KITP Santa Barbara Program “Quantum Dynamics,” Oct 17, 2012

# Topological orbital phases beyond standard optical lattices

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[View of Pittsburgh---source: PittsburghSkyline.com]

# Acknowledgement

***Collaborators:***

**Sankar Das Sarma (Maryland)**
**Andreas Hemmerich (experiment/Hamburg)**
**Xiaopeng Li (student, Pittsburgh)**
**Kai Sun (Maryland → Michigan)**
**Erhai Zhao (postdoc, now GMU)**

*Pittsburgh→ George Mason University, Fairfax, VA / Assistant Professor*

***Our work:***

- **Background and perspective (news & views): Nature Physics 7, 101 (2011) [with M. Lewenstein]**

- **PRL 100, 160403 (2008)**
- **Nature Physics 8, 67 (2012)**
- **arXiv:1205.0254, + 1210.1859**

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And ***NSF of China*** Overseas Scholar Collaboration Program (sponsor: Peking Univ/**Biao Wu**)

# Two main thrusts in theoretical cold atom research

- Use cold atoms/optical lattices to quantum simulate important condensed matter problems: such as Mott-Superfluid in Bose-Hubbard model, high Tc superconductors, spin liquid models with ring exchange, … [extensive work by many theory groups worldwide]

- As a new type of quantum matter of no prior analogue in CM physics:
  - New many-body regimes: Fermi gases in unitarity (…, many groups …), large effective Zeeman splitting (order of E_Fermi), …

    **This talk**
  - Quantum particles (especially bosons) in the excited "higher orbital" bands of optical lattices. Beyond the s: p, d, …
  - Quantum dynamics: great potential of studying fast dynamics in a ultracold (slow) system.

# Outline

1. Introduction
   - Optical lattices. What is the p-orbital band? Why?

2. Experimental Progress

3. Quantum 120$^o$ model of "spinless" p-band fermions
   - Strong anisotropy of p orbits – new feature → direction-dependent orbit exchange

4. Z-class topological Insulator in an optical orbital ladder
   - Ladder reduced from Hamburg experimental system
   - Due to hybridization of opposite parity s and p orbitals
   - Transition to non-topological Mott insulator by interaction

5. Conclusion

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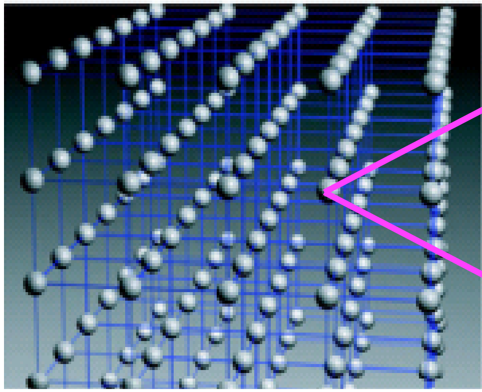
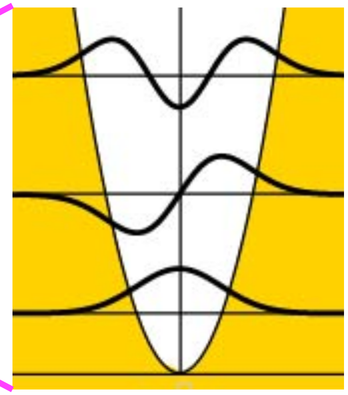
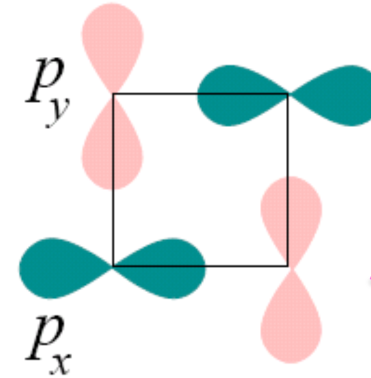
# p-band of an optical lattice (illustration)



[optical lattice picture from web]

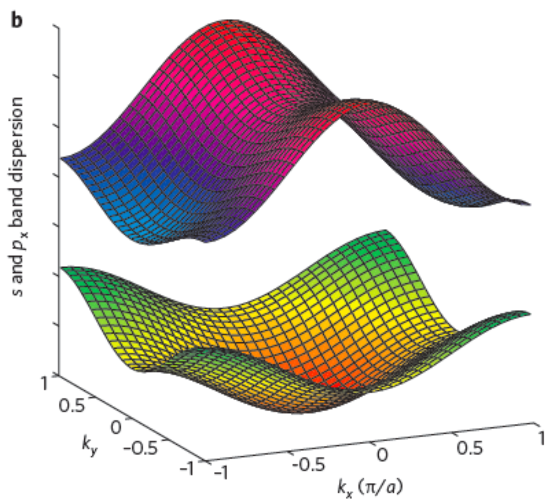



Top view




Lattice spacing $\lambda_L \sim 400\text{nm}$
Level spacing $\sim 1\text{MHz} \sim 50\mu\text{K}$

With tunneling, discrete levels become Bloch bands.




p-band

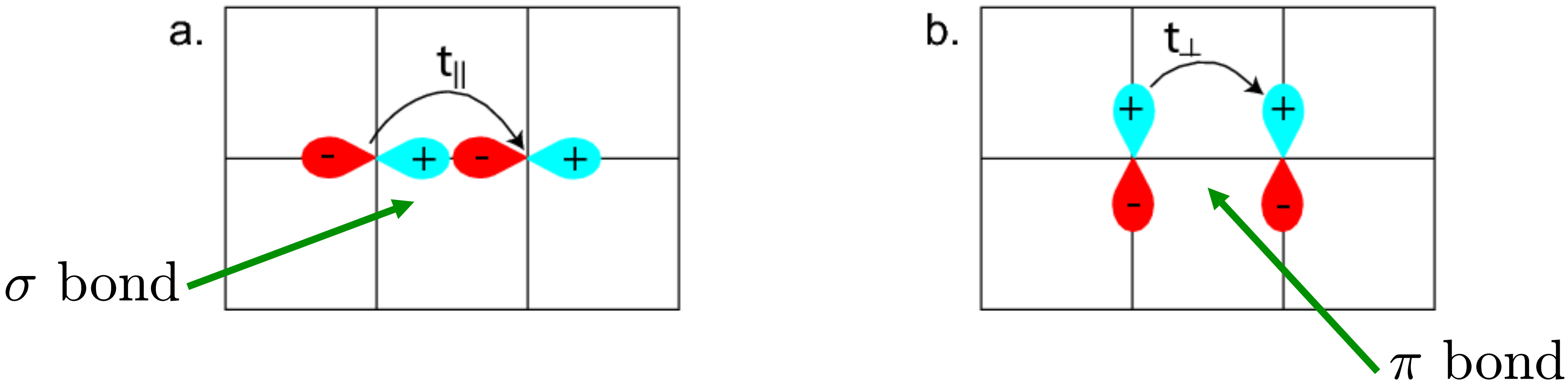
s-band

- This talk
- Compared with s: orbital degeneracy → emergent symmetry

[M. Lewenstein & WVL, Nature Phys. 7, 101 (2011)]

# Features of p-orbital Hubbard model: illustration with bosons in 3D cubic lattice [WVL and C. Wu, PRA (2006)]

$$H = \sum_{\mathbf{r}\mu} [t_{\|}\delta_{\mu\nu} - t_{\perp}(1-\delta_{\mu\nu})] \left( b^{\dagger}_{\mu,\mathbf{r}+a\mathbf{e}_{\nu}} b_{\mu\mathbf{r}} + h.c. \right) + \frac{1}{2} U \sum_{\mathbf{r}} \left[ n_{\mathbf{r}}^2 - \frac{1}{3} \mathbf{L}_{\mathbf{r}}^2 \right]$$




$\mu, \nu = x, y, z$ or $p_x, p_y, p_z$

Density field operator: $n_{\mathbf{r}} = \sum_{\mu} b^{\dagger}_{\mu\mathbf{r}} b_{\mu\mathbf{r}}$

Angular momentum operator: $L_{\mu\mathbf{r}} = -i \sum_{\nu\lambda} \epsilon_{\mu\nu\lambda} b^{\dagger}_{\nu\mathbf{r}} b_{\lambda\mathbf{r}}$

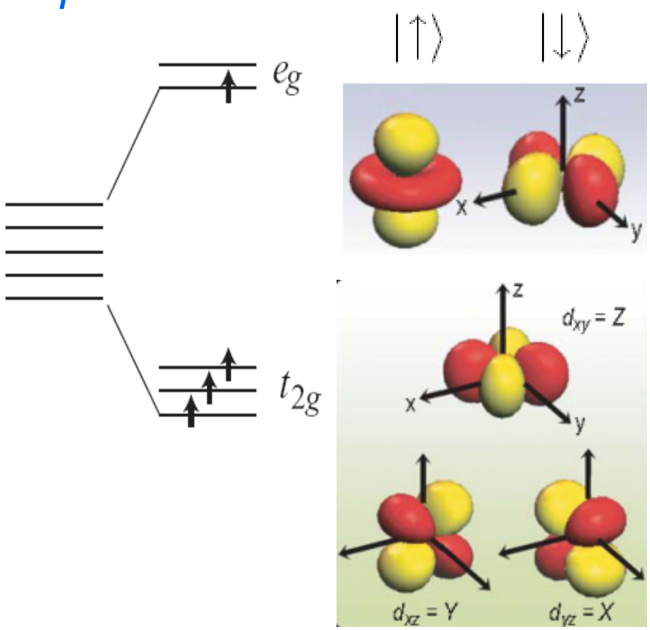
# Orbital ordering of d-electrons

[see, for example, review by Tokura and Nagaosa, science 288, 462, (2000).]

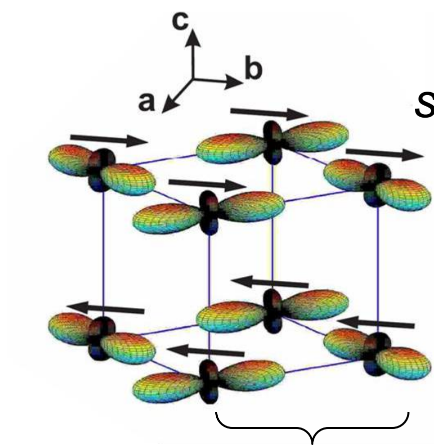
***orbital: shape of the electron cloud***

Orbital degeneracy in transition metal oxides:

*pseudospin 1/2:*




$LaMnO_3$




superposition of $e_g$ orbitals

Kugel-Khomskii superexchange

$$H = \sum_{\langle i,j \rangle} \hat{J}_{ij}^{(\gamma)} (\vec{S}_i \vec{S}_j + \frac{1}{4}),$$

$$\hat{J}_{ij}^{(\gamma)} = J(T_i^{(\gamma)} T_j^{(\gamma)} - \frac{1}{2} T_i^{(\gamma)} - \frac{1}{2} T_j^{(\gamma)} + \frac{1}{4})$$

$$T_i^{(a/b)} = \frac{1}{4}(-\sigma_i^z \pm \sqrt{3}\sigma_i^x), \qquad T_i^{(c)} = \frac{1}{2}\sigma_i^z$$

- Charge, spin, orbital, and lattice degree of freedom entangled together
- Complex phase diagrams

# *A new direction*: p-orbital physics in optical lattices

- **Orbital degeneracy** (*px, py, pz orbitals*) is considerably less understood in comparison. Implies emergent symmetry.
- **Similar to spin physics** but is different fundamentally.
- **Strong anisotropy**: Anisotropy is an interesting new feature, not a problem!
- ***p*-orbitals are different than *d*-orbital in solids:** Parity ODD. New possibility---*p*-orbital bosons as opposed to *d*-electrons (fermions) in solids
- Unique to cold atom systems, "non-standard" condensed matter systems. For instance, Orbital physics of bosons has no prior analogue in CM physics?!

⇒ **unique quantum phases (a main motivation of our study).**

# Early theoretical studies on the excited bands (pre 2010)

An incomplete list!! *[red=WVL involved]*

## On multi-orbital

- V. Scarola, S. Das Sarma, *Phys. Rev. Lett.* (2005)
- O. E. Alon, A. I. Streltsov, L. S. Cederbaum, *Phys Rev Lett* (2005)
- …

## On p-orbital

- A. Isacsson and S. Girvin, *Phys. Rev. A* 72, 053604 (2005).
- A. B. Kuklov, *Phys. Rev. Lett.* (2006)
- WVL and C. Wu, *Phys. Rev. A* (2006)
- C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. I*
- A. F. Ho, arXiv:cond-mat/0603299
- C. Xu and M. P. A. Fisher, *Phys. Rev. B* 75, 10442
- C. Wu, D. Bergman, L. Balents, and S. Das Sarma
- A. Kantian, A. J. Daley, P. Törmä and P. Zoller, *Ne*
- L. Guo, Y. Zhang, and S. Chen, *Phys. Rev. A (200*
- E. Zhao and WVL, *Phys. Rev. Lett.* (2008);
- R. O. Umucallar and M. Ö. Oktel, *Phys. Rev. A* (
- K. Wu and H. Zhai, *Phys. Rev. B* (2008)
- L. Wang, X. Dai, S. Chen, X. C. Xie, *arXiv:0805.*
- R. M. Lutchyn, S. Tewari, S. Das Sarma, *arXiv:08*
- V. Stojanovic, C. Wu, WVL and S. Das Sarma, *Ph*
- …
- K. Sun. E. Zhao, WVL, *Phys. Rev. Lett.* (2010)
- Z. Zhang, H. H. Hung, C.M. Ho, E. Zhao, WVL,
- A. Collin, J. Larson, and J.-P. Martikainen, *Phys. I*
- …

### *Our p-orbital work*

- Phys. Rev. **A**74, 013607 (2006).
- Phys. Rev. Lett. 97, 190406 (2006).
- Phys. Rev. Lett. 100, 160403(2008)
- Phys. Rev. Lett. 101, 125301 (2008)
- Phys. Rev. Lett. 104, 165303 (2010).
- Phys. Rev. A82, 033610 (2010)
- Phys. Rev. A83, 063626 (2011)
- Phys. Rev. A 85, 053606 (2012)
- Nature Phys. 7, 101 (2011)
- Nature Phys. 8, 67 (2012)
- Phys. Rev. Lett. 108, 175302 (2012)
- arXiv:1205.0254, …:1210.1859.

Bose Condensate → Mott → 1D p-orbitals
→fermions → topological

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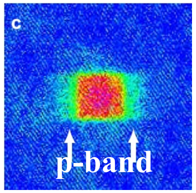
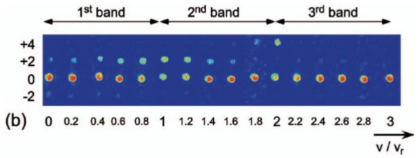
# Experiments: atoms observed in higher orbital bands

## *Fermions on p-band*

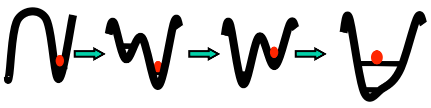



- Fermions transferred into p-band by sweeping across Feshbach resonance, i.e., by strong interaction. [M. Köhl et al, PRL **94**, 080403 (2005)]
- Band cross through Dirac point [ETH/Esslinger et al, Nature 483, 302 (2012)]

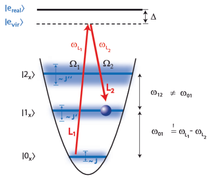
## *Bosons on p-band*



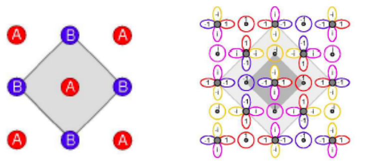

- By moving lattices [A. Browaeys, W. D. Phillips, et al, PRA **72**, 053605 (2005)]



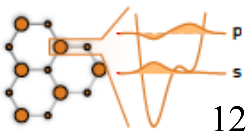
- Dynamically deforming the double-well super-lattice [NIST Porto/ Phillips groups: PRA (2006); J. Phys. B (2006); PRL 2007; Nature 2007; …]



- Pumping bosons by Raman transition [T. Mueller, I. Bloch et al., PRL 2007]



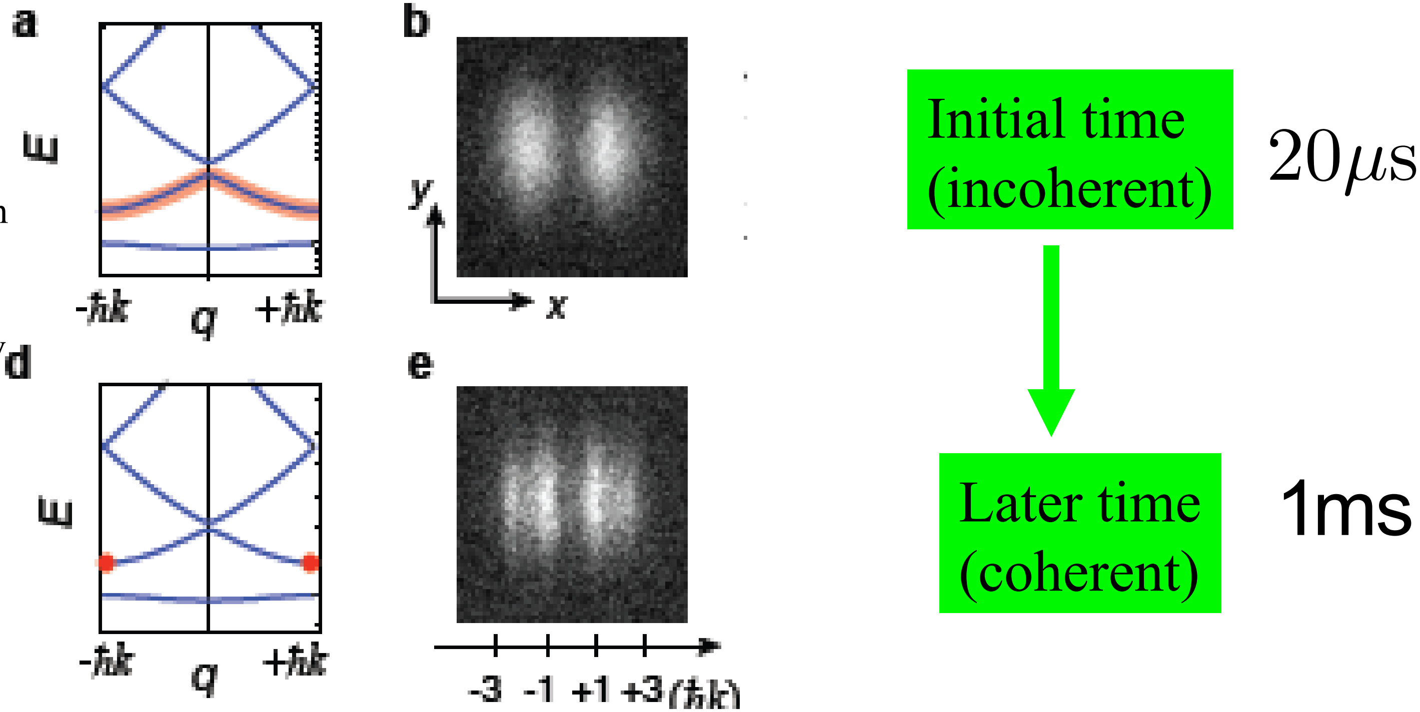
- p-band in (symmetric) double-well lattice [G. Wirth. M. Olschlager, A. Hemmerich, *Nature Physics* 2011]; f-band [PRL, 2011]; avoided band-crossing & Landau-Zener [PRL (2012)]




- "Unconventional (complex) multi-orbital superfluidity" on hexagonal double-well 'super-lattice' [P. Soltan-Panahi, …, K. Sengstock, Nature Physics (online Nov 2011)]

# Pioneer Experimental observation: finite momentum BEC
## by the Mainz/Bloch group [Mueller, Bloch, et al, PRL, 2007]

In Experiment:
- selected one direction
- hence only one p sub-band,
- no orbital degeneracy




Mueller, Bloch et al [PRL 2007] and theoretical prediction
[Isacsson-Girvin, 2005; Kuklov, 2006; WVL, Wu 2006] **agrees**!

# Experiments: atoms observed in higher orbital bands

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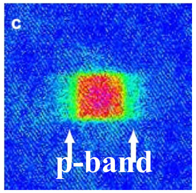


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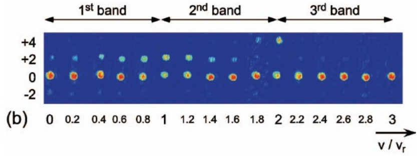


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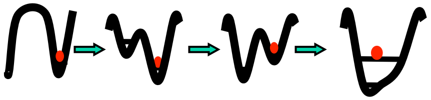
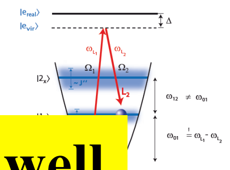
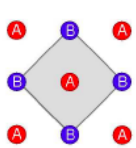


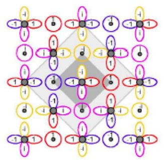
- Pumping bosons by Raman transition [T. Mueller, I. Bloch et al., PRL 2007]



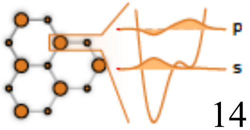
**Double well**

- p-band in (symmetric) double-well lattice [G. Wirth. M. Olschlager, A. Hemmerich, *Nature Physics* 2011]; f-band [PRL, 2011], avoided band-crossing & Landau-Zener [PRL (2012)]

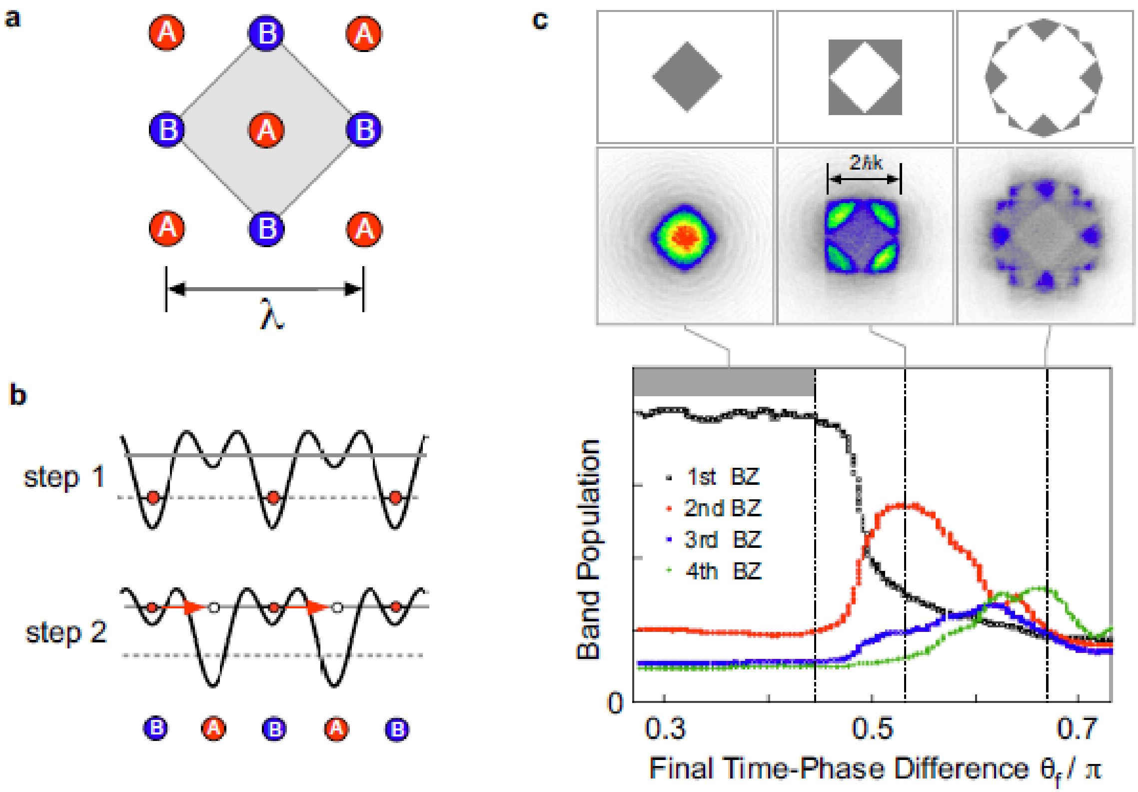


- "Unconventional (complex) multi-orbital superfluidity" on hexagonal double-well 'super-lattice' [P. Soltan-Panahi, …, K. Sengstock, Nature Physics (online Nov 2011)]



# Boson: p- and f-band experiments – double well lattices




**Hamburg/**
**A. Hemmerich group**

Remark:
First observation of p-band BEC with C4 symmetry and hence orbital degeneracy

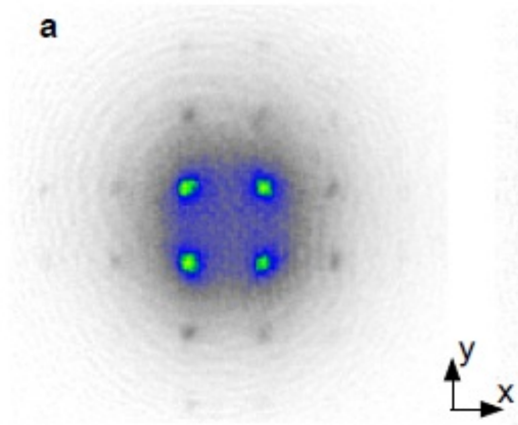
- "P-band superfluidity+orbital order in chequerboard (double well) lattice", long life time [G. Wirth. M. Olschlager, A. Hemmerich, *Nature Physics* 2011]
- "F-band" [M. Olschlager, G. Wirth, A. Hemmerich, PRL 2011]
- Avoided band-crossing & Landau-Zener [Olschlager, Hemmerich, et al, PRL (2012)]
- Interacting chiral p+ip order [Hemmerich group, new preprint (private/unpublished)]

# Experimental observation: complex p-orbital superfluids

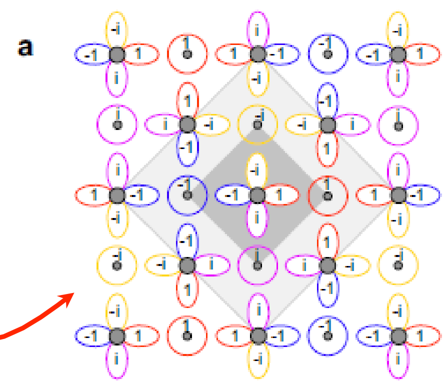
G. Wirth, M. Olschlager, A. Hemmerich,
Nature Phys. 2011

Isotropic lattice

***Observed momentum distribution***

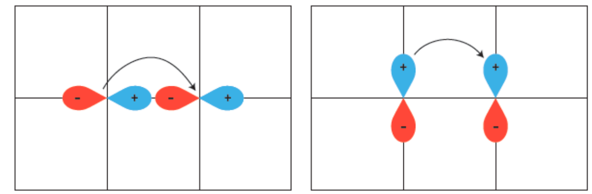


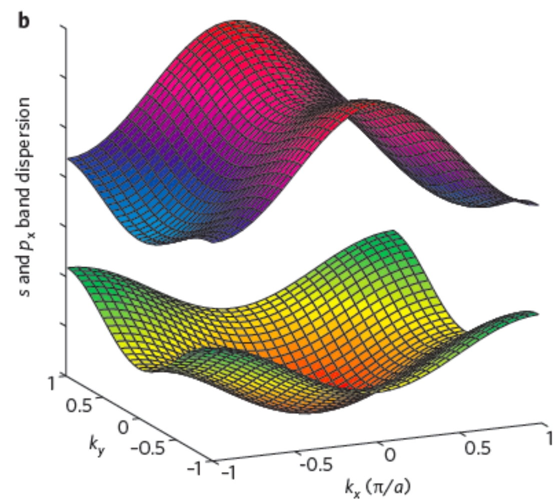

***Interpretation:***
***Nature of orbital order***




complex $p_x \pm ip_y$

**Theoretical understanding**






[M. Lewenstein & WVL, Nature Phys. 7, 101 (2011)]

Experimental finding consistent with prediction by [WVL, C. Wu, PRA 2006]

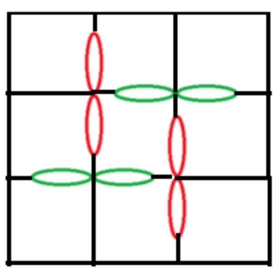
# p-orbital band phase diagram

[X. Li, E. Zhao and WVL, PRA 83, 063626 (2011)]
[See also early Mott phase results by A. Isacsson and S. M. Girvin, PRA 72, 053604 (2005); and A. Collin, J. Larson, and J.-P. Martikainen, Phys. Rev. A 81, 023605 (2010)]

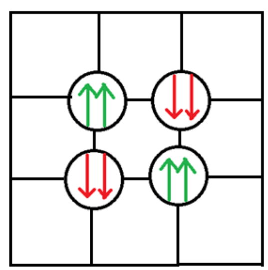
**-Mott phases at commensurate filling**

1. filling =1
2. filling >1

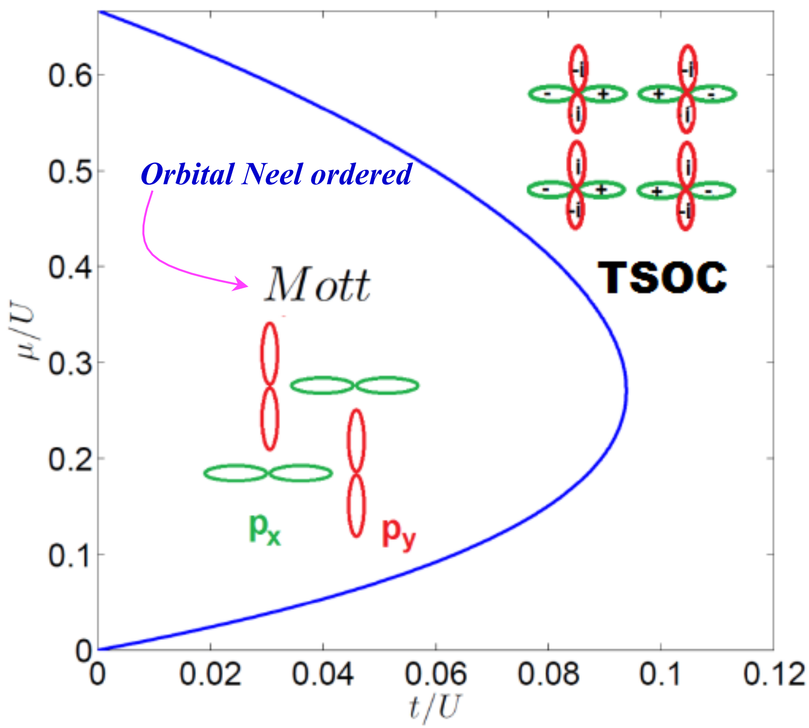
$(\updownarrow) = p_x \pm i p_y$



*Orbital Neel order*



**-phase boundary of Mott with one particle per site**



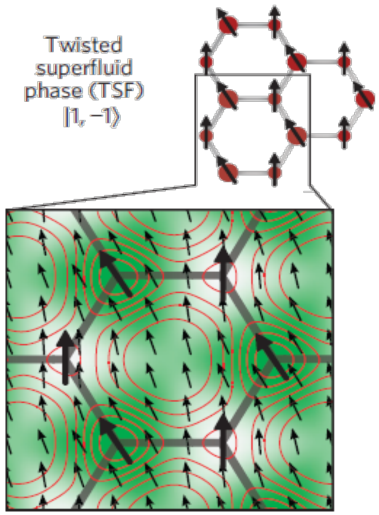

## *Compare Mott results*

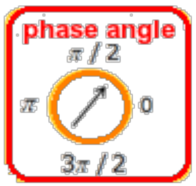
- Our theory agrees with Isacsson-Girvin MFT for filling=1
- Differs Isacsson-Girvin at higher integer fillings [who predicted $p_x$-$p_y$ alternating]
- Collin-Larson-Martikainen disagrees with both Isacsson-Girvin and us

**Compare: the usual s-band Mott phase is featureless.**

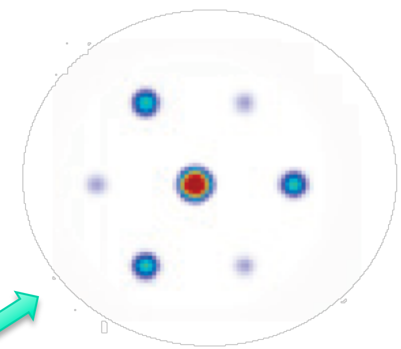
# Observation II: complex multi-orbital superfluidity

[P. Soltan-Panahi, Lühmann, Struck, Windpassinger, K. Sengstock, Nat. Phys. 8, 71 (2012)]

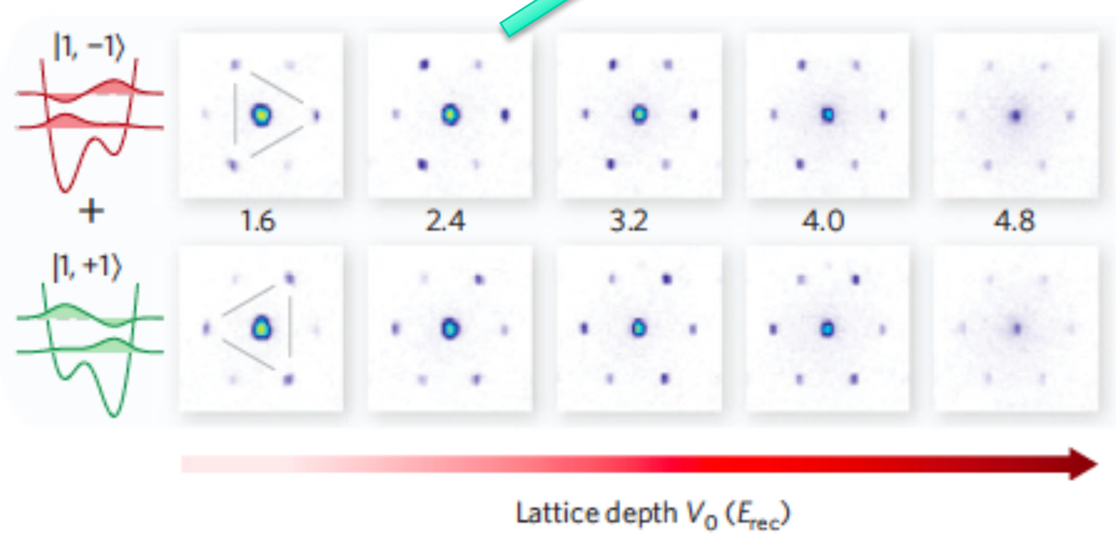



$$\sqrt{n_s}\,|s\rangle + e^{i\theta}\sqrt{n_p}\,|p\rangle$$

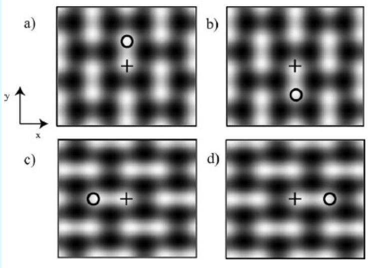


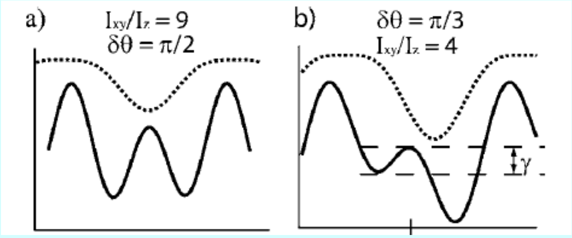
Phase twisting superfluid



# Other early and recent experiments of double-well superlattices

## NIST group (Porto/ Phillips)



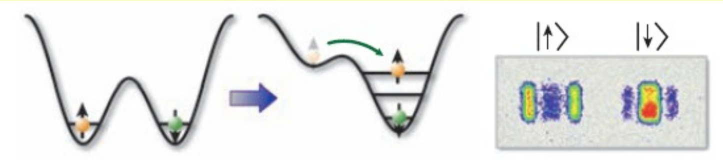

[J. Sebby-Strabley, et al, PRA (2006); M. Anderlini, et al, J. Phys. B (2006); Nature (2007); …]

Does not preserved original geometric symmetry

## Bloch group

[S. Trotzky et al, Science (2008); P. Cheinet et al, Phys. Rev. Lett. 101, 090404 (2008) ; Y. Chen et al, arXiv: 1003.4956 (2010); …]

**Time-Resolved Observation and Control of Superexchange Interactions with Ultracold Atoms in Optical Lattices**
S. Trotzky, *et al.*
*Science* **319**, 295 (2008);




## Stamper-Kurn group

[G. Jo et al, Phys. Rev. Lett. 108, 045305 (2012)]

Superlattice (red and blue lattices overlapped) → Kagome

# *Part 3 and 4:*

- **Quantum 120$^o$ orbital only model in optical lattices**
- **Topological insulator (Z invariant) of sp-orbital ladder**

---

***Work done (in collaboration) with:***

Sankar Das Sarma, Andreas Hemmerich, Xiaopeng Li, Kai Sun, Erhai Zhao.

***References:***

- **Background and perspective (news & views):**
  **Nature Physics 7, 101 (2011) [with M. Lewenstein]**

- **PRL 100, 160403 (2008)**
- **Nature Physics 8, 67 (2012)**
- **arXiv:1205.0254**

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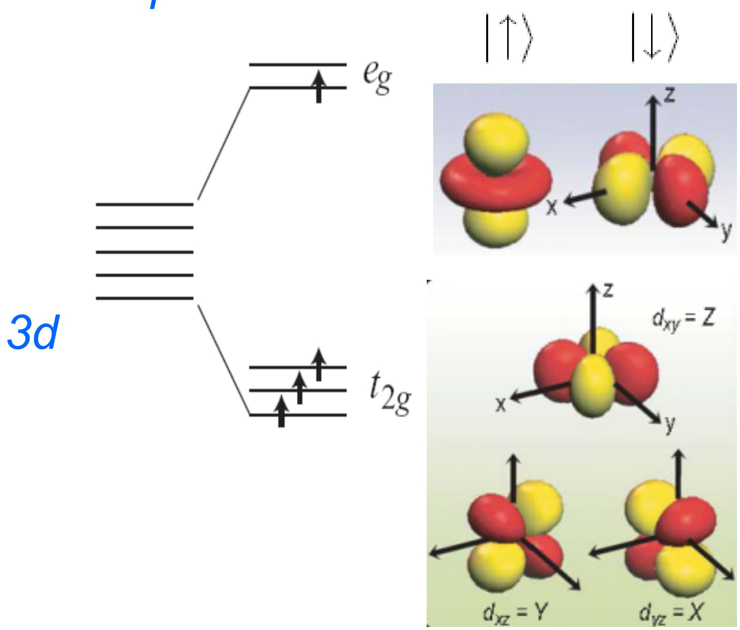
# Orbital ordering of d-electrons

[see, for example, review by Tokura and Nagaosa, science 288, 462, (2000).]

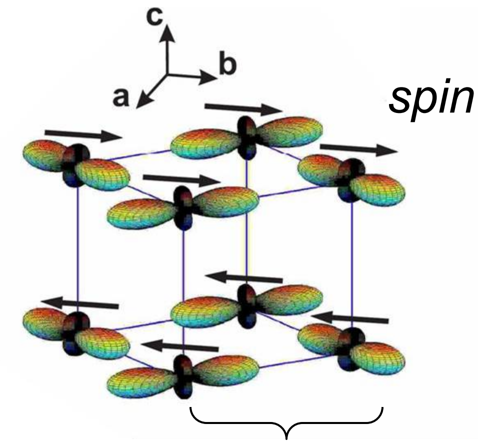
***orbital: shape of the electron cloud***

Orbital degeneracy in transition metal oxides:

*pseudospin 1/2:*




$LaMnO_3$




superposition of $e_g$ orbitals

Kugel-Khomskii superexchange

$$H = \sum_{\langle i,j \rangle} \hat{J}_{ij}^{(\gamma)} (\vec{S}_i \vec{S}_j + \frac{1}{4}),$$

$$\hat{J}_{ij}^{(\gamma)} = J(T_i^{(\gamma)} T_j^{(\gamma)} - \frac{1}{2} T_i^{(\gamma)} - \frac{1}{2} T_j^{(\gamma)} + \frac{1}{4})$$

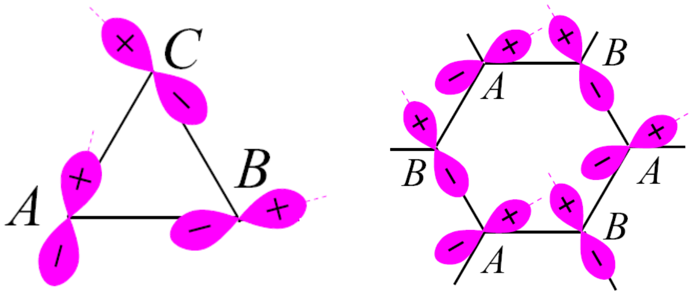
$$T_i^{(a/b)} = \frac{1}{4}(-\sigma_i^z \pm \sqrt{3}\sigma_i^x), \qquad T_i^{(c)} = \frac{1}{2}\sigma_i^z$$

- Charge, spin, orbital, and lattice degree of freedom entangled together
- Complex phase diagrams

# Orbital quantum 120° model

[E. Zhao and WVL, PRL (2008); See also independent work by C. Wu, PRL 2008.]

Pseudo-spin operators on frustrated lattices (triangular, honeycomb, Kagome, …)

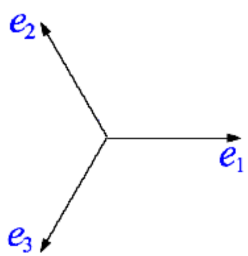



$|p_x\rangle$ $|p_y\rangle$

$$T_\mu = \frac{1}{2}\left(c_x^\dagger, c_y^\dagger\right)\sigma_\mu \begin{pmatrix} c_x \\ c_y \end{pmatrix}$$

$$\mu, \nu = x, y, z$$

The orbital exchange Hamiltonian is:




$$H_{120} = J_z \sum_{\mathbf{R},j} T_j(\mathbf{R}) T_j(\mathbf{R}+\hat{e}_j)$$

lattice sites

$j$ =1,2,3

where

$$T_1 = T_z,\ T_2 = -\tfrac{1}{2}T_z - \tfrac{\sqrt{3}}{2}T_x$$

$$T_3 = -\tfrac{1}{2}T_z + \tfrac{\sqrt{3}}{2}T_x$$

This quantum 120° model is closely related to the compass model and Kitaev model.

Quantum 120° model of electron $e_g$ orbitals: van den Brink, New J. Phys. 6, 201 (2004).

# Outline

1. Introduction
   - Optical lattices. What is the p-orbital band? Why?

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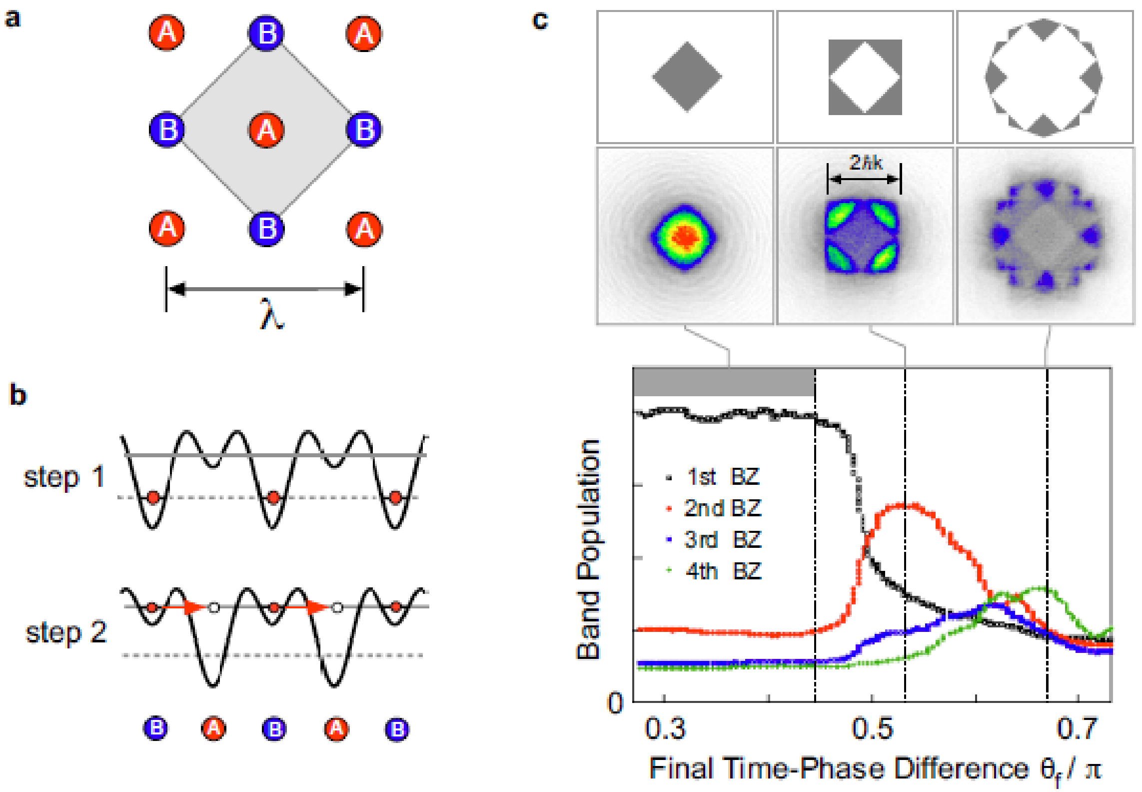
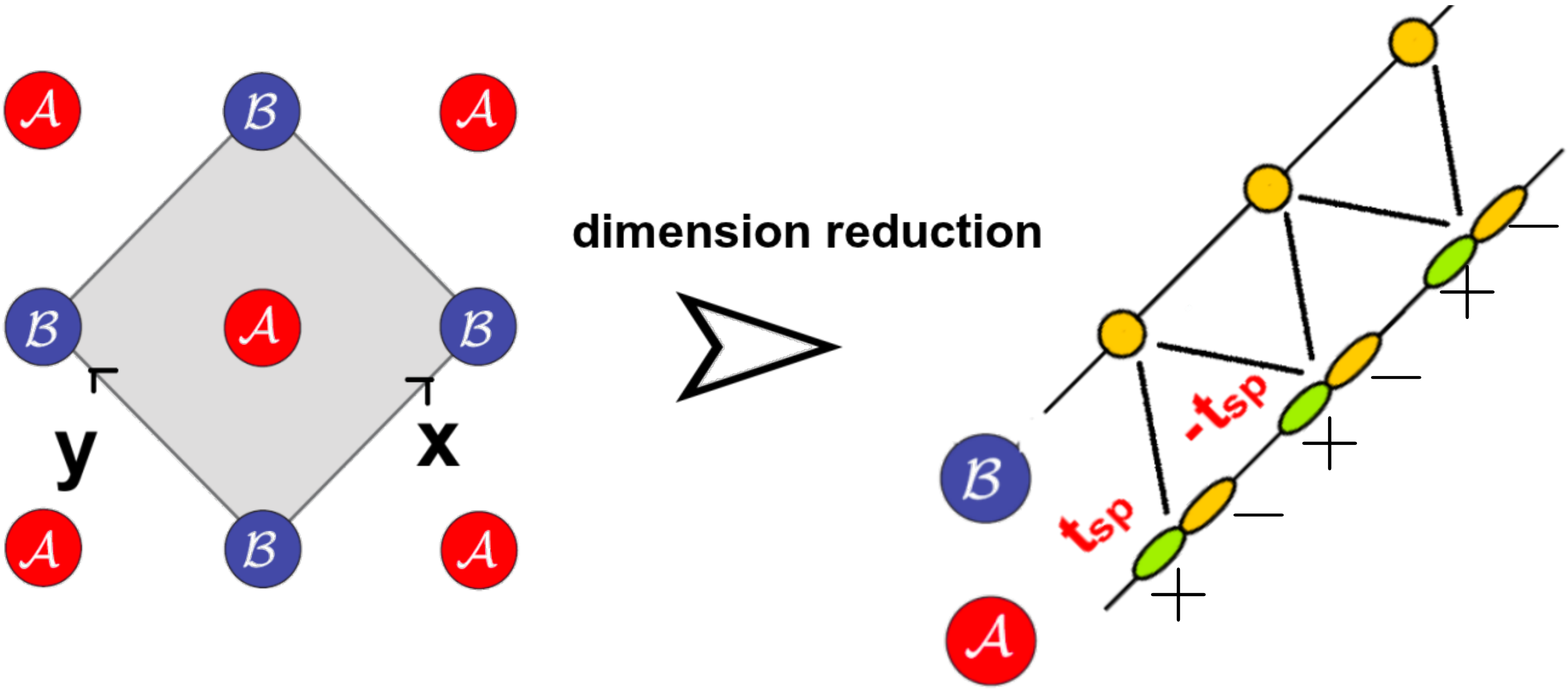
# Review: p- and f-band experiments – double well lattices



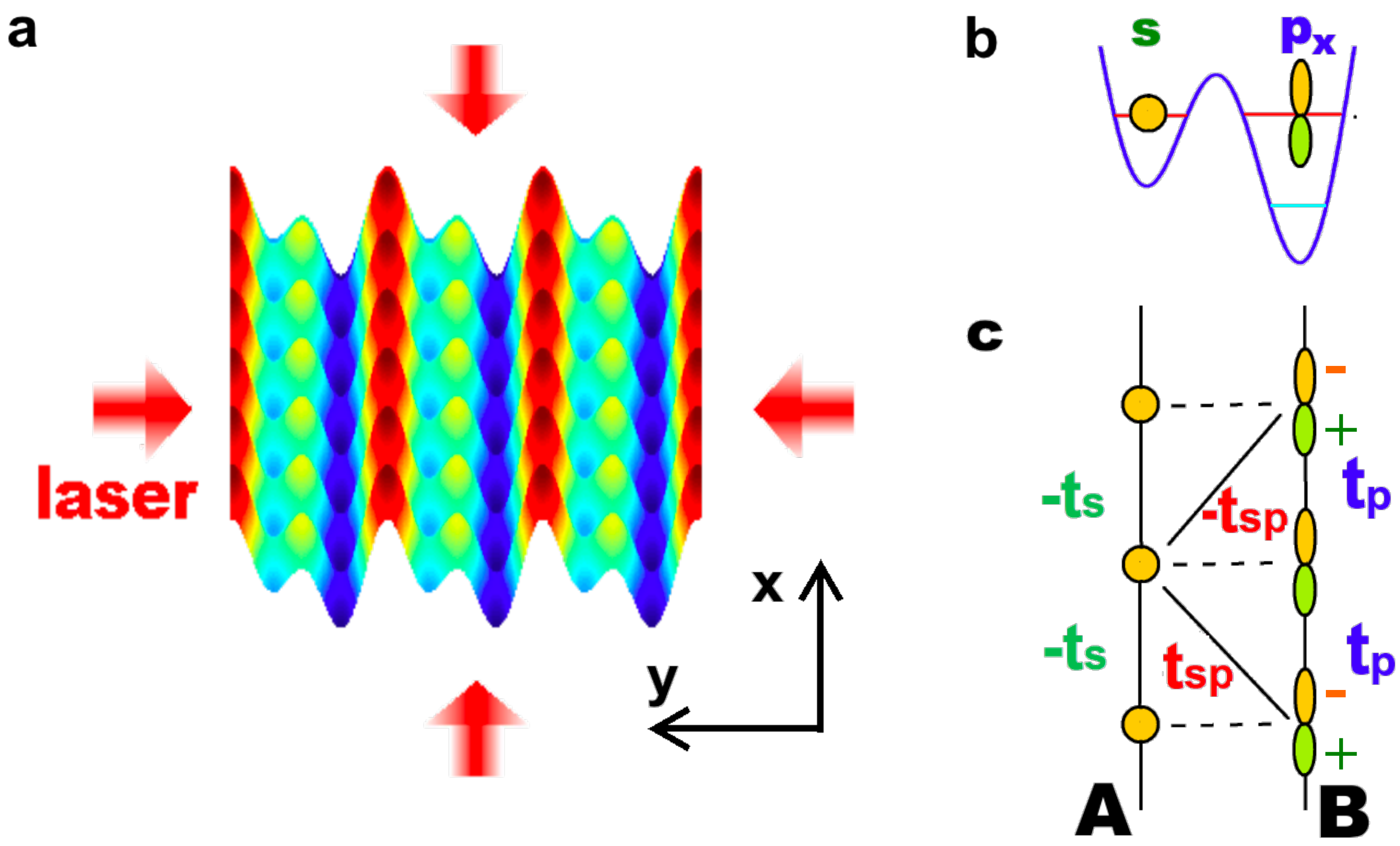

**Hamburg/
A. Hemmerich group**

- "P-band superfluidity+orbital order in chequerboard (double well) lattice", long life time [G. Wirth. M. Olschlager, A. Hemmerich, *Nature Physics* 2011]
- "F-band" [M. Olschlager, G. Wirth, A. Hemmerich, PRL 2011]

# Hamburg 2D double-well lattice → sp-orbital Ladder



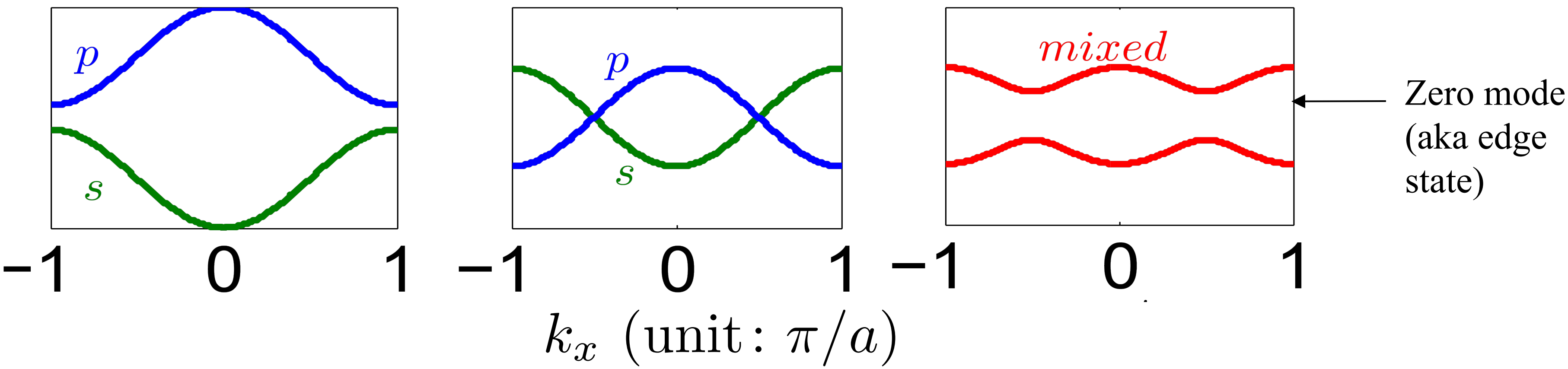
# Topological orbital ladders




Xiaopeng Li, Erhai Zhao and WVL, arXiv:1205.0254

# *sp*-mixing and topological nature

## -*sp*-mixing

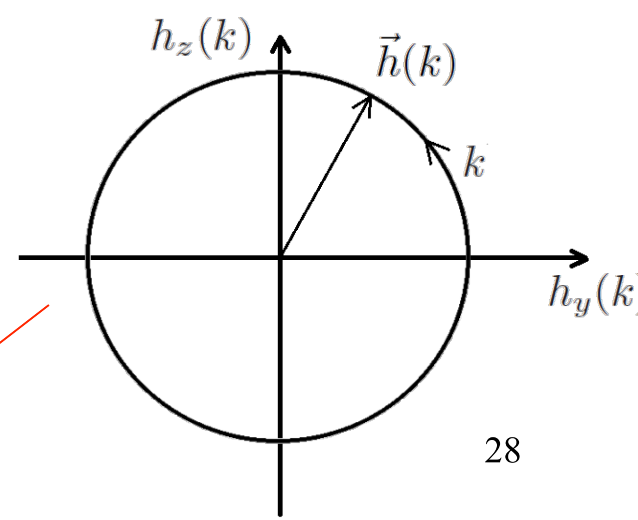


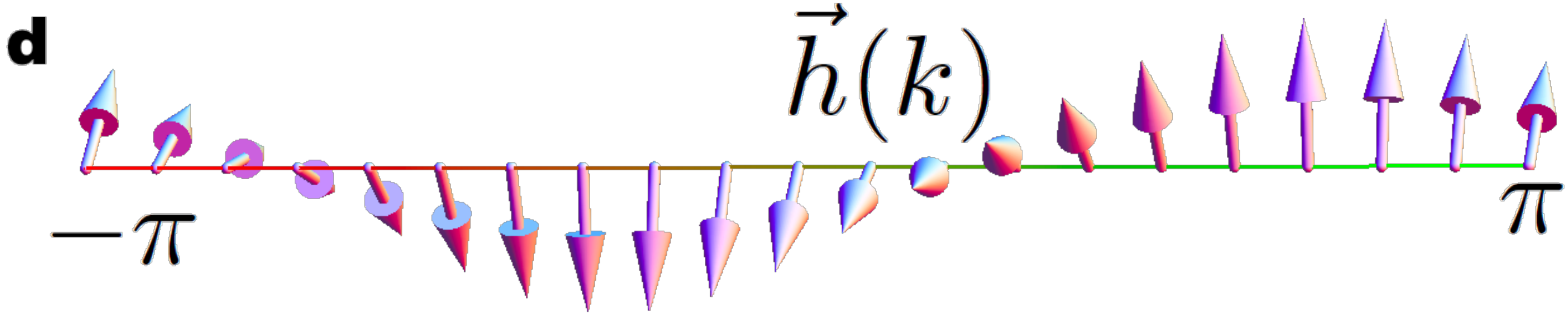

## -Hamiltonian in momentum space

$$\mathcal{H}(k) = \begin{bmatrix} -2t_s \cos k & -2it_{sp} \sin k \\ 2it_{sp} \sin k & 2t_p \cos k \end{bmatrix}$$

$$\mathcal{H}(k) = h_0(k)\mathbb{I} + \vec{h}(k) \cdot \vec{\sigma}$$

Berry phase is $\pi$.






*Momentum k*

*(lattice constant a =1)*

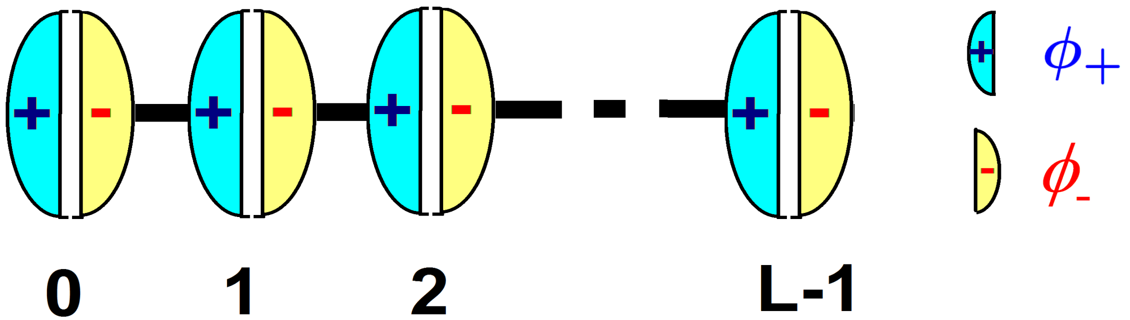
# Edge states

**-flat band limit** (easy to show)

$$t_s = t_p = t_{sp} = t \qquad E(k) = \pm 2t$$

$$H_0 \to 2t \sum_j \phi_-^\dagger(j)\phi_+(j+1) + h.c. \qquad \phi_\pm = [a_p \pm a_s]/\sqrt{2}$$

**Edge states are completely localized**



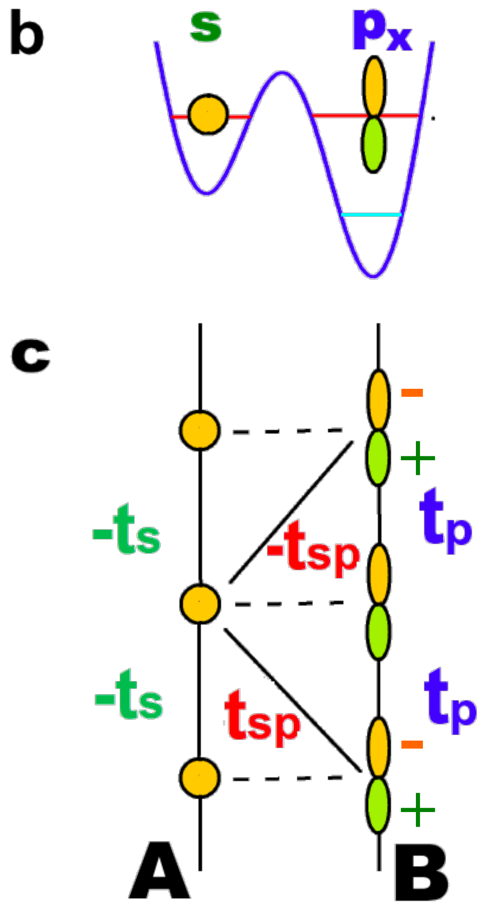

**-general case**

$$H_0 = \sum_j C_j^\dagger \left[\frac{t_p - t_s}{2}\mathbb{I} - \frac{t_p + t_s}{2}\sigma_z - i t_{sp}\sigma_y\right] C_{j+1} + h.c. \qquad C_j = \begin{bmatrix} a_s(j) \\ a_p(j) \end{bmatrix}$$

Edge states decay with a width $\xi = 2/\log(|(\sqrt{t_s t_p} + t_{sp})/(\sqrt{t_s t_p} - t_{sp})|)$

# Summary: topological insulator from odd parity

1. Topological insulator (index group Z class) at half filling.
2. Compare with spin-orbit coupling generated by artificial gauge field in cold atoms/Pioneer experiments:
   A. Bosons: NIST (I. Spielman et al), USTC (S.Chen, J. Pan et al) …
   B. Fermions: Shanxi U (J. Zhang et al), MIT (M. Zwierlein et al)
3. This model: No spin, but orbit. Resembles spin-orbit coupling if (s, p) space viewed pseudo-spin-1/2.
4. New result: topological phase not requiring any of previously known mechanisms: rotation, gauge field, p-wave pairing,…




Xiaopeng Li, Erhai Zhao and WVL, arXiv:1205.0254

# *Emergent Effective Spin-orbit coupling*

**-Hamiltonian in momentum space**

$$\sim \sigma_y \sin k_x \approx \sigma_y k_x$$

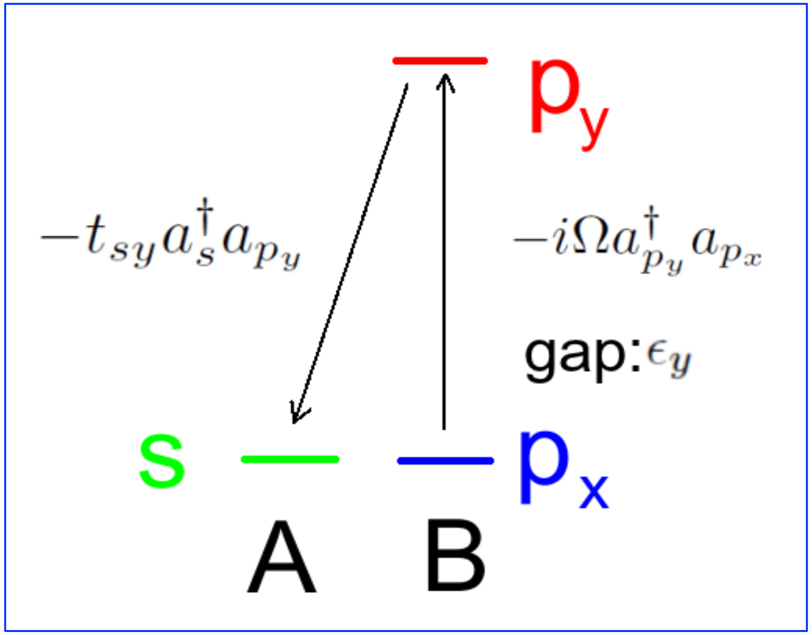
$$\mathcal{H}(k) = \begin{bmatrix} -2t_s \cos k & -2it_{sp} \sin k \\ 2it_{sp} \sin k & 2t_p \cos k \end{bmatrix}$$

$$\mathcal{H}(k) = h_0(k)\mathbb{I} + \vec{h}(k) \cdot \vec{\sigma}$$

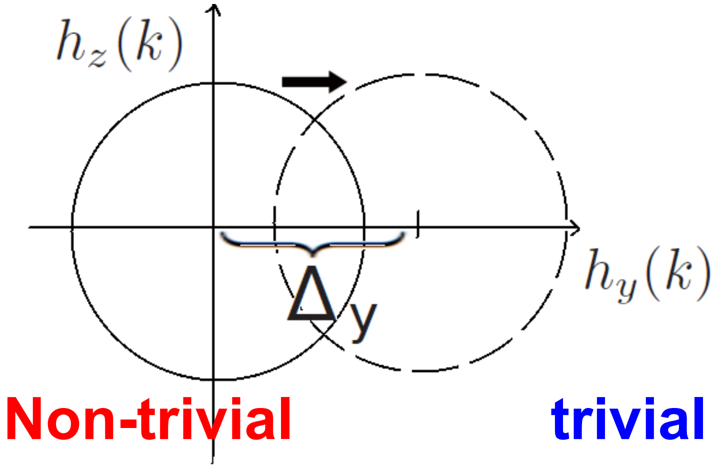
# Topological phase transition – driven by rotation

**-rotating individual lattice sites**

$$\delta H = \frac{\Omega^2}{\epsilon_y} C_j^\dagger \sigma_y C_j \quad \Delta_y = \frac{\Omega^2}{\epsilon_y}$$




**-phase transition**




$$\Delta_y^c = 2t_{sp}$$

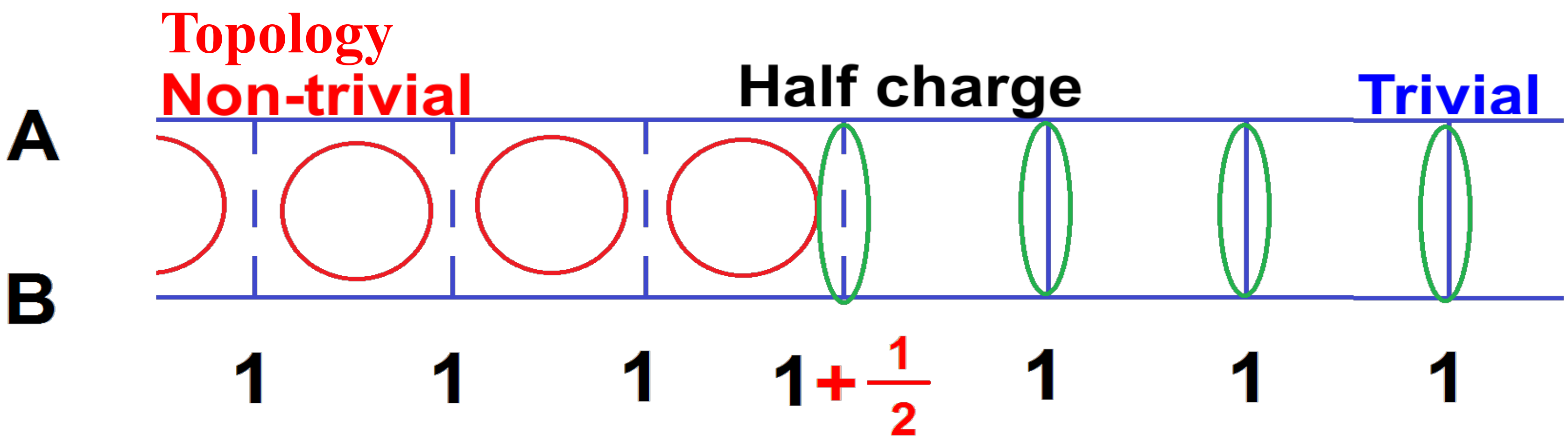
N. Gemelke et al, arXiv:1007.2677

Xiaopeng Li, Erhai Zhao and WVL, arXiv:1205.0254

# Domain Wall Fractional Charge

$$H_\eta = H + \frac{\Delta_y}{2}\sum_j \left[1 - \cos\eta(j)\right] C_j^\dagger \sigma_y C_j$$

$$\eta(j = -\infty) = 0 \qquad \eta(j = +\infty) = \pi$$



# Firm Computation of Fractional Charge:
## Background (auxiliary) gauge field method

- **Introduce background gauge field:** $(A_\tau, A_x)\,, \qquad \tau = it$

$$
\begin{aligned}
\mathcal{D}_\tau &= \partial_\tau + iA_\tau(j,\tau), \\
\mathcal{T}_{j\,j+1} &= e^{iA_x(j+1/2,\tau)} T_{j\,j+1}, \\
T_{j\,j+1} &= \begin{bmatrix} -t_s & -t_{sp} \\ t_{sp} & t_p \end{bmatrix}, \\
\mathcal{T}_{j\,j+1} &= \mathcal{T}^\dagger_{j+1\,j}.
\end{aligned}
$$

Berry phase

- **Effective action** $\tilde{S}_{\text{eff}}[A_\mu] = \int dx dt (A_x \partial_t \eta - A_t \partial_x \eta) \frac{1}{2\pi} \partial_\eta \gamma(\eta)$

- **Charge**

$$Q = \int \frac{\delta \tilde{S}_{\text{eff}}}{\delta A_t} = -\frac{1}{2\pi} \int dx \partial_x \eta \partial_\eta \gamma(\eta) = -\int \frac{d\eta}{2\pi} \partial_\eta \gamma(\eta)$$

Find: $Q = \frac{1}{2} \bmod 1$

# Effects of interaction, beyond half filling, etc.

- Topological to (non-topological) Ferro-orbital Mott insulator transition, driven by interaction [X. Li, E. Zhao, WVL, arXiv: 1205.0254]

- Away from half filling, find interesting phases: orbital density wave, pair density wave, and especially quasi-1D superconductivity with repulsive interaction, …, by RG/ Bosonization [X. Li, WVL, arXiv: 1210.1859]

**Brief note on a Related Theoretical Result:**

nature physics

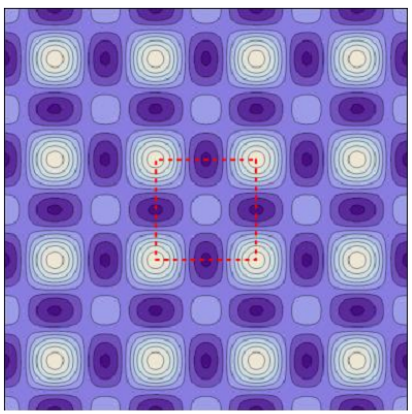
LETTERS

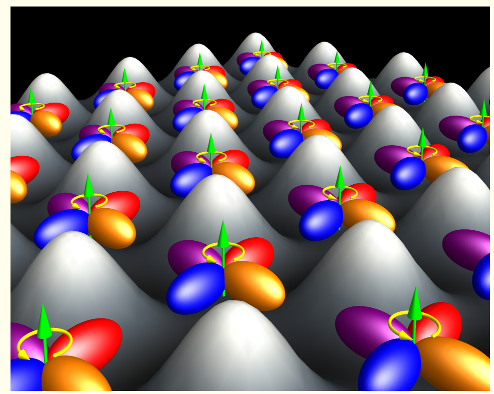
PUBLISHED ONLINE: 20 NOVEMBER 2011 | DOI: 10.1038/NPHYS2134

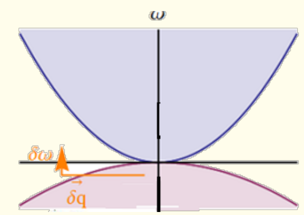
# Topological semimetal in a fermionic optical lattice

Kai Sun[1], W. Vincent Liu[2,3,4]*, Andreas Hemmerich[5] and S. Das Sarma[1]

System: p-orbital fermions on Double-well lattice








Ordered (topological insulator)

Normal(topological semimetal): winding number =2, quadratic dispersion, different than Graphene

0 $T_c$ T

**Ordered phase**

- Breaks Time-reversal symmetry
- Topological Insulator
- New mechanism – interaction driven -- Differs from the previously known's: artificial gauge field, rotation, spin-orbit coupling, …

# Conclusion---Optical Lattice Orbital Physics

## Frustrated orbital 120º model

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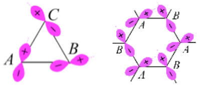


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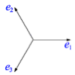

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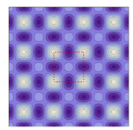

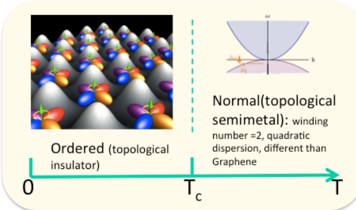


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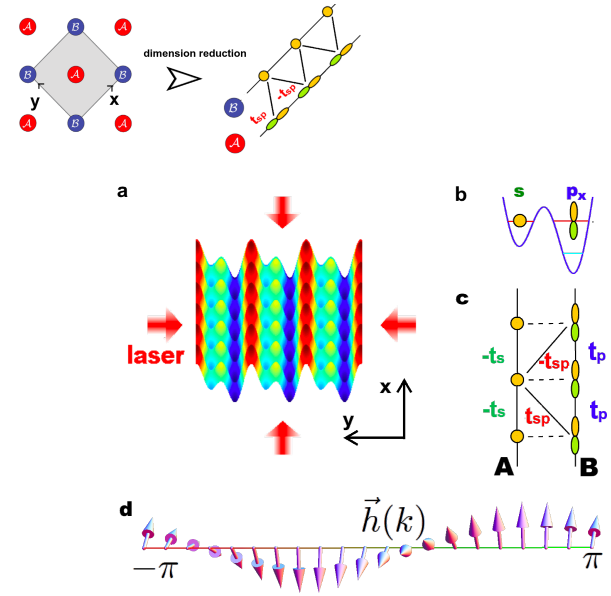
## Topological orbital ladder

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OPTICAL LATTICES

**[Nature Physics 7, 101 (Feb 2011)]**

# Orbital dance

Emulating condensed-matter physics with ground-state atoms trapped in optical lattices has come a long way. But excite the atoms into higher orbital states, and a whole new world of exotic states appears.

Maciej Lewenstein and W. Vincent Liu