Synthetic gauge fields for ultracold atoms
Strong Correlation Effects and Dynamics

Arun Paramekanti (University of Toronto)

M. Killi, S. Trotzky, A. Hall (Toronto), J. S. Bernier (UBC)
W. S. Cole, S. Zhang, N. Trivedi (Ohio State University)
A. Dhar, M. Maji, T. Mishra, R. V. Pai, S. Mukerjee (Bangalore)

References
M. Killi and AP, PRA 85, (R)061606 (2012)
W. S. Cole, S. Zhang, AP, and N. Trivedi, PRL 109, 085302 (2012)
A.Dhar, M. Maji, T. Mishra, R. V. Pai, S. Mukerjee, and AP, Phys. Rev. A 85, (R)041602 (2012)
M. Killi, S. Trotzky, AP (manuscript in preparation)

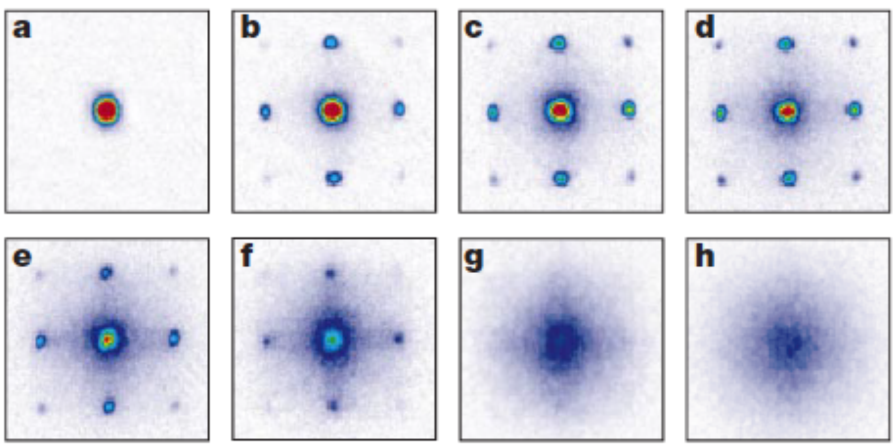
KITP, Quantum Dynamics (6 Sept 2012)

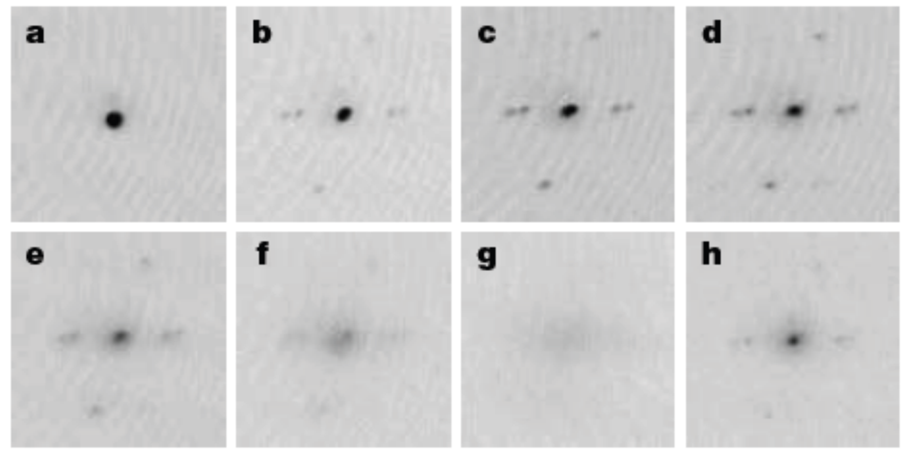
Funding: NSERC CRSNG CIFAR CANADIAN INSTITUTE for ADVANCED RESEARCH

Strong Correlations and Quantum Phase Transitions in Optical Lattices

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner[*], Olaf Mandel[*], Tilman Esslinger[†], Theodor W. Hänsch[*] & Immanuel Bloch[*]





Evidence for superfluidity of ultracold fermions in an optical lattice

J. K. Chin[1], D. E. Miller[1], Y. Liu[1], C. Stan[1]†, W. Setiawan[1], C. Sanner[1], K. Xu[1] & W. Ketterle[1]

Recent Experimental Progress

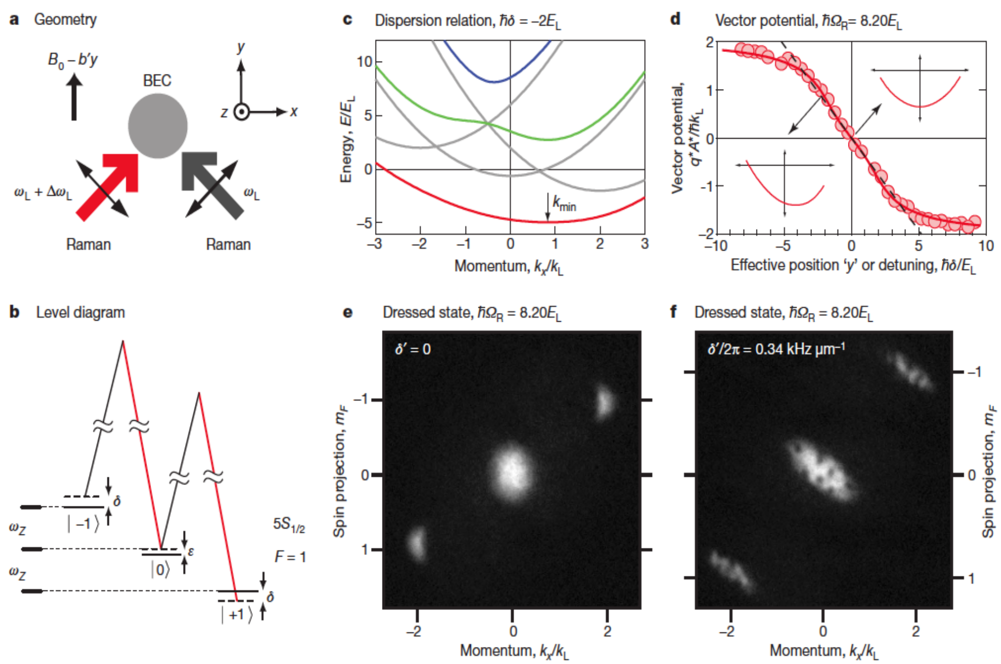
- **Strong effective orbital magnetic fields on the lattice**
- **Tunable spin-orbit coupling (Rashba + Dresselhaus)**

Interesting topological states of matter arise in the presence of magnetic fields or spin-orbit coupling

- **Quantum Hall and Chern insulators**
- **Topological insulators or semimetals**

Synthetic magnetic fields: Experimental Progress

Raman induced “gauge field” for atoms in the continuum

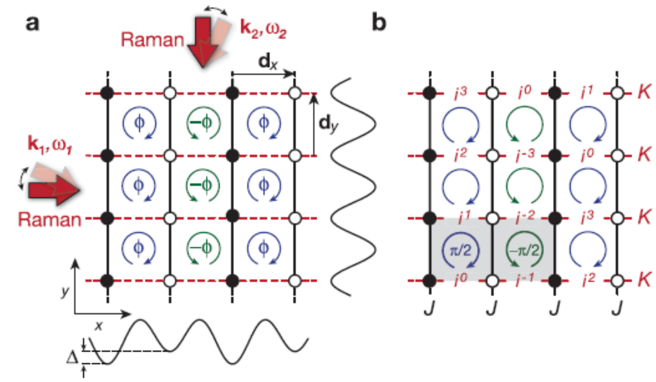


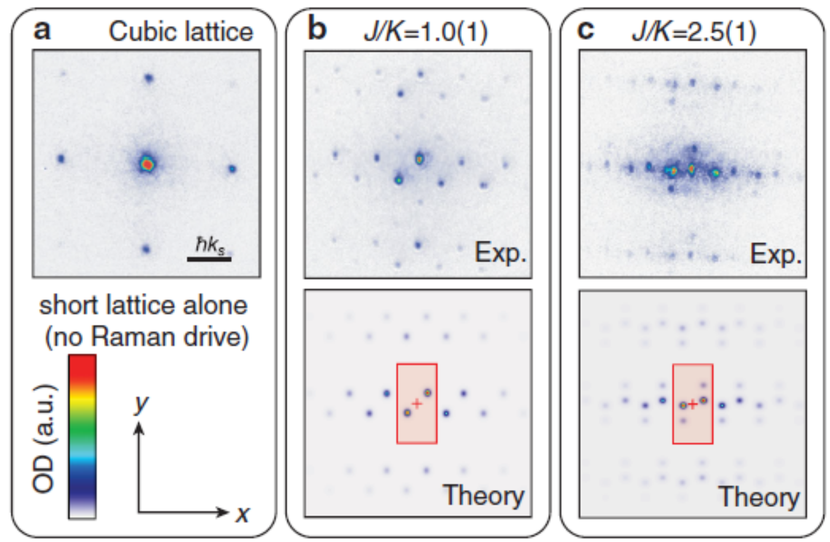
Vol 462 | 3 December 2009 | doi:10.1038/nature08609

Synthetic magnetic fields for ultracold neutral atoms

Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

Raman assisted tunneling in a lattice





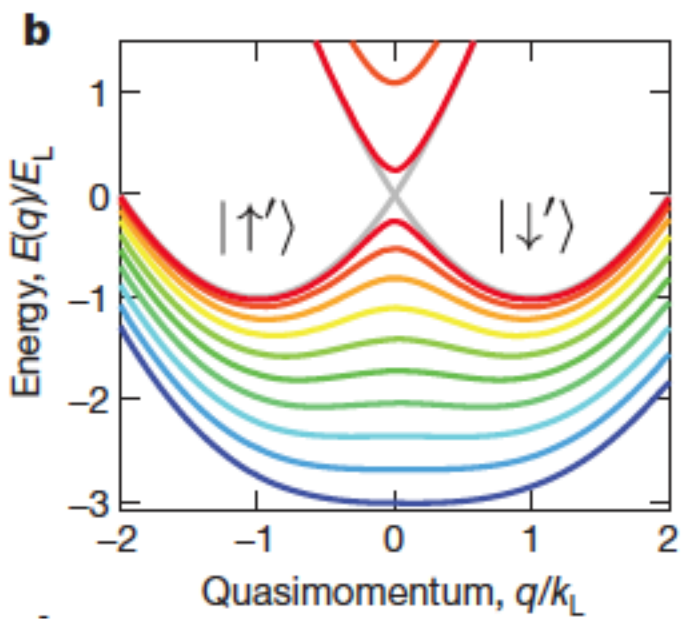
Large staggered fluxes in an optical lattice
M. Aidelsburger, et al (PRL 2011)

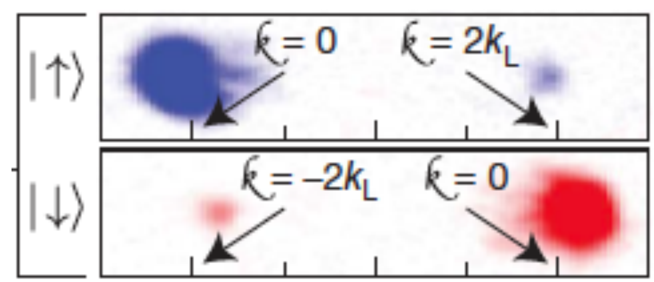
Synthetic spin-orbit coupling: Experimental Progress

LETTER
doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]





$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \check{1} + \frac{\Omega}{2} \check{\sigma}_z + \frac{\delta}{2} \check{\sigma}_y + 2\alpha \hat{k}_x \check{\sigma}_y$$

Rashba: $k_x \sigma_y - k_y \sigma_x$

Dresselhaus: $k_x \sigma_y + k_y \sigma_x$

Equal Rashba/Dresselhaus

More general SO coupling possible?

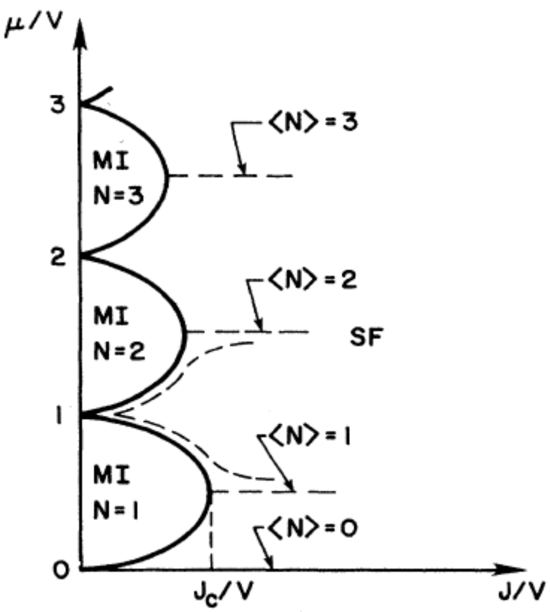
Some important issues

- Strong correlations and synthetic magnetic fields?
- Strong correlations and synthetic spin-orbit coupling?
- Probing equilibrium mass currents induced by gauge fields?

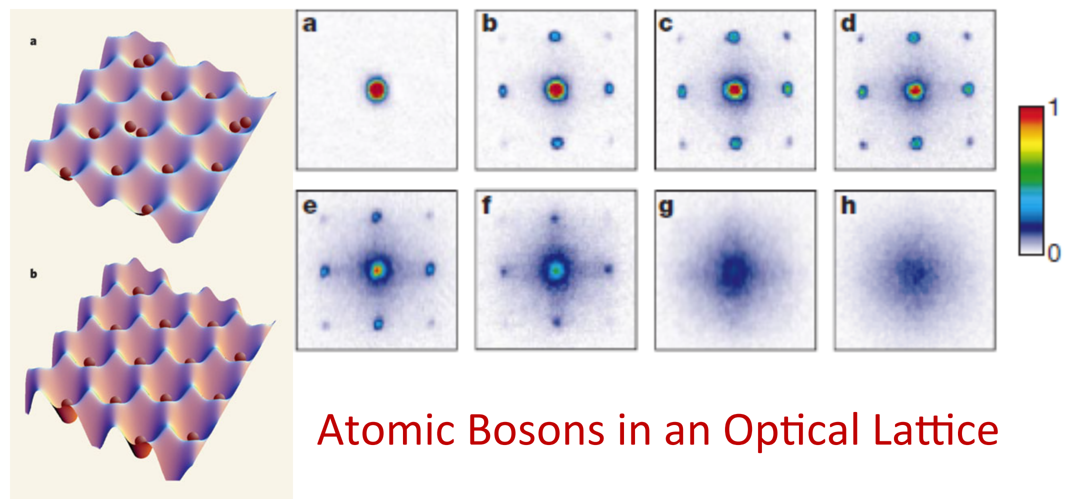
Part 1: Bose Hubbard model in a synthetic magnetic field

Bose Hubbard Model and conventional Mott transition

$$\hat{H} = -\sum_{\langle i,j \rangle} \hat{a}_i^\dagger t_{ij} \hat{a}_j - \mu \sum_i \hat{a}_i^\dagger \hat{a}_i + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i$$



Fisher, Weichman, Grinstein, Fisher (PRB 1989)

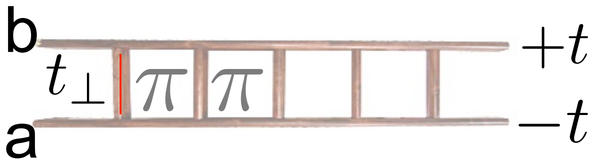


Atomic Bosons in an Optical Lattice

Proposal: D. Jaksch, et al (PRL 1998)
Experiment: M. Greiner, et al (Nature 2002)

What happens in the presence of magnetic flux?

The “fully frustrated” Bose Hubbard ladder

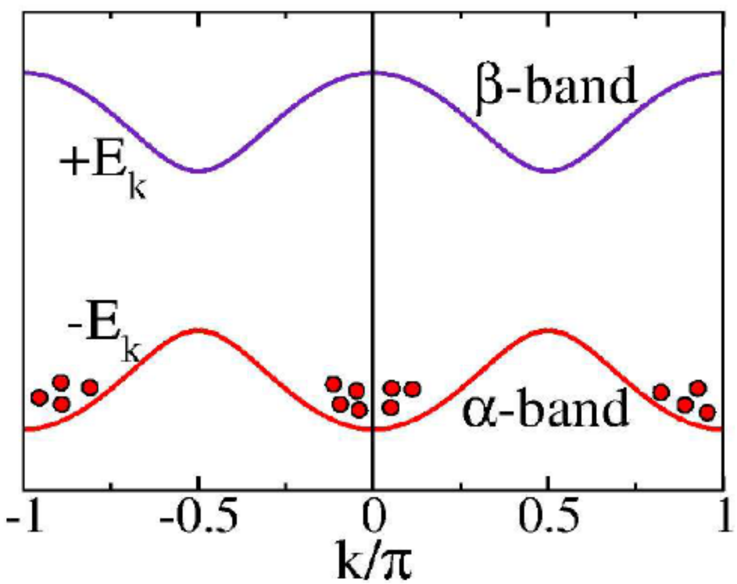


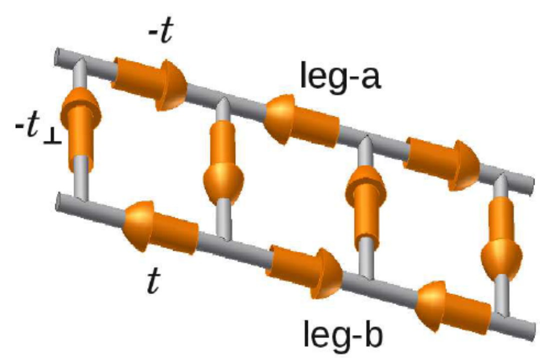
$$H = -t\sum_x \underbrace{(a_x^\dagger a_{x+1} + a_{x+1}^\dagger a_x) + t\sum_x (b_x^\dagger b_{x+1} + b_{x+1}^\dagger b_x)}_{\text{Intrachain hopping terms}} - t_\perp \sum_x \underbrace{(a_x^\dagger b_x + b_x^\dagger a_x)}_{\text{Interchain hopping}} + \overbrace{\frac{U}{2}\sum_x (n_{a,x}^2 + n_{b,x}^2)}^{\text{Hubbard repulsion}}$$

- Quantum version of FF Josephson junction arrays
- Ladder: numerically exact phase diagram via DMRG

Weak correlation limit of the FFBH ladder model

Weak Correlations: Landau theory





$$E_{\rm low}^{\rm mft} = (-E_0 - \mu) \sum_{i=0,\pi} |\varphi_i|^2 + U(u_0^4 + v_0^4) \sum_{i=0,\pi} |\varphi_i|^4 + 8U u_0^2 v_0^2 |\varphi_0|^2 |\varphi_\pi|^2$$
$$+ 2U u_0^2 v_0^2 (\varphi_0^{*2} \varphi_\pi^2 + \varphi_\pi^{*2} \varphi_0^2)$$

Umklapp-type term which favors relative +/- pi/2 phase and breaks time reversal

Mean field theory

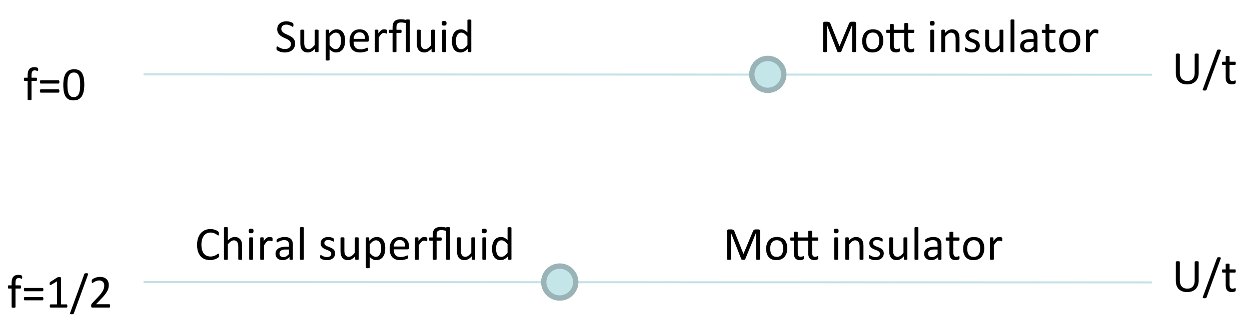
Generalize usual superfluid-Mott transition mean field theory (effective single-site approximation)

Ladder mean field theory

$$\frac{1}{\sqrt{4t^2+t_\perp^2}} = \frac{n}{\mu - U(n-1)} + \frac{n+1}{Un-\mu}$$

$$\frac{U^{\pi-\text{flux}}_{c,ladder}}{U^{0-\text{flux}}_{c,ladder}} = \frac{\sqrt{4t^2+t_\perp^2}}{2t+t_\perp} < 1$$

1. Continuous Chiral Superfluid to Mott transition
2. U_c for the Mott transition suppressed



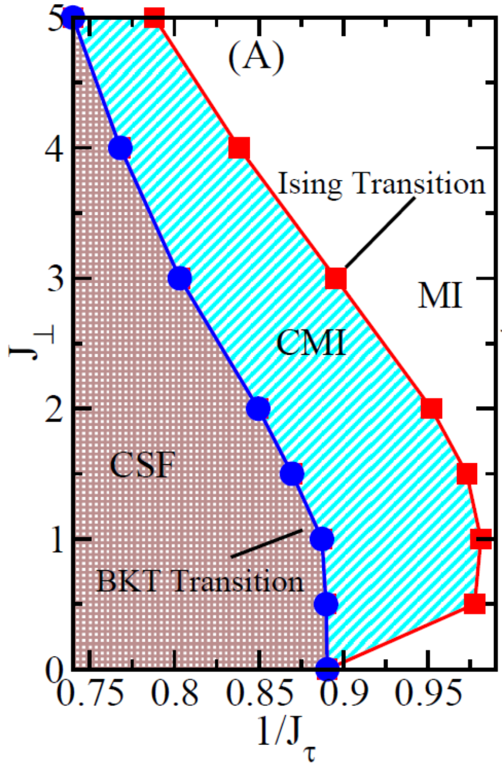
Recent example: Lim, et al (PRL 2008)
More general point: Critical theory involves 2 bosonic modes

We go **beyond mean field theory** using two numerical approaches

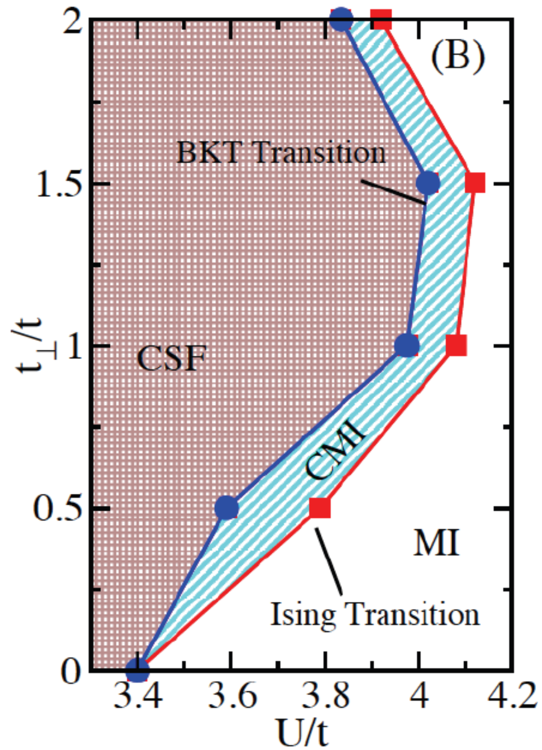
- Simulate1+1=2 dimensional frustrated classical XY model (Bilayer model)
- Directly simulate the ladder Hubbard model using DMRG methods

Phase Diagram of Fully Frustrated Bose Hubbard Ladder

(d+1) classical XY model



DMRG result



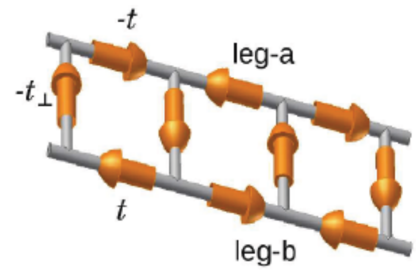
Three Phases

Chiral Superfluid: Luttinger superfluid, long range loop current order

Conventional Mott Insulator: charge gap, no superfluidity, no current order

Chiral Mott Insulator: charge gap, no superfluidity, long range current order

Physical Pictures for the Chiral Mott Insulator
Starting from the Superfluid



Chiral superfluid = Vortex Crystal
. Flux nucleates vortex or antivortex
. Vortex-vortex interaction is repulsive
. Equal number of V/AV
. "Antiferromagnetic" crystal

Regular Mott insulator = Vortex Superfluid (*D. Haldane; Halperin/Dasgupta; Fisher/Lee*)
. Dual - proliferated quantum phase slips

Chiral Mott insulator = Vortex Supersolid
. Defect in crystal: Extra vacancy/interstitial vortex/antivortex
. Proliferating and condensing dilute defects: Vortex superfluid
. Background current pattern preserved: Vortex crystal

Physical Pictures for the Chiral Mott Insulator

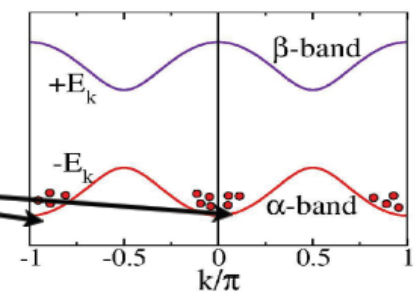
Starting from the trivial Mott insulator

Excitations of a Conventional Mott insulator

. Gapped Particles: "double occupancy"
. Gapped Holes: "vacancy"
. Dispersing particles/holes: Like a "semiconductor"

Excitations of a Conventional Mott insulator with flux

. Gapped Particles: "double occupancy"
. Gapped Holes: "vacancy"
. Dispersing particles/holes **with multiple minima**
Like a "semiconductor" with multiple valleys

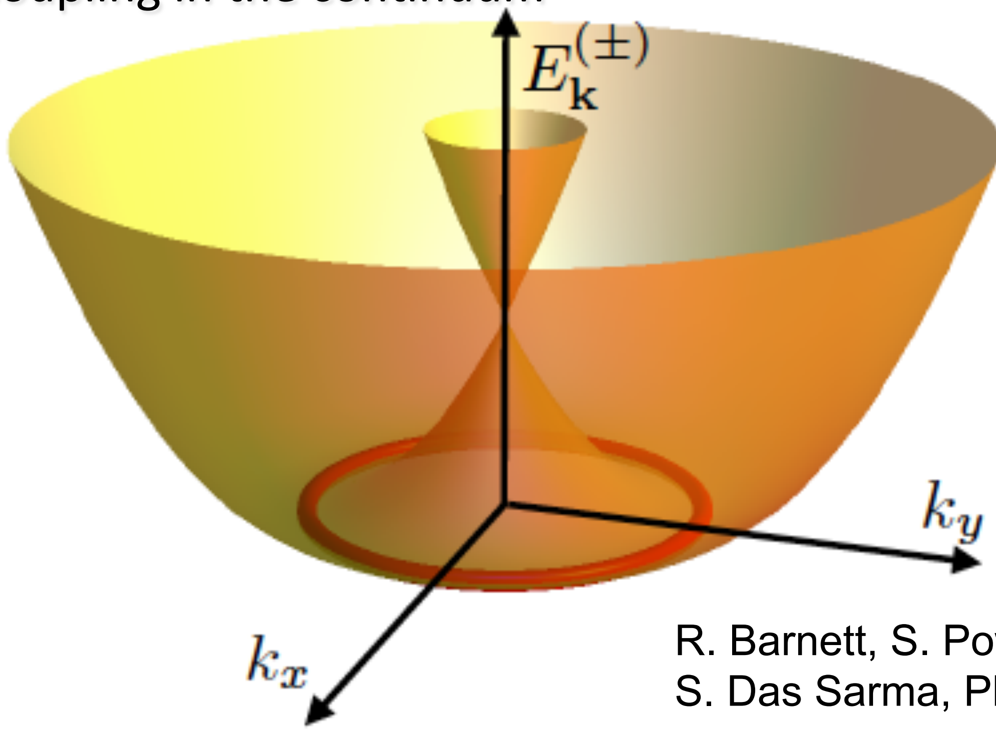


Semiconductors can have excitons and exciton condensation: Halperin-Rice

Chiral Mott insulator: Indirect exciton condensate

Part 2: Strong Interactions and Spin-Orbit Coupling

Rashba-Coupling in the continuum



R. Barnett, S. Powell, T. Grass, M. Lewenstein, S. Das Sarma, Phys. Rev. A 85 023615 (2012)

Ring of low energy states:

- "Quantum order by disorder" appears to select single-K condensate (but only partial study)
- Long wavelength fluctuations: Thermal "log-divergence" in 3D

Hubbard Model of Rashba-Coupled Bosons

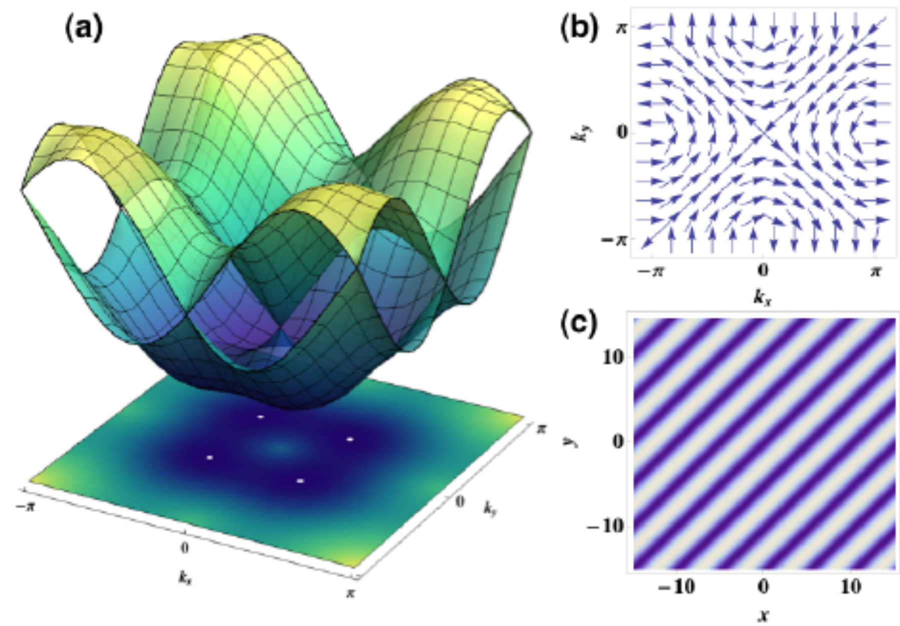
$$H = -t\sum_{\langle ij\rangle}(\psi_i^\dagger \mathcal{R}_{ij}\psi_j + \text{h.c.}) + \frac{1}{2}\sum_{i\sigma\sigma'} U_{\sigma\sigma'} a_{i\sigma}^\dagger a_{i\sigma'}^\dagger a_{i\sigma'} a_{i\sigma}$$

$$\mathcal{R}_{ij} \equiv \exp[i\vec{A}\cdot(\vec{r}_i - \vec{r}_j)]$$

$$\vec{A} = (\alpha\sigma_y, \beta\sigma_x, 0)$$

Spin-orbit coupling is like
a "non-abelian" gauge field

Lattice Rashba coupling: $\beta = -\alpha$
Lower degeneracy, more stable



W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012

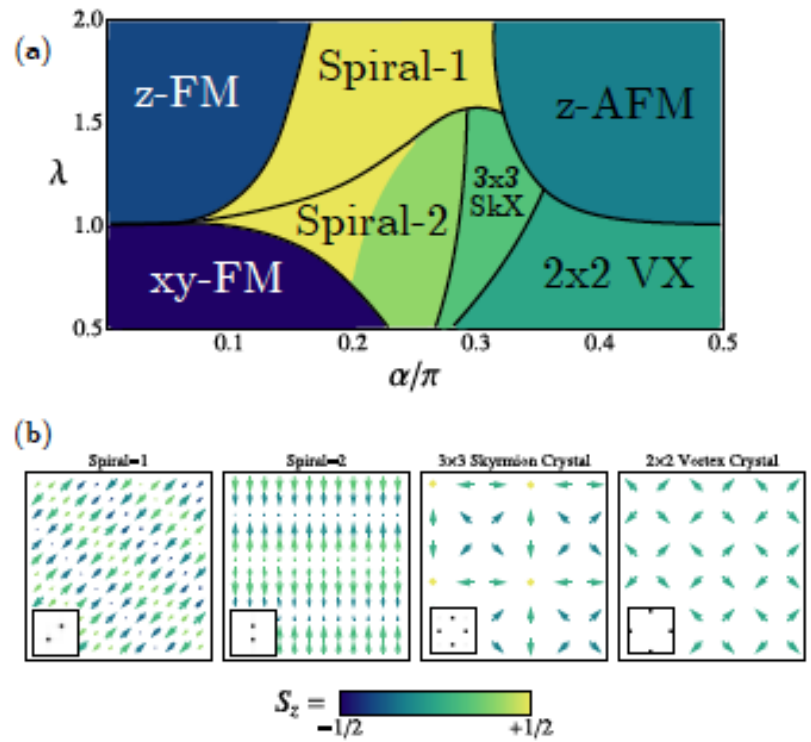
Strong Coupling Mott insulator of Rashba-Coupled Bosons

$$H = -t\sum_{\langle ij\rangle}(\psi_i^\dagger \mathcal{R}_{ij}\psi_j + \text{h.c.}) + \frac{1}{2}\sum_{i\sigma\sigma'} U_{\sigma\sigma'} a_{i\sigma}^\dagger a_{i\sigma'}^\dagger a_{i\sigma'} a_{i\sigma}$$

Effective spin Hamiltonian at O(t²/U)

$$H_{\text{spin}} = \sum_{i,\delta=\hat{x},\hat{y}} \left\{ \sum_{a=x,y,z} J_\delta^a S_i^a S_{i+\delta}^a + \vec{D}_\delta \cdot (\vec{S}_i \times \vec{S}_{i+\delta}) \right\}$$

$J_{\hat{x}}^x = -\frac{4t^2}{\lambda U}\cos(2\alpha)$	$J_{\hat{y}}^x = -\frac{4t^2}{\lambda U}$
$J_{\hat{x}}^y = -\frac{4t^2}{\lambda U}$	$J_{\hat{y}}^y = -\frac{4t^2}{\lambda U}\cos(2\alpha)$
$J_{\hat{x}}^z = -\frac{4t^2}{\lambda U}(2\lambda - 1)\cos(2\alpha)$	$J_{\hat{y}}^z = -\frac{4t^2}{\lambda U}(2\lambda - 1)\cos(2\alpha)$
$\vec{D}_{\hat{x}} = -\frac{4t^2}{U}\sin(2\alpha)\hat{y}$	$\vec{D}_{\hat{y}} = \frac{4t^2}{U}\sin(2\alpha)\hat{x}$



Rich classical magnetic phase diagram

W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012
J. Radic, A. Di Ciolo, K. Sun, V. Galitski, PRL 2012
Z. Cai, X. Zhou, C. Wu, PRA 2012

Magnetic Superfluids and Slave Boson Mean Field Theory

W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, arXiv: 1205.2319

$$H=-t\sum_{\langle ij\rangle}(\psi_i^\dagger \mathcal{R}_{ij}\psi_j+\text{h.c.})+\frac{1}{2}\sum_{i\sigma\sigma'}U_{\sigma\sigma'}a_{i\sigma}^\dagger a_{i\sigma'}^\dagger a_{i\sigma'}a_{i\sigma}$$

Usual mean field theory: Decoupling of the hopping term

Leads to a superfluid with inhomogeneous magnetic order and various non-uniform current patterns

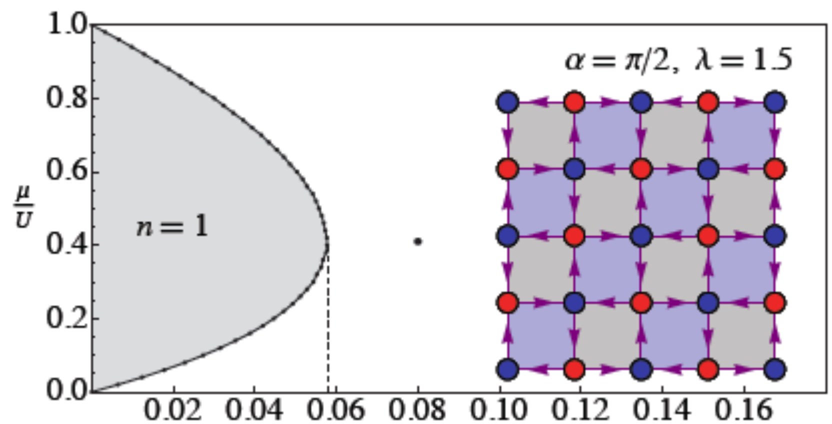
Carry out a slave boson decomposition: $a_{\mathbf{r},\sigma} = \underbrace{b_{\mathbf{r}}}_{\text{Chargon}}\,\underbrace{f_{\mathbf{r},\sigma}}_{\text{Spinon}}$

$$H_b = -t\sum_{\mathbf{r},\delta=\hat{x},\hat{y}}(R_\delta^{\mu\nu}\Phi_{\mathbf{r},\mu}^*\Phi_{\mathbf{r}+\delta,\nu}b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\delta}+\text{h.c.})+\frac{U}{2}\sum_{\mathbf{r}}b_{\mathbf{r}}^\dagger b_{\mathbf{r}}^\dagger b_{\mathbf{r}}b_{\mathbf{r}}$$

$$H_f = -t\sum_{\mathbf{r},\delta=\hat{x},\hat{y}}(R_\delta^{\mu\nu}B_{\mathbf{r},\mathbf{r}+\delta}\Phi_{\mathbf{r},\mu}^*\Phi_{\mathbf{r}+\delta,\nu}+\text{c.c.})+U(\lambda-1)\sum_{\mathbf{r}}|\Phi_{\mathbf{r}\uparrow}|^2|\Phi_{\mathbf{r},\downarrow}|^2$$

In magnetically ordered phases: “Chargons” see an abelian gauge field!

Example

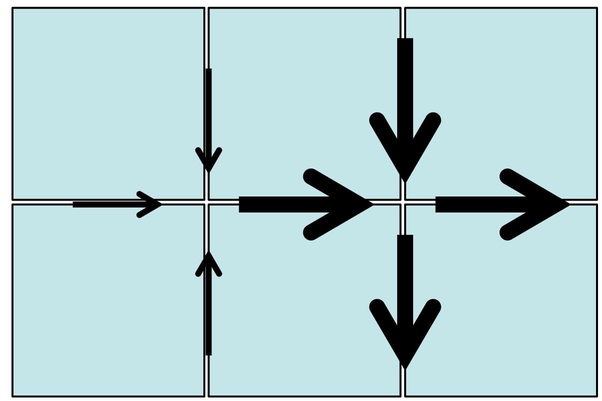


W.S. Cole, S. Zhang, A. Paramekanti, N. Trivedi, PRL 2012

- For zAFM: Going into the superfluid induces spontaneous loop currents
- Slave boson description shows **abelian "pi"-flux for chargons**

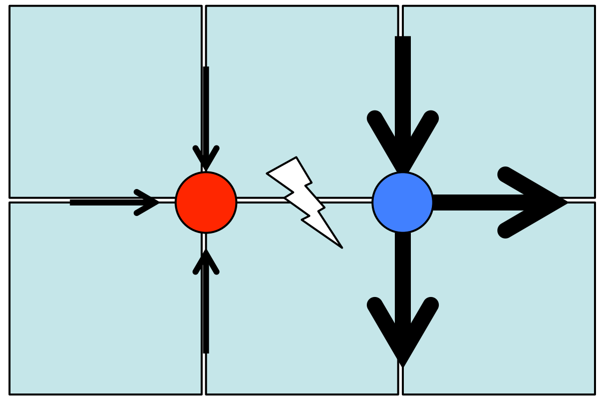
- Slave boson description also useful for analyzing SF-MI phase transitions
- Full self-consistent calculations of other phases underway

Part 3: Quantum Quenches: Tool to Probe Atom Mass Currents



In equilibrium, the currents satisfy a steady state condition: Current-In = Current-Out

Quantum Quenches: Tool to Probe Atom Mass Currents

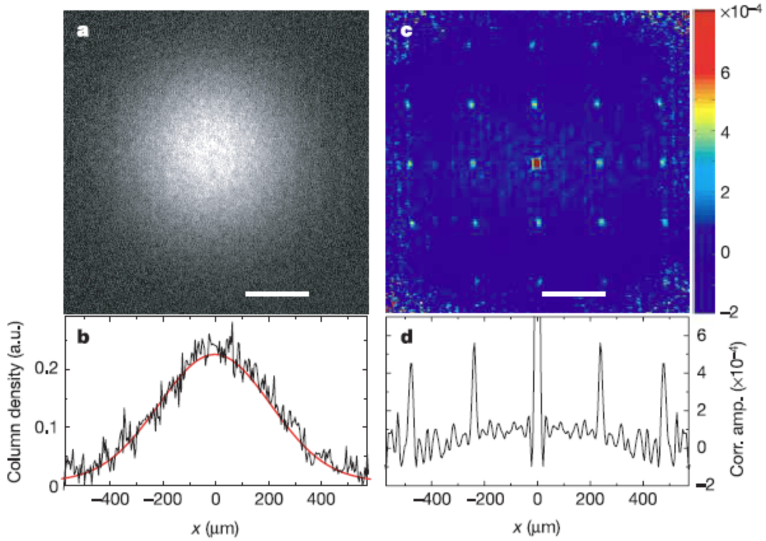


In equilibrium, the currents satisfy a steady state condition: Current-In = Current-Out

If we disrupt this condition, get current imbalance at the nodes

Continuity Equation tells us that this will lead to a density change: Measuring the induced densities after a short interval of time tells us about the underlying initial currents

Experimental Probes of Atom Density



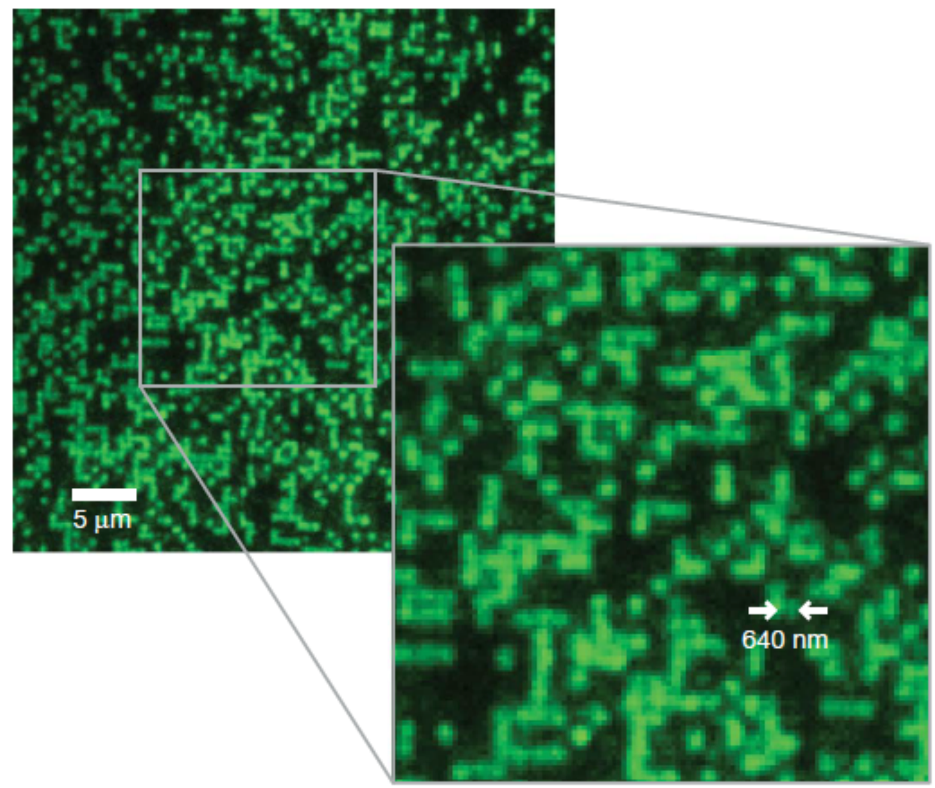
Spatial quantum noise interferometry in expanding ultracold atom clouds

Simon Fölling, Fabrice Gerbier, Artur Widera, Olaf Mandel, Tatjana Gericke & Immanuel Bloch

Nature **434**, 481 (2005)

A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice

Waseem S. Bakr[1], Jonathon I. Gillen[1], Amy Peng[1], Simon Fölling[1] & Markus Greiner[1]



nature Vol 462|5 November 2009

Coherent Bragg scattering

C. Weitenberg, et al (I. Bloch group)
Phys. Rev. Lett. ***106****, 215301 (2011)*

General scheme to probe currents

Imagine starting from a d-dimensional system

Let us quench the hopping in (d-1) “transverse dimensions”

1D continuity equation at short time:

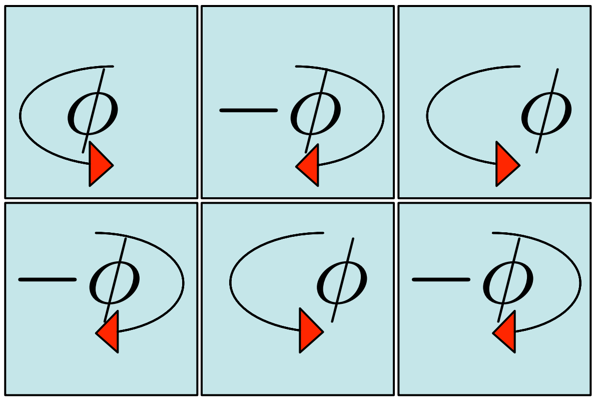
$$\delta n(\vec{q}, t = \epsilon) \approx -i q_x \epsilon J_x(\vec{q}, t = 0)$$

$$q \neq 0 \implies J_x(\vec{q}, t = 0) \approx i \frac{\delta n(\vec{q}, t=\epsilon)}{q_x \epsilon}$$

$$q = 0 \implies \text{Center of mass displacement}$$

Checkerboard Staggered Flux Superfluid

$$H = -\sum_{\vec{r},\vec{r}'} J_{\vec{r},\vec{r}'} B^{\dagger}_{\vec{r}} B_{\vec{r}'} + \frac{U}{2}\sum_{\vec{r}} B^{\dagger}_{\vec{r}} B^{\dagger}_{\vec{r}} B_{\vec{r}} B_{\vec{r}}$$



$$J_{\mathbf{r},\mathbf{r}+\hat{x}} = J_x \text{ and } J_{\mathbf{r},\mathbf{r}+\hat{y}} = J_y \exp(i(-1)^{x+y}\phi/2)$$

Let Jx=Jy initially

Imagine quenching Jx from initial value using GP equation to study dynamics

Low energy modes at k=(0,0) and k=(π,π)
Set condensate wavefunction: Ψ(x,y,t)=A(t) + B(t) $(-1)^{(x+y)}$

$$i\frac{dA}{dt} = \tilde{\epsilon}_0 A - i\gamma_0 B + U(A|A|^2 + 2A|B|^2 + B^2 A^*)$$
$$i\frac{dB}{dt} = i\gamma_0 A - \tilde{\epsilon}_0 B + U(B|B|^2 + 2B|A|^2 + A^2 B^*)$$

Staggered density

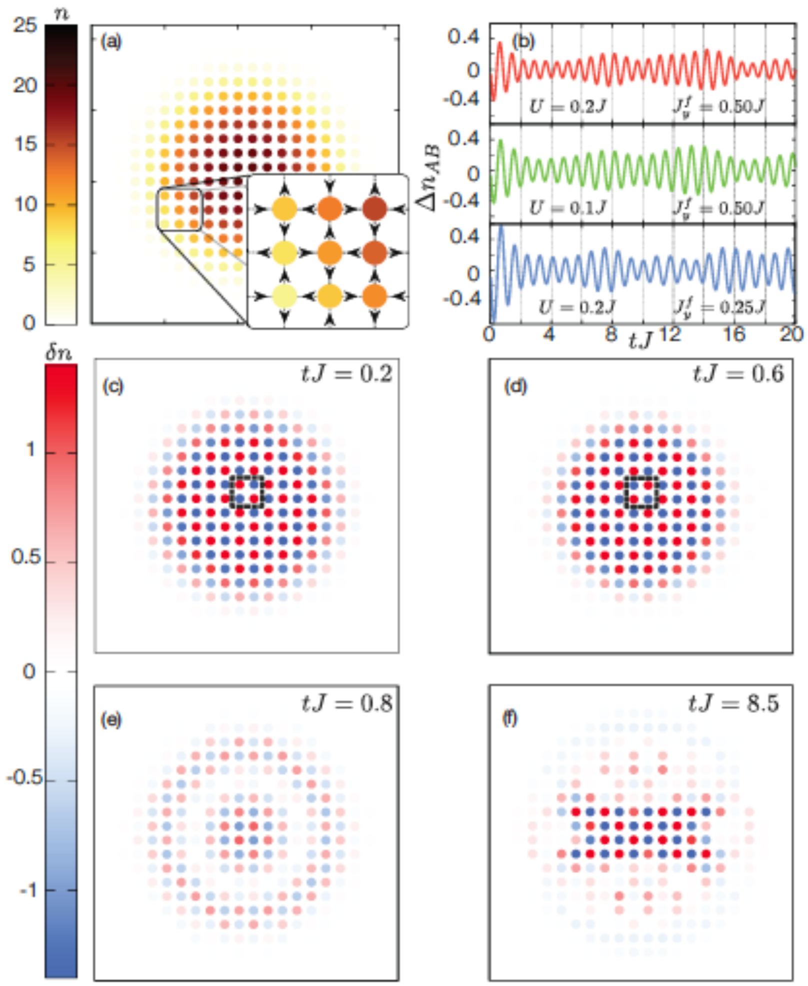
$$\Delta n(t) = 2(\frac{\delta J}{J})\frac{\mathcal{I}}{\Omega}\sin(\Omega t)$$

M. Killi, A. Paramekanti, PRA (2012)

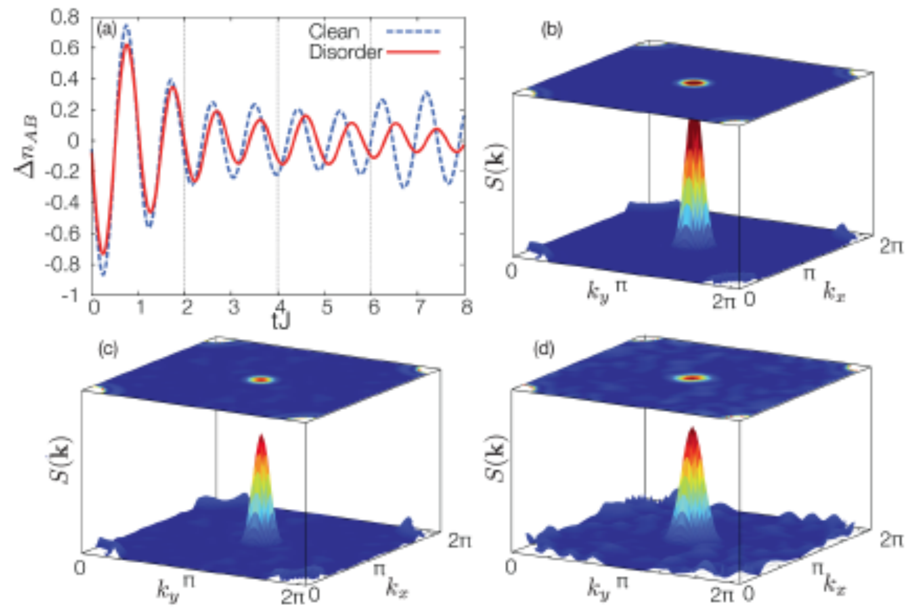
Checkerboard Staggered Flux Superfluid

Add harmonic trap / “fluctuations”

M. Killi, S. Trotzky, AP (preprint)

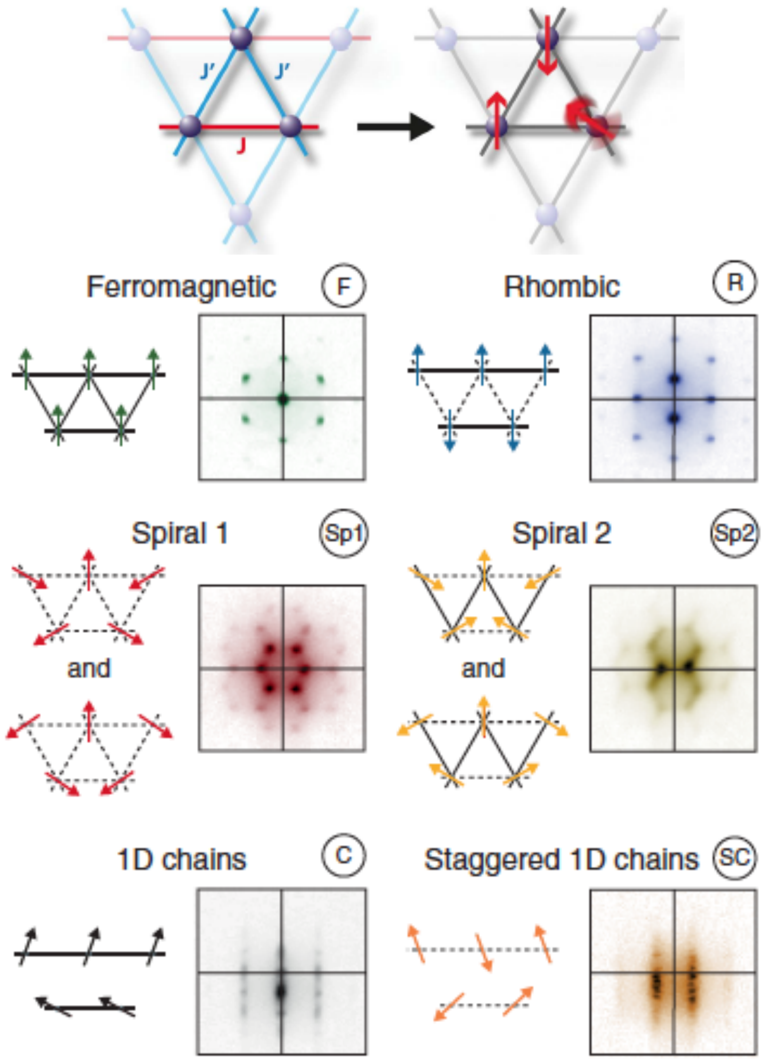


Real space

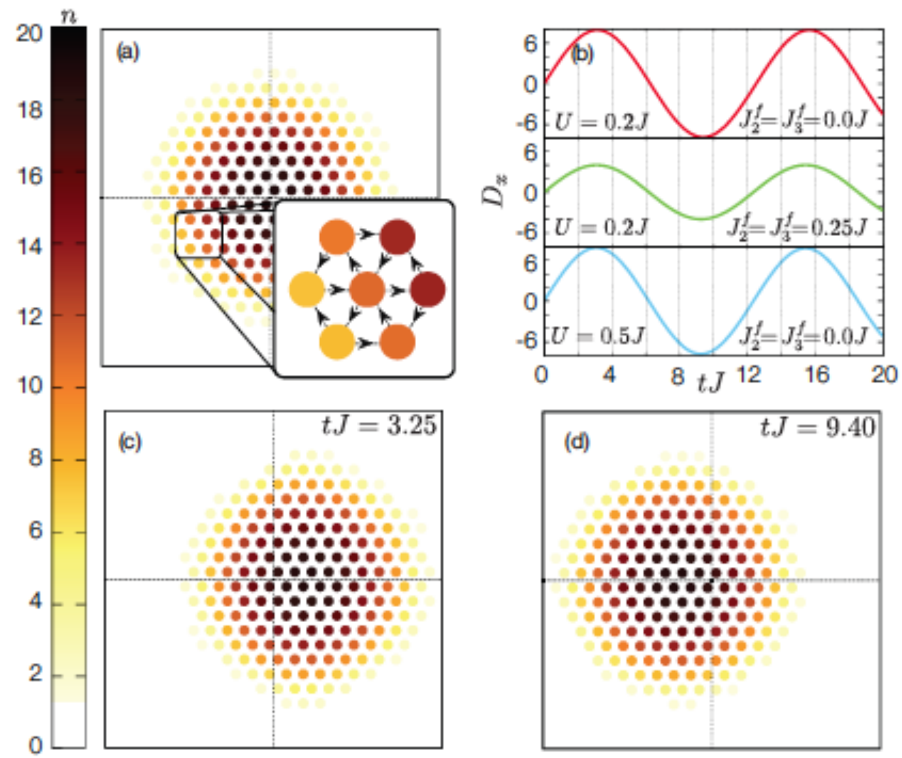


Momentum space

Triangular lattice frustrated superfluid



J. Struck, et al, Science (2011)



Quench two directions
Get Dipole oscillations

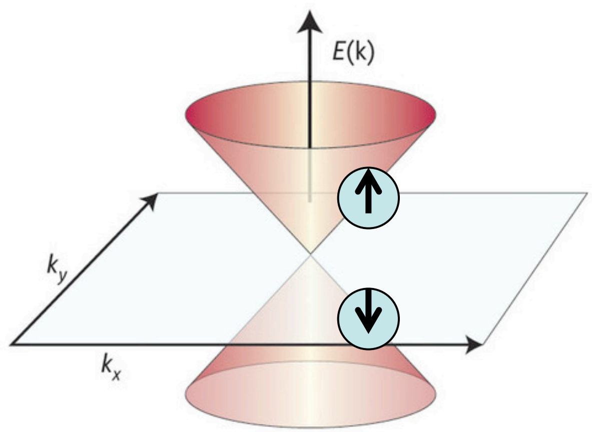
M. Killi, S. Trotzky, AP (preprint)

Checkerboard Staggered Flux Fermi Gas

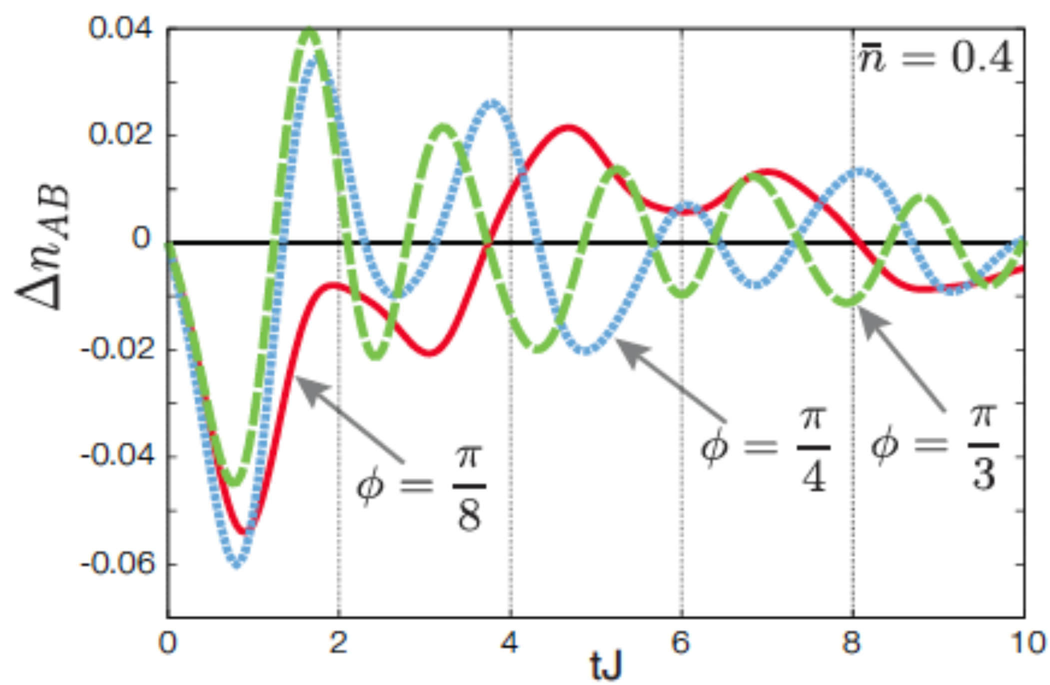
$$H = -\sum_{r,r'} J_{r,r'} C_r^\dagger C_{r'}$$

Fluxes ϕ, −ϕ

$$H = \sum_{\mathbf{k}}{}' \begin{pmatrix} f_{\mathbf{k}}^\dagger & f_{\mathbf{k}+\mathbf{Q}}^\dagger \end{pmatrix} (\varepsilon_{\mathbf{k}}\tau^z + \gamma_{\mathbf{k}}\tau^y) \begin{pmatrix} f_{\mathbf{k}} \\ f_{\mathbf{k}+\mathbf{Q}} \end{pmatrix}$$



M. Killi, A. Paramekanti, PRA (2012)



"Spin Precession Dynamics" when J_y is quenched

More complex structure (like dHvA): But dominant frequency over range of fillings

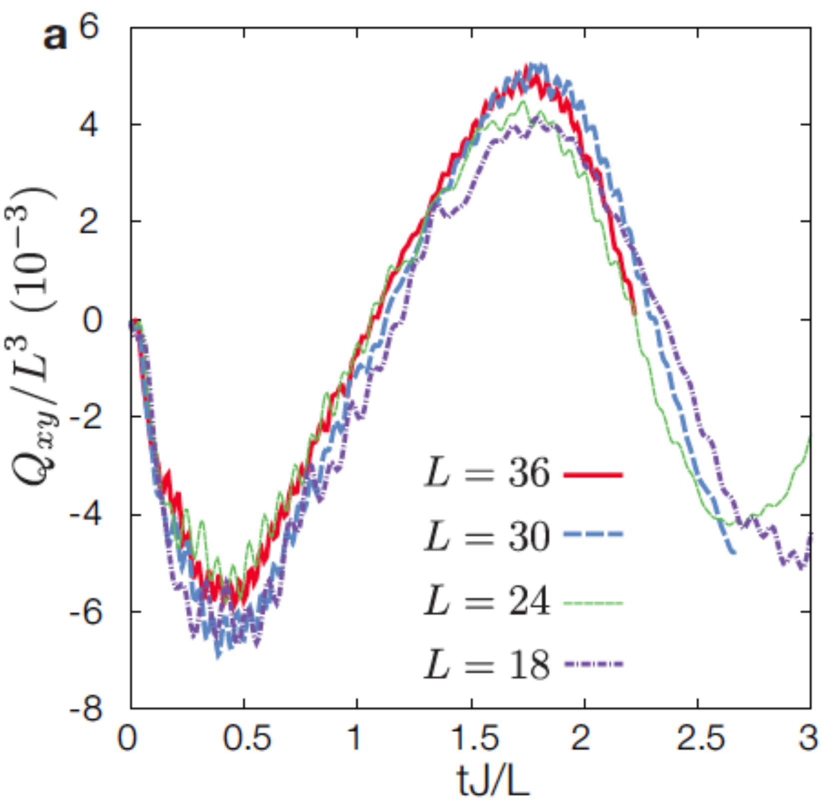
$$\tilde{\Omega}^* \approx 2\sqrt{J^2 + (J_y^f)^2 - 2JJ_y^f \cos\frac{\phi}{2}}$$

Lattice integer quantum Hall state of fermions

$$H = -\sum_{r,r'} J_{r,r'} C_r^\dagger C_{r'}$$

Hofstadter model with flux 2π/3 per plaquette

M. Killi, A. Paramekanti, PRA (2012)



Study the dynamics of the quadrupole moment of the system with open boundary conditions

$$Q_{xy}(t) = \sum_{x,y} xy\delta n(x,y,t)$$

Motivated by:
L. J. LeBlanc, et al, arXiv:1201.5857 (Superfluid Hall Effect)

"Quadrupolar current oscillations" when J_y is quenched

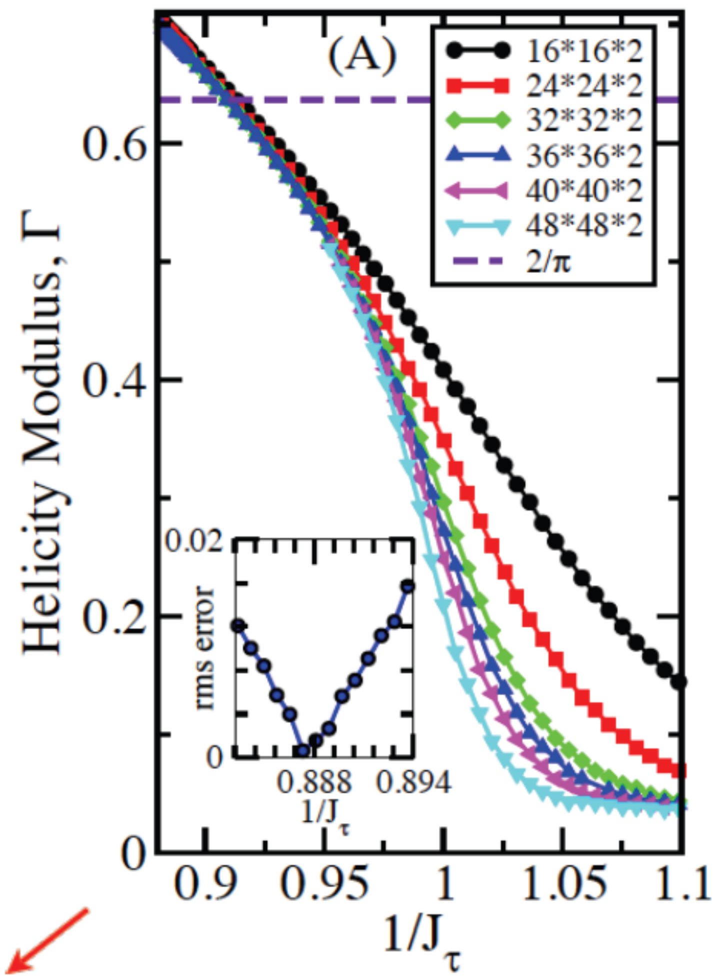
Summary

- Magnetic flux introduces "frustration" and makes the system more susceptible to forming a Mott insulator
- On a ladder, Hubbard repulsion induces a "chiral Mott Insulator" state at intermediate correlation
- Rashba coupling and strong correlations induces various exotic magnetically ordered Mott insulators and superfluids
- Slave boson theory of strongly correlated magnetic superfluid – shows how magnetic orders imprint various "abelian fluxes" on charge degrees of freedom
- **Quantum quench dynamics is an effective tool to uncover mass current patterns**

Classical XY model and Monte Carlo study

To detect the vanishing of superfluidity
we compute the Helicity Modulus

Helicity modulus ~ superfluid density
How much free energy cost to twist the phase?
How much kinetic energy of superflow?



Looks like a Berezinskii Kosterlitz Thouless
transition at which Helicity Modulus jumps

To locate the transition and check if it
is of the BKT type, use finite size scaling
form derived from RG equations

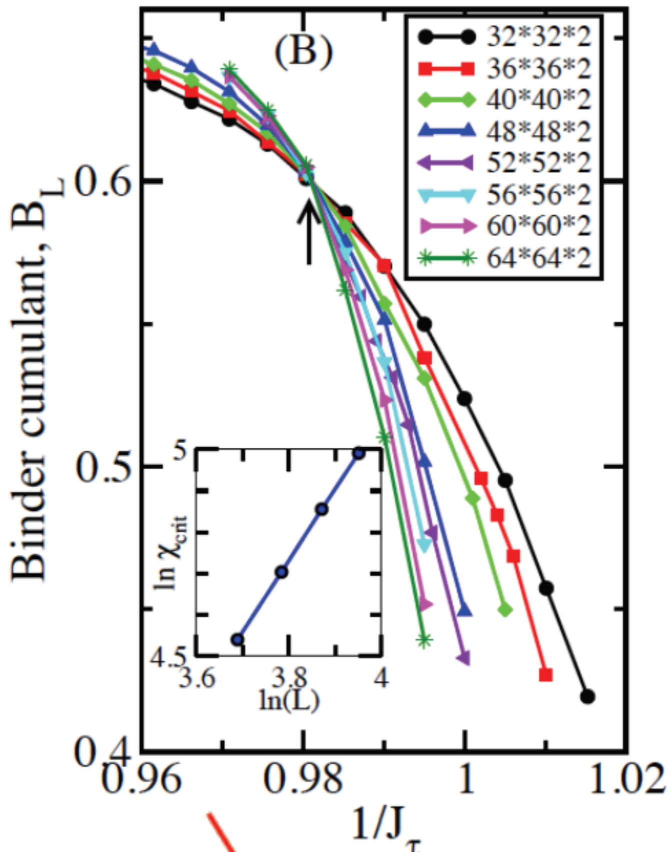
$$\boxed{\Gamma(L) = A\left(1 + \frac{1}{2}\frac{1}{\log L + C}\right)}$$

Weber/Minnhagen (PRB 1993)
Olsson (PRL 1995)
S. Mukerjee, et al (PRL 2006)

We find A $= 2/\pi$ as expected
for a BKT transition

Classical XY model and Monte Carlo study

To detect the staggered current order, we use the method of Binder cumulants



$$B_L = \left(1 - \frac{\langle m^4 \rangle_L}{3\langle m^2 \rangle_L^2}\right)$$

m: Staggered current order parameter

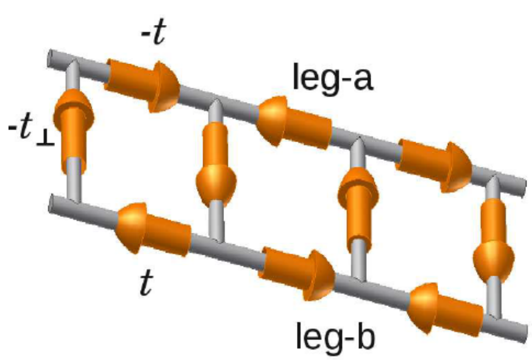
Basic idea: Fixed points have universal order parameter distributions

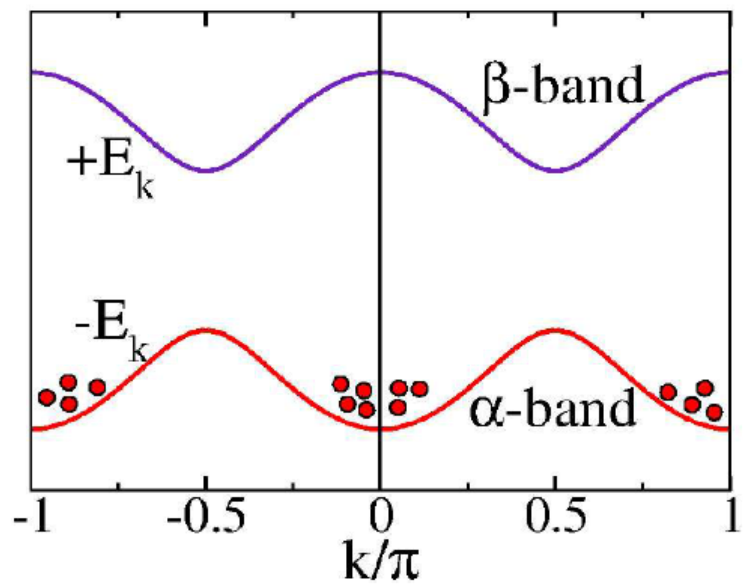
. Crossing point for all L indicates a critical point

. Susceptibility scaling consistent with 2D Ising transition

What happens with half-flux-quantum and Hubbard repulsion?

Weak Correlations: Landau theory





Fixing the relative phase leads to

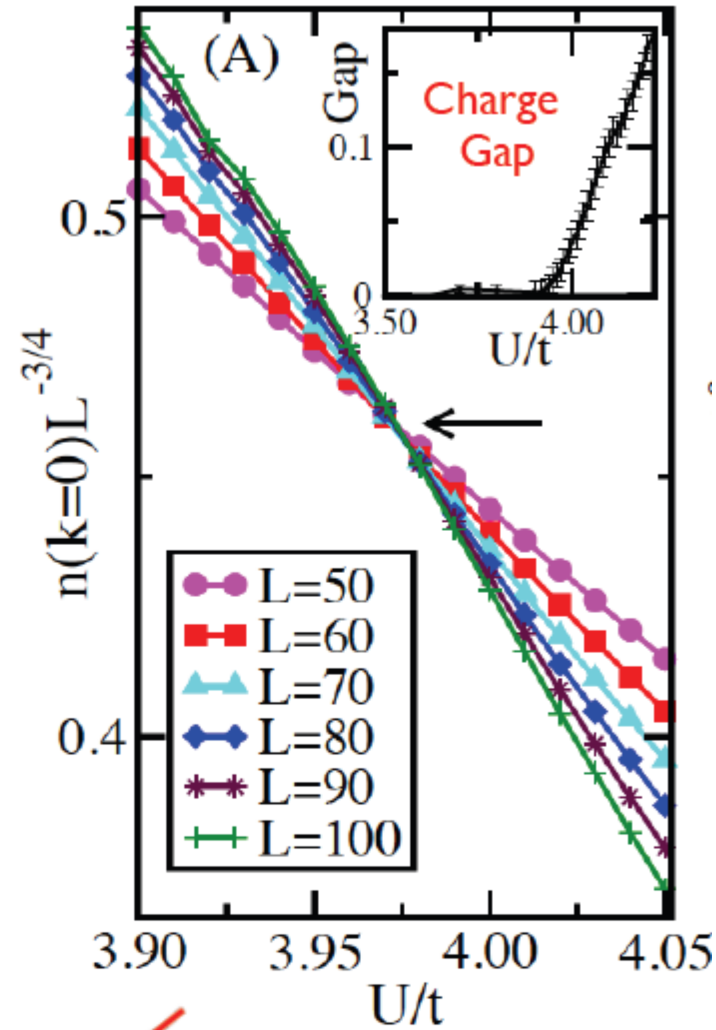
$$E^{\mathrm{mft}}_{\mathrm{low}} = (-E_0 - \mu) \sum_{i=0,\pi} |\varphi_i|^2 + U(u_0^4 + v_0^4)(|\varphi_0|^2 + |\varphi_\pi|^2)^2 - 2U(u_0^2 - v_0^2)^2|\varphi_0|^2|\varphi_\pi|^2$$

Favors equal amplitude condensate

DMRG study of the Bose Hubbard Ladder

To detect the onset of insulating behavior, we compute the charge gap

To detect the vanishing of superfluidity, plot the scaled momentum distribution at k=0

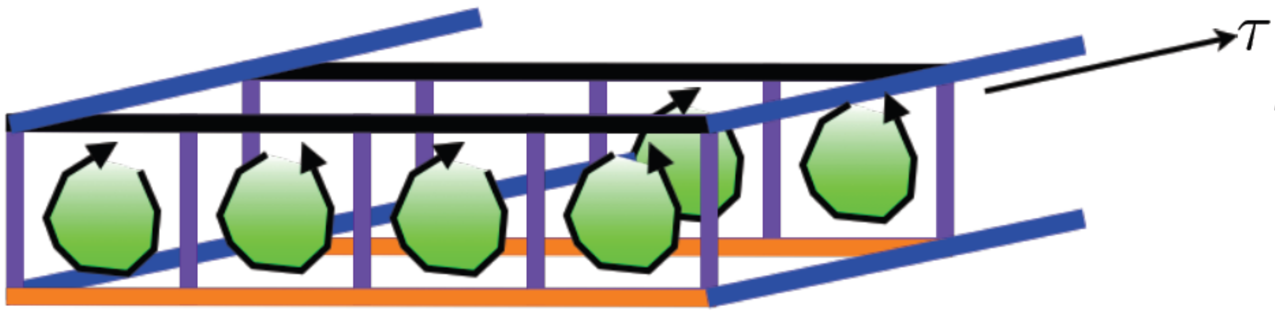


For a BKT transition, the power law decay of phase correlations at the transition is 1/r^{1/4}

This translates into a L^{3/4} divergence for n(k=0)

Onset of charge gap coincides with identification of BKT transition point

Classical XY model and Monte Carlo study

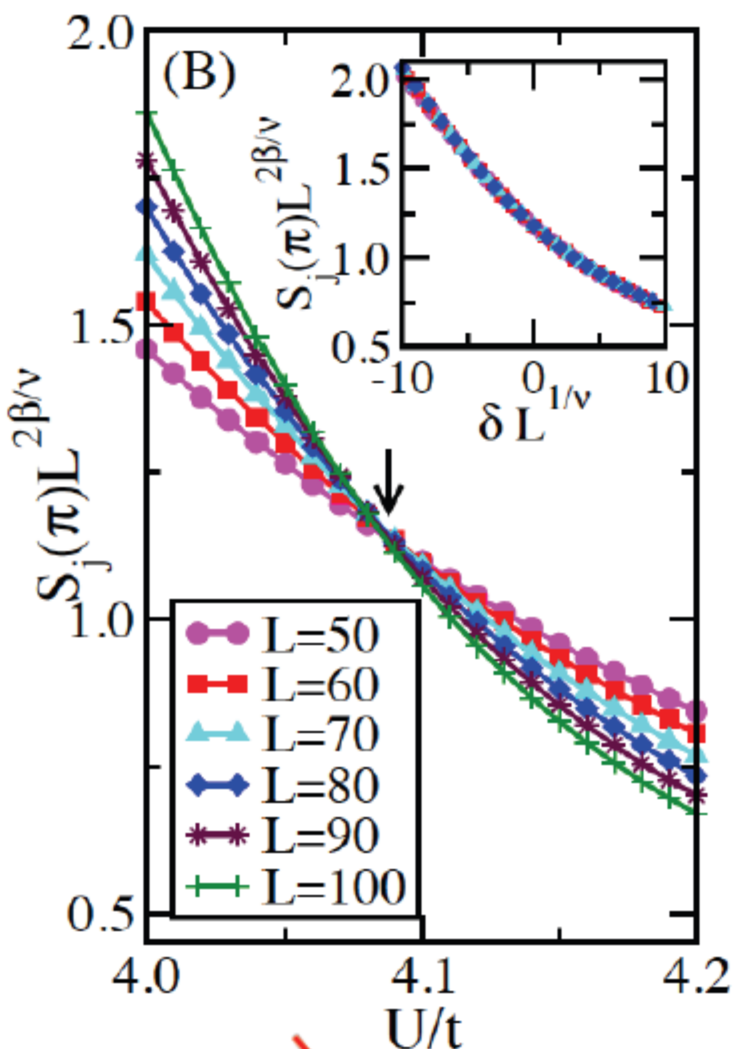


$$S_{\rm cl}^{1+1} = -\sum_{x\tau}\left[J_{\|}\cos(\varphi^a_{x+1,\tau}-\varphi^a_{x,\tau}) - J_{\|}\cos(\varphi^b_{x+1,\tau}-\varphi^b_{x,\tau}) + J_{\perp}\cos(\varphi^a_{x,\tau}-\varphi^b_{x,\tau})\right]$$
$$- \; J_\tau\sum_{x\tau}\left[\cos(\varphi^a_{x,\tau+1}-\varphi^a_{x,\tau}) + \cos(\varphi^b_{x,\tau+1}-\varphi^b_{x,\tau})\right]$$

$2\epsilon\tilde{t} = J_{\|},\; 2\epsilon\tilde{t}_{\perp} = J_{\perp}$: Spatial couplings are like interchain hopping amplitudes

$1/\epsilon U = J_\tau$: Imaginary time coupling is like 1/U

DMRG study of the Bose Hubbard Ladder



To detect the staggered current order, look at the scaling of the structure factor

Using 2D Ising exponents, find the transition point where loop current order vanishes

Observe scaling collapse of the data for various system sizes

Find consistent crossing point and data collapse for 2D Ising exponents