) = (a_{\mathbf{k}}^{x}(\lambda), a_{\mathbf{k}}^{y}(\lambda), a_{\mathbf{k}}^{z}(\lambda))$

Examples: Ising, XY in 1D, Kitaev model in 1D, 2D

General Auxiliary Hamiltonian in Fourier space

$$\mathscr{H}_{1} = \lambda'(t) \sum_{\mathbf{k}} \frac{1}{2\varepsilon_{\mathbf{k}}^{2}} \psi_{\mathbf{k}}^{\dagger} \left[\left(\vec{a}_{\mathbf{k}}(\lambda) \times \partial_{\lambda} \vec{a}_{\mathbf{k}}(\lambda) \right) \cdot \vec{\sigma}_{\mathbf{k}} \right] \psi_{\mathbf{k}}$$

Quantum Ising Chain

$$\begin{array}{ll} \text{Ising chain hamiltonian} & \mathscr{H}_0 = -\sum_{n=1}^N (\sigma_n^x \sigma_{n+1}^x + g \sigma_n^z) \\ \text{Critical point} & g = 1 & |\uparrow\uparrow\uparrow\dots\uparrow\rangle \\ g \gg 1 & |\to\to\to\dots\to\rangle & |\downarrow\downarrow\downarrow\dots\downarrow\rangle \end{array} g \ll 1 \\ & |\downarrow\downarrow\downarrow\dots\downarrow\rangle \end{array}$$

Jordan Wigner transformation+Fourier transform

$$\mathscr{H}_0 = 2\sum_{k>0} \psi_k^{\dagger} \left[\sigma_k^z (g - \cos k) + \sigma_k^x \sin k \right] \psi_k$$
$$\mathscr{H}_1 = -g'(t) \sum_{k>0} \frac{1}{2} \frac{\sin k}{g^2 + 1 - 2g \cos k} \psi_k^{\dagger} \sigma_k^y \psi_k$$

Auxiliary Hamiltonian in real space

Auxiliary Hamiltonian

$$\begin{aligned} \mathscr{H}_{1} &= -g'(t) \left[\sum_{m=1}^{N/2-1} h_{m}(g) \mathscr{H}_{1}^{[m]} + \frac{1}{2} h_{N/2}(g) \mathscr{H}_{1}^{[N/2]} \right] \\ \mathscr{H}_{1}^{[m]} &= 2i \sum_{n=1}^{N} \left(c_{n} c_{n+m} + c_{n}^{\dagger} c_{n+m}^{\dagger} \right) \\ h_{m} &= \frac{1}{8} \left\{ \begin{array}{c} g^{m-1} & \text{for } |g| < 1 \\ g^{-m-1} & \text{for } |g| > 1 \end{array} \right. \end{aligned}$$

A time dependent long-range interaction!

In spin representation

$$\mathscr{H}_1^{[m]} = \sum_{n=1}^N \left(\boldsymbol{\sigma}_n^x \boldsymbol{\sigma}_{n+1}^z \cdots \boldsymbol{\sigma}_{n+m-1}^z \boldsymbol{\sigma}_{n+m}^y + \boldsymbol{\sigma}_n^y \boldsymbol{\sigma}_{n+1}^z \cdots \boldsymbol{\sigma}_{n+m-1}^z \boldsymbol{\sigma}_{n+m}^x \right)$$

Truncated Auxiliary Hamiltonian

Transverse field: linear quench of through critical point $g(t) = g_c - \upsilon t$

Truncated Auxiliary Hamiltonian





Summary

Recipes for "Fast-good" dynamics

I- Self-similar scaling lawsII- Inhomogeneous KZMIII-Transitionless quantum driving & QPT



New techniques

Design of experiments

Quantum speed limits?

1945 Mandelstam and Tamm: isolated systems 2012 AdC et al. arXiv:1209.1737 all systems, also coupled to an environment

STA and thermodynamics, work statistics





Collaborators

@ LANL	
M. Boshier	Quantum
M. M. Rams (now at Vienna)	@ LANL
W. H. Zurek	
Invariants of motion	
X. Chen (Bilbao)	
D. Guery-Odelin (Tolouse)	
J. G. Muga (Bilbao)	
A. Ruschhaupt (Hannover/Cork	<)
Inhomogeneous KZM	
M. B. Plenio (Ulm)	G. De Chiara (Belfast)
A. Retzker (Jerusalem)	G. Morigi (Saarland)
Speed limits	
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M. B. Plenio (Ulm)	
S. F. Huelga (Ulm)	

Quantum Lunch Seminar series @ LANL