Relaxation, pre-thermalization and diffusion in a noisy Quantum Ising Chain

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The model and the out of equilibrium protocol

$$H = H_0 + V$$

$$H_0 = -J\sum_i \sigma_i^x \sigma_{i+1}^x + g\sigma_i^z \qquad \Delta = |g-1|$$

Two gapped phases: quantum paramagnetic and ferromagnetic separated by a quantum critical point at g=1

$$V = \sum_i \delta g(t)\sigma_i^z \qquad \langle \delta g(t)\delta g(t')\rangle = \frac{\Gamma}{2}\delta(t-t')$$

We perform a quantum quench from the paramagnetic phase at g_0 to the paramagnetic phase at g and we compute observables of physical interest during the dynamics

Dynamics of Thermalization

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$$\frac{\Gamma}{\Delta} << 1$$

- ***Prethermalization***
  drfits towards GGE (*Kollar et al.* 2010)

- ***Dephasing*** $t \approx \frac{1}{\Gamma}$: coherences killed

- ***Thermalization*** and population heating towards $T \to \infty$

Fig.1

Fig.2

Order parameter correlations

$$\left\langle m_x(r,t)\, m_x(0,t)\right\rangle = r^{-1/2} e^{-\frac{r}{\xi(t)}}, \qquad \frac{1}{\xi(t)} = \frac{1}{\xi} + \frac{\Gamma t}{2(1+\Delta)^2}$$

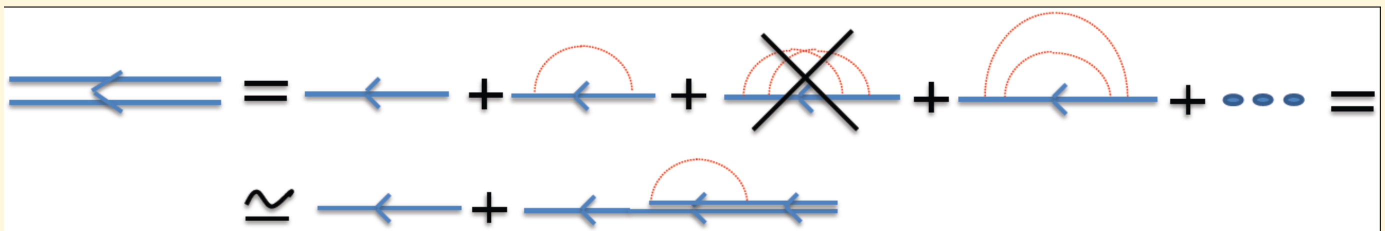
No Quench

Different signatures in different observables.

A bit of details ...

$$\rho_k = \begin{pmatrix} \langle \gamma_k^\dagger \gamma_k \rangle & \langle \gamma_k^\dagger \gamma_{-k}^\dagger \rangle \\ \langle \gamma_{-k} \gamma_k \rangle & \langle \gamma_{-k} \gamma_{-k}^\dagger \rangle \end{pmatrix}$$

$$\gamma = \frac{\Gamma}{\Delta} << 1$$



Keldysh

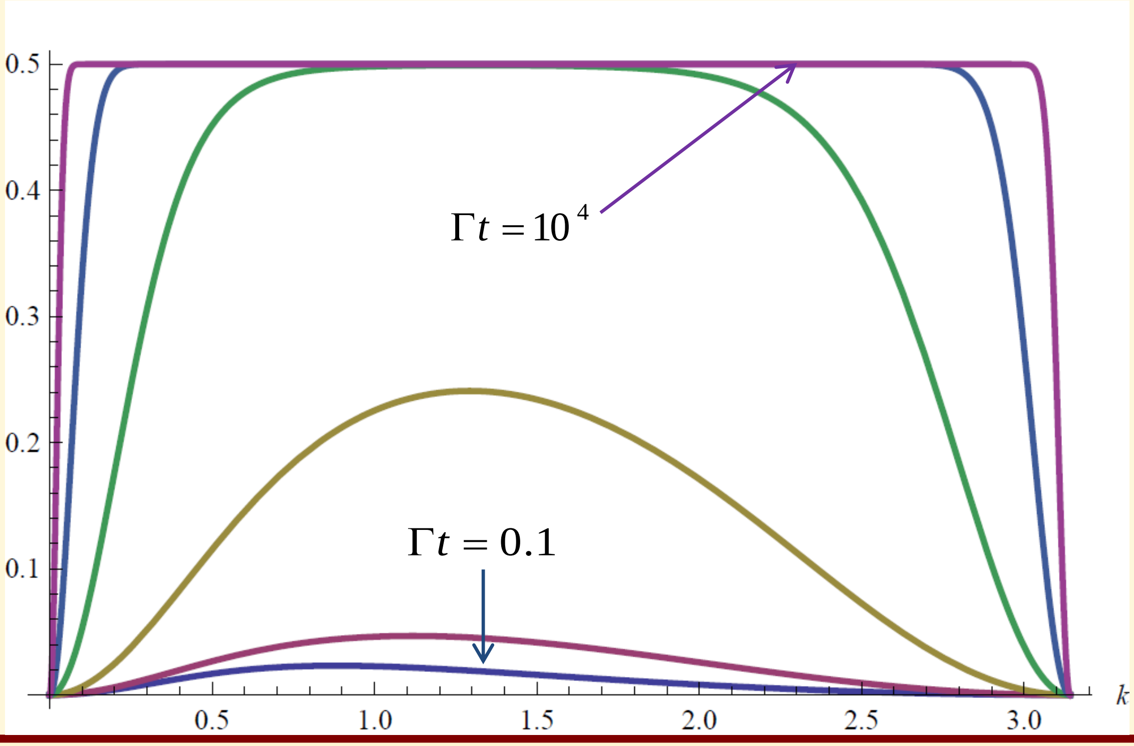
$$\partial_t \rho_k = -i[H_k^0, \rho_k] + \frac{\Gamma}{2}(\sigma \rho_k \sigma - \rho_k)$$

$$\sigma \equiv \cos 2\theta_k \sigma_z + \sin 2\theta_k \sigma_y$$

Populations

$$\left\langle \gamma_k^+(t)\gamma_k(t)\right\rangle = \frac{1}{2} + \left(\sin^2(\Delta\alpha_k) - \frac{1}{2}\right)e^{-\Gamma \sin^2\theta_1 t}$$ *Populations*

$$\theta(g) = \frac{1}{2}\arctan\left(\frac{\sin k}{g - \cos k}\right) \quad \Delta\alpha_k = \theta(g_1) - \theta(g_0)$$



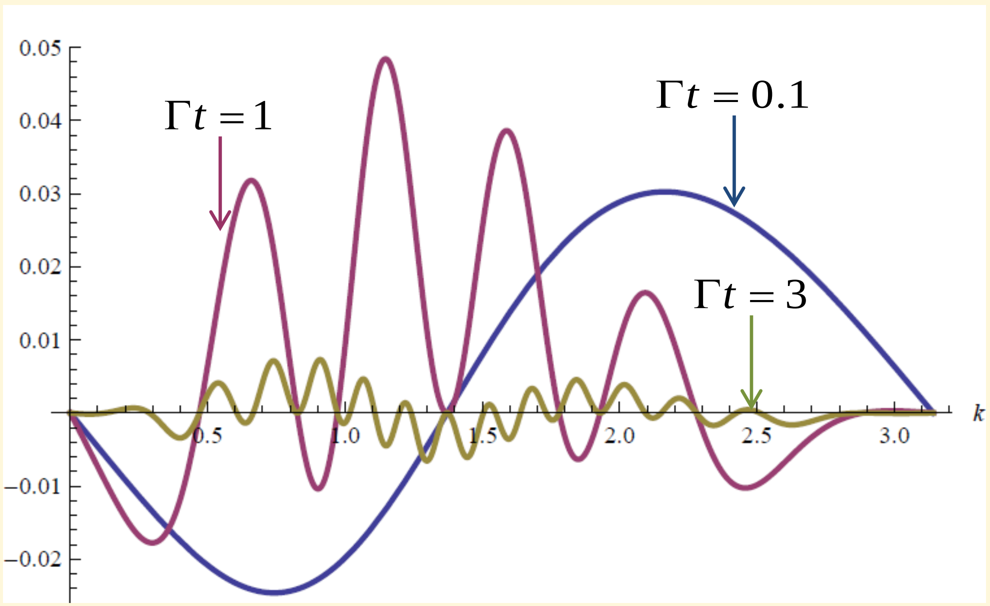
The relaxation rates tend to vanish close to the band edges $(k = 0, \pm\pi)$ for $\Gamma t >> 1$

$$\Gamma_k \approx_{(k \approx 0)} \Gamma \frac{k^2}{\Delta^2}$$

Most of the modes relax on time scales of the order of $1/\Gamma$

Coherences

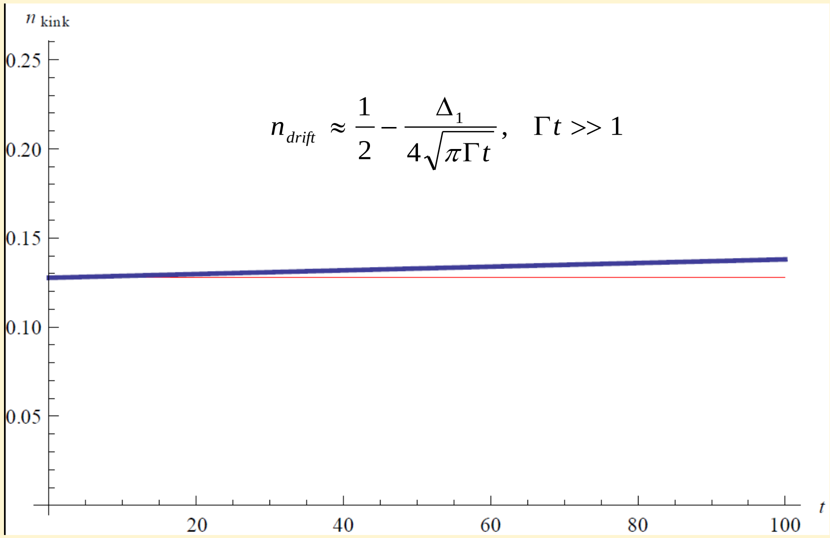
$$\left\langle \gamma_k^+(t)\gamma_{-k}^+(t) \right\rangle$$

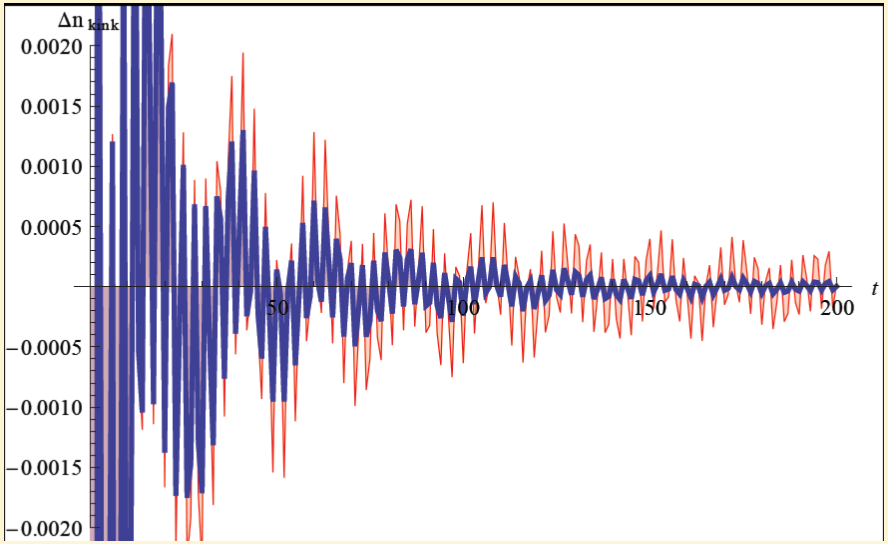


Coherences decay exponentially fast as $e^{-\Gamma t}$, and they are subleading compared to populations for $\Gamma t >> 1$.

Number of kinks

$$n_{kink}(t) = n_{drift}(t) + \Delta n(t)$$





Number of excitations $$N = \int_0^{\pi} dk \left\langle \gamma_k^+ \gamma_k \right\rangle \approx \frac{\pi}{2} - \frac{\alpha}{\sqrt{\Gamma t}}$$

Conclusions

- We study the dynamics of thermalization resulting from a time-dependent noise in a Quantum Ising Chain subject to a sudden quench of g.

- For weak noises the dynamics shows a pre-thermalized state at intermediate time scales, eventually drifting towards an asymptotic inifinite temperature state, charatezied by a diffusive behaviour in the transverse magnetization correlations.

Future issues

- Different times correlation functions?
- Fluctuation-dissipation theorem?
- Effect of spatial inhomogeneous noise? $V = \sum_i \delta g_i(t)\sigma_i^z$
- Integrability breaking in a closed system?