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M.N.Kiselev

Kondo Force in NEM Devices: Dynamical Probe for a Kondo Cloud



Quantum Dynamics in Far from Equilibrium Thermally Isolated Systems
KITP, Santa Barbara, September, 26 2012

[arXiv:1206.4435](https://arxiv.org/abs/1206.4435)

Outline

- Experiment in NEM
- Kondo effect in- and out- of equilibrium
- Odd-N shuttle at weak coupling
- Odd-N shuttle at strong coupling
- Kondo forces and retardation time

Nanomechanical Resonator Shuttling Single Electrons at Radio Frequencies

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²Center for NanoScience and Sektion Physik, Ludwig-Maximilians-Universität, Theresienstrasse 37, 80333 München, Germany

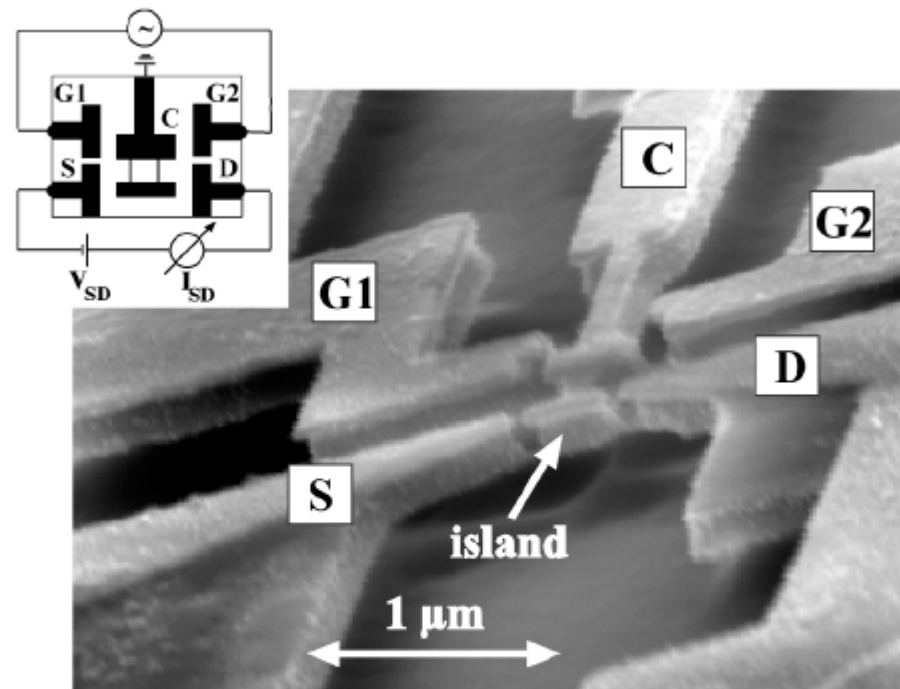
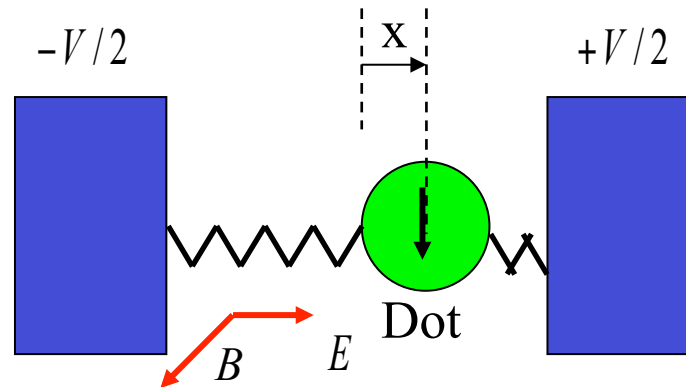
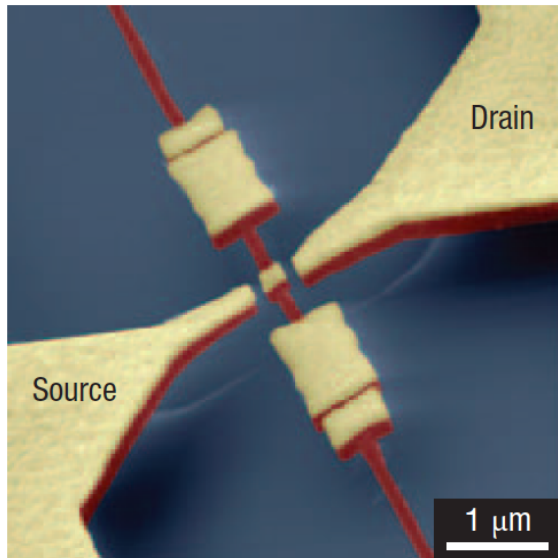
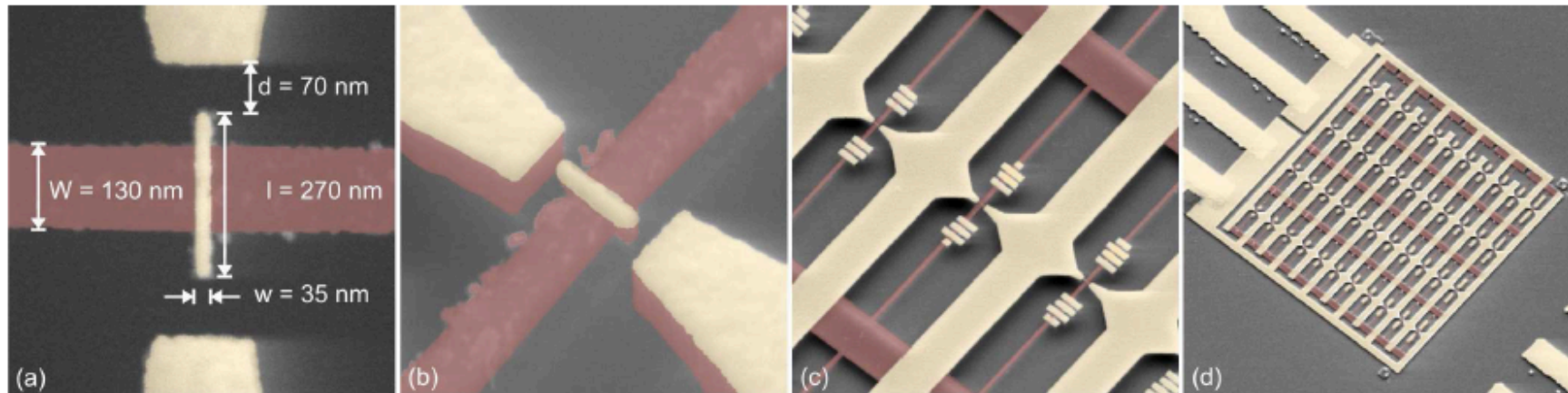


FIG. 1. Electron micrograph of the quantum bell: The pendulum is clamped on the upper side of the structure. It can be set into motion by ac power, which is applied to the gates on the left- and right-hand sides (G1 and G2) of the clapper (C).

Nano-electro-mechanical shuttling

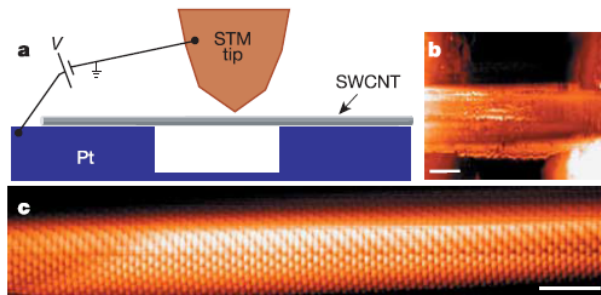
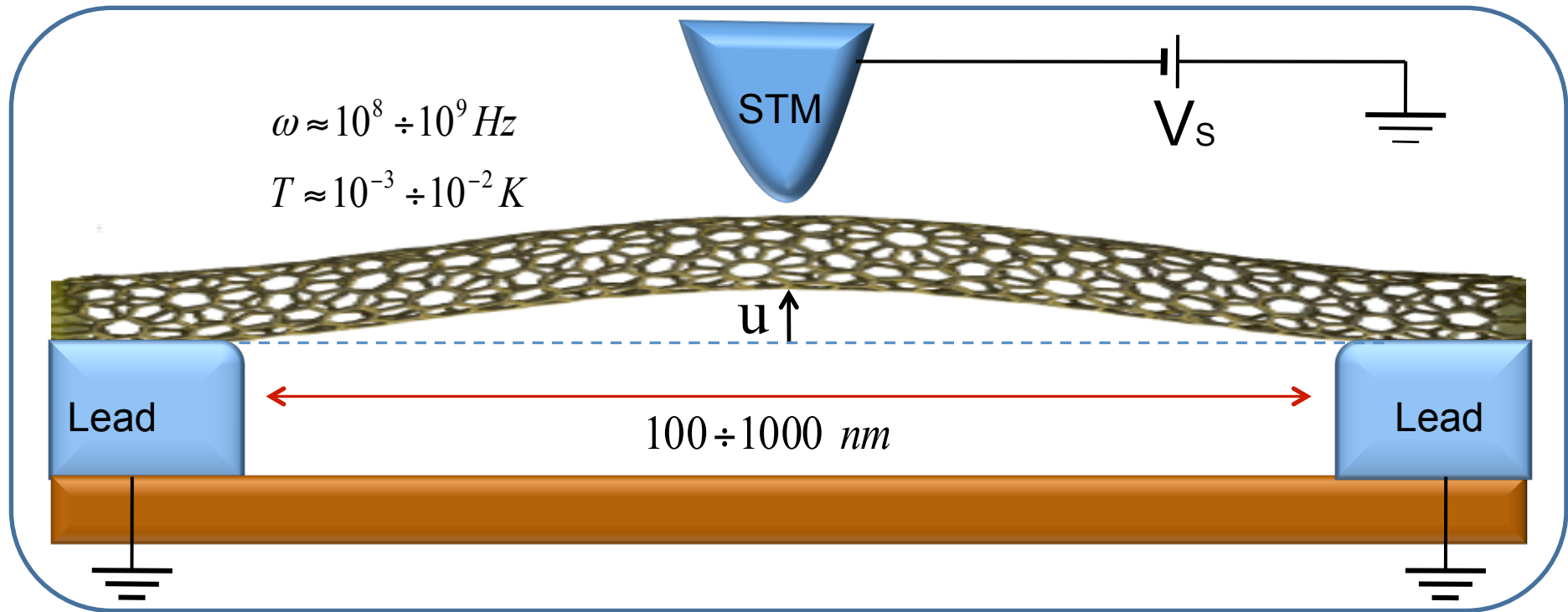


J. Kotthaus et al, Nature Nanotechnology 2008

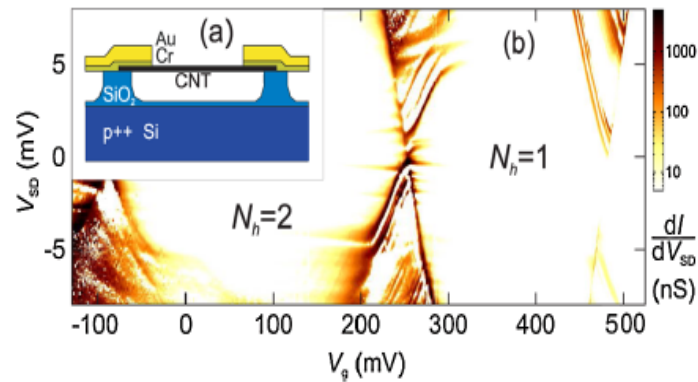


D.Koenig and E.Weig ArXiv: 1207.4313

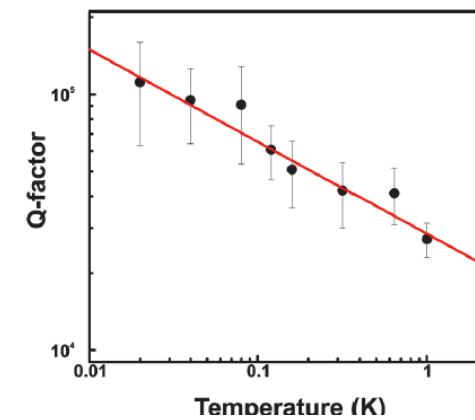
System



B.J.LeRoy, et al. Nature, **432**, 371 (2004)

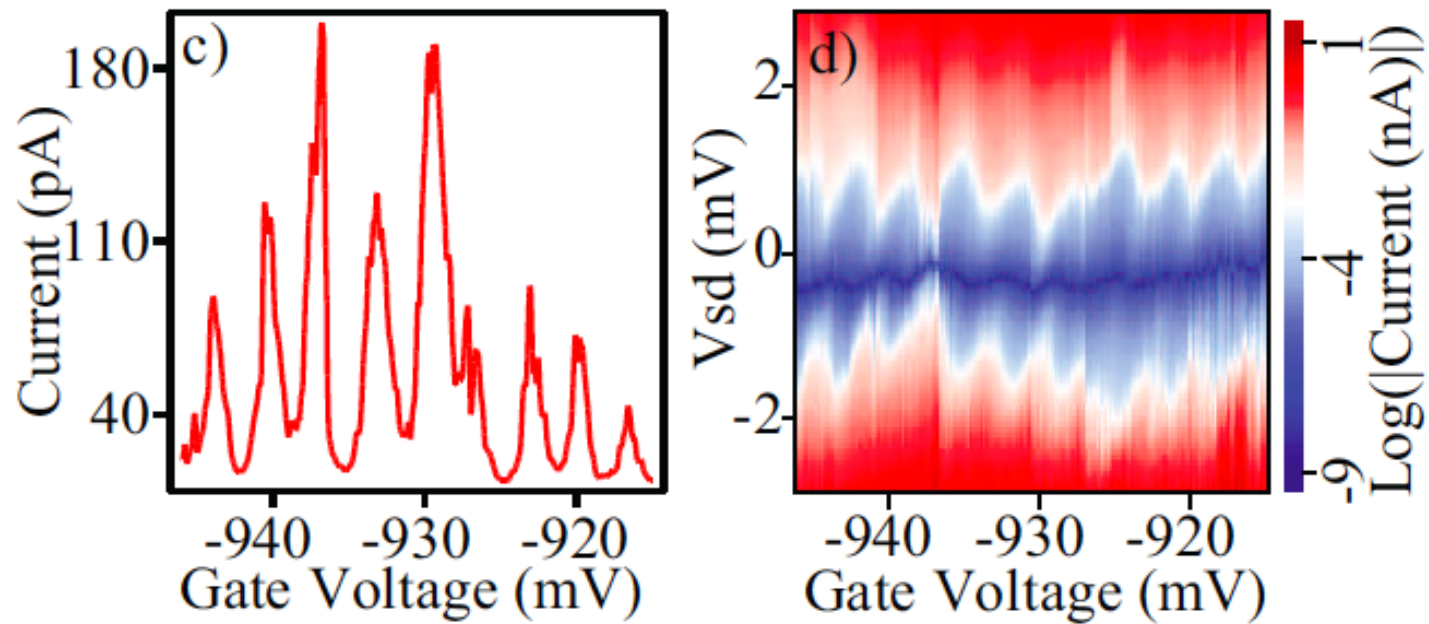
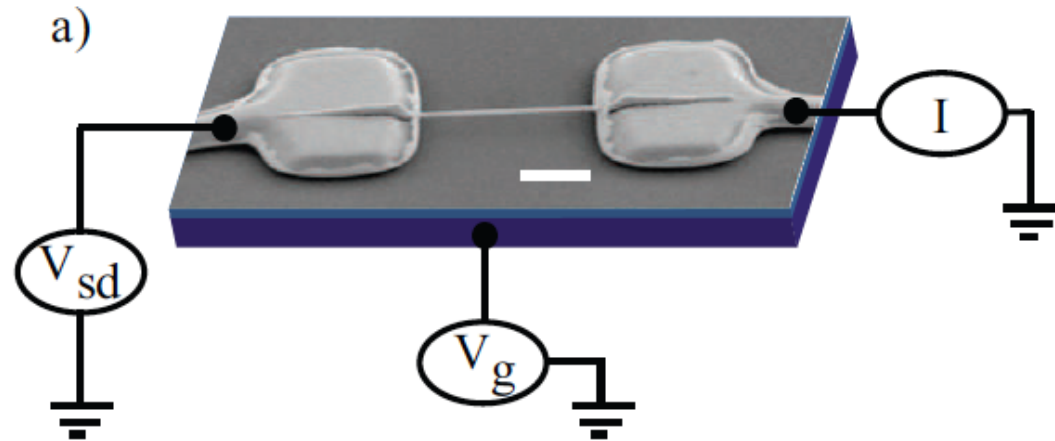


Huttel AK, et al. PHYS. REV. LETT. **102**, 225501, (2009)

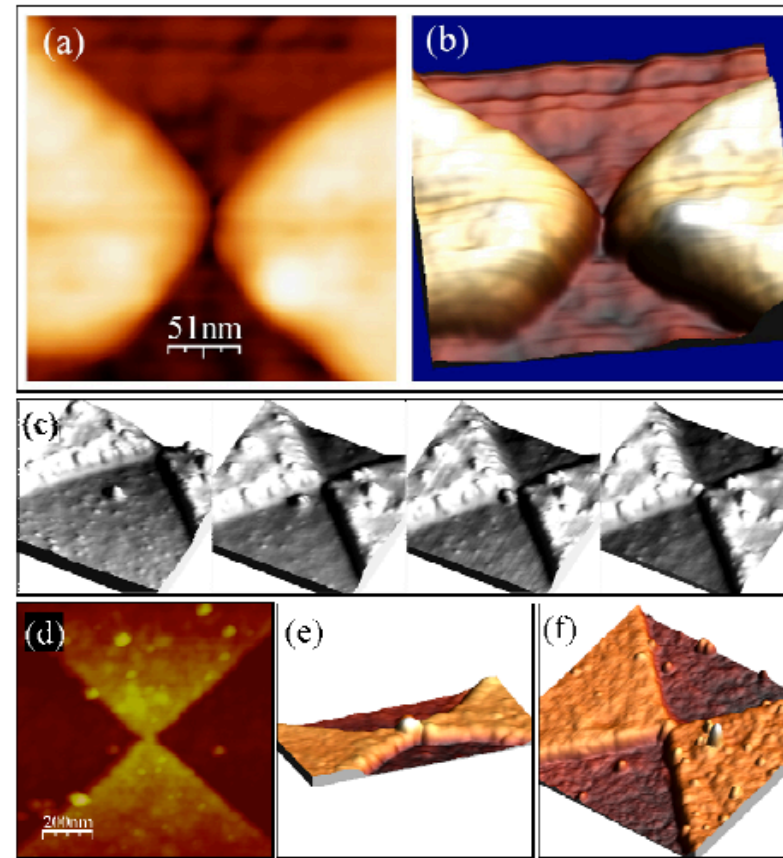
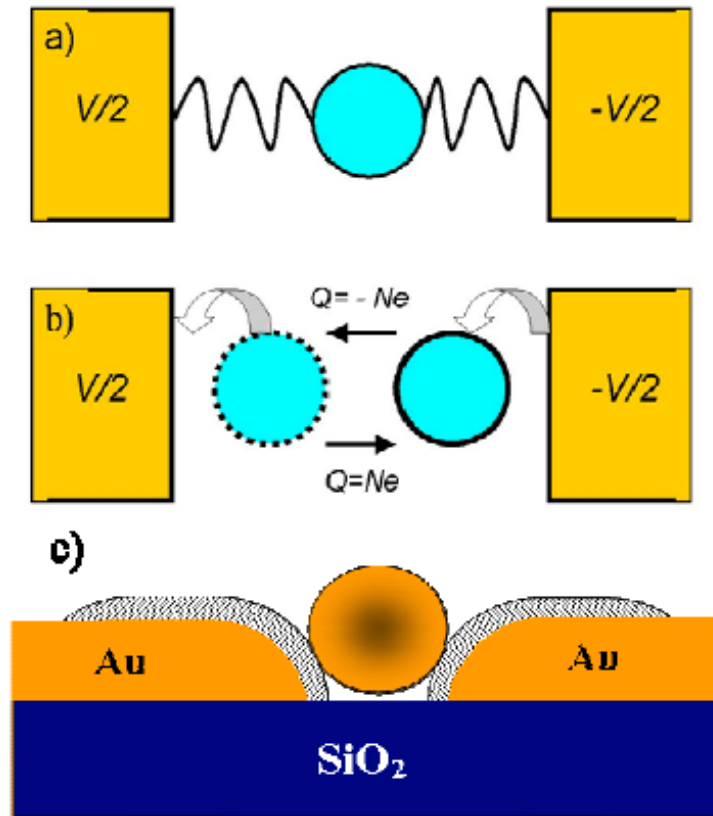


Huttel AK, et al. Nano letters **9**, 2547, (2009)

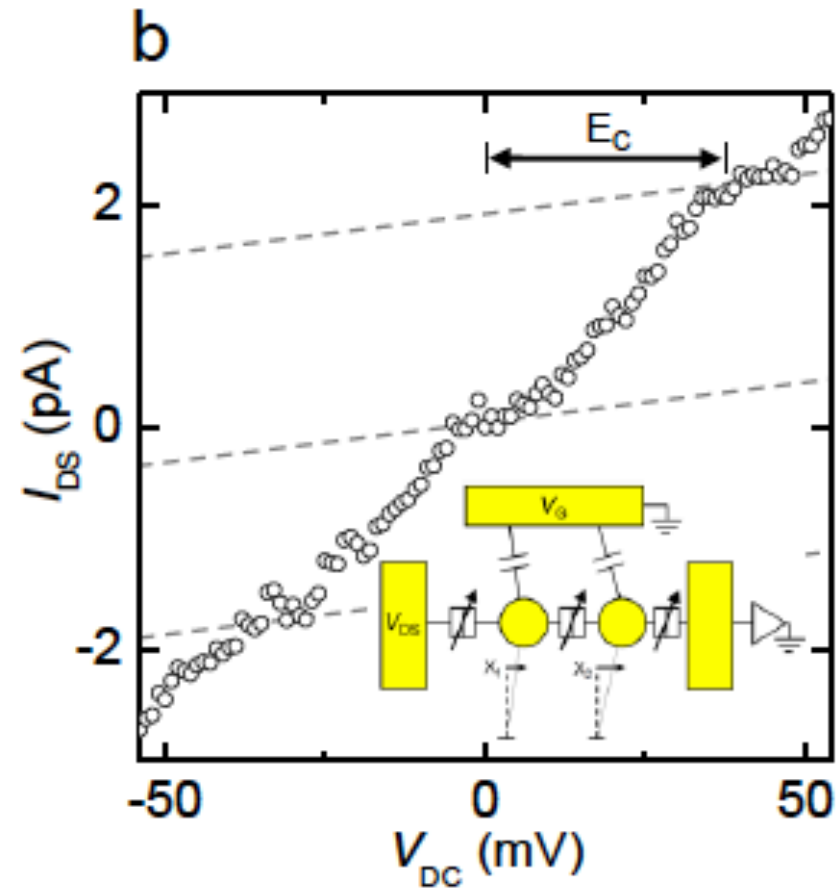
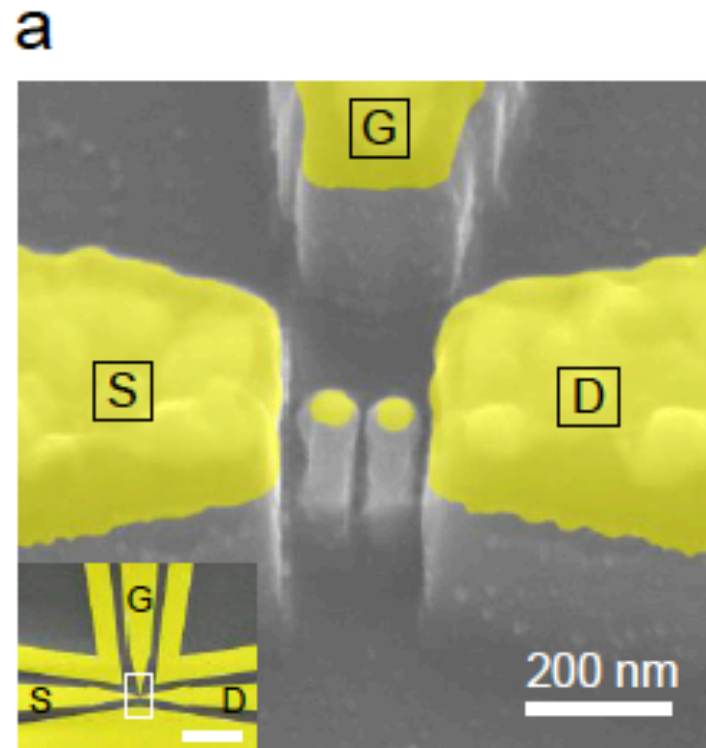
InAs nano-wire



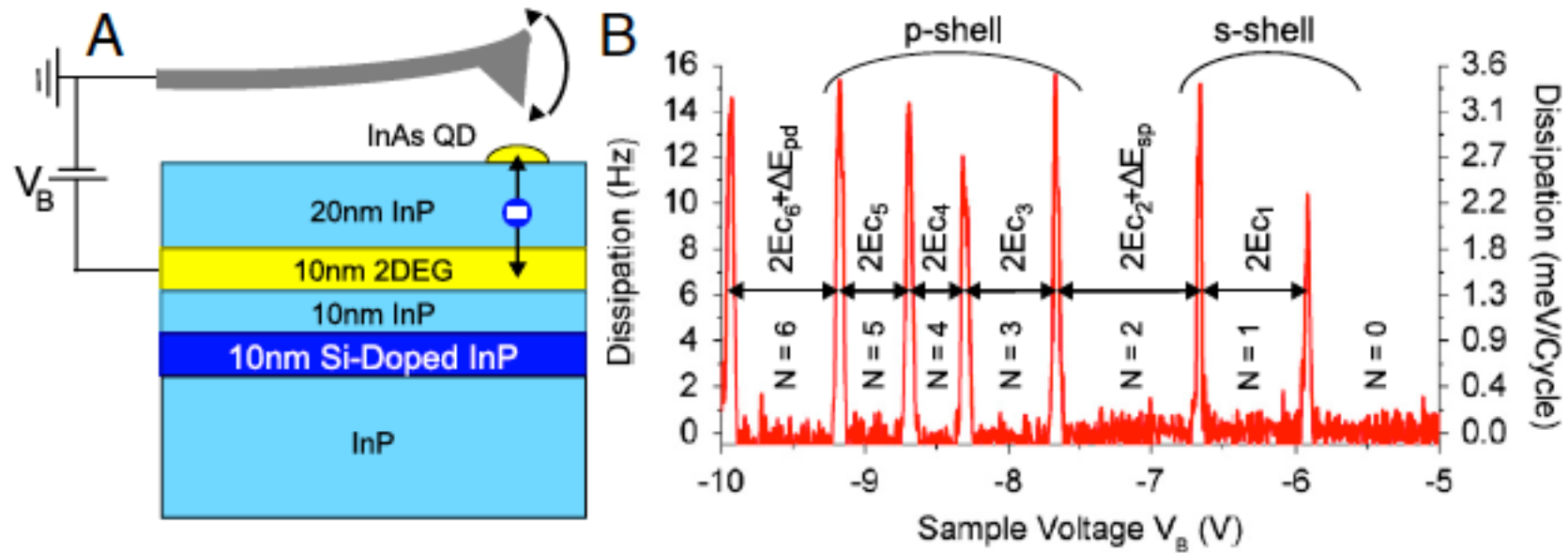
C_{60} setup



Silicon - on - insulator setup

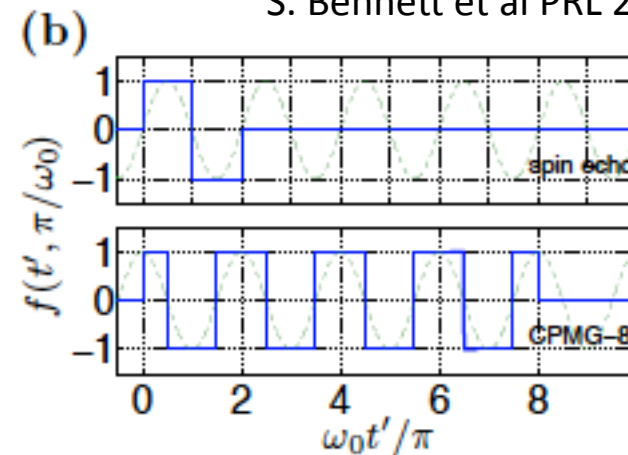
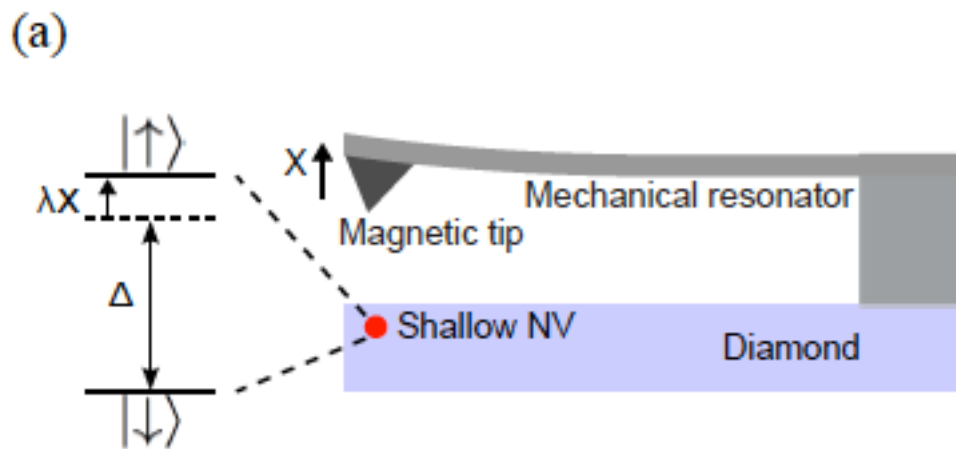


Cantilever



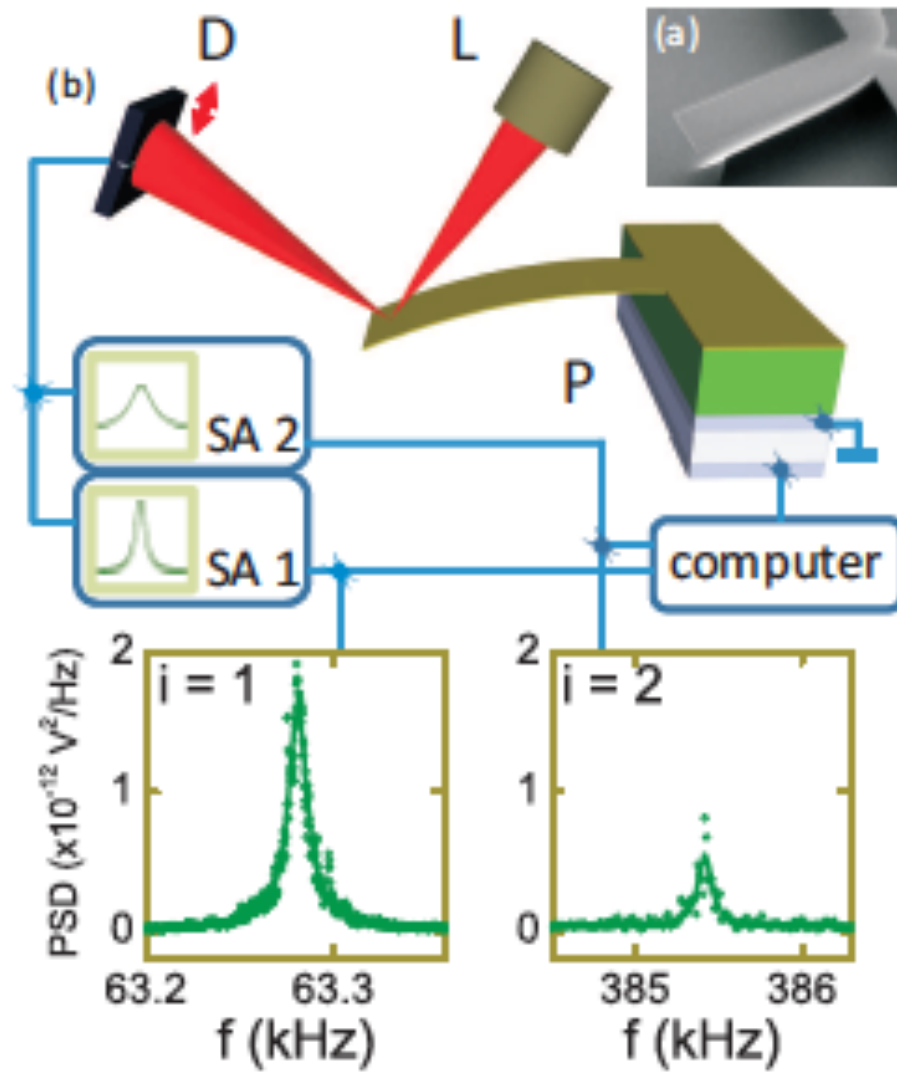
L. Cockins et al PNAS 2010

S. Bennett et al PRL 2010

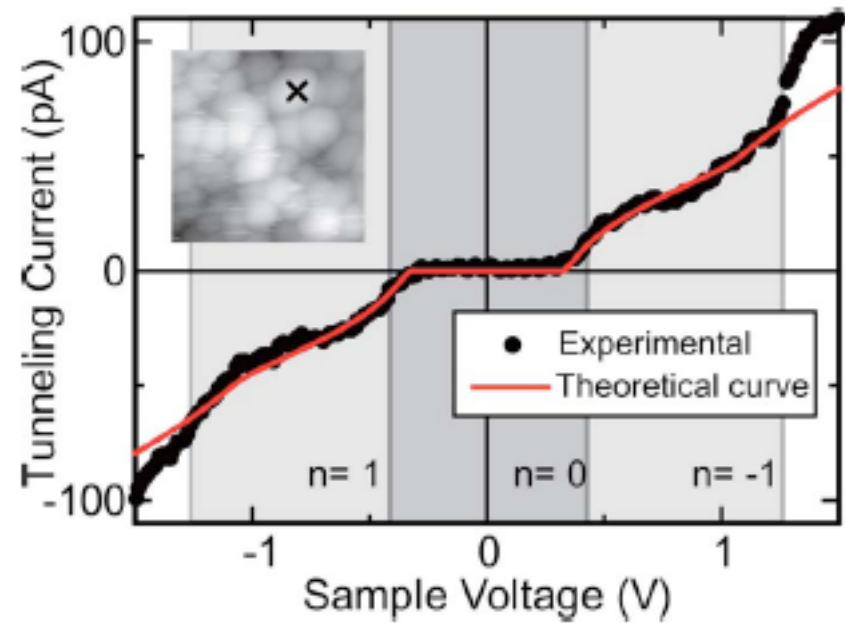
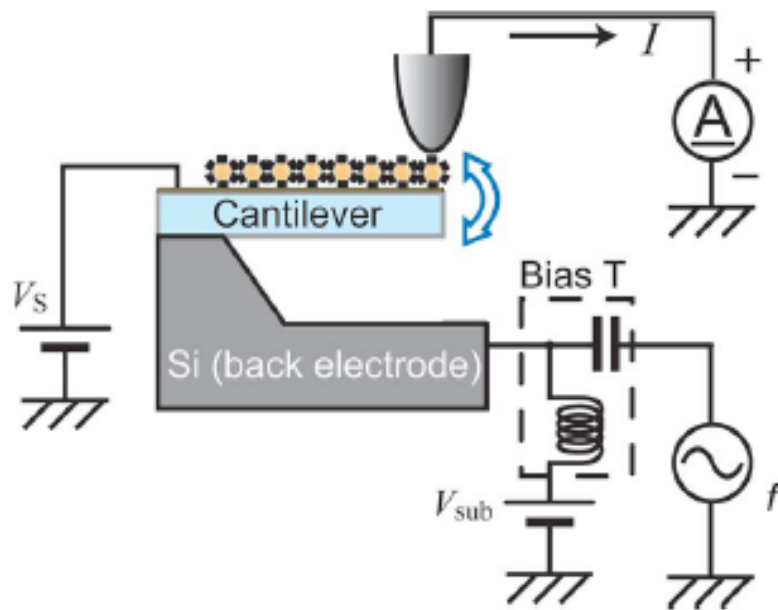


S. Bennett et al ArXiv 1205.6704

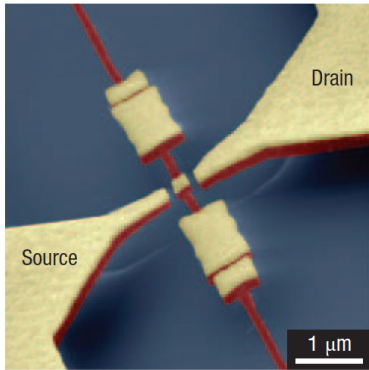
Displacement measurements



Coulomb Blockade in NEM

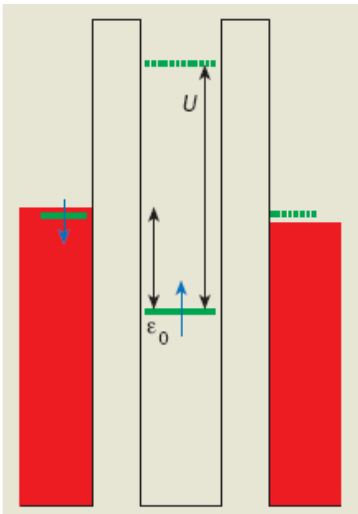


Single orbital level coupled to two leads



$$H = H_{leads} + H_{tun} + H_{dot}$$

$$H_{leads} = \sum_{k, \sigma \alpha=L,R} [\epsilon_k - \mu_\alpha] c_{k, \sigma \alpha}^\dagger c_{k, \sigma \alpha}$$



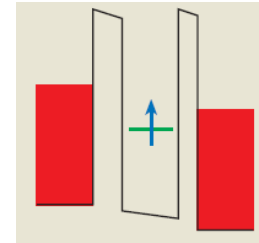
$$H_{tun} = \sum_{k, \sigma \alpha} [V_\alpha(t) c_{k, \sigma \alpha}^\dagger d_\sigma + H.c.]$$

$$H_{dot} = \sum_{\sigma} \epsilon_0 d_\sigma^\dagger d_\sigma + U(n - N)^2$$

Tunneling width

$$\Gamma_\alpha(t) = \pi \rho |V_\alpha|^2(t)$$

Single orbital level coupled to two leads



Time-dependent Glazman-Raikh rotation

$$\begin{pmatrix} c_{k\sigma L} \\ c_{k\sigma R} \end{pmatrix} = U_t \begin{pmatrix} c_{k\sigma+} \\ c_{k\sigma-} \end{pmatrix} \quad U_t = \begin{pmatrix} \cos \theta_t & -\sin \theta_t \\ \sin \theta_t & \cos \theta_t \end{pmatrix}$$

$$\tan \theta_t = \left| \frac{V_R(t)}{V_L(t)} \right| \quad |V|^2(t) = |V_L|^2(t) + |V_R|^2(t)$$

$$H_{Berry} = \sum_{k, \sigma \gamma = \pm} \left(c_{k, \sigma+}^\dagger + c_{k, \sigma-}^\dagger \right) \underbrace{\left[-i U_t^{-1} \frac{\partial U_t}{\partial t} \right]}_{a_t} \begin{pmatrix} c_{k, \sigma+} \\ c_{k, \sigma-} \end{pmatrix}$$

Gauge potential

$$a_t = \frac{d\theta_t}{dt} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

From Anderson model to Kondo model

$$H' = H_{dot} + H_{leads} + H_{tun}$$

$$H_K = W H' W^\dagger \quad W = \exp(V)$$

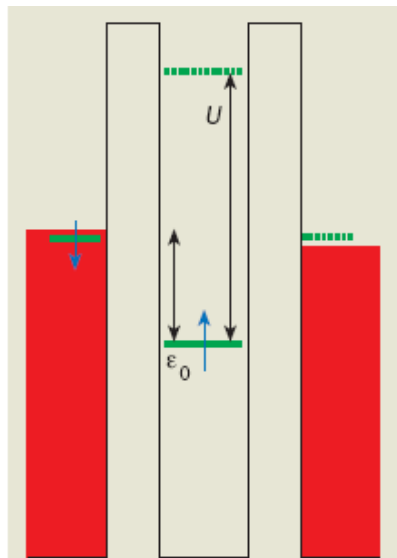
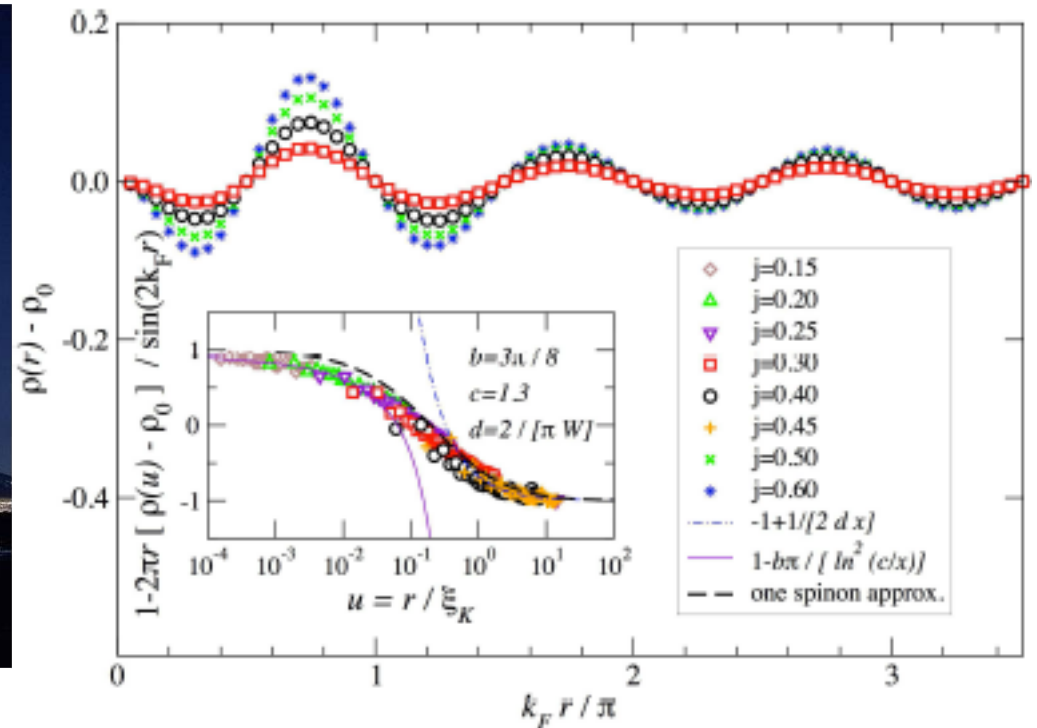
$$V = \sum_{k\sigma\alpha} \left[\left(w_{k\alpha}^{(1)} (1 - n_{-\sigma}) + w_{k\alpha}^{(2)} n_{-\sigma} \right) d_\sigma^\dagger c_{k\sigma\alpha} + h.c. \right]$$

$$0 = H_{tun} + [V, H_{dot} + H_{leads}] - i\hbar \frac{\partial V}{\partial t}$$

$$H_K = \sum_{k\alpha\sigma, k'\alpha'\sigma'} \mathcal{J}_{\alpha\alpha'}(t) \left[\vec{\sigma}_{\sigma\sigma'} \cdot \vec{S} + \frac{1}{4} \delta_{\sigma\sigma'} \right] c_{k\sigma,\alpha}^\dagger c_{k'\sigma',\alpha'}$$

$$\mathcal{J}_{\alpha,\alpha'}(t) = \sqrt{\Gamma_\alpha(t) \Gamma_{\alpha'}(t)} / (\pi \rho_0 E_d(t))$$

Kondo cloud

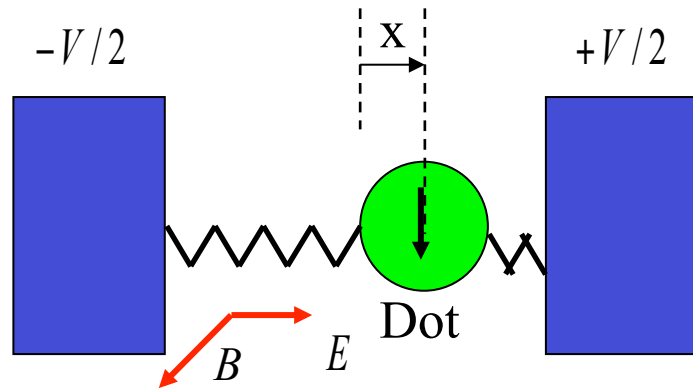


$$T_K = \frac{1}{2} (\Gamma U)^{1/2} \exp\left(\pi \epsilon_0 \frac{\epsilon_0 + U}{\Gamma U}\right)$$

Kondo cloud is exponentially big!

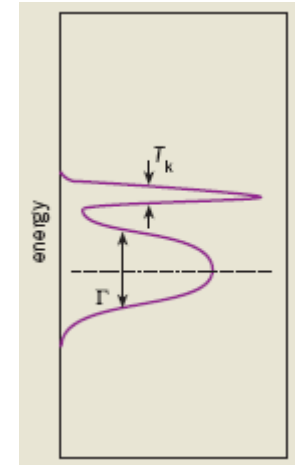
How to detect it?

Odd-N Kondo shuttle $T \gg T_K$



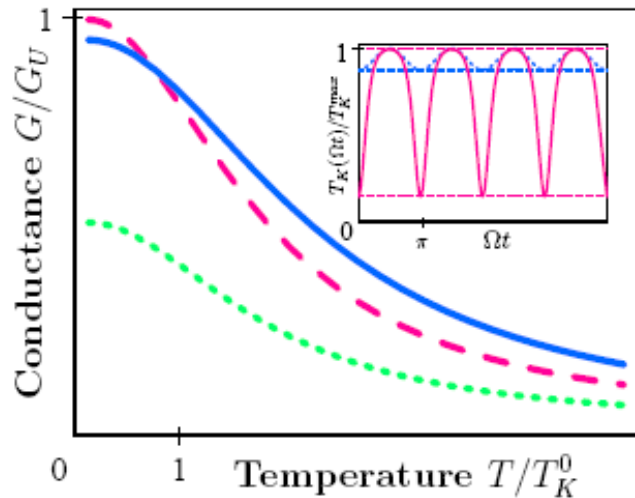
Competition between

Breit-Wigner Resonance



$$G = \frac{2e^2}{h} \left\langle \frac{4\Gamma_L(t)\Gamma_R(t)}{(\Gamma_L(t)+\Gamma_R(t))^2} \right\rangle$$

Abrikosov-Suhl Resonance



$$G(T) = \frac{3\pi^2}{16} G_U \left\langle \frac{4\Gamma_L(t)\Gamma_R(t)}{(\Gamma_L(t)+\Gamma_R(t))^2} \frac{1}{[\ln(T/T_K(t))]^2} \right\rangle$$

$$\langle T_K \rangle = T_K^0 \left\langle \exp \left[\frac{\pi U}{4\Gamma_0} \frac{\sinh^2(x(t)/\lambda_0)}{1+2\sinh^2(x(t)/\lambda_0)} \right] \right\rangle$$

$$\frac{\delta G_K}{G_K^0} = \frac{G(T) - G_K^0}{G_K^0} = 2 \frac{\delta T_K}{T_K^0} \frac{1}{\ln(T/T_K^0)}$$

Adiabaticity $\hbar\Omega \ll T_K \ll \Gamma$

Kondo effect at strong coupling $T \ll T_K$

Scattering phase $\delta_{\uparrow} + \delta_{\downarrow} = 0.$ $\delta_{\uparrow} - \delta_{\downarrow} = \pi.$ $\delta_s = s \frac{\pi}{2}$

Effective strong coupling Hamiltonian

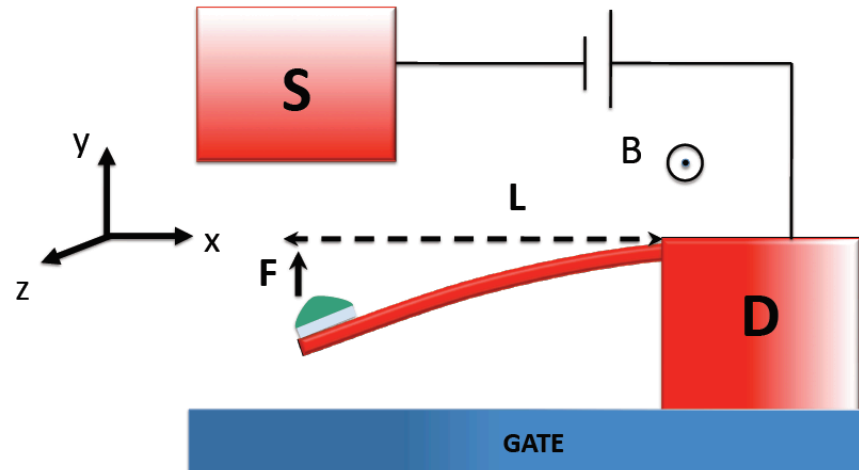
$$H_{\text{fixed point}} = \sum_{ks} \xi_k \varphi_{ks}^{\dagger} \varphi_{ks} - \sum_{kk's} \frac{\xi_k + \xi_{k'}}{2\pi\nu T_K} \varphi_{ks}^{\dagger} \varphi_{k's} + \frac{1}{\pi\nu^2 T_K} \rho_{\uparrow} \rho_{\downarrow}.$$

$$-\pi\nu \tilde{T}_{in}(\omega) = i \frac{\omega^2 + \pi^2 T^2}{2T_K^2}, \quad -\pi\nu \text{Im} T_s(\omega) = 1 - \frac{3\omega^2 + \pi^2 T^2}{2T_K^2}.$$

Low temperature conductance

$$G = G_0 \left[1 - (\pi T / T_K)^2 \right], \quad T \ll T_K$$

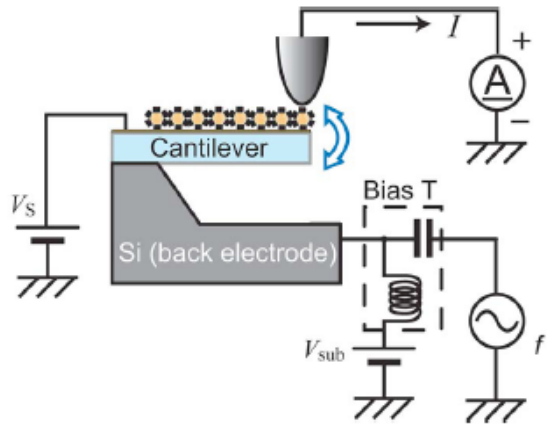
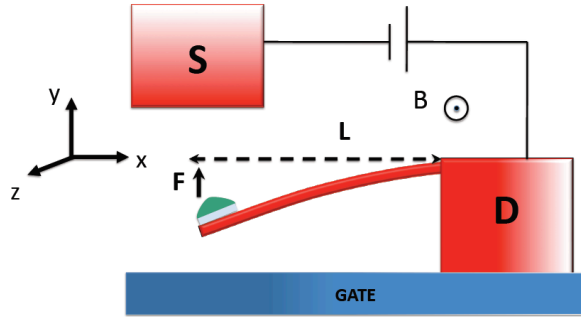
Kondo Force in NEM setup



$$\ddot{\vec{u}} + \frac{\omega_0}{Q_0} \dot{\vec{u}} + \omega_0^2 \vec{u} = \frac{1}{m} \vec{F}$$

$$F(\vec{u}, t) = F_0(t) + \vec{I}_t \cdot B \cdot L + F_{\text{emf}}$$

Odd-N Kondo shuttle $T \ll T_K$



$$H' = H_{\text{lead}} + H_B + H_{\text{ex}} + \delta H$$

$$H_{\text{lead}} = \sum_{a=1,2} \sum_{k\sigma} \xi_{k\alpha} \psi_{ak\sigma}^\dagger \psi_{ak\sigma},$$

$$H_B = i\hbar \frac{d\vartheta_t}{dt} \sum_{k\sigma} \left(\psi_{1k\sigma}^\dagger \psi_{2k\sigma} - \psi_{2k\sigma}^\dagger \psi_{1k\sigma} \right),$$

$$H_{\text{ex}} = \frac{J}{4} \sum_{kk', \sigma\sigma', m, m'} \psi_{1k\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} \psi_{1k'\sigma'} d_m^\dagger \vec{\sigma}_{mm'} d_{m'}$$

$$\tan \vartheta_t = \sqrt{\frac{\Gamma_r}{\Gamma_l(t)}} \quad \frac{\Gamma_r}{\Gamma_l(0)} = \exp\left(\frac{2y_0}{\lambda}\right)$$

$$\delta H = \frac{eV_{\text{bias}}}{2} \left[(N_2 - N_1) \cos 2\vartheta_t + \sum_{k\sigma} (\psi_{1k\sigma}^\dagger \psi_{2k\sigma} + h.c.) \sin 2\vartheta_t \right]$$

$$\sin^2 2\vartheta_t = \frac{4\Gamma_l\Gamma_r}{(\Gamma_l + \Gamma_r)^2} = \frac{1}{\cosh^2 \frac{[y(t) - y_0]}{\lambda}}$$

Rotating frame basis

$$\hat{\mathcal{I}} = \frac{e}{2} \frac{d}{dt} \left(\hat{N}_r - \hat{N}_l \right) = \frac{d}{dt} \hat{Q}$$

Glazman – Raikh rotation $(r, l) \rightarrow (1, 2)$

$$\hat{Q} = \hat{Q}_t + \hat{q}_t$$

$$\hat{Q}_t = \frac{e}{2} \cos 2\vartheta_t (\hat{N}_1 - \hat{N}_2) \rightarrow S^z$$

$$\hat{q}_t = -\frac{e}{2} \sin 2\vartheta_t \sum_{k\sigma} \left(\psi_{1k\sigma}^\dagger \psi_{2k\sigma} + \psi_{2k\sigma}^\dagger \psi_{1k\sigma} \right) \rightarrow S^x$$

$$H_B = i\hbar \frac{d\vartheta_t}{dt} \sum_{k\sigma} \left(\psi_{1k\sigma}^\dagger \psi_{2k\sigma} - \psi_{2k\sigma}^\dagger \psi_{1k\sigma} \right) \rightarrow S^y$$

Emergent SU(2) algebra

Friedel Phase

$$N_1 - N_2 = \frac{\delta_t}{\pi}$$

$$\delta_\sigma = \sigma \frac{\pi}{2} + \frac{\varepsilon}{T_K} - \sigma \frac{B}{T_K} + O\left(\left(\frac{\varepsilon}{T_K}\right)^3, \sigma \left(\frac{B}{T_K}\right)^3\right) \quad \text{Nozieres FL theory}$$

$$\delta_t = \delta_\uparrow + \delta_\downarrow = \frac{2\varepsilon}{T_K} \quad \delta_a = \delta_\uparrow - \delta_\downarrow = \pi - \frac{2B}{T_K}$$

$$T_K = D_0 \exp\left[-\frac{\pi E_c}{4(\Gamma_l + \Gamma_r)}\right]$$

Friedel Phase and Glazman - Raikh angle are not independent

$$\frac{1}{\delta_t} \frac{d\delta_t}{dt} = \frac{\pi E_c}{4\Gamma_0} \sin 2\vartheta_t \frac{d\vartheta_t}{dt}$$

Tunnel current

$$\hat{I} = \frac{d}{dt} \hat{Q}_t + \frac{d}{dt} \hat{q}_t$$

AC component $\bar{I}_0(t) = \frac{e}{2\pi} \cos 2\vartheta_t \cdot \frac{d\delta_t}{dt}$

DC component $\bar{I}_{\text{int}}(t) = \frac{e^2}{\hbar} V_{\text{bias}} \frac{d}{dt} \left[\sin 2\vartheta_t \int_{-\infty}^t dt' \sin 2\vartheta_{t'} \Pi_{t-t'}^R \right]$

$$\Pi_t^R = -\frac{i}{2} \sum_{k\sigma} \sum_{\alpha \neq \gamma} [G_{\alpha k\sigma}^R(t) G_{\gamma k\sigma}^K(-t) + G_{\alpha k\sigma}^K(t) G_{\gamma k\sigma}^A(-t)]$$

Friedel phase does not have its own kinetics and adiabatically follows the displacement

$$\frac{d\delta_t}{dt} = \frac{d\delta_t}{dy} \cdot \dot{y} \quad F \sim I \sim \dot{y} \quad \text{Kondo Friction?}$$

Key assumptions

Adiabaticity

$$\hbar\omega_0 \ll T_K \ll E_c$$

Integer valency - Kondo effect

$$\Gamma \ll E_c$$

Smallness of inelastic corrections

$$T \ll T_K$$

Weak non-equilibrium

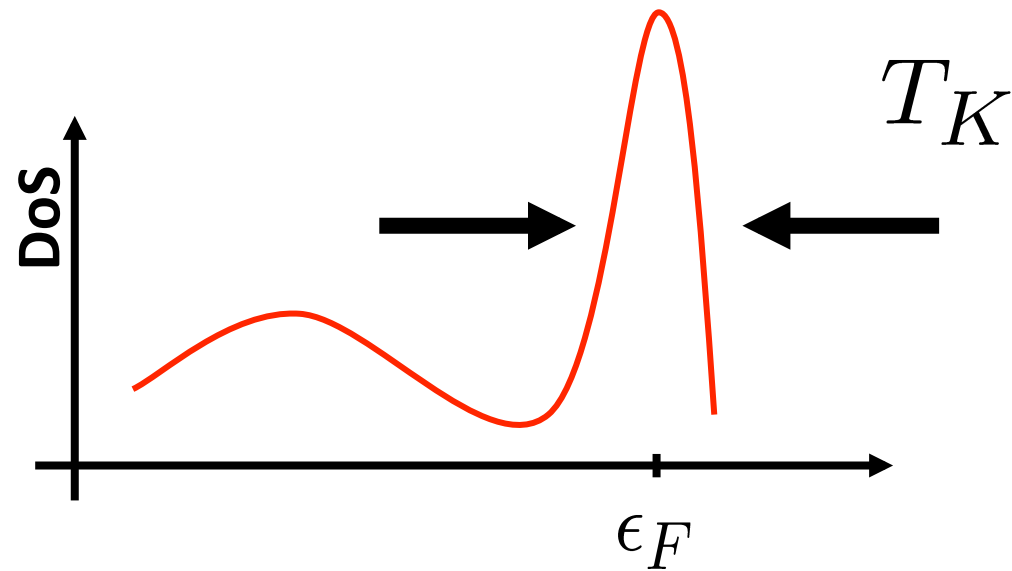
$$eV_{\text{bias}} \ll T_K$$

Small magnetization effects

$$B \ll T_K$$

Conjecture: one parametric scaling still holds at weak non-equilibrium

Tunnel current associated with Friedel phase

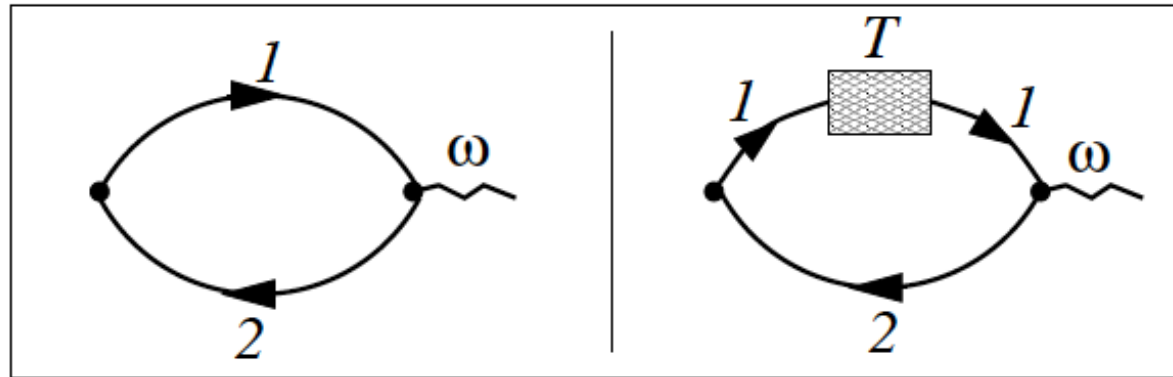


$$\bar{I}_0(t) = \frac{\dot{y}}{\lambda} \frac{eE_c}{8\Gamma_0} \cdot \frac{eV_{\text{bias}}}{k_B T_K(t)} \cdot \frac{\tanh\left(\frac{y-y_0}{\lambda}\right)}{\cosh^2\left(\frac{y-y_0}{\lambda}\right)}$$

$$\delta F = -G_0 V_{\text{bias}} B L \frac{\pi E_c}{16\Gamma_0} \frac{\hbar}{T_K(t)} \frac{d}{dt} \cosh^{-2} \frac{(y - y_0)}{\lambda}$$

Tunnel current: Ohmic contribution I

$$\bar{I}_{\text{int}}(t) = -\frac{e}{2} \left\langle \frac{d}{dt} \left[\sin 2\vartheta_t \sum_{k\sigma} \left(\psi_{1k\sigma}^\dagger \psi_{2k\sigma} + \psi_{2k\sigma}^\dagger \psi_{1k\sigma} \right) \right] \right\rangle$$



$$\omega \cdot \text{Im}\Pi^R(\omega) = \int d\epsilon \frac{\partial n}{\partial \epsilon} \left(-\pi \rho_0 \sum_{\sigma} \text{Im}T_{\sigma}(\epsilon) \right)$$

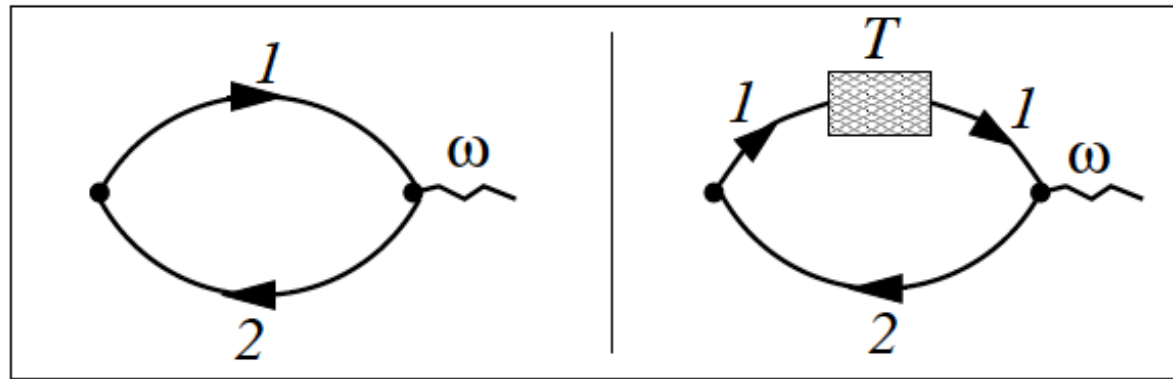
$$\bar{I}_{\text{int}}(t) = G_0 V_{\text{bias}} \sin^2 2\vartheta_t \sum_{\sigma} \sin^2 \delta_{\sigma}$$

$$F_{ad} = 2G_0 V_{\text{bias}} BL \cosh^{-2} \frac{[y(t) - y_0]}{\lambda}$$

Tunnel current: Ohmic contribution II

Inelastic processes

$$\bar{I}_{\text{int}}(t) = \frac{e^2}{\hbar} V_{\text{bias}} \frac{d}{dt} \left[\sin 2\vartheta_t \int_{-\infty}^t dt' \sin 2\vartheta_{t'} \Pi_{t-t'}^R \right]$$



$$\omega \cdot \text{Im} \Pi^R(\omega) = \int d\epsilon \frac{\partial n}{\partial \epsilon} \left(-\pi \rho_0 \sum_{\sigma} \text{Im} T_{\sigma}(\epsilon) \right)$$

$$-\pi \rho_0 \text{Im} T_{\sigma}(\epsilon) = 1 - \frac{3\epsilon^2 + \pi^2 T^2}{2T_K^2}$$

Tunnel current: Ohmic contribution III

Finite temperatures

$$G = G_0 \sin^2 2\vartheta_t \left(1 - (\pi T / T_K(t))^2 \right)$$

Non-linear in B- effects

$$G = G_0 \sin^2 2\vartheta_t \left(1 - (B / T_K(t))^2 \right)$$

Non-linear conductance

$$\frac{dI}{dV_{\text{bias}}} = G_0 \sin^2 2\vartheta_t \left(1 - \frac{3}{2} (eV_{\text{bias}} / T_K(t))^2 \right)$$

Retardation time

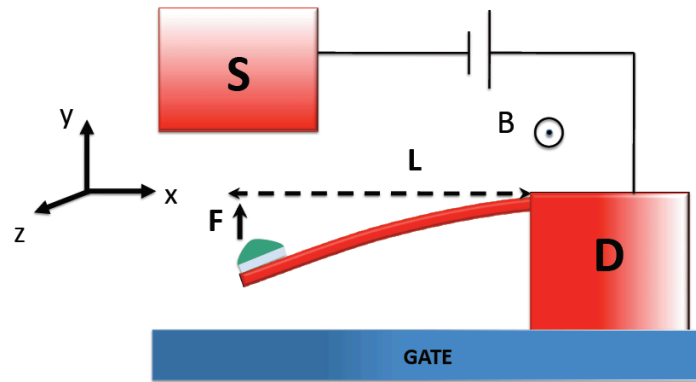
$$F_{ad} = 2G_0 V_{\text{bias}} BL \cosh^{-2} \frac{[y(t) - y_0]}{\lambda}$$

$$\delta F = -G_0 V_{\text{bias}} BL \frac{\pi E_c}{16\Gamma_0} \frac{\hbar}{T_K(t)} \frac{d}{dt} \cosh^{-2} \frac{(y - y_0)}{\lambda}$$

$$F_L = F_{ad}(y(t)) - \dot{y} \frac{dF_{ad}}{dy} \frac{\hbar\pi E_c}{16\Gamma_0 k_B T_K^{(0)}}$$

$$\tau = \frac{\hbar\pi E_c}{16\Gamma_0 k_B T_K^{(0)}} = \frac{1}{2} \left| \frac{Q^{-1}(B) - Q^{-1}(-B)}{\omega(B) - \omega(-B)} \right|$$

Electromotive Force



$$\ddot{\vec{u}} + \frac{\omega_0}{Q_0} \dot{\vec{u}} + \omega_0^2 \vec{u} = \frac{1}{m} \vec{F}$$

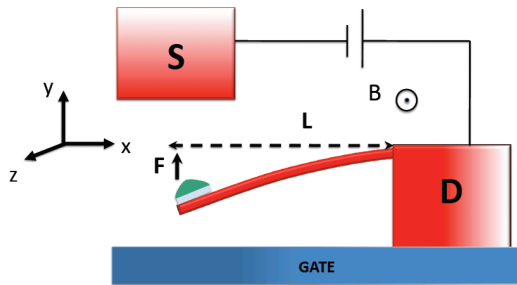
$$F(\vec{u}, t) = F_0(t) + \bar{\mathcal{I}}_t \cdot B \cdot L + F_{\text{emf}}$$

$$F_{\text{emf}} \sim \dot{y} (B \cdot L)^2 G_0$$

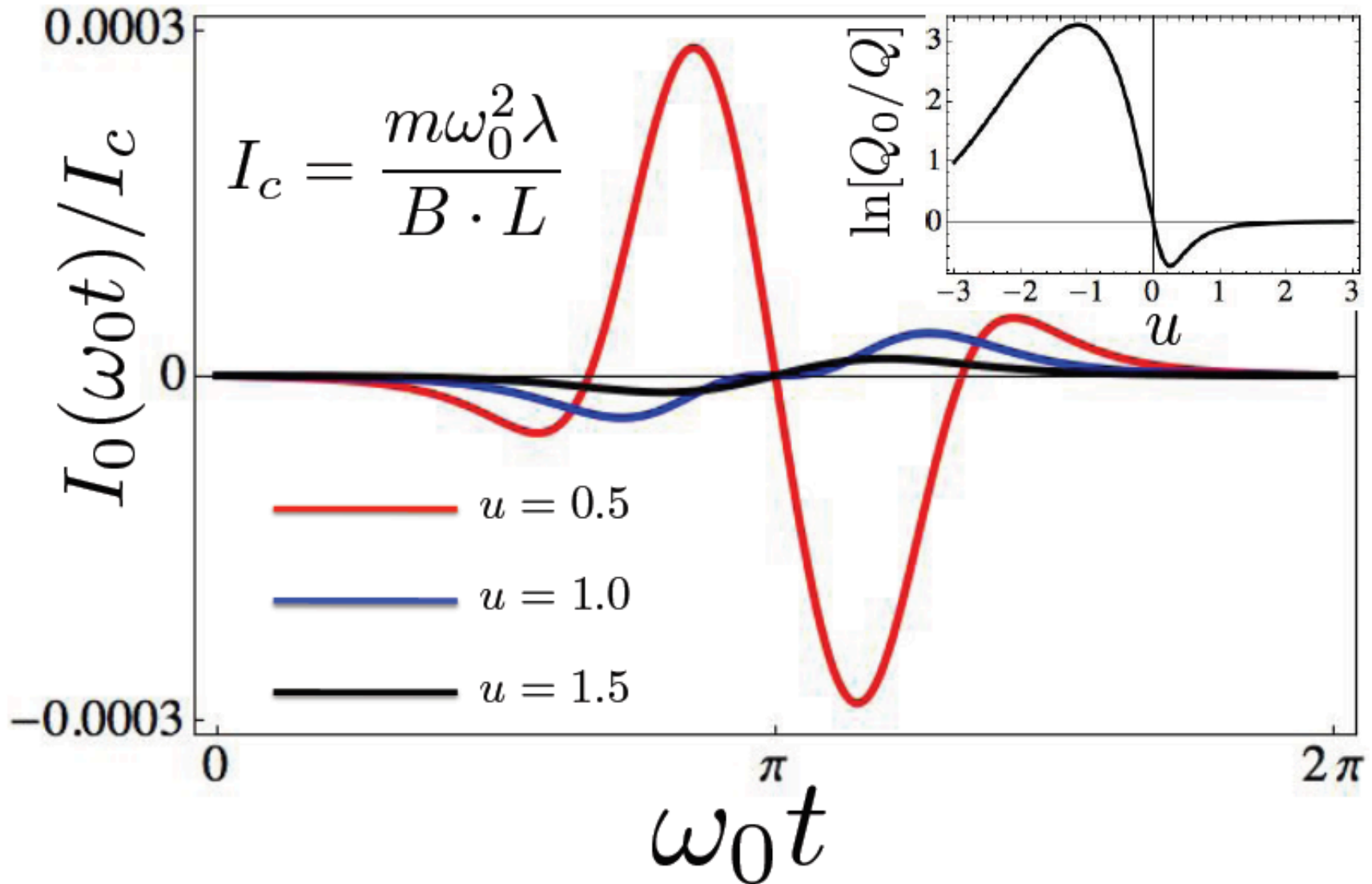
$$\Phi / \Phi_0 \cdot L / \lambda < E_c / \Gamma_0 \cdot |eV_{\text{bias}}| / (k_B T_K)$$

$$\Phi = B \cdot \mathcal{S}_\lambda \quad \Phi_0 = h/e$$

$$B < 10T$$



Exponential sensitivity



This work is done in collaboration with



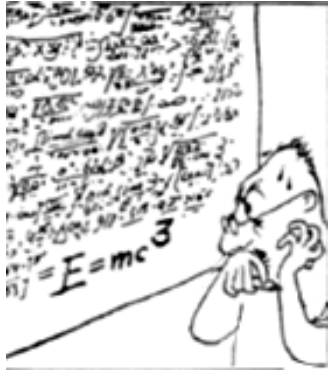
Konstantin Kikoin,
TAU, Israel



Leonid Gorelik,
Chalmers, Sweden



Robert Shekhter,
Gothenburg, Sweden



Conclusions

- Study of Kondo-NEM phenomenon gives an additional (as compared with a standard conductance measurements in a non-mechanical device) information on retardation effects in formation of many-particle cloud accompanied the Kondo tunneling.
- Measuring the nanomechanical response on Kondo-transport in nanomechanical single-electronic device enables one to study kinetics of Kondo effect and offers a new approach for studying nonequilibrium Kondo phenomena
- Kondo effect provides a possibility for super high tunability of the mechanical dissipation as well as super sensitive detection of mechanical displacement.